# Pigouvian Taxes: A Strategic Approach

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## Abstract

This paper analyzes the problem of designing mechanisms to implement efficient solutions in economies with externalities. We provide two simple mechanisms implementing the Pigouvian Social Choice Correspondence in environments in which coalitions can or cannot be formed.

## 1. Introduction

This paper studies the problem of implementing *efficient allocations* in economies with externalities. In these environments, it is well known that competitive equilibrium fails to be Pareto efficient. Pigou (1920) proposed solving this problem by establishing a *tax system* (called Pigouvian taxes) in such a way that when agents are price and tax takers a Pareto efficient allocation is attained.[1]

A classical criticism of Pigou's approach is that the computation of Pigouvian taxes requires precise knowledge of utility and production

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[1] Other approaches dealing with the problem of externalities include the classical contributions by Coase (1960) and Arrow (1970).

functions—information that is not in the hands of the regulator. This is the typical problem dealt with in the Theory of Implementation. In this framework Varian (1994) presents a two-stage *compensation mechanism* implementing in subgame perfect Nash equilibria (SPE) the Pigouvian Social Choice Correspondence (PSCC), that is, the mapping between the set of admissible economies and the allocations that are a Pigouvian equilibrium for the corresponding economy. Varian's compensation mechanism for the two-agent case is such that at the first stage each agent announces the Pigouvian tax for the other agent. The agent generating the externality is penalized whenever the sum of the Pigouvian taxes is not balanced. At the second stage, this agent selects the output level.

The compensation mechanism provides a nice solution to the problem of implementing the PSCC when agents' preferences are strictly convex. However, this mechanism presents two difficulties. First, in economies in which one agent has linear preferences, there are SPE of this mechanism yielding allocations that are not Pareto efficient (see Examples 2.8 and 2.9 below). Second, and more important, the solution concept used by Varian, namely SPE, cannot be easily extended to environments in which coalitions can be formed. We believe that an extension to these environments is crucial because of their relevance to potential applications.[2]

In this paper we present two "simple" mechanisms that implement, even for linear economies, the PSCC for different equilibrium concepts.

The first mechanism implements the PSCC in strong Nash equilibria. An important difference between Varian's mechanism and ours is that output is not decided by a single individual. As we will see in Examples 2.8 and 2.9, the reason that Pareto inefficient allocations can be supported by SPE in Varian's compensation mechanism is that output is decided by a single individual. In order to avoid this problem, our mechanism determines the output level through the interaction of all agents. In order to interpret this procedure, let us think of the level of externality as pollution. The quantity of pollution must equal the quantity of pollution permits, which are determined by the interaction of all agents. Assuming that the quantity of pollution can be monitored without cost, and that pollution and output are in fixed proportions, it follows that output is (indirectly) determined by the quantity of pollution permits.

[2] In the case considered in this paper, Pigouvian taxes can be interpreted as Lindahl prices of the pollution. Therefore, a Pigouvian equilibrium can be seen as a Lindahl equilibrium. Nakamura (1988) presents a feasible and continuous mechanism (at the cost of the simplicity of the mechanism) implementing Pigouvian allocations in Nash equilibria but not in strong equilibria. Walker (1981) presents a mechanism implementing Lindahl allocations in Nash equilibria but not in strong equilibria. Peleg (1996) presents a mechanism implementing the Lindahl correspondence in Nash and strong equilibria for the case in which all strategically active agents have strictly increasing preferences in all goods. In our model, agent 0 has strictly increasing preferences in the public good but preferences for the rest of the agents are strictly decreasing in this good. Thus, Peleg's mechanism cannot be directly applied to our framework.

The second mechanism doubly implements the PSCC in Nash and strong Nash equilibria. This mechanism is more complicated than the first one but it has the additional advantage that implementation occurs in both Nash and strong Nash equilibria. Therefore, the implementation of the desired result does not depend on the possibilities of coalition formation.

The two mechanisms are simple. Agents announce allocations and prices, and the mechanisms mimic the role of competitive markets. The first mechanism is continuous but there are Nash equilibria yielding allocations that are not Pareto efficient. The second mechanism is discontinuous but it can be made continuous at the cost of complicating the mechanism. Thus, the trade-off between the two mechanisms is that of simplicity versus robustness of the equilibrium concept.

The rest of the paper is organized as follows. Section 2 presents the model. Implementation of Pigouvian SCC in strong Nash equilibria using a continuous mechanism is reported in Section 3. Section 4 presents a mechanism that doubly implements the Pigouvian SCC in Nash and strong Nash equilibria.

## 2. The Model

Let us consider the following externality problem involving $n + 1$ agents. Let $I = \{0,1,\ldots,n\}$ be the set of agents. Agent 0 consumes $q$ units of a good generating an external effect on the other agents. Agents' preferences are representable by a utility function, $u_i$, which depends on two variables. The first one, $T_i \in \mathbb{R}$, plays the role of (a transfer of) numéraire,[3] whereas the second, $q \in \mathbb{R}_+$, measures the consumption of the good that generates the externality. For simplicity, we assume that each $u_i$ is quasilinear, but all of our results are still valid in the case of nonquasilinear utility functions. Thus, agent $i$'s utility function can be expressed as

$$u_i(T_i, q) = T_i + v_i(q),$$

where $v_i$ is concave, strictly increasing for agent 0, and strictly decreasing for the other agents.[4] Alternatively, for interpretative convenience, we

[3] In order to simplify the presentation, we assume that agents can incur unbounded debts. An alternative assumption is that $T_i$ is nonnegative and agents prefer any feasible allocation to any unfeasible allocation. In any case, the mechanisms presented in this paper can be made feasible at the cost of some extra complications.

[4] As was pointed out by Starrett (1972), externalities introduce nonconvexities that may preclude the existence of a Pigouvian equilibrium. In our case, for $q$ large enough, agents $1,\ldots,n$ would suffer a damage larger than any possible payoff they could obtain. In such a case these agents would like to shut down business (or move to a different location) and buy an infinite amount of pollution permits. The existence of a Pigouvian equilibrium can be restored if pollution cannot exceed a suitably chosen upper bound for which agents $1,\ldots,n$ wish to remain active. See Boyd and Conley (1997) for an alternative interpretation of this issue.

may write $T_i = t_i q$. In such a case, $t_i$ is interpreted as a per-unit tax. An *economy* is a list $e = (u_0, u_1,.., u_n)$. Let $\mathcal{E}$ denote the set of all admissible economies. Given an economy $e$, we say that an *allocation* $z = (T_0, T_1, \ldots, T_n, q)$ is *feasible* for $e$ if $q \in \mathbb{R}_+$ and $\sum_{i \in I} T_i = 0$. Let $Z$ denote the set of all feasible allocations. In order to simplify notation, we extend preferences (and utility functions) over allocations in the following way. We say that agent $i$ weakly prefers allocation $z = (T_0, \ldots, T_n, q)$ to $z' = (T'_0, \ldots, T'_n, q')$ iff $u_i(T_i, q) \geq u_i(T'_i, q')$. In this case, we will also write $u_i(z) \geq u_i(z')$.

The properties we introduce next are minimal conditions that should be satisfied by any solution to be considered satisfactory. The minimal requirements for an allocation are

1. *Pareto efficiency*. Given an economy $e$, we say that a feasible allocation $z$ is efficient if there is no feasible allocation $z'$ that is weakly preferred to $z$ by all the individuals, and strictly preferred by at least one of the agents. That is, $z \in Z$ is Pareto efficient if there is no $z' \in Z$ such that, $u_i(z') \geq u_i(z)$ for all $i \in I$, and $u_j(z') > u_j(z)$ for some $j \in I$.
2. *Individual rationality*. Given an economy $e$, we say that a feasible allocation $z$ is individually rational if it yields no fewer payoffs to all agents than the allocation where neither exchange nor production occurs. An allocation $z \in Z$ is individually rational whenever $u_i(z) \geq u_i(0,0)$ for all $i \in I$.

We are interested in social choice correspondences satisfying these two minimal conditions. It is well known that, in a competitive equilibrium, the level of output chosen by agent 0 fails to be Pareto efficient and individually rational. Pigou (1920) pointed out a solution to this problem—intervention by a regulator who imposes a tax system. The idea of establishing a tax system to solve the problem of externalities is credited to Pigou; however, he did not address the problem of distributing the income generated by these taxes.

Definition 2.1: *The Pigouvian Social Choice Correspondence (PSCC), $\phi_p : \mathcal{E} \to \mathbb{R}^n \times \mathbb{R}_+$, associates to each economy, $e = (u_0, u_1,.., u_n) \in \mathcal{E}$, a tax/transfer system, $(t_i^*)_{i \in I} \in \mathbb{R}^n$, and an output $q^* \in \mathbb{R}_+$ such that both of the following hold:*

1. *$q^*$ maximizes $u_i(T_i, q) = u_i(t_i^* q, q) = t_i^* q + v_i(q)$ for all $i \in I$*
2. $\sum_{i \in I} t_i^* = 0$.

Note that the quantity $q$ may be interpreted as a public good produced by a firm (agent 0) and paid by all agents. The only difference with the standard public good case is that utility functions of agents 1 to $n$ are decreasing on $q$. In such a case, given an economy $e = (u_0, u_1,.., u_n) \in \mathcal{E}$, $t_i$ is agent $i$'s individualized price of the public good, and condition 2 in Definition 2.1 represents the usual way of defining the price at which the firm sells the public good.

The PSCC has good properties. For instance, it is Pareto efficient and individually rational. However, in many cases the regulator does not have access to the information needed to compute the Pigouvian taxes. This is where implementation theory comes into the picture. The following concepts are standard in the literature.

DEFINITION 2.2: *A mechanism* $\Gamma$, *is a list of strategy spaces,* $M = (M_i)_{i \in I}$, *and an outcome function,* $f: \times_{i \in I} M_i \to \mathbb{R}^n \times \mathbb{R}_+$.

Given $e = (u_0, u_1, .., u_n) \in \mathcal{E}$, let $(\Gamma, e)$ denote a *normal form game.* Let $\chi$ be an equilibrium concept (i.e., Nash equilibrium, etc.) and $\chi(\Gamma, e)$ be the set of $\chi$-equilibria of the game $(\Gamma, e)$. In this paper we focus our attention on Nash and strong Nash equilibria.

DEFINITION 2.3: *A profile of messages* $m = (m_0, m_1, \ldots, m_n) \in M$ *is a Nash equilibrium of the game* $(\Gamma, e)$ *if for all* $i \in I$ *and for all* $m'_i \in M_i$, *we have that* $u_i(f(m_i, m_{-i})) \geq u_i(f(m'_i, m_{-i}))$. *Let* $N(\Gamma, e)$ *be the set of Nash equilibria of the game* $(\Gamma, e)$.

DEFINITION 2.4: *A profile of messages* $m = (m_0, m_1, \ldots, m_n) \in M$ *is a strong Nash equilibrium of the game* $(\Gamma, e)$ *if there is no* $S \subseteq I$ *and* $m'_S \in \times_{i \in S} M_i$, *satisfying* $u_i(f(m'_S, m_{-S})) > u_i(f(m_S, m_{-S}))$ *for all* $i \in S$. *Let* $S(\Gamma, e)$ *be the set of strong Nash equilibria of the game* $(\Gamma, e)$.

DEFINITION 2.5: *The mechanism* $\Gamma$ *implements the PSCC* $\phi_P: \mathcal{E} \to \mathbb{R}^n \times \mathbb{R}_+$ in $\chi$-*equilibrium if for all* $e \in \mathcal{E}$ *with* $\chi(\Gamma, e) \neq \emptyset$, $f(\chi(\Gamma, e)) = \phi_P(e)$.

Let $\chi$ and $\chi'$ be two concepts of equilibrium, and $\chi(\Gamma, e)$ (respectively, $\chi'(\Gamma, e)$) be the set of $\chi$-equilibria (respectively, $\chi'$-equilibria) of the game $(\Gamma, e)$.

DEFINITION 2.6: *The mechanism* $\mathbf{\Gamma}$ *doubly implements the PSCC* $\phi_P: \mathcal{E} \to \mathbb{R}^n \times \mathbb{R}_+$ *in* $\chi$-*equilibrium and* $\chi'$-*equilibrium if for all* $e \in \mathcal{E}$ with $\chi\ (\Gamma, e) \neq \emptyset$, *and* $\chi'(\Gamma, e) \neq \emptyset$, $f(\chi(\Gamma, e)) = f(\chi'(\Gamma, e)) = \phi_P(e)$.

Let us introduce Varian's (1994) compensation mechanism for the two agents case.

DEFINITION 2.7 (The Compensation Mechanism; Varian 1994):

- ***Announcement stage****: Agents 0 and 1 simultaneously announce the magnitude of the appropriate Pigouvian per-unit tax; denote the announcement of firm 0 by* $m_0$ *and the announcement of agent 1 by* $m_1$.
- ***Choice stage****: The regulator makes side payments to the agents so that they face the following payoff functions:*

$$u_0(q, m_0, m_1) = m_1 q + v_0(q) - \alpha(m_0 + m_1)^2$$

$$u_1(q, m_0, m_1) = m_0 q + v_1(q).$$

*The parameter* $\alpha > 0$ *is of arbitrary magnitude. Agent 0 selects the level of output* $q$.

Thus, both individuals simultaneously select Pigouvian taxes at the first stage and agent 0 selects the output level at the second stage. Varian (1994) shows that this mechanism implements in SPE the Pigouvian Social Choice Correspondence when agents' preferences are strictly convex. However, we provide two examples which show that Varian's results are only valid when preferences are convex but not strictly convex.

*Example 2.8:* Agent 0 has linear preferences (constant returns to scale). Consider the following profit functions

$$v_0(q) = pq - cq, \qquad p > c > 0$$

$$v_1(q) = -\frac{d}{2}q^2 \qquad d > 0.$$

The Pigouvian allocation is $q^* = (p - c)/d$, $t_0^* = c - p$, $t_1^* = p - c$. Note that $q^* = (p - c)/d$, $m_0 = t_1^*$, $m_1 = t_0^*$ is an SPE of Varian's compensation mechanism yielding the Pigouvian allocation. However, setting $\hat{q} = 0$, $\hat{m}_0 = p - c$, $\hat{m}_1 = c - p$ is a subgame perfect equilibrium because agent 0's payoff is identically zero in the second stage.

*Example 2.9* (Linear externality): Consider the following functions

$$v_0(q) = pq - Aq^c, \qquad c > 1,\, p, A > 0$$

$$v_1(q) = -dq, \qquad d > 0$$

The Pigouvian allocation is $q^* = [(p + d)/cA]^{1/(c-1)}$, $t_0^* = -t_1^* = -d$. Then agent 1's payoff is zero. Note that $q^*$, $m_0 = t_1^*$, $m_1 = t_0^*$ is an SPE of Varian's compensation mechanism yielding the Pigouvian allocation. However, there are SPE where agent 1 chooses any arbitrary $m_1$ (which yields an undesirable outcome in the second stage) because agent 1 makes zero payoff in the second stage.

We have seen that the compensation mechanism, in the case of economies in which exactly one agent has linear preferences, does not solve the problem of finding a mechanism that implements the Pigouvian SCC. More seriously, coalition formation cannot be considered because an extension of SPE to deal with this possibility does not exist. Thus, we look for new mechanisms in which the problem of coalition formation can be dealt with.

## 3. A Continuous Mechanism Implementing the Pigouvian Correspondence in Strong Nash Equilibria

This section presents a simple and continuous mechanism that implements, in strong Nash equilibria, the Pigouvian Social Choice Correspondence. For the sake of concreteness, think of the classical example of the papermill (agent 0) and the fishermen (agents $1, \ldots, n$). In this context, it is reasonable to assume that agents know the damages and profits caused by the papermill output and that they can meet together and discuss the quantity of pollution permits. Let us introduce the mechanism $\Gamma^S$ that formalizes the rules under which discussions are organized. Let $M_i = \mathbb{R}^2$ be the strategy space for each $i \in I$. A strategy for agent $i \in I$ is a pair $m_i = (m_i^1, m_i^2) \in \mathbb{R}^2$. The first component is interpreted as her proposed Pigouvian tax, $m_i^1$. The second component, $m_i^2$, is interpreted as the incremental level of pollution permits. A profile of strategies is a list $m = (m_0, m_1, \ldots, m_n) \in \mathbb{R}^{2(n+1)}$. The outcome function is

$$f(m_0, m_1, \ldots, m_n) = (t_0(m), \ldots, t_n(m), q(m)), \tag{3.1}$$

where $q(m) = \max\{0, \Sigma_{i=0}^n m_i^2\}$ and, for each $i$ in $I$, $t_i(m) = \min\{m_i^1, -\Sigma_{j \neq i} m_j^1\}$.

In this mechanism each agent $i \in I$ receives a compensation (or pays a tax) of $T_i(m) = t_i(m) q(m)$. Her payoff is $t_i(m) q(m) + v_i(q(m))$. Thus, in mechanism $\Gamma^S$ the quantity of pollution permits is unanimously decided by all agents and each agent's tax is given by the taxes announced by the others, except when she states a "high" tax (or a "low" subsidy) for herself. A strong Nash equilibrium of the game formed by this mechanism and the payoff functions of each agent yields a self-enforcing agreement that no group of agents would like to renegotiate.[5]

We first state a result connecting strong Nash and Nash equilibria for this mechanism.

LEMMA 3.1: *$\hat{m} = (\hat{m}_0, \hat{m}_1, \ldots, \hat{m}_n)$ is a strong Nash equilibrium of the above mechanism if and only if* (i) *$\hat{m}$ is a Nash equilibrium and* (ii) $\Sigma_{i=0}^n \hat{m}_i^1 = 0$.

*Proof:* To prove the necessary condition, let $\hat{m} = (\hat{m}_0, \hat{m}_1, \ldots, \hat{m}_n)$ be a strong Nash equilibrium of $\Gamma^S$ which does not satisfy condition (ii). First, consider the case where $q(\hat{m}) > 0$. Then consider the case where each agent plays the strategy $\widetilde{m}_i = (\hat{m}_i^1 - \Sigma_{i \in I}[t_i(\hat{m})/(n+1)], \hat{m}_i^2)$. Since the transfer received by each agent is larger than before and the level of $q$ is unchanged, all agents will benefit from this deviation. Notice that, when condition (ii) is not satisfied, $\Sigma_{i \in I} t_i(\hat{m})$ is negative. Second, suppose that $q(\hat{m}) = 0$. Consider the following agents' strategies. For agent 0, $\widetilde{m}_0 = (-\Sigma_{i=1}^n t_i, q)$ and for each $i = 1, \ldots, n$, $\widetilde{m}_i = (\Sigma_{j \neq i} t_j, 0)$, where $q$ and $t_i, i = 1, \ldots, n$, are positive real numbers satisfying $q \Sigma_{i=1}^n t_i < v_0(q) - v_0(0)$. Notice that, by construction, all agents prefer the outcome associated with these strategies rather that the equilibrium outcome. A contradiction.

In order to prove the sufficient condition, let $\hat{m}$ be a Nash equilibrium satisfying condition (ii) in Lemma 3.1. Suppose this is not a strong Nash equilibrium, so there is a coalition $S \subseteq N$ and strategies

[5] Notice that we do not assume the outcome of the meeting can be written in the form of a contract that can be enforced by courts, as in the famous Coase Theorem.

$\widetilde{m}_s \in M_S$ such that $u_i(T_i(\widetilde{m}_s, \hat{m}_{-s}), q(\widetilde{m}_s, \hat{m}_{-s})) > u_i(T_i(\hat{m}), q(\hat{m}))$ for each $i \in S$. Let us consider the following cases:

(a) $t_i(\widetilde{m}_s, \hat{m}_{-s}) = t_i(\hat{m})$ for some $i \in S$. Then, $u_i(t_i(\widetilde{m}_s, \hat{m}_{-s})q, q) = u_i(t_i(\hat{m})q, q)$ for all $q \in \mathbb{R}_+$, so $q(\hat{m}) \in \arg\max_q u_i(t_i(\widetilde{m}_s, \hat{m}_{-s})q, q)$ if, and only if $\hat{q} \in \arg\max_q u_i(t_i(\hat{m})q, q)$. A contradiction.

(b) $t_i(\widetilde{m}_s, \hat{m}_{-s}) \neq t_i(\hat{m})$ for all $i \in S$. Since $\sum_{j \in I} t_j(\widetilde{m}_s, \hat{m}_{-s}) \leq 0 = \sum_{j \in I} t_j(\hat{m})$, there should be an agent $i \in S$ such that $t_i(\widetilde{m}_s, \hat{m}_{-s}) < t_i(\hat{m})$. Therefore, $u_i(t_i(\hat{m})q(\widetilde{m}_s, \hat{m}_{-s}), q(\widetilde{m}_s, \hat{m}_{-s})) \geq u_i(t_i(\widetilde{m}_s, \hat{m}_{-s}) \times q(\widetilde{m}_s, \hat{m}_{-s}), q(\widetilde{m}_s, \hat{m}_{-s})) > u_i(t_i(\hat{m})q(\hat{m}), q(\hat{m}))$. Note that for agent $i$ there is a strategy $\overline{m}_i$ such that $q(\overline{m}_i, \hat{m}_{-i}) = q(\widetilde{m}_s, \hat{m}_{-s})$. Since $\hat{m}$ is a Nash equilibrium, we get a contradiction. ■

The main result in this section is given in the following theorem.

THEOREM 3.2: *The mechanism $\Gamma^S$ implements in strong Nash equilibria the Pigouvian Social Choice Correspondence.*

*Proof:* Let $e = (u_0, u_1, .., u_n) \in \mathcal{E}$ be an economy. We first prove that every Pigouvian equilibrium for $e$ can be supported by a strong Nash equilibrium of the game $(\Gamma, e)$.

Let $(T_0^*, \ldots, T_n^*, q^*) \in \mathbb{R}^n \times \mathbb{R}_+$ be a Pigouvian allocation for $e$. We show that the strategy profile $\overline{m}$, where $\overline{m}_i = (t_i^*, (q^*/n+1))$ and $t_i^* = T_i^*/q^*$ for each $i \in I$, is a strong Nash equilibrium for $(\Gamma^S, e)$ yielding this allocation. Note that, given other agents' strategies, no unilateral deviation by an agent can force a lower tax (higher subsidy) $t_i$ for her. Therefore, each agent $i \in I$ selects her incremental output level $m_i^2$ so to maximize her payoff $t_i^*(m_i^2 + [nq^*/(n+1)]) + v_i(m_i^2 + [nq^*/(n+1)])$. Since $(T_0^*, \ldots, T_n^*, q^*)$ is a Pigouvian allocation for $e$, $\overline{m}_i^2$ maximizes the function above. Thus $\overline{m}$ is a Nash equilibrium of $(\Gamma^S, e)$. Since $\sum_{i \in I} \overline{m}_i^1 = 0$, by Lemma 3.1, $\overline{m}$ is a strong Nash equilibrium of $(\Gamma^S, e)$.

On the other hand, let $\overline{m} \in \mathbb{R}^{2(n+1)}$ be a strong Nash equilibrium of $(\Gamma^S, e)$. By Lemma 3.1, $\overline{m}$ is a Nash equilibrium of $(\Gamma^S, e)$ and $\sum_{i \in I} \overline{m}_i^1 = 0$. Since $\overline{m}$ is a Nash equilibrium of $(\Gamma^S, e)$, $q(\overline{m})$ maximizes $t_i(\overline{m})q + v_i(q)$ for each $i \in I$. Given that $\sum_{i \in I} \overline{m}_i^1 = 0$, each agent $i \in I$ will behave as a price-taker because she can not decrease her own tax. Therefore, $(t_1(\overline{m})q(\overline{m}), \ldots, t_n(\overline{m})q(\overline{m}), q(\overline{m}))$ is a Pigouvian allocation for $e$. ■

## 4. Double Implementation of the Pigouvian Correspondence

This section introduces a mechanism that doubly implements the Pigouvian Social Choice Correspondence in Nash and strong Nash equilibria. It is related to the one presented by Corchón and Wilkie (1996). The motivation for this mechanism is that in some cases the planner lacks information not only about agents' preferences but also about the coalition formation possibilities. Thus we need a mechanism that copes with cases in which coalitions can or cannot be formed.

To simplify the presentation we introduce a mechanism that is discontinuous. Nevertheless, the mechanism can be made continuous by making the proper modifications (see Corchón and Wilkie 1996). Thus the tradeoff between the mechanism presented in Section 3 and the one in this section is that of simplicity versus a more robust concept of implementation.

We introduce the mechanism $\Gamma^H$. Let $M_i = \mathbb{R}^2$ be the strategy space for each $i \in I$. A strategy for agent $i \in I$ is a pair $m_i = (m_i^1, m_i^2) \in \mathbb{R}^2$. For each $i \in I$, $m_i^1$ is the tax (or subsidy) she proposes for herself, and $m_i^2$ is an incremental quantity on the production level. A profile of strategies is written as $\widetilde{m} = (\widetilde{m}_0, \widetilde{m}_1, \ldots, \widetilde{m}_n) \in \mathbb{R}^{2(n+1)}$. The outcome function is given by

$$f(\widetilde{m}) = (T_0(\widetilde{m}), \ldots, T_n(\widetilde{m}), q(\widetilde{m})) \tag{4.1}$$

with $q(\widetilde{m}) = \max\{0, \Sigma_{i=0}^n m_i^2\}$ if $\Sigma_{j\in I} m_j^1 = 0$ and $q(\widetilde{m}) = 0$ in any other case, and $T_i(\widetilde{m}) = m_i^1\, q(\widetilde{m})$ if $\Sigma_{j\in I} m_j^1 = 0$ and $-\gamma|\Sigma_{i=0}^n m_i^1|$ otherwise, where the parameter $\gamma > 0$ can be chosen to be arbitrarily small.

A natural interpretation can be provided for this mechanism. Each agent announces her proposed Pigouvian tax, $m_i^1$. The second component for each agent's strategy, $m_i^2$, is the incremental level of pollution permits. The output level is $q(\widetilde{m})$ and she receives a compensation (or pays a tax) of $T_i(\widetilde{m})$. So her payoff will be $T_i(\widetilde{m}) + v_i(q(\widetilde{m}))$. This compensation depends on two factors. First, if the announced taxes balance the sum of transfers among agents, each agent's compensation is $q(\widetilde{m})$ times the tax she announced. Second, if such a balance does not hold, no output is produced and each agent is penalized.

We next introduce our main result of this section.

THEOREM 4.1: *The mechanism $\Gamma^H$ doubly implements in Nash and strong Nash equilibria the Pigouvian Social Choice Correspondence.*

*Proof:* Let $e = (u_0, u_1,.., u_n) \in \mathcal{E}$ be an economy. We will first prove that every Pigouvian equilibrium can be supported by a strong Nash equilibrium of the game $(\Gamma^H, e)$.

Let $(T_0^*, \ldots, T_n^*, q^*) \in \mathbb{R}^n \times \mathbb{R}_+$ be a Pigouvian allocation for $e$. We show that the strategy profile $\overline{m}$, where $\overline{m}_i = (t_i^*, [q^*/(n{+}1)])$ and $t_i^* = (T_i^*/q^*)$ for each $i \in I$, is a strong Nash equilibrium of $(\Gamma^H, e)$ yielding such an allocation.

First, it is clear that such strategies yield the desired outcome. Now we show that $\overline{m} \in S(\Gamma^H, e)$. Consider a coalition $S \subseteq I$ and a deviation $\hat{m}_S$ of agents in this coalition. If $\overline{m}_i^1 = \hat{m}_i^1$ for all $i \in S$, any agent in $S$ can get any $q' \in \mathbb{R}_+$ at a price $t_i^*$. By definition of a Pigouvian allocation the agents cannot improve payoffs with any $q' \neq q^*$. If $\overline{m}_i^1 \neq \hat{m}_i^1$ for some $i \in S$, we should consider two cases.

(i) If $\sum_{i\in S} \hat{m}_i^1 + \sum_{j\in I\setminus S} \overline{m}_j^1 \neq 0$, then $q(\hat{m}_S, \overline{m}_{-S}) = 0$. Suppose that agents in $S$ obtain a higher payoff. But then for all $i \in S$

$$v_i(0) > -\gamma \left| \sum_{i\in S} \hat{m}_i^1 + \sum_{j\in I\setminus S} \overline{m}_j^1 \right| + v_i(0) > t_i^* q^* + v_i(q^*),$$

contradicting that $(T_0^*, \ldots, T_n^*, q^*)$ is a Pigouvian allocation for $e$.

(ii) If $\sum_{i\in S} \hat{m}_i^1 + \sum_{j\in I\setminus S} \overline{m}_j^1 = 0$, then $\sum_{i\in S} \hat{m}_i^1 = \sum_{i\in S} \overline{m}_i^1$. In this case, there should be an agent $i \in S$ such that $\hat{m}_i^1 < \overline{m}_i^1$. Suppose that $i$ gets a higher payoff. Since $\hat{m}_i^1 < \overline{m}_i^1$, we have that

$$\begin{aligned} u_i(\overline{m}_i^1 q(\hat{m}_S, \overline{m}_{-S}), q(\hat{m}_S, \overline{m}_{-S})) &\geq u_i(\hat{m}_i^1 q(\hat{m}_S, \overline{m}_{-S}), q(\hat{m}_S, \overline{m}_{-S})) \\ &> u_i(\overline{m}_i^1 q(\overline{m}), q(\overline{m})), \end{aligned}$$

which contradicts that $(T_0^*, \ldots, T_n^*, q^*)$ is a Pigouvian allocation for $e$. Thus, $\overline{m}$ is a strong Nash equilibrium and hence a Nash equilibrium.

On the other hand, let $\hat{m} = (\hat{m}_0, \hat{m}_1, \ldots, \hat{m}_n) \in N(\Gamma^H, e)$. We show that this yields a Pigouvian allocation for $e$. Consider two cases.

*Case A:* $\sum_{i\in I} \hat{m}_i^1 \neq 0$. In such a case, for each agent there is a strategy, namely $\overline{m}_i = (-\sum_{j\neq i} \hat{m}_j^1, -\sum_{j\neq i} \hat{m}_j^2)$ yielding $q(\overline{m}_i, \hat{m}_{-i}) = T_i(\overline{m}_i, \hat{m}_{-i}) = 0$, a contradiction.

*Case B:* $\sum_{i\in I} \hat{m}_i^1 = 0$. But then, given $\hat{m}_{-i}$, each agent is a price-taker so the outcome associated with the Nash equilibrium is a Pigouvian allocation for $e$.

To conclude the proof, let $\hat{m} = (\hat{m}_0, \hat{m}_1, \ldots, \hat{m}_n) \in S(\Gamma^H, e)$. Since $S(\Gamma^H, e) \subseteq N(\Gamma^H, e)$, $\hat{m}$ is a Nash equilibrium of this game. Therefore, its outcome is a Pigouvian allocation for $e$. ■

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