2022-10

Working paper. Economics

ISSN 2340-5031

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Asymmetric Information with multiple risks: the case of the Chilean Private Health Insurance Market

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Abstract

Abstract: We extend Rothshild and Stiglitz (1976) model to two sources of risk to better proxy real-world health insurance markets. This extension produces an interesting theoretical possibility: Take individuals A and B, who are low risks in one dimension but A is riskier in the other dimension. Then, A may enjoy less coverage than B in the former dimension (coverage reversal). The existence of this reversal determines which individuals are more likely to suffer adverse selection. We adapt Chiappori and Salanié (2000) positive correlation test to account for this multi-dimensionality and apply it to individual-level claims data for the privately insured in Chile.

Keywords: *Health insurance, adverse selection, advantageous selection, insurance markets, positive correlation test, competitive multidimensional screening.*
JEL Classification: *I13, L13, D82*

1 Introduction

Recent work on insurance markets focuses on finding plausible explanations for why insured risk and coverage do not show a positive correlation in certain markets. The idea that higher risk types enjoy higher coverage (adverse selection, AS henceforth) originates in the seminal work of Rothshild and Stiglitz (1976) (RS henceforth), and was later made

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popular by Chiappori and Salanié (2000) with the Positive Correlation Test (PCT). The lack of universal support for such correlation has been attributed by some to the multi-dimensionality and nature of private information. Individuals would not only be informed about their own risk, leading to the typical AS result, but also about other characteristics, such as their own risk aversion or their cognitive ability. These characteristics could increase their demand for coverage in tandem with a reduction of their risk, a phenomenon referred to as propitious or advantageous selection (PS henceforth).[1] In the context of health insurance markets—historically characterized by AS (e.g. Cutler and Zeckhauser, 2000, Cohen and Siegelman, 2010, Bolhaar et al., 2012, Olivella and Vera-Hernández, 2013, Sapelli and Vial, 2003, Panthöfer, 2016)— a few papers show empirical evidence in favor of cognitive ability as the source of propitious selection rather than risk aversion (e.g. Fang et al., 2008, Einav and Finkelstein, 2011).[2]

We propose a different source of multi-dimensionality that lies in multiple sources of risk, namely, the risk of needing inpatient care and the risk of needing outpatient care.[3] For this purpose, we extend the competitive separating equilibrium of RS to allow for two sources of risk and two-dimensions of coverage contracts, reflecting the inpatient and outpatient dichotomy.

Our three main contributions are the following. First, we characterize the separating equilibrium in RS with multidimensional risk. Second, we show that to fully account for this multi-dimensionality, it is not enough to carry out two PCTs, one for the inpatient dimension and another for the outpatient dimension. Third, we illustrate this insight by carrying our PCTs to individual-level claims data for the privately insured in Chile.

The recognition that insured risk is multi-dimensional, as reflected in most health insurance contracts, is important in empirical studies because the two risks are usually

[1]Key references are: Hemenway (1990), De Meza and Webb (2001), Keane and Stavrunova (2016), Fang and Wu (2018).

[2]Dedonder and Hindricks (2009) theoretically show that, in equilibrium, a high degree of risk aversion does not necessarily imply a higer precautionary effort, which suggest that risk aversion is unlikely to be the source of propitious selection.

[3]Technically, notice that these dimensions are not only directly relevant to the insuree (like cognitive ability or risk preference are) but also to the insurer. In this sense one could say that while risk is a "common value", cognitive ability and risk tolerance are "private values". Our multi-dimensionality is more symmetric in the sense that both sources of risk are common value. As we will see, this symmetry makes the characterization of the separating equilibrium a bit simpler.

covered to different degrees of generosity as in, for example, Medicare parts A and B. One obvious reason for having such differences in coverage is the existence of moral hazard, which has been shown to affect outpatient services significantly more than inpatient services;[4] but there may be other reasons. For example, it is conceivable that clients' private information regarding one of the risks is relatively more precise, leading to a larger degree of AS in that dimension. In the opposite direction, insurers may be able to condition on observables that are more informative of the actual risks in one dimension than in the other, which would dampen AS in the former dimension.

We propose an additional issue. Namely, the need to screen individuals who are privately informed in these two dimensions cannot be achieved with two independent contracts (one for inpatient and one for outpatient coverage). Indeed, the contract that is aimed at attracting individuals that are low risks in both dimensions (say type LL) must be distorted in order to dissuade not one but two types of individuals from taking this contract. Namely, these two types are "HL", that is, high risks in the inpatient dimension and low risks in the outpatient dimension, and type "LH", with the opposite configuration of risks. This double role of the "distortion at the bottom" brings two interrelated insights.

First, although we show that RS's main result of AS also applies to the two-dimensional case, i.e., a higher risk in any dimension results in higher coverage in that dimension, under some conditions the following phenomenon could be observed: Individuals who are low risk in both dimensions (type LL) could actually enjoy more coverage in the inpatient dimension than individuals who are also low risks in the inpatient dimension but are high risk in the outpatient dimension (type LH). We refer to this phenomenon as "coverage reversal". Now, this might seem to contradict the positive correlation between risk and coverage, as an individual with average higher risk (type LH) is enjoying less coverage in one of the dimensions than an individual with lower average risk (type LL). However, this perception is incorrect. Notice that we are comparing individuals who have the same (low) risk in the inpatient dimension and therefore the PCT is silent about their coverage

[4]Some references are Manning et al. (1987); Chiappori et al. (1998); Sapelli and Vial (2003); Gardiol et al. (2005); Freeman et al. (2008), Olivella and Vera-Hernández (2013). Inpatient services, however, may also suffer from moral hazard as some papers show, e.g., Card et al. (2008), Anderson et al. (2012), and Finkelstein et al. (2012).

in that dimension.

Second, to test for AS, as mentioned above, it is not enough to carry two separate analyses of how coverage correlates with risk on each dimension, as done for example in Bardey and Buitrago (2017) and Olivella and Vera-Hernández (2013). To see this, let us focus (without loss of generality) on the inpatient dimension. By carrying a single test, we would be mixing individuals who, in the outpatient dimension, are low risks (say group "•L") and individuals who are high risks (say group "•H") in the same sample. Let us refer to this exercise as the "pool sample" test. We show that the correct approach is to conduct two separate tests —even within the inpatient dimension: one with only individuals who are high risk in the outpatient dimension i.e., the "•H sample", and another with only individuals who are low risk in the outpatient dimension, i.e., the "•L sample". In doing this, it is possible to find significant differences in the magnitude of AS across the three samples. Among the possible scenarios, is one where there is evidence of AS in only one of the subsamples, say the •L sample, and no evidence of AS in either the pool sample or the •H sample. Interestingly, whether AS is more substantial in the •L sample or in the •H sample crucially depends on the absence or presence (respectively) of coverage reversal in the inpatient dimension. Thus, a different source of multi-dimensionality, namely, the presence of two sources of risk, could help explain why the empirical literature finds mixed evidence of the presence of AS in some insurance markets.

Needless to say, by symmetry, the same results hold if one focuses on the outpatient dimension: one should not carry a pool sample test in the outpatient dimension, but rather two separate tests, one for the H• sample (only individuals who are high risks in the inpatient dimension are included) and one for the L• sample (only individuals who are low risk in the inpatient dimension are included). Depending on the absence or presence of coverage reversal in the outpatient dimension, again respectively, the L• sample or the H• sample will show a larger degree of AS than the pool sample.

There are very few theoretical works dealing with multiple sources of risk. Fluet and Pannequin (1997) study, as we do, a model with two sources of risk but they restrict their analysis to the two intermediate types (HL and LH in our notation).[5] Janssen and Karamychev (2008) also study two sources of risk but most of their results are derived for the case where there are no individuals that are low risk in both dimensions.[6] Crocker and Snow (2011) is yet another example with multiple sources of risk (or "perils") but there are only two types, a high and a low risk, who differ in the loss probability distribution across these multiple perils.[7] Hence our theoretical contribution is to analyze the full cartesian product of types across the two sources or risk.

The data, provided by the Chilean regulatory agency, contain individual-level information on the universe of the privately insured in Chile for 2007 and 2008. In those years, roughly 15% of the Chilean population was privately insured. We restrict the sample to the largest regional market, the metropolitan region of Santiago (hereafter Santiago), where private insurers, designated as "ISAPREs" (from the Spanish acronym of Instituciones de Salud Previsional), have around 63% of their clients. This restriction guarantees that all individuals in our sample have access to all ISAPREs and plans.

Besides relevant claim information e.g., services usage and copayments, the data contain information on individual characteristics that ISAPREs can legally use for underwriting, namely gender and age. In addition, there is information on municipality of residence and income, neither of which can be used to set premiums. It is also possible to identify the clients of each ISAPRE and, within ISAPRE, which plan each individual holds. Finally, the data contain the ISAPREs' reported plan-specific "average" inpatient and outpatient coverages, which are solicited by the regulator.

We categorize all individuals into the gender-age cells that ISAPREs can use to price discriminate. These are 5-year groups by gender, adding to 14 cells altogether. The theoretical model assumes the existence of four risk-types—namely HH, HL, LH and LL—, which leads to four different coverage profiles per cell. Accordingly, we divide the

[5]Moreover, their objective is very different from ours, namely, they show that efficiency is lost when the coverages for each of the two risks are offered by different firms rather than have the two coverages bundled in a single contract at all firms (as it is the case in our paper).

[6]Incidentally, these authors prove that in such a market the zero profit condition as well as the no distortion at the top must hold in a competitive equilibrium. We prove this result to the full set of types (see Proposition1).

[7]These authors show that by bundling coverage of different losses across these perils, insurers can reduce the distortions required to screen individuals.

risk and coverage spaces into four categories for each cell.

The main purpose of our empirical analysis is to test for AS. In order to do so using the PCT, we require measures of coverage and risk. As measures of coverage, we use the plan-specific average inpatient and outpatient coverages reported by the ISAPREs to the regulator. Concerning measures of risk, we use the individuals' average expenditures across the two years in each dimension as a measure of their risk in that dimension. We estimate bivariate probit models where the two dependent variables are a dummy for high or low coverage in a given dimension and a dummy for high or low risk in the same dimension. In these regressions we control for all combinations of gender and 5-year age groups. A negative correlation between the unobserved components of risk and coverage informs us of the presence of PS. A positive correlation, however, may be the result of AS, moral hazard or both. Because inpatient services are less prone to moral hazard than outpatient services, a positive correlation in the former dimension would more likely indicate the presence of AS.[8]

We start by performing the PCT in each dimension by pooling all individuals with the same risk in that dimension i.e., the "pool sample". This means that high risks in, say, the inpatient dimension are those of type HH as well as those of type HL, and all the remainder types are considered low risk. Continuing with the inpatient dimension, we then perform two additional PCTs: one involving individuals with H risk in the outpatient dimension (the "•H sample"); and another involving individuals with L risk in the outpatient dimension (the "•L sample"). We repeat the analogous and symmetric three tests for the outpatient dimension: "pool sample", "H• sample", and "L• sample".

In order to assign individuals to •H and •L samples, we need to know individual's risk types. Because we do not observe types directly, there are at least two options. Option (a) is to assume individuals behave according to the predictions of our theoretical model such that individuals of type HH buy high coverage in both dimensions, or plans hh; individuals of type HL buy high coverage only for inpatient services, or plans hl; and so forth. Then, we take advantage of the two-year data panel to infer types directly from

[8]See footnote 4 above for more on this topic.

individual's choice of plans in the first year. Option (b), which we explain below, does not require assuming that individuals behave according to our theory and will be used as a robustness check.

Correlation estimates for the outpatient dimension are always positive and significantly larger than those for the inpatient dimension. This is a result consistent with the more likely presence of moral hazard in the former dimension. More importantly, results also show that PCTs from pool samples differ, in a statistically significant way, from those obtained with at least one of the other two samples. In the inpatient dimension, for example, we obtain a result consistent with AS in the pool sample when in fact the two sub-samples show opposite results: a negative correlation indicative of PS in the •H sample and a positive correlation, consistent with AS, in the •L sample. As explained above, our theory suggests that this reflects the absence of coverage reversal in the inpatient dimension. The pool sample estimate also over(under)-estimates in a statistically significant way the correlations obtained in the outpatient dimension for the H• (L•)-types. Again according to our theory, this would reflect the fact that there is no coverage reversal in the outpatient dimension either.

We extend the initial set of controls in our regressions to include what the literature as referred to as *unused variables* i.e., variables that are not used for underwriting but that may or may not be observed by the ISAPRE. In our case, these include controls for income and municipality and are observed by the ISAPRE. Although, in general the confidence intervals of the results overlap with those obtained before, the initial negative correlation in the inpatient dimension in the •H sample becomes non-statistically different from zero, while the values of the correlations for the outpatient dimension decrease but remain positive and very statistically significant. Controlling for income proxies did not affect the main conclusion i.e., that there is evidence of AS in inpatient only among the •L sample and the positive correlations in outpatient are consistent with AS, moral hazard or both.

To gauge the robustness of these results, we then conduct all the previous PCTs by gender-age cell. The resulting correlations are now less precisely estimated but some finer conclusions can be drawn. For example, the AS in inpatient among the •L sample is mostly driven by women. In the case of the outpatient dimension, the estimated correlations among the L• types are statistically larger than estimates for the H• types for men and older women. There is evidence of coverage reversal in very few gender and age combinations but they are never statistically significant.

Finally, and as an additional robusteness check, we re-estimate the PCT by gender-age cell but inferring risk types not from individuals' choices of coverage in 2007 but from their expenditures in years 2007 and 2008.[9] This alternative classification is what we refer to as option (b). It implies that H-types (L-types) in a given dimension are now those individuals whose expenditures in that dimension are higher (lower) than the median of their gender-age cell. Albeit slightly different, the correlation estimates obtained with option (b) are highly correlated with those obtained with option (a) and, therefore, the main conclusions remain valid.

To sum-up, we extend the basic RS model to two sources of risk and two coverages to better proxy the reality of health insurance markets. This more complex model, produces interesting theoretical possibilities such as "coverage reversal". The theory also shows that results from the PCT may differ not only by dimension but also within a given dimension across individuals, depending on their risk in the other dimension. The existence or absence of coverage reversal determines which set of individuals is more likely to suffer AS. Regarding our empirical findings, we find that pooling individuals, just as others before us have done, leads to evidence of AS in the inpatient dimension. However, we show that this result is misleading because it hides the existence of PS among the •H sample i.e. among those with high risk in the outpatient dimension, especially if inpatient also suffers from some moral hazard. Results for the outpatient dimension show a positive correlation between risk and coverage for all sets of individuals, which is consistent with AS, moral hazard, or both. Lastly, we do not find strong evidence in favor of coverage reversal in our data.

[9] *A priori* we should use only 2007 expenditures to characterize types because they are not a function of coverage choices made in 2008. However, expenditures in a single year are a noisy measure of risk, particularly in inpatient, due to the low number of individuals with positive expenditures in a given year. Hence, we decided to use the average expenditure over the two years as a better proxy of risk.

The rest of the paper is structured as follows: Section 2 contains the theoretical model and results thereof; Section 3 describes the Chilean health system; Section 4 describes our dataset; Section 5 describes the empirical methodology and results; In Section 6 we present some conclusions and discuss some policy implications of our findings. Appendices A and B contains all proofs. Appendix C contains additional tables and figures.

2 The Model

2.1 The players

The players are a set of insurers and a set of individuals. There are 4 states of the world: Not ill, ill needing inpatient services only, ill needing outpatient service only, and ill needing both types of services. We represent these 4 states of the world as, respectively, $\Omega = \{\emptyset, i, o, b\}$. We assume for simplicity that the needs of inpatient and outpatient services are independent.[10] This means that it suffices to define two probabilities: the probability that the individual needs inpatient services, p_i and the probability that the individual needs outpatient services, p_o. The distribution of probabilities in the set Ω is given by $\{(1-p_i)(1-p_o), p_i(1-p_o), p_o(1-p_i), p_i p_o\}$. Also for simplicity, we assume that insurance companies offer coverage in each of the services, inpatient and outpatient, independently. This is indeed the case for the Chilean case. For instance, they do not condition the coverage of outpatient services on whether the patient also uses inpatient services.[11] Therefore, an insurance contract establishes a premium P, a fixed level of coverage for inpatient services c_i, and a fixed level of coverage for outpatient services c_o.

Individuals are characterized by two sets of variables. The first set contains the variables that are publicly observable and the law allows insurers to use for underwriting (in the case of Chile, gender and age). The second set contains the variables that are either not observable (like private information on health risks) or are observable but insurers are not allowed to condition either premia or coverage (in Chile, income and location).

[10]More on this assumption in footnote 13.

[11]Janssen and Karamychev (2008) show that this assumption is innocuous if only types HH, HL,and LH are assumed to exist.

We conduct our theoretical analysis for a fixed cell where cell is defined by the first set of observables, for example, for 37-year old women. We also assume that any heterogeneity across individuals except for differences in risk (that is, in p_o and p_i) is completely eliminated once one conditions on cell. Therefore, the loss an uninsured individual suffers when needing inpatient (outpatient) services ℓ_i (ℓ_o) and the initial wealth w are the same for all individuals in the same cell.[12]

The expected utility of an individual who accepts an insurance package (P, c_i, c_o) and has probabilities p_i and p_o is given by

$$\begin{gathered} Eu\left(P, c_i, c_o\right) = \left(1-p_i\right)\left(1-p_o\right) u\left(w-P\right) + p_i\left(1-p_o\right) u\left(w-P-\ell_i+c_i\right) + \\ p_o\left(1-p_i\right) u\left(w-P-\ell_o+c_o\right) + p_i p_o u\left(w-P-\ell_i-\ell_o+c_i+c_o\right). \end{gathered} \tag{1}$$

We assume that there are two possible levels in the probability of inpatient (respectively, outpatient) services, $p_i^L < p_i^H$ (respectively, $p_o^L < p_o^H$). Therefore there are four types: (p_i^H, p_o^H), (p_i^H, p_o^L), (p_i^L, p_o^H), and (p_i^L, p_o^L); which we denote by HH, HL, LH, and LL.[13] Let $T = \{HH, HL, LH, LL\}$ represent the set of all possible types. We use $IJ \in T$ to represent a generic type. Hence the first letter in type IJ refers to inpatient risk and the second to outpatient risk. We sometimes refer to the group of individuals with types HH and LH (HL and LL), whose risk in the outpatient dimension is H (L), as group "•H" ("•L"). Analogously, we refer to the group formed by types HL and HH (LH and LL) as group "H•" ("L•").

The expected utility of an agent of type $x \in T$ accepting the contract aimed at the

[12]There are three reasons why differences in income within a cell are not of special concern. First, differences in income across the whole sample are relatively small since only individuals in the upper two deciles in the income distribution opt out of the public system. Second, ISAPREs may circumvent the law and condition their offers on income, for instance through selective advertising. Since we have data on income we can also condition our empirical analysis on income. Finally, the evidence on the sign of the effects of income on both risk and the willingness to pay for coverage is far from clear (see Panthöfer, 2016, for example).

[13]Suppose that the events "needing inpatient services" and "needing outpatient services" were not independent. This would require establishing 3 distinct probabilities, and therefore $2^3 = 8$ types would exist even in the simplest case of dichotomous types: LLL, LLH, LHL,...,HHH. As far as we know, no theoretical work exists dealing with such a situation at the time of writing this paper.

agent of type $y \in T$ is given by

$$\begin{aligned} Eu\left(x,y\right) = \left(1-p_o^x\right)\left(1-p_i^x\right)u\left(w-P^y\right)+ \\ p_i^x\left(1-p_o^x\right)u\left(w-P^y-\ell_i+c_i^y\right)+p_o^x\left(1-p_i^x\right)u\left(w-P^y-\ell_o+c_o^y\right)+ \\ p_i^x p_o^x u\left(w-P^y-\ell_i-\ell_o+c_i^y+c_o^y\right). \end{aligned} \tag{2}$$

To simplify notation, we let $Eu\left(x,x\right)=[x]$ and $Eu(x,y)=[x,y]$.

A generic incentive compatibility constraint (ICC henceforth) requires that a type x is not willing to pick the contract aimed to type y. We express this by $[x] \geq [x,y]$. Whenever this expression is satisfied with equality, that is, if this incentive compatibility constraint is binding, we write $[x]=[x,y]$.

Notice that the equilibrium needs to specify a contract for each type. Since there are 3 components in each contract and we have 4 types, we have 12 endogenous variables, namely $\left\{P^{IJ},c_o^{IJ},c_i^{IJ}\right\}_{i,j=L,H}$. We sometimes denote this triplet as contract C^{IJ}, $I,J=H,L$. We concentrate in the fully separating equilibrium candidate. We will show that this candidate is fully determined by four zero profit conditions; three "no distortion at the top" conditions; and four (downward adjacent) incentive compatibility constraints.[14]

2.2 Constructing a fully separating equilibrium

In a fully separating equilibrium, all variables $\left\{P^{IJ},c_o^{IJ},c_i^{IJ}\right\}_{i,j=L,H}$ are potentially different. Hence we need 12 equations to determine the equilibrium. The next proposition drastically reduces the number of equations needed and constitutes the main characterization result in the paper. In words, if a type has high risk in some dimension, the individual of this type should enjoy full insurance in that dimension. In addition, all contracts, regardless of the type at which they are aimed, should yield zero profits, which implies that the premium exactly covers the type's expected coverage. Formally,

[14]As in Wambach (2000), which also addresses a multidimensional screening (although very different) model, we content ourselves in characterizing the necessary conditions for a separating equilibrium. Whether such equilibrium is robust to deviations is left for further research. Let us point out however, that, as shown by Jack (2006) and Olivella and Vera-Hernández (2007), introducing a slight horizontal differentiation does not alter the characterization of the separating contracts whereas deviations (i.e. by offering less distorted insurance contracts) cease to be profitable. In any case, under perfect competition, a sufficient condition for existence of a separating equilibrium is that the proportions of low risks in each dimensions be sufficiently small (RS).

Proposition 1: *At any separating equilibrium candidate, the following conditions must hold:*

(i) (No distortion at the top in each dimension) $c_i^{HH} = c_i^{HL} = \ell_i$*;* $c_o^{HH} = c_o^{LH} = \ell_o$

(ii) (Zero profits at each contract) $P^{HH} = p_i^H \ell_i + p_o^H \ell_o$*;* $P^{HL} = p_i^H \ell_i + p_o^L c_o^{HL}$*;* $P^{LH} = p_i^L c_o^{LH} + p_o^H \ell_o$*; and* $P^{LL} = p_i^L c_i^{LL} + p_o^L c_o^{LL}$*.*

Before we provide some intuition for these results, several remarks are due here. First, the proposition says nothing about the relative size of either c_o^{HL} and c_o^{LL} or c_i^{LH} and c_i^{LL}. Second, notice that $P^{HH} = p_i^H \ell_i + p_o^H \ell_o \geq \left\{ \begin{array}{c} p_i^H \ell_i + p_o^L c_o^{HL} = P^{HL} \\ p_i^L c_i^{LH} + p_o^H \ell_o = P^{LH} \end{array} \right\}$. However, since it can be the case that $c_o^{HL} < c_o^{LL}$ ($c_i^{LH} < c_i^{LL}$), the comparative size of P^{HL} and P^{LL} (P^{LH} and P^{LL}) is undetermined.

The intuition behind each of the statements in our first proposition is not simple. Let us start with the most complex one, namely, the fact that type HL obtains full coverage in the inpatient dimension. Assume the contrary, that is, $c_i^{HL} < \ell_i$. We show that a unilateral deviation exists where a firm offers an alternative contract aimed at type HL which, as compared to the existing contract for HL, (i) is preferred by type HL; (ii) is dis-preferred by type HH; and (iii) yields more profits. We also prove that these three properties also ensure that neither type LH nor type LL would switch to the new contract. In other words, the new contract preserves incentive compatibility and yields additional profits. Let us describe the main features of this new contract. First, and again as compared to the original contract aimed at type HL, the new contract is (1) slightly more expensive (this makes it relatively unattractive for all types but ensures profits are higher), (2) it offers a slightly larger coverage in inpatient services (this ensures that HL and HH would be willing to accept it); and (3) it offers a slightly smaller coverage in outpatient services (which pushes away type HH). Of course the three objectives (i)-(iii) listed above are at first glance in conflict, but thanks to the difference in the probabilities among types one is able to reconcile them.

To prove that the contract aimed at type HH shows no distortion at the top in inpatient is also by contradiction. Hence, assume that coverage is not full in inpatient services. Then an insurer can deviate by offering an alternative contract aimed at HH

with a higher premium and a higher coverage in the inpatient service. Notice also that any argument in reference to the inpatient dimension will also hold, by symmetry, for the outpatient dimension.

Finally, to show that the separating candidate yields zero profits at all contracts we use the usual deviation where the premium is decreased slightly and coverages are modified in order to preserve incentive compatibility.

Once we take the 8 equations given in Proposition 1 into account, we only need 4 additional equations to determine the 12 unknown. We proceed as follows. We posit that the only binding incentive compatibility constraints are (1) $[HH] = [HH, HL]$; (2) $[HH] = [HH, LH]$; (3) $[HL] = [HL, LL]$; and (4) $[LH] = [LH, LL]$. We prove that these 4 equations, together with the 8 equations in Proposition 1 yield a unique candidate for a separating equilibrium.[15]

The first thing we notice when studying the system of equations is that coverage c_o^{HL} is determined solely by the equation $[HH] = [HH, HL]$. Indeed this equation can be written as

$$[HH] \equiv u(w - P^{HH}) = (1 - p_o^H)u(w - P^{HL}) + p_o^H u(w - P^{HL} - \ell_o + c_o^{HL}) \equiv [HH, HL].$$

Using the zero profit conditions in Proposition 1, we get

$$\begin{gathered} u(w - p_i^H \ell_i - p_o^H \ell_o) = \\ (1 - p_o^H)u(w - p_i^H \ell_i - p_o^L c_o^{HL}) + p_o^H u(w - p_i^H \ell_i - p_o^L c_o^{HL} - \ell_o + c_o^{HL}), \end{gathered} \tag{3}$$

which has a single unknown, c_o^{HL}. Notice also that the equation is independent of p_i^L. This is summarized in the following Lemma:

Lemma 1a: *Coverage c_o^{HL} is determined solely by $[HH] = [HH, HL]$ and is independent of p_i^L.*

[15]We have not been able to analytically prove, for the general case, that the four constraints used to construct the separating equilibrium imply that all the other incentive compatibility constraints are also satisfied. However, we have been able to prove this for the case where types are the same in one dimension (Proposition 2.3 below) and we do check that all ICCs are satisfied in the simulations reported below.

We also prove that the equation has a unique solution c_o^{HL*} that provides some, but only partial, coverage except in the case where $p_o^L = p_o^H$, where $c_o^{HL*} = \ell_o$ (full coverage) is the unique solution. Formally,

Lemma 1b: *Coverage c_o^{HL*} is unique. Moreover, it satisfies $0 < c_o^{HL*} < \ell_o$ if $p_o^L < p_o^H$* and $c_o^{HL*} = \ell_o$ *if* $p_o^L = p_o^H$.

By symmetry, we have the next two lemmas:

Lemma 2a: *Coverage c_i^{LH} is determined solely by $[HH] = [HH, LH]$ and is independent of p_o^L.*

Lemma 2b: *Coverage c_i^{LH*} is unique. Moreover, it satisfies $0 < c_i^{LH*} < \ell_i$ if $p_i^L < p_i^H$* and $c_i^{LH*} = \ell_i$ *if* $p_i^L = p_i^H$.

For further reference, the equation $[HH] = [HH, LH]$ is, also by symmetry, given by

$$[HH] \equiv u(w - P^{HH}) = (1 - p_i^H)u(w - P^{LH}) + p_i^H u(w - P^{LH} - \ell_i + c_i^{LH}) \equiv [HH, LH]. \quad (4)$$

The only remaining unknowns to be determined are c_i^{LL}, c_o^{LL}. For this we use binding constraints $[HL] = [HL, LL]$ and $[LH] = [LH, LL]$, as well as the zero profit condition in Proposition 1. This leads to a system of two equations and two unknowns, as we show next. We first define $P^{HL*} = p_i^H \ell_i + p_o^L c_o^{HL*}$, $P^{LH*} = p_i^L c_i^{LH*} + p_o^H \ell_o$, and $P^{LL} = p_i^L c_i^{LL} + p_o^L c_o^{LL}$. Then, $[HL] = [HL, LL]$ implies:

$$\begin{gathered}[HL] \equiv (1 - p_o^L)u(w - P^{HL*}) + p_o^L u(w - P^{HL*} - \ell_o + c_o^{HL*}) = \\ (1 - p_o^L)(1 - p_i^H)u(w - P^{LL}) + p_i^H(1 - p_o^L)u(w - P^{LL} - \ell_i + c_i^{LL}) + \\ p_o^L(1 - p_i^H)u(w - P^{LL} - \ell_o + c_o^{LL}) + \\ p_i^H p_o^L u(w - P^{LL} - \ell_i - \ell_o + c_i^{LL} + c_o^{LL}) \equiv [HL, LL],\end{gathered} \quad (5)$$

Similarly, $[LH] = [LH, LL]$ implies

$$
\begin{aligned}
[LH] \equiv & (1-p_i^L)u(w-P^{LH*}) + p_i^L u(w-P^{LH*}-\ell_i+c_i^{LH*}) = \\
& (1-p_i^L)(1-p_o^H)u(w-P^{LL}) + p_i^L(1-p_o^H)u(w-P^{LL}-\ell_i+c_i^{LL}) + \\
& p_o^H(1-p_i^L)u(w-P^{LL}-\ell_o+c_o^{LL}) + \\
& p_i^L p_o^H u(w-P^{LL}-\ell_i-\ell_o+c_i^{LL}+c_o^{LL}) \equiv [LH, LL].
\end{aligned} \tag{6}
$$

Coverages $\left\{c_i^{LL}, c_o^{LL}\right\}$ are computed by solving the system of equations given by (5) and (6) after substituting $P^{LL} = p_i^L c_i^{LL} + p_o^L c_o^{LL}$ into (5) and (6).

To sum up, the equilibrium candidate can be computed in 6 steps. In Step 1, substitute the (full) coverages $c_i^{HH} = c_i^{HL} = \ell_i$ and $c_o^{HH} = c_o^{LH} = \ell_o$, as established in part (i) of Proposition 1, into the premia given in part (ii) of Proposition 1. In Step 2, substitute again $c_i^{HH} = c_i^{HL} = \ell_i$ and $c_o^{HH} = c_o^{LH} = \ell_o$, as well as the premia resulting from step 1, into the incentive compatibility constraint ensuring that type HH does not wish to mimic type LH. This gives an equation that only depends on c_i^{LH}. In Step 3, solve this equation to compute c_i^{LH}. In Step 4 and 5, proceed as in steps 2 and 3 but now in reference to the other intermediate type, HL.[16] It is technically interesting to notice that the two equations in Steps 2 and 4 are mutually independent. In Step 6, obtain c_o^{LL} and c_i^{LL} by solving the system of two binding incentive compatibility constraints: one ensures that the intermediate type HL does not wish to mimic type LL, the other ensures that the other intermediate type, LH, does not wish to mimic type LL either.

It is possible to sign some comparative statics that will be useful in deriving further results. First, dimension by dimension, an increase in the low type's risk brings about an increase in the corresponding intermediate type coverage. In other words, the inpatient coverage for type LH increases with p_i^L and the outpatient coverage for type HL increases with p_o^L. Mathematically,

Proposition 2. $\frac{\delta c_i^{LH}}{\delta p_i^L} > 0$ and $\frac{\delta c_o^{HL}}{\delta p_o^L} > 0$.

It is also useful to study the special case where there is full information in one of the dimensions. We address the case where this occurs in the inpatient dimension. Due to the symmetry of our model, this is without loss of generality.

[16] Namely, in Step 4, substitute $c_i^{HH} = c_i^{HL} = \ell_i$ and $c_o^{HH} = c_o^{LH} = \ell_o$, as well as the premia resulting from Step 1, into the incentive compatibility constraint ensuring that type HH does not wish to mimic type HL. This gives an equation that only depends on c_o^{HL}. In Step 5, solve this equation to compute c_o^{HL}.

2.3 A special case: Symmetric information in one of the dimensions

We are now assuming that $p_i^L = p_i^H = p_i$. Unsurprisingly, we obtain that all types receive full insurance in the inpatient dimension. Also unsurprisingly, types LL and HL obtain the same (partial but positive) coverage in the outpatient dimension, as these two types coincide in this dimension and there is actually no heterogeneity in the inpatient dimension. In other words, if $p_i^L = p_i^H$, we are back to the RS (uni-dimensional screening) model. Formally,

Proposition 3: *If $p_i^L = p_i^H = p_i$ but $p_o^L < p_o^H$, then $c_i^{IJ*} = \ell_i$ for all $IJ \in T$ and $0 < c_o^{LL*} = c_o^{HL*} = x < \ell_o$ is a solution to the system of all incentive compatibility constraints, where x is the unique solution to the equation*

$$u(w - p_i\ell_i - p_o\ell_o) = (1 - p_o^H)u(w - p_i\ell_i - p_o^L x) + p_o^H u(w - p_i\ell_i - p_o^L x - \ell_o + x).$$

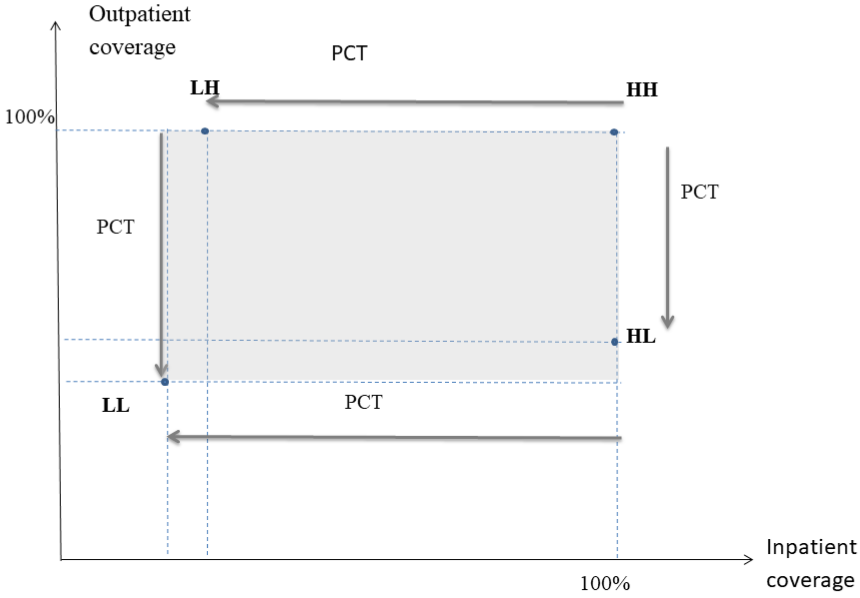
The natural question is what happens when types are very similar, but not the same. This leads to one important insight of our paper. We however present the more general results first.

2.4 General predictions: A graphical rendition

Our theoretical model predicts the configuration of coverages that is depicted in Figure 1 for the case where $c_i^{LL} < c_i^{LH}$ and $c_o^{LL} < c_o^{HL}$. The distance between coverages in that figure will be determined by the four probabilities $\{p_s^k\}_{s=i,o}^{k=L,H}$, as well as the losses $\{\ell_s\}_{s=i.o}$ and income w.

However, and as we will show in the next subsection by means of a numerical example, for certain values of the probabilities and losses, it is possible that the theoretical prediction shows an apparent reversal, whereby type LH enjoys a lower coverage in the inpatient dimension than type LL, that is, $c_i^{LL} > c_i^{LH}$. This situation is depicted in Figure

Figure 1: The theoretical prediction without reversal



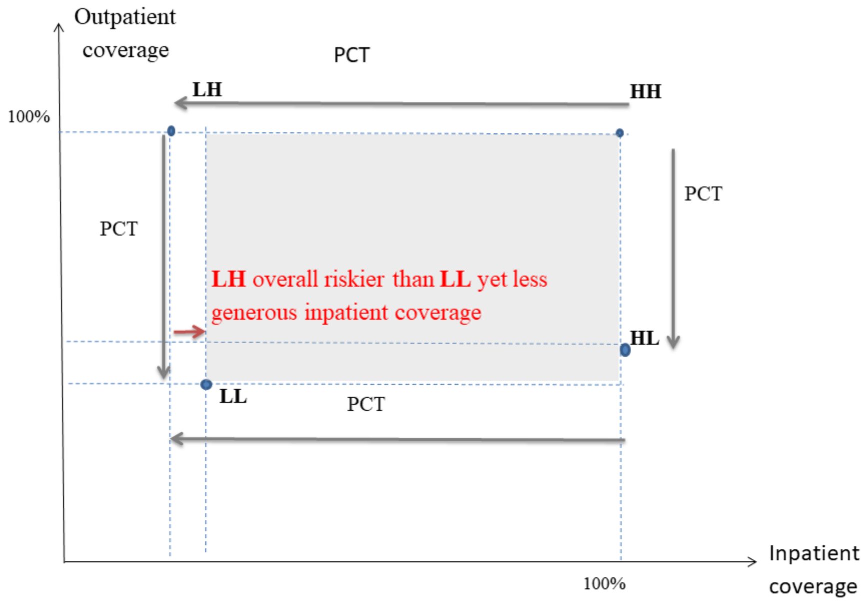
2 below. Notice that this reversal does not contradict the PCT, as this test is silent when one compares individuals with the same risk type in one dimension, as it is the case when we compare LH and LL: both are low risk in the inpatient dimension. However, if one were to build an uni-dimensional index of risk for these individuals, type LH would indeed be riskier than LL.

Symmetrically, one could also have that coverage reversal is observed in the outpatient dimension, meaning $c_o^{HL} < c_o^{LL}$. (We do not provide the corresponding figure, as it is just a symmetric analog of Figure 2.)

Finally, we have proven (see Proposition 2.3) that if information in one of the dimensions was symmetric, then our model would be the same as in RS. For instance, if observables used by insurers completely and uniquely determine the probability of needing outpatient (inpatient) services, then the theoretical figure would have outpatient (inpatient) always with full coverage while for the other dimension there would be a low and a full coverage level.

We now address the issue of coverage reversal, where we provide the conditions under which such phenomenon can occur, provide some intuition, and briefly discuss its empirical

Figure 2: The theoretical prediction with reversal in the inpatient dimension



consequences. Let us advance here that one obtains coverage reversal in one dimension when types are very close in the other dimension. Hence the title of our next subsection.

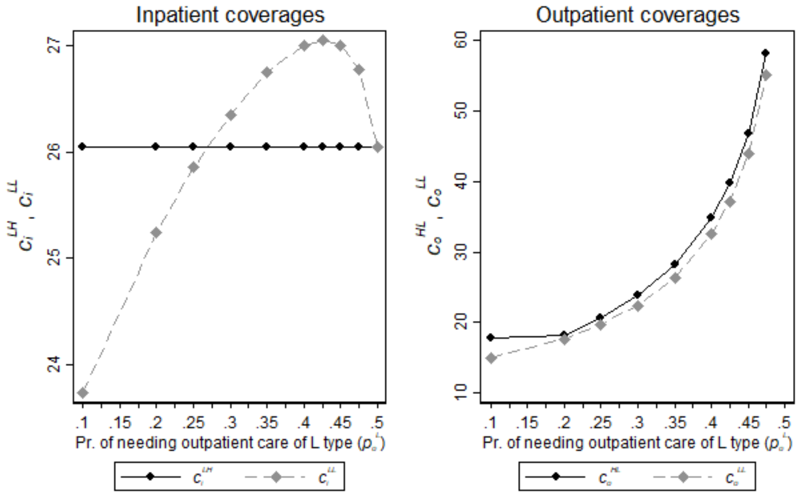
2.4.1 Almost symmetric information in one of the dimensions

Suppose types in one of the dimensions, say outpatient, are close, whereas types in the other dimension, inpatient, are far apart. Then the coverage for inpatient services for type LL, c_i^{LL}, will be more generous than the coverage for the same services for type LH, c_i^{LH}. In other words, although type LH is (seemingly) riskier than type LL, the former enjoys less coverage in the inpatient dimension. As mentioned above, the symmetric situation ($c_o^{LL} > c_o^{HL}$ when p_i^L is close to p_i^H) is also possible.

The numerical example showing this result is based on the following functional form and parameter values: $u(z) = \ln(z)$; $w = 0.8$; $\ell_i = 0.5$; $\ell_o = 0.3$; $p_i^H = 0.4$; $p_i^L = 0.1$; $p_o^H = 0.5$. In Figure 3, we plot the coverages in the inpatient dimension (left panel) and in the outpatient dimension (right panel) as p_o^L increases from 0.1 to 0.5 in the left panel, and from 0.1 to 0.475 in the right panel.[17] The difference in the domain is in order to make

[17]The values of p_o^L used are 0.100; 0.200; 0.250; 0.300; 0.350; 0.400; 0.425; 0.450; 0.475; 0.500.

Figure 3: Simulated equilibrium coverages



In this simulation, w=0.8, l_i=0.5, l_o=0.3, p_i^H=0.4, p_i^L=0.1, p_o^H=0.5. In the r.h.s graph, p_o^L<0.5 to make the difference between the two coverages more visible. Utility function is $U(z)=ln(z)$. Equilibrium values of coverages were obtained with ©Mathematica.

the difference between c_o^{HL} and c_o^{LL} in the right panel clearer since, when $p_o^L = p_o^H = 0.5$, both coverages will coincide at $c_o^{HL} = c_o^{LL} = \ell_o$ (full coverage), as shown in Proposition 3.

As can be seen in Figure 3, we have coverage reversal in the inpatient dimension when p_o^L is sufficiently close to p_o^H, namely, for $p_o^L \geq 0,275$, approximately. Notice also that Lemma 2a is confirmed in this simulation, as c_i^{HL} does not depend on p_o^L, and that another result in Proposition 3 is also confirmed, as c_i^{LH} coincides with c_i^{LL} when $p_o^L = p_o^H = 0.5$. Finally, since types in the inpatient dimension are relatively far apart ($p_i^H = 0.4$, $p_i^L = 0.1$), we do not observe coverage reversal in the outpatient dimension.

2.5 Intuition and empirical implications

In order to provide some intuition for these results, we discuss two cases separately. In Case 1, assume types in the outpatient dimension are close whereas types in the inpatient dimension are far apart. In Case 2, types are far apart in both dimensions.

Start with Case 1. Consider the ICC ensuring that type HH accepts contract C^{HH} rather than contract C^{HL}. Notice that these two types only differ in the outpatient dimension. When types are very similar in this dimension, a small downwards distortion in the outpatient coverage offered to HL suffices. Hence, c_o^{HL} is relatively generous. Let us turn now to the ICC ensuring that type HH accepts contract C^{HH} rather than C^{LH}.

Since types in the inpatient dimension are far apart, a large distortion in the inpatient dimension for type LH is needed. Hence coverage c_i^{LH} is meager. Consider now the ICC ensuring that type HL accepts contract C^{HL} rather than contract C^{LL}. These two types only differ in the inpatient dimension, and therefore it is the coverage in this dimension that matters most in preserving incentive compatibility. Since types are far apart in the inpatient dimension, the distortion in c_i^{LL} will have to be important. Recall, however, that c_o^{HL} was relatively generous, which means that the distortion in c_i^{LL} will not have to be that large. Hence c_i^{LL} could still be relatively generous. To sum up, c_i^{LL} could be larger than c_i^{LH} in Case 1. This explains the possibility of coverage reversal.

Let us now turn to Case 2, where types are far apart in both dimensions. We proceed as before. The ICC ensuring that HH accepts contract C^{HH} and not C^{HL} involves types that only differ in the outpatient dimension. However, since types in this dimension are now far apart, the downward distortion in c_o^{HL} will have to be large. Hence c_o^{HL} is meager. Consider now the ICC ensuring that type HL accepts C^{HL} and not C^{LL}. These types only differ in the inpatient dimension. Now there are two reasons why c_i^{LL} must be heavily distorted. One is that types in the inpatient dimension are also far apart. The other is that coverage c_o^{HL} was meager to start with. In sum, c_i^{LL} will be smaller than c_i^{LH}. Hence coverage reversal will not occur in Case 2.

The main empirical consequence of the varying sign of the difference between c_o^{HL} and c_o^{LL} is that under coverage reversal (negative difference), we have that $c_i^{HL} - c_i^{LL} = \ell_i - c_i^{LL} < \ell_i - c_i^{LH} = c_i^{HH} - c_i^{LH}$, while the difference between risks is the same in either side of the inequality, and equal to $p_i^H - p_i^L$. This implies that the correlation between coverage and risk will be larger and, hence, the PCT likely more statistically significantly different from zero when the individuals are high risks in the outpatient dimension than when individuals are low risk in that dimension. If the opposite of coverage reversal occurs, then the opposite can be said about the magnitude of the correlation and the relative significance of the PCT. The main implication of these observations is that, when testing for positive correlation in the inpatient dimension, mixing in the same sample individuals with different types in the outpatient dimension may lead to conclusions that are different from when the test is only carried with individuals who are high (or only low) risks in the outpatient dimension. Needless to say, the same discussion applies to testing for AS in the outpatient dimension: We further develop these ideas in Section 5.

3 The Chilean Health System

Health insurance is mandatory for all employees and retirees but they can choose between public and private insurance. In the period 2007-2008, around 70% of the Chilean population had public coverage and roughly 15% were privately covered.[18] Private insurers, designated as "ISAPREs" (from the Spanish acronym of *Instituciones de Salud Previsional*), can be of two types depending on whether their clients are drawn from the entire population ("open" ISAPRE) or come from a job-related group ("closed" ISAPRE).[19] During 2007-2008, there were 6 closed ISAPREs, and 8 open ISAPREs. In 2008, 96.7% of private policyholders were clients of open ISAPREs.[20] Our theory and empirical analysis focus on open ISAPREs only as they are the ones competing for clients in the market.

In 2007-2008, the pricing of private plans was subject to some mild regulation. ISAPREs would set a "reference premium" for each plan which they could modify annually and unilaterally. The premium paid by each policyholder depended on the reference premium and some risk-rating factors, which were only functions of the beneficiaries' age, gender, and insurance status (policyholder or dependent).[21] We restrict our analysis to policyholders without dependents, which means that, for a given plan, ISAPREs must charge the same premium to policyholders of the same age and gender. Since 2009, an increasing number of policyholders have sued their ISAPRE for unjustified annual increases

[18]Source: (http://www.fonasa.cl/wps/wcm/connect/internet/sa-general/informacion+corporativa/estadisticas+institucionales/estadisticas+institucionales).

[19]Most, if not all, closed ISAPREs were created by worker unions that offered medical insurance to their affiliates before the approval of the law of ISAPREs in March 1981 (*Decreto con Fuerza de Ley* N° 3). After the enactment of the law, these organizations were restructured as ISAPREs to continue functioning.

[20]Source: https://www.supersalud.gob.cl/documentacion/666/w3-article-5293.html.

[21]Some exceptions are family plans that have lower effective premiums for dependents (children and spouses) when the dependents' income is below the minimum legal wage. Other examples are plans that offer reduced coverage for specific events such as childbirth, and physician visits. For these events the coinsurance rate may reach 75%, but the plan is cheaper than a comparable plan without reduced coverage.

in their plan's premium (Candia, 2017). In most cases, judges ruled against the ISAPREs since they were not able to fully justify the rationale behind the intended changes (as established in the law DFL N$^{\text{o}}$1, passed by the Chilean Ministry of Health in 2005). Hence, since 2009, the premium of a given plan does not only depend on gender and age but, for those who renew a plan, potentially on a court ruling. For this reason, we use data up to the year 2008 to avoid having to model this particular feature of the Chilean private health insurance system.[22]

4 Data

The data was provided to us by the Chilean regulatory agency (Superintendencia de Salud del Gobierno de Chile). It constitutes a unique individual-level dataset compiled from administrative records from all private health insurers in Chile. The data contain all claims to ISAPREs during 2007 and 2008. Insurance claims made to ISAPREs are mostly filled automatically upon service usage, which virtually eliminates the problem of many private systems where the decision to file a claim is endogenous (Chiappori and Salanié, 2000; Cohen and Siegelman, 2010).

Crucial to our analysis, the dataset contains all the variables that ISAPREs are allowed to use for pricing, i.e., age, gender, and the existence of dependents for each insured individual. Other individual characteristics such as whether the individual is the main policyholder, the labor market status of the policyholder (classified as employed, self-employed, retired, or voluntary contributor), the individual's region of residence and, when the individual is employed, a proxy of income,[23] are also available in the dataset. There is also information about the chosen plan, such as the name of the ISAPRE, the type of plan (individual, family, or collective), whether the plan has a preferred provider,[24]

[22]Other aspects of the Chilean private health insurance system using the same dataset were studied by, for example, Atal (2019) and Duarte (2012).

[23]Income is inferred from the policyholder's "mandatory contribution" up to a cap. The cap per month is 179.73 USD in 2007 and 195.76 in 2008. The mandatory contribution refers to the 7% of wage retained by the policyholder's employer to pay for her health insurance premium. If the premium exceeds this amount, the policyholder will top it up. If, on the contrary, the premium is lower than 7% of the policyholder's wage, the ISAPRE will keep hold of the amount paid in excess to pay for copayments and other health-related expenditures. We will address this issue in Section 5.

[24]When a provider is stated in the contract as "preferred provider" it implies lower copayments when

and the premium paid to the ISAPRE. We only observe the plan that was chosen by each individual and lack information on the subset of plans, or menu, offered to them and from which the chosen plan was selected.

Hereafter, we restrict our sample to employees between 25 and 59 years old without dependents, who live in Santiago de Chile in 2008 and who hold a contract with an open ISAPRE in 2007 and 2008. We also drop the smallest of the open ISAPREs, which has only 0.8% of its clients in Santiago. We restrict the sample to employees because insurance is mandatory for them. We drop individuals with dependents because they may take the risk of their dependents into account instead of their own when choosing an insurance plan. We restrict the analysis to Santiago, where 63.5% and 63.3% of the beneficiaries lived in 2007 and 2008, respectively, and where all ISAPREs have a large share of customers. This makes Santiago a market where the assumptions of the theoretical model are more likely to hold since any potential client may buy a plan from any ISAPRE. Hence, just as in our model, all policyholders in the data have access to the same set of ISAPRES and therefore to the same set of plans, which may not occur in other smaller regional markets.[25] Finally, we restrict the sample to individuals who changed plans at most once within the year. The tiny percentage of policyholders who hold three or more consecutive plans in a given year are dropped from the sample. We are left with 361,044 observations, representing 180,522 policyholder observed in 2007 and 2008.

Measures of Coverage

As measures of a plan's inpatient and outpatient coverage generosity, we use its average coverage in these dimensions as reported by the ISAPRE to the regulator. These reported coverages reflect the plan's average coverage across all services in each dimension, irrespectively of their usage by the policyholder. Hence, when two individuals in

using that provider. When the preferred provider is an expensive clinic, however, the contract premium may be higher. The addition of a preferred provider to a contract is consistent with the theoretical model. Simply interpret a plan with a preferred provider as one with lower coverage and cheaper premium, aimed at attracting lower risks.

[25]Table A.1 in Appendix B shows the distribution by region and ISAPRE of policyholders who are employees, have no dependents, are aged 25-59 and who changed plans at most once within the year in year 2008. A very similar distribution is obtained for 2007. The unequal presence of ISAPREs in regional markets may affect the availability of plans in each regional market and the way consumers choose among available plans. Our restriction to the Santiago market (region 13) avoids possible confounding effects coming from this heterogeneity.

our sample have the same plan, they also face the same average coverage.[26] Among the different plans bought by at least one policyholder in the sample, there are 84 different combinations of inpatient and outpatient coverages.[27] The average plan is more generous in the inpatient dimension, with an average coverage of 89.9%, than in the outpatient dimension, with an average coverage of 77.4%.

Because ISAPREs can price discriminate by gender and age, we categorize all policyholders into gender-age cells.[28] Figures A.3 and A.4 show the distribution of all active plans across age groups for women and men. The distribution of coverages is skewed to the right, i.e., high coverages, for both genders and all age groups. The most popular combination of average coverages across all age groups, which attracts 50.9% of individuals in 2008, is 90% of inpatient care and 70% of outpatient care. From hereafter, we refer to the modal coverages 90-70 as "the most popular combination".

In equilibrium, our theoretical model predicts the existence of four coverage profiles offered: $hh = (\ell_i, \ell_o)$; $hl = (\ell_i, c_o^{HL})$, $lh = (c_i^{LH}, \ell_o)$, and $ll = (c_i^{LL}, c_o^{LL})$. Thus both the theoretical model and our empirical strategy in Section 5 demand discretizing the coverage space into four categories, i.e., $\{hh, hl, lh, ll\}$, for each gender-age cell. We construct binary variables in both dimensions using the most popular combination —90% of inpatient and 70% of outpatient care—as cutoff values. Hence, a plan is hh when its inpatient coverage is higher than 90% and its outpatient coverage is higher than 70%. Since around half of our sample holds a plan with the (90%, 70%) combination, the assignment of this combination to either high or low coverage could make our results too sensitive to the cutoffs used. We opted to drop from the sample all individuals who hold the most popular combination (90%,70%). Our final sample has 88,667 individuals observed in both 2007 and 2008. This final sample has higher percentage of men (61.8

[26]Individuals with the same plan will face different *realized* coverages since their usage and choice of providers will most likely differ, in which case they may also face different caps on the coverage. Realized coverages are only observed for those health services actually used by individuals. Hence, our measure of coverage, which reflects potential coverage, is preferred.

[27]Plans differ in other elements such as preferred providers, copayment structure by provider, and service caps. We do not explicitly take these differences into account but to the extent they reflect differences in coverage, they should be embedded in our measure of coverage.

[28]ISAPREs use gender and age-related risk factors to price their plans. These age-related factors partition age into 5-year intervals. We use the same age partition.

versus 56.0%) and a slightly younger average policyholder (38.3 years old versus 38.7). The average inpatient coverage, however, remains 89.9% while the average outpatient coverage increases to 85.0%.

Table 1 shows how average coverage varies substantially by gender and not much by age. On average, men's inpatient and outpatient coverages are 10 and 6 percentage points higher than women's. The difference between genders is highest for the 25-29 group and it decreases monotonically with age. Gender differences in premiums are likely the main cause. For example, in 2008 women paid 22.5% higher premiums compared to men while enjoying lower coverages. The difference in average premiums across genders also decreases with age, from 38.9% at 25-29 to 2.3% at 55-59. The obvious reason for this pattern is the potential usage of expensive services by women of childbearing age. An additional reason for the apparent over-insurance, particularly amongst young men, is provided by the particular billing procedure of health insurance premiums (see footnote 23). Because young policyholders, especially young men, face relatively low premiums, they may prefer to buy more coverage than needed rather than see their money being hold up by the ISAPRE.

Table 1: Average coverage in 2008

		Age groups						
	All	25-29	30-34	35-39	40-44	45-49	50-54	55-59
Inpatient:								
All	89.9	90.6	88.5	89.5	90.7	91.1	90.9	89.6
Men	93.7	94.6	93.3	93.4	93.6	93.9	94.0	92.6
Women	83.7	79.1	78.6	82.4	86.4	87.9	88.5	87.5
Outpatient:								
All	85.0	87.2	85.0	84.4	84.6	84.5	84.3	83.3
Men	87.3	89.4	87.7	86.5	86.3	86.4	86.2	85.2
Women	81.2	80.9	79.5	80.6	81.9	82.4	82.7	81.9

Note: Numbers show average inpatient and outpatient 2008 coverage in each gender-age cell after dropping from the sample the most common plan with 90% inpatient coverage and 70% outpatient coverage. The coverages correspond to the last plan held in 2008.

Measures of Risk

We use inpatient and outpatient expenditures to characterize individual's risk in these two dimensions of care. Expenditures are the amount paid to the health provider upon a given service, equivalent to the sum of copayments, paid by the policyholder, and the amount covered by the ISAPRE.[29] Because the average over 2007 and 2008 is more informative about risk than a single year's value, we use the average expenditures over the two years in each dimension. One reason why averaging is more informative is because it dilutes expenditures related to isolated acute episodes. For example, a high inpatient expenditure level in 2007 may reflect the occurrence of an acute episode, say appendicitis, which is likely uncorrelated with inpatient risk in 2008. Another reason why averaging expenditures is preferred is because it makes it less likely to miss relevant expenditures. For example, both inpatient and outpatient expenditure data are characterized by a large number of zero expenditures, especially inpatient. To be precise, while only 7.1% and 7.7% of the policyholders have positive inpatient expenditures in 2007 and 2008, the number reaches 13.4% when we average both years' expenditures.

Overall, around 86% of policyholders are users, i.e., they filed at least one claim in either 2007 or 2008. Figure 4 shows the distribution excluding outliers of the average—over 2007 and 2008—outpatient (left panel) and positive inpatient (right panel) expenditures for each gender-age group.[30] The median outpatient expenditures are between 2.8 and 4.1 times higher for women than for men. It grows significantly with age, reaching a 60% increase between ages 25-29 and 55-59 for men, and 35% for women.[31] Differently from outpatient, the median inpatient expenditure is always zero for both genders and all age groups.[32] Conditional on being positive (see right panel of Figure 4), the median inpatient

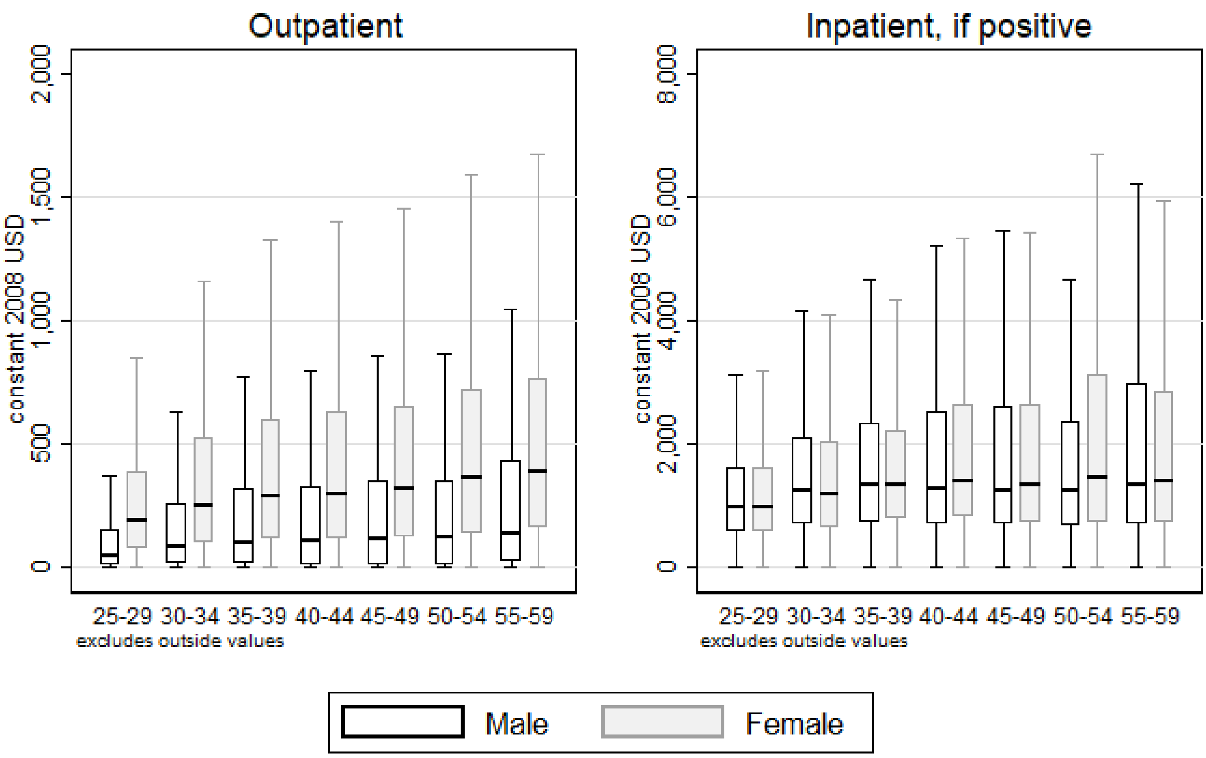
[29] Individual risk may alternatively be measured by usage such as hospitalizations and doctor visits. Hospitalizations are a rare event in general, with only 5.8% (6.4%) of the individuals hospitalized in 2007 (2008). Doctor visits are relatively frequent—with an average of 3.2 (3.3) annual visits in 2007 (2008). From the point of view of the ISAPREs, however, risk is better captured by expenditures than by usage because: i) both hospitalizations and outpatient episodes vary in terms of costs; ii) most likely ISAPREs face different costs for the same services. Moreover, the use of expenditures fits better into our theoretical model of Section 2. For all these reasons, we use expenditures rather than usage to categorize individuals risk types in our empirical application.

[30] We removed the outliers from Figure 4 to avoid the boxes to be too compressed, that means that around 8.6 and 7.9% of the observations are left out. Figure A.5 shows the distribution when outliers are included. When including outliers, the average inpatient and outpatient expenditures conditional on usage are relatively high, namely 4,200 and 435 USD in 2007 and 4,329 and 465 in 2008 (all measured in 2008 USD).

[31] If, instead, we compare the outpatient expenditures across genders using their mean, we get a similar pattern. The mean outpatient expenditures are between 1.5 and 2.2 times higher for women than for men. It grows significantly with age, reaching a 85% increase between ages 25-29 and 55-59 for men, and 58% for women.

[32] The mean of the inpatient expenditures is between 1 (ages 25-29 and 50-54) and 2 times higher for women than for men. Between ages 55-59 and 25-29, the mean inpatient expenditure grows 34% for men and 111% for women.

Figure 4: Distribution of Expenditures



Box-plots exclude outside values, which leaves out aproximately 8.6% and 7.9% of the inpatient and outpatient observations respectively. Left panel shows the distribution of the average 2007 and 2008 outpatient expenditures. The right panel shows the 2007 and 2008 average inpatient expenditures conditional on being positive because the 75 percentile is zero for all gender and age groups.

expenditure grows with age although it shows close to no difference between genders.

5 Testing for Adverse Selection

Chiappori and Salanié (2000) propose the following reduced-form model to test for asymmetric information. Consider a latent variable reflecting insurance coverage (c_i^*) of policyholder i and a latent variable (r_i^*) denoting her risk. The two variables are modeled as:

$$\begin{cases} c_i^* = X_{1i}\theta_c + \varepsilon_{ci} \\ r_i^* = X_{1i}\theta_r + \varepsilon_{ri} \end{cases} \tag{7}$$

where $\begin{pmatrix} \varepsilon_{ci} \\ \varepsilon_{ri} \end{pmatrix} \sim N(0, \Sigma)$ with correlation coefficient ρ. X_{1i} denotes characteristics of policyholder i which are observed by the insurer and can be used for pricing purposes, in our case, only gender and age. In the absence of moral hazard, a positive ρ is evidence of AS as unobservables that increase the probability of buying higher coverage also increase risk. This simple test is referred to as the PCT. The presence of moral hazard also leads to positive correlation between risk and coverage, hence, a positive ρ does not necessarily imply AS. A negative ρ, however, is always indicative of PS. de Meza and Webb (2017) criticize the PCT as a test of asymmetric information because the absence of correlation is not a property of symmetric information. Indeed, our theoretical model of Section 2 predicts a degenerate distribution equal to full coverage in the presence of symmetric information, which would preclude the estimation of model (7). Hence, while we acknowledge this criticism to the PCT, we take the variety of existing coverages in Figures A.3 and A.4 as consistent with the existence of asymmetric information in our data and, just as others have done, use the PCT to inform about the extent of AS/PS.

An application of the PCT to a two-dimensional setting, calls for separate estimation of the system (7) in each dimension. However, there are two possible approaches in doing so. Take first the inpatient dimension. The first approach is to pool all individuals of high inpatient risk, i.e., types HH and HL in the notation of our model, and contrast their risk and coverage with the pool of all individuals of low inpatient risk, i.e., types LH and LL.

We denote the results from this approach as the "pool sample". The other is to perform two independent PCTs even within the inpatient dimension: one test involving individuals with types HH and LH (i.e., compare high and low inpatient risk for individuals with high outpatient risk), consistently with the model the "•H sample", and another test involving individuals with types HL and LL (i.e., high and low inpatient risk and low outpatient risk), again consistently with the model the "•L sample". We have shown theoretically and will show empirically that the second approach is preferable, since the extent of AS may differ across samples.

The direction of differences across samples depend on whether the phenomenon of coverage reversal is present or not. Under coverage reversal, we have $c_i^{LH} < c_i^{LL}$, which in turn implies that $c_i^{HL} - c_i^{LL} = \ell_i - c_i^{LL} < \ell_i - c_i^{LH} = c_i^{HH} - c_i^{LH}$. In this case (since $0 < p_i^{HL} - p_i^{LL} = p_i^H - p_i^L = p_i^{HH} - p_i^{LH}$), the estimates of the correlation coefficient ρ should be larger when the individual has a high risk in the outpatient dimension than when he has a low risk in that dimension. If the opposite of reversal occurs, then the estimated ρ in the inpatient dimension is larger when the individual is a low risk in the outpatient dimension than when he is a high risk in that dimension. Needless to say, the same discussion applies to testing for AS in the outpatient dimension: one should perform separate tests for, on the one hand, individuals with types HH and HL ("H• sample") and, on the other hand, individuals with types LH and LL ("L• sample").

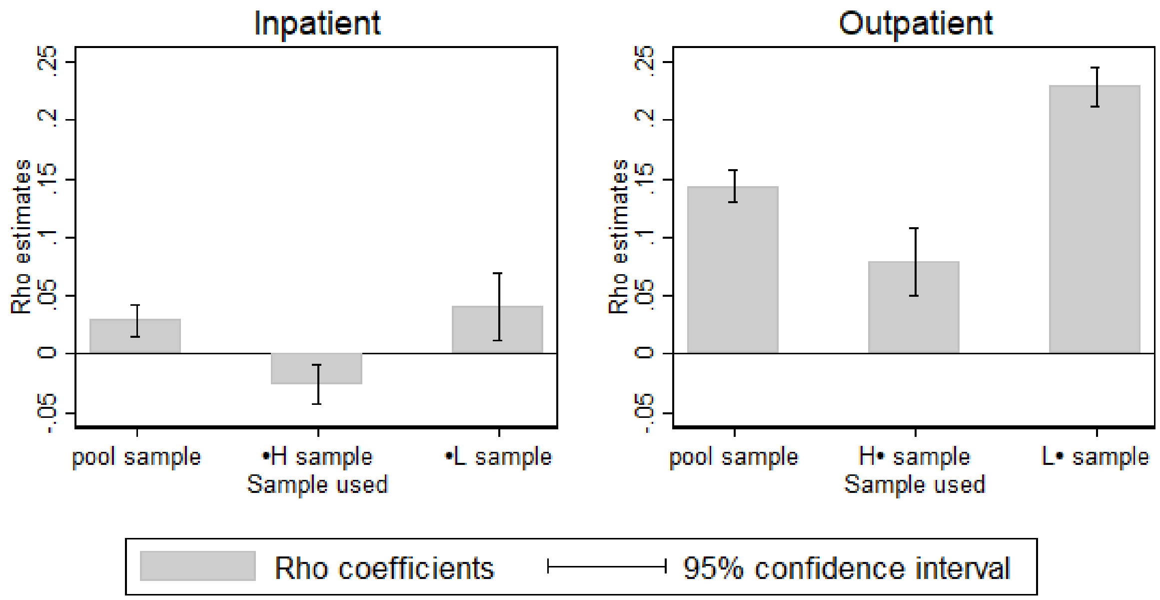
To perform PCTs keeping the risk level of individuals fixed in one dimension, requires assigning individuals to types. Because types are not directly observed, we start by assuming individuals behave according to the predictions of our theoretical model, such that individuals of type HH buy high coverage in both dimensions, or plans hh, individuals of type HL buy high coverage for inpatient services only, or plans hl, and so forth. We can then infer types directly from individual's insurance coverage in the previous year. In this case, restricting the sample to, for example, types HH and HL is equivalent to restricting the sample to individuals who hold plans hh and hl in 2007.

We estimate model (7) for each dimension controlling for all combinations of gender-age group as variables X_1. Figure 5 shows the values of the estimates of ρ by dimension across three samples. The "pool sample" represents the dimension-specific estimate of ρ obtained when pooling all individuals. The label "$\bullet$H sample" refers to the ρ estimate for the inpatient dimension when the estimating sample is restricted to individuals of type H in the outpatient dimension i.e., types HH and LH, and "$\bullet$L sample" when the sample is restricted to types L in the outpatient dimension i.e., types HL and LL. Similarly, for the outpatient dimension, "H$\bullet$ sample" and "L$\bullet$ sample" show results when restricting the estimating samples to individuals of H- and L-types in the inpatient dimension.

The results in Figure 5 warrant at least the following comments: (i) correlation estimates differ statistically significantly between the two sub-samples in both dimensions. More specifically, the correlation estimate is always statistically larger amongst those with low risk in the opposite dimension i.e., it is larger among the $\bullet$L sample than among the $\bullet$H sample and it is larger among the L$\bullet$ sample than among the H$\bullet$ sample. (ii) The pool estimate differs statistically significantly from the sub-samples, with the exception of the $\bullet$L sample. Based on the former estimate alone, for example, we would ascertain the existence of AS in the inpatient dimension when this can only be established on the $\bullet$L sample. The pool sample estimate would also over(under)-estimate the level of asymmetric information (possibly a combination of AS and moral hazard) in the outpatient dimension for the H(L)$\bullet$ sample. (iii) Estimates for the outpatient dimension are larger than the ones for the inpatient dimension and always positive, which is consistent with the more likely presence of moral hazard in the former dimension. (iv) There is no evidence of coverage reversal, a theoretically possible phenomenon, which would lead to larger estimates among $\bullet$H sample than among $\bullet$L sample and, similarly, for the outpatient dimension, larger estimates among H$\bullet$ sample than among L$\bullet$ sample. (v) There is evidence of PS in inpatient among the $\bullet$H sample.

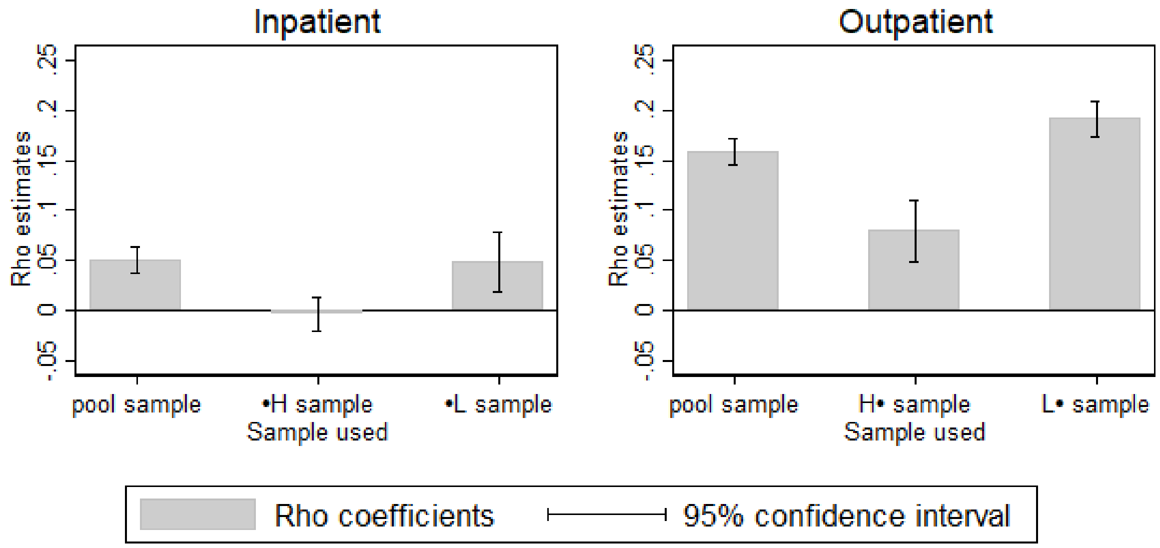
We extend the specification in (7) to include a set of unused variables in underwriting, X_2, in both equations as potential sources of AS or PS (Finkelstein and Poterba, 2014). We include controls for our 2008 income proxy, a dummy for missing income, a dummy for income higher than 33,558 2008 USD (approximately the level of income above which the mandatory contribution annual cap value in 2008 is binding), and municipality dummies

Figure 5: Positive Correlation Tests



ρ estimates obtained from bivariate probit models when controlling for gender and age-group combinations. for different samples. Robust s.e.. Coverage dependent variable dummies are constructed from inpatient (outpatient) coverage of the last plan held in 2008. High inpatient coverage if >90%. High outpatient coverage if >70%. Risk dependent variable takes value 1 if 2007 and 2008 average inpatient (outpatient) expenditures are larger than the median of their gender-age group. Types HH, HL, LH and LL are inferred from the 2007 coverage choices. Plan (90,70) dropped from sample. «pool sample» means estimates using all individuals. «•H sample» means estimation is restricted to H types in the outpatient dimension. «•L sample» means estimation is restricted to L types in the outpatient dimension. «H• sample» and «L• sample» means estimation is restricted to H- and L-types, respectively, in the inpatient dimension.

Figure 6: Positive Correlation Tests controlling for unused variables



ρ estimates obtained from bivariate probit models when controlling for gender and age-group combinations. In addition we controll for income and dummies for: missing income, income higher than 33558 (approximately the income above which the cap on the mandatory contribution in 2008 is binding) and municipality. Robust s.e.. Coverage dependent variable dummies are constructed from inpatient (outpatient) coverage of the last plan held in 2008. High inpatient coverage if coverage>90%. High outpatient coverage if coverage >70%. Risk dependent variable takes value 1 if the 2007 and 2008 average inpatient (outpatient) expenditures is larger than the gender-age group median. Types HH, HL, LH and LL are inferred from 2007 coverage choices. Plan (90,70) dropped from sample. «pool sample» means estimates using all individuals. «•H sample» means estimation is restricted to H types in the outpatient dimension. «•L sample» means estimation is restricted to L types in the outpatient dimension. «H• sample» and «L• sample» means estimation is restricted to H- and L-types, respectively, in the inpatient dimension.

(which in Chile and, in particular in Santiago, also act as income proxies). Except for the L• sample, all confidence intervals of the new estimated correlations overlap with those presented in Figure 5. Therefore, there is no strong evidence that income is acting as a source of PS or AS. We do, however, find that when controlling for all these income proxies the ρ estimate is no longer negative in inpatient among the •H sample (see Figure 6). Still, were moral hazard to exist at all in inpatient services, a zero correlation as the one found for the •H sample in Figure 6 would be interpreted as evidence of PS.

Finally, we show separate estimates of ρ across gender and age by dimension without and with controls for unused variables. Although standard errors are understandably much larger, there are still significant differences in ρ across sub-samples for a given cell and across gender-age cells. Figure A.6 shows that the negative correlation observed in the aggregate in Figure 5 among the •H sample is exclusively driven by young men and that, again, it disappears once proxies of income are included as controls (see Figure A.7).[33]

[33] In fact, the negative correlation among the •H sample also disappears and becomes zero when we

The disaggregated estimates also show that the evidence of AS in inpatient among the •L sample from Figure 6 is mostly driven by women. The estimates for the outpatient dimension, reveal that the ρ's among the L• sample are statistically larger than estimates for the H• sample among men and older women. Noteworthy, there are a few cases of coverage reversal e.g., in outpatient for women aged 35-39, and in inpatient for women aged 40-44 but none is statistically significant.

So far, we carried out PCTs by dimension on different samples by assigning individuals to risk types according to the predictions of our theoretical model. We denote this classification as option (a). As a robustness check, we now carry out PCTs but infer individual's risk types from their inpatient and outpatient expenditures. We denoted this as option (b). H-types in a given dimension are now those individuals whose expenditures in that dimension are larger than the median of their gender-age cell. The median inpatient expenditure is always zero for both genders and all age groups due to the large number of zeros. Hence, any individual with positive inpatient expenditures is a high-risk in the inpatient dimension. In contrast, in the outpatient dimension, the median expenditure is always positive, substantially higher for women than for men, and it grows with age.

Both option (a) and (b) have advantages and drawbacks. Option (a) has the advantage of relying on the individual's private information to inform us about types through revealed preference. The main drawback arises from deviations from the theoretical model. For example, the *de facto* subsidization of young policyholders' insurance premiums, which likely leads to over-insurance. By using individual expenditures to infer types, option (b) circumvents this potential mismatch between coverage and types. Over-insured young individuals with, say, *hh* plans, would no longer be classified as types HH when their expenditures are lower than the median of their group. A priori, only 2007 expenditures should be used to characterize types because they are not a function of coverage choices made in 2008 (the dependent variable c_i^*). However, as we explain above, the average expenditure over the two years (in 2008 USD) is a better proxy of risk and, therefore, we

repeat the exercise in Figure 5 only for ages of 35 and above. It is possible that the billing procedure of health insurance premiums (see footnote 23), which leads to a *de facto* subsidization for relatively wealthy young men, by incentivizing over-insurance is causing the negative correlation.

use it to classify types.

Figure A.8 in the Appendix shows the estimated ρ under option (b). The estimates corresponding to the pool sample are the same as those in Figure A.6 by definition because they use the pool of all individuals. As expected, the previous evidence on PS for young individuals disappears once we use expenditures to classify risk types. It is now more common to find evidence of AS in the inpatient dimension, especially for women but the values of ρ are slightly lower for the outpatient dimension in the case of men.[34] In any case, the ρ estimates obtained under options (a) and (b) are relatively highly correlated, as Table A.2 shows. As individuals grow older, the method used to infer types matters less as types inferred from 2007 plans' choice and from expenditures become more aligned. The alignment with age is caused by: (i) lower incentives to over-insure as premiums become more expensive and, consequently, the mandatory contribution of 7% of wage is more likely to bind; (ii) Less abundant number of zero expenditures as individuals age making expenditures more informative about types.

6 Conclusion

Our extension of the basic RS model to two sources of risk and the corresponding two dimensions of coverage provides several insights. One of the most interesting is the possibility of coverage reversal. Coverage reversal in a dimension is more likely the closer risk types are in the other dimension. If coverage reversal is present in one dimension, say inpatient, the PCT for inpatient services will yield more substantial AS in a sample where only higher risks in the outpatient dimension are included than in a sample where all individuals are included. In contrast, in the absence of coverage reversal in inpatient, there will be more substantial AS in the sample where only the low risks in outpatient are included. In both cases, performing the PCT in the aforementioned sub-samples provides unbiased information on the degree of AS selection for different groups of individuals.

Empirically, we do not find evidence consistent with coverage reversal, but the data

[34] Because only a small percentage of the population has positive inpatient expenditures, the estimates for the H• sample are obtained with small sample sizes, which translates into the large standard deviations for these samples seen in Figure A.8.

is consistent with PS in the inpatient dimension when the sample only includes the high risks in the outpatient dimension whereas it is consistent with AS when the sample only includes the low risks in the outpatient dimension. This may be due, as pointed out by the literature, to other dimensions of unobservable heterogeneity among individuals, cognitive ability being one possibility. As for the outpatient dimension, we recognize that the significantly positive results of the PCT in both the pool sample and the two sub-samples may be due to a moral hazard bias. However, the fact that the sub-sample of high risks (low risks) in the inpatient dimension yields significantly lower (higher) AS estimates than the pool sample should not be affected by the moral hazard bias (unless some interactions between AS and moral hazard are present).

Our results are useful to review regulations aimed at correcting the distortions that insurers use to screen individuals. For instance, minimum coverage legislation (coupled with mandatory enrollment) is part of the Affordable Care Act in the US (McFadden et al. (2015)). In general, if such a policy is aimed at reducing coverage distortions in, say, the inpatient dimension, this policy should be aimed at the low risks in that dimension. However, our results suggest such interventions affect individuals very differently depending on their risk profiles in both dimensions. Hence, it is not enough to tailor such intervention to each dimension of coverage but should also distinguish between high and low risks in the other dimension. Let us for instance continue with the inpatient dimension. Within the low risks in this dimension, whether it is the individuals who are also low risks in the outpatient dimension or the ones who are high risks in the latter dimension that suffer more distortions in the *laissez faire*, depends on whether coverage reversal is absent or present, respectively. In our case, the data suggest that reversal is never present, which implies that individuals who are low risks in all dimensions are the ones who should be the main target of these interventions.

Acknowledgments

We thank the Superintendencia de Salud del gobierno de Chile for granting the access to the data, and for the support provided. We would like to thank comments and suggestions from Mathias Kiffman and Helena Perrone, who discussed our paper, and to Philippe De Donder, Roger Feldman, Sebastian Panthöfer, Mar Reguant and Martí Mestieri as well as audiences at iHEA world congress, The Barcelona GSE Summer Forum, Oligoworkshop, Jornadas de Economía Industrial, EARIE Annual Conference, Universidad Javeriana, Universidad Alberto Hurtado, Pontificia Universidad Católica (Chile), Universidad Adolfo Ibáñez, and UQAM. We acknowledge financial support from Ministerio de Ciencia, Innovación y Universidades grants PID2019-108576RB-I00 and CAM-EPUC3M11 (V PRICIT) (Machado) and the Ministerio de Economía y Competitividad and Feder (through the Severo Ochoa Programme for Centres of Excellence in R&D, CEX2019-000915-S, project PGC2018-094348-B-I00) and The ONCE Foundation (Olivella). Olivella is member of CODE and MOVE. Finally, Carolina Concha and Valentina Reig helped with the initial stages of the gathering of the dataset.

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A APPENDIX: Proof of Proposition 1

Part I: Profits derived from the contract $C^{LL} = \left(P^{LL}, c_i^{LL}, c_o^{LL}\right)$ aimed to attracting type LL individuals cannot be positive in equilibrium

Suppose by contradiction that C^{LL} delivers positive profits. Consider the following unilateral deviation by one of the insurers, say I:

- **Reduce** premium: $P^{LL\prime} = P^{LL} - \varepsilon$.
- If inpatient care is needed, **reduce** coverage from c_i^{LL} to $c_i^{LL} - \delta_i \varepsilon$.
- If outpatient care is needed, **reduce** coverage from c_o^{LL} to $c_o^{LL} - \delta_o \varepsilon$.

Let us call this deviation contract C_ε^{LL}. Of course, at $\varepsilon = 0$, $C_0^{LL} = C^{LL}$, the *status quo*. We are going to prove that there exist $\delta_i > 0$ and $\delta_o > 0$ such that, for some small enough $\varepsilon > 0$, LL's payoff increases (and therefore I monopolizes all individuals of this type thus deriving higher profits under incentive compatibility) without violating any of the incentive compatibility constraints.

Notice first that, for a fixed contract $C = (P, c_i, c_o)$ and a given state of the world, the final wealth of all types is the same. This is because individuals only differ in the probability distribution over the states of the world. Hence, let us define the following auxiliary functions:

i) $\beta(\varepsilon, C) = w - (P - \varepsilon + \ell_i + \ell_o - c_i - c_o + (\delta_i + \delta_o)\varepsilon)$, which is the final wealth in the state "both services are needed";

ii) $\iota(\varepsilon, C) = w - (P - \varepsilon + \ell_i - c_i + \delta_i \varepsilon)$, which is the final wealth in the state "only inpatient services are needed";

iii) $\omega(\varepsilon, C) = w - (P - \varepsilon + \ell_o - c_o + \delta_o \varepsilon)$, which is the final wealth in the state "only outpatient services are needed";

iv) $\theta(\varepsilon, C) = w - (P - \varepsilon)$, which is the final wealth in the state "healthy".

Let also

v) $b(C) = u'(\beta(0, C))$, $i(C) = u'(\iota(0, C))$, $o(C) = u'(\omega(0, C))$, and $\varnothing(C) = u'(\theta(0, C))$.

The following lemma will be useful later on.

Lemma I-1 *For any (status quo) contract C with full or less-than-full coverage, we have that $b(C) \geq i(C) \geq \varnothing(C)$ and $b(C) \geq o(C) \geq \varnothing(C)$*

Proof: Final wealth when both services are needed is at most as large as final wealth when only outpatient services are needed. Since u' is decreasing, we can write $b = u'(\beta(0, C)) \geq u'(\omega(0, C)) = o$. Similarly, final wealth when only outpatient services are needed is at most as large as final wealth when no services are needed. Hence $o(C) \geq \varnothing(C)$. Symmetrically, substitute "outpatient" by "inpatient" to obtain $b(C) \geq i(C) \geq \varnothing(C)$. QED

Returning to the main argument, we can express type LL's payoff when truthful as:

$$E(LL, LL|\varepsilon, \delta_i, \delta_o) = \begin{array}{c} p_i^L p_o^L u\left(\beta\left(\varepsilon, C^{LL}\right)\right) + p_i^L\left(1 - p_o^L\right) u\left(\iota\left(\varepsilon, C^{LL}\right)\right) + \\ \left(1 - p_i^L\right) p_o^L u\left(\omega\left(\varepsilon, C^{LL}\right)\right) + \left(1 - p_i^L\right)\left(1 - p_o^L\right) u\left(\theta\left(\varepsilon, C^{LL}\right)\right) \end{array} \tag{A.1}$$

Notice that this payoff coincides with the status quo if $\varepsilon = 0$. Therefore LL gains with the deviation at some $\varepsilon > 0$ if and only if $\mathcal{LL} \equiv \frac{\partial E(LL, LL|\varepsilon, \delta_i, \delta_o)}{\partial \varepsilon}|_{\varepsilon=0} > 0$, where:

$$\mathcal{LL} = \begin{array}{c} p_i^L p_o^L (1 - \delta_i - \delta_o) b\left(C^{LL}\right) + p_i^L\left(1 - p_o^L\right)(1 - \delta_i) i\left(C^{LL}\right) + \\ \left(1 - p_i^L\right) p_o^L (1 - \delta_o) o\left(C^{LL}\right) + \left(1 - p_i^L\right)\left(1 - p_o^L\right) \varnothing\left(C^{LL}\right) \end{array} \tag{A.2}$$

We can express type HL's payoff when accepting the new contract (that is, when mimicking type LL) as:

$$E(HL, LL|\varepsilon, \delta_i, \delta_o) = \begin{array}{c} p_i^H p_o^L u\left(\beta\left(\varepsilon, C^{LL}\right)\right) + p_i^H\left(1 - p_o^L\right) u\left(\iota\left(\varepsilon, C^{LL}\right)\right) + \\ \left(1 - p_i^H\right) p_o^L u\left(\omega\left(\varepsilon, C^{LL}\right)\right) + \left(1 - p_i^H\right)\left(1 - p_o^L\right) u\left(\theta\left(\varepsilon, C^{LL}\right)\right) \end{array} \tag{A.3}$$

If in the *status quo* type HL was not mimicking type LL (i.e. not buying contract C^{LL}), he will neither mimic LL by choosing C_ε^{LL} if his payoff when accepting the latter is smaller than when accepting C^{LL}. Since the new contract coincides with the status quo if $\varepsilon = 0$, HL will not mimic LL at some $\varepsilon > 0$ if the following two conditions hold:

1) $\mathcal{HL} \equiv \frac{\partial E(HL, LL|\varepsilon, \delta_i, \delta_o)}{\partial \varepsilon}|_{\varepsilon=0} = 0$, where

$$\begin{array}{rl}\mathcal{HL} = & p_i^H p_o^L b\left(C^{LL}\right)\left(1-\delta_i-\delta_o\right)+p_i^H\left(1-p_o^L\right) i\left(C^{LL}\right)\left(1-\delta_i\right)+ \\ & \left(1-p_i^H\right) p_o^L o\left(C^{LL}\right)\left(1-\delta_o\right)+\left(1-p_i^H\right)\left(1-p_o^L\right) \varnothing\left(C^{LL}\right)\end{array} \tag{A.4}$$

and

2) $\frac{\partial^2}{\partial\varepsilon^2} E\left(HL, LL|\varepsilon, \delta_i, \delta_o\right)<0$ ($E\left(HL, LL|\varepsilon, \delta_i, \delta_o\right)$ is strictly concave in ε).

Symmetrically, we can express type LH's payoff when accepting the new contract as

$$\begin{array}{rr}E\left(LH, LL|\varepsilon, \delta_i, \delta_o\right)= & p_i^L p_o^H u\left(\beta\left(\varepsilon, C^{LL}\right)\right)+p_i^L\left(1-p_o^H\right) u\left(\iota\left(\varepsilon, C^{LL}\right)\right)+ \\ & \left(1-p_i^L\right) p_o^H u\left(\omega\left(\varepsilon, C^{LL}\right)\right)+\left(1-p_i^L\right)\left(1-p_o^H\right) u\left(\theta\left(\varepsilon, C^{LL}\right)\right)\end{array} \tag{A.5}$$

As before, LH will not mimic LL at some $\varepsilon>0$ if

3) $\mathcal{LH} \equiv \frac{\partial E(LH, LL|\varepsilon, \delta_i, \delta_o)}{\partial \varepsilon}|_{\varepsilon=0}=0$, where

$$\begin{array}{rl}\mathcal{LH} = & p_i^L p_o^H b\left(C^{LL}\right)\left(1-\delta_i-\delta_o\right)+p_i^L\left(1-p_o^H\right) i\left(C^{LL}\right)\left(1-\delta_i\right)+ \\ & \left(1-p_i^L\right) p_o^H o\left(C^{LL}\right)\left(1-\delta_o\right)+\left(1-p_i^L\right)\left(1-p_o^H\right) \varnothing\left(C^{LL}\right)\end{array} \tag{A.6}$$

and

4) $\frac{\partial^2 E(LH, LL|\varepsilon, \delta_i, \delta_o)}{\partial \varepsilon^2}<0$.

Let us prove that (2) and (4) hold.

$\frac{\partial^2}{\partial\varepsilon^2} E\left(HL, LL|\varepsilon, \delta_i, \delta_o\right)$ can be expressed as

$$\begin{array}{c}p_i^H p_o^L u''\left(\beta\left(\varepsilon, C^{LL}\right)\right)\left(-\left(-1+\delta_i+\delta_o\right)\right)^2+ \\ p_i^H\left(1-p_o^L\right) u''\left(\iota\left(\varepsilon, C^{LL}\right)\right)\left(-\left(-1+\delta_i\right)\right)^2+ \\ \left(1-p_i^H\right) p_o^L u''\left(\omega\left(\varepsilon, C^{LL}\right)\right)\left(-\left(-1+\delta_o\right)\right)^2+ \\ \left(1-p_i^H\right)\left(1-p_o^L\right) u''\left(\theta\left(\varepsilon, C^{LL}\right)\right)(-(-1))^2\end{array} \tag{A.7}$$

which is negative. Condition (4) holds as well by symmetry.

We now find δ_i and δ_o such that conditions (1) and (3) are satisfied. It will also be useful for later on to notice that (1) ad (2) (respectively, (3) and (4)) also imply that HL (respectively, LH) is not better off by mimicking LL if $\varepsilon<0$, that is, if one increases premium and coverage in both services.

Finally, type HH's payoff when accepting the new contract is

$$E\left(HH, LL|\varepsilon, \delta_i, \delta_o\right) = \begin{array}{r} p_i^H p_o^H u\left(\beta\left(\varepsilon, C^{LL}\right)\right) + p_i^H\left(1-p_o^H\right) u\left(\iota\left(\varepsilon, C^{LL}\right)\right) + \\ \left(1-p_i^H\right) p_o^H u\left(\omega\left(\varepsilon, C^{LL}\right)\right) + \left(1-p_i^H\right)\left(1-p_o^H\right) u\left(\theta\left(\varepsilon, C^{LL}\right)\right) \end{array} \tag{A.8}$$

Type HH will not mimic LL at the deviation if $\mathcal{HH} \equiv \frac{\partial E(HH,LL|\varepsilon,\delta_i,\delta_o)}{\partial\varepsilon}|_{\varepsilon=0} < 0$, where

$$\mathcal{HH} \equiv \begin{array}{l} p_i^H p_o^H \left(1-\delta_i-\delta_o\right) b\left(C^{LL}\right) + p_i^H\left(1-p_o^H\right)\left(1-\delta_i\right) i\left(C^{LL}\right) + \\ \left(1-p_i^H\right) p_o^H\left(1-\delta_o\right) o\left(C^{LL}\right) + \left(1-p_i^H\right)\left(1-p_o^H\right) \varnothing\left(C^{LL}\right). \end{array} \tag{A.9}$$

In sum, the required conditions are

$$\left\{\mathcal{LL} > 0, \mathcal{HL} = 0, \mathcal{LH} = 0, \mathcal{HH} < 0\right\}. \tag{A.10}$$

The next step is to solve the system of equations $\{\mathcal{HL} = 0, \mathcal{LH} = 0\}$ for (δ_i, δ_o). We do this using Mathematica© to get δ_i^* and δ_o^*.[35]

Also using this software we substitute these values for (δ_i, δ_o) into the expressions $\mathcal{LL} > 0$ and $\mathcal{HH} < 0$ and simplify the resulting expressions.

Expression $\mathcal{LL} > 0$ at (δ_i^*, δ_o^*) simplifies to (the dependency of b, i, o, and $\varnothing$ on C^{LL} is omitted in the reminder of Part I):

Figure A.1: Figure A1: Expression $\mathcal{LL} > 0$ at (δ_i^*, δ_o^*)

```
((piH - piL)
   (b i o piH poH + ety (i (o (-1 + piH) - b piH) (-1 + poH) - b o (-1 + piH) poH))
   (poH - poL)) / (b o (piH - piL) poH poL + i (b piH piL (poH - poL) +
      o (piL (-1 + poH) poL + piH (piL poL - poH (-1 + piL + poL))))) > 0
```

The numerator can be rewritten as:

$$\begin{array}{c} \left(p_i^H - p_i^L\right)\left(p_o^H - p_o^L\right) \times \\ \left\{p_i^L p_o^L bio + \left[i\left(o\left(1-p_i^L\right) + bp_i^L\right)\left(1-p_o^L\right) + bo\left(1-p_i^L\right) p_o^L\right] \varnothing\right\}, \end{array} \tag{A.11}$$

which is positive.

[35]The code is available upon request from the authors.

The denominator can be rewritten as

$$\underbrace{\left(p_i^H - p_i^L\right) p_o^H p_o^L b o}_{B} + i\underbrace{\left(b p_i^H p_i^L \left(p_o^H - p_o^L\right) + o\left\{p_i^L\left(p_o^H-1\right)p_o^L + p_i^H\left[p_i^L p_o^L - p_o^H\left(p_i^L+p_o^L-1\right)\right]\right\}\right)}_{A}$$

Since B and i are positive, we only need to show that A is positive. Recall that $b \geq o$ by Lemma I-1. Moreover, the factor multiplying b, i.e., $p_i^H p_i^L \left(p_o^H - p_o^L\right)$ is positive. Therefore, it suffices to show that

$$\begin{aligned}\hat{A} = \; & o p_i^H p_i^L \left(p_o^H - p_o^L\right) + o\left\{p_i^L\left(p_o^H-1\right)p_o^L + p_i^H\left[p_i^L p_o^L - p_o^H\left(p_i^L+p_o^L-1\right)\right]\right\} = \\ & o\underbrace{\left\{p_i^H p_i^L \left(p_o^H - p_o^L\right) + p_i^L\left(p_o^H-1\right)p_o^L + p_i^H\left[p_i^L p_o^L - p_o^H\left(p_i^L+p_o^L-1\right)\right]\right\}}_{M}\end{aligned} \tag{A.12}$$

is positive. Since o is positive, it again suffices to show that M is positive. The requirement $M > 0$ simplifies, after some algebra, to $p_i^H p_o^H > p_i^L p_o^L$, which is true. That is, $\mathcal{LL} > 0$ at (δ_i^*, δ_o^*).

Expression $\mathcal{HH} < 0$ at (δ_i^*, δ_o^*) is given by[36]

Figure A.2: Expression $\mathcal{HH} < 0$ at (δ_i^*, δ_o^*) is given by

```
((piH - piL)
    (b i o piH poH + ety (i (o (-1 + piH) - b piH) (-1 + poH) - b o (-1 + piH) poH))
    (poH - poL)) / (b o (piH - piL) poH poL + i (b piH piL (poH - poL) +
      o (piL (-1 + poH) poL + piH (piL poL - poH (-1 + piL + poL))))) > 0
```

It is easy to check that this expression is the same as the one given in Figure A1 above, which we already proved to hold. This concludes this step.

Part II Profits derived from the HH contract cannot be positive in equilibrium

Suppose by contradiction that $\mathcal{C}^{HH}$ delivers positive profits. Consider the following deviation for the contract aimed at attracting type-HH individuals:

- Let $\varepsilon < 0$ and **Increase** premium: $P^{HH\prime} = P^{HH} - \varepsilon$.

[36]The Mathematica© program changed the sign of the inequality when simplifying the expression for $\mathcal{HH} < 0$ evaluated at (δ_i^*, δ_o^*).

- If inpatient care is needed, **increase** coverage from c_i^{HH} to $c_i^{HH} - \delta_i \varepsilon$.
- if outpatient care is needed, **increase** coverage from c_o^{HH} to $c_o^{HH} - \delta_o \varepsilon$.

Using the auxiliary functions (i)-(iv) defined at the outset, here we use $\beta\left(\varepsilon, C^{HH}\right)$, $\iota\left(\varepsilon, C^{HH}\right)$, $\omega\left(\varepsilon, C^{HH}\right)$ and $\theta\left(\varepsilon, C^{HH}\right)$. That is, we compute final wealths at the contract aimed at attracting HH instead of that attracting LL. Let also, using (v), $b\left(C^{HH}\right) = u'\left(\beta\left(0, C^{HH}\right)\right)$, $i\left(C^{HH}\right) = u'\left(\iota\left(0, C^{HH}\right)\right)$, $o\left(C^{HH}\right) = u'\left(\omega\left(0, C^{HH}\right)\right)$, and $\varnothing\left(C^{HH}\right) = u'\left(\theta\left(0, C^{HH}\right)\right)$.

Then type LL's payoff when mimicking HH is

$$\begin{aligned} E\left(LL, HH|\varepsilon, \delta_i, \delta_o\right) = \quad & p_i^L p_o^L u\left(\beta\left(\varepsilon, C^{HH}\right)\right) + p_i^L\left(1 - p_o^L\right) u\left(\iota\left(\varepsilon, C^{HH}\right)\right) + \\ & \left(1 - p_i^L\right) p_o^L u\left(\omega\left(\varepsilon, C^{HH}\right)\right) + \left(1 - p_i^L\right)\left(1 - p_o^L\right) u\left(\theta\left(\varepsilon, C^{HH}\right)\right). \end{aligned} \tag{A.13}$$

We can say that LL does not want to mimic HH if, recalling that we are now using $\varepsilon < 0$, $\mathcal{LL}_{II} \equiv \frac{\partial E(LL,HH|\varepsilon,\delta_i,\delta_o)}{\partial \varepsilon}|_{\varepsilon=0} > 0$, where

$$\begin{aligned} \mathcal{LL}_{II} = \quad & p_i^L p_o^L \left(1 - \delta_i - \delta_o\right) b\left(C^{HH}\right) + p_i^L\left(1 - p_o^L\right)\left(1 - \delta_i\right) i\left(C^{HH}\right) + \\ & \left(1 - p_i^L\right) p_o^L \left(1 - \delta_o\right) o\left(C^{HH}\right) + \left(1 - p_i^L\right)\left(1 - p_o^L\right) \varnothing\left(C^{HH}\right). \end{aligned} \tag{A.14}$$

As for type HH's payoff when truthful, it is given by

$$\begin{aligned} E\left(HH, HH|\varepsilon, \delta_i, \delta_o\right) = \quad & p_i^H p_o^H u\left(\beta\left(\varepsilon, C^{HH}\right)\right) + p_i^H\left(1 - p_o^H\right) u\left(\iota\left(\varepsilon, C^{HH}\right)\right) + \\ & \left(1 - p_i^H\right) p_o^H u\left(\omega\left(\varepsilon, C^{HH}\right)\right) + \left(1 - p_i^H\right)\left(1 - p_o^H\right) u\left(\theta\left(\varepsilon, C^{HH}\right)\right) \end{aligned} \tag{A.15}$$

Therefore, HH gains with the deviation at $\varepsilon = 0$ if and only if, again recalling that $\varepsilon < 0$ in the deviation, $\mathcal{HH}_{II} \equiv \frac{\partial E(HH,HH|\varepsilon,\delta_i,\delta_o)}{\partial \varepsilon}|_{\varepsilon=0} < 0$, where

$$\begin{aligned} \mathcal{HH}_{II} = \quad & p_i^H p_o^H \left(1 - \delta_i - \delta_o\right) b\left(C^{HH}\right) + p_i^H\left(1 - p_o^H\right)\left(1 - \delta_i\right) i\left(C^{HH}\right) + \\ & \left(1 - p_i^H\right) p_o^H \left(1 - \delta_o\right) o\left(C^{HH}\right) + \left(1 - p_i^H\right)\left(1 - p_o^H\right) \varnothing\left(C^{HH}\right). \end{aligned} \tag{A.16}$$

Compute now type HL's payoff when mimicking HH:

$$\begin{aligned} E\left(HL, HH|\varepsilon, \delta_i, \delta_o\right) \quad & = p_i^H p_o^L u\left(\beta\left(\varepsilon, C^{HH}\right)\right) + p_i^H\left(1 - p_o^L\right) u\left(\iota\left(\varepsilon, C^{HH}\right)\right) + \\ & \left(1 - p_i^H\right) p_o^L u\left(\omega\left(\varepsilon, C^{HH}\right)\right) + \left(1 - p_i^H\right)\left(1 - p_o^L\right) u\left(\theta\left(\varepsilon, C^{HH}\right)\right). \end{aligned} \tag{A.17}$$

The first derivative w.r.t. ε is

$$\frac{\partial}{\partial\varepsilon}E\left(HL,HH|\varepsilon,\delta_i,\delta_o\right)= \begin{array}{c} p_i^H p_o^L u'\left(\beta\left(\varepsilon,C^{HH}\right)\right)\left(1-\delta_i-\delta_o\right)+ \\ p_i^H\left(1-p_o^L\right)u'\left(\iota\left(\varepsilon,C^{HH}\right)\right)\left(1-\delta_i\right)+ \\ \left(1-p_i^H\right)p_o^L u'\left(\omega\left(\varepsilon,C^{HH}\right)\right)\left(1-\delta_o\right)+ \\ \left(1-p_i^H\right)\left(1-p_o^L\right)u'\left(\theta\left(\varepsilon,C^{HH}\right)\right). \end{array} \tag{A.18}$$

The proof that the second derivative is negative is the same given in Part I: despite the fact that β, ι,ω,θ were evaluated at contract $\mathcal{C}^{LL}$ instead of $\mathcal{C}^{HH}$, this does not invalidate the proof as it is based on the concavity of u at any contract and the fact that the chain rule implies this second derivative is multiplied by a squared term. Therefore, for some $\varepsilon<0$ small enough, HL does not want to mimic HH if $\mathcal{HL}_{II}\equiv\frac{\partial(E(HL,HH|\varepsilon,\delta_i,\delta_o))}{\partial\varepsilon}|_{\varepsilon=0}=0$, where

$$\mathcal{HL}_{II}= \begin{array}{c} p_i^H p_o^L b\left(C^{HH}\right)\left(\left(1-\delta_i-\delta_o\right)\right)+p_i^H\left(1-p_o^L\right)i\left(C^{HH}\right)\left(1-\delta_i\right)+ \\ \left(1-p_i^H\right)p_o^L o\left(C^{HH}\right)\left(1-\delta_o\right)+\left(1-p_i^H\right)\left(1-p_o^L\right)\varnothing\left(C^{HH}\right) \end{array} \tag{A.19}$$

A similar argument allows us to state that, for some $\varepsilon<0$ small enough, LH does not want to mimic HH if $\mathcal{LH}_{II}\equiv\frac{\partial(E(LH,HH|\varepsilon,\delta_i,\delta_o))}{\partial\varepsilon}|_{\varepsilon=0}=0$, where

$$\mathcal{LH}_{II}= \begin{array}{c} p_i^L p_o^H\left(1-\delta_i-\delta_o\right)b\left(C^{HH}\right)+p_i^L\left(1-p_o^H\right)\left(1-\delta_i\right)i\left(C^{HH}\right)+ \\ \left(1-p_i^L\right)p_o^H\left(1-\delta_o\right)o\left(C^{HH}\right)+\left(1-p_i^L\right)\left(1-p_o^H\right)\varnothing\left(C^{HH}\right)=0. \end{array} \tag{A.20}$$

The proof concludes by noticing that the set of conditions

$$\left\{\mathcal{LL}_{II}>0,\mathcal{HL}_{II}=0,\mathcal{LH}_{II}=0,\mathcal{HH}_{II}<0\right\}.$$

coincides with the set conditions (A.10) , which were shown to hold in Part I (i.e., in the proof of zero profits at LL), except that in Part I the values b, i, o, and $\varnothing$ were evaluated at the contract C^{LL} aimed to attract type LL. This does not invalidate the argument since only the rankings $b\geq i\geq\varnothing$ and $b\geq o\geq\varnothing$ established in Lemma I-1 were used, and these rankings hold at any contract with full or less than full insurance. This concludes the proof.

Part III Profits derived from the HL contract cannot be positive in equilibrium

Suppose, by contradiction, that profits coming from an HL type are positive. Consider the following deviation:

Take $\varepsilon > 0$, $\delta_i > 0$, and $\delta_o > 0$ sufficiently small.

- Increase the premium so that it becomes $P^{HL\prime} = P^{HL} + \varepsilon$.
- Increase inpatient coverage by setting $c_i^{HL\prime} = c_i^{HL} + \delta_i \varepsilon$.
- Decrease outpatient coverage by setting $c_o^{HL\prime} = c_o^{HL} - \delta_o \varepsilon$.

Similarly to the procedure in parts I and II, define final wealths as follows (we need to redefine the auxiliary functions since the deviation is quite different). Hence

$$\begin{array}{rc} \beta(\varepsilon) = & w - \left(P^{HL} + \varepsilon + \ell_i + \ell_o - c_i^{HL} - c_o^{HL} - (\delta_i - \delta_o)\,\varepsilon\right) \\ \iota(\varepsilon) = & w - \left(P^{HL} + \varepsilon + \ell_i - c_i^{HL} - \delta_i \varepsilon\right) \\ \omega(\varepsilon) = & w - \left(P^{HL} + \varepsilon + \ell_o - c_o^{HL} + \delta_o \varepsilon\right) \\ \theta(\varepsilon) = & w - \left(P^{HL} + \varepsilon\right). \end{array} \tag{A.21}$$

and $b = u'(\beta(0))$, $i = u'(\iota(0))$, $o = u'(\omega(0))$, and $\varnothing = u'(\theta(0))$.

The proof proceeds in several steps.

Step 1. Compute type HL's payoff when she is truthful:

$$\begin{array}{rc} E(HL, HL|\varepsilon, \delta_i, \delta_o) = & p_i^H p_o^L u(\beta(\varepsilon)) + p_i^H \left(1 - p_o^L\right) u(\iota(\varepsilon)) + \\ & \left(1 - p_i^H\right) p_o^L u(\omega(\varepsilon)) + \left(1 - p_i^H\right)\left(1 - p_o^L\right) u(\theta(\varepsilon)). \end{array} \tag{A.22}$$

Type HL gains with the deviation at $\varepsilon = 0$ if and only if

$$\begin{array}{rc} \frac{\partial E(HL,HL|\varepsilon,\delta_i,\delta_o)}{\partial \varepsilon}\Big|_{\varepsilon=0} = & (\delta_i - \delta_o - 1)\, p_i^H p_o^L b + (\delta_i - 1)\, p_i^H \left(1 - p_o^L\right) i - \\ & \left(1 - p_i^H\right) p_o^L (1 + \delta_o)\, o - \left(1 - p_i^H\right)\left(1 - p_o^L\right) \varnothing > 0. \end{array} \tag{A.23}$$

This can be rewritten as

$$\begin{array}{c} \delta_i > \\ 1 + \dfrac{\delta_o p_o^L \left(p_i^H b + \left(1 - p_i^H\right) o\right) + \left(1 - p_i^H\right) p_o^L o + \left(1 - p_i^H\right)\left(1 - p_o^L\right) \varnothing}{p_i^H \left(p_o^L b + \left(1 - p_o^L\right) i\right)} \equiv \\ \delta_i^{\min,HL}, \end{array} \tag{A.24}$$

that is, if the increase in coverage is large enough.

Step 2. Compute type HH's payoff when mimicking HL

$$E\left(HH,HL|\varepsilon,\delta_i,\delta_o\right)= \begin{array}{c} p_i^H p_o^H u\left(\beta\left(\varepsilon\right)\right)+p_i^H\left(1-p_o^H\right)u\left(\iota\left(\varepsilon\right)\right)+ \\ \left(1-p_i^H\right)p_o^H u\left(\omega\left(\varepsilon\right)\right)+\left(1-p_i^H\right)\left(1-p_o^H\right)u\left(\theta\left(\varepsilon\right)\right). \end{array} \tag{A.25}$$

The deviation diminishes HH's incentives to mimic HL at $\varepsilon=0$ if and only if

$$\frac{\partial E(HH,HL|\varepsilon,\delta_i,\delta_o)}{\partial\varepsilon}|_{\varepsilon=0}= \begin{array}{c} \left(\delta_i-\delta_o-1\right)p_i^H p_o^H b+\left(\delta_i-1\right)p_i^H\left(1-p_o^H\right)i- \\ \left(1-p_i^H\right)p_o^H\left(1+\delta_o\right)o-\left(1-p_i^H\right)\left(1-p_o^H\right)\varnothing<0, \end{array} \tag{A.26}$$

which can be rewritten as

$$\begin{array}{c} \delta_i< \\ 1+\dfrac{\delta_o p_o^H\left(p_i^H b+\left(1-p_i^H\right)o\right)+\left(1-p_i^H\right)p_o^H o+\left(1-p_i^H\right)\left(1-p_o^H\right)\varnothing}{p_i^H\left(p_o^H b+\left(1-p_o^H\right)i\right)}\equiv \\ \delta_i^{MAX,HH} \end{array} \tag{A.27}$$

Step 3. Condition $\delta_i<\delta_i^{MAX,HH}$ is compatible with $\delta_i>\delta_i^{\min,HL}$ if and only if

$$\begin{array}{c} \dfrac{\delta_o p_o^H\left(p_i^H b+\left(1-p_i^H\right)o\right)+\left(1-p_i^H\right)p_o^H o+\left(1-p_i^H\right)\left(1-p_o^H\right)\varnothing}{p_o^H b+\left(1-p_o^H\right)i}> \\ \dfrac{\delta_o p_o^L\left(p_i^H b+\left(1-p_i^H\right)o\right)+\left(1-p_i^H\right)p_o^L o+\left(1-p_i^H\right)\left(1-p_o^L\right)\varnothing}{p_o^L b+\left(1-p_o^L\right)i}, \end{array}$$

which can be rewritten (after a large amount of algebra) as

$$\delta_o>\frac{\frac{b\varnothing}{io}-1}{\frac{p_i^H b}{\left(1-p_i^H\right)o}+1}\equiv\delta_o^{\min HL,HH}. \tag{A.28}$$

Step 4 We now compute type LH's payoff when mimicking HL**:**

$$E\left(LH,HL|\varepsilon,\delta_i,\delta_o\right)= \begin{array}{c} p_i^L p_o^H u\left(\beta\left(\varepsilon\right)\right)+p_i^L\left(1-p_o^H\right)u\left(\iota\left(\varepsilon\right)\right)+ \\ \left(1-p_i^L\right)p_o^H u\left(\omega\left(\varepsilon\right)\right)+\left(1-p_i^L\right)\left(1-p_o^H\right)u\left(\theta\left(\varepsilon\right)\right). \end{array} \tag{A.29}$$

Therefore the deviation diminishes LH's incentives to mimic HL at some sufficiently small ε if and only if

$$\begin{array}{rc} \frac{\partial E(LH,HL|\varepsilon,\delta_i,\delta_o)}{\partial \varepsilon}|_{\varepsilon=0} = & \begin{array}{c} (\delta_i - \delta_o - 1) p_i^L p_o^H b + (\delta_i - 1) p_i^L \left(1 - p_o^H\right) i - \\ \left(1 - p_i^L\right) p_o^H (1 + \delta_o) o - \left(1 - p_i^L\right)\left(1 - p_o^H\right) \varnothing < 0, \end{array} \end{array} \tag{A.30}$$

or

$$\begin{array}{c} \delta_i < \\ 1 + \frac{\delta_o p_o^H \left(p_i^L b + \left(1 - p_i^L\right) o\right) + \left(1 - p_i^L\right) p_o^H o + \left(1 - p_i^L\right)\left(1 - p_o^H\right) \varnothing}{p_i^L \left(p_o^H b + \left(1 - p_o^H\right) i\right)} \equiv \\ \delta_i^{MAX,LH}. \end{array} \tag{A.31}$$

We now prove that this condition is already implied by the condition that ensures that HH does not want to mimic HL after the deviation. Formally,

Lemma III-1 *Condition* $\delta_i < \delta_i^{MAX,HH}$ *implies condition* $\delta_i < \delta_i^{MAX,LH}$.

Proof: Notice that $\delta_i < \delta_i^{MAX,HH}$ implies $\delta_i < \delta_i^{MAX,LH}$ if and only if $\delta_i^{MAX,LH} > \delta_i^{MAX,HH}$, or, using the previous definitions and after some algebra,

$$\left(p_i^H - p_i^L\right) \delta_o p_o^H u_o' + \left(p_i^H - p_i^L\right) p_o^H u_o' + \left(p_i^H - p_i^L\right)\left(1 - p_o^H\right) u_\varnothing' > 0. \tag{A.32}$$

Using $p_i^H > p_i^L$, the condition is equivalent to

$$\delta_o p_o^H u_o' + p_o^H u_o' + \left(1 - p_o^H\right) u_\varnothing' > 0, \tag{A.33}$$

which holds since $\delta_o > 0$. QED

Step 5. Type LL's payoff when mimicking HL is

$$\begin{array}{rc} E(LL, HL|\varepsilon, \delta_i, \delta_o) = & \begin{array}{c} p_i^L p_o^L u(\beta(\varepsilon)) + p_i^L \left(1 - p_o^L\right) u(\iota(\varepsilon)) + \\ \left(1 - p_i^L\right) p_o^L u(\omega(\varepsilon)) + \left(1 - p_i^L\right)\left(1 - p_o^L\right) u(\theta(\varepsilon)). \end{array} \end{array} \tag{A.34}$$

Therefore the deviation diminishes LL's incentives to mimic HL for a sufficiently small ε

if and only if

$$\frac{\partial E(LL,HL|\varepsilon,\delta_i,\delta_o)}{\partial\varepsilon}|_{\varepsilon=0} = \begin{array}{c}(\delta_i-\delta_o-1)\,p_i^L p_o^L b + (\delta_i-1)\,p_i^L\left(1-p_o^L\right)i- \\ \left(1-p_i^L\right)p_o^L\left(1+\delta_o\right)o - \left(1-p_i^L\right)\left(1-p_o^L\right)\varnothing < 0.\end{array} \tag{A.35}$$

This is equivalent to, after some algebra,

$$\begin{array}{c}\delta_i < \\ 1+\dfrac{\delta_o p_o^L\left(p_i^L u_b' + \left(1-p_i^L\right)u_o'\right)+\left(1-p_i^L\right)p_o^L u_o' + \left(1-p_i^L\right)\left(1-p_o^L\right)u_\varnothing'}{p_i^L\left(p_o^L u_b' + \left(1-p_o^L\right)u_i'\right)} \equiv \\ \delta_i^{MAX,LL}\end{array} \tag{A.36}$$

Step 6. Condition $\delta_i < \delta_i^{MAX,LL}$ is compatible with $\delta_i > \delta_i^{\min,HL}$ if and only if, after some algebra,

$$\delta_o > -\frac{p_o^L u_o' + \left(1-p_o^L\right)u_\varnothing'}{p_o^L u_o'}, \tag{A.37}$$

which is satisfied since $\delta_o > 0$ and the RHS is negative.

To sum up, suppose $\delta_o > \min\left\{0, \frac{\frac{u_b' u_\varnothing'}{u_i' u_o'}-1}{\frac{p_i^H u_b'}{\left(1-p_i^H\right)u_o'}+1}\right\}$. Then $\delta_i^{MAX,HH} > \delta_i^{\min,HL}$ and for all $\delta_i \in \left(\delta_i^{\min,HL}, \delta_i^{MAX,HH}\right)$, we have that, with the deviation, type HL is better off whereas types LH, HH, and LL are worse off. By having $\varepsilon > 0$ small enough, profits coming from the contract aimed at type HL will suffer a marginal change and continue to be positive, whereas the ISAPRE will monopolize all HL-types. Since the incentive compatibility constraints for types HH, LH, and LL will be preserved, the profits accruing from the contracts aimed to these types remain the same. Hence we have found a successful deviation.

Part IV Profits derived from the LH contract cannot be positive in equilibrium

The proof is symmetric to Part III.

Part V Type HH's contract fully covers inpatient expenditures, that is, $c_i^{HH} = \ell_i$

Suppose, by contradiction, that $c_i^{HH} < \ell_i$. It will be useful for later to notice that this implies that $u_i' > u_\varnothing'$ and that $u_b' > u_o'$. Then consider the following deviation. Take $\varepsilon > 0$, $\delta_p > 0$, $\delta_i > 0$.

- Let the premium become $P^{HH\prime} = P^{HH} + \delta_p \varepsilon$.
- Let inpatient coverage become $c_i^{HH\prime} = c_i^{HH} + \delta_i \varepsilon$, that is, increase inpatient coverage.
- Let outpatient coverage stay the same $c_o^{HH\prime} = c_o^{HH}$.

The proof proceeds in several steps.

Step 1. The (deviant) ISAPRE's payoff at the contract aimed at HH when HH is truthful becomes

$$\begin{aligned} \pi^{HH}(\varepsilon, \delta_i, \delta_p) = \quad & P^{HH} + \delta_p \varepsilon - \{p_i^H p_o^H \left(c_i^{HH} + \delta_i \varepsilon + c_o^{HH}\right) + \\ & p_i^H \left(1 - p_o^H\right)\left(c_i^{HH} + \delta_i \varepsilon\right) + \left(1 - p_i^H\right) p_o^H c_o^{HH}\}. \end{aligned} \tag{A.38}$$

Hence

$$\begin{aligned} \tfrac{\partial \pi}{\partial \varepsilon} = \quad & \delta_p - p_i^H p_o^H \delta_i - p_i^H \left(1 - p_o^H\right) \delta_i = \\ & \delta_p - \left[p_i^H \left(1 - p_o^H\right) + p_i^H p_o^H\right] \delta_i = \delta_p - p_i^H \delta_i. \end{aligned} \tag{A.39}$$

Now $\frac{\partial \pi}{\partial \varepsilon} > 0$ if and only if

$$\delta_i < \frac{\delta_p}{p_i^H} \equiv \delta_i^{MAX,ISAPRE}, \tag{A.40}$$

that is, if the increase in coverage is small enough, as expected.

Step 2. Type HH's payoff when accepting HH's contract after the deviation is

$$\begin{aligned} E\left(HH, HH|\varepsilon, \delta_p, \delta_i\right) = \quad & p_i^H p_o^H u\left(w - \left(P^{HH} + \delta_p \varepsilon + \ell_i + \ell_o - c_i^{HH} - c_o^{HH} - \delta_i \varepsilon\right)\right) + \\ & p_i^H \left(1 - p_o^H\right) u\left(w - \left(P^{HH} + \delta_p \varepsilon + \ell_i - c_i^{HH} - \delta_i \varepsilon\right)\right) + \\ & \left(1 - p_i^H\right) p_o^H u\left(w - \left(P^{HH} + \delta_p \varepsilon + \ell_o - c_o^{HH}\right)\right) + \\ & \left(1 - p_i^H\right)\left(1 - p_o^H\right) u\left(w - \left(P^{HH} + \delta_p \varepsilon\right)\right). \end{aligned} \tag{A.41}$$

The partial derivative with respect to ε evaluated at $\varepsilon = 0$ is positive iff

$$\begin{aligned} \tfrac{\partial}{\partial \varepsilon} E\left(HH, HH|\varepsilon, \delta_p, \delta_i\right)|_{\varepsilon=0} = \quad & p_i^H p_o^H u_b' \left(\delta_i - \delta_p\right) + p_i^H \left(1 - p_o^H\right) u_i' \left(\delta_i - \delta_p\right) - \\ & \left(1 - p_i^H\right) p_o^H u_o' \delta_p - \left(1 - p_i^H\right)\left(1 - p_o^H\right) u_\varnothing' \delta_p > 0, \end{aligned} \tag{A.42}$$

or

$$\delta_i > \delta_p + \frac{\delta_p \left(1 - p_i^H\right) \left(p_o^H u_o' + \left(1 - p_o^H\right) u_\varnothing'\right)}{p_i^H \left(p_o^H u_b' + \left(1 - p_o^H\right) u_i'\right)} \equiv \delta_i^{\min,HH} \left(\overset{+}{\delta_p}\right)$$

Step 3 $\delta_i > \delta_i^{\min,HH} \left(\overset{+}{\delta_p}\right)$ is compatible with $\delta_i < \delta_i^{MAX,ISAPRE}$ if and only if (after some algebra)

$$p_o^H \left(u_o' - u_b'\right) + \left(1 - p_o^H\right) \left(u_\varnothing' - u_i'\right) < 0, \tag{A.43}$$

which is true since $u_i' > u_\varnothing'$ and $u_b' > u_o'$.

Step 4. Show that profits coming from types HL, LH, and LL do not decrease with the deviation.

There are two possibilities:

i) The deviation does not break incentive compatibility for any type in $\{HL, LH, LL\}$.

ii) The deviation breaks incentive compatibility and some type mimics HH.

In case (i) we are done since all types stay at their original contracts so profits coming from these contracts do not change.

In case (ii), we make use of the next three lemmas.

Lemma V-1 *Suppose that*

$$\delta_p \varepsilon + \left(p_o^H - p_o^L\right) c_o^{HH} \geq p_i^H \delta_i \varepsilon. \tag{A.44}$$

If, after the deviation, HL mimics HH, profits coming from HL do not decrease.

Proof: Take any menu of contracts $\{(c_i^x, c_o^x)\}_{x \in \{LL,LH,HL,HH\}}$. The general expression for profits coming from type $x \in \{LL, LH, HL, HH\}$ when accepting the contract aimed at type $y \in \{LL, LH, HL, HH\}$ are

$$\begin{aligned} &P^y - p_i^x p_o^x \left(c_i^y + c_o^y\right) - p_i^x \left(1 - p_o^x\right) c_i^y - p_o^x \left(1 - p_i^x\right) c_o^y = \\ &= P^y - p_i^x c_i^y - p_o^x c_o^y \equiv \Pi\left[x, y\right]. \end{aligned} \tag{A.45}$$

The profits coming from HL when HL mimics HH at the deviation are

$$\Pi\left[HL,HH\right]\equiv\underbrace{P^{HH}-p_i^Hc_i^{HH}}_{A}+\delta_p\varepsilon-p_i^H\delta_i\varepsilon-p_o^Lc_o^{HH}. \tag{A.46}$$

Now, since the profits coming from HL when HL does not mimic are zero by Part III, our aim is to prove that $\Pi\left[HL,HH\right]\geq 0$.

We also know that profits coming from type HH at the candidate when HH does not mimic are zero as well, from Part II. That is, $P^{HH}-p_i^Hc_i^{HH}-p_o^Hc_o^{HH}=0$, or

$$\underbrace{P^{HH}-p_i^Hc_i^{HH}}_{A}=p_o^Hc_o^{HH}. \tag{A.47}$$

Substituting A in $\Pi\left[HL,HH\right]\geq 0$ by the RHS of the last expression we obtain

$$p_o^Hc_o^{HH}+\delta_p\varepsilon-p_i^H\delta_i\varepsilon-p_o^Lc_o^{HH}\geq 0, \tag{A.48}$$

which, after taking the 3rd term to the RHS and pooling the terms in c_i^{HH}, give us the condition in the lemma .QED

Lemma V-2 *Suppose that*

$$\delta_p\varepsilon+\left(p_i^H-p_i^L\right)c_i^{HH}\geq p_i^L\delta_i\varepsilon. \tag{A.49}$$

If, after the deviation, LH mimics HH, profits do not decrease.

Proof: Use the symmetric argument. QED

Lemma V-3. *Suppose that the conditions in Lemmas V-1 and V-2 hold. If, after the deviation, LL mimics HH, profits do not decrease.*

Proof: Profits coming from LL when LL mimics HH at the deviation are $\Pi\left[LL,HH\right]\equiv\delta_p\varepsilon+P^{HH}-p_i^Lc_i^{HH}-p_o^Lc_o^{HH}-p_i^L\delta_i\varepsilon$. Hence our aim is to show that $\Pi\left[LL,HH\right]\geq 0$. We know that profits coming from HH at the candidate when HH does not mimic are zero.

Hence $P^{HH} = p_i^H c_i^{HH} + p_o^H c_o^{HH}$. Substitute into $\Pi[LL, HH] \geq 0$ to obtain

$$\delta_p \varepsilon + p_o^H c_o^{HH} + p_i^H c_i^{HH} - p_i^L c_i^{HH} - p_o^L c_o^{HH} - p_i^L \delta_i \varepsilon \geq 0. \tag{A.50}$$

Now, the sum of (A.44) and (A.49) implies that

$$2\delta_p \varepsilon + \left(p_o^H - p_o^L\right) c_o^{HH} + \left(p_i^H - p_i^L\right) c_i^{HH} \geq p_i^L \delta_i \varepsilon + p_i^H \delta_i \varepsilon, \tag{A.51}$$

where the RHS is larger than $2p_i^L \delta_i \varepsilon$ and the LHS is smaller than $2\delta_p \varepsilon + 2\left[\left(p_o^H - p_o^L\right) c_o^{HH} + \left(p_i^H - p_i^L\right) c_i^{HH}\right]$, therefore we know that

$$2\delta_p \varepsilon + 2\left[\left(p_o^H - p_o^L\right) c_o^{HH} + \left(p_i^H - p_i^L\right) c_i^{HH}\right] > 2p_i^L \delta_i \varepsilon, \tag{A.52}$$

which, after dividing both sides by 2, yields

$$\delta_p \varepsilon + \left(p_o^H - p_o^L\right) c_o^{HH} + \left(p_i^H - p_i^L\right) c_i^{HH} > p_i^L \delta_i \varepsilon, \tag{A.53}$$

which is equivalent to (A.50), the desired result. QED

We prove now that the conditions in Lemmas V-1 and V-2 have already been imposed above, so lemmas V-1, V-2, and V-3 apply (which concludes Step 4). Indeed, recall that we imposed that $\delta_i < \frac{\delta_p}{p_i^H} \equiv \delta_i^{MAX,ISAPRE}$, and hence $\delta_p \varepsilon > p_i^H \delta_i \varepsilon$. Now, both $\left(p_o^H - p_o^L\right) c_o^{HH}$ and $\left(p_i^H - p_i^L\right) c_i^{HH}$ are positive. Therefore the previous inequality implies that

1) $\delta_p \varepsilon + \left(p_o^H - p_o^L\right) c_o^{HH} > p_i^H \delta_i \varepsilon$, which is the condition in Lemma V-1, and

2) $\delta_p \varepsilon + \left(p_i^H - p_i^L\right) c_i^{HH} > p_i^H \delta_i \varepsilon$, which is the condition in Lemma V-2.

In sum, we have shown that if and only if $c_i^{HH} < \ell_i$ then $\delta_i^{\min HH} < \delta_i^{\max ISAPRE}$ and for $\delta_i \in \left(\delta_i^{\min HH} < \delta_i^{\max ISAPRE}\right)$ and $\delta_p > 0$, the deviant ISAPRE makes additional profits from type HH when truthful, type HH is indeed truthful, and profits coming from types HL, LH, and LL do not decrease.

Part V Type HH's contract fully covers outpatient expenditures, that is, $c_o^{HH} = \ell_o$

This holds by symmetry.

Part VI Type HL's contract fully covers inpatient expenditures, that is, $c_i^{HL} = \ell_i$

Suppose, by contradictions, that $c_i^{HL} < \ell_i$. Then consider the following deviation. Take $\varepsilon > 0$, sufficiently small, and $\delta_i > 0$, and δ_o (no sign constraints)

- Let the premium become $P^{HL\prime} = P^{HL} + \varepsilon$.
- Let inpatient coverage become $c_i^{HL\prime} = c_i^{HL} + \delta_i \varepsilon$, that is, increase inpatient coverage.
- Let outpatient coverage become $c_o^{HL\prime} = c_o^{HL} - \delta_o \varepsilon$, where δ_o could be either positive or negative. That is, outpatient coverage may be increased or decreased.

The proof proceeds in several steps.

Step 1. The (deviant) ISAPRE's payoff coming from the contract aimed at type HL when she is truthful must increase. A necessary and sufficient condition is

$$\delta_i < \frac{1 + p_o^L \delta_o}{p_i^H} \equiv \delta_i^{MAX,ISAPRE}. \tag{A.54}$$

Step 2. Type HL's payoff when she is truthful increases (this preserves incentive compatibility for this type). A necessary and sufficient condition is

$$\delta_i > \tag{A.55}$$

$$1 + \frac{\delta_o p_o^L \left(p_i^H u_b' + \left(1 - p_i^H\right) u_o'\right) + \left(1 - p_i^H\right) p_o^L u_o' + \left(1 - p_i^H\right)\left(1 - p_o^L\right) u_\varnothing'}{p_i^H \left(p_o^L u_b' + \left(1 - p_o^L\right) u_i'\right)} \equiv$$

$$\delta_i^{\min,HL}.$$

Step 3. Condition (A.55) is compatible with (A.54), that is, with $\delta_i < \delta_i^{MAX,ISAPRE}$

if and only if

$$\delta_o p_o^L \left[p_i^H u_b' + \left(1-p_i^H\right) u_o' - p_o^L u_b' - \left(1-p_o^L\right) u_i'\right] < \qquad \text{(A.56)}$$
$$\left(1-p_i^H\right) \left[p_o^L \left(u_b' - u_o'\right) + \left(1-p_o^L\right) \left(u_i' - u_\varnothing'\right)\right]$$

Notice that we cannot proceed unless we sign the term in brackets in the LHS of expression (A.56), that is, unless we sign

$$p_i^H u_b' + \left(1-p_i^H\right) u_o' - p_o^L u_b' - \left(1-p_o^L\right) u_i' \equiv A\left(c_i^{HL}\right). \qquad \text{(A.57)}$$

The (ancillary) function $A\left(c_i^{HL}\right)$ depends on c_i^{HL} since both u_i' and u_b' depend on this coverage.

CASE 1: $A\left(c_i^{HL}\right) > 0$

In this case the condition for both the ISAPRE and HL to improve is that

$$\delta_o < \frac{\left(1-p_i^H\right) \left[p_o^L \left(u_b' - u_o'\right) + \left(1-p_o^L\right) \left(u_i' - u_\varnothing'\right)\right]}{p_o^L A\left(c_i^{HL}\right)} \equiv \delta_o^{MAX,ISAPRE,HL,1} > 0 \qquad \text{(A.58)}$$

Incidentally, note that in this case $\delta_o = 0$, i.e. not changing outpatient coverage, satisfies the condition.

CASE 2: $A\left(c_i^{HL}\right) \leq 0$

Suppose first that $A\left(c_i^{HL}\right) = 0$. Then the condition becomes:

$$\delta_o p_o^L \underbrace{A\left(c_i^{HL}\right)}_{0} < \left(1-p_i^H\right) \left[p_o^L \underbrace{\left(u_b' - u_o'\right)}_{positive} + \left(1-p_o^L\right) \underbrace{\left(u_i' - u_\varnothing'\right)}_{positive}\right], \qquad \text{(A.59)}$$

which implies that it is satisfied *for any* δ_o (including $\delta_o = 0$).

Suppose now that $A\left(c_i^{HL}\right) < 0$. Then the condition becomes

$$\delta_o > \frac{\left(1-p_i^H\right) \left[p_o^L \left(u_b' - u_o'\right) + \left(1-p_o^L\right) \left(u_i' - u_\varnothing'\right)\right]}{p_o^L A\left(c_i^{HL}\right)} \equiv \delta_o^{\min,ISAPRE,HL,2} < 0. \qquad \text{(A.60)}$$

Once more, $\delta_o = 0$ satisfies the requirement.

Step 4. HH will not mimic HL if

$$\begin{aligned} &\delta_i < \\ &1+\frac{\delta_o p_o^H\left(p_i^H u_b' + \left(1-p_i^H\right)u_o'\right) + \left(1-p_i^H\right)p_o^H u_o' + \left(1-p_i^H\right)\left(1-p_o^H\right)u_\varnothing'}{p_i^H\left(p_o^H u_b' + \left(1-p_o^H\right)u_i'\right)} \equiv \\ &\delta_i^{MAX,HH} \end{aligned} \tag{A.61}$$

Step 5. This is compatible with $\delta_i > \delta_i^{\min,HL}$ if and only if

$$\delta_o > \frac{\frac{u_b' u_\varnothing'}{u_i' u_o'} - 1}{\frac{p_i^H u_b'}{\left(1-p_i^H\right)u_o'} + 1} \equiv \delta_o^{\min HL,HH} \tag{A.62}$$

Incidentally, it is not clear whether $\delta_o = 0$ satisfies the condition.

Step 6. Recall that we have two cases: Case 1, where $A\left(c_i^{HL}\right) > 0$, and Case 2, where $A\left(c_i^{HL}\right) \leq 0$.

In Case 2, the two requirements that are we left with are: $\delta_o > \delta_o^{\min HL,HH}$ and $\delta_o > \delta_o^{\min,ISAPRE,HL,2}$, which go in the same direction. Hence we have Corollary 2:

Corollary 2: *When $A\left(c_i^{HL}\right) \leq 0$ the only requirement for HL and the ISAPRE to be better-off with the deviation and for HH not to mimic HL is $\delta_o > \max\left\{\delta_o^{\min HL,HH}, \delta_o^{\min,ISAPRE,HL,2}\right\}$.*

In Case 1, the two requirements go in opposite directions: $\delta_o > \delta_o^{\min HL,HH}$ and $\delta_o < \delta_o^{MAX,ISAPRE,HL,1}$. These are compatible if $\delta_o^{MAX,ISAPRE,HL,1} > \delta_o^{\min HL,HH}$.

Notice that:

$$\begin{aligned} \delta_o^{MAX.ISAPRE,HL,1} \equiv\ & \frac{\left(1-p_i^H\right)\left[p_o^L u_b' + \left(1-p_o^L\right)u_i' - p_o^L u_o' - \left(1-p_o^L\right)u_\varnothing'\right]}{p_o^L\left[\left(p_i^H - p_o^L\right)u_b' - (1-p_o^L)u_i' + \left(1-p_i^H\right)u_o'\right]} = \\ & \frac{\left(1-p_i^H\right)\left[p_o^L u_b' + \left(1-p_o^L\right)u_i' - p_o^L u_o' - \left(1-p_o^L\right)u_\varnothing'\right]}{p_o^L\left[p_i^H u_b' - p_o^L u_b' + \left(1-p_i^H\right)u_o' - (1-p_o^L)u_i'\right]} = \\ & \frac{\left(1-p_i^H\right)\left[p_o^L u_b' + \left(1-p_o^L\right)u_i' - p_o^L u_o' - \left(1-p_o^L\right)u_\varnothing'\right]}{p_o^L A\left(c_i^{HL}\right)} \end{aligned} \tag{A.63}$$

so that we are certain in case 1 that the denominator is positive. Using this and after some algebra, one can express $\delta_o^{MAX,ISAPRE,HL,1} > \delta_o^{\min HL,HH}$ as

$$\left(p_i^H u_b' + (1-p_i^H)u_o'\right)\underbrace{\left[u_i' - u_\varnothing'\right]}_{+} - p_o^L u_o' u_i' > -p_o^L u_b' u_\varnothing'. \tag{A.64}$$

Adding and subtracting $p_o^L u_o' u_\varnothing'$ leads to:

$$\Leftrightarrow \underbrace{\left(p_i^H u_b' + (1-p_i^H)u_o' - p_o^L u_o'\right)}_{L} \underbrace{[u_i' - u_\varnothing']}_{+} > p_o^L \underbrace{[u_o' - u_b']}_{-} u_\varnothing' \tag{A.65}$$

Since the RHS is negative and $u_i' - u_\varnothing' > 0$, it suffices to show that the term L in the LHS is positive Indeed, since we are in case 1, we have $A\left(c_i^{HL}\right) = p_i^H u_b' + \left(1-p_i^H\right)u_o' - p_o^L u_b' - \left(1-p_o^L\right)u_i'$ is positive and hence $p_i^H u_b' + \left(1-p_i^H\right)u_o' - p_o^L u_b'$ is positive as well. Moreover, since $u_b' > u_o'$, $L \equiv p_i^H u_b' + \left(1-p_i^H\right)u_o' - p_o^L u_o'$ is also positive.

This allows us to formulate the following lemma.

Lemma VI-1. *If* $A\left(c_i^{HL}\right) > 0$, *then* $\delta_o^{\min HL,HH} < \delta_o^{MAX,ISAPRE,HL,1}$.

Corollary4: Using Corollaries 2 and 3 and Lemma VI-1, we have that both for $A\left(c_i^{HL}\right) > 0$ and for $A\left(c_i^{HL}\right) \leq 0$, there exist δ_o and δ_i such that the deviation is profitable, improves HL, and worsens HH when mimicking HL.

Step 7 Ensuring that LH does not want to mimic HL requires

$$\delta_i < \tag{A.66}$$
$$1 + \frac{\delta_o p_o^H \left(p_i^L u_b' + \left(1-p_i^L\right)u_o'\right) + \left(1-p_i^L\right)p_o^H u_o' + \left(1-p_i^L\right)\left(1-p_o^H\right)u_\varnothing'}{p_i^L\left(p_o^H u_b' + \left(1-p_o^H\right)u_i'\right)} \equiv$$
$$\delta_i^{MAX,LH}.$$

Lemma VI-5 *Condition* $\delta_i < \delta_i^{MAX,HH}$ *implies condition* $\delta_i < \delta_i^{MAX,LH}$.

Proof: Recall that

$$\delta_i^{MAX,HH} \equiv$$
$$1 + \frac{\delta_o p_o^H \left(p_i^H u_b' + \left(1-p_i^H\right)u_o'\right) + \left(1-p_i^H\right)p_o^H u_o' + \left(1-p_i^H\right)\left(1-p_o^H\right)u_\varnothing'}{p_i^H\left(p_o^H u_b' + \left(1-p_o^H\right)u_i'\right)}$$

Therefore $\delta_i < \delta_i^{MAX,HH}$ implies $\delta_i < \delta_i^{MAX,LH}$ if and only if $\delta_i^{MAX,LH} > \delta_i^{MAX,HH}$, or

$$\frac{\delta_o p_o^H \left(p_i^L u_b' + \left(1 - p_i^L\right) u_o'\right) + \left(1 - p_i^L\right) p_o^H u_o' + \left(1 - p_i^L\right)\left(1 - p_o^H\right) u_\varnothing'}{p_i^L} > \tag{A.67}$$
$$\frac{\delta_o p_o^H \left(p_i^H u_b' + \left(1 - p_i^H\right) u_o'\right) + \left(1 - p_i^H\right) p_o^H u_o' + \left(1 - p_i^H\right)\left(1 - p_o^H\right) u_\varnothing'}{p_i^H}.$$

Since $p_i^L < p_i^H$, a sufficient condition is that

$$\begin{gathered}\delta_o p_o^H \left(p_i^L u_b' + \left(1 - p_i^L\right) u_o'\right) + \left(1 - p_i^L\right) p_o^H u_o' + \left(1 - p_i^L\right)\left(1 - p_o^H\right) u_\varnothing' > \\ \delta_o p_o^H \left(p_i^H u_b' + \left(1 - p_i^H\right) u_o'\right) + \left(1 - p_i^H\right) p_o^H u_o' + \left(1 - p_i^H\right)\left(1 - p_o^H\right) u_\varnothing'\end{gathered} \tag{A.68}$$

or, after some algebra,

$p_o^H u_o' + \left(1 - p_o^H\right) u_\varnothing' > \delta_o p_o^H \underbrace{(u_b' - u_o')}_{+}$, which can be rewritten as

$$\delta_o < \frac{p_o^H u_o' + \left(1 - p_o^H\right) u_\varnothing'}{p_o^H \underbrace{(u_b' - u_o')}_{+}}, \tag{A.69}$$

where the RHS is positive. Hence if $\delta_o \leq 0$, we are done.

Suppose now that $\delta_o > 0$. Let us return to the necessary and sufficient condition (A.67), which can be rewritten as, after some algebra,

$$\left[p_i^H - p_i^L\right] \delta_o p_o^H u_o' + \left[p_i^H - p_i^L\right] p_o^H u_o' + \left[p_i^H - p_i^L\right]\left(1 - p_o^H\right) u_\varnothing' > 0. \tag{A.70}$$

Using $\left[p_i^H - p_i^L\right] > 0$, the condition is

$$\delta_o p_o^H u_o' + p_o^H u_o' + \left(1 - p_o^H\right) u_\varnothing' > 0, \tag{A.71}$$

which holds since we are now assuming that $\delta_o > 0$. QED

Step 8. Ensuring that LL does not want to mimic HL requires

$$\delta_i < \tag{A.72}$$

$$1 + \frac{\delta_o p_o^L \left(p_i^L u_b' + \left(1-p_i^L\right) u_o'\right) + \left(1-p_i^L\right) p_o^L u_o' + \left(1-p_i^L\right)\left(1-p_o^L\right) u_\varnothing'}{p_i^L \left(p_o^L u_b' + \left(1-p_o^L\right) u_i'\right)} \equiv$$

$$\delta_i^{MAX,LL}$$

Lemma VI-3. *If*

$$\delta_o > -1 - \frac{\left(1-p_o^L\right) u_\varnothing'}{p_o^L u_o'} \equiv \delta_o^{\min,HL,LL},$$

condition $\delta_i < \delta_i^{MAX,LL}$ *is compatible with* $\delta_i > \delta_i^{\min,HL}$.

Proof: Recall that

$$\delta_i^{\min,HL} = 1 + \frac{\delta_o p_o^L \left(p_i^H u_b' + \left(1-p_i^H\right) u_o'\right) + \left(1-p_i^H\right) p_o^L u_o' + \left(1-p_i^H\right)\left(1-p_o^L\right) u_\varnothing'}{p_i^H \left(p_o^L u_b' + \left(1-p_o^L\right) u_i'\right)} \tag{A.73}$$

Therefore, $\delta_i < \delta_i^{MAX,LL}$ is compatible with $\delta_i > \delta_i^{\min,HL}$ iff

$$\frac{\delta_o p_o^L \left(p_i^L u_b' + \left(1-p_i^L\right) u_o'\right) + \left(1-p_i^L\right) p_o^L u_o' + \left(1-p_i^L\right)\left(1-p_o^L\right) u_\varnothing'}{p_i^L \left(p_o^L u_b' + \left(1-p_o^L\right) u_i'\right)} > \frac{\delta_o p_o^L \left(p_i^H u_b' + \left(1-p_i^H\right) u_o'\right) + \left(1-p_i^H\right) p_o^L u_o' + \left(1-p_i^H\right)\left(1-p_o^L\right) u_\varnothing'}{p_i^H \left(p_o^L u_b' + \left(1-p_o^L\right) u_i'\right)} \tag{A.74}$$

This can be rewritten as the condition in the lemma. QED

Step 9. We now check that condition $\delta_o > -1 - \frac{\left(1-p_o^L\right) u_\varnothing'}{p_o^L u_o'} \equiv \delta_o^{\min,HL,LL}$ is compatible with all the conditions imposed in the previous analysis, namely:

In case 1, where $A\left(c_o^{HL}\right) > 0$, we imposed
$\delta_o < \frac{\left(1-p_i^H\right)\left[p_o^L\left(u_b'-u_o'\right)+\left(1-p_o^L\right)\left(u_i'-u_\varnothing'\right)\right]}{p_o^L A\left(c_i^{HL}\right)} \equiv \delta_o^{MAX,ISAPRE,HL,1}$
and
$\delta_o > \frac{\frac{u_b' u_\varnothing'}{u_i' u_o'} - 1}{\frac{p_i^H u_b'}{\left(1-p_i^H\right) u_o'} + 1} \equiv \delta_o^{\min HL,HH}$
Hence only compatibility with the first of these must be checked. We have

Lemma VI-4 $\delta_o^{\min,HL,LL} < \delta_o^{MAX,ISAPRE,HL,1}$

Proof

$\delta_o^{\min,HL,LL} < \delta_o^{MAX,ISAPRE,HL,1}$ holds if and only if

$$-1-\frac{\left(1-p_o^L\right)u'_\varnothing}{p_o^L u'_o}<\frac{\left(1-p_i^H\right)\left[p_o^L\left(u'_b-u'_o\right)+\left(1-p_o^L\right)\left(u'_i-u'_\varnothing\right)\right]}{p_o^L\left[p_i^H u'_b+\left(1-p_i^H\right)u'_o-p_o^L u'_b-\left(1-p_o^L\right)u'_i\right]}, \tag{A.75}$$

or

$$\frac{\left(1-p_i^H\right)\left[p_o^L\overbrace{\left(u'_b-u'_o\right)}^{+}+\left(1-p_o^L\right)\overbrace{\left(u'_i-u'_\varnothing\right)}^{+}\right]}{p_o^L\left[p_i^H u'_b+\left(1-p_i^H\right)u'_o-p_o^L u'_b-\left(1-p_o^L\right)u'_i\right]}>-1-\frac{\left(1-p_o^L\right)u'_\varnothing}{p_o^L u'_o} \tag{A.76}$$

where $p_i^H u'_b+\left(1-p_i^H\right)u'_o-p_o^L u'_b-\left(1-p_o^L\right)u'_i\equiv A\left(c_i^{HL}\right)>0$ so

$$\frac{\left(1-p_i^H\right)\left[p_o^L\overbrace{\left(u'_b-u'_o\right)}^{+}+\left(1-p_o^L\right)\overbrace{\left(u'_i-u'_\varnothing\right)}^{+}\right]}{p_o^L\underbrace{\left[p_i^H u'_b+\left(1-p_i^H\right)u'_o-p_o^L u'_b-\left(1-p_o^L\right)u'_i\right]}_{+}}>\underbrace{-1-\frac{\left(1-p_o^L\right)u'_\varnothing}{p_o^L u'_o}}_{-}. \tag{A.77}$$

Hence the LHS is positive whereas the RHS is negative. QED

As a consequence, in case 1 we only need to impose

$$Max\left\{\delta_o^{\min,HL,LL},\delta_o^{\min HL,HH}\right\}<\delta_o<\delta_o^{MAX,ISAPRE,HL,1}, \tag{A.78}$$

where $\delta_o^{\min,HL,LL}<\delta_o^{MAX,ISAPRE,HL,1}$ was just established in Lemma VI-4 and $\delta_o^{\min HL,HH}<\delta_o^{MAX,ISAPRE,HL,1}$ was established in Lemma VI-1.

In **case 2** we imposed

$\delta_o>\frac{\left(1-p_i^H\right)\left[p_o^L\left(u'_b-u'_o\right)+\left(1-p_o^L\right)\left(u'_i-u'_\varnothing\right)\right]}{p_o^L A\left(c_i^{HL}\right)}\equiv\delta_o^{\min,ISAPRE,HL,2}$

and

$\delta_o>\frac{\frac{u'_b u'_\varnothing}{u'_i u'_o}-1}{\frac{p_i^H u'_b}{\left(1-p_i^H\right)u'_o}+1}\equiv\delta_o^{\min HL,HH}$

Therefore $\delta_o>-1-\frac{\left(1-p_o^L\right)u'_\varnothing}{p_o^L u'_o}\equiv\delta_o^{\min,HL,LL}$ cannot contradict either of these conditions. We just need to impose

$\delta_o>Max\left\{\delta_o^{\min,ISAPRE,HL,2},\delta_o^{\min HL,HH},\delta_o^{\min HL,LL}\right\}$.

To sum up, we have the following

Lemma VI-8. *(I) Suppose $A\left(c_i^{HL}\right)>0$. Then there exist $\delta_o^{\min,HL,LL}<\delta_o^{MAX,ISAPRE,HL,1}$*

and $\delta_o^{\min HL,HH} < \delta_o^{MAX,ISAPRE,HL,1}$ *such that, for*

$$Max\left\{\delta_o^{\min,HL,LL},\delta_o^{\min HL,HH}\right\} < \delta_o < \delta_o^{MAX,ISAPRE,HL,1},$$

(i) there exist $\delta_i^{MAX,LL} > \delta_i^{\min,HL}$ *and* $\delta_i^{MAX,HH} > \delta_i^{\min,HL}$ *such that for* $\delta_i \in \left(\delta_i^{\min,HL}, Min\left\{\delta_i^{MAX,LL},\delta_i^{MAX,HH}\right\}\right)$*, HL gains with the deviation, whereas LL, LH, and HH do not gain by mimicking HL after the deviation.*

(ii) There exist $\delta_i^{\min,HL} < \delta_i^{MAX,ISAPRE}$*, such that for all* $\delta \in \left(\delta_i^{\min,HL},\delta_i^{MAX,ISAPRE}\right)$*, both HL and the ISAPRE gain with the deviation.*

(II) Suppose $A\left(c_i^{HL}\right) \leq 0$*. Then for*

$$\delta_o > Max\left\{\delta_o^{\min,ISAPRE,HL,2},\delta_o^{\min HL,HH},\delta_o^{\min HL,LL}\right\},$$

(i) there exist $\delta_i^{MAX,ISAPRE} > \delta_i^{\min,HL}$*, such that for all* $\delta_i \in \left(\delta_i^{\min,HL},\delta_i^{MAX,ISAPRE}\right)$ *we have that both HL and the ISAPRE gain with the deviation;*

(ii) there exist $\delta_i^{MAX,HH} > \delta_i^{\min,HL}$*, such that, for all* $\delta_i \in \left(\delta_i^{\min,HL},\delta_i^{MAX,HH}\right)$*, we have that neither HH nor LH gain by mimicking HL after the deviation; and*

(iii) there exist $\delta_i^{MAX,LL} > \delta_i^{\min,HL}$*, such that LL does not gain by mimicking HL after the deviation.*

This concludes the proof, as we have found a deviation where the ISAPRE is better-off and no incentive compatibility constraints are violated. QED

Part VII Type LH's contract fully covers outpatient expenditures, that is, $c_o^{LH} = \ell_o$

This is just by symmetry with respect to Part VI.

B APPENDIX: Other Proofs

Proof of Lemma 1a: derived in the text.

Proof of Lemma 1b: Throughout this proof, we use $w_i' = w - p_i^H \ell_i$

Equation (3) is

$$u(w_i' - p_o^H \ell_o) = (1 - p_o^H) u(w_i' - p_o^L c_o^{HL}) + p_o^H u(w_i' - p_o^L c_o^{HL} - \ell_o + c_o^{HL})$$

Case 1. $p_o^L < p_o^H$

The first thing to note is that the RHS is strictly increasing in c_o^{HL} for $c_o^{HL} < \ell_o$ and $p_o^L < p_o^H$. Indeed,

$$\frac{\partial\left((1-p_o^H)u(w_i'-p_o^L c_o^{HL})+p_o^H u(w_i'-\ell_o+c_o^{HL}(1-p_o^L))\right)}{\partial c_o^{HL}} =$$
$$(1 - p_o^H) u'(w_i' - p_o^L c_o^{HL})(-p_o^L) + p_o^H u'(w_i' - p_o^L c_o^{HL} + c_o^{HL} - \ell_o)(1 - p_o^L),$$

which, if $c_o^{HL} < \ell_o$ is strictly greater than $(1 - p_o^H) u'(w_i' - p_o^L c_o^{HL})(-p_o^L) + p_o^H u'(w_i' - p_o^L c_o^{HL})(1-p_o^L)$, which, taking $u'(w_i' - p_o^L c_o^{HL})$ as common factor, is equal to $\left[p_o^H - p_o^L\right] u'(w_i' - p_o^L c_o^{HL})$, which is positive.

Suppose $c_o^{HL} = \ell_o$ and still $p_o^L < p_o^H$. Then the RHS of the equation becomes

$$(1 - p_o^H) u(w_i' - p_o^L \ell_o) + p_o^H u(w_i' - \ell_o + \ell_o(1 - p_o^L)) = u(w_i' - p_o^L \ell_o)$$

This is larger than the LHS $u(w_i' - p_o^H \ell_o)$, since $p_o^L < p_o^H$. Therefore the ICC is violated with the RHS larger than the LHS.

If instead $c_o^{HL} = 0$ then the RHS of the equation becomes $(1 - p_o^H) u(w_i') + p_o^H u(w_i' - \ell_o)$. Now, using Jensen's inequality we can say that the latter expression is strictly smaller than $u[(1 - p_o^H) w_i' + p_o^H (w_i' - \ell_o)]$, which is equal to $u(w_i' - p_o^H \ell_o)$, i.e., smaller than the LHS. Again the ICC is violated, but now with the LHS larger than the RHS.

By continuity, this proves that there must exist a unique c_o^{HL} strictly between zero and ℓ_o that solves the ICC.

Case 2: $p_o^L = p_o^H = p_o$.

Suppose now that $p_o^L = p_o^H = p_o$. Then Equation (3) becomes

$$u(w_i' - p_o\ell_o) = (1-p_o)u(w_i' - p_o c_o^{HL}) + p_o u(w_i' - p_o c_o^{HL} - \ell_o + c_o^{HL})$$

Suppose now that $c_o^{HL} = \ell_o$. Then the RHS becomes $(1-p_o)u(w_i' - p_o\ell_o) + p_o u\left(w_i' - p_o\ell_o\right)$, which coincides with the LHS. Hence $c_o^{HL} = \ell_o$ is *a* solution for the ICC. The derivative of the RHS with respect to c_o^{HL} is now given by

$(1-p_o)u'(w_i' - p_o c_o^{HL})(-p_o) + p_o u'(w_i' - p_o c_o^{HL} + c_o^{HL} - \ell_o)(1-p_o)$, or, taking $(1-p_o)p_o$ as common factor,

$p_o(1-p_o)\left[u'(w_i' + c_o^{HL}(1-p_o) - \ell_o) - u'(w_i' - p_o c_o^{HL})\right]$, which is positive iff

$u'(w_i' + c_o^{HL}(1-p_o) - \ell_o) > u'(w_i' - p_o c_o^{HL})$, or since $u'' < 0$, iff

$c_o^{HL}(1-p_o) - \ell_o < -p_o c_o^{HL}$, or $c_o^{HL} < \ell_o$. Hence the RHS is increasing in c_o^{HL} for all $c_o^{HL} < \ell_o$ and coincides with the LHS at $c_o^{HL} = \ell_o$.

This shows that, if $p_o^L = p_o^H = p_o$ then $c_o^{HL} = \ell_o$ is the unique solution for the ICC.

QED

Proof of lemmas 2a and 2b: These follow directly from lemmas 1a and 1b, by symmetry.

Proof of Proposition 2 $\frac{\delta c_i^{LH}}{\delta p_i^L} > 0$ and $\frac{\delta c_o^{HL}}{\delta p_o^L} > 0$.

We just show the first statement. The other follows by symmetry. By Lemma 2a, c_i^{LH} is fully determined by

$$u(w - P^{HH}) = (1-p_i^H)u(w - P^{LH}) + p_i^H u(w - P^{LH} - \ell_i + c_i^{LH}).$$

Differentiate totally with respect to p_i^L, bearing in mind that $P^{LH} = p_i^L c_i^{LH} + p_o^H \ell_o$. We get

$$\begin{aligned} 0 \ &= \ (1-p_i^H)u'(w - P^{LH})\left(-c_i^{LH} - p_i^L \frac{\partial c_i^{LH}}{\partial p_i^L}\right) + \\ &\quad p_i^H u'(w - P^{LH} - \ell_i + c_i^{LH})\left(-c_i^{LH} - (p_i^L - 1)\frac{\partial c_i^{LH}}{\partial p_i^L}\right). \end{aligned}$$

Let $n^{LH} = w - P^{LH}$ and $a^{LH} = w - P^{LH} - \ell_i + c_i^{LH}$. The previous expression can then be rewritten as:

$$\frac{\partial c_i^{LH}}{\partial p_i^L} = \frac{-c_i^{LH}\left((1-p_i^H)u'(n^{LH}) + p_i^H u'(a^{LH})\right)}{p_i^L(1-p_i^H)u'(n^{LH}) - (1-p_i^L)p_i^H u'(a^{LH})}.$$

The last expression is positive if and only if $(1-p_i^H)p_i^L u'(n^{LH}) - (1-p_i^L)p_i^H u'(a^{LH}) < 0$, or $(1-p_i^H)p_i^L u'(n^{LH}) < (1-p_i^L)p_i^H u'(a^{LH})$ or

$$\frac{(1-p_i^H)p_i^L}{(1-p_i^L)p_i^H} < \frac{u'(a^{LH})}{u'(n^{LH})},$$

where the LHS is less than one since $p_i^L < p_i^H$ and the RHS is larger than one since $a^{LH} < n^{LH}$ but u' is decreasing. **QED**

Proof of Proposition 3

The fact that $c_i^{HH*} = c_i^{HL*} = \ell_i$ and that $c_o^{LH} = c_o^{HH} = \ell_o$ was already established in Proposition 1 and it holds for any probability vector. The fact that $c_i^{LH*} = \ell_i$ whenever $p_i^L = p_i^H = p_i$ was established in Lemma 2b. Suppose that $c_i^{LL} = \ell_i$ and $c_o^{LL} = c_o^{HL} = x$. Then we can write $P^{HH} = p_i\ell_i + p_o^H\ell_o$; $P^{HL} = p_i\ell_i + p_o^L x$; $P^{LH} = p_i\ell_i + p_o^H\ell_o$; and $P^{LL} = p_i\ell_i + p_o^L x$. We can substitute these findings into the general formula (2) to obtain $[x]$ and $[x,y]$ for all $x, y \in \{LL, LH, HL, HH\}$. After straight simplification of these expression it is easy to check that all incentive compatibility constraints, except

i) the (binding) ICC $[HH] = [HH, HL]$, given by (3)

ii) the (binding) ICC $[LH] = [LH, LL]$, given by (6).

are satisfied.

We analyze $[HH] = [HH, HL]$ next.

Expression $[HH]$ can be rewritten as $u(w - p_i^H\ell_i - p_o^H\ell_o)$. Expression $[HH, HL]$ is given by $(1-p_o^H)u(w - p_i\ell_i - p_o^L x) + p_o^H u(w - p_i\ell_i - p_o^L x - \ell_o + x)$. Hence we are left with the equation that determines x:

$$u(w - p_i\ell_i - p_o^H\ell_o) = (1-p_o^H)u(w - p_i\ell_i - p_o^L x) + p_o^H u(w - p_i\ell_i - p_o^L x - \ell_o + x). \quad \text{(A.79)}$$

We now prove that the solution x lies strictly between zero and full coverage and is unique. Notice that if $x = \ell_o$ then the RHS becomes $u(w - p_i\ell_i - p_o^L\ell_0)$, which is larger than the LHS since $p_o^H > p_o^L$. Notice also that if $x = 0$ then the RHS becomes $(1 - p_o^H)u(w - p_i\ell_i) + p_o^H u(w - p_i\ell_i - \ell_o)$, which, by Jensen's inequality, is smaller than $u\left((1 - p_o^H)(w - p_i\ell_i) + p_o^H(w - p_i\ell_i - \ell_o)\right)$. This expression, after simplification becomes $u\left(w - p_i\ell_i - p_o^H\ell_o\right)$, which coincides with the LHS. By continuity, the solution must satisfy $0 < x < \ell_o$. Uniqueness obtains by showing that the RHS is increasing in x. Indeed, it's derivative with respect to x is given by $(1 - p_o^H)u'(w - p_i\ell_i - p_o^L x)\left(-p_o^L\right) + p_o^H u'(w - p_i\ell_i - p_o^L x - \ell_o + x)\left(1 - p_o^L\right)$, which is positive if and only if $\frac{p_o^H\left(1-p_o^L\right)}{p_o^L(1-p_o^H)} > \frac{u'(w-p_i\ell_i-p_o^L x)}{u'(w-p_i\ell_i-p_o^L x-\ell_o+x)}$, where the LHS is larger than 1 (since $p_o^H > p_o^L$) whereas the RHS is smaller than 1 as long as $x < \ell_o$ (since $w - p_i\ell_i - p_o^L x > w - p_i\ell_i - p_o^L x - \ell_o + x$ but u' is decreasing).

Let us turn now to constraint $[LH] = [LH, LL]$.

Expression $[LH]$ becomes $(1 - p_i)u(w - p_i\ell_i - p_o^H\ell_o) + p_i u(w - p_i\ell_i - p_o^H\ell_o)$, which simplifies to $u(w - p_i\ell_i - p_o^H\ell_o)$. Expression $[LH, LL]$ becomes

$$\begin{array}{c} \left(1 - p_o^H\right)(1 - p_i)\,u(w - p_i\ell_i - p_o^L x) + p_i(1 - p_o^H)u(w - p_i\ell_i - p_o^L x) + \\ p_o^H(1 - p_i)u(w - p_i\ell_i - p_o^L x - \ell_o + x) + p_i p_o^H u(w - p_i\ell_i - p_o^L x - \ell_o + x) \end{array},$$

which simplifies to $(1 - p_o^H)u(w - p_i\ell_i - p_o^L x) + p_o^H u(w - p_i\ell_i - p_o^L x - \ell_o + x)$. Notice that equating the simplified $[LH]$ to the simplified $[LH, LL]$ leads to the same equation as (A.79).**QED**

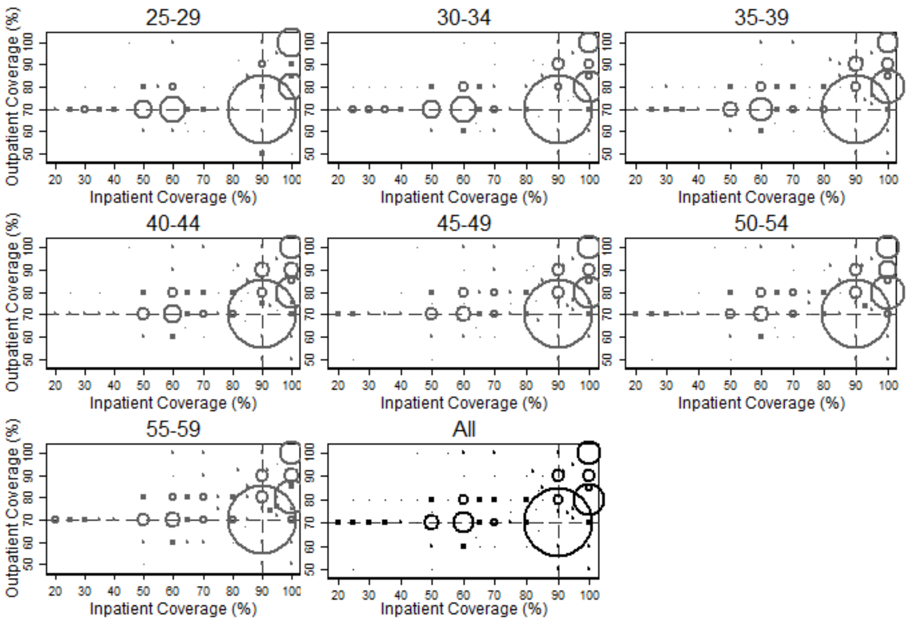
C APPENDIX: Tables and Figures

Table A.1: *Distribution of Beneficiaries by ISAPRE and Region (2008)*

Region	1		2		3		4		5		6		7		%Pop
	N	%	N	%	N	%	N	%	N	%	N	%	N	%	%
1	1056	1.2	1623	2.0	2751	3.5	456	0.9	1170	2.2	13	0.1	4	0.1	1.9
2	1440	1.6	1685	2.1	2504	3.1	1427	2.7	2875	5.4	16	0.1	0	0.0	2.6
3	700	0.8	375	0.5	509	0.6	332	0.6	1331	2.5	6	0.0	0	0.0	0.9
4	1757	2.0	808	1.0	956	1.2	647	1.2	1038	1.9	18	0.1	0	0.0	1.4
5	4710	5.2	5249	6.5	5689	7.1	2152	4.1	4671	8.7	2367	11.2	238	8.0	6.6
6	1634	1.8	2450	3.1	1543	1.9	1057	2.0	3807	7.1	56	0.3	14	0.5	2.8
7	1823	2.0	1444	1.8	1982	2.5	2338	4.5	1613	3.0	375	1.8	9	0.3	2.5
8	3057	3.4	3504	4.4	5933	7.4	1595	3.0	11781	22.0	1568	7.4	147	4.9	7.3
9	1212	1.3	2138	2.7	2817	3.5	1079	2.1	3266	6.1	840	4.0	102	3.4	3.0
10	1423	1.6	2642	3.3	3357	4.2	1142	2.2	5192	9.7	1234	5.8	11	0.4	3.9
11	234	0.3	423	0.5	390	0.5	159	0.3	263	0.5	8	0.0	4	0.1	0.4
12	961	1.1	565	0.7	1090	1.4	522	1.0	1542	2.9	6	0.0	0	0.0	1.2
13	68713	76.4	56462	70.3	47351	59.4	38576	73.5	12759	23.8	14324	67.5	2449	82.2	63.3
14	473	0.5	467	0.6	2112	2.7	569	1.1	1913	3.6	390	1.8	0	0.0	1.6
15	703	0.8	450	0.6	687	0.9	413	0.8	324	0.6	3	0.0	0	0.0	0.7
Total	89896	100	80285	100	79671	100	52464	100	53545	100	21224	100	2978	100	100
Share	23.7%		21.1%		21.0%		13.8%		14.1%		5.6%		0.8%		

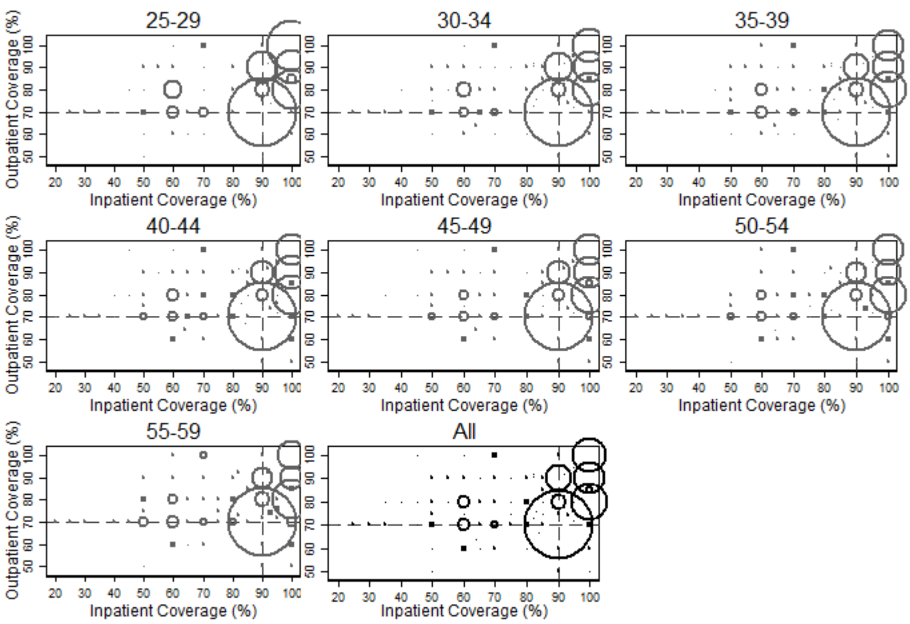
Notes: This table shows the distribution in 2008 by region and ISAPRE of all beneficiaries aged 25-59 who are employees, do not have dependents and who signed at most 2 different contracts during 2008. The region of Santiago (region 13) holds on average 63.3% of this sample and will be the focus of the paper. The remainder of the paper will restrict further the sample to include only those clients who are present in both 2007 and 2008.

Figure A.3: Plans bought by Women by Age



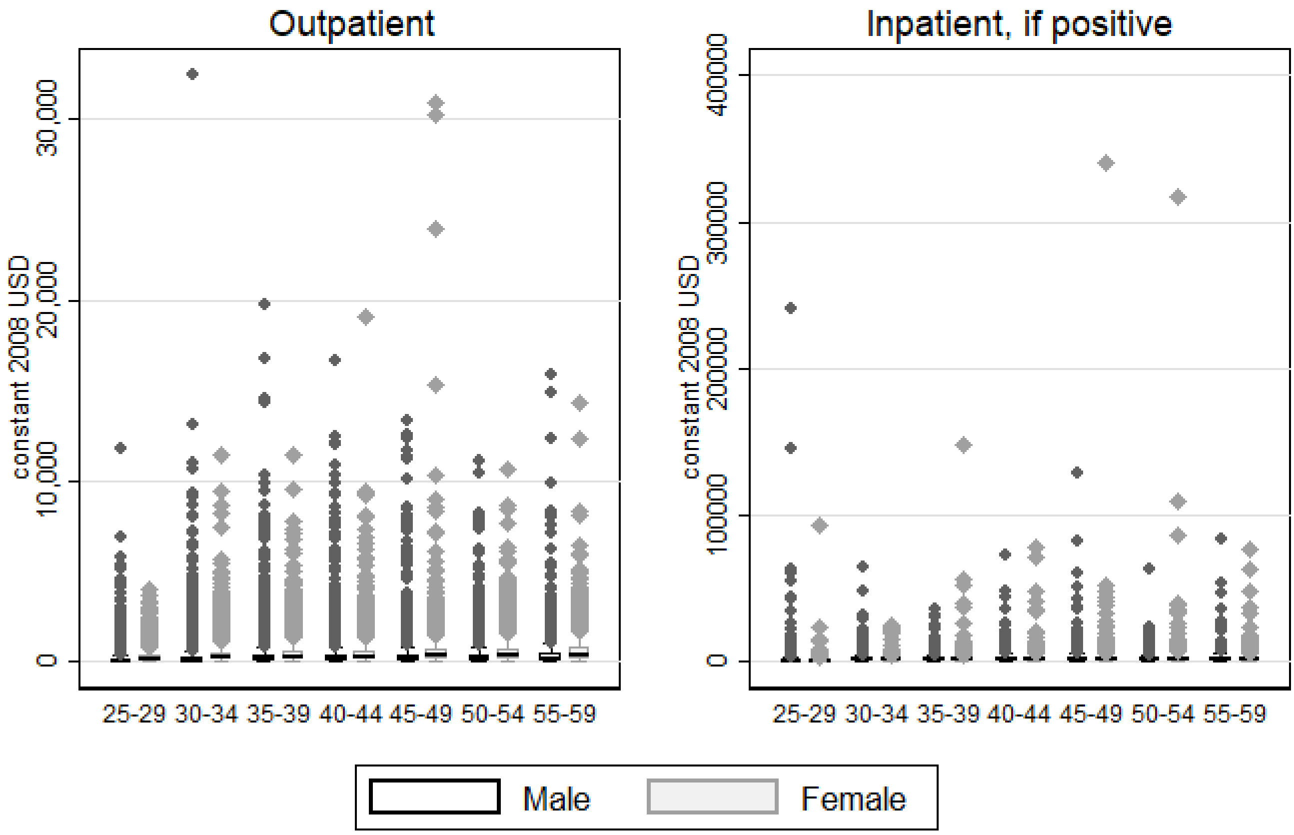
Note: Graph refers to last plan held in 2008.

Figure A.4: Plans bought by Men by Age



Note: Graph refers to last plan held in 2008.

Figure A.5: Distribution of Expenditures with outliers



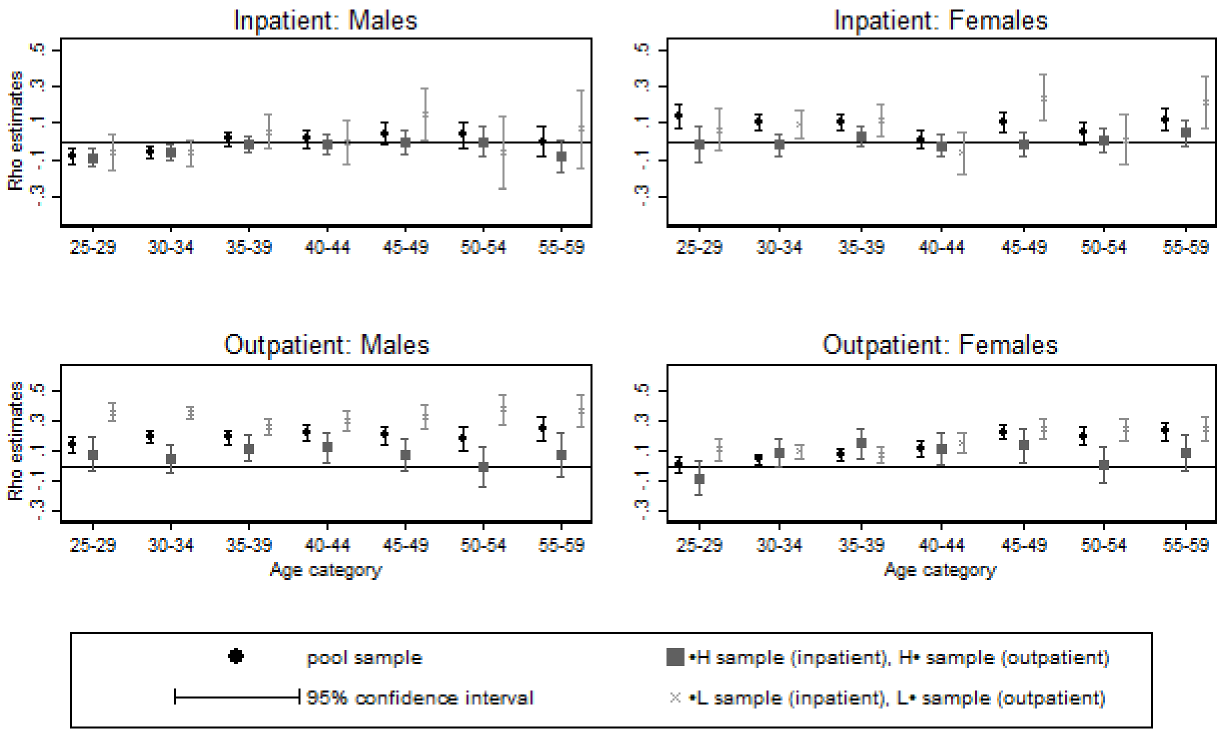
Box-plots include outliers. Left panel shows the distribution of the average 2007 and 2008 outpatient expenditures. The right panel shows the 2007 and 2008 average inpatient expenditures conditional on being positive because the 75 percentile is zero for all gender and age groups.

Table A.2: Correlations between ρ estimates when types are inferred from 2007 coverages and from expenditures

	Men		Women	
	HH and LH	HL and LL	HH and LH	HL and LL
Inpatient: correlation of ρ estimates	0.923**	0.258	0.303	0.769*
	HH and HL	LH and LL	HH and HL	LH and LL
Outpatient: correlation of ρ estimates	0.017	0.482	0.252	0.666
Observations	7	7	7	7

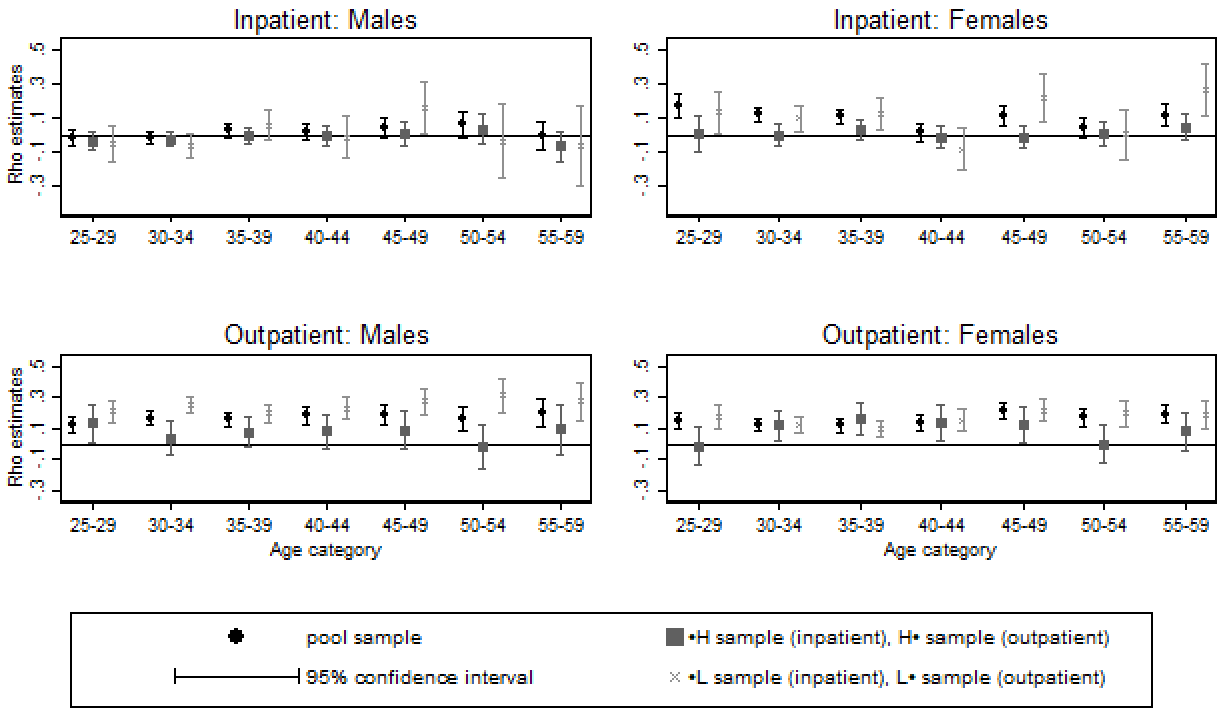
Note: Correlations between ρ estimates obtained when types are inferred from 2007 coverages and when they are inferred from the average of 2007 and 2008 expenditures (measured in 2008 USD). Columns "HH and HL" show correlations when the sample is restricted to types HH and HL, i.e types whose inpatient risk is high. And "LH and LL" shows correlations when the sample is restricted to types LH and LL, i.e. low inpatient risk. Likewise "HH and LH" shows correlations when the sample is restricted to types HH and LH, i.e types whose outpatient risk is high. And "HL and LL" shows correlations when the sample is restricted to types HL and LL.

Figure A.6: Positive Correlation Tests by Gender and Age



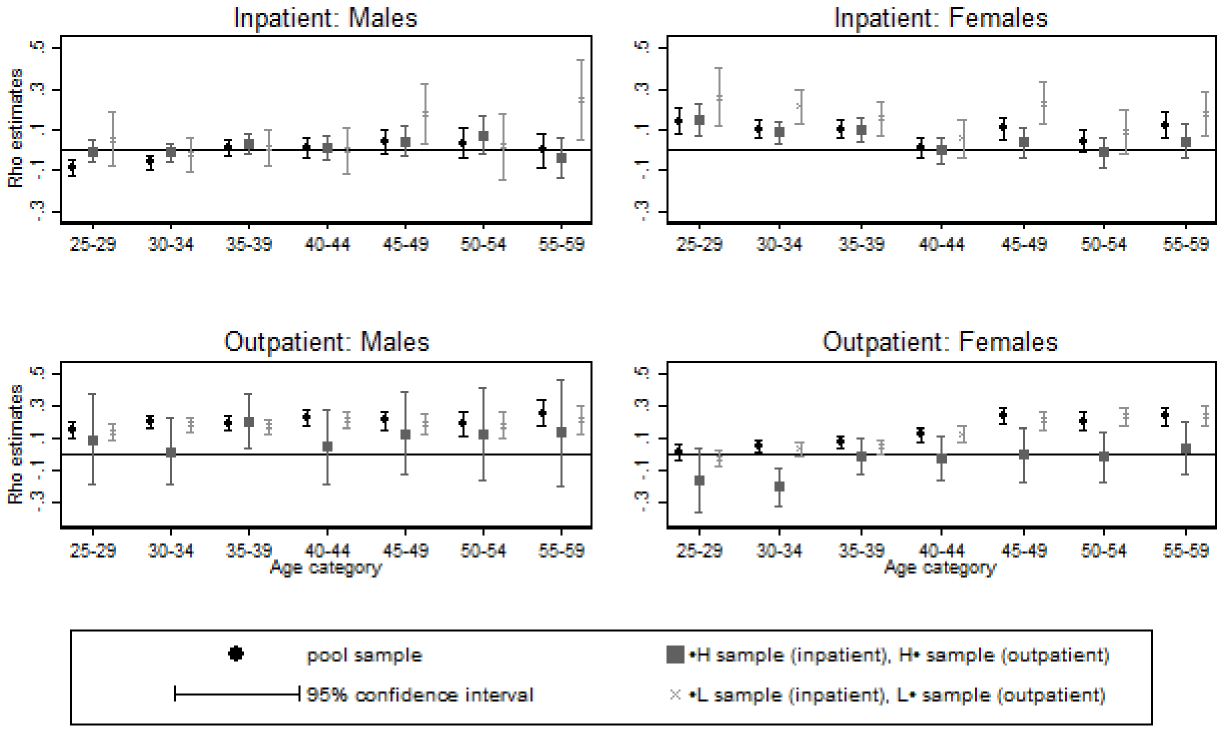
ρ estimates obtained from bivariate probit models by gender and age-group. Robust s.e.. Coverage dependent variable is constructed from the last plan held in 2008. High inpatient coverage if coverage>90%. High outpatient coverage if coverage>70%. Risk dependent variable takes value 1 if the 2007 and 2008 average inpatient (outpatient) expenditures is larger than the gender-age group median. Types HH, HL, LH and LL inferred from 2007 plans. Plan (90,70) dropped from sample. «pool sample» means estimates using all individuals. «•H sample» means estimation is restricted to H types in the outpatient dimension. «•L sample» means estimation is restricted to L types in the outpatient dimension. «H• sample» and «L• sample» means estimation is restricted to H- and L-types, respectively, in the inpatient dimension.

Figure A.7: Positive Correlation Tests controlling for unused variables



ρ estimates obtained from bivariate probit models controlling for income and dummies for: missing income, income higher than 33500 (aproximately the cap value) and municipality. Robust s.e.. Coverage dependent variable is constructed from inpatient (outpatient) coverage of the last plan held in 2008. High inpatient coverage if coverage>90%. High outpatient coverage if coverage>70%. Risk dependent variable takes value 1 if the 2007 and 2008 average inpatient (outpatient) expenditures is larger than the gender-age group median. Types HH, HL, LH and LL are inferred from 2007 plans. Plan (90,70) dropped from sample. «pool sample» means estimates using all individuals. «•H sample» means estimation is restricted to H types in the outpatient dimension. «•L sample» means estimation is restricted to L types in the outpatient dimension. «H• sample» and «L• sample» means estimation is restricted to H- and L-types, respectively, in the inpatient dimension.

Figure A.8: Positive Correlation Tests (types inferred from expenditures)



Notes: ρ estimates obtained from bivariate probit models (no covariates). Robust s.e.. Coverage dependent variables is constructed inpatient (outpatient) coverage of the last plan held in 2008. High inpatient coverage if coverage>90%. High outpatient coverage if coverage >70%. Risk dependent variable takes value 1 if the 2007 and 2008 average inpatient (outpatient) expenditures is larger than the gender-age group median. Types HH, HL, LH and LL inferred from the average of 2007 and 2008 expenditures. Plan (90,70) dropped from sample. «pool sample» means estimates using all individuals. «•H sample» means estimation is restricted to H types in the outpatient dimension. «•L sample» means estimation is restricted to L types in the outpatient dimension. «H• sample» and «L• sample» means estimation is restricted to H- and L-types, respectively, in the inpatient dimension.