



Working Paper 06-52  
Economics Series 19  
August 2006

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**CORRIGENDUM TO “EXISTENCE OF EQUILIBRIUM IN SINGLE AND  
DOUBLE PRIVATE VALUE ACTIONS”**

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***Abstract***

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This is a corrigendum to Theorem 15 of Jackson and Swinkels (2005) [Existence of Equilibrium in Single and Double Private Value Auctions, *Econometrica*, 73, 93-140], which proves the existence of equilibrium with positive probability of trade for private value auctions.

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I am grateful to Diego Moreno and Myrna Wooders for helpful conversations and to Matthew O. Jackson and Jeroen M. Swinkels for kind and valuable comments

Jackson and Swinkels (2005) proved the existence of equilibrium with positive probability of trade for private value auctions (Theorem 15). This important theorem was established with great ingenuity, but there is a slight error in the last part of its proof, on p. 137. In the penultimate inequality, a  $\delta$  appears on the right hand side which is absent in the previous inequality. Thus, the term  $\frac{\omega}{k}$  should be dropped from the right in the last inequality. This is not yet sufficient to break the argument, but the observation that  $\zeta$  can be bounded above by  $2M\frac{\omega}{k}$  is.

The following modification in the proof is sufficient. The definition of the modified auction  $\mathcal{A}^x$ , for  $x \in \{3, 4, \dots\}$  is changed to the following: with probability  $1/x$ , a non-strategic player  $n + 1$  has endowment  $e_{n+1} = \ell$  and submits  $\ell$  sell offers which are all equal to a random variable uniform on  $[\underline{w}, \bar{w}]$ ; with probability  $1/x$ ,  $e_{n+1} = 0$  and  $n + 1$  submits  $\ell$  buy offers which are all equal to a random variable uniform in  $[\underline{w}, \bar{w}]$ . For such a game, most of the arguments given in the original proof work without changes.<sup>1</sup> The modification is in what follows.

Since player in  $i^x \in H$  is (occasionally) a buyer, there is a probability  $\zeta > 0$  that such a player has an endowment of at most  $\ell - 1$  units.<sup>2</sup> Define  $E_1''$  as the event where  $Q_{B,n+1}^x > 0$  and  $i^x$  has endowment of at most  $\ell - 1$  units. Define  $E_1'$  and  $E_1$  as before. Again, we have  $\Pr_x(E_1) \geq \zeta \hat{\mu}_x$ .<sup>3</sup>

If  $E_1 = E_1'$ ,  $i^x$  has no sell bids at or below  $\bar{w} - 2\delta$  and there is at least one buy bid above  $\bar{w} - 2\delta$ . If  $E_1 = E_1''$ ,  $i^x$  has at most  $\ell - 1$  sell bids at or below  $\bar{w} - 2\delta$  (because she has only  $\ell - 1$  units), while there are at least  $\ell$  buy bids above  $\bar{w} - 2\delta$ . Then, under  $E_1 \cap E_2 \cap E_{3j}$ ,  $j$  sells at least one extra object by  $d_j$ .

The rest of the argument works.<sup>4</sup>

## REFERENCE

JACKSON, M. O. AND J. M. SWINKELS (2005): "Existence of Equilibrium in Single and Double Private Value Auctions," *Econometrica*, 73, 93-140.

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<sup>1</sup>The limitation to the probability  $\Pr_x(Q_B^x > \ell)$  was based on the argument that this event will occur only if more than two players bid above  $\bar{w} - 2\delta$ . This remains true.

<sup>2</sup>Such  $\zeta$  cannot be limited by  $2M\frac{\omega}{k}$  as before.

<sup>3</sup>This limitation is also sufficient to use the consequences of (6). I thank Prof. Swinkels for this observation.

<sup>4</sup>There is a typo in (5): the  $n$  in the right hand side should be  $n + 1$ . Similar replacements should be done in its consequences.