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LIQUIDITY CONSTRAINTS AND TIME NON-SEPARABLE PREFERENCES: SIMULATING MODELS WITH LARGE STATE SPACES

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Abstract

This paper presents an alternative method for the stochastic simulation of nonlinear and possibly non-differentiable models with large state spaces. We compare our method to other existing methods, and show that the accuracy is satisfactory. We then use the method to analyze the features of an intertemporal optimizing consumption-saving model, when the utility function is time non-separable and when liquidity constraints are imposed. Two non-separabilities are studied, habit persistence and durability of the commodity. As the model has no closed-form solution, we compute deterministic and stochastic solution paths. It enables us to compare income and consumption volatility, and to describe the density of consumption under the different hypotheses on the utility function.

Key Words
Stochastic simulations, Liquidity constraints, Habit formation, Durability.

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1 Introduction

Simulation of stochastic dynamic non-linear models is commonly done now in economics, as closed-form solutions for these models are beyond reach. Several methods have been proposed and most of them are presented in Taylor and Uhlig (1990), applied to stochastic growth models. Other examples include Zeldes (1989b), who simulates rational expectation models of consumption with non-quadratic utility functions, Deaton (1991) who analyses the effect of liquidity constraints on consumption and savings, or Novales (1990) who simulates stochastic equilibrium model of interest rates. Two of the most popular methods are probably linearization or value function iterations. The first method is not desirable in non-linear models and impossible to implement when the models are non-differentiable, which occur in several economic problems as liquidity constraints or commodity price models with competitive storage. The second method uses a recursive fixed-point method to determine the optimal value function of the problem. This method requires the discretisation of the state variables, which, in practice, limits the problem to one or two state variables. Thus, the study of large models or of rich dynamics are beyond the scope of this method, as it would require days of CPU time to obtain some reasonable accurate solution.

The purpose of this paper is to present an alternative method of simulation for large non-linear models. This method is based on the relaxation algorithm using Newton-Raphson iterations described by Laffargue (1990) and Boucekkine (1995). [1] It has been adapted to stochastic simulations by approximating rational expectation by perfect foresight. The method present two main advantages over the value function method. First, when the value function method can accommodate for hardly more than two state variables, we are able to simulate models with up to six state variables. Second, since the value functions depend on the parameter of the model, they have to be recalculated each time the parameters vary. This is clearly a drawback when the simulations are imbedded in an estimation procedure, as in Lee and Ingram (1991) or in Deaton and Laroque (1995). The method proposed here accommodate changes in the values of the parameters easily. Of course, as this method is not an "exact" method, some care has to be taken for

[1] The simulation method is available upon request. The deterministic part has been programmed in GAUSS, under the name DYNARE, and is available by anonymous ftp at cepremap.msh-paris.fr in repertory /pub/cepremap.

its application. The paper present therefore some comparisons (when it is possible) with value function methods.

We apply this method to various time non-separable models of consumption with liquidity constraints, which are typically non-differentiable and imply a large number of state variables. We study the implication of habit formation and of durability on the allocation between consumption and savings. These issues have been extensively considered separately in econometric analysis, to explain the departure between the data and the random walk model presented by Hall (1978).

Flavin (1985) test whether the empirical rejection of the PIH are to be attributed to a myopic behavior of the agent- i.e. the marginal propensity to consume out of current income is non-zero- or to a capital market failure, generating liquidity constraints. She concludes for the second hypothesis. Many other authors, as Hall and Mishkin (1982), Żeldes (1989a), Mariger (1987) and Campbell and Mankiw (1989) have shown that a significant part of the population (around 15 % to 20%) faces liquidity constraints, which means that they are not allowed to borrow.

To explain the departure between Hall's theory and empirical studies, econometicians have also introduced habit persistence. Habit persistence occurs when past consumption alter the choice of present consumption directly through the subutility function. The subutility function is then not separable between current consumption and lagged one consumption. It can be either increasing in both current and past consumption or increasing in current consumption and decreasing in past consumption. This type of models have been tested by Muellbauer (1988).

Finally, Hayashi (1985) emphasizes the role of the durability of commodities. A distinction has to be made between sheer consumption and expenditure. The utility derived from consumption can be the result of several periods of expenditure, or can last for some time after the purchase.

We compute deterministic and stochastic simulations of intertemporal optimizing consumption saving models. Given an exogenous stochastic process for the labor income, we numerically derive the optimal path for consumption and savings under borrowing restriction. We study the effect of time non-separable preferences. We find that durability and habit persistent notably distort the results of Deaton. Even if assets still acts as a buffer against good draws of income, they do not always ensure consumption smoothness. When the income process becomes more persistent and when durability is present, consumption becomes much more noisy than income. On the contrary, when habit persistence occurs, consumption smoothness increases and the asymmetry in the consumption time series, noticed by Deaton, tends to disappear.

The layout of this paper is as follows. We first present the intertemporal model. In the following section, we explain the simulation method and we provide accuracy comparisons with value function iteration methods. Section 4 presents deterministic simulation results. Stochastic simulations results are provided in Section 5. Section 6 concludes.

2 The Model

The consumer maximizes his total utility subject to a budget constraint and a borrowing restriction,

$$U = E_t\left[\sum_{\tau=t}^{+\infty} \beta^{\tau-t} \nu\left(\frac{c_\tau - \alpha c_{\tau-1}}{1-\alpha}\right)\right] \tag{1}$$

$$\begin{aligned} s.t.\quad a_t &= (1+r)a_{t-1} + y_t - k_t \\ c_t &= k_t + (1-\delta)k_{t-1} \\ a_t &\geq 0 \end{aligned}$$

where $\beta < 1$ is the rate of time preference, and $\nu(c_t, c_{t-1})$ is the instantaneous subutility function. The first constraint is the usual budget constraint involving real assets a_t, labor income, y_t, the real interest rate, r, assumed to be constant, and expenditure on the good, k_t. The second constraints allows for durability in the model. Current consumption is a function of current and lagged expenditures. The notion of durability is the one introduced by Hayashi (1985), and can refer even to non-durable goods. Often, the utility of consumption is not only derived at the expenditure date, but can last fore more than that [2].

The last constraint is the liquidity constraint which takes here a simple form, but we could have assumed as well that assets would be superior to any fixed limit.

The subutility depends not only on current consumption but also on lagged consumption. The parameters α and δ belong to $[0,1[\text{x}[0,1]$. The

[2]Think of a very good wine, or a holiday, for instance.

first parameter is a measure of the importance of lagged consumption for the consumer. The parameter δ indicates the durability of the good for the agent. The consumer is, in this model, infinitely lived. The only source of uncertainty is an exogenous labor income modeled as an AR(1) process

$$y_t = \rho y_{t-1} + (1-\rho) y_{LT} + \epsilon_t$$

where ρ is the coefficient of persistence of the income process, y_{LT} is the long run value of income and ϵ_t is an i.i.d. white noise which is assumed normally distributed. We have chosen a non quadratic subutility function of the CRRA form as

$$\nu(c_t) = \frac{c_t^{1-\gamma}}{1-\gamma}$$

γ is the coefficient of relative risk aversion. The consumer have a precautionary demand for savings for two reasons. First, there is uncertainty about future earnings. Second, he is subject to liquidity constraints and he has to save to level out bad draws of income, knowing that borrowing will not be permitted.

Our model embodies four specific models with liquidity constraints,

1. $\alpha = 0$, $\delta = 1$, the basic intertemporal optimization model of consumption with liquidity constraints, as in Deaton (1991). We call it from now on M_1.

2. $0 < \alpha < 1$, $\delta = 1$, the model with habit persistence (M_2), studied by Muellbauer (1988) and Novales (1990).

3. $\alpha = 0$, $0 < \delta < 1$, the model with a durable good (M_3), studied by Hayashi (1985).

4. $0 < \alpha < 1$, $0 < \delta < 1$, the general model with both durability and habit formation (M_4).

The difference between habit formation and durability depend on the sign of the derivative of the utility function. Habit formation is present if $\partial\nu_t/\partial c_{t-1}$ is negative, and durability is defined by a positive derivative. In the models considered here, M_3 has a positive derivative, while the sign of the derivative for model M_2 depends on the sign of α. However, for $\alpha < 0$, the model has essentially the same structure as M_3, so we restrict α to be in [0,1[.

Let us now examine the first order condition of the above program. They can be written as

$$\begin{cases} \lambda_t = \text{Max}\left[\tilde{\lambda}_t, (1+r)/\beta \text{E}_t \lambda_{t+1}\right] \\ \lambda_t = \frac{1}{1-\alpha}\left[\nu'_t + (1-\delta-\alpha)\beta^{-1}\nu'_{t+1} - \beta^{-2}\alpha(1-\delta)\nu'_{t+2}\right] \\ \nu'_t = \left[k_t + (1-\delta-\alpha)k_{t+1} - \alpha(1-\delta)k_{t+2}\right]^{-\gamma} \end{cases} \quad (2)$$

where λ_t is the Lagrange multiplier associated with the budget constraint. $\tilde{\lambda}_t$ is the Lagrange multipliers associated with the budget constraint, when the borrowing constraint is binding at period t. We are able to calculate it by using the budget constraint at period t, $t+1$, conditionally to $a_t = 0$.

In the long run, it can be easily checked that consumption equals y_{LT}, the long run value of income. The Euler equations show non-linearity and non-differentiability, due to the *Max* and the expectation operators. It is then not possible to derive closed-form decision rules for optimal consumption and savings. We therefore solve numerically the optimal paths.

3 The Simulation Method

A quick look at the structure of Euler equation (2) is sufficient to capture the difficulties of simulating our models. First, all models M_i $i = 1, .., 4$ are non differentiable, due to the inequality constraint appearing in the general optimization problem (1). Moreover, beginning with model M_2, the consumption variable exhibits a dynamic of order greater than 2 (it is equal to 6 in the case of model M_4 !), a feature inherent to time non separable preferences. The combination of non-differentiability and high dynamics order makes highly problematic the implementation of "exact" stochastic simulation methods (in the sense of Taylor and Uhlig (1990)). Whereas it is possible to solve the first order dynamics model M_1 with an arbitrary accuracy -see Deaton (1991), this seems quite impossible to solve in the case of model M_4, for feasibility considerations. Fixed point methods on discretized Euler equations appear, indeed, especially adapted to first order dynamics models, completed by stationary first order Markovian specification for the innovation terms. If the dynamics order of a considered model is higher, the latter numerical framework could be questioned. Especially, when the dynamics order is equal to 6, as for model M_4, this setting seems somewhat intractable, even when searching for time independent solutions. In this case, and regardless the special difficulty of setting explicit mathematical proofs for fixed-point methods convergence on such models, computing the stationary solutions by these methods on discretized state spaces will involve a huge numerical cost, and it seems really difficult to control efficiently the precision of the solutions.

Given the latter remark, we use directly an approximation solution method for all models, to give a unified numerical framework. There are two main simulation methods. A first approach consists in linearizing the model (or equivalently in adopting a linear quadratic setting as in Christiano (1990) or Heaton (1993)). This device is not possible in the presence of non differentiabilities. The other strategy uses non linear deterministic solvers within a Monte Carlo experiment, as in Gagnon (1990) and Boucekkine (1995). We implement this device to deal with nondifferentiabilities. The following subsection gives some elements of this method. Subsection (3.2) studies its numerical precision on model M_1, comparing it with the "exact" method suggested by Deaton (1991).

3.1 The Stochastic Experiment

The stochastic simulation method is in line with the usual Monte Carlo experiments conducted in economics and in econometrics. First, we focus on the law of motion of the exogenous labor income, that is

$$y_t = \rho y_{t-1} + (1-\rho) y_{LT} + \epsilon_t \tag{3}$$

for $t \geq 1$. Instead of introducing an ad-hoc Markovian stationary distribution for the innovation term ϵ_t, we adopt the following perfect foresight approximation.

$$\epsilon_1 \leadsto \mathcal{N}(0, \sigma^2) \tag{PFA}$$

$$\epsilon_t = 0 \qquad \forall t > 1$$

Once we assume the perfect foresight approximation,the model becomes deterministic and can therefore be solved by any deterministic algorithm.

At this time, we have to deal with the usual problems related to the resolution of structurally infinite time support equations

$$\begin{cases} a_t = (1+r)a_{t-1} + y_t - k_t \\ k_t = c_t + (1-\delta)c_{t-1} \\ \lambda_t = \text{Max}\left[\tilde{\lambda}_t, (1+r)\beta^{-1}\text{E}_t\lambda_{t+1}\right] \\ \lambda_t = \frac{1}{1-\alpha}\left[\nu'_t + (1-\delta-\alpha)\beta^{-1}\nu'_{t+1} - \beta^{-2}\alpha(1-\delta)\nu'_{t+2}\right] \\ \nu'_t = [k_t + (1-\delta-\alpha)k_{t+1} - \alpha(1-\delta)k_{t+2}]^{-\gamma} \end{cases} \tag{DP}$$

Computing the exogenous sequence $(y_t)_{t\geq 1}$ with assumption (PFA) requires the initialization of the law of motion (3), i.e. a value for y_0. The problem (DP) is structurally under-determined. To solve it, we approximate it by a finite time two boundary system. As usual, (see Fisher (1992) and Boucekkine (1995)), this is done by initializing the time lagged variables (i.e. by assigning a value for $c_0, a_0, k_0, ...$) and by fixing the forward variables at their corresponding deterministic long run values at a conveniently chosen period T, called the solution time horizon. The model is then solved over the periods t=1,2,...,T, with two boundary values mentioned above.

The solution of the model (DP) depends thus on the choice of the initial values. Hereafter, we denote it by $DP(y_0, a_0, c_0, k_0, ...)$. It is straightforward to show that the deterministic long run values of the model are :

$$a_s = 0 \qquad k_s = y_{LT} \qquad c_s = \frac{y_{LT}}{2-\delta} \tag{4}$$

Of course, for each draw of ϵ_1, the solution time horizon T must be chosen such that these long run values are reached within the selected horizon; otherwise, the finite time approximation will generate important numerical errors. We solve the two boundary values deterministic system with the relaxation algorithm described by Laffargue (1990) using control parameter specified by Boucekkine (1995).

To generate a pseudo sample $(c_1, c_2, ..., c_N)$ for consumption we use the following device:

i) Set $y_0 = Y_{LT}$, $a_0 = a_s$, $c_0 = c_s$ and $k_0 = k_s$, where the variable indexed by s are the value at the steady state. Draw ϵ_1 from N(0,σ^2) and assume (PFA). Set $y^p = y_1 = y_{LT} + \epsilon_1$. Store y^p.

ii) Solve the deterministic problem DP($y_{LT}, a_s, c_s, k_s, ...$). Hereafter denote by (a_1^*, c_1^*, k_1^*) the solution values at $t = 1$ for the endogenous variables.

Set $c_1 = c_1^*$. This value constitute the first value of our consumption pseudo-sample series. Set $c^p = c_1^*$, $a^p = a_1^*$ and $k^p = k_1^*$. Store c^p, a^p and k^p, in order to initialize the process at next step.

iii) Draw a new ϵ_1 from N(0,σ^2) and assume (PFA). Solve the deterministic problem DP($y^p, a^p, c^p, k^p, ...$). Set $c_h = c_1^*$, $1 < h \leq H$, with H the length of the pseudo-sample. Store c_h, . Set $y^p = y_1 = \rho y^p + (1 - \rho) y_{LT} + \epsilon_1$, $c^p = c_1^*$, $a^p = a_1^*$ and $k^p = k_1^*$. Store y^p, c^p, a^p and k^p.

iv) Repeat step iii) until having reached the desired length for the consumption pseudo- samples H.

As one can observe, the stochastic generator consists in replicating in a usual way an approximate stationary consumption rule. For a given draw ϵ_1, determining the life-time labor income profile under (PFA) and given the initial values of the predetermined variables, the consumption decision rule is computed as the solution at the first period of the corresponding deterministic problem. The approach clearly underestimates future uncertainty. The subsequent important question concerns the magnitude of the generated bias. In the following section, we show that the induced departure from the rational expectations hypothesis can be controlled through an adequate choice of the innovation variance.

3.2 Checking the accuracy

To perform stochastic simulations, the method requires equating future innovations to their mean for each replication. This might introduce some departures from the "exact" simulation path. However, if the standard deviation of the income shocks is not to big, the model is only slightly non-linear and our method would then give a good estimate of the exact solution. In order to check the accuracy, we compare the results with an alternative simulation method involving value function iterations, as in Deaton (1991). This method can be considered as an "exact" method, since the approximation error can be made as small as one want, by increasing the number of points on the grid. However, computationally, this method is limited by the size of the state space. These simulations can only reasonably be performed for one or two state variables, that is for the simplest model we present here (M_1). The method require solving the functional equation formed by the first order condition (using the notation of Deaton (1991))

$$\begin{aligned} p(x,y) &= \text{Max}\,(\lambda(x), \zeta(x,y)) \\ \zeta(x,y) &= \beta \int p\,[(1+r)\,(x - \lambda^{-1}p(x,y)) + \rho y + y_{LT}(1-\rho) + \epsilon, \\ &\qquad \rho y + y_{LT}(1-\rho) + \epsilon]\, dF(\epsilon) \end{aligned} \tag{5}$$

where $\lambda(x) = u'^{-1}(x)$ and $p(x,y) = \lambda(f(x,y))$ is the marginal utility of money.

To solve this problem, the function $p(.,.)$ is discretized over a two dimensional grid. We start with a first guess, $p_0(.,.) = \text{Max}(\lambda(x), 0)$. This is the marginal utility of money at the terminal period, when every thing is spend. We update the function using

$$\begin{aligned} p_{n+1}(x,y) &= \text{Max}\,(\lambda(x), \zeta_n(x,y)) \\ \zeta_n(x,y) &= \beta \int p_n\,[(1+r)\,(x - \lambda^{-1}p_n(x,y)) + \rho y + y_{LT}(1-\rho) + \epsilon, \\ &\qquad \rho y + y_{LT}(1-\rho) + \epsilon]\, dF(\epsilon) \end{aligned} \tag{6}$$

and we stop the iterations when $p_n(x,y)$ is close enough to $p_{n+1}(x,y)$. [3] The calculation of the conditional expectation require the function $p_n(.,.)$ to be integrated with respect to y. This is done by replacing the continuous process by a discrete one, as developed in Tauchen and Hussey (1991), and used in Deaton (1991). We used a 50 by 15 grid for all the simulations.

We have performed both methods for model M_1, with the same set of parameters and the same sequence of innovations. The persistence of income takes the values 0, 0.15 and 0.75. The standard error of the innovations takes the values 3%, 5% and 10%. The value function iteration method can be considered as the reference. The comparison between both methods are displayed in Tables 1 and 2. Table 1 compare point to point both consumption trajectories by comparing both stream of utility generated by both simulated consumption paths. We focus on the relative percentage error, calculated as $err = (U(c_I) - U(c_{II}))/U(c_I) * 100$, where I refers to the value function method and II to our method. First statistic is the average percentage departure between both method, calculated as the mean over the sample of the percentage error in absolute values. The second statistic is the maximum departure (in absolute value) between both method ($\max_t |err_t|$).

[3] The iterations were stopped when the maximum distance between two successive functions were less than 0.1% (about 10^{-8}).

The third line is the standard deviation of the error. Last statistic is the concentration of the errors around zero, expressed as the frequency of errors less than two standard deviation of the errors.

Table 1: Evaluating the accuracy

s.d.		ρ=0	ρ=0.15	ρ=0.75
$\sigma = 3\%$	Average departure	0.25	0.31	0.45
	Maximum departure	0.98	1.23	4.29
	Standard deviation	0.33	0.42	0.45
	Prob($\|err\| \leq 2$ s.d.)	95.4	93.8	92.2
$\sigma = 5\%$	Average departure	0.48	0.60	0.98
	Maximum departure	2.46	2.80	8.40
	Standard deviation	0.64	0.82	1.63
	Prob($\|err\| \leq 2$ s.d.)	94.8	95.6	92.8
$\sigma = 10\%$	Average departure	1.59	1.79	3.29
	Maximum departure	9.28	10.03	17.66
	Standard deviation	2.18	2.45	4.63
	Prob($\|err\| \leq 2$ s.d.)	93.20	94.39	94.00

Notes : All figures are in percentage of relative deviation. Simulations were performed with $\gamma = 2$, r=.05, $\beta = .95$ and $y_{LT} = 100$. Statistics on a sample of 500 periods.

The bias is increasing in the size and in the persistence of the shock. The average departure is small, even for larger shocks, ranging from 0.25% to 3.3%. The concentration of the errors around zero are also quite large, around 94%. However, when the income gets persistent and more noisy, extreme outliers can appear, as for the case $\sigma = 10\%$. To give an idea of the magnitude of the bias, we have also computed the value function iteration with a smaller grid, which introduces a bias compared to the large grid. The new grid is a 10 by 10 grid, the one used in Deaton (1991). For $\sigma = 3\%$ and $\rho = 0.75$, the statistics measuring the departure between the method with larger grid and the one with coarser grid are : average departure 0.80%, maximum departure 3.67%, standard deviation 1.11, and concentration around zero 92.6%. For $\sigma = 10\%$ and $\rho = 0.75$, the statistics are : average departure 1.17%, maximum departure 5.6%, standard deviation 1.56, and concentration around zero 94%.

The bias is comparable than the bias introduced by our method. In practice, a 50 by 15 grid is burdensome, and one might be tempted to reduce its size. In this light, the accuracy of the method proposed in the paper seems very satisfactory.

Table 2 presents the volatility ratio of consumption versus income, determined by both methods. The volatility ratio is an important economic issue for consumption theory and we shall mainly focus on this statistic in a further section. As for the volatility ratio, our simulations perform well.

For larger models (M_2, M_3, and M_4), we have no other method to compare to. However, the introduction of habit persistence or of durability does not introduce further non-linearities, but increases the state-space. The non-linearity due to the *max* operator is already present for model M_1. We can hope, in regard to the result for M_1, that the results are satisfactory for the other models as well. [4]

To eliminate bias as much as possible, we are keeping the innovation shocks small (3%). To our knowledge there is no way to evaluate the bias.

Table 2: Evaluating the accuracy : Volatility of Consumption

	ρ=0		ρ=0.15		ρ=0.75	
Method	*I*	*II*	*I*	*II*	*I*	*II*
$\sigma = 3\%$	0.85	0.91	0.88	0.94	1.00	1.00
$\sigma = 5\%$	0.79	0.85	0.83	0.89	1.00	1.00
$\sigma = 10\%$	0.66	0.78	0.71	0.82	0.96	0.97

Note : I : Value function iteration method. II : Our method. Simulations were performed with $\gamma = 2$, r=.05, $\beta = .95$ and $y_{LT} = 100$. Statistics on a sample of 500 periods.

[4]Another test of the validity of the method could be the test developped by Den Haan and Marcet (1994). However, this test is better adapted to test one method against another than a direct test of the accuracy.

4 Simulation Results

Although liquidity constraints, habit persistence and durability have been extensively tested on data, there have been no theoretic attention on the implications on savings and consumption profiles, except Heaton (1993) who studies the consumption behavior in a continuous-time linear-quadratic model with time-non separable preferences. What happens if one introduces in a basic model of intertemporal optimization with liquidity constraints, as studied by Deaton (1991), habit persistence or durability ? Does habit persistence generate the same features of consumption and savings as a model with durability ?

To answer these question, we present in a first time, deterministic simulation, to analyze the response of the model to a change in income. We present in a second section stochastic simulations.

4.1 Deterministic Simulation Results

We start the analysis of the different models presented in section 2 by performing deterministic simulations. This amounts to perturb the model at the first period (the shock is then unanticipated by the agent), and to put to zero all further shocks. The exogenous variable is the income, and we focus on positive shocks. As liquidity constraints are imposed, a negative shock on income induces only a one to one decrease in consumption and has no effect on savings. A positive shock has a lasting effect on both consumption and savings, and the duration depends on the nature of the non separabilities in the utility function. We focus on the expenditure, rather than on consumption. The distinction is only important for models M_3 and M_4, as for the other models expenditure and consumption are equal.

The simulations are carried out for the value of the parameters:

$$\begin{array}{ccc} \gamma = 2 & \beta = 0.95 & r = 0.05 \\ \delta = 0.1, 0.5, 1 & \alpha = 0, 0.5, 0.9 & \text{shock} = 3\% \end{array}$$

We start with a quick comparison between the four models. The results are shown in Figure 1. Model M_1 exhibits the less persistence. The effects of a positive shock on income only last for six periods. All other models appear to be more persistent. Habit persistence alone (M_2) and habit persistence together with durability introduces much more persistence. Note that model

M_4 is not a model "in between" M_2 and M_3. Durability introduces oscilations in the consumption path, when habit formation generates smoothness.

The first period response of expenditures to a positive shock on income is highest when durability is present and lowest in presence of habit formation.

Durability. The results for model M_3 are displayed in Figure 2, for different values of the coefficient δ. As durability becomes more important, consumption shows more persistence, and the response to an income shock is higher. Note that Figure 2 presents the results for expenditures and not consumption. As consumption is the sum of current and lagged purchases, it is smoother. However, some wriggles appear when durability is more important. This confirms the earlier result of Heaton (1993). With this specification, the agent can tolerate a fluctuating consumption because he actually consumes an average of past consumption, which is smoother.

The savings path shows the inverted features, as consumption and savings are closely linked. When durability is important, savings is depressed as consumption becomes higher.

Habit persistence. Model M_2 exhibits consumption paths which are smoother than model M_1, (see Figure 3). With habit persistence, the agent finds disutility to vary the level of consumption from one date to another, because lagged consumption enters directly his subutility function. He will therefore try to smooth as much as possible his expenditure. As a result, the more habit persistence there is, the more savings and the less consumption there will be.

Durability and Habit persistence. When durability and habit persistence are both present, the model shows a greater smoothness, which is much more than model M_2 alone. is not an average of models M_2 and M_3. The consumption and savings paths take a long time to return to the long run value.

4.2 Stochastic Simulation Results

The stochastic simulations are carried out for different parameter values, but for a same sequence of innovations, in order to be able to compare the different series. We focus once again on the effect of durability and habit persistence, as well as the effect of the persistence of the income process, measured through the parameter ρ. We set

$$\gamma = 2 \qquad \beta = 0.95 \qquad r = 0.05$$
$$\delta = 0.15, 0.75 \quad \alpha = 0.15, 0.75 \quad \rho = 0.15, 0.75$$
$$\text{s.d. shock} = 3\%$$

We obtain 18 pseudo time-series of consumption and savings. We try to screen the particular characteristics of each model using several statistics. If we first turn to a qualitative description of the consumption series, we find the same results as Deaton (1991) for model M_1, as can be seen in Figure 6. The effect of liquidity constraints are easy to visualize. The series are clearly asymmetric and the downward spikes are much more intense than the upward ones. It shows that assets are much more successful to level out good draws of income than bad ones, since the consumer is not able to borrow against future income. The assets show frequent zero-level. When the liquidity constraints is binding, consumption equals income for models M_1 and M_2. For the other models, the consumption which is a weighted function of current and past expenditure, does equal income only if the agent experiences two periods of binding constraints. For the different ranges of the parameters, the ratios of consumption and income volatility are reported in Table 3 we find that most of the time, consumption is smoother than income. The results for model M_1 are comparable with those obtained by Deaton (1991). Introducing habit persistence make the consumption even smoother, a fact that we had already noticed in the previous section. On the contrary, durability produce less smoothness, or even a consumption with much more noise than income. As a general rule, smoothness increases when the persistence of the income process or the durability decreases, or when habit persistence increases.

Table 4 report the persistence of consumption. In the case studied by Hall (1978), consumption follows a unit root, whatever the income process. In the case studied here, as liquidity constaints are sometimes bounding, the persistence of consumption is linked to the persistence of income. Even low persistence in the income process can generate high persistence in consumption.

We report in Table 5 the mean and the volatility of assets. The average level of assets is most often comparable across models for a same value of ρ. When the parameter ρ increases, the average level of assets and its volatility increase as well.

We also look at the consumption density functions for different parameter values. They allow to characterize the asymmetry of the consumption profiles. All density functions have been estimated by a non parametric method using an epanetchikov kernal. Figure 8a display the density functions for a persistence of income of $\rho = 0.15$ for the four models. The densities are asymmetric with a huge peak and a thick left tail. This left tail represents the downward peaks observed in Figure 6. The asymmetry occurs because there is no upward peaks, as the agent saves good draws of income. As the perisistence of income increases, the density tends to be more symmetric. Durability (model M_3), gives a flatter density.

5 Conclusion

In the previous sections, we have shown the feasibility of the simulation method. Although there is some bias, especially when the variance of the innovations is large, this method is more adapted than value function iterations when the state space is large. Even though the latter method can be made arbitrarily exact, in practice and in a reasonable amount of time, the bias would not be small when dealing with large models. Moreover, when the simulations are imbedded in an estimation framework, the policy functions have to be recalculated at each step, slowing down the method, so a smaller grid is even more likely to be chosen.

Starting from the results of Deaton (1991), we show that they can be notably distorted when we introduce durability or habit persistence. We characterize the consumption and savings paths for each of our four models. Durability introduces erratic fluctuations toward the long run steady state and the response to income shocks is higher. The stochastic implication is that consumption can be much nosier than income, a fact that contradict the PIH. Habit persistence introduces more smoothness in consumption, and thus a higher assets level. In term of volatility, consumption is much more smoother than income, and the asymmetry of the consumption density, noticed by Deaton, tends to disappear.

References

ALLAN BAR-ILLIAN AND A. S. BLINDER. Life-Cycle Permanent- Income Model and Consumer Durables. 1987. NBER Working Paper 2149.

RAOUF BOUCEKKINE. An Alternative Methodology for Solving Nonlinear Forward-Looking Models. *Journal of Economic Dynamics and Control*, 19:711–734, 1995.

J. CAMPBELL AND G. MANKIW. Consumption, Income and Interest Rates : Reinterpreting the Time Series Evidence. In *NBER Macroeconomic Annual 1989*, pages 1–50, Olivier Blanchard and Stanley Fischer, eds. Chicago University Press, 1989.

L. CHRISTIANO. Solving the Stochastic Growth Model by Linear-Quadratic Approximation and by Value-Function Iteration. *Journal of Business and Economic Statistics*, 8:–, 1990.

ANGUS DEATON AND GUY LAROQUE. Estimating a Nonlinear Rational Expectations Model with Unobserved State Variables. 1995. mimeo, CREST-INSEE 9501.

ANGUS DEATON. Savings and Liquidity Constraints. *Econometrica*, 59:1221–1248, January 1991.

WOUTER DEN HAAN AND ALBERT MARCET. Accuracy in Simulations. *Review of Economic Studies*, 61:3–18, January 1994.

PAUL FISHER. *Rational Expectations in Macroeconomic Models.* Kluwer Academic Publishers, London, 1992.

MAJORIE FLAVIN. Excess Sensitivity of Consumption to Current Income : Liquidity Constraints or Myopa ? *Canadian Journal of Economics*, 18:177–136, 1985.

J. GAGNON. Solving the Stochastic Growth Model by Deterministic Extended Path. *Journal of Business and Economic Statistics*, 8:–, 1990.

ROBERT HALL AND FREDERICK MISHKIN. The Sensitivity of Consumption to Transitory Income : Estimates from Panel Data on Household. *Econometrica*, 50:461–481, March 1982.

ROBERT E. HALL. Stochastic Implications of the Life Cycle- Permanent Income Hypothesis : Theory and Evidence. *Journal of Political Economy*, 86:971–987, 1978.

FUMIO HAYASHI. The Permanent Income Hypothesis and Consumption Durability : Analysis Based on Japanese Panel Data. *Quarterly Journal of Economics*, 100:1083–1113, November 1985.

JOHN HEATON. The Interaction between Time-Nonseparable Preferences and Time Aggregation. *Econometrica*, 55:553–557, 1993.

JEAN-PIERRE LAFFARGUE. Résolution d'un modèle à anticipations rationnelles. *Annales d'Economie et de Statistique*, 17:97–119, 1990.

BONG-SOO LEE AND BETH FISHER INGRAM. Simulation Estimation of Time-Series Models. *Journal of Econometrics*, 47:197–205, 1991.

R. P. MARIGER. A Life Cycle Consumption Model with Liquidity Constraints : Theory and Empirical Results. *Econometrica*, 55:553–557, 1987.

JOHN MUELLBAUER. Habits, Rationality and Myopa in the Life Cycle Consumption Function. *Annales d'Economie et de Statistique*, 9:47–70, 1988.

ALPHONSOS NOVALES. Solving Nonlinear Rationnal Expectations Models : A Stochastic Equilibrium Model of Interest Rates. *Econometrica*, 58:93–111, 1990.

G. TAUCHEN AND ROBERT HUSSEY. Quadrature-Based Methods for Obtaining Approximate Solutions to Nonlinear Asset Pricing Models. *Econometrica*, 59:371–396, March 1991.

J. B. TAYLOR AND H. UHLIG. Solving Nonlinear Stochastic Growth Models : A Comparison of Alternative Solution Methods. *Journal of Business and Economic Statistics*, 8:–, 1990.

STEPHEN P. ZELDES. Consumption and Liquidity Constraints : An Empirical Investigation. *Journal of Political Economy*, 97:305–346, 1989.

STEVEN ZELDES. Optimal Consumption with Stochastic Income : Deviations from Certainty Equivalence. *Quarterly Journal of Economics*, 104:275–298, 1989.

Table 3: Consumption-Income Volatility Ratio

ρ	δ	α	M_4	M_3	M_2	M_1
0.15	0.15	0.15	0.93	0.95	0.63	0.65
0.15	0.15	0.75	0.96	0.95	0.55	0.65
0.15	0.75	0.15	0.71	0.72	0.63	0.65
0.15	0.75	0.75	-	0.72	0.55	0.65
0.75	0.15	0.15	-	1.56	0.86	0.87
0.75	0.15	0.75	-	1.56	-	0.87
0.75	0.75	0.15	1.06	1.06	0.86	0.87
0.75	0.75	0.75	-	1.06	-	0.87

Notes : M_1 : Liquidity constraints alone. M_2 : Habit persistence and liquidity constraints. M_3 : Durability and liquidity constraints. M_4 : Durability, habit persistence and liquidity constraints.

Table 4: Consumption Persistence

ρ	δ	α	M_3	M_2	M_1
0.15	0.15	0.15	0.72	0.45	0.38
0.15	0.15	0.75	0.72	0.81	0.38
0.15	0.75	0.15	0.53	0.45	0.38
0.15	0.75	0.75	0.53	0.81	0.38
0.75	0.15	0.15	0.94	0.88	0.91
0.75	0.15	0.75	0.94		0.91
0.75	0.75	0.15	0.91	0.88	0.91
0.75	0.75	0.75	0.91		0.91

Notes : Persistence is defined as the coefficient of AR(1) process.

Table 5: Mean and Volatility of Assets x 10^{-2}

ρ	δ	α	M_3		M_2		M_1	
0.15	0.15	0.15	6.6	4.6	6.4	3.6	4.6	3.4
0.15	0.15	0.75	5.2	4.6	6.4	7.5	6.8	3.4
0.15	0.75	0.15	3.6	5.1	3.6	4.6	3.4	4.5
0.15	0.75	0.75	3.6	5.1	7.5	6.8	3.4	4.5
0.75	0.15	0.15	26.8	34.7	15.6	23.5	14.8	22.8
0.75	0.15	0.75	26.8	34.7	-	-	14.8	22.8
0.75	0.75	0.15	18.5	26.5	15.6	23.5	14.8	22.8
0.75	0.75	0.75	18.5	26.5	-	-	14.8	22.8

Figure : 1
Consumption Path for the Four Models

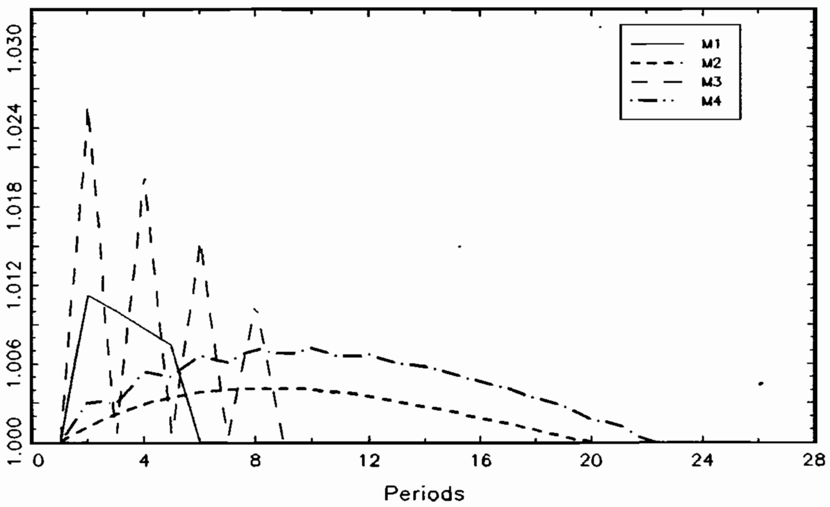

Figure : 2
Consumption Path for Durability Model (M3)

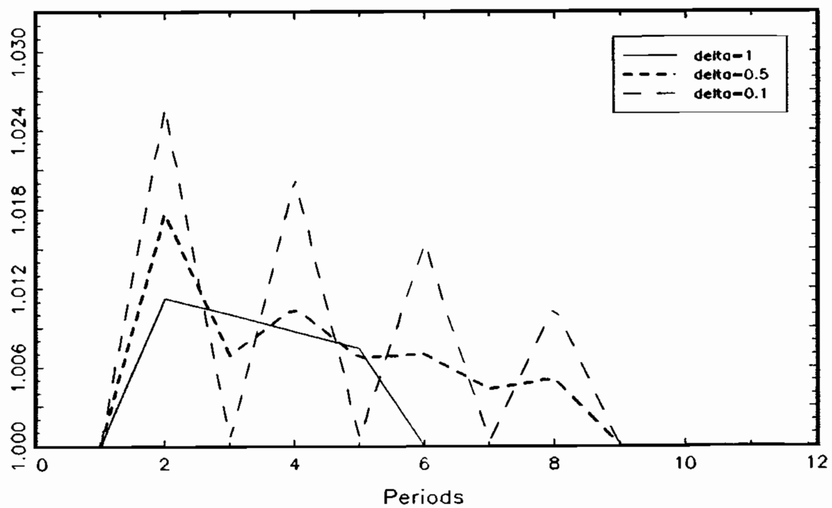

Figure : 3

Consumption Path for Habit Formation Model (M2)

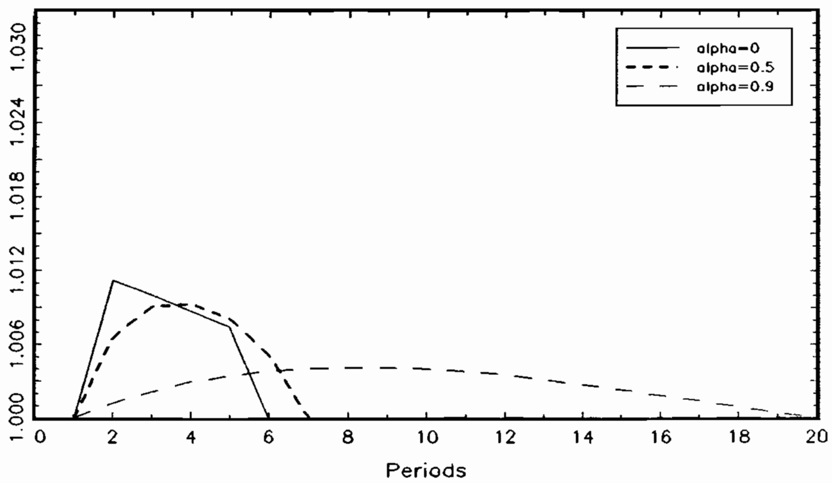

Figure : 4

Consumption Path for Habit Formation versus Durability (M2/M3)

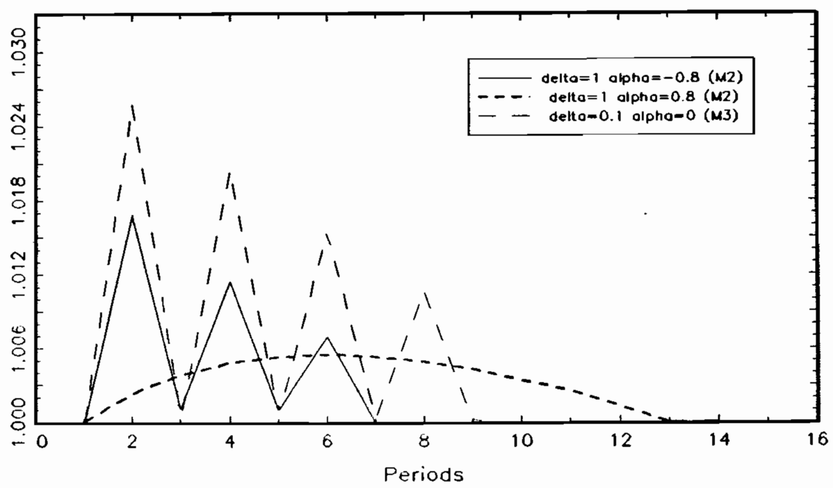

Figure : 5a
Consumption Path for Model M1

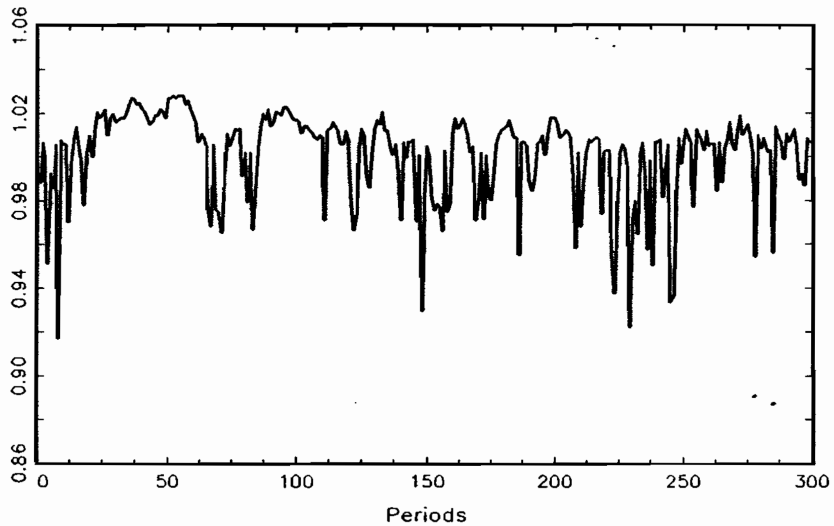

Figure : 5b
Consumption Path for Model M2

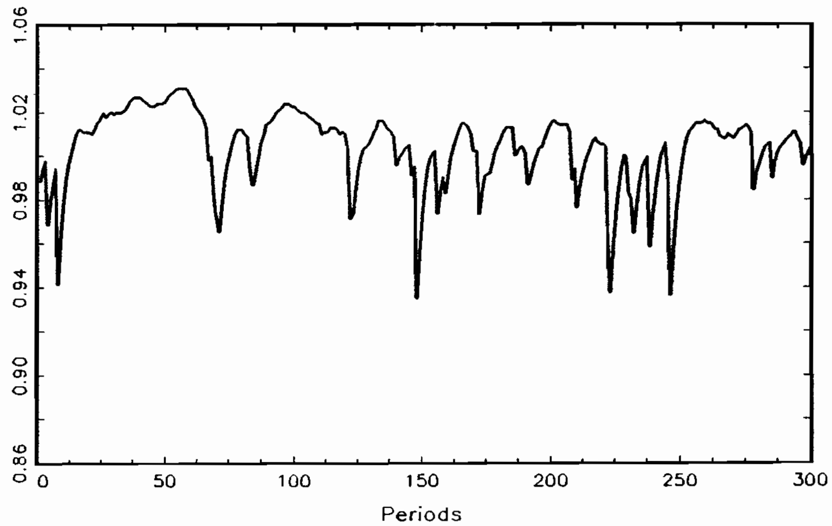

Figure : 5c
Consumption Path for Model M3

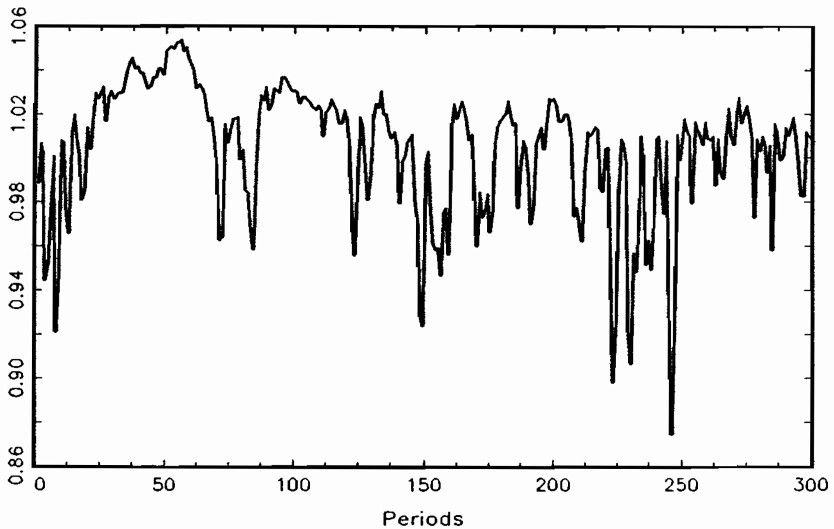

Figure : 5d
Consumption Path for Model M4

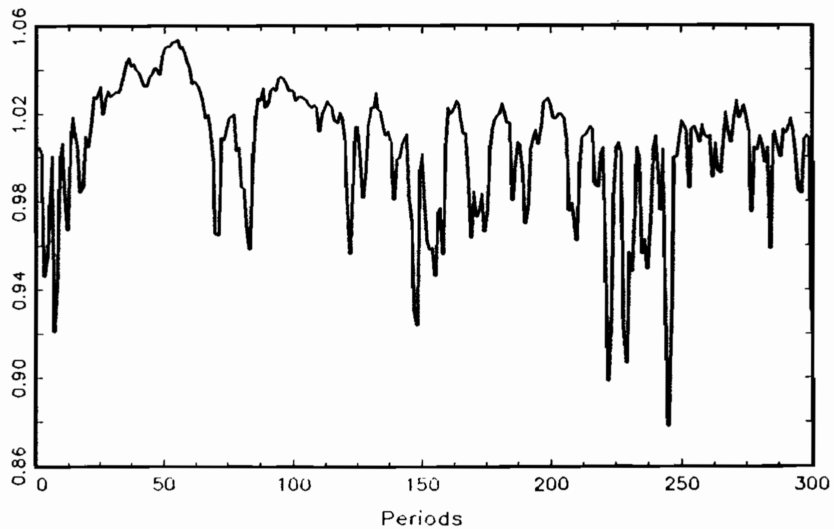

Figure : 6a
Savings Path for Model M1

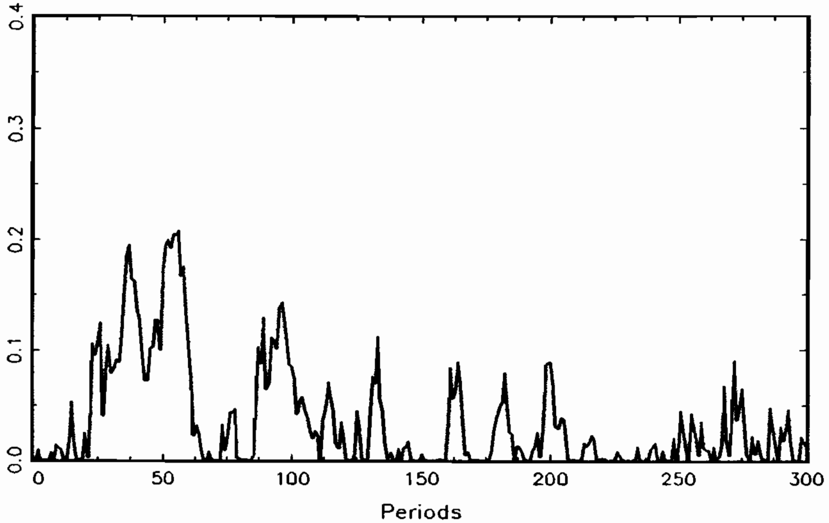

Figure : 6b
Savings Path for Model M2

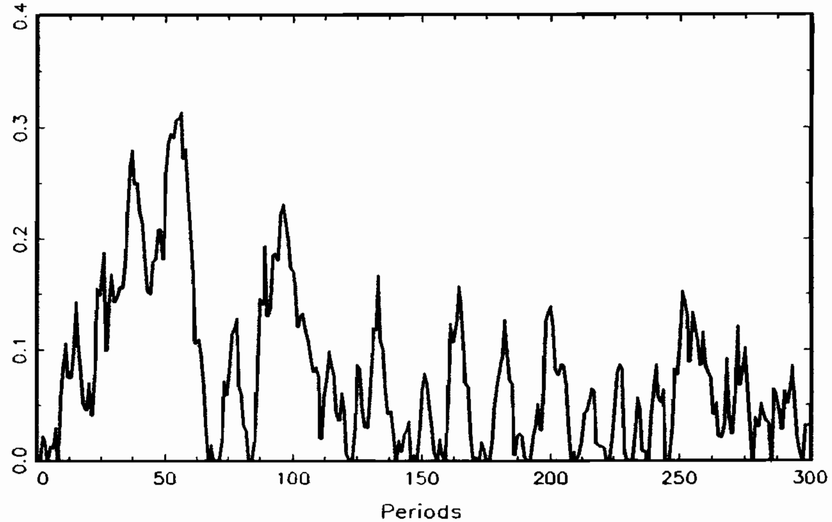

Figure : 6c
Savings Path for Model M3

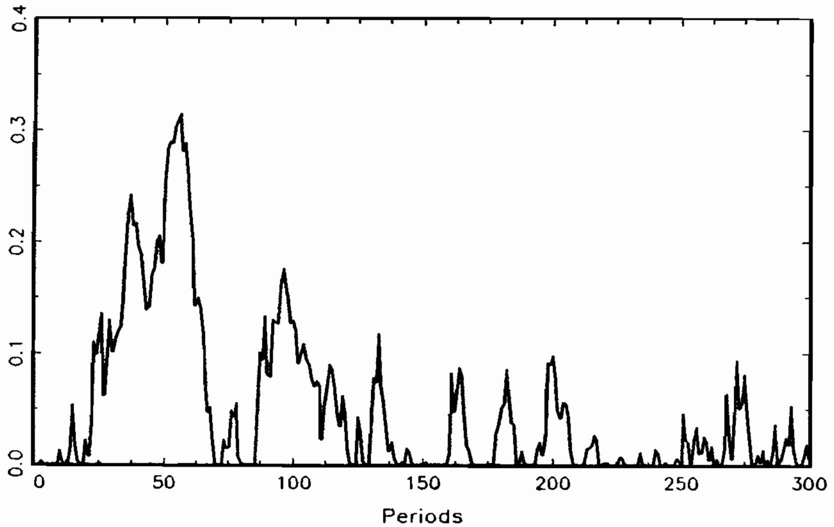

Figure : 6d
Savings Path for Model M4

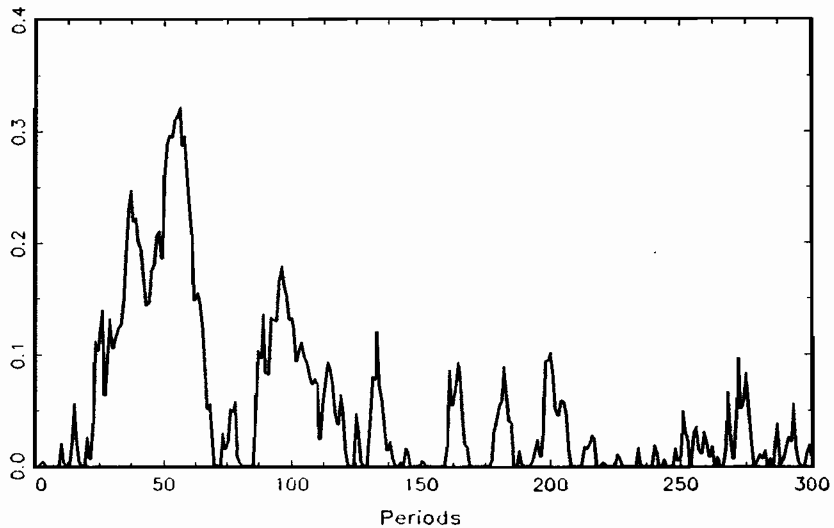

Figure : 7a
Density of Consumption, rho=0.15

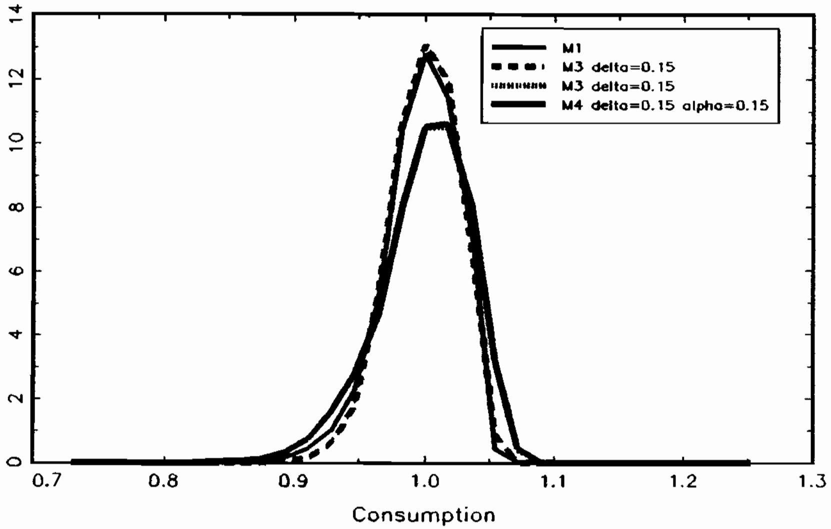

Figure : 7b
Density of Consumption rho=0.75

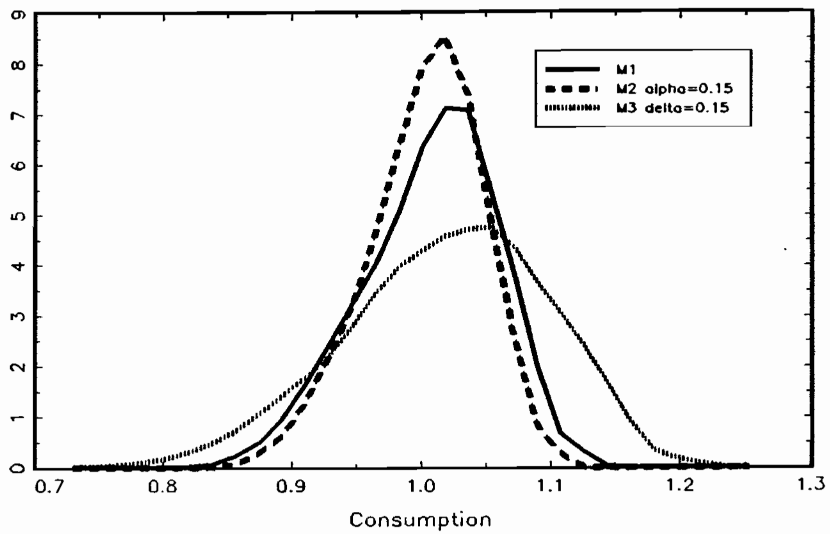

Figure : 7c
Density of Consumption