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Frequency-Modulated OFDM: a new Waveform for High-Mobility Wireless Communications

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Abstract—This paper introduces a novel waveform for wireless communication, denoted as Frequency-Modulated Orthogonal Frequency-Division Multiplexing (FM-OFDM). By frequency-modulating an OFDM signal and further defining a suitable cutoff subcarrier, the resulting constant-envelope waveform (ensuring a low Peak to Average Power Ratio) exhibits strong robustness not only to Doppler, but also to phase noise and carrier frequency offsets. These impairments introduce an additive error term at the information-bearing subcarriers that is mostly concentrated at the lowest part of the spectrum. No inter-carrier interference is thus present, and impairments can be easily overcome by mapping the information above a cutoff subcarrier. Theoretical expressions and numerical results prove the superiority of FM-OFDM over the state of the art, particularly in high-speed Rayleigh channels and/or high phase noise.

Index Terms—Frequency modulation, Frequency division multiplexing, Doppler effect, Phase noise.

I. INTRODUCTION

WIRELESS communications are at a turning point in the evolution from the current Fourth and Fifth Generation (4G and 5G respectively) to the next Sixth Generation (6G) of mobile communication technologies [1]–[3]. Increased attention is being paid to new requirements set by use cases like Industrial Internet of Things (IIoT), Ultra Reliable Low-Latency Communications (URLLC), Joint Communications and Sensing (JCAS), to name a few. These new services also bring new challenges, from improved energy efficiency to better support of Doppler and hardware impairments.

Cyclic Prefix-Orthogonal Frequency Division Multiplexing (CP-OFDM) [4], [5] is one of the most used waveforms in wireless communication systems. It possesses a great flexibility to multiplex different users and/or services in the frequency domain and is able to exploit the low-complexity one-tap equalizer to compensate the effects of the channel. However, these advantages come at the expense of facing some drawbacks. One of the most well-known issues is its high Peak to Average Power Ratio (PAPR), which requires a significant back-off to keep the High-Power Amplifiers (HPA) far from their non-linear region [6], hence limiting the maximum transmit power. Several PAPR reduction techniques exist to avoid this issue such as clipping, selective mapping, or tone reservation (among others), at the expense of added distortion, increased complexity or reduced data efficiency.

For these reasons, Discrete Fourier Transform-spread OFDM (DFT-s-OFDM) is standardized in the uplink of 4G [7] and 5G [8]. It minimizes the PAPR as compared to CP-OFDM, but its advantage reduces when subcarriers are not localized in frequency [9]. Another important weakness of CP-OFDM is its sensitivity to Carrier Frequency Offset (CFO) and Phase Noise stemming from imperfect oscillators [10]–[12].

Constant-envelope waveforms have received considerable attention because they allow power amplifiers to operate close to their saturation point. Continuous-Phase Modulation (CPM) [13] is an example where the phase carries the information as an integral of a train of modulating pulses that ensures phase continuity, thereby exhibiting high spectral efficiency at the cost of higher decoding complexity. However, CPM lacks flexibility to allocate data and control resources. Constant-Envelope OFDM (CE-OFDM) overcomes this issue by adopting OFDM as the modulating waveform that drives the phase of a complex exponential [14]–[18]. The low PAPR and bandwidth-performance control allowed by CE-OFDM makes it a very interesting candidate for applications with energy constraints. However, its performance is degraded when the channel is time-varying.

Dealing with the mobility issue, Orthogonal Time-Frequency-Space (OTFS) modulation has been proposed to cope with doubly-dispersive channels in high-Doppler conditions [19]–[21]. OTFS makes channel estimation more stable than in CP-OFDM because it works in the delay-Doppler domain. Multiple receiver structures have already been proposed, making the study of OTFS a very active area of research. Another waveform called Frequency-Domain Multiplexing with Frequency-Domain Cyclic Prefix (FDM-DFCP) was introduced in [22] as a time-frequency dual of CP-OFDM, capable to overcome Doppler spread in absence of a delay spread. The receiver in this case must estimate and compensate the Doppler profile through a dual version of frequency-domain equalization. Other works focus on the design of Doppler-tolerant sequences for CP-OFDM [23]. However, none of these alternatives provide a fully satisfactory solution to the challenges ahead, either because of PAPR or complexity.

In this work, a new waveform called frequency-modulated OFDM (FM-OFDM) is developed mainly targeting high-mobility scenarios for beyond-5G and 6G systems. The OFDM information-bearing signal modulates the differences in the phase rather than the phases themselves. This way of encoding the information makes it more robust to phase noise, Doppler and frequency offsets than CE-OFDM or CP-OFDM, because the instantaneous frequency tends to vary much more slowly than the instantaneous phase under such impairments. More-
over, we prove that impairments cause an additive, instead of multiplicative, error term that appears at the lowest subcarriers of the instantaneous frequency spectrum. As a result, no inter-carrier interference (ICI) appears, impairments are easier to overcome and, contrary to CP-OFDM, the subcarrier spacing (SCS) does not need to be increased to cope with phase noise or Doppler. A novel concept is introduced, the cutoff subcarrier, aimed to avoid data from being affected by phase/frequency impairments by simply selecting its value to be higher than their spectral widths. As discussed in the Numerical Results section, time variability has a beneficial effect thanks to the increased Doppler diversity enabled by FM-OFDM. This work shows that channel estimation and equalization can be avoided in frequency-flat channels while reaching excellent performance up to speeds that are typical of non-Terrestrial Networks. Finally, the behavior in frequency-selective channels proves to be similar to, or even better than, the flat-fading case provided that the equalizer removes the effect of multipath, with similar techniques to those applied with conventional multicarrier waveforms. To the best knowledge of the authors, no other waveform benefits from these properties at such low complexity. Similarly to CE-OFDM, FM-OFDM also enjoys the constant envelope property, the ability to trade off performance against bandwidth, and the flexibility to freely allocate users and control information into subcarriers. It also shares some properties with conventional multicarrier waveforms. To the best knowledge of the authors, no other waveform benefits from these properties at such low complexity. The main contributions of this paper are summarized as follows:

- A novel waveform is proposed where the instantaneous frequency is modulated by an OFDM information-bearing signal for improved robustness to phase/frequency impairments.
- Baseband processing blocks for transmission and reception are described and the impact of Doppler, phase noise, and CFO on demodulation performance is analyzed.
- It is proved that phase/frequency impairments in FM-OFDM exhibit an additive, instead of multiplicative, error term at the lowest subcarriers of the instantaneous frequency spectrum, thereby avoiding inter-carrier interference. Channel estimation and equalization are proved to be unnecessary in flat-fading channels thus simplifying receiver implementations.
- The concept of cutoff subcarrier is introduced to minimize the effect of impairments and lower down the error floor of bit error rates to very small values. Practical rules for selection of the cutoff subcarrier are given. In most typical terrestrial scenarios, the cutoff subcarrier can be set to zero therefore not incurring any additional overhead.
- A comparison is performed with CE-OFDM in regard to the bandwidth, spectral efficiency, signal-to-noise ratio (SNR) in Additive White Gaussian Noise (AWGN) channel, and complexity. Novel expressions are derived for the range of modulation indices that allows unambiguous detection in FM-OFDM and CE-OFDM.
- The numerical results confirm the validity of the theoretical expressions and show the superiority of the proposed waveform over the state of the art in high-Doppler scenarios. Results in flat-fading Rayleigh channels show an excellent performance up to Doppler spread values as high as 138.88 kHz under proper choice of the cutoff subcarrier, yet with no need of channel estimation or equalization. Results in frequency-selective Rayleigh channels are even better with the use of equalizers that are capable to remove multipath in time-varying channels, like those based on partitioned convolutions, thanks to the extra multipath diversity.

The rest of the paper is organized as follows. Section II introduces the system model. Section III describes the main processing blocks involved in the proposed FM-OFDM waveform. Section IV analyzes FM-OFDM in high-speed wireless channels with impairments. Section V studies the characteristics of FM-OFDM in terms of bandwidth, spectral efficiency, SNR and complexity, and compares them with other waveforms like CE-OFDM and CP-OFDM. Section VI is devoted to the numerical results, and finally Section VII concludes the work.

Notation: \( j \triangleq \sqrt{-1} \) is the imaginary constant. For scalars, \( \lfloor \cdot \rfloor \) denotes the rounding towards zero operation. For a periodic signal, \( (\cdot)_T \) denotes its time average over a period \( T \). For a complex vector, \( \| \cdot \| \) denotes its norm, \( (\cdot)^\dagger \) is the complex conjugate, \( \Re \{ \cdot \} \) and \( \Im \{ \cdot \} \) are its real and imaginary part respectively, and \( \mathcal{W}[n] = a[n] - a[n - 1] \) is the backwards difference operator. \( b_k \) denotes an \( N \)-point vector with components \( b_0, b_1, \ldots, b_{N-1} \). \( d[n] \) represents any signal obtained after equalization. When applied to complex vectors, \( \text{DFT}_N \{ s[n] \} \) and \( \text{IDFT}_N \{ X[k] \} \) denote \( N \)-point normalized Discrete Fourier Transform and Inverse Discrete Fourier Transform operators, respectively, defined by \( \text{DFT}_N \{ s[n] \} = \sum_{n=0}^{N-1} s[n] \exp(-j2\pi kn/N) \) and \( \text{IDFT}_N \{ X[k] \} = \frac{1}{N} \sum_{n=0}^{N-1} X[k] \exp(j2\pi kn/N) \). \( \mathcal{W} \) represents the expectation operator, and \( \delta[n] \) is the discrete Dirac delta function. Finally, \( CN(\eta, \sigma^2) \) is a circularly symmetric Gaussian random process with mean \( \eta \) and variance \( \sigma^2 \).

II. System Model

The system model considers the transmission of a block of complex modulated symbols using a multicarrier waveform over the duration of a symbol. Transmission suffers the impact of a doubly-dispersive channel with delay spread and Doppler spread. A receiver comprising non-ideal oscillators introduces impairments over the phase and frequency characteristics of the received signal, and is aimed to estimate the original modulated symbols with the best possible error performance.

A. Considered Scenario

In every \( T_c \)-seconds block interval, a block of \( N_c \) complex modulated \( M \)-QAM (Quadrature Amplitude Modulation) symbols \( \{ X_i \} \) is generated. We assume that \( T_c \) can be comparable
to, or higher than, the channel’s coherence time. For simplicity, we will consider that \( N_C = 1 \). The complex modulated symbols are carried by a generic multicarrier waveform \( p[n] \) characterized by a subcarrier spacing \( S = 1/T_s \) and a sampling frequency \( f_s = N/T_s \). A mathematical transformation \( \mathcal{F} \) then converts the multicarrier waveform into a different complex baseband signal \( \hat{b}[n] \) that possesses better PAPR characteristics than the multicarrier waveform, to which a cyclic prefix (CP)\(^1\) is prepended with a length \( N_{CP} \) equal or higher than the maximum delay of the channel expressed in samples \( i[n] = \mathcal{F}\{p[n]\} \).

\[
x[n] = \begin{cases} 1 & n = 0, \ldots, N - N_{CP} - 1 \\ -1 & n = N_{CP}, \ldots, N + N_{CP} - 1. 
\end{cases}
\]

By way of examples, an identity transformation \( \mathcal{F} = I \) would yield a CP-OFDM waveform, whereas a CE-OFDM waveform with modulation index \( m \) and \( N_0 \) active subcarriers \([15]\) could be obtained by a transformation (with suitable parameters \( A_s \) and \( \theta \))

\[
\mathcal{F}\{p[n]\} = A_s \exp\left(2\pi m \sum_{n=0}^{N-1} \frac{n}{N} p[n] + \theta \right).
\]

The signal \( x[n] \) is transmitted over a doubly-dispersive wireless channel that exhibits both delay spread and Doppler fading. A receiver with non-ideal hardware introduces some uncertainty in the estimation of the channel impulse response obtained via channel estimation\(\). Finally, the inverse of the transformation performed by the transmitter is applied to the equalized signal \( \hat{x}[n] \) to obtain an estimate of the original multicarrier waveform \( p[n] = \mathcal{F}^{-1}\{\hat{x}[n]\} \), from which estimates of the original complex modulated symbols \( \{X[k]\} \) can be obtained.

### B. Channel Model

The channel considered in this work is a doubly-dispersive wireless channel \([26]\), \([27]\). Its discrete-time channel impulse response can be written as a superposition of \( L \) uncorrelated scatterers (taps) whose multipath components are characterized by discrete delays and time-varying complex amplitudes. For convenience, we separate the time-varying phase term caused by Doppler \( \psi_d,z[n] \) from the rest of the channel impulse response, in the form

\[
h[n, z] = \sum_{k=0}^{L-1} h_k[n] e^{j2\pi f_d k z} e^{j2\pi f_d k z} h(z - z_k),
\]

where \( h_k[n] \) represents the time-varying real amplitude of the \( k \)-th tap, \( \zeta_k \) denotes its initial phase, and \( z_k = \{\gamma_k\} \) is the discrete delay corresponding to the continuous-time delay \( \gamma \) after rounding to the nearest integer. This channel model can correspond to either pure non-line-of-sight (NLOS) dominated by Rayleigh scattering, or line-of-sight (LOS) with a direct ray and some Rayleigh fading caused by scatterers. Large-scale fading is not considered as it runs over time scales of seconds to minutes (much larger than the duration of practical symbols) depending on the environment, and only small-scale fading is assumed to have an impact on performance \([27]\).

### III. PROPOSED FM-OFDM WAVEFORM

The block diagram for transmission and reception of a single-user baseband FM-OFDM waveform is sketched in Fig. 1, illustrating a transmitter and a receiver linked by a doubly-dispersive wireless channel. Unshaded blocks are common to CE-OFDM and FM-OFDM, while shaded blocks are new.

#### A. Transmitter

Over the duration of a symbol, a QAM modulator takes \( N_0 \) input bits \( \{b_i\} \) and delivers a block of \( N_0 \) complex modulated M-QAM symbols \( \{X[k]\} \), where \( N_0 = N_0/N_{QAM} \). The number of bits per complex constellation symbol \( K_{QAM} \) is equal to \( \log_2 M \). The complex modulated symbols are then mapped to subcarriers after skipping those up to the cutoff subcarrier \( k_0 \), where \( 0 \leq k_0 < N/2 \). The resulting vector is padded with zeroes up to \( N \) thus yielding a conjugate symmetric vector \( X[k] = \begin{cases} X[k - k_0] & k = k_0 + 1, \ldots, k_0 + N_0 - N, \\
X[k - (N - k_0)] & k = N - N_0 - (k_0 + 1), \ldots, N - 2 - k_0 \\
0 & \text{elsewhere}
\end{cases} \)

The number of active subcarriers is \( N_A = 2N_0 \), with \( 0 < N_A \leq N - 2(k_0 + 1) \). The IDFT block performs a normalized \( N \)-point inverse DFT on \( X[k] \) thus yielding the following instantaneous frequency signal

\[
f[n] = \frac{1}{\sqrt{N}} \sum_{k=0}^{N-1} X[k] e^{j2\pi k n / N},
\]

where \( m \) is called the modulation index. As the DC subcarrier is skipped by the mapping in \((4)\), \( |f[n]| = 0 \), and the variance of the instantaneous frequency is \( \sigma_f^2 = \left(\frac{1}{N} \sum_n |f[n]|^2\right)^2 = m^2 \) independent of the number of active subcarriers.

The frequency modulator block yields the instantaneous phase signal as a cumulative sum

\[
\psi[n] = \psi_0 + 2\pi \sum_{n=0}^{N} f[n].
\]
where \( \phi_0 \) is an arbitrary constant. As in CE-OFDM, \( \phi_0 \) can be zero or equal to the last phase in the previous symbol to smoothen the signal’s variations [15]. For simplicity, we will consider \( \phi_0 = 0 \). The rationale for carrying the information in the phase differences stems from the increased robustness to phase/frequency impairments that a differential encoding can bring, because frequency tends to exhibit much slower variations than phase when faced to such impairments. The phase \( \phi[n] \) can fall outside the interval \([−\pi, \pi]\) as a result of the summation in (6), but \( f[n] \) must lie within \([-0.5, 0.5]\) to avoid ambiguities at the boundaries \( \pm \pi \). This limits the range of allowed modulation indices as

\[
\frac{N}{N_0} \left| s[n] \right|_{\text{max}} < 0.5, \quad (7)
\]

where \( \left| s[n] \right|_{\text{max}} \) is the peak value of the modulating signal \( s[n] = \text{IDFT}_N \{ X[k] \} \). A similar expression can be derived for CE-OFDM that involves differences in the phases, as

\[
\frac{N}{N_0} \left| s[n] - s[n-1] \right|_{\text{max}} < 0.5. \quad (8)
\]

Notice that the maximum modulation indices are different for FM-OFDM and CE-OFDM and depend on \( N, N_0 \) and the modulation order.

The complex phasor block then yields the constant-envelope signal with amplitude \( A \),

\[
\hat{s}[n] = A e^{j \hat{\phi}[n]}. \quad (9)
\]

Finally, a CP block as in (1) appends a replica of the last \( N_{CP} \) samples to the beginning of the symbol to allow the use of FDE techniques, when needed.

**B. Receiver**

Ignoring the CP, and substituting (9) and (3) into (2), the received signal can be written as

\[
r[n] = A_k \sum_{m=0}^{L-1} k_0[n-m] \exp \left( j \psi_m[n] \right) \exp \left( j \phi[n] \right) + \hat{w}[n], \quad (10)
\]

where \( \phi = -2\pi f_c T_s + a_k \) and \( f_c \) is the carrier frequency. The Equalizer in Fig. 1 is aimed to overcome the effects of multipath and, with the aid of an estimated channel frequency response, yields an equalized signal \( \hat{f}[n] \) that can be written without loss of generality as

\[
f[n] = \hat{h}[n] x[n] \exp \left( j \hat{\phi}[n] + \hat{\psi}[n] \right). \quad (11)
\]

In (11), imperfect equalization is represented by an additive error term \( \hat{w}[n] \), a residual real amplitude \( \hat{h}[n] \), and a residual phase \( \hat{\phi}[n] + \hat{\psi}[n] \). The time-varying part of the residual phase, \( \hat{\phi}[n] \), contains contributions from Doppler, phase noise and CFO after equalization, i.e., \( \hat{\phi}[n] = \hat{\phi}_D[n] + \hat{\phi}_N[n] + \hat{\phi}_C[n] \), as discussed in Section IV, flat-fading channels do not require equalization or channel estimation and these blocks are depicted with dashed lines in Fig. 1.

The phase extraction block then acquires the instantaneous phase

\[
\hat{\phi}_D[n] = \arg \left\{ \hat{f}[n] \right\} = \text{atan} \left( \frac{\Im \{ \hat{f}[n] \}}{\Re \{ \hat{f}[n] \}} \right), \quad n = 0, \ldots, N-1.
\]

To avoid ambiguities, the atan operation must be followed by a phase unwrapper that adds or subtracts multiples of \( 2\pi \) until the difference between two consecutive phases lies within \((-\pi, \pi)\).

The frequency extraction block then performs a backwards difference operation \( \nabla \) to yield the instantaneous frequency signal

\[
\hat{f}[n] = \frac{1}{2\pi} \nabla \hat{\phi}_D[n],
\]

with an initial frequency value defined by

\[
\hat{f}[0] = \frac{1}{2\pi} \left( \hat{\phi}_D[0] - \hat{\phi}_D[N-1] \right).
\]

Given that \( \hat{f}[N-1] = 0 \) (ignoring the noise), \( \hat{\phi}_D[N-1] = \phi_0 \) and

\[
\hat{f}[0] = \frac{1}{2\pi} \left( \hat{\phi}_D[0] - \hat{\phi}_D[N-1] \right).
\]

In other words, the phase behaves as a circular signal when obtaining the instantaneous frequency \( \hat{f}[n] \) regardless of the value of \( \phi_0 \).

An \( N \)-point DFT is performed over \( \hat{f}[n] \) to yield the subcarriers bearing the received data as

\[
X[k] = \frac{1}{N} \sum_{n=0}^{N-1} \hat{f}[n] e^{-j2\pi nk/N}.
\]
Afterwards, the Subcarrier demapping block skips those up to \( k_0 \) and picks the \( N_z \) positive subcarriers containing the received \( M \)-QAM symbols \( \{ X_j \} \):

\[
\{ \hat{X}_j \} = \hat{X} [k_0 + j], \quad j = 1, \ldots, N_z.
\]

Finally, QAM demodulation yields the \( N_z \) received bits \( \{ \hat{b}_j \}, \quad j = 0, \ldots, N_0 - 1 \).

IV. ANALYSIS OF FM-OFDM IN TIME-VARIANT CHANNELS WITH IMPAIRMENTS

A. Analysis of flat-fading channels. Cutoff subcarrier.

It is insightful to particularize the analysis to a frequency-flat fading channel with \( L = 1 \) and generalize it later to the multi-tap case. Let us first focus on the received signal in (10) prior to equalization. After dropping the subscript \( i \) and assuming perfect time and symbol synchronization, \( r(n) \) takes the form

\[
r[n] = b[n]e^{j \phi[n]} + \sum_{k=1}^{N_0-1} \hat{b}_k e^{j \phi[n]} + w[n].
\]

With a time-varying impairment term \( \varphi[n] \) containing contributions from Doppler, phase noise and CFO,

\[
\varphi[n] = \varphi_0[n] + \sum_{k=1}^{N_0-1} \hat{b}_k \varphi[n] + \psi[n].
\]

Taking into account the noise analysis in Appendix A, we can write

\[
\frac{1}{2} \Im \{ \hat{X}_j \} f[n] = \frac{1}{2} \Im \{ \sum \varphi[n] \} f[n] + \frac{1}{2} \Im \{ \sum \varphi[n] + \varphi_0[n] \} f[n] = 0.
\]

Equation (15), together with the fact that the initial phase \( \psi \) does not have any impact on detection, renders the important conclusion that FM-OFDM can overcome phase and frequency impairments without any channel estimation or equalization in flat-fading channels.

It is important to note that, although (15) provides a first approximation to \( k_0 \), the value that yields the best performance depends on the spillover of Doppler and phase noise impairments over the data subcarriers in the instantaneous frequency spectrum. Numerical assessment of the optimum value of \( k_0 \) can then be performed by starting from the value yielded by (15) and further refining it via simulation, as is discussed in Section VI-G.

B. Analysis of frequency-selective channels

All considerations made so far about the cutoff subcarrier can now be extended to frequency-selective channels. The Equalizer in Fig. 1 is needed to overcome the effect of the channel’s delay spread on the signal. However, it is generally not capable of perfectly aligning multipath components in a coherent way, and residual signal contributions may remain after equalization. Notice that (11) resembles the output of a flat-fading channel with complex amplitude \( \hat{b}_0 e^{j \phi[n]} + \psi \). The additive error \( \tilde{w}[n] \) can contain signal contributions as

\[
\tilde{w}[n] = \sum_{k=1}^{N_0-1} \hat{b}_k e^{j \phi[n]} + w[n].
\]
where \( \hat{h}_{n}[\zeta] \) denotes a residual impulse response by imperfect equalization. By denoting \( \ell[n] = \arg\left[\sum_{\nu=0}^{N-1} \hat{h}_{n}[\nu] \nu i[n] \right] \), we can write
\[
\frac{1}{2\pi} \arg\{f[n]\} = f[n] + \frac{1}{2\pi} \arg\{\hat{\phi}_n[n] + \hat{\phi} + \ell[n] + \Delta\phi[n]\} .
\]
While \( \hat{\phi}_n[n] \) only contains residual phases from Doppler, phase noise and CFO impairments, \( \ell[n] \) contains residual signal contributions. Denoting by \( W_r \) the spectral width of the impairment in (16), we can write without loss of generality \( W_r \geq f_T \), then the cutoff subcarrier can be obtained as a generalization of (15),
\[
k_o \geq \left[ \max\{W_r, \frac{W_{RN}}{5CS}\} \right] .
\]

The extent to which multipath is removed from the received signal determines the amount of residual signal contributions whose phase variations \( \ell[n] \) need to be absorbed by the cutoff subcarrier. The more effectively the equalized signal resembles the output of a one-tap fading channel, the closer \( W_r \) will be to \( f_T \) in (17). The problem of transmission of FM-OFDM in doubly-dispersive channels thus reduces to the design of FDE techniques whose equalized signal resembles the output of a one-tap residual channel, however fast it changes within the symbol. In other words: the equalizer shall remove the delay spread from the received signal, but does not need to take care of the Doppler spread as the latter can be absorbed by the cutoff subcarrier.

Following [36], one possible technique to equalize the time-varying channel involves the use of a stepwise approximation of the channel’s impulse response over \( N \) intervals with the aid of so-called partitioned convolutions. Convolutions can be replaced by FFTs when a ZP is appended to the symbols (generally different to the first ones because of Doppler). Intersection of the received block and the channel is guaranteed to be invertible [25]. A bank of piecewise MMSE equalizers can therefore be employed whose time-domain outputs are windowed and concatenated to yield the equalized signal. However, an estimation for \( W_r \) to use in (17) can be difficult to obtain. One simple criterion is to assume that \( W_r \approx f_T \), in order to obtain a first approximation to \( k_o \) from (17), and further refine it via simulation. This is the approach taken in the results of Section VI-G.

V. CHARACTERISTICS OF FM-OFDM

In this Section, we present a detailed analysis of the characteristics of FM-OFDM in terms of bandwidth, spectral efficiency, SNR and complexity, and compare them with other waveforms like CE-OFDM [15], [16], [18] and CP-OFDM.

A. Bandwidth and Spectral Efficiency

The bandwidth containing \( \geq 98\% \) of the FM-OFDM signal’s power can be approximately obtained by applying Carson’s rule, which for frequency-modulated analog signals reads [13]
\[
B_{FM} \approx 2 \left( \frac{|f(t)|_{\text{max}}}{f_T} + \frac{W}{2} \right) .
\]

where \( |f(t)|_{\text{max}} \) represents the maximum frequency excursion obtained from the derivative of the continuous-time phase \( \varphi(t) \), and \( W \) is the double-sided bandwidth of the OFDM modulating signal \( N_s/f_T \). Relating it with the discrete-time domain,
\[
|f(t)|_{\text{max}}=\frac{1}{2\pi} \left| \frac{d\varphi(t)}{dt} \right|_{\text{max}} + \frac{1}{2\pi} \left| \hat{\phi}_n[n] + \hat{\phi} + \ell[n] + \Delta\phi[n] \right|_{\text{max}} = \frac{1}{2\pi} \left| \frac{d\varphi(t)}{dt} \right|_{\text{max}} + \frac{1}{f_T} \int_{-f_T}^{f_T} \frac{W_{RN}}{5CS}.
\]

Given the frequent peaks in the OFDM modulating signal it may be more convenient to use the RMS frequency excursion \( \frac{|f(t)|_{\text{rms}}}{f_T} \), rather than its peak value. The approximate RMS bandwidth is therefore
\[
B_{FM} \approx 2 \left( \frac{|f(t)|_{\text{rms}}}{f_T} + \frac{N_s}{2T_s} \right) = 2 \left( m f_s + \frac{N_s}{2T_s} \right) .
\]

The RMS bandwidth of a CE-OFDM signal can be taken from [15] which, after following the notation in this paper, reads
\[
B_{CE} = \max\{2m, 1\} \frac{N_s}{T_s} .
\]

Given that the rate is \( R = N_s K_{QAM}/(2T_s) \) for both waveforms, their respective spectral efficiencies \( e_{FM}, e_{CE} \) can be written as
\[
e_{FM} = \frac{R}{B_{FM}} = \frac{N_s K_{QAM}}{4T_s m f_s + \frac{N_s}{2T_s}} ,
e_{CE} = \frac{R}{B_{CE}} = \frac{K_{QAM}}{2\max\{2m, 1\}} .
\]

At very low modulation indices, \( e_{FM} = e_{CE} = K_{QAM}/2 \) and the spectral efficiencies are the same. As shown in Section VI-A, this happens also for their maximum modulation indices.

B. SNR in AWGN channels with no impairments

The SNR is now studied over an ideal AWGN channel with no transceiver impairments. The received FM-OFDM signal can be written after discarding the CP as
\[
r[n] = A_n \cos \varphi \cos n + w[n] .
\]

As shown in Appendix A, the SNR of the instantaneous frequency subcarriers \( SNR^M \) is
\[
SNR^M = \frac{2\pi^2 a^2 N}{N_s \left[ \text{arctan}^2 w[n] \right]} \left( 1 - \cos \frac{2\pi^2}{a^2} \right) ,
\]

where \( w[n] \triangleq \frac{w[n]}{a^2} \) is a function of the radial and normal noise components \( w_r[n] \) and \( w_a[n] \) respectively. Notice the noise reduction in the lowest subcarriers because of the \( 1 - \cos(\cdot) \) term in the denominator of (20).

Similarly, the SNR of the instantaneous phase subcarriers in CE-OFDM is derived in Appendix B as
\[
SNR^CE = \frac{4\pi^2 a^2 N}{N_s \left[ \text{arctan}^2 w[n] \right]} .
\]
The subcarrier SNR improvement of FM-OFDM compared to CE-OFDM is thus

\[
\frac{SNR_{FM}^2}{SNR_{CE}^2} = \frac{1}{2 \left(1 - \cos \frac{\pi m}{N_c}\right)} SNR_{IS},
\]

and similarly,

\[
SNR_{FM}^2 = \frac{16 \pi^2 m^2 N_c^2}{N_c} SNR_{IS},
\]

where \(SNR_{IS}\) is the input signal-to-noise ratio which is related to the bit energy-to-noise density ratio \(E_b/N_0\) (Appendix A),

\[
SNR_{IS} = \frac{A_f^2/2}{N_0 T_s} \frac{N_c K_{QAM} E_b}{2N} N_0.
\]

C. Receiver Complexity

Complexity is now calculated as the number of floating-point multiplications per symbol in a typical receiver implementation with ideal channel estimation and MMSE equalizer. Table I shows it for the CP-OFDM, CE-OFDM, and FM-OFDM waveforms after excluding from the analysis the subcarrier demapping and demodulator steps, which are common. \(C_{\text{CE}}\) denotes the complexity of the arctan operation. The table shows that CE-OFDM and FM-OFDM roughly spend two more DFT steps than CP-OFDM in a frequency-selective channel. In flat-fading channel conditions, FM-OFDM complexity is lower than in CE-OFDM, and comparable to CP-OFDM if efficient CORDIC (Coordinate Rotation Digital Computer) implementations are used for the arctan operation [31].

---

### Table I

<table>
<thead>
<tr>
<th>Waveform</th>
<th>Relative complexity (flat channel)</th>
<th>Relative complexity (non-flat channel)</th>
</tr>
</thead>
<tbody>
<tr>
<td>CP-OFDM</td>
<td>(N_0 T_s + N_c)</td>
<td>(N_0 T_s + N_c)</td>
</tr>
<tr>
<td>CE-OFDM</td>
<td>(N_0 T_s + N_c \times \frac{N_c}{2})</td>
<td>(N_0 T_s + N_c \times \frac{N_c}{2})</td>
</tr>
<tr>
<td>FM-OFDM</td>
<td>(N_0 T_s + N_c \times \frac{N_c}{2})</td>
<td>(N_0 T_s + N_c \times \frac{N_c}{2})</td>
</tr>
</tbody>
</table>

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### VI. NUMERICAL RESULTS

In this Section, all the relevant theoretical expressions are numerically verified and the bit error rate performance of FM-OFDM is assessed against CP-OFDM and CE-OFDM, in both AWGN and Rayleigh fading channels. Numerical results for Rayleigh channels with very high Doppler spread are also presented to illustrate the behavior of FM-OFDM in high-speed scenarios. The simulation assumptions are gathered in Table II.

#### A. RMS and occupied bandwidth

In order to check the validity of the bandwidth expressions of Section V-A, the spectrum of FM-OFDM and CE-OFDM given by the Welch’s method is depicted in Fig. 2 for \(N = 512\) and \(N_0 = 64\), as well as the RMS bandwidths obtained by application of (18) and (19). A minimum representative value of \(m\) was taken as 0.01/2\(\pi\), and the maximum \(m\) was found via an exhaustive search (by increasing it until any further increase yielded an error floor) to avoid phase ambiguities on transmission for each waveform (0.6/2\(\pi\) and 2.5/2\(\pi\), respectively). As seen in the table, (18) and (19) are closer to the theoretical RMS bandwidths (OBW) values yielded by the Welch’s method for the two values of \(m\). As seen in the table, (18) and (19) are closer to the 99.999% bandwidth when the modulating index is small, because the small contribution of the modulating signal to the overall bandwidth (from the small value of \(m\)) makes it imperceptible unless a high fraction of the power is sought in
the OBW. At moderate and high values of $m$, Carson’s rule yields a more realistic value close to the 90% OBW. Despite these minor discrepancies, (18) provides a sufficiently accurate estimation of the actual OBW, and comparison with (19) proves that both waveforms are equivalent in bandwidth and spectral efficiency, the only difference being their maximum $m$ values given by (7) and (8).

### B. SNR in AWGN channel with no phase/frequency impairments

The subcarrier SNR of FM-OFDM is now obtained by simulation for an AWGN channel with no other impairments, and compared with the high-SNR approximation in (23). It is reasonable to assume that the impact of phase unwrap errors can be minimized by considering very small values of $m$, and a high input SNR. Hence, a value $m = 0.001/2\pi$ is selected in this subsection. Fig. 3 shows the simulated and theoretical values of the subcarrier SNR as a function of the subcarrier index $k$ and input $E_b/N_0$ for $15\ kHz$ SCS. The small $m$ motivates the very small SNR values seen in the figures, and realistic modulation indices would yield much higher values of the subcarrier SNR. Very good match is obtained between theory and simulation for $E_b/N_0 \geq 15\ dB$, in line with the threshold $E_b/N_0$ in this case ($16\ dB$).

The subcarrier SNR improvement of FM-OFDM vs. CE-OFDM is now numerically obtained as a function of $k$ for an AWGN channel and compared with the theoretical expression (22). The improvement is calculated by obtaining the difference in the input $E_b/N_0$ values of FM-OFDM and CE-OFDM that reaches BER = $10^{-4}$ assuming no phase unwrap errors. Fig. 4 shows the SNR improvement for $m = 0.001/2\pi$, $N = 512$, $N_s = 2$, and $m = 0.001/2\pi$.

Despite this SNR gain, it is shown in the next subsection that FM-OFDM and CE-OFDM have equivalent AWGN performances in absence of other impairments. For the same $m$, FM-OFDM presents a lower SNR and a higher bandwidth than CE-OFDM. When optimum values of $m$ are considered in both waveforms, their SNRs for a given BER are very similar and their performances comparable in an AWGN channel.

![Fig. 2. Power spectrum of FM-OFDM and CE-OFDM averaged after 100 slots, and RMS bandwidths $B_{FM}$, $B_{CE}$ given by (18) and (19) respectively, for $N = 512$, $N_s = 64$, and QPSK modulation. The spectra are shown for a very small modulation index value ($0.01/2\pi$) and for their maximum values $m_{FM} = 0.6/2\pi$ and $m_{CE} = 2.5/2\pi$.](image1)

![Fig. 3. Simulated values of the subcarrier SNR of FM-OFDM in AWGN channel as a function of the subcarrier position $k$ and input $E_b/N_0$, and comparison with the theoretical expression (23), for $N = 512$, $N_s = 128$, $m = 0.001/2\pi$, and QPSK modulation. Threshold is at $E_b/N_0 = 16\ dB$.](image2)

![Fig. 4. Simulated subcarrier SNR improvement of FM-OFDM vs. CE-OFDM in AWGN channel at the first subcarrier positions $k \in [1, 20]$, and comparison with the theoretical expression (22), for BER = $10^{-4}$, $N = 512$, $N_s = 2$, and $m = 0.001/2\pi$.](image3)
C. Impact of the modulation index in AWGN channel with no phase/frequency impairments

Fig. 5 compares the bit error rates of CP-OFDM, CE-OFDM and FM-OFDM in AWGN channel and bQAM modulation with $N = 512$ and $N_a = 64$, for different modulation indices. Threshold is at $E_b/N_0 = 14.25$ dB.

D. Impact of the time variability of the channel with zero cutoff subcarrier

Figures 6 and 7 compare the bit error rates of FM-OFDM, CE-OFDM and CP-OFDM in a flat-fading Rayleigh channel at different modulations and user speeds, with $N = 512$, $N_a = 128$, $k_0 = 0$, and $m = 0.6/2\pi$. MMSE equalization is considered in CE-OFDM and CP-OFDM with ideal per-symbol channel estimation, whereas no equalization is present in FM-OFDM. As expected, BER saturates above $10^{-3}$ in CP-OFDM and CE-OFDM. FM-OFDM in contrast shows remarkable robustness to mobility. Notice that the cutoff subcarrier is zero in these curves.

In QPSK, mobility is even beneficial for FM-OFDM performance as a consequence of the increased Doppler diversity. Doppler diversity [34], [35] is a beneficial consequence of spreading the data over the symbol’s duration as per the frequency modulation step in Fig. 1. When the channel varies significantly in a symbol, a deep fade in parts of it may sometimes be compensated by a less deep fade in others, and the loss of some samples because of fading can eventually be compensated by other samples with less fading contributing to the same subcarrier. This leads to extra Doppler diversity and improved BER compared to lower speeds with the advantage that, in contrast to other works [36], none of the Doppler components need to be estimated in FM-OFDM. The higher sensitivity to errors of 16QAM modulation may slightly hinder the benefit from Doppler diversity and make it perform worse at higher speeds, but the performance difference between 250 km/h and 1,000 km/h is very small (less than 2 dB).

E. Impact of phase noise with zero cutoff subcarrier

Fig. 8 compares the bit error rates of FM-OFDM, CE-OFDM and CP-OFDM with phase noise in AWGN channel and 64QAM modulation at different carrier frequencies, with
SCS = 120 kHz. FM-OFDM performance is remarkably close to the ideal case at 30 GHz and 200 GHz carrier frequencies, and also very good at 300 GHz, despite not requiring any equalization. In contrast, CP-OFDM and CE-OFDM with MMSE equalization become largely unusable unless mitigation techniques are applied to compensate for phase noise impairments [10].

F. Performance in Rayleigh frequency-selective channels with zero cutoff subcarrier

Let us first focus on a low-speed frequency-selective channel to show that the behavior of FM-OFDM is similar to CE-OFDM, and in some cases superior to CP-OFDM, with MMSE equalization. Fig. 9 illustrates the performance of the three waveforms in this case at 3 km/h, with 64QAM and two different modulation indices. At this speed, FDE can perfectly cope with multipath and FM-OFDM and CE-OFDM show a similar performance for their highest m (0.6/2𝜋 and 1.2/2𝜋, respectively). Notice that the maximum m is different to that in Fig. 5 because multipath gives rise to phase unwrap errors more frequently than in the AWGN case, thus reducing the available range of modulation indices. Both waveforms outperform CP-OFDM above the threshold for their maximum m and exhibit a steeper BER slope as a result of the higher multipath diversity enabled by the spreading of data symbols in the frequency domain, as is characteristic of angular modulations [13].

Let us now focus on the results for a high-speed frequency-selective channel. Fig. 10 illustrates the numerical results with TDL-C 300 m channel model at 120 km/h for QPSK and 16QAM, using for FM-OFDM a piecewise MMSE equalizer [30] with Ns = 4, and traditional MMSE equalizers for the other two waveforms. FM-OFDM, in addition to showing excellent performance, exhibits a significant Eb/N0 gain with respect to the flat fading case thanks to the extra multipath diversity in the TDL-C channel. In contrast, CP-OFDM and CE-OFDM show error floors that are similar to those in the flat-fading case (Fig. 6).

G. Impact of the cutoff subcarrier in highly time-variant channels with phase noise

Table IV shows a first approximation of the cutoff subcarrier k0 given by (17) as a function of SCS and speed v. As discussed in Section IV-B, we consider for simplicity W0 = f0. Phase noise impact is only accounted for at 30 GHz and 300 GHz carrier frequencies and is considered negligible at 6 GHz. It is apparent that k0 is small in most cases except at very high speeds and/or lower SCS values. Speeds of 1.000 km/h and 2.000 km/h are representative of airborne communications, and the extreme case at 25,000 km/h (typical of Low Earth Orbit, LEO, satellites) is gaining some attention in 3GPP as part of the study of non-terrestrial networks [37]. The cases with 120 kHz SCS follow the model in [29] and show the reduced impact of phase noise. The cases with smaller SCS are
shown to illustrate the superiority of FM-OFDM in scenarios where CP-OFDM and CE-OFDM fall short due to the high ratio $f_0/SCS > 1$. The influence of $k_0$ in such cases is illustrated in Fig. 11 for a Rayleigh flat fading channel at 1,000 km/h and 2,000 km/h, and a SCS of 3.75 kHz. Curves show a significant reduction in the error floor of 16QAM and 64QAM for increased $k_0$. The error floor is caused by the spillover of Doppler impairment over the data subcarriers in the instantaneous frequency spectrum. When $k_0$ is increased, the spillover is reduced and performance is improved at the higher modulations. QPSK, inherently less sensitive to errors, already exhibits a remarkably good performance at these speeds and shows little influence from $k_0$.

Another example characteristic of LEO satellites is shown in Fig. 12 with a ground speed of 25,000 km/h and 15 kHz SCS. Doppler spread reaches 138.8 kHz, but QPSK performance is very good for $k_0 \geq 10$ and reaches an error floor below $10^{-6}$ at $k_0 = 20$. CP-OFDM and CE-OFDM are basically useless at these speeds unless Doppler compensation techniques are employed. Compared with the QPSK results at 1,000 km/h in Fig. 6, there is an $E_b/N_0$ gain of almost 5 dB at BEIR $= 10^{-3}$ as a result of Doppler diversity, that increases with $k_0$.

VII. CONCLUSIONS

In this paper, a new FM-OFDM waveform is presented that fills the gaps of previous works on modulations in terms of PAPR and detection complexity when faced with high mobility and/or phase noise conditions. By encoding the information in the instantaneous frequency instead of the phase, extra resilience to Doppler, phase noise and frequency offsets can be enjoyed at a low receiver complexity. It is shown how these impairments exhibit an additive error term whose energy is mostly confined at the lowest subcarriers of the instantaneous frequency spectrum. The concept of cutoff subcarrier is introduced that pushes the error floor down to very low values even at extremely high Doppler or phase noise. No channel estimation or equalization is needed in channels with a large coherence bandwidth compared to the signal bandwidth, hence simplifying receiver implementations. FM-OFDM also yields excellent performance in frequency-selective channels by employing equalizers capable to remove multipath in time-varying channels. Use at higher frequencies can also benefit from its constant envelope nature and better support of Doppler varying channels. Use at higher frequencies can also benefit from its constant envelope nature and better support of Doppler and phase noise. Numerical results confirm the theoretical derivations and show the superiority of the proposed waveform in presence of high Doppler spread and/or phase noise. Doppler diversity is also verified to yield beneficial effects with speed at low-order modulations.

APPENDIX A

SNR OF FM-OFDM IN AWGN CHANNEL

Defining the input signal-to-noise ratio (neglecting the overhead from the cyclic prefix) as

$$SNR_{in} = \frac{A^2}{N_0 f_s}.$$ 

$SNR_{in}$ can be related to the bit energy-to-noise density ratio $E_b/N_0$ by

$$\frac{E_b}{N_0} = \frac{A^2}{N_0 f_s N_c K_{QAM}}. \quad \text{(25)}$$

This expression already takes into account the net amount of information contained in one symbol. The signal’s power spectral density $S_f[k]$ is equal to $m^2 |\Delta f|^2$ for the active subcarriers, and 0 otherwise. The total signal power $\mathbb{E} [|f[n]|^2]$
Given finitely small subcarrier spacing, the cosine function can be approximated by its argument, hence
\[ \Delta f[n] = \arctan \frac{w[n]}{\Delta f_s / 2} \] (26)
The noise in the instantaneous frequency is then
\[ \Delta f[n] = \frac{1}{2\pi} \arctan \frac{w[n]}{\Delta f_s / 2} - \arctan \frac{w[n-1]}{\Delta f_s / 2} \].

Given that \( w[n] \) and \( w[n] \) are wide-sense stationary random processes, so is \( \Delta f[n] \) and its autocorrelation \( R_{\Delta f}[p] \) only depends on the time difference \( p \). Direct calculation for \( p = 0 \) and \( p = 1 \) yields, denoting for simplicity \( w'[n] = \frac{w[n]}{\Delta f_s / 2} \)
\[ R_{\Delta f}[0] = \frac{1}{4\pi^2} \mathbb{E} \left[ \arctan^2 w'[n] \right] \]
\[ R_{\Delta f}[1] = \frac{1}{4\pi^2} \mathbb{E} \left[ \arctan w'[n] \arctan w'[n+1] \right] - \frac{1}{\pi^2} \mathbb{E} \left[ \arctan^2 w'[n+1] \right] - \frac{1}{\pi^2} \mathbb{E} \left[ \arctan^2 w'[n-1] \right] \]

The latter follows from the fact that \( \arctan \) is an odd function and \( \arctan \) yields values that are symmetrically distributed around 0. An identical result is obtained for \( p = -1 \).

Therefore,
\[ R_{\Delta f}[p] = \frac{1}{4\pi^2} \mathbb{E} \left[ \arctan w'[n] \right] \times \left[ \delta[p] - \frac{1}{\pi^2} \delta[p - 1] - \frac{1}{\pi^2} \delta[p + 1] \right] \] (27)

Given that the phase \( \varphi[n] \) is a circular signal (Section III-A), an \( N \)-point DFT can be performed on (27) to yield the power spectral density of the noise in the instantaneous frequency \( S_{\Delta f}[k] \).
\[ S_{\Delta f}[k] = \frac{1}{2\pi^2} \mathbb{E} \left[ \arctan^2 w'[n] \right] \left[ 1 - \cos \frac{2\pi k}{N} \right] \]

In the limit of an infinite number of subcarriers with infinitely small subcarrier spacing, the cosine function can be approximated by a Taylor expansion around the origin, then \( \cos f = 1 - f^2/2 \) and \( S_{\Delta f} \propto f^2 \), as in analog FM. The subcarrier SNR at the active subcarriers is thus
\[ SNR_M^{\Delta f} = \frac{S_{\Delta f}[k]}{S_{\Delta f}[k]} \cdot \frac{2\pi^2 m^2 N}{N\pi \mathbb{E} \left[ \arctan^2 w'[n] \right] \left[ 1 - \cos \frac{2\pi k}{N} \right]} \]

At high signal-to-noise conditions, \( SNR_M^{\Delta f} \) and the arctan function can be approximated by its argument, hence \( \mathbb{E} \left[ \arctan^2 w'[n] \right] \approx N \delta_{k,0} / (2\pi^2) \) and
\[ SNR_M^{\Delta f} = \frac{8\pi^2 m^2 N}{N\pi} SNR_{\Delta f} \]

APPENDIX B

SNR of CE-OFDM in AWGN Channel

According to [15], the baseband representation of a CE-OFDM waveform can be given by
\[ s[n] = A_e \exp j(\varphi[n] + \theta) \]
where \( \theta \) is an arbitrary phase offset and \( \varphi[n] \) is the instantaneous phase carrying the information. An analysis similar to that in FM-OFDM yields, for the subcarrier SNR,
\[ SNR_{CE}^{\text{OFDM}} = \frac{S_{\Delta f}[k]}{S_{\Delta f}[k]} \cdot \frac{4\pi^2 m^2 N}{N\pi \mathbb{E} \left[ \arctan^2 w'[n] \right] \left[ 1 - \cos \frac{2\pi k}{N} \right]} \]

At high signal-to-noise conditions, \( SNR_{CE}^{\text{OFDM}} \) is in line with equation (16) of [15] after noting that their study considers PAM-based subcarriers, with half the power of the QAM subcarriers in this work.

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REFERENCES


