96-38

UNIVERSITY OF CALIFORNIA, SAN DIEGO

DEPARTMENT OF ECONOMICS

INVESTIGATING THE RELATIONSHIP BETWEEN GOLD AND SILVER PRICES

BY

ALVARO ESCRIBANO

AND

CLIVE W.J. GRANGER

DISCUSSION PAPER 96-38
NOVEMBER 1996

INVESTIGATING THE RELATIONSHIP BETWEEN GOLD AND SILVER PRICES

by

Alvaro Escribano*
Department of Statistics and Econometrics
Universidad Carlos III de Madrid
and
Clive W.J. Granger
Department of Economics
University of California, San Diego

October 1996

Abstract:
This paper analyze the long-run relationship between gold and silver prices. The three main questions addressed are: the influence of a large bubble from 1979:9 to 1980:3 on the cointegration relationship, the extent to which by including error correction terms in a nonlinear way we can beat the random walk model out-of sample and, the existence of a strong simultaneous relationship between the rates of return of gold and silver. Different efficient single equation estimation techniques are required for each of the three questions and this is explained within a simple bivariate cointegrating system. With monthly data from 1971 to 1990, it is found that cointegration could have occurred during some periods and specially during the bubble and post-bubble period. However, dummy variables for the intercept of the long-run relationships are needed during the full sample. For the price of gold the nonlinear models perform better than the random walk in-sample and out-of sample. In-sample nonlinear models for the price of silver perform better than the random walk but this predictive capacity is lost out-of sample, mainly due to the structural change that occurs (reduction) in the variance of the out-of sample models. The in-sample and out-of sample predictive capacity of the nonlinear models is reduced when the variables are in logs. Clear and strong evidence is found for a simultaneous relationship between the rates of return of gold and silver. In the three type of relationships that we have analyzed between the prices of gold and silver, the dependence is less out-of sample, possibly meaning that the two markets are becoming separated.

Key words:
Gold and silver prices, cointegration, bubble, nonlinear error-correction, efficient market hypothesis, out-of sample forecast, forecast encompassing.

* This paper was written while the first author was visiting the Department of Economics at UCSD and he acknowledge financial support from the Spanish Ministry of Education and Science grants PR94-137 and PB93-0234. The second author was supported by NSF grant SBR 93-08295.

1. Introduction

Gold and silver have been actively traded for thousands of years and remain important, closely observed markets. Traditionally the ratio of gold to silver prices lay between eight and twenty, suggesting a fairly close long-run relationship. Here monthly prices are analyzed from the end of 1971, when both price series were deregulated, until mid 1990 using some recently developed time series techniques, including cointegration and linear and non-linear error-correction models. Data after June 1990 are used to evaluate models. The main objective is to see if there is any evidence of a stable or semi-stable long-run relationship between these prices that can be useful for forecasting gold and silver prices. We are interested in estimating the contemporaneous relationships between the prices of gold and silver in levels, in logs, in rates of return and in first differences. The simple economics of the situation is not clear, as gold and silver have both distinct and important commercial uses for which there are no substitutes, suggesting that the two markets should be separated. However, elsewhere they do act as quite close substitutes, such as for jewelry and as investments that are used to reduce certain types of risks in portfolios, particularly high inflation risks. These prices are determined in clearly speculative markets and so can be expected to be unit root processes. If they are then co-integrated, the extent to which either can be forecast will be expected to be limited, due to the standard efficient market hypothesis. We check the departures from the pure efficient hypothesis by analyzing the dynamic linear and nonlinear structure of the first difference of the series with the class of error correction models.

There is a feature of this data that makes it particularly interesting, which is the widely known and well documented "bubble" in silver prices from roughly June 1979 to March 1980. The Hunt brothers, of Texas, and others, appeared to try to corner the silver available for speculation, so that investors who sold short had difficulty buying silver to deliver at the end of the contract. By August 1979 the Hunt brothers and their collaborators may have owned or had rights to $2 billion worth of silver, representing over 250 million ounces. The price of silver rose from $6 in 1978 to $10.61 on August 31, 1979, peaking at $48.70 on January 7, 1980 and falling back to $10.80 on March 28, 1980. The eventual price reduction occurred after substantial changes in market trading rules. A rather journalistic account of the period can be found in the book "Beyond Greed" by S. Fay (1982). For convenience this period just will be called "the bubble" in this paper although it does not completely correspond to the concept of bubble found in the financial literature as its period of existence and the reason for ocurrence are known with some accuracy. A plot of the prices against time, as used in this analysis, is shown in Figures 1a and 1c. It is seen that gold prices do increase during and around the bubble, even though the Hunts do not seem to have undertaken any special trading in gold in this period. However, the gold price movement is less spectacular. A further objective of the paper is to investigate the effects of the substantial bubble on the long-run relationship and evaluate to what extent nonlinear error correction models (NEC) can account for the rest.

Figure 2a plots the price of gold against the price of silver. The bubble corresponds to the six points in the upper right quadrant. Apart from these points, the remainder do generally lie around lines of a similar slope, although there does seem to be a possible change in intercept pre- and post the bubble period. In the following analysis the "full sample" period 1971:1 to 1990:6, with 224 observations is analyzed and also the "post-bubble" period 1980:4 to 1990:6, having 111 observations. Log prices and price levels are analyzed separately. The data is taken from the I.M.F, *International Financial Statistics*, the price of gold is $ per fine ounce, London and the price of silver per troy ounce, New York.

The post-sample period is 1990:7 to 1994:6, contains 48 terms. The choice of 1990:7 as the starting date is accidental; when the first version of this paper was prepared, only about 15 terms were preserved for the post sample, but delays in completion of the analysis has allowed this post-sample size to increase. It does allow for a methodological opportunity, which we have not seen investigated before. In many time series modeling exercises, alternative models are compared by their post-sample forecasting ability. One can use forecast encompassing, for example, or the combination of forecasts. However, a possible difficulty that this procedure of post-sample evaluation faces is that the generating mechanism for the process could have changed between the in and out of sample, so that a regime shift had occurred. For this data set, we have sufficient post-sample data to analyze it and to thus compare the model found with in-sample models. The forecasts from the in-sample models can then be compared with these post-sample models.

The following notation is used: *PG*, *PS* for the price of gold and silver; *LX* for log of *X*, ΔX for difference of *X*, $X_{-k} \equiv X_{t-k}$ i.e., X_t lagged *k* time units. Thus, ΔLPG_{-3} is ΔLog Price Gold$_{t-3}$. Three Dummies are used in presenting the results.

$DB = 1$ if $t = 1979{:}9$ to $1980{:}3$ 0 otherwise.

$D2 = 1$ if $t = 1980{:}4$ to $1986{:}4$, 0 otherwise.

$D3 = 1$ if $t = 1986{:}5$ to $1990{:}6$, 0 otherwise.

Thus DB represents the bubble period, $D2$ the immediate post bubble period and $D3$ a later period when a further change in the intercept of the cointegrating relationship (if any) may have occurred. The dummies that mainly affects the intercepts of the cointegrating relationships are shown in Figures 1a and 1c. The three periods used for the dynamic (linear and nonlinear) analysis are shown in the other Figures 1b to 1f.

The analysis performed in this paper is quite different to that of two earlier papers that consider gold and silver prices. Chan and Mountain (1988) analyze weekly data, plus and interest rate series for the early 1980's and are concerned with causality questions, using linear models and without consideration of cointegration. They claim to have found a feedback relationship between gold and silver prices and models that out-forecast random walks, although this latter statement is not formally tested. Akgiray, Booth, Hatem, and Mustafa (1991) look at daily returns for gold and silver for the period 1975 to 1986, where return is the "natural logarithm of the ratio of two successive daily spot prices." They find no forecastability in the means of returns but temporal structure in the variance, which is modeled as a GARCH process. Our results, presented below, are rather different, but are difficult to compare as we use different techniques, time periods, and monthly data. MacDonald and Taylor (1988) do consider cointegration between three monthly metal prices: tin, lead, and zinc, and find none, but do not look at gold and silver prices.

The structure of the paper is the following. Section 2, presents the results of estimating the cointegration relationships between the prices of gold and silver in levels and in logs. Section 3, discusses the selected linear and nonlinear error correction models for the prices of gold and silver. The same class of models but estimated for the rates of return of gold and silver are presented in section 4. Section 5, presents the estimated contemporaneous relationships between the first differences of the two prices and between the two rates of return. The economic intuition of these results is explained in terms of the implications on a normalized portfolio. The conclusions are in section 6. Finally, in the "Appendix" there is a discussion about efficient estimation procedures of the three different types of parameters of interest.

2. Long-Run Relationships

The variables *PG*, *PS*, *LPG* and *LPS* were all tested using Dickey-Fuller tests in all sample sizes and in all cases the null of I(1) was not rejected, but details are not shown. The plots in 2a and 2c are of *PG* against *PS* and *LPG* against *LPS* indicated apparent linear relationships, but with occasional switches in level (intercept). These switches correspond to the periods captured by the dummies defined above.

Table 1 shows the long-run estimated regressions used to investigate the possible presence of cointegration. The first column has *PG* as its dependent variable and this is related to *PS* plus a constant, the dummies defined in the previous section and a product of the bubble dummy and *PS*, the second column now has *PS* as the dependent variable, with *PS* replaced by *PG* as the explanatory variable and the same dummies. The final two columns investigate similar long-run relationships between the logs of the prices of gold and silver. The question of greatest interest is whether or not the residuals are I(1). If this "equilibrium regression" contained r explanatory I(1) variables, critical values for the Dickey-Fuller test, with or without a linear trend, are given by MacKinnon (1991) for $r \leq 5$. However, our regression is not a traditional one, as each equation contains one regular I(1) variable, two or three dummies and possibly a dummy multiplying the I(1) variable. It is suggested that the equivalent number of explanatory variables will be either between 1 and 5 for the *PG* and *PS* equations and either 1, 2 or 3 for the *LPG* and *LPS* equations. Using the MacKinnon tables, the 95% critical values for 1, 2, 3, 4, and 5 explanatory variables are -3.37, -3.80, -4.17, -4.49 and -4.77 respectively for H_0: the residual is I(1). For each of the residuals, the null is rejected, in the direction of the residual being I(0). The results thus suggest several features of the data:

(i) cointegration appears to be found for the full sample, both for levels and for logs.

(ii) the intercept-dummies greatly strengthen the cointegration results. Without their use, cointegration is marginal. Less dummies are needed for the logs of prices than for the prices. The dummies are largely used to explain the bubble period and its impact on the post-bubble period.

If there is cointegration between *PG* and *PS*, there will be just a single equilibrium relationship, and so it should be possible to solve for the second column of Table 1 from the first column, by switching the sides of the equation of *PG* and *PS*. This relation would hold exactly if $R^2 = 1$. Suppose that the first equation is written as

$$PG = c + \theta PS + \beta_1 DB + \lambda DB.PS + \beta_2 D2 + \beta_3 D3 + e$$

which solves out as

$$PS = -\theta^{-1} \frac{[c + \beta_1 DB + \beta_2 D2 + \beta_3 D3 + PG]}{(1 + \lambda / \theta \cdot DB)}$$

From Table 1, it is seen that θ=26.6, λ=-12.8, etc. For the period when *DB*=0, it is possible to compare the values given by this formula with those obtained from the full-sample regressions, given in Table 1.

Regressor	**Full-Sample**	**Derived Estimate**
Constant	-0.52	-1.13
PG	0.03	0.04
D2	-3.55	-4.68
D3	-7.07	-8.21

and for the bubble period:

DB	-10.3	-10.4
*DB**PG	0.04	0.04

A similar exercise with the final two columns of Table 1, deriving an Equation for *LPS* from the *LPG* equation gave the following

Regressor	**Full-Sample Regressor**	**Derived LPG eqn**
Constant	-4.28	-4.8
LPG	1.16	1.27
D2	-.47	-.57
D3	-.90	-1.01

The bubble period was not specifically involved with the log price equations as the regressions found the bubble dummies insignificant. The derived equations appear adequately to approximate the values found by direct estimation. As these equations have residuals that are not white noises, confidence intervals on the coefficient estimates are not reported, so a formal comparison is not attempted. However, this analysis gives added confidence that the apparent cointegration was actually present at least during part of the in-sample period.

Table 1 also shows the values found for Dickey-Fuller tests applied to the errors (parameters are not estimated out-of sample) of the equilibrium models when applied to the out-of-sample period. With 48 terms, the MacKinnon (1991) table suggests a 95% critical value of -1.94 for the Dickey-Fuller test with no constant or trend and -3.42 for this test with a trend for a single series and with no explanatory series, so that the first pair of residual series reject I(1) but the residuals from the log prices do not do so clearly. There is some evidence that the cointegration continues into the post-sample, but the evidence is weak for the log price series, compare figures 2e and 2f.

Chow-forecast test results for parameter constancy are also shown in Table 1, finding no further evidence of changes in parameter values for the long run relationships for the levels of the prices of gold and silver, but there are significant changes in the log price relationships, compare also figures 2e and 2f. A possible explanation for the results of the test is that the variances of the residuals for the in-sample and out-of-sample periods are quite different, as will be documented in later sections.

3. Error-Correction Models for the Prices of Gold and Silver

In this section a number of alternative error correction models are considered with ΔPG. and ΔPS as the dependent variables, where Δ denotes difference. Models are estimated over the three time periods identified in the first section, the full sample (1972:12 to 1990:6), the post-bubble period (1981:04 to 1990:6), and the post sample period (1990:7 to 1994:6), then the models for the first two periods are used to provide one-step forecasts over the post-sample period.

Seven different types of model specification were considered. In models 1 to 4, lags of ΔPG and ΔPS were included for consideration, up to lag 10. These terms were not used in models 5 to 7. In models 1, 3, 4 and 7 the error-correction terms Z_{t-1} entered the model non-linearly.

Model 5 is the simplest in form, with no explanatory variable and so corresponds to the random walk model. Model 2 is the standard linear error-correction model, using lagged price differences, Model 6 is similar but without the lagged terms. Model 1 uses a cubic in Z_{t-1}, as used previously by Escribano (1986), see figure 3a. Model 3 includes terms $Z_{t-1}D(Z_{t-1} > 0)$ and $Z_{t-1}D(Z_{t-1} \leq 0)$ where $D(Z > 0)$ is a dummy that is one if Z is positive, zero otherwise, which corresponds to putting Z_{t-1} and its absolute value into the model, see figure 3c. This nonlinear form of the error-correction model has been used by Granger and Lee (1989), for example.

Models 4 and 7 use terms $Z_{t-1}D(\Delta Z_{t-1} > 0)$ and , $Z_{t-1}D(\Delta Z_{t-1} \leq 0)$, which is a form previously used by Escribano and Pfann(1990), see figure 3d. In every model and for each time period the term Z_t is defined as the residual in the corresponding full-sample equilibrium model given in Table 1. In-sample, five specification tests are provided for each model, to test for autocorrelations in the residuals, up to lag seven, ARCH up to order 7, normality, heteroskedasticity and the Reset test for linearity. Each equation is estimated individually by OLS. As essentially the same explanatory variables are used in both Model 1 for ΔPG and ΔPS, no gain in efficiency would occur from estimating the system, and similarly for other pairs. The coefficients on the lags of ΔPG, ΔPS (or the lags) plus their *t*-values, are not shown to save space. All *t*-values shown are heteroskedastic robust. However, the *t*-values on *Z*-terms are non-standard under a null of no cointegration.

Table 2a shows the estimates of the seven models for the error correction model having ΔPG as dependent variable, using the full sample period on the top, and below the same specifications are used over the post-sample period. Table 2b similarly shows the estimates for these models for the post-bubble period at the top, and underneath the same specification for out-of-sample. Thus, if a particular group of variables are reported in the model in the top panel, they will be used again in the bottom panel. Tables 2c, 2d show the identical tables using ΔPS as the dependent variable. Table 2e summarizes forecasting evaluations of the four models in-sample (full and post-bubble) models. Clearly these tables contain many results, those to which we wish to draw particular attention are:

1. There does appear to be some evidence of non-linearity in error-correction terms in models 1 and 3 for ΔPG (Table 2a) and possible for ΔPS (Table 2c) for the full sample. However, no corresponding evidence of non-linearity is found in the post-bubble and out-of-sample period models. Models in these latter two periods generally pass the specification tests (except normality) whereas for the full sample most models failed the ARCH and heteroskedasticity tests.

2. For ΔPG, Model 1 produced the lowest value of σ in both the full and post-bubble periods, whereas Model 7 produced the lowest σ value out-of-sample. For ΔPS, Model 1 gave the lowest σ in the full period, Models 1 and 3 were equal best in the post-bubble period and Model 6 was best in the out-of-sample period. Some of the σ^2 values obtained were not significantly different, as reported below. The non-linear error-

correction terms should be considered as local approximations to the true non-linear specification if it occurs. In particular, if Z_{t-1} enters a cubic it would produce a non-stable difference equation for x_t, since for large values Z_{t-1} the cubic polynomial is unbounded, and so would not be appropriate as this series is supposed to be I(0). However, as an approximation to the unknown nonlinear function the cubic polynomial is very informative since it can encompass large types of nonlinear adjustments toward the equilibrium, compare figure 3a with 3c.

3. There are clear differences between σ values across time periods. To summarize the data, the following shows the median σ value over the seven models for different periods

	ΔPrice Gold	ΔPrice Silver	ΔLog Price Gold	ΔLog Price Silver
	23.07	1.50	0.0613	0.0953
Post-bubble Sample	19.04	0.61	0.0487	0.0736
Out-of Sample period	9.64	0.23	0.0278	0.0507
Forecasts from Full Sample	10.24	0.29	0.0284	0.0501
Forecasts from Post-bubble models	10.32	0.28	0.0289	0.0519

As the price of gold is substantially higher than that of silver, it is hardly surprising that the standard deviations of the residuals for the ΔPG equations are much larger than those for ΔPS. It is less clear why this inequality is reversed for the log prices. The full sample includes the bubble, which is not completely captured by the simple dummies used in the long run relationship, and so again it is not surprising that σ for the full sample is larger than for other periods. However, it is also clear from these results, and also visually from figures 1a to 1d, that volatility is less during the out-of-sample period than previous periods.

4. The results of the out-of-sample one-step forecasting exercises using the models in Tables 2a,b,c, and d are shown in Table 2e. The figures show that in terms of producing low σ values from these errors, Models 6 and 7 do best for *PG*, with the post-sample model superior to the full-sample model. Similarly Models 5, 6, and 7 are superior for both periods for *PS*. Thus, the models that fit best in-sample do not forecast the best. The table also shows the result of testing if the errors from the apparently best forecasting model have a significantly lower variance than the errors form the random walk model (5). The test used is that discussed in Granger and Newbold (1986), Chapter 7, and more recently by Diebold and Mariano (1995). If the two sets of errors to be compared are e_{1t}, e_{2t}, one forms $s_t = e_{1t} + e_{2t}$ and $d_t = e_{1t} - e_{2t}$ and asks if s_t, d_t are correlated. In our

case the regression of s_t on d_t was run, the results in Table 2e show the coefficient found and its t-value. It is seen that for Gold the full sample and the post-bubble period Model 7 forecasts significantly better than the random walk, but no such result is found for silver.

5. The results of these last two comments suggest that there could have been a change in parameter values for the models from in- to out-of-sample, including a volatility change, supporting the idea that the two markets are becoming more separated.

4. Error-Correction Models for the Log Prices of Gold and Silver.

A similar modeling process was conducted using log prices and the results for ΔLPG dependent variable are shown in Tables 3a and 3b and with ΔLPS as dependent variable in Tables 3c and 3d with the forecast evaluations shown in Table 3e. Specification, estimation, and evaluation details are as in the previous section. Some noteworthy features of these results are:

1. All of the models for ΔLPG have no apparent significant long-run structure as all terms involving Z_{t-1} have low t-values. For the ΔLPS models, there is evidence that Z_{t-1} enters significantly, either linearly or possibly non-linearly, at least in the full-sample. Taken at it's face value, these results would be interpreted as saying that *LPG* and *LPS* are cointegrated with *LPG* as the common stochastic trend. The statistical results for the post-bubble and the out-of-sample periods are less clear but are not contradictory to such a conclusion. If *LPG* is the common stochastic trend, an implication is that there is evidence of a long-run causality from log gold prices to log silver prices, according to Granger and Lin (1995). It is possible that this causality was present around the bubble period but not in more recent periods.

2. In terms of σ values, the first four models generally fitted best for the three sample periods for both *LPG* and *LPS*, so that using lags of *LPG*, *LPS* produced an apparently superior model, contrary to a random walk theory. The forecasting results are less clear, for log gold prices Models 5 and 6 or 7 provide the best forecasts but there is actually very little to choose between the various models results. For log silver prices, Models 2 or 3 provide the best forecasts and generally, Models 5, 6 and 7, which use no lag terms, all rank low in their forecasting performance. The significance of the best forecasting

model compared to the random walk was tested using the same sum and difference of errors procedure described in the previous section. For *LPG* no comparison was required as the random walk was the best model for both the full and post-bubble periods. For *LPS* the best models were not found to be significantly better at forecasting than the random walk.

3. Comparing the σ values of the residuals of the out-of-sample models with those of the one step forecasts shows that there is little gained from the out-of-sample modeling process, i.e. with this data. Put another way, there seems to be little temporal structure in the post-sample period for either *LPG* or *LPS*, so that both are rather well described as random walks, probably without cointegration. This appears to be a different model from that found for the in-sample periods, at least for the *LPS*.

The tests for parameter constancy were applied to all of the error-correction models, either for the full or post-sample bubble sample periods compared to the post-sample period. These tests typically assume that the variance of the residuals is the same in all periods. However, this seems not to be true here, and if the residual variance is less out-of-sample than in-sample, as found here, the test is biased towards not rejecting the null of no change of parameter values. In all cases, this null was not rejected but this is thought to be a consequence of the test rather than a property of the data in some cases.

Generally, the results for the log prices are quite different, and usually simpler, than those for the levels of prices. The log gold prices are nearly a random walk, and are the long-run cause of log silver prices for at least part of the full period. Evidence of a non-linear error-correction model for ΔLPS is possible but the evidence is not strong. For price levels, there is stronger evidence for non-linear error correction terms, the direction of long-run causality is now more likely to be from *PS* to *PG* but non-linearly. There seems to be clear evidence of time varying coefficients and of volatility through the data period being considered. As the prices get further from the bubble period, volatility decreases, prices become nearer to random walks and cointegration is reduced, possibly lost. The advantage of having a long out-of-sample period is indicated, as an appropriate time-varying coefficient test can be applied. Without this, the problem of the best in-sample models producing relatively inferior forecasts would have been unresolvable.

5. Simultaneous Relationships

The models discussed in the previous sections have been concerned with temporal relationships and not much of one variable can be explained by its own lags and the lags of other variables. If one takes the best dynamic model for *PG* and best for *PS*, for some period, then the resulting residuals will be correlated white noises. The extend to which the residuals are correlated could be evidence of the presence of some unobserved "common factor or feature" that affects both price series during the time-span between observations. This factor is often characterized as "news" in the financial literature.

It is frequent practice to use the return on an asset as the standard measure, defined as $(P_t - P_{t-1}) / P_{t-1}$ and approximated by $\log P_t - \log P_{t-1}$. The approximation is satisfactory provided that the size of the price change is small compared to the level of the price, but using monthly data and over a volatile period there are several occasions when the approximation is not acceptable, see figures 4a to 4d. Modeling just returns also means that the information in the cointegration between prices will not be properly used.

Tables 4a and 4b show these results. Using residuals of ΔPG or ΔLPG, as dependent variables, either for the best model (1 for *PG* or 2 for *LPG*) or the random walk (model 5) against the corresponding residual for ΔPS or ΔLPS as the explanatory variable using either the best model (3 for *PS* or 1 for *LPS*) or the random walks (model 5). The results are for the full sample and the post-bubble period in Table 4a and the forecasting period in Table 4b. Each table shows the coefficient value and associated *t*-value in the regression, the achieved σ-value, denoted σ_1, and the σ-value from the original model for the dependent variable, plus the ratio of these last two quantities. The square of this ratio is the amount of the change in the variance achieved; it is roughly 50% in most cases. The results can be interpreted as saying that the common factor represents approximating 50% of the variance of *PG* or *LPG* in all periods, but the size of the regression coefficient is seen to vary substantially. Generally, the regressions have satisfactory Durbin-Watson Statistic values.

An alternative interpretation can be given for these results using the log price changes and using σ-values as measures of risk. If R_{1t}, R_{2t} are a pair of return series, consider a normalized portfolio $\alpha_1 R_{1t} + \alpha_2 R_{2t}$, where $\alpha_1^2 + \alpha_2^2 = 1$, and where a negative α_i means going short. The regressions have considered portfolios of the form $LPG + \lambda LPS$ which transform into a normalized portfolio by taking $\alpha_1 = 1/\theta$ and $\alpha_2 = \lambda/\theta$ where $\theta^2 = 1 + \lambda^2$. As $\hat{\lambda}$ is negative, there is a sign interdeterminency, but the case α_1 positive, α_2 negative will be assumed. If the white noise residuals are taken as returns, with zero expectations and if risk is measured by variance, it follows that for the forecasting period, for instance, the risk for the residual of the random model in *LPG is* 0.00076, for *LPS* is 0.00080 and for the portfolio is 0.00029. As the expected return is still zero, there is seen to be a substantial reduction in risk from buying gold and going short on silver, for the full sample. The risk is further reduced to 0.00021 if the best in-sample models are used (model 1 for gold, 2 for silver).

6. Conclusions

In this paper we have analyzed the relationships between gold and silver prices particularly from the forecasting viewpoint. We have studied the influence of the large bubble from 1979:9 to 1980:3 on the cointegration relationship and found evidence of cointegration but with different intercepts during the bubble and after the bubble periods. For the prices of gold and silver we have studied if alternative nonlinear error correction formulations can beat the random walk, in terms of out of sample forecastability. Furthermore, we have studied the simultaneous relationship between the rates of return of gold and silver and between the first difference of the two prices. Different efficient estimation techniques are required for each of the three questions and this was explained in the "Apendix" with a simple bivariate cointegrating system.

With monthly data from 1971 to 1990, it is found that cointegration could have occurred during some periods and specially during the bubble and post-bubble period. In-sample nonlinear models for silver perform better than the random walk but this predictive capacity is lost out-of sample, mainly due to the structural change that occur with a reduction in the variance of the models during the out-of sample period. For gold the nonlinear models perform better in-sample and out-of sample. The in-sample and out-of sample predictive capacity of the nonlinear models is reduced when the models are logs.

That gold and silver prices have been strongly related, is evident from their behavior during the bubble period. The long-run relationship appears to be complicated and one that varies at particular dates. One gets a rather different view of how the series are linked by looking at the levels or the logs of the prices.

Table 4e shows the strong simultaneous relationship found by a simple linear regression using the actual return of gold as the dependent variable and a constant and the return on silver as the explanatory variable, for each of the three periods. The regression coefficient is clearly lower in the out-of-sample period and the standard deviation of the equations residual is the same as for the final column of Table 4b, suggesting that in this last period the change in log prices is a close approximation to the return, see figures 4e and 4f.

If one assume that a particular model, such as for log price of silver, is correctly specified and has Gaussian residuals, it is possible to derive the model for the price of silver, and hence its error-correction model. However, the usual presence of non-normal residuals, the frequent presence of heteroskedasticity and the reality of possible model mis-specification makes such exercises of little value. It does seem that the bubble period had a lasting influence on cointegration, on the short-run dynamics and possibly on the non-linearity of the relationship. The most recent period in the data set has been the least volatile, follows models most closely agreeing with the efficient market theory and has the ratio of gold to silver prices, at over 60 at the time of writing, at historically high levels, possibly suggesting that some separation of the two markets is occurring, see figures 1e and 1f.

Many econometricians argue that a post-sample evaluation of models is potentially a useful exercise, and we support that proposition. However, if the structure of the model is changing through time, it is more difficult to evaluate the relevance of the models derived in sample, and this does seem to be a property of the data being analyzed in this paper. This also makes forecasting particularly difficult. We are convinced that this is an interesting data set to be used as a benchmark for comparing different methodologies and different nonlinear models. Furthermore, we believe that to consider the possibility of having nonlinear cointegration relationships seem to be promising with this data set but this suggestion opens interesting and difficult questions for future research.

References

Akgiray, V. G.G. Booth, J.J. Hatem, and C. Mustafa (1991), "Conditional Dependence in Precious Metal Prices, " *The Financial Review* 26, 367-386.

Chan, M.W.L. and D.C. Mountain (1988), "The Interative and Causal Relationships Involving Precious Metal Price Movements," *Journal of Business and Economic Statistics*, 6, 69-77.

Diebold, F.X. and R.S. Mariano (1995), "Comparing Predictive Accuracy, " *Journal of Business and Economic Statistics*, 13, 253-64.

Escribano, A. (1986), "Nonlinear Error Correction: The Case of the Money Demand in the U.K. (1878-1970),". PhD. Dissertation UCSD, Chapter IV, 110-169.

Escribano, A. and G. Pfann (1991), "Nonlinear Error Correction, Asymmetric Adjustment and Cointegration". W. P. Universidad Carlos III de Madrid, 91-20.

Fay, S. (1982) Beyond Greed, Viking Press, N.Y.

Granger, C.W.J. and T.-W. Lee (1989), "Investigation of Production, Sales, and Inventory Relationships Using Multicointegration and Nonsymmetric Error Correction Models," *Journal of Applied Econometrics* 4, S145-S159.

Granger, C.W.J. and J. Lin(1995), "Causality in the Long Run", *Econometric Theory*, 11, 530-36.

Granger, C.W.J. and P. Newbold (1986), Forecasting Economic Time Series, 2nd Edition. Academic Press, San Diego.

MacDonald, R. and M. Taylor (1988), "Metal Prices, Efficiency and Cointegration: Some Evidence From the London Metal Exchange," *Bulletin of Economic Research* 40, 235-39.

MacKinnon, J. (1991), "Critical Values For Cointegration Tests." Chapter 13 of Long-Run Economic Relationships, edited by R.F. Engle and C.W.J. Granger, Oxford University Press.

Table 1

Price of Gold (PG) and Price of Silver (PS): Long Run Relationships from 1971:11 to 1990:6

Regressors	Dependent Variable is PG	Dependent Variable is PS	Dependent Variable is log(PG)	Dependent Variable is log(PS)
constant	30.1	-0.52	3.8	-4.28
PS	26.6			
PG		0.03		
DB	138.7	-10.3		
DB*PS	-12.8			
DB*PG		0.04		
D2	124.6	-3.55	0.45	-0.47
D3	218.4	-7.07	0.80	-0.90
log(PS)			0.79	
log(PG)				1.16
Sample Size	224	224	224	224
R^2	0.98	0.97	0.97	0.95
DW	0.50	0.66	0.25	0.26
DF (Unit Root Test on Residuals)	-5.59 (-4.78)*	-6.58 (-4.78)	-4.25 (-3.78)	-4.5 (-3.78)

* In parenthesis are the 5 % critical values for this particular sample size, obtained from Mackinnon(1991) by counting each regressor, but the intercept, as a new variable.

Tests of Long Run Parameter Constancy of Full Sample Models (1971:11 - 1990:6)
Forecasting Period is from 1990:7 to 1994:6

Forecast Chi^2 (48)	11.94 p-value 1.0	11.08 p-value 1.0	96.96 **p-value 0.0**	118.2 **p-value 0.0**

Unit Root Test on Long Run Errors:
Out of Sample Period (1990:7 - 1994:6)

DF (no constant and no trend)	-3.4 (-1.95)**	-3.3 (-1.95)		
DF (with constant and trend)			-3.5 (-3.5)	-3.3 (-3.5)

** In parenthesis are the 5 % critical values obtained from Mackinnon(1991), which are the Dickey-Fuller critical values but adjusted for this particular sample size.

Table 2a
Dependent Variable: First Difference of the Price of Gold (ΔPG)

Full Sample Models (1972:12 - 1990:6)

Regressors	**Model 1**	**Model 2**	**Model 3**	**Model 4**	**Model 5**	**Model 6**	**Model 7**
Constant	4.7 (2.5)*	1.5 (0.9)	3.6 (1.9)	1.7 (1.1)	1.4 (0.8)	1.4 (0.8)	1.5 (0.8)
Lags of ΔPG and ΔPS**	yes	yes	yes	yes	no	no	no
$Z_{-1} \times 10$		-0.6 (0.9)				-0.3 (0.4)	
$Z^2_{-1} \times 10^2$	-0.7 (3.4)						
$Z^3_{-1} \times 10^4$	-0.6 (2.7)						
$Z_{-1}\,D(Z_{-1}>0)$			-0.2 (2.2)				
$Z_{-1}\,D(Z_{-1}\leq 0)$							
$Z_{-1}\,D(\Delta Z_{-1}>0)$				-0.1 (1.3)			-0.04 (0.4)
$Z_{-1}\,D(\Delta Z_{-1}\leq 0)$							
Sample Size	211	211	211	211	211	211	211
R^2	0.29	0.26	0.27	0.26	0.0	0.0	0.0
DW	2.04	2.03	2.02	2.03	1.61	1.6	1.6
σ	**22.54**	23.07	22.88	23.05	26.21	26.26	26.26
AR(7), F(7, --)	0.24	0.43	0.47	0.42	**3.47**	**3.47**	**3.46**
ARCH(7), F(7, --)	**2.2**	**2.4**	**2.3**	**2.4**	2.0	**2.06**	2.04
Normality, Chi^2 (2)	**114.0**	**125.7**	**121.9**	**120.9**	**173.2**	**172.7**	**171.7**
Heter. x_i^2 , F(--, --)	**3.35**	**3.6**	**3.5**	**3.5**		0.92	0.04
Reset, F(1, --)	0.04	0.20	0.21	0.10		3.30	2.83

Out of Sample Models (1990:7 - 1994:6)

Regressors	**Model 1**	**Model 2**	**Model 3**	**Model 4**	**Model 5**	**Model 6**	**Model 7**
Constant	0.5 (0.3)*	1.1 (0.1)	2.7 (1.6)	1.0 (0.8)	0.7 (0.4)	0.1 (0.1)	1.0 (0.8)
Lags of ΔPG and ΔPS**	yes	yes	yes	yes	no	no	no
Z_{-1}		-0.3 (2.7)				-0.3 (2.8)	
$Z^2_{-1} \times 10^4$	0.6 (0.0)						
$Z^3_{-1} \times 10^3$	-0.7 (2.3)						
$Z_{-1}\,D(Z_{-1}>0)$			-0.5 (2.2)				
$Z_{-1}\,D(Z_{-1}\leq 0)$							
$Z_{-1}\,D(\Delta Z_{-1}>0)$				-0.6 (3.9)			-0.6 (3.4)
$Z_{-1}\,D(\Delta Z_{-1}\leq 0)$							
Sample Size	48	48	48	48	48	48	48
R^2	0.27	0.30	0.26	0.40	0.0	0.15	0.22
DW	2.09	2.00	1.92	1.83	1.76	1.72	1.70
σ	9.80	9.49	9.73	8.78	10.32	9.64	9.22
AR(7), F(7,--)	0.48	0.41	0.44	0.82	1.04	1.15	1.15
ARCH(7), F(7,--)	1.3	0.66	0.95	0.34	1.11	2.28	**3.13**
Normality, Chi^2 (2)	**11.7**	**13.4**	**14.3**	**9.3**	5.8	4.16	4.5
Heter. x_i^2 , F(--,--)	0.54	0.56	0.49	0.68		0.07	0.82
Reset, F(1,--)	0.03	0.9	1.29	2.05		0.01	0.31

* In parenthesis are the absolute values of the t-ratios of the coefficients. When homoskedasticity is rejected heteroskedasticity consistent standard errors (HCSE) are used in the t-ratios, White (1980).

** The terms not reported are the coefficients of ΔPG_{-7}, ΔPG_{-10}, ΔPS_{-1}, ΔPS_{-2}, ΔPS_{-3}, ΔPS_{-8}, ΔPS_{-10}. Those coefficients are significant in the full sample but many of them are not in the out of sample period.

Table 2b
Dependent Variable: First Difference of the Price of Gold (ΔPG)

Regressors							
			Post Bubble Models (1981:04 - 1990:6				
Regressors	**Model 1**	**Model 2**	**Model 3**	**Model 4**	**Model 5**	**Model 6**	**Model 7**
Constant	1.3 (0.58)*	-1.8 (1.0)	1.8 (0.8)	-2.4 (1.3)	-1.3 (0.7)	-1.3 (0.7)	-1.3 (0.7)
Lags of ΔPG and ΔPS**	yes	yes	yes	yes	no	no	no
Z_{-1}		-0.2 (1.9)				-0.06 (0.8)	
$Z^2_{-1} \times 10^2$	-0.6 (2.5)						
$Z^3_{-1} \times 10^4$	-0.9 (2.6)						
$Z_{-1} D(Z_{-1}>0)$			-0.37 (2.7)				
$Z_{-1} D(Z_{-1}\le 0)$							
$Z_{-1} D(\Delta Z_{-1}>0)$				-0.09 (0.8)			-0.08 (0.8)
$Z_{-1} D(\Delta Z_{-1}\le 0)$				-0.27 (2.0)			-0.05 (0.4)
Sample Size.	111	111	111	111	111	111	111
R^2	0.17	0.13	0.16	0.14	0.0	0.0	0.0
DW	1.95	1.96	1.96	1.93	1.84	1.83	1.82
σ	**18.66**	19.04	18.72	19.03	19.55	19.57	19.66
AR(7), F(7,--)	0.32	0.35	0.31	0.35	1.22	1.27	1.24
ARCH(7), F(7,--)	1.43	1.54	1.67	1.36	**3.95**	**4.35**	**4.34**
Normality, Chi^2 (2)	**6.35**	**7.20**	**8.02**	5.81	**24.7**	**28.8**	**28.3**
Heter. x_i^2, F(--,--)	0.62	0.78	0.69	0.72		0.07	0.16
Reset, F(1,--)	0.33	1.73	1.10	0.79		**5.04**	3.54
			Out of Sample Models (1990:7 - 1994:6)				
Regressors	**Model 1**	**Model 2**	**Model 3**	**Model 4**	**Model 5**	**Model 6**	**Model 7**
Constant	0.5 (0.2)*	0.1 (0.1)	2.5 (1.3)	0.9 (0.5)	0.7 (1.4)	0.1 (0.1)	0.9 (0.6)
Lags of ΔPG and ΔPS**	yes	yes	yes	yes	no	no	no
Z_{-1}		-0.3 (2.3)				-0.3 (2.8)	
$Z^2_{-1} \times 10^4$	-0.8 (0.0)						
$Z^3_{-1} \times 10^3$	-0.8 (2.0)						
$Z_{-1} D(Z_{-1}>0)$			-0.5 (1.8)				
$Z_{-1} D(Z_{-1}\le 0)$							
$Z_{-1} D(\Delta Z_{-1}>0)$				-0.6 (3.0)			-0.6 (3.3)
$Z_{-1} D(\Delta Z_{-1}\le 0)$				-0.1 (0.6)			0.1 (0.6)
Sample Size.	48	48	48	48	48	48	48
R^2	0.22	0.24	0.20	0.31	0.0	0.15	0.22
DW	2.11	1.98	1.95	1.88	1.76	1.72	1.67
σ	10.27	10.03	10.28	9.66	10.32	9.64	9.31
AR(7), F(7,--)	1.34	0.75	0.89	1.02	1.04	1.15	1.26
ARCH(7), F(7,--)	1.04	1.48	1.23	**2.76**	1.11	2.28	**3.11**
Normality, Chi^2 (2)	3.14	2.66	2.11	0.84	5.78	4.16	4.52
Heter. x_i^2, F(--,--)	0.42	0.33	0.39	0.51		0.07	0.77
Reset, F(1,--)	0.09	1.49	2.69	1.05		0.01	0.40

* In parenthesis are the absolute values of the t-ratios of the coefficients. When homoskedasticity is rejected heteroskedasticity consistent standard errors (HCSE) are used in the t-ratios, White(1980).

** The terms not reported are the coefficients of ΔPG_{-1}, ΔPG_{-4}, ΔPG_{-5}, ΔPG_{-6}, ΔPG_{-8}, ΔPS_{-1}, ΔPS_{-4} ΔPS_{-8}. Those coefficients are significant in the post bubble sample but many of them are not in the out of sample period.

Table 2c

Dependent variable: First Difference of the Price of Silver (ΔPS)

Regressors	Model 1	Model 2	Model 3	Model 4	Model 5	Model 6	Model 7
Full Sample Models (1972:12 - 1990:6)							
Constant x 10	2.0 (2.4)*	0.2 (0.2)	4.0 (2.6)	1.0 (0.9)	0.1 (0.1)	0.2 (0.1)	5.0 (2.7)
Lags of ΔPS and ΔPG**	yes	yes	yes	yes	no	no	no
Z_{-1}	-0.1 (0.4)	-0.05 (0.4)				0.1 (0.4)	
Z^2_{-1}	-0.3 (3.7)						
Z^3_{-1}	0.04 (0.6)						
$Z_{-1}\,D(Z_{-1}>0)$			-0.6 (2.3)				-0.55 (2.0)
$Z_{-1}\,D(Z_{-1}\leq 0)$			0.6 (1.9)				0.9 (1.6)
$Z_{-1}\,D(\Delta Z_{-1}>0)$				-0.4 (2.0)			
$Z_{-1}\,D(\Delta Z_{-1}\leq 0)$				0.23 (1.0)			
Sample Size	211	211	211	211	211	211	211
R^2	0.41	0.33	0.37	0.35	0.0	0.0	0.06
DW	1.85	2.02	1.94	2.00	1.35	1.36	1.38
σ	1.41	1.50	**1.45**	1.48	1.789	1.790	1.74
AR(7), F(7,--)	**2.36**	0.16	0.42	0.18	**11.55**	**11.0**	**11.76**
ARCH(7), F(7,--)	**4.11**	**10.4**	**5.94**	**8.21**	**8.68**	**8.32**	**6.12**
Normality, Chi^2 (2)	**433.9**	**235.3**	**158.9**	**149.9**	**883.4**	**823.0**	**577.4**
Heter. x_i^2, F(--,--)	**2.63**	**3.66**	**3.00**	**3.14**		**4.59**	1.99
Reset, F(1,--)	**4.44**	**12.02**	**9.70**	**12.48**		**29.28**	**27.87**
Out of Sample Models (1990:7 - 1994:6)							
Regressors	Model 1	Model 2	Model 3	Model 4	Model 5	Model 6	Model 7
Constant x 10	0.6 (1.3)*	0.2 (0.6)	1.0 (1.5)	0.4 (1.2)	0.1 (0.3)	0.2 (0.7)	0.6 (1.0)
Lags of ΔPS and ΔPG**	yes	yes	yes	yes	no	no	no
Z_{-1}	-0.05 (0.3)	0.06 (0.6)				0.08 (1.2)	
Z^2_{-1}	-0.3 (1.3)						
Z^3_{-1}	-0.03 (0.1)						
$Z_{-1}\,D(Z_{-1}>0)$			-0.29 (1.1)				-0.1 0.8)
$Z_{-1}\,D(Z_{-1}\leq 0)$			0.24 (1.5)				0.2 (1.7)
$Z_{-1}\,D(\Delta Z_{-1}>0)$				-0.1 (1.1)			
$Z_{-1}\,D(\Delta Z_{-1}\leq 0)$				0.2 (1.9)			
Sample Size	48	48	48	48	48	48	48
R^2	0.21	0.14	0.19	0.21	0.0	0.02	0.04
DW	1.94	2.04	2.01	2.00	1.81	1.84	1.89
σ	0.2322	0.2348	0.2318	0.2288	0.2287	0.2285	0.2292
AR(7), F(7,--)	0.57	0.39	0.41	1.12	0.67	0.71	0.27
ARCH(7), F(7,--)	2.22	1.95	2.04	**2.46**	**3.16**	**2.93**	**2.61**
Normality, Chi^2 (2)	**14.69**	**10.9**	**13.60**	**20.20**	**7.95**	**8.52**	**10.48**
Heter. x_i^2, F(--,--)	1.39	1.53	1.48	1.28		1.35	1.24
Reset, F(1,--)	0.01	0.22	0.00	0.04		1.05	0.29

* in parenthesis are the absolute values of the t-ratios of the coefficients. When homoskedasticity is rejected heteroskedasticity consistent standard errors (HCSE) are used in the the t-ratios, White(1980).

** the terms not reported are the coefficients of ΔPS_{-1}, ΔPS_{-2}, ΔPS_{-5}, ΔPS_{-7}, ΔPS_{-8}, ΔPG_{-1}, ΔPG_{-7}. ΔPG_9. Those coefficients are significant in the full sample but many of them are not in the out of sample period.

Table 2d

Dependent Variable: First Difference of the Price of Silver (ΔPS)

Post Bubble Models (1981:04 - 1990:6)

Regressors	**Model 1**	**Model 2**	**Model 3**	**Model 4**	**Model 5**	**Model 6**	**Model 7**
Constant x 10	0.3 (0.4)*	-0.4 (0.7)	0.6 (0.7)	-0.3 (0.5)	-1.0 (1.0)	-1.0 (1.1)	1.0 (1.0)
Lags of ΔPS and ΔPG**	yes	yes	yes	yes	no	no	no
Z_{-1}	-0.2 (1.6)	-0.01 (1.4)				-0.1 (1.2)	
Z^2_{-1}	-0.1 (1.8)						
Z^3_{-1}	0.04 (1.1)						
$Z_{-1}D(Z_{-1}>0)$			-0.2 (2.0)				-0.3 (1.8)
$Z_{-1}D(Z_{-1}\leq 0)$			0.05 (0.4)				0.1 (1.2)
$Z_{-1}D(\Delta Z_{-1}>0)$				-0.15 (1.2)			
$Z_{-1}D(\Delta Z_{-1}\leq 0)$				-0.1 (0.9)			
Sample Size	111	111	111	111	111	111	111
R^2	0.34	0.32	0.34	0.32	0.0	0.03	0.07
DW	1.88	1.92	1.89	1.91	1.93	1.89	1.81
σ	0.6051	0.6088	**0.6051**	0.6113	0.7017	0.6956	0.6846
AR(7), F(7,--)	0.46	0.51	0.51	0.51	**2.41**	**2.46**	**2.49**
ARCH(7), F(7,--)	0.87	0.86	0.89	0.88	**3.49**	**4.67**	**5.54**
Normality, Chi^2 (2)	**17.62**	**21.48**	**20.44**	**21.18**	**41.70**	**34.80**	**28.70**
Heter. x_i^2, F(--,--)	1.61	1.83	1.54	1.66		**3.36**	2.45
Reset, F(1,--)	0.23	0.01	0.00	0.00		**4.67**	0.15

Out of Sample Models (1990:7 - 1994:6)

Regressors	**Model 1**	**Model 2**	**Model 3**	**Model 4**	**Model 5**	**Model 6**	**Model 7**
Constant x 10	0.5 (1.1)*	0.2 (0.5)	1.0 (1.2)	0.3 (1.0)	0.1 (0.3)	0.2 (0.7)	1.0 (1.2)
Lags of ΔPS and ΔPG**	yes	yes	yes	yes	no	no	no
Z_{-1}	-0.03 (0.2)	0.05 (0.4)				0.1 (1.2)	
Z^2_{-1}	-0.3 (1.1)						
Z^3_{-1}	-0.1 (0.4)						
$Z_{-1}D(Z_{-1}>0)$			-0.3 (0.9)				-0.1 (0.8)
$Z_{-1}D(Z_{-1}\leq 0)$			0.2 (1.0)				0.2 (1.7)
$Z_{-1}D(\Delta Z_{-1}>0)$				-0.2 (1.4)			
$Z_{-1}D(\Delta Z_{-1}\leq 0)$				-0.3 (1.8)			
Sample Size	48	48	48	48	48	48	48
R^2	0.18	0.14	0.17	0.27	0.0	0.02	0.04
DW	1.94	1.92	1.95	1.95	1.81	1.84	1.89
σ	0.2430	0.2419	0.2411	0.2261	0.2287	0.2285	0.2292
AR(7), F(7,--)	0.60	0.19	0.45	0.90	0.67	0.70	0.68
ARCH(7), F(7,--)	1.75	1.90	1.76	1.40	**3.16**	**2.93**	**2.61**
Normality, Chi^2 (2)	**7.22**	3.70	**6.25**	**7.16**	**7.95**	**8.52**	**10.48**
Heter. x_i^2, F(--,--)	0.24	0.22	0.26	0.18		1.35	1.24
Reset, F(1,--)	0.29	1.56	0.39	0.38		1.05	0.29

* In parenthesis are the absolute values of the t-ratios of the coefficients. When homoskedasticity is rejected heteroskedasticity consistent standard error (HCSE) are used in the t-ratios, White(1980).

** The terms not reported are the coefficients of ΔPS_{-3}, ΔPS_{-4}, ΔPS_{-7}, ΔPS_{-8}, ΔPG_{-1}, ΔPG_{-3}, ΔPG_{-4} ΔPG_{-5}, ΔPG_{-6} ΔPG_{-8}. Those coefficients are significant in the full sample but many of them are not in the out of sample period.

Table 2e

Forecasting Evaluation of Models for ΔPG and ΔPS
Based on their 1-Step Forecast Errors from 1990:7 to 1994:6

Forecasting ΔPG with the Full Sample Models of Table 2a
(1972:12 to 1990:6)

Forecasting Error Criteria	**Model 1**	**Model 2**	**Model 3**	**Model 4**	**Model 5**	**Model 6**	**Model 7**
σ	10.40	10.36	10.19	10.24	10.32	10.17	**10.16**
SC	4.74	4.74	4.70	4.71	4.73	4.70	**4.70**
HQ	4.72	4.71	4.68	4.69	4.70	4.67	**4.67**
FPE	110.41	109.59	105.98	107.13	108.73	105.61	**105.40**
Forecast test: Compares Model 7 and 5							**-26.5 (3.5)**

Forecasting ΔPG with the Post Bubble Models of Table 2b
(1981:04 to 1990:6)

Forecasting Error Criteria	**Model 1**	**Model 2**	**Model 3**	**Model 4**	**Model 5**	**Model 6**	**Model 7**
σ	11.26	10.74	10.81	11.17	10.32	10.05	**10.01**
SC	4.90	4.81	4.82	4.89	4.73	4.67	**4.67**
HQ	4.88	4.78	4.80	4.86	4.70	4.65	**4.64**
FPE	129.38	117.72	119.29	127.41	108.73	103.10	**102.24**
Forecast test: Compares Model 7 and 5							**-10.03 (2.9)**

Forecasting ΔPS with the FullSample Models of Table 2c
(1972:12 to 1990:6)

Forecasting Error Criteria	**Model 1**	**Model 2**	**Model 3**	**Model 4**	**Model 5**	**Model 6**	**Model 7**
σ	0.26	0.30	0.29	0.30	0.23	**0.23**	0.29
SC	-2.65	-2.36	-2.44	-2.34	-2.89	**-2.91**	-2.43
HQ	-2.68	-2.39	-2.46	-2.37	-2.92	**-2.93**	-2.46
FPE	0.07	0.09	0.08	0.09	0.05	**0.05**	0.08
Forecast test: Compares Model 6 and 5						**-0.41 (0.3)**	

Forecasting ΔPS with the Post Bubble Models of Table 2d
(1981:04 to 1990:6)

Forecasting Error Criteria	**Model 1**	**Model 2**	**Model 3**	**Model 4**	**Model 5**	**Model 6**	**Model 7**
σ	0.29	0.29	0.28	0.28	**0.23**	0.24	0.23
SC	-2.44	-2.45	-2.52	-2.48	**-2.89**	-2.78	-2.88
HQ	-2.46	-2.47	-2.55	-2.50	**-2.92**	-2.81	-2.91
FPE	0.08	0.08	0.08	0.08	**0.05**	0.06	0.05

Table 3a
Dependent Variable: First Difference of the Logprice of Gold (ΔLPG)

Full Sample Models (1972:12 - 1990:6)

Regressors	Model 1	Model 2	Model 3	Model 4	Model 5	Model 6	Model 7
Constant x 10^2	1.0 (1.0)*	0.4 (0.9)	0.4 (0.8)	0.4 (0.9)	0.8 (1.8)	0.8 (1.8)	1.0 (1.6)
Lags of ΔLPG and ΔLPS**	yes	yes	yes	yes	no	no	no
Z_{-1} x 10^2	1.0 (0.1)	-3.0 (0.6)				0.04 (0.01)	
Z^2_{-1}	-0.1 (0.6)						
Z^3_{-1}	-1.2 (0.7)						
Z_{-1} D(Z_{-1}>0)			-0.04 (0.6)				
Z_{-1} D(Z_{-1}≤0)			-0.02 (0.2)				
Z_{-1} D(ΔZ_{-1}>0)				-0.05 (0.8)			0.02 (0.4)
Z_{-1} D(ΔZ_{-1}≤0)				-0.007 (0.1)			-0.03 (0.4)
Sample Size	211	211	211	211	211	211	211
R^2	0.19	0.19	0.19	0.19	0.0	0.0	0.00
DW	2.03	2.02	2.02	2.03	1.41	1.41	1.42
σ	0.0613	**0.0611**	0.0613	0.0612	0.0666	0.0667	0.0668
AR(7), F(7,--)	0.43	0.45	0.48	0.50	**4.56**	**4.55**	**4.56**
ARCH(7), F(7,--)	1.72	1.82	1.79	1.76	1.30	1.29	1.23
Normality, Chi^2 (2)	**43.03**	**42.21**	**42.31**	**43.33**	**47.53**	**47.54**	**45.96**
Heter. x_i^2 , F(--,--)	**1.81**	**1.91**	**1.82**	1.65		1.16	0.66
Reset, F(1,--)	0.12	0.26	0.23	0.17		0.42	0.86

Out of Sample Models (1990:7 - 1994:6)

Regressors	Model 1	Model 2	Model 3	Model 4	Model 5	Model 6	Model 7
Constant x 10	0.3 (1.4)*	0.3 (2.0)	0.3 (2.0)	0.2 (1.8)	0.02 (0.5)	0.2 (1.5)	0.1 (1.2)
Lags of ΔLPG and ΔLPS**	yes	yes	yes	yes	no	no	no
Z_{-1}	-0.6 (0.9)	-0.16 (2.0)				-0.1 (1.4)	
Z^2_{-1}	5.4 (0.9)						
Z^3_{-1}	-16.1 (1.1)						
Z_{-1} D(Z_{-1}>0)			-0.16 (2.0)				
Z_{-1} D(Z_{-1}≤0)							
Z_{-1} D(ΔZ_{-1}>0)				-0.2 (2.0)			-0.1 (1.5)
Z_{-1} D(ΔZ_{-1}≤0)				-0.1 (1.3)			-0.05 (0.6)
Sample Size	48	48	48	48	48	48	48
R^2	0.22	0.17	0.17	0.18	0.00	0.04	0.06
DW	2.1	2.07	2.07	2.11	1.74	1.82	1.83
σ	0.02765	0.02781	0.02781	0.02795	0.02817	0.02783	0.02791
AR(7), F(7,--)	1.27	1.24	1.23	1.03	1.06	1.25	1.07
ARCH(7), F(7,--)	1.89	1.68	1.68	1.76	1.00	1.16	1.21
Normality, Chi^2 (2)	5.74	4.00	4.00	4.65	4.91	4.44	4.34
Heter. x_i^2 , F(--,--)	0.70	0.73	0.73	0.67		0.80	0.99
Reset, F(1,--)	0.20	0.46	0.46	0.26		0.62	0.56

* In parenthesis are the absolute values of the t-ratios of the coefficients. When homoskedasticity is rejected heteroskedasticity consistent standard errors (HCSE) are used in the t-ratios, White(1980).

** The terms not reported are the coefficients of ΔLPG_{-1}, ΔLPG_{-6}, ΔLPG_{-7}, ΔLPG_{-11}, ΔLPS_{-2}, ΔLPS_{-11}. Those coefficients are significant in the full sample but many of them are not in the out of sample period.

Table 3b

Dependent Variable: First Difference of the Logprice of Gold (ΔLPG)

Post Bubble Models (1981:4 - 1990:6)

Regressors	**Model 1**	**Model 2**	**Model 3**	**Model 4**	**Model 5**	**Model 6**	**Model 7**
Constant x 10^2	-0.3 (0.6)*	-0.4 (0.8)	-0.3 (0.4)	-0.3 (0.6)	-0.3 (0.7)	-0.3 (0.7)	-0.3 (0.6)
Lags of ΔLPG**	yes	yes	yes	yes	no	no	no
$Z_{\_1}$ x 10	0.08 (0.1)	0.2 (0.5)				0.04 (0.8)	
$Z^2_{\_1}$ x 10	-0.1 (0.2)						
$Z^3_{\_1}$	0.5 (0.3)						
$Z_{\_1}$ D($Z_{\_1}$>0)			0.01 (0.2)				
$Z_{\_1}$ D($Z_{\_1}$≤0)			0.03 (0.4)				
$Z_{\_1}$ D($\Delta Z_{\_1}$>0)				0.001 (0.02)			0.02 (0.3)
$Z_{\_1}$ D($\Delta Z_{\_1}$≤0)				0.06 (0.6)			0.08 (0.6)
Sample Size	111	111	111	111	111	111	111
R^2	0.04	0.04	0.04	0.04	0.0	0.0	0.0
DW	1.93	1.93	1.93	1.94	1.83	1.84	1.83
σ	0.0488	0.0484	0.0486	0.0485	**0.0487**	0.0488	0.0490
AR(7), F(7,--)	0.68	0.74	0.74	0.58	1.19	1.16	1.06
ARCH(7), F(7,--)	**2.93**	**2.99**	**2.99**	**2.75**	**4.51**	**4.23**	**3.86**
Normality, Chi^2 (2)	3.41	3.33	3.33	2.65	**16.36**	**14.02**	**11.44**
Heter. x_i^2 , F(--,--)	**2.26**	**3.48**	**2.54**	**2.87**		0.08	1.85
Reset, F(1,--)	0.42	0.31	0.37	0.00		0.05	1.14

Out of Sample Models (1990:7 - 1994:6)

Regressors	**Model 1**	**Model 2**	**Model 3**	**Model 4**	**Model 5**	**Model 6**	**Model 7**
Constant x10	0.2 (1.0)*	0.2 (1.4)	0.2 (1.4)	0.1 (1.2)	0.02 (0.5)	0.15 (1.5)	0.1 (1.2)
Lags of ΔLPG**	yes	yes	yes	yes	no	no	no
$Z_{\_1}$	-0.5 (0.7)	-0.1 (1.4)				-0.1 (1.5)	
$Z^2_{\_1}$	4.6 (0.8)						
$Z^3_{\_1}$	-13.2 (1.0)						
$Z_{\_1}$ D($Z_{\_1}$>0)			-0.1 (1.4)				
$Z_{\_1}$ D($Z_{\_1}$≤0)							
$Z_{\_1}$ D($\Delta Z_{\_1}$>0)				-0.09 (1.4)			-0.1 (1.5)
$Z_{\_1}$ D($\Delta Z_{\_1}$≤0)				-0.05 (0.5)			-0.05 (0.6)
Sample Size	48	48	48	48	48	48	48
R^2	0.08	0.05	0.05	0.06	0.00	0.04	0.06
DW	1.95	1.92	1.92	1.92	1.74	1.82	1.83
σ	0.02853	0.02838	0.02838	0.02848	0.02817	0.02783	0.02791
AR(7), F(7,--)	1.32	1.29	1.29	1.13	1.06	1.25	1.07
ARCH(7), F(7,--)	0.86	0.96	0.96	1.03	1.00	1.16	1.21
Normality, Chi^2 (2)	4.25	3.75	3.75	3.79	4.91	4.44	4.34
Heter. x_i^2 , F(--,--)	0.97	0.35	0.35	0.56		0.80	0.99
Reset, F(1,--)	1.97	0.00	0.01	0.00		0.62	0.56

* In parenthesis are the absolute values of the t-ratios of the coefficients. When homoskedasticity is rejected heteroskedasticity consistent standard errors (HCSE) are used in the t-ratios, White(1980).

** The terms not reported are the coefficients of $\Delta LPG_{\_1}$, $\Delta LPG_{\_6}$. Those coefficients are significant in the full sample but none of them are significant in the out of sample period.

Table 3c
Dependent Variable: First Difference of the Logprice of Silver (ΔLPS)

Full Sample Models (1972:12 - 1990:6)

Regressors	**Model 1**	**Model 2**	**Model 3**	**Model 4**	**Model 5**	**Model 6**	**Model 7**
Constant x 10	0.1 (1.3)*	0.01 (0.1)	0.1 (1.0)	0.01 (0.2)	0.05 (0.6)	0.04 (0.5)	0.02 (0.3)
Lags of ΔLPS and ΔLPG**	yes	yes	yes	yes	no	no	no
Z_{-1}	-0.13 (1.4)	-0.18 (2.8)				-0.1 (1.6)	
Z^2_{-1}	-0.5 (1.6)						
Z^3_{-1}	-0.89 (0.7)						
$Z_{-1}\,D(Z_{-1}>0)$			-0.26 (2.1)				
$Z_{-1}\,D(Z_{-1}\leq 0)$			-0.1 (1.1)				
$Z_{-1}\,D(\Delta Z_{-1}>0)$				-0.2 (2.0)			-0.05 (0.5)
$Z_{-1}\,D(\Delta Z_{-1}\leq 0)$				-0.16 (2.1)			-0.17 (1.9)
Sample Size	211	211	211	211	211	211	211
R^2	0.23	0.22	0.22	0.22	0.0	0.02	0.03
DW	2.04	2.00	2.02	2.01	1.44	1.41	1.44
σ	**0.0948**	0.0951	0.0951	0.0953	0.1051	0.1042	0.1041
AR(7), F(7,--)	0.85	0.80	0.81	0.82	**4.50**	**4.81**	**4.75**
ARCH(7), F(7,--)	**7.71**	**7.81**	**7.95**	**7.70**	**9.58**	**11.15**	**11.25**
Normality, Chi^2 (2)	**93.3**	**93.3**	**92.1**	**94.3**	**204.0**	**190.8**	**173.0**
Heter. x_i^2 , F(--,--)	**3.81**	**4.72**	**4.22**	**4.10**		**5.32**	**2.70**
Reset, F(1,--)	1.12	2.05	1.53	2.23		1.86	**5.36**

Out of Sample Models (1990:7 - 1994:6)

Regressors	**Model 1**	**Model 2**	**Model 3**	**Model 4**	**Model 5**	**Model 6**	**Model 7**
Constant x 10	0.1 (0.3)*	-0.3 (1.5)	-0.3 (1.5)	-0.4 (1.6)	0.2 (0.3)	-0.1 (0.4)	-0.1 (0.9)
Lags of ΔLPS and ΔLPG**	yes	yes	yes	yes	no	no	no
Z_{-1}	1.6 (1.5)	-0.2 (1.6)				-0.04 (0.6)	
Z^2_{-1}	13.0 (1.8)						
Z^3_{-1}	25.2 (1.8)						
$Z_{-1}\,D(Z_{-1}>0)$							
$Z_{-1}\,D(Z_{-1}\leq 0)$			-0.2 (1.6)				
$Z_{-1}\,D(\Delta Z_{-1}>0)$				-0.3 (1.9)			-0.16 (1.6)
$Z_{-1}\,D(\Delta Z_{-1}\leq 0)$				-0.17 (1.4)			-0.04 (0.5)
Sample Size	48	48	48	48	48	48	48
R^2	0.30	0.23	0.23	0.25	0.00	0.00	0.06
DW	2.06	2.03	2.03	2.01	1.81	1.78	2.03
σ	0.0498	0.0507	0.0507	0.0507	0.0520	0.0525	0.0516
AR(7), F(7,--)	0.76	1.26	1.26	0.58	0.62	0.60	0.54
ARCH(7), F(7,--)	0.89	1.12	1.12	1.52	**3.17**	**2.98**	**2.87**
Normality, Chi^2 (2)	1.63	4.33	4.33	5.06	5.95	5.45	4.78
Heter. x_i^2 , F(--,--)	1.06	1.57	1.57	1.21		0.73	0.97
Reset, F(1,--)	0.05	0.01	0.01	0.03		0.04	0.21

* In parenthesis are the absolute values of the t-ratios of the coefficients. When homoskedasticity is rejected heteroskedasticity consistent standard errors (HCSE) are used in the t-ratios, White(1980).

** The terms not reported are the coefficients of ΔLPS_{-1}, ΔLPS_{-2}, ΔLPS_{-3}, ΔLPS_{-7}, ΔLPS_{-8}, ΔLPS_{-10}, ΔLPG_{-3}, ΔLPG_{-7}. Those coefficients are significant in the full sample but many of them are not in the out of sample period.

Table 3d

Dependent Variable: First Difference of the Logprice of Silver(ΔLPS)

Regressors	Model 1	Model 2	Model 3	Model 4	Model 5	Model 6	Model 7
			Post Bubble Models (1981:4 - 1990:6)				
Constant x 10^2	0.07 (0.08)*	-0.6 (0.8)	0.6 (0.6)	-0.2 (0.3)	-0.8 (1.2)	-1.0 (1.3)	-0.7 (0.9)
Lags of ΔLPS and ΔLPG**	yes	yes	yes	yes	no	no	no
Z_{-1}	-0.2 (1.5)	-0.12 (1.8)				-0.11 (1.5)	
Z^2_{-1}	-0.5 (1.5)						
Z^3_{-1}	0.7 (0.4)						
$Z_{-1} D(Z_{-1}>0)$			-0.3 (1.9)				
$Z_{-1} D(Z_{-1}\leq 0)$			0.01 (0.1)				
$Z_{-1} D(\Delta Z_{-1}>0)$				-0.29 (2.4)			-0.2 (1.6)
$Z_{-1} D(\Delta Z_{-1}\leq 0)$				-0.05 (0.7)			-0.04 (0.5)
Sample Size	111	111	111	111	111	111	111
R^2	0.19	0.17	0.19	0.19	0.0	0.03	0.04
DW	1.89	1.90	1.89	1.93	1.81	1.76	1.70
σ	0.0732	0.0736	0.0733	**0.0730**	0.0782	0.0775	0.0772
AR(7), F(7,--)	0.89	0.85	1.00	0.77	1.69	1.90	1.80
ARCH(7), F(7,--)	2.09	1.87	**2.14**	1.44	**2.26**	**2.29**	**2.18**
Normality, Chi^2 (2)	**18.0**	**20.2**	**18.9**	15.01	**17.9**	**12.0**	**9.33**
Heter. x_i^2 , F(--,--)	0.81	1.00	0.83	0.94		1.29	0.91
Reset, F(1,--)	0.08	1.13	0.02	0.00		2.30	0.33

Regressors	Model 1	Model 2	Model 3	Model 4	Model 5	Model 6	Model 7
			Out of Sample Models (1990:7 - 1994:6)				
Constant x 10	0.2 (0.4)*	-0.3 (1.2)	-0.3 (1.2)	-0.3 (1.3)	0.02 (0.7)	-0.06 (0.3)	-0.2 (0.9)
Lags of ΔLPS and ΔLPG**	yes	yes	yes	yes	no	no	no
Z_{-1}	1.4 (1.3)	-0.16 (1.4)				-0.04 (0.6)	
Z^2_{-1}	10.6 (1.5)						
Z^3_{-1}	19.8 (1.5)						
$Z_{-1} D(Z_{-1}>0)$							
$Z_{-1} D(Z_{-1}\leq 0)$			-0.16 (1.4)				
$Z_{-1} D(\Delta Z_{-1}>0)$				-0.2 (1.7)			-0.16 (1.6)
$Z_{-1} D(\Delta Z_{-1}\leq 0)$				-0.1 (1.1)			-0.04 (0.5)
Sample Size	48	48	48	48	48	48	48
R^2	0.24	0.20	0.20	0.22	0.00	0.00	0.06
DW	2.06	1.96	1.96	1.99	1.81	1.78	2.03
σ	0.0503	0.0505	0.0505	0.0505	0.0520	0.0525	0.0516
AR(7), F(7,--)	1.19	1.76	1.76	0.83	0.62	0.60	0.54
ARCH(7), F(7,--)	0.15	0.40	0.40	0.51	**3.17**	**2.98**	**2.86**
Normality, Chi^2 (2)	2.05	4.34	4.34	4.60	5.95	5.45	4.78
Heter. x_i^2 , F(--,--)	0.31	0.30	0.30	0.56		0.73	0.97
Reset, F(1,--)	0.26	0.13	0.13	0.08		0.04	0.21

* In parenthesis are the absolute values of the t-ratios of the coefficients. When homoskedasticity is rejected heteroskedasticity consistent standard errors (HCSE) are used in the t-ratios, White(1980).

** The terms not reported are the coefficients of ΔLPS_{-1}, ΔLPS_{-3}, ΔLPS_{-4}, ΔLPG_{-4}, ΔLPG_{-5}, ΔLPG_{-7}. Those coefficients are significant in the full sample but many of them are not in the out of sample period.

Table 3e
Forecasting Evaluation of Models for ΔLPG and ΔLPS based on their 1-Step Forecast Errors from 1990:7 to 1994:6

Forecasting ΔLPG with the Full Sample Models of Table 3a (1972:12 to 1990:6)

Forecasting Error Criteria	Model 1	Model 2	Model 3	Model 4	Model 5	Model 6	Model 7
σ	0.0284	0.0285	0.0284	0.0284	**0.0282**	0.0282	0.0292
SC	-7.06	-7.06	-7.06	-7.07	**-7.08**	-7.08	-7.01
HQ	-7.09	-7.08	-7.08	-7.09	**-7.10**	-7.10	-7.03
FPE	0.0008	0.0008	0.0008	0.0008	**0.0008**	0.0008	0.0009

Forecasting ΔLPG with the Post Bubble Models of Table 3b (1981:04 to 1990:6)

Forecasting Criteria	Model 1	Model 2	Model 3	Model 4	Model 5	Model 6	Model 7
σ	0.0291	0.0291	0.0290	0.0287	**0.0282**	0.0288	0.0282
SC	-7.01	-7.01	-7.02	-7.05	**-7.08**	-7.04	-7.08
HQ	-7.04	-7.04	-7.05	-7.07	**-7.10**	-7.06	-7.10
FPE	0.0009	0.0009	0.0009	0.0008	**0.0008**	0.0008	0.0008

Forecasting ΔLPS with the Full Sample Models of Table 3c (1972:12 to 1990:6)

Forecasting Criteria	Model 1	Model 2	Model 3	Model 4	Model 5	Model 6	Model 7
σ	0.0501	**0.0491**	0.0495	0.0494	0.0520	0.0523	0.0569
SC	-5.93	**-5.97**	-5.95	-5.96	-5.85	-5.84	-5-67
HQ	-5.95	**-5.99**	-5.97	-5.98	-5.88	-5.87	-5.70
FPE	0.0026	**0.0025**	0.0025	0.0025	0.0028	0.0028	0.0028
Forecast test: Compares Model 7 and 5		**-0.46 (0.8)**					

Forecasting ΔLPS with the Post Bubble Models of Table 3d (1981:04 to 1990:6)

Forecasting Criteria	Model 1	Model 2	Model 3	Model 4	Model 5	Model 6	Model 7
σ	0.0519	0.0510	**0.0506**	0.0521	0.0520	0.0522	0.0510
SC	-5.86	-5.89	**-5.91**	-5.85	-5.85	-5.85	-5.89
HQ	-5.88	-5.92	**-5.93**	-5.87	-5.88	-5.87	-5.92
FPE	0.0027	0.0026	**0.0026**	0.0028	0.0028	0.0028	0.0027
Forecast test: Compares Model 2 and 5			**-0.47 (0.5)**				

Table 4a
Short Run Relationships Between the Prices of Gold and Silver

Full Sample Period (1972:12 - 1990:6)

Regressors	**Residuals of Model 1 (ΔPG)**	**Residuals of Model 5 (ΔPG)**	**Residuals of Model 2 (ΔLPG)**	**Residuals of Model 5 (ΔLPG)**
Residuals of Model 3 of ΔPS	12.15 (18.04)			
Residuals of Model 5 for ΔPS		11.90 (20.2)		
Residuals of Model 1, ΔLPS			0.45 (13.8)	
Residuals of Model 5, ΔLPS				0.47 (15.9)
Sample Size	211	211	211	211
R^2	0.61	0.66	0.48	0.55
σ_1	13.81	15.29	0.0434	0.0448
σ (Models with only lagged variables)	22.54 (model 1)	26.21 (model 5) Random Walk	0.0611 (model 2)	0.0666 (model 5) Random Walk
ratio (σ_1/σ)	.61	.58	.71	.67

Post-Bubble Period (1981:04 - 1990:6)

Regressors	**Residuals of Model 1 (ΔPG)**	**Residuals of Model 5 (ΔPG)**	**Residuals of Model 5 (ΔLPG)**	**Residuals of Model 5 (ΔLPG)**
Residuals of Model 3 of ΔPS	20.2 (8.9)			
Residuals of Model 5 for ΔPS		19.77 (10.6)		
Residuals of Model 4, ΔLPS			0.49 (10.4)	
Residuals of Model 5, ΔLPS				0.45 (11.2)
Sample Size	111	111	111	111
R^2	0.41	0.50	0.49	0.53
σ	13.56	13.78	0.0346	0.0333
σ (Models with only lagged variables)	18.66 (model 1)	19.55 (model 5) Random Walk	0.0487 (model 5) Random Walk	0.0487 (model 5) Random Walk
ratio (σ_1/σ)	.73	.70	.71	.66

Table 4b
Short Run Relationships Between the Prices of Gold and Silver

Forecasting Period (1990:07 - 1994:6)

	Dependent Variable			
Regressors	**Residuals of Model 1 (ΔPG)**	**Residuals of Model 5 (ΔPG)**	**Residuals of Model 1 (ΔLPG)**	**Residuals of Model 5 (ΔLPG)**
Residuals of Model 5 of ΔPS	24.80 (5.4)			
Residuals of Model 5 for ΔPS		31.10 (6.5)		
Residuals of Model 2, ΔLPS			0.40 (7.2)	
Residuals of Model 5, ΔLPS				0.37 (6.4)
Sample Size	48	48	48	48
R^2	0.39	0.47	0.53	0.46
σ_1	7.15	7.48	0.0171	0.021
σ	9.80	10.32	0.0276	0.0282
(Models with only lagged variables)	(model 1) Table 2a	(model 5) Random Walk	(model 1) Table 2a	(model 5) Random Walk
ratio (σ_1/σ)	.73	.72	.62	.74

Table 4c
Relationship Between the Rate of Return of Gold (RRG) and the Rate of Return of Silver (RRS)

	Dependent Variable		
	RRG (1971:12 - 1990:06)	**RRG (1981:04 - 1990:06)**	**RRG (1990:07 - 1994:06)**
Regressors			
RRS	0.49 (17.4)	0.45 (11.0)	0.37 (6.3)
Sample Size	211	111	48
R^2	0.59	0.53	0.46
σ_1	0.0455	0.0338	0.0210

Appendix

Efficient Parameter Estimation in a Bivariate Cointegrated System

Consider the following error-correction (EC) data generating process (DGP)

$$\Delta PG_t = a_{11}\,\Delta PG_{t-1} + a_{12}\,\Delta PS_{t-1} - \delta_g\,(PG_{t-1} - \beta\,PS_{t-1}) + \varepsilon_{gt} \qquad \text{(A1.a)}$$

$$\Delta PS_t = a_{21}\,\Delta PG_{t-1} + a_{22}\,\Delta PS_{t-1} - \delta_s\,(PS_{t-1} - \beta^{-1}\,PG_{t-1}) + \varepsilon_{st} \qquad \text{(A1.b)}$$

where the 2x1 vector ε is i.i.d.N(0, Ω).

Alternatively we can say that

$$\varepsilon_{gt} = \alpha\,\varepsilon_{st} + \upsilon_t \qquad \text{(A1.c)}$$

with orthogonal zero mean elements, $\text{cov}(\varepsilon_{st}, \upsilon_t) = 0$.

The cointegrating vector (with long-run parameters) is unique and equal to (1, -β), the contemporaneous relationship (short-run parameter) is measured by α and the coefficients of the dynamic terms have all the information about Granger-causality in the short-run (a_{12}, a_{21}) and Granger-causality in the long-run (δ_g, δ_s).

The above system can be written in two interesting equivalent ways that would allow us to discuses alternative well known estimation procedures and to emphasize the advantages and disadvantages of each of them.

By direct substitution of (A1.c) in (A1.a) one gets the following system of equations with orthogonal errors,

$$\Delta PG_t = a_{11}\,\Delta PG_{t-1} + a_{12}\,\Delta PS_{t-1} - \delta_g\,(PG_{t-1} - \beta\,PS_{t-1}) + \alpha\,\varepsilon_{st} + \upsilon_t \qquad \text{(A2.a)}$$

$$\Delta PS_t = a_{21}\,\Delta PG_{t-1} + a_{22}\,\Delta PS_{t-1} - \delta_s\,(PS_{t-1} - \beta^{-1}\,PG_{t-1}) + \varepsilon_{st} \qquad \text{(A2.b)}$$

Alternatively one can form a linear combination of the two first equations multiplying (A1.a) and (A1.b) by the vector (1, -α) giving the following two equation system,

$$\Delta PG_t = a^*_{11}\,\Delta PG_{t-1} + a^*_{12}\,\Delta PS_{t-1} - \delta^*_g\,(PG_{t-1} - \beta\,PS_{t-1}) + \alpha\,\Delta PS_t + \upsilon_t \qquad \text{(A3.a)}$$

$$\Delta PS_t = a_{21}\,\Delta PG_{t-1} + a_{22}\,\Delta PS_{t-1} - \delta_s\,(PS_{t-1} - \beta^{-1}\,PG_{t-1}) + \varepsilon_{st} \qquad \text{(A3.b)}$$

where $a^*_{11} = (a_{11} - \alpha a_{21})$ and $a^*_{12} = (a_{12} - \alpha a_{22})$.

In what follows, fully efficient estimation methods will be briefly discussed. The first possibility is to use system of equation methods, like full information maximum likelihood (FIML) on equations (A1.a) and (A1.b), see Johansen(1988). Notice that since there is a parameter restriction (β) between the two equations of the system that invalidates estimating it by 1-step single equation methods (like NLS) even when all of the equations have the same regressors.

Engle and Granger(1987) 2-step estimator with the later improvement of Engle and Yoo(1991) 3-step estimator solved the inefficiency problem of 1-step single equation methods.

In the first-step, the cointegrating vector (1, -β) is super-consistently estimated (although not efficiently) by OLS in the equation,

$$PG_t = \beta\ PS_t + u_t \qquad \text{(A4)}$$

In the second-step, the lagged residuals are introduced in (A1.a) and (A1.b) since by doing that we are imposing the cross-equation parameter restriction and it is now, when the system has the same regressors in all the equations, that system of equations estimation methods are reduced to single equation ones (OLS in each equation of the second step).

In the third-step of Engle and Yoo(1991), the OLS estimator of β is made efficient, and less biased, by correcting it using the estimated coefficient (c_1) obtained from the OLS-regression of,

$$\varepsilon_{gt} = c_1\ PS_{t-1} + \omega_t\ . \qquad \text{(A5)}$$

In the empirical application used by Engle and Yoo(1991) they estimate the second-step in the system (A3.a) and (A3.b). However, is important to realize that by doing that the economic interpretation of the coefficients of the equation (A3.a) might change. Those changes can be specially important if in equation (1.a) the coefficient $a_{12} = 0$ and in (1.b) $a_{22} \neq 0$, because we can even get the wrong direction of the short-run Granger-causality.

To avoid that problem one has two alternative and equivalent single equation procedures. First as it is usual, estimate the cointegrating vector by OLS in (A4). Take those residuals lagged once and form the error correction terms of equations (A1.a) and (A1.b). Second, estimating the parameters of those equations (a_{11} , a_{12} , a_{21} , a_{22} , δ_g ,and δ_g) by OLS is fully efficient since all the equations have same regressors. Third, estimate the parameter α of equation (A1.c) by substituting the unknown errors (ε_{gt} and ε_{st}) by the residuals of the regressions from second step (this is the estimation procedure implemented in this paper). Fourth, calculate the efficient estimator of the cointegrating vector by transforming its OLS estimate by the coefficient (c_1) obtained by running the regression (A5) but with the residuals υ_t as the dependent variable.

The other equivalent procedure is to estimate the first step as usual and in the second-step, estimate only the parameters of equation (A2.b) to get the residuals ε_{st} . In the third step, one estimates equation (A2.a) with the residuals, ε_{st}, of (A2.b) as a new regressor. In the fourth and final step, equation (A5) is estimated to obtain the correction of the OLS estimated cointegrating vector.

The main advantages of these last two *fully efficient* estimation procedures are that they:
1) always estimate directly the parameters of interest,
2) directly get from the usual econometric packages their corresponding standard errors,
3) avoid getting wrong short-run Granger-causality conclusions,
4) always use single equation methods that do not depend on weak exogeneity conditions,
5) do not need to get into the specification of the full dynamic systems of equations.

Price of Gold (PG) and Price of Silver (PS)

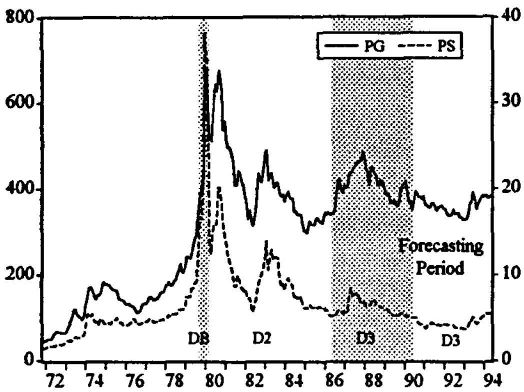

Figure 1a

First differences of the Prices of Gold (DPG) and Silver (DPS)

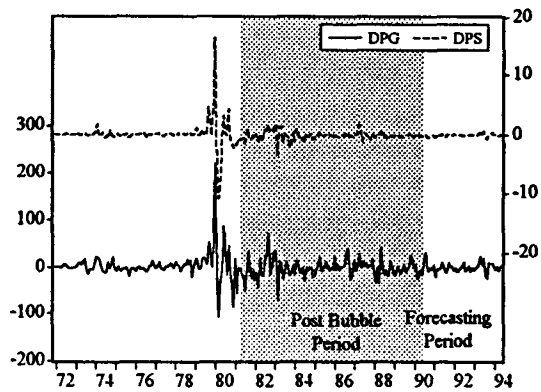

Figure 1b

Log. of Price of Gold (LPG) and Log. of Price of Silver (LPS)

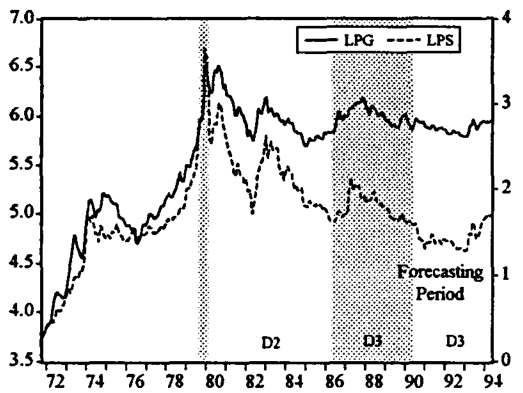

Figure 1c

First Differences of the Logprices of Gold (DLPG) and Silver (DLPS)

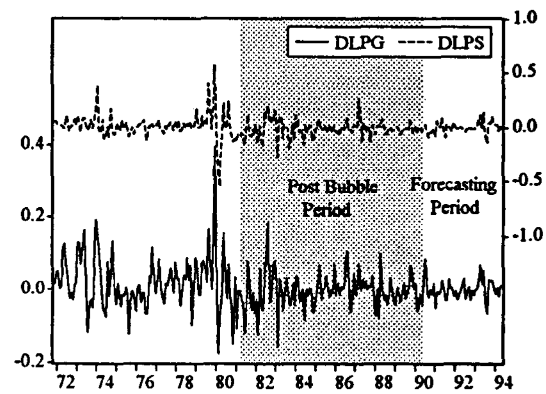

Figure 1d

Ratio of the Prices of Gold and Silver (RGS)

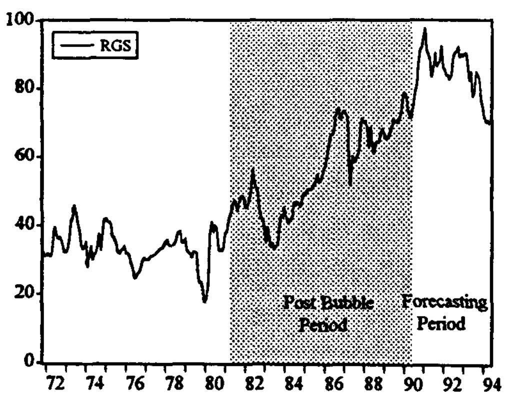

Figure 1e

Log of the Ratio of the Price of Gold and Silver (LRGS)

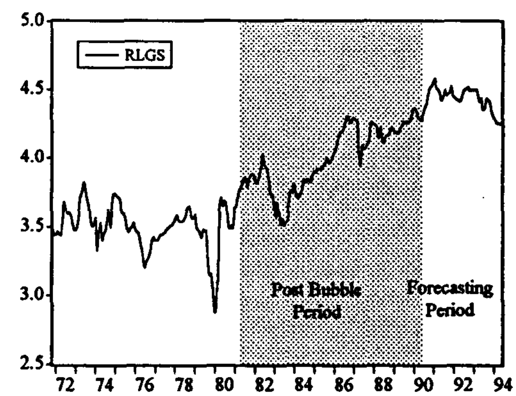

Figure 1f

Cross-plot of the Price of Gold (PG) and the Price of Silver (PS) from 1971:11 to 1990:06

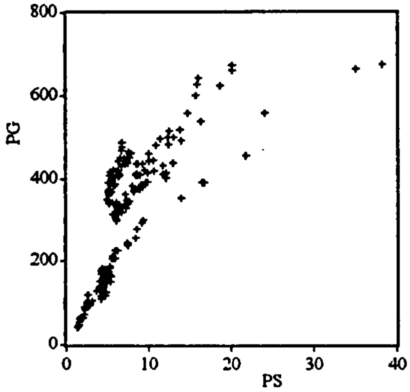

Figure 2a

Cross-plot of the Fitted values of PG against the Price of Silver (PS) from 1971:11 to 1990:06

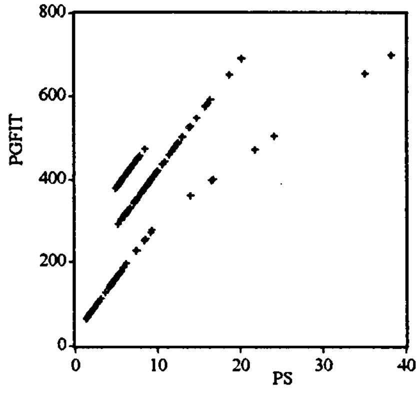

Figure 2b

Cross-plot of the Logprices of Gold (LPG) against Silver (LPS) from 1971:11 to 1990:6

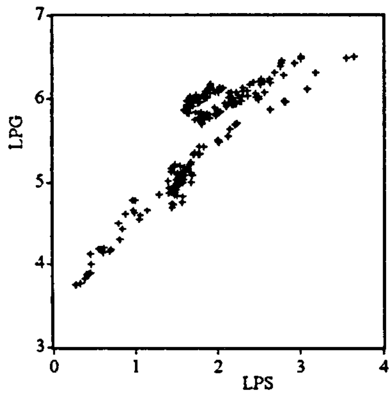

Figure 2c

Cross-plot of the Fitted values of LPG against the Logprices of Silver (LPS) from 1971:11 to 1990:06

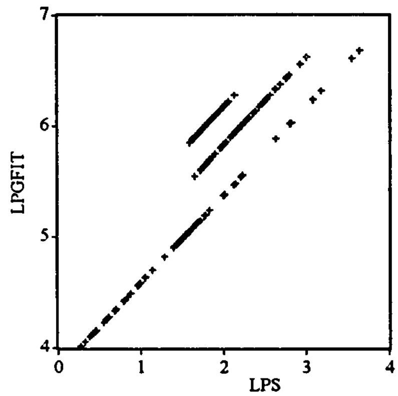

Figure 2d

Long-run Relationship Between the Prices of Gold and Silver Price of Gold (PG), Fitted Values (PGFit) and Residuals (R1G)

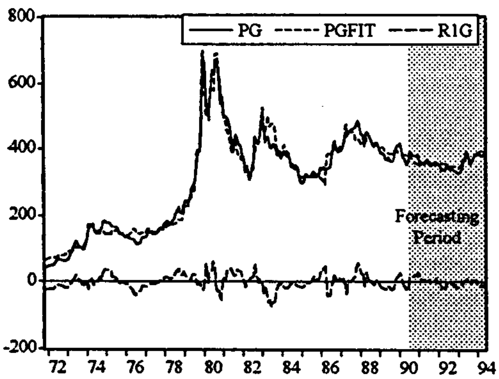

Figure 2e

Long-run Relationship Between the Logprices of Gold and Silver Logprice of Gold (LPG), Fitted Values (LPGFIT) and Residuals (R1LG)

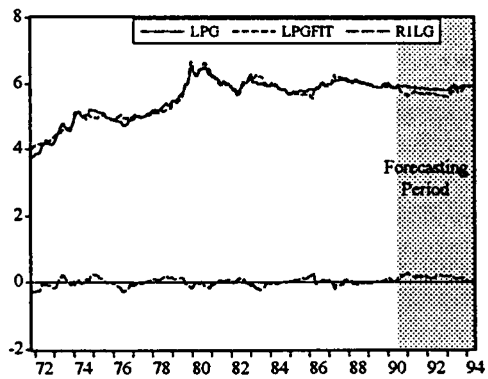

Figure 2f

Nonlinear Error Correction (NEC1): Escribano(1986)
Model 1 of Table 5a

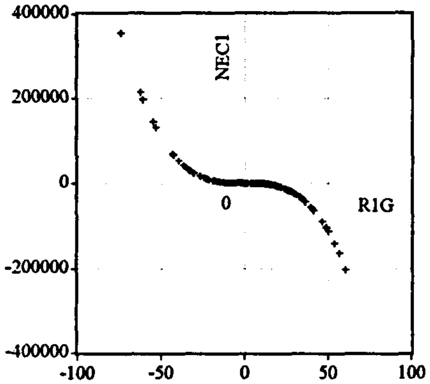

Figure 3a

Linear Error Correction (LEC): Engle and Granger(1987)
Model 2 of Table 5a

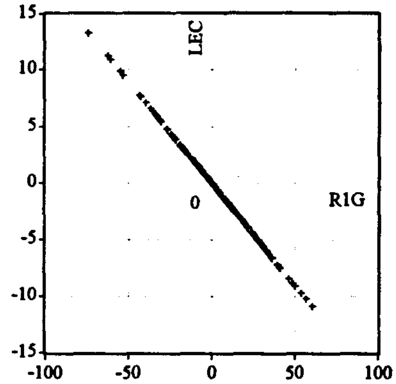

Figure 3b

Nonlinear Error Correction (NEC3): Granger and Lee(1989)
Model 3 of Table 5a

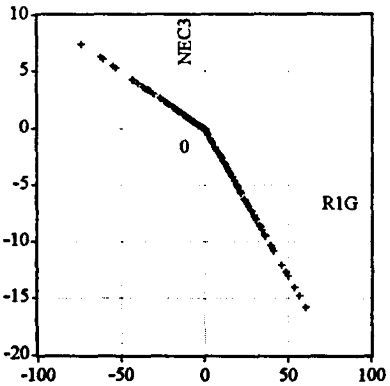

Figure 3c

Nonlinear Error Correction (NEC4): Escribano and Pfann(1990)
Model 4 of Table 5a

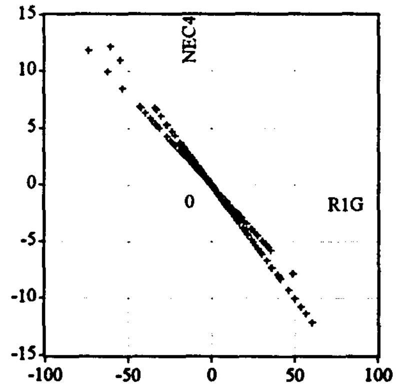

Figure 3d

Rate of Return of Gold (RRG) and the First Difference of the Logprice of Gold (DLPG)

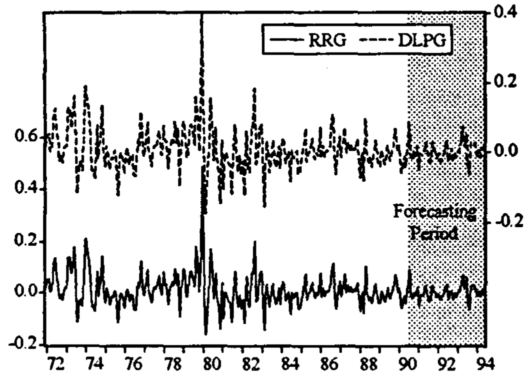

Figure 4a

Rate of Return of Silver (RRS) and the First Difference of the Logprice of Silver (DLPS)

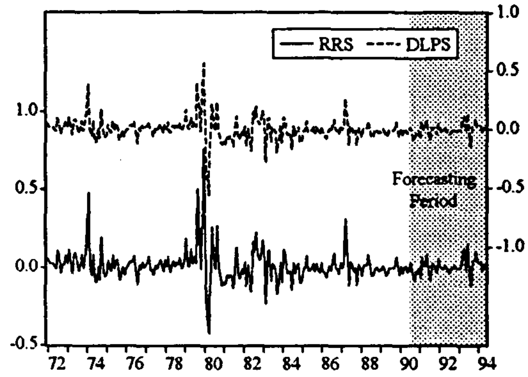

Figure 4b

Cross-plot of the Rate of Return of Gold (RRG) and the First Difference of the Logprice of Gold (DLPG) from 1971:12 to 1990:06

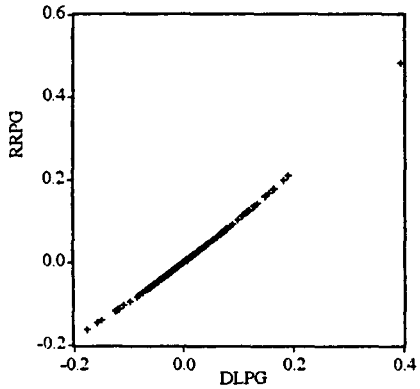

Figure 4c

Cross-plot of the Rate of Return of Silver (RRS) and the First Difference of the Logprice of Silver (DLPS) from 1971:12 to 1990:06

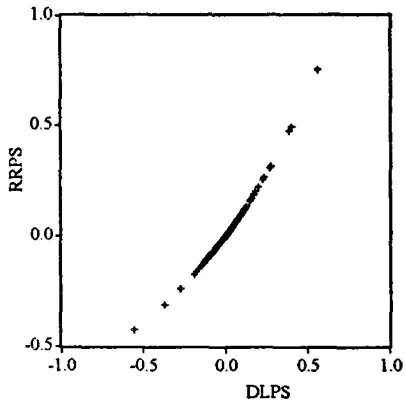

Figure 4d

Cross-plot of the Rate of Return of Gold (RRG) and the First Difference of the Logprice of Gold (DLPG) from 1990:07 to 1994:06

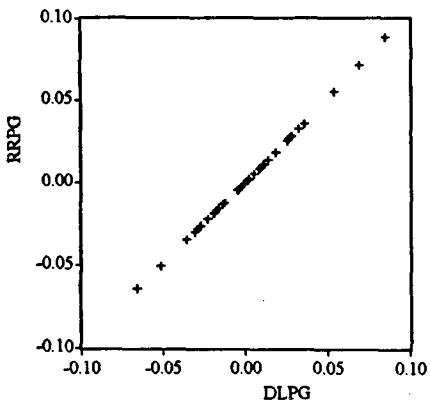

Figure 4e

Cross-plot of the Rate of Return of Silver (RRS) and the First Difference of the Logprice of Silver (DLPS) from 1990:07 to 1994:06

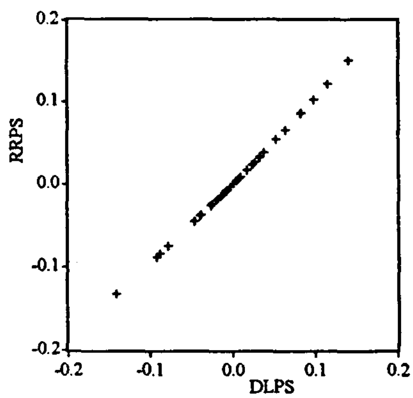

Figure 4f