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Social Connectivity, Media Bias, and Correlation Neglect*

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Short Title: Social Connectivity and Correlation Neglect

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Abstract

A biased newspaper aims to persuade voters to vote for the government. Voters are uncertain about the government’s competence. Each voter receives the newspaper’s report as well as independent private signals about the competence. Voters then exchange messages containing this information on social media and form posterior beliefs, neglecting correlation among messages. We show that greater social connectivity increases the probability of an efficient voting outcome if the prior favours the government; otherwise, efficiency decreases. The probability of an efficient outcome remains strictly below one even when connectivity becomes large, implying a failure of the Condorcet jury theorem.

Keywords: correlation neglect; social media; voting; information aggregation; media bias; Bayesian persuasion; deliberation

JEL codes: D72, D83, D85, D91
1 Introduction

“Repetition does not transform a lie into a truth.”

Franklin Delano Roosevelt

“Want to make a lie seem true? Say it again. And again. And again.”

Emily Dreyfuss, Wired

Digital social media platforms have increased individual social connectivity immensely in recent years by enabling individuals to connect with an almost arbitrary number of people. For example, in 2016 79% of Americans used Facebook[1] and the average user had 338 Facebook friends.[2] Individuals receive a substantial amount of political information from their social media connections. For instance, 62% of US adults receive their news via social media[3] and a 26-country study has shown that over 50% of all web users use social media for news each week.[4]

The effect of social media on political outcomes has been the focus of a number of recent studies.[5] Our paper proposes a novel channel through which social media can have an effect. Specifically, we explore the effect of increasing social connectivity on news reporting by biased media, and through it – on voting outcomes.

As connectivity increases, so does the number of people with whom a voter can exchange politically relevant information. A voter thus gains access to a larger number of information

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4The Guardian, June 15, 2016, “Facebook’s rise as news source hits publishers’ revenues.”
sources. At the same time, information received from different social media contacts can be partly based on a common news source, and hence correlated. This would have no effect on voters’ beliefs if they fully realised this correlation. Crucially, however, evidence suggests that individuals exhibit correlation neglect: they treat correlated signals received from different sources as independent signals. This leads them to overvalue the informativeness of these signals. A biased information provider, such as a media outlet, can exploit this correlation neglect to manipulate voters’ beliefs. Thus, greater connectivity can make voters better informed, while at the same time making it easier for biased media to persuade them. The main aim of this paper is to analyse how increased social connectivity affects information aggregation and voting outcomes in the presence of a biased provider of information.

To address these questions, we model a population of voters choosing whether or not to vote for the government, and a biased newspaper that aims to maximise the likelihood that the government wins the election. Each voter is connected to a number of other voters on social media. There is a binary state of the world. All voters prefer voting for the government in the high state, and voting against the government in the low state. The newspaper commits to an editorial policy that, conditional on the realisation of the state, sends a report to all voters. Each voter observes the newspaper’s report, as well as an independent continuous signal about the state. Each voter then communicates this information to her social media friends. Hence, each voter observes a number of messages which are not identical (since they contain independent signals), but are correlated, as they are partly based on the newspaper’s report. Voters, however, fail to realise that these messages are correlated when forming their beliefs.

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posterior beliefs.

We analyse what happens if the distribution of the number of connections shifts in a way that increases the number of connections that voters tend to have. In a benchmark case when voters correctly perceive correlation, such an increase in connectivity exposes each voter to a higher number of independent signals. Hence, voters become better informed. As connectivity becomes very large, in the limit, voters learn the state perfectly. They then almost surely make the correct decision – that is, re-elect the government if and only if the state is high. Hence, a version of the Condorcet jury theorem for this setting would hold.

However, because voters neglect correlation, an increase in connectivity has two effects. On the one hand, each voter is exposed to a larger number of independent signals, making it harder for the newspaper to manipulate her beliefs. On the other hand, each voter receives more realisations of the newspaper’s report, which she now perceives as independent messages. Hence, the newspaper’s report becomes more persuasive, counteracting the first effect.

We show that as connectivity increases, the accuracy of the newspaper’s reports and the probability that voters make an efficient collective decision can both increase and decrease, depending on the prior belief. If the prior is such that without additional information voters are willing to vote for the government, then greater connectivity increases the probability of an efficient collective decision. On the other hand, if the prior is low enough that voters are ex ante predisposed to vote against the government, then an increase in connectivity makes an efficient voting outcome less likely. Thus, greater connectivity – for example, as a result of greater social media penetration – increases the efficiency of the electoral process if voters ex ante prefer the same outcome as the information provider. If, on the other hand, voters
ex ante prefer the opposite outcome, increased connectivity reduces efficiency.

As connectivity becomes arbitrarily large, in the limit each voter observes infinitely many independent signals about the state. However, we show that even in this case, correlation neglect enables the newspaper to ensure that voters do not fully learn the state. Thus, the probability of an efficient collective decision remains below one, implying a failure of the Condorcet jury theorem.

The rest of this section discusses the related literature. Section 2 introduces the model. Section 3 characterises the newspaper’s optimal editorial policy. Section 4 uses this characterisation to analyse the effect of connectivity on the efficiency of the voting outcome. Section 5 extends the model to consider partial correlation neglect, arbitrary voting rules, and an imperfectly informed newspaper. Section 6 concludes. The appendix contains the proofs.

Related literature. Our paper contributes to the literature studying informational efficiency of voting (see e.g. Austen-Smith and Banks, 1996; Feddersen and Pesendorfer, 1998). Within this literature, a number of papers have looked at the role of information exchange. Gerardi and Yariv (2007) extend the classic Condorcet jury theorem framework by allowing voters to exchange their private signals before taking a vote. In a Bayesian framework, they show that all veto-free electoral rules yield the same equilibrium outcomes. In subsequent experimental work, Goeree and Yariv (2011) show that public communication increases efficiency of the vote, while Buechel and Mechtenberg (2019) find that private voter-to-voter communication can reduce efficiency. Pogorelskiy and Shum (2019) analyse individuals who

In a setting without voting or strategic senders, Levy and Razin (2018) examine information exchange on a social network by agents who may neglect correlation, while Acemoglu et al. (2010) analyse exchange of information when agents follow a non-Bayesian updating heuristic and posteriors are computed by averaging priors.
receive private signals from media outlets with exogenous biases, and communicate them prior to voting. They find evidence of failure of Bayesian updating – individuals do not account for bias in the signals they share – which implies that social media may reduce efficiency of collective decisions. In our paper, individuals also deviate from Bayesian updating when exchanging information. However, rather than failing to recognise the bias in the signals they receive, they fail to realise that signals are correlated.\(^8\) Levy and Razin (2015a) model a private value election in which voters receive unbiased correlated signals and neglect correlation between them. They show that correlation neglect can improve efficiency of collective decisions. Unlike the literature discussed above, we study information aggregation in a setting in which some of the information is provided by a strategic sender. In this setting we show that increased connectivity can both improve and degrade information aggregation, which remains imperfect even in the limit.

Our model uses the Bayesian persuasion framework introduced by Kamenica and Gentzkow (2011). More specifically, we add to the literature that studies Bayesian persuasion in a voting setup. Alonso and Câmara (2016) examine the problem of a sender who designs a policy experiment to persuade a heterogeneous group of voters to vote in a particular way. Wang (2015), Bardhi and Guo (2018), and Chan et al. (2019) study persuasion of heterogeneous voters when the sender is able to design different experiments for different groups of voters. Heese and Lauermann (2019) study persuasion of voters who, as in our model, receive exogenous private signals in addition to the sender’s signal.\(^9\) The distinguishing feature of our

\(^8\)In a related experimental paper, Kawamura and Vlasvars (2017) show that voters overweight an unbiased public signal even in a setting without deliberation (and hence without observing multiple realisations of the same message). Our paper suggests that information exchange reinforces this effect, implying that a when the signal comes from a biased newspaper, the bias would increase when connectivity is larger.

\(^9\)More generally, a number of papers have studied persuasion of heterogeneous receivers in non-voting setups (Kolotilin et al. 2017, Kolotilin 2018, Ginzburg 2019).
paper relative to that literature is that we consider information exchange on a social network
by receivers who neglect correlation.

Levy et al. (2018, forthcoming) also study Bayesian persuasion of receivers who neglect
correlation. In both papers, the sender has multiple signals at her disposal and can choose
correlation between them. The former paper constructs a general model of information
design, showing that the sender can obtain a payoff arbitrarily close to her first-best when
the number of available signals is large. The latter describes a setting in which two competing
senders aim to persuade voters by choosing correlation between signals of fixed precision. Our
setup is different in two ways. First, in our model, the multiplicity of signals emerges from
information exchange between voters on social media. Hence, the sender can choose neither
the number of signals each voter observes – it is determined exogenously by the number
of the voter’s social media connections – nor the correlation between signals. Instead, the
sender can choose signal precision. Second, in our model voters receive independent signals
in addition to the signal from the newspaper. This underlies our analysis of informational
efficiency of the election and, moreover, prevents the newspaper from achieving its first-best
even when the number of signals is large.

Finally, our paper is related to the research on media bias or slant, specifically, to the
literature that models biased media as committing to a reporting strategy ex ante (Gehlbach
and Sonin, 2014, Gentzkow et al., 2015, Boleslavsky and Kim, 2018). We contribute to this
literature by providing the analysis of how media bias is affected by information exchange
among receivers who neglect correlation.

See, for example, Bernhardt et al. (2008), Gentzkow and Shapiro (2010), Duggan and Martinelli (2011),
Oliveros and Vardy (2015), Piolatto and Schnett (2015), and others.
2 Model

Players, network, and information. There are two groups of players: a continuum of voters whose mass is 1, and a newspaper. Each voter is connected to \( n \in \{0, 1, ..., N\} \) other citizens on social media. Our model is agnostic about the actual structure of the network – the only object of interest is the distribution of the number of connections across voters. For all \( n \in \{0, 1, ..., N\} \), let \( \gamma_n \) be the share of voters with \( n \) social media connections. Let \( \gamma \equiv (\gamma_0, ..., \gamma_N) \) be the distribution of the number of connections, that is, the degree distribution of the network.

There is an unknown binary state of the world \( \theta \in \{0, 1\} \). The state reflects the government’s competence, with \( \theta = 1 \) representing a more competent government. Let \( q \equiv \Pr(\theta = 1) \) denote the common prior probability that the government is competent. The newspaper observes the state, and sends a binary report \( r \in \{0, 1\} \).

In addition, each voter \( i \) receives a private signal \( s_i \in \mathbb{R} \). Conditional on the state, signals are independent across voters. In state 1, \( s_i \) is drawn from a normal distribution with mean \( \mu > 0 \) and variance that is normalised to 1. In state 0 the signal is drawn from a normal distribution with mean \( -\mu \) and variance 1. In the following \( \Phi \) and \( \phi \) will refer to, respectively, the c.d.f. and p.d.f. of the standard normal distribution. Note that these signal distributions satisfy the monotone likelihood ratio property, and a higher realisation of the signal indicates that the state is more likely to equal 1. A higher value of \( \mu \) corresponds to a more precise signal, while \( \mu = 0 \) corresponds to a signal that carries no information.

\[^{11}\]This is without loss of generality, as the report can always be interpreted as a recommendation. See, for example, Alonso and Câmara (2016).
Preferences and actions. The newspaper commits ex ante to a strategy that specifies for each state $\theta \in \{0, 1\}$ the probability $p_0$ of sending report 1; report 0 is sent with the complementary probability. We will refer to the pair $(p_0, p_1)$ as the newspaper’s editorial policy. Without loss of generality, we will assume that $p_0 \leq p_1$ – thus, report 1 induces a weakly higher posterior belief that $\theta = 1$. After the exchange of information (described below), each voter $i$ chooses an action $a_i \in \{0, 1\}$ where the action $a_i = 1$ ($a_i = 0$) corresponds to voting for (against) the government.

A voter who chooses the action 0 receives a payoff of 0. A voter who chooses the action 1 receives a payoff of $\frac{1}{2}$ if $\theta = 1$, and a payoff of $-\frac{1}{2}$ if $\theta = 0$. Thus, voters prefer voting for the government in state 1, and voting against it in state 0. We assume that each voter who is indifferent chooses action 1.

The newspaper receives a payoff of 1 if at least half of the voters vote for the government, and zero otherwise. The newspaper thus aims to maximize the probability that the government wins the election.

Timing, information exchange, and belief formation. The sequence of events is depicted in Figure 1. At the beginning of the game, the newspaper chooses an editorial policy $(p_0, p_1)$. Then, Nature draws the state $\theta$ according to the common prior $q$. For each voter $i$, Nature also draws her number of social media connections from distribution $\gamma$, as well as her private signal. Upon observing $\theta$, the newspaper sends report 1 with probability $p_\theta$, and
report 0 with the complementary probability $1 - p_0$. Each voter observes the newspaper’s report $r$ and her private signal $s_i$. Based on these two sources of information, she then forms a likelihood ratio $x_i = \frac{\Pr(\theta=0|r, s_i)}{\Pr(\theta=1|r, s_i)}$, and truthfully reveals $x_i$ to her friends on social media. Thus, a voter with $n$ social media connections observes $n+1$ such messages (those of her $n$ friends, plus her own). These $x_i$’s are correlated, as they are based on identical realisations of the newspaper’s report, in addition to independent signals. Voters, however, fail to realise this correlation. Specifically, we assume that when updating their beliefs, voters perceive $n$ fully correlated signals as $k(n)$ independent signals. The main part of our analysis focuses on the case of full correlation neglect, in which $k(n) = n$. We will compare this to the benchmark case of Bayesian voters, for whom $k(n) = 1 \forall n$. Partial correlation neglect, in which $k(n)$ is an arbitrary function, will be considered in Section 5.

After updating her belief, each voter $i$ chooses an action $a_i \in \{0, 1\}$. After this, payoffs are realised.

**Discussion of the assumptions.** The model outlined above describes the choice of editorial policy as a Bayesian persuasion problem. This means, in particular, that the newspaper commits to a reporting strategy before learning the state. One explanation is that a change in the editorial policy requires a change of the editorial board – for example, by replacing existing staff with people with more pronounced pro-government views, the newspaper can increase the values of $p_0$ and $p_1$.\footnote{Hiring decisions by the media are in fact often seen as adjustments to editorial policy. See, for example, Vox, February 2019, “CNN hires GOP operative with no journalism experience to coordinate its 2020 coverage”, for a discussion of one such case.} Hiring and firing staff members entails a fixed cost,\footnote{These can include search costs to hire staff members, severance payment for fired journalists, or efficiency losses incurred when workers are replaced.} which helps create commitment power. Additionally, if the newspaper cares about its reputation,
an outcome equivalent to one in a setting with commitment can emerge as an equilibrium in a repeated cheap talk game (see Mathevet et al. 2019 or Best and Quigley 2020).

While the sender in the model is framed as a newspaper, the analysis can be applied to other types of senders. Consider, for example, special interests, such as super PACs in the United States, that finance scientific studies. Since a study must meet scientific standards, a special interest group cannot freely choose the message that it sends. Rather, funding a study by a researcher or a think tank with a particular political slant can be seen as committing to an experiment.\[14\]

In the model we assume that each voter observes the newspaper’s report, and a single independent normally distributed signal. The latter represents all information a voter receives from sources other than the newspaper. Note that the model can easily be generalised to allow each voter to observe multiple independent signals. Suppose each voter $i$ observes a vector of $m$ independent signals $(s_i^1, ..., s_i^m)$, such that each signal $s_i^j$ is normally distributed with mean $\mu^j > 0$ in state 1 and mean $-\mu^j$ in state 0, and with variance $(\sigma^j)^2$. The Bayesian posterior distribution of the state conditional on observing this vector of signals is identical to the posterior distribution of the state conditional on observing a single signal $s_i = \sum_{j=1}^{m} s_i^j$ that is normally distributed with mean $\mu = \sum_{j=1}^{m} \mu^j$ in state 1 and mean $-\mu$ in state 0, and variance $\sigma^2 = \sum_{j=1}^{m} (\sigma^j)^2$. Our model thus nests this more general formulation, with $\sigma^2$ normalised to one. Note that this normalisation is without loss of generality, because only the ratio $\mu/\sigma^2$ matters for the analysis – thus, both an increase in $\mu$ and a decrease in $\sigma^2$ represent an increase in the informativeness of independent signals.

Communication between voters happens simultaneously, and only once. Because of this,\[14\]

\[14\]We thank an anonymous referee for suggesting this example.
a voter only observes the information shared by voters to whom she is directly connected. One implication of this is that the number of connections is the only feature of the network that affects a voter’s belief updating. Hence, only the degree distribution of the network, and not its overall structure, determines the equilibrium.

We assume that a voter who votes for the government receives a payoff of $\frac{1}{2}$ in state 1, and a payoff of $-\frac{1}{2}$ in state 0. This can easily be extended to, for example, allowing each voter to receive a payoff of $1 - \lambda$ in state 1, and a payoff of $\lambda$ in state 0, with $\lambda \in (0, 1)$. All results continue to hold with this generalisation.\footnote{See the working paper version (Denter, Dumav and Ginzburg, 2020).}

Voters are assumed to have preferences over their actions – each voter prefers matching her vote to the state – and not over the outcome of the election. Note that since the model posits a continuum of voters, the probability of a given voter being pivotal is zero. Hence, even if we allow voters’ payoffs to also depend on the voting outcome, this would not change their equilibrium behaviour. For the same reason, it is an equilibrium strategy for each voter to report her information truthfully to her friends.

Overall, the assumption that the set of voters is a continuum makes the model appropriate for the analysis of political processes in large voting bodies – for example, of a media outlet attempting to ensure that the government wins an election, or that the number of citizens participating in a protest does not exceed a certain margin. It is less appropriate for analysing persuasion of small committees.

In the setting of our paper only one strategic newspaper exists, with all other information being captured by the independent signals. This setup applies best to situations in which a single outlet or owner dominates the media market. As Djankov et al. (2003) show, in a
number of European countries, state-owned television channels enjoy a dominant position in the market. Media monopolies can be particularly strong in countries where, in addition, members of the ruling party have control over private media outlets. One example is Italy, where Silvio Berlusconi directly or indirectly controlled around 90 percent of Italy’s television, in addition to other media outlets [Ragnedda and Muschert 2010]. In other countries, media concentration is often high at the sub-national level. For example, in the US roughly 50 percent of counties, including some counties with hundreds of thousands of residents, are served by exactly one local newspaper [Abernathy 2018]. A similar situation exists in the UK, where 43 percent of local authority districts in England, Scotland, and Wales are served by a single publisher, and in 69 percent a single publisher controls over 70 percent of the local newspaper circulation [Ramsay and Moore 2016]. Since national media is unlikely to report on a particular region regularly, this means that elections at this level can be influenced by media monopolies. The model is less well-suited to analyse media markets with several large competing outlets, such as the nationwide media reporting on national politics in the US.

3 Optimal Editorial Policy

Consider the last stage of the game. Let \( \pi \) be the probability that the government is competent, i.e., that \( \theta = 1 \). If a voter has this posterior belief, her expected utility from voting for the government equals 
\[
\pi \frac{1}{2} - (1 - \pi) \frac{1}{2} = \pi - \frac{1}{2}.
\]
She thus votes for the government if and only if \( \pi \geq \frac{1}{2} \).

A possible strategy for the newspaper is to select an editorial policy \( p_0 = p_1 \) — that is, to send an uninformative report that does not affect the voters’ behaviour. All such babbling
strategies are payoff-equivalent for the newspaper. We will thus without loss of generality exclude editorial policies $(0, 0)$ and $(1, 1)$. Since we have assumed that $p_0 \leq p_1$, this means that we restrict attention without loss of generality to editorial policies in which $p_1 > 0$ and $p_0 < 1$.

Suppose the newspaper chooses an editorial policy $(p_0, p_1)$. Consider a voter with $n$ social media connections. This voter observes $n + 1$ messages $x_1, ..., x_{n+1}$. For the purposes of belief updating, this is equivalent to observing $n + 1$ independent signals $s_1, ..., s_{n+1}$, as well as $n + 1$ realisations of $r$ which the voter perceives as $k (n + 1)$ independent realisations. If the newspaper sends report $r = 1$, this voter will form a posterior belief

$$
\pi (p_0, p_1 \mid r = 1) = \frac{q p_1^{k(n+1)} \prod_{i=1}^{n+1} \phi (s_i - \mu)}{q p_1^{k(n+1)} \prod_{i=1}^{n+1} \phi (s_i - \mu) + (1 - q) p_0^{k(n+1)} \prod_{i=1}^{n+1} \phi (s_i + \mu)}
$$

$$
= \frac{1}{1 + \frac{1-q}{q} \left( \frac{p_0}{p_1} \right)^{k(n+1)} e^{-2S\mu}}
$$

where $S \equiv \sum_{i=1}^{n+1} s_i$. We will refer to $S$ as independent evidence. It summarises all new information that is not based on the newspaper’s report. A higher value of $S$ indicates that the state is more likely to be 1. $S$ is distributed normally with mean $(n + 1) \mu$ in state 1 and $- (n + 1) \mu$ in state 0, and with variance $(n + 1)$.

The voter will then vote for the government if and only if $\pi (p_0, p_1 \mid r = 1) \geq \frac{1}{2}$. This is always true if $p_0 = 0$. If $p_0 > 0$, the voter votes for the government if and only if her independent evidence is sufficiently convincing, that is, if and only if

$$
S \geq S_n (p_0, p_1)
$$

(1)
where
\[ S_n(p_0, p_1) \equiv \frac{1}{2\mu} \ln \frac{1 - q}{q} + \frac{1}{2\mu} k(n + 1) \ln \left( \frac{p_0}{p_1} \right) \]

Note that the threshold \( S_n(p_0, p_1) \) is decreasing in the prior \( q \): the higher \( q \) is, the less independent evidence is necessary to convince a voter to vote for the government. Furthermore, the threshold is increasing in \( \frac{p_0}{p_1} \): intuitively, an increase in this ratio means that the editorial policy is less informative, and hence more convincing independent evidence is needed for a voter to vote for the government after report 1.

At the same time, consider the case when the newspaper sends report \( r = 0 \). Then a voter with \( n \) connections who observed \( S \) will form a posterior belief:
\[
\pi(p_0, p_1 \mid r = 0) = \frac{(1 - p_1)^{k(n+1)}}{(1 - p_1)^{k(n+1)} + \frac{1 - q}{q} (1 - p_0)^{k(n+1)} e^{-2\mu}.}
\]

This voter votes for the government if and only if \( \pi(p_0, p_1 \mid r = 0) \geq \frac{1}{2} \). When \( p_1 = 1 \), this condition is never satisfied. Otherwise, the voter votes for the government if and only if
\[
S \geq S_n(1 - p_0, 1 - p_1) \tag{2}
\]

Equations (1) and (2) imply that given report \( r \), a voter will vote for the government if she observes independent evidence \( S \) that is sufficiently large. This is more likely to happen in state 1. Hence, given \( r \), the government receives a larger share of votes in that state. In addition, \( p_1 \geq p_0 \) implies that \( S_n(p_0, p_1) \leq S_n(1 - p_0, 1 - p_1) \). Then (1) and (2) imply that given evidence \( S \), a voter is more likely to vote for the government when the newspaper sends report 1. Thus, \( r = 1 \) gives the government a weakly larger vote share than \( r = 0 \).
When $q$ is sufficiently large, the majority of voters vote for the government in both states even without the newspaper’s report. In this case, the newspaper can achieve its first-best payoff by babbling, that is, by setting $p_0 = p_1$. On the other hand, if $q$ is low enough, the newspaper needs to reveal some information. The next result characterises the optimal editorial policy of the newspaper, distinguishing these two cases:

**Proposition 1.** For each $\gamma$ there exists a unique $\bar{q}(\gamma) > \frac{1}{2}$ such that

(i.) if $q \geq \bar{q}(\gamma)$, babbling is an equilibrium strategy;

(ii.) if $q < \bar{q}(\gamma)$, then there exists a unique optimal editorial policy $(p_0^*, p_1^*)$ such that $p_1^* = 1$ and $p_0^* \in (0, 1)$, where $p_0^*$ is uniquely determined by

$$
\sum_{n=0}^{N} \gamma_n \left(1 - \Phi \left[ \frac{S_n (p_0^*, 1) + (n + 1) \mu_1}{\sqrt{n + 1}} \right] \right) = \frac{1}{2}.
$$

Furthermore, $\bar{q}(\gamma)$ is defined by the condition

$$
\sum_{n=0}^{N} \gamma_n \left(1 - \Phi \left[ \frac{\frac{1}{2} \ln \frac{1-\bar{q}(\gamma)}{\bar{q}(\gamma)} + (n + 1) \mu_1}{\sqrt{n + 1}} \right] \right) = \frac{1}{2}. \quad (3)
$$

In plain words, $\bar{q}(\gamma)$ is the prior belief at which exactly $\frac{1}{2}$ of voters vote for the government in state 0. When the newspaper cannot achieve its first-best by babbling, it adopts a partially informative editorial policy. Specifically, it always sends report 1 in state 1, and in state 0 it sends report 1 with some probability $p_0^*$ that is distinct from zero and from one. We can think of $p_0^*$ as a measure of bias: the higher it is, the more likely the newspaper is to misreport the state. The bias is such that after report 1 exactly $\frac{1}{2}$ of all voters vote for the government when the state is 0. On the other hand, $p_1^* = 1$ implies that upon
receiving report 0, all voters know that \( \theta = 0 \) with certainty. Thus, the optimal editorial policy is analogous to a result in [Kamenica and Gentzkow (2011)], extended to a setting with independent signals and correlation neglect. In particular, Proposition \( \square \) implies that the newspaper gains from persuasion whenever \( q < \bar{q}(\gamma) \) – that is, the newspaper can increase the government’s reelection probability when \( q \) is sufficiently low.

If \( q \geq \bar{q}(\gamma) \), the government wins the election in each state. Otherwise, in state 1 it wins the election with certainty, while in state 0 it receives no votes with probability \( 1 - p_0^* \), and exactly \( \frac{1}{2} \) of all votes with probability \( p_0^* \). Hence, if \( q < \bar{q}(\gamma) \), a competent government always wins the election, while an incompetent government wins the election with probability \( p_0^* \). Let \( \Psi \) be the equilibrium probability that the election produces the correct decision. It then equals

\[
\Psi = \begin{cases} 
1 - (1 - q) p_0^* & \text{if } q < \bar{q}(\gamma), \\
q & \text{if } q \geq \bar{q}(\gamma).
\end{cases} \tag{4}
\]

This is monotone decreasing in the newspaper’s bias \( p_0^* \) – intuitively, the less informative the newspaper’s report is, the lower is the probability that the collective decision is correct. Furthermore, holding the prior constant, \( \Psi \) is always larger in any non-babbling equilibrium than in a babbling equilibrium. The newspaper’s expected equilibrium payoff is \( q + 1 - \Psi \). Thus, at the equilibrium, the newspaper’s payoff is lower when the probability of the correct collective decision is higher.
4 The Effect of Connectivity

In this section we will look the effect of an increase in connectivity on the equilibrium. Specifically, by an increase in connectivity we will mean a shift from connectivity distribution \( \gamma \) to another distribution \( \tilde{\gamma} \) that first-order stochastically dominates \( \gamma \). How will such a shift affect the probability of the correct decision \( \Psi \)? Because \( \Psi \) depends on the optimal bias of the newspaper, we need to establish whether an increase in connectivity will move the equilibrium between babbling and non-babbling. For this, the following result will be useful:

**Lemma 1.** *If \( \tilde{\gamma} \) first-order stochastically dominates \( \gamma \), then \( \bar{q}(\tilde{\gamma}) > \bar{q}(\gamma) \).*

The lemma implies that an increase in connectivity can induce the newspaper to move from babbling to a partially informative editorial policy, but not the other way around. Intuitively, when the newspaper babbles, increased connectivity implies that voters observe more precise independent evidence. Therefore, a higher prior \( q \) is needed for \( \frac{1}{2} \) of them to vote for the government in state 0.

For any pair of \( \gamma \) and \( \tilde{\gamma} \) such that \( \tilde{\gamma} \) first-order stochastically dominates \( \gamma \), if \( q \geq \bar{q}(\tilde{\gamma}) \), the newspaper babbles under both distributions. Then a shift from \( \gamma \) to \( \tilde{\gamma} \) has no effect on the probability of a correct collective decision \( \Psi \) regardless of whether voters neglect correlation. In the case when \( q < \bar{q}(\tilde{\gamma}) \), we will first analyse a benchmark setting of no correlation neglect, before examining the effect of an increase in connectivity when voters neglect correlation.

4.1 A Benchmark: Bayesian Voters

If voters do not neglect correlation, then \( k(n) = 1, \forall n \). Then we have the following result:

\[16\] Many standard network models use first-order stochastic dominance to rank degree distributions. See e.g. [Erdös and Rényi](1959) or [Jackson and Rogers](2007).
**Proposition 2.** Suppose $q < \tilde{q}(\tilde{\gamma})$. If voters do not neglect correlation, then an increase in connectivity increases the probability of a correct decision.

Intuitively, when the number of connections tends to increase, each voter observes more independent signals, and thus her information becomes more precise. In particular, in state 0 voters become more pessimistic about the government’s competence. To ensure that report $r = 1$ induces sufficiently many voters to vote for the government when $\theta = 0$, the newspaper needs to make this report a stronger signal. Hence, it reduces the bias, which in turn increases the probability $\Psi$ of the correct decision.

We can also look at what happens in the limit as connectivity becomes arbitrarily large. Formally, suppose that $\gamma_N = 1$, that is, all voters have $N$ connections. Additionally, let $N \to \infty$, i.e., the number of social media connections of each voter approaches infinity. The next result shows that in the limit the probability of a correct decision approaches 1 as the number of connections becomes arbitrarily large:

**Proposition 3.** Suppose $\gamma_N = 1$. If voters do not neglect correlation, then $\lim_{N \to \infty} \Psi = 1$.

Propositions 2 and 3 say that when voters do not neglect correlation, a version of the Condorcet jury theorem holds: as each voter observes more independent signals from her friends on social media, she becomes increasingly more informed. Hence, it becomes increasingly more likely that the outcome of the vote corresponds to voters’ preferences. The presence of the biased newspaper does not change this, since the newspaper optimally reveals more information as voters become more informed. In the limit, voters observe infinitely many independent signals, which overpowers any biased signal by the newspaper.
4.2 The Effect of Correlation Neglect

We will now derive the main results of the paper: comparative statics for the case when voters neglect correlation. In that case, \( k(n) = n \). Then we have the following result:

**Proposition 4.** Suppose \( q < \tilde{q}(\gamma) \). If voters neglect correlation, then an increase in connectivity increases the probability of a correct decision if \( q > \frac{1}{2} \), reduces it if \( q < \frac{1}{2} \), and leaves it unchanged if \( q = \frac{1}{2} \).

Hence, when the prior is such that voters are ex ante willing to vote for the government, an increase in connectivity makes them more likely to reach the correct decision, as in the case without correlation neglect. However, when voters are ex ante predisposed against the government, higher connectivity makes the correct decision less probable.

We can also look at voting outcomes in the limit, as connectivity becomes arbitrarily large. As before, suppose that \( \gamma_n = 1 \), and let \( N \to \infty \). The next result shows that in this case the probability of a correct collective decision remains strictly below one in the limit:

**Proposition 5.** Suppose \( \gamma_N = 1 \). If voters neglect correlation, then \( \lim_{N \to \infty} \Psi = 1 - (1 - q) e^{-2\mu^2} < 1 \).

To see the intuition behind these results, consider first Proposition 5. When connectivity becomes very large, voters receive a large number of independent signals. All else equal, this should make them almost perfectly informed. On the other hand, voters also observe a large number of realisations of the newspaper’s report, which they treat as independent. Hence, the report becomes more persuasive. Crucially, as connectivity increases, the number of the newspaper’s reports perceived by voters is increasing at the same rate as the number
of independent signals – hence, unlike the case without correlation neglect, the newspaper’s persuasive power does not disappear as connectivity becomes large. Hence, in the limit, the probability $\Psi$ that the election produces the correct outcome is bounded away from one.

The intuition behind Proposition 4 follows from this. Recall that the probability that the election produces the correct decision equals 1 in state 1, and $1 - p^*_0$ in state 0. At each connectivity level, the newspaper selects its bias $p^*_0$ to overcome the prior and the independent signals in state 0, ensuring that the government wins exactly $\frac{1}{2}$ of all votes in that state after report 1. In the limit, voters observe infinitely many independent signals, which eliminate any effect that the prior can have on their beliefs. Hence, in the limit $p^*_0$ is independent of the prior, and so is the probability of the correct decision in state 0. When connectivity is low, however, the prior has an effect. Specifically, when $q < \frac{1}{2}$, voters are ex ante unwilling to vote for the government. To induce voters to do so, the newspaper needs to send a sufficiently credible report, that is, to select a low bias. Hence, when the prior is unfavourable to the government, the bias $p^*_0$ converges to its limit from below. Therefore, the probability $\Psi$ of a correct decision decreases as connectivity grows. On the other hand, when $q > \frac{1}{2}$, the newspaper can select a high bias when connectivity is low – thus, as connectivity increases, the bias converges to the limit from above, and hence $\Psi$ increases.

The magnitude of the probability to choose the correct outcome in the limit can be seen in Figure 2. When both the prior $q$ and the independent signal precision $\mu$ are small, the probability that voters collectively take the correct decision approaches zero, although voters receive informative and independent signals.

The results in this section imply that the Condorcet jury theorem fails when voters neglect correlation, as the probability of an incorrect decision remains positive even when
Figure 2: \( \lim_{N \to \infty} \Psi \) when \( \gamma_N = 1 \) as a function of \( q \in [0, 1] \) and \( \mu \in [0, 1] \). When \( q \) and \( \mu \) are small, the probability \( \Psi \) that voters collectively choose correctly approaches zero.

Each voter observes a large number of signals. Note that this result contrasts with a result obtained by Levy and Razin (2015a), who show that correlation neglect may actually improve information aggregation when voters have private values and signals are unbiased. In that paper, correlation neglect leads voters to overvalue the information, which might help to overcome ideological bias. In our paper, correlation neglect impedes information aggregation when some information is provided by a biased sender.

Note that if \( \mu = 0 \) – that is, if independent signals carry no information – then \( \Psi \) converges to \( q \) as \( N \to \infty \). Hence, the newspaper can achieve its first-best utility in the limit if independent signals are uninformative.\(^{17}\) In general, however, the existence of independent signals puts a bound on the maximum utility that the newspaper can attain.

\(^{17}\)Similar results emerge in Levy et al. (2018), where independent signals do not exist, and the sender can approach her first-best when the number of messages she sends approaches infinity.
5 Extensions

5.1 Partial Correlation Neglect

In our model of correlation neglect we have assumed that \( k(n) = n \) – that is, that voters perceive any number of perfectly correlated signals as an equivalent number of independent signals. One can argue that this may not always be the case. For example, the strength of correlation neglect may decrease when the number of signals becomes larger.

As connectivity becomes large, the equilibrium in the limit depends on the shape of \( k(n) \). If \( k \) becomes flat when \( n \) is large – that is, if voters cease to perceive additional reports as independent when they observe many of them – then for large \( n \) a further increase in connectivity only increases the number of independent signals that voters receive, without increasing the newspaper’s persuasive power. Therefore, a result similar to the one in the case with no correlation neglect (see Propositions 2 and 3) emerges: an increase in connectivity increases the probability of a correct collective decision, which converges to one as connectivity becomes arbitrarily large.

On the other hand, if \( k \) remains an increasing function when \( n \) is large, then an increase in connectivity continues to increase the newspaper’s ability to manipulate beliefs. Hence, the probability of a correct collective decision remains distinct from one even in the limit, and results from Section 4.2 continue to qualitatively hold.

The following proposition captures this intuition:

**Proposition 6.** Suppose \( \gamma_N = 1 \). If \( \lim_{N \to \infty} \frac{k(N)}{N} = L \) for some \( L > 0 \), then \( \lim_{N \to \infty} \Psi = 1 - (1 - q) e^{-\frac{2n^2}{\tau}} < 1. \) If \( \lim_{N \to \infty} \frac{k(N)}{N} = 0 \), then \( \lim_{N \to \infty} \Psi = 1. \)
Note that $\Psi$ in the limit is decreasing in $L$, because the greater correlation neglect is in the limit, the more persuasion power the newspaper has.

### 5.2 Generic Voting Rule

So far we have assumed that the newspaper aims to maximise the probability that at least $\frac{1}{2}$ of voters vote for the government. Suppose instead that the newspaper receives a payoff of 1 if and only if the share of voters who vote for the government is at least $\tau \in (0, 1)$\(^{18}\). For example, some voters may be partisans, who are willing to support or oppose the government regardless of the state. The newspaper then needs to persuade a sufficient fraction of the remaining voters to vote for the government.

Using the same logic as in the proof of Proposition 1, we can show that the equilibrium in this setting has the same structure as the one described in Section 3. Specifically, there exists a threshold value of $q$ such that if $q$ is larger than that value, babbling is an optimal strategy. If $q$ is lower than the threshold, the equilibrium editorial policy involves $p_1^* = 1$ and $p_0^* \in (0, 1)$ such that the government receives exactly fraction $\tau$ of votes in state 0 after report 1.

How does connectivity affect efficiency of the collective decision? We can show that the following result holds:

**Proposition 7.** Suppose babbling is not optimal. If voters neglect correlation, then an increase in connectivity increases the probability of a correct collective decision if $q > \frac{1}{2}$ and

---

\(^{18}\)It is straightforward to show that if $\tau = 0$, any strategy that does not perfectly reveal the state guarantees the newspaper a payoff of 1, as a positive mass of voters will always receive sufficiently high signals. If $\tau = 1$, the only way for the newspaper to attain a payoff of 1 with positive probability is to reveal the true state when it equals 1 – hence, truthful revelation is an optimal strategy.
\[ \tau \leq \frac{1}{2}, \text{ and decreases it if } q < \frac{1}{2} \text{ and } \tau \geq \frac{1}{2}. \]

This result parallels the result derived in Proposition 4. The latter showed that an increase in connectivity reduces \( \Psi \) if the prior is unfavourable to the government, and vice versa. Proposition 7 shows that an increase in connectivity reduces \( \Psi \) if the prior is unfavourable and, in addition, the newspaper has to overcome an unfavourable voting rule. Similarly, an increase in connectivity increases \( \Psi \) if the prior and the voting rule favour the government.

When the prior and the voting rule affect the newspaper in opposite ways – that is, if \( q > \frac{1}{2} \) and \( \tau > \frac{1}{2} \), or if \( q < \frac{1}{2} \) and \( \tau < \frac{1}{2} \) – the effect of connectivity on the probability of a correct collective decision is, in general, non-monotone. Nevertheless, we can show that for any voting rule, as connectivity becomes arbitrarily large, \( \Psi \) converges to the same limit:

**Proposition 8.** Suppose \( \gamma_N = 1 \). If voters neglect correlation, then for all voting rules \( \tau \in (0, 1) \), \( \lim_{N \to \infty} \Psi = 1 - (1 - q)e^{-2q^2} \).

Hence, the result of Proposition 5 holds regardless of the voting rule.

### 5.3 Newspaper Receives Imprecise Signals

In our analysis so far the newspaper perfectly observed the true state \( \theta \). This enabled it to choose an editorial policy with arbitrary precision, and hence it was always able to persuade voters. But the assumption of a newspaper perfectly observing the state is strict. We will now study implications of a newspaper receiving a signal \( \zeta \) with precision \( \alpha = Pr[\zeta = \theta] \in (\frac{1}{2}, 1) \).

Receiving an imprecise signal constrains the newspaper’s editorial policy’s maximum precision, and hence may also impede the newspaper’s ability to persuade a majority of voters. Intuitively, when \( \alpha \) is small, even truthfully revealing \( \zeta \) may not be sufficient to persuade
voters, if \( q < \frac{1}{2} \). If \( \mu \) is small, this may be true even independent of the state. For larger \( \mu \), however, the newspaper might be able to persuade the electorate in state 1, while in state 0 this remains impossible. The reason is that the voters’ independent signals shift the distribution of beliefs by more when \( \mu \) is larger. Finally, when \( \alpha \) is large, persuasion is possible in both states as before. In this section we focus on the case where persuasion is generally possible, meaning that independent of the true state and the network structure the newspaper is able to persuade a majority of voters by choosing an appropriate editorial policy.\(^{19}\) The following lemma establishes under which conditions this is the case:

**Lemma 2.** If voters neglect correlation, persuasion is generally possible if and only if

\[
\max \left \{ 1, \frac{1-q}{q} \right \} \frac{1-\alpha}{\alpha} \leq e^{-2\mu^2}. \quad (5)
\]

Intuitively, persuasion is generally possible when the newspaper’s signal is sufficiently precise, that is, when \( \alpha \) is close to 1. Moreover, as precision of voters’ independent signals increases, precision of the newspaper’s signal may need to increase as well for the newspaper to remain able to persuade a majority of voters. Note that the condition under which babbling is optimal is independent of the quality of the newspaper’s signal \( \alpha \), because when the newspaper babbles it does not transmit any information. The next proposition shows that all of our earlier results qualitatively remain valid when persuasion is generally possible:

\(^{19}\)If persuasion is not generally possible, two distinct cases can emerge. First, persuasion is generally not feasible. This is the case when \( \alpha \) is very small. In this case any editorial policy yields the same outcome of zero utility to the newspaper and hence also any editorial policy is optimal. Second, it is possible that the newspaper can only persuade a majority in state \( \theta = 1 \), while it is impossible when \( \theta = 0 \). The reason is that when \( \mu > 0 \) and \( \theta = 0 \) the newspaper needs to be more persuasive than when \( \theta = 1 \), but it cannot send any signal with precision greater than \((1-\alpha)/\alpha > 0\). In this case the newspaper is indifferent between all sufficiently informative editorial policies, i.e., those policies that help the government to win if \( \theta = 1 \). One of these optimal editorial policies is truthfully revealing its signal \( \zeta \), i.e., \( p^*_1 = 1 \) and \( p^*_0 = 0 \).
Proposition 9. Assume that persuasion is generally possible, i.e., \((5)\) holds, and that \(q < \bar{q}(\bar{\gamma})\). If voters neglect correlation, then an increase in connectivity increases \(\Psi\) if \(q > \frac{1}{2}\), decreases \(\Psi\) if \(q < \frac{1}{2}\), and leaves \(\Psi\) unchanged if \(q = \frac{1}{2}\). If \(\gamma_N = 1\), then \(\lim_{N \to \infty} \Psi = 1 - (1 - q) \frac{e^{\beta \mu (1-\alpha) \gamma}}{(1-\alpha) - e^{2\beta \mu}} \in (0,1)\).

Note that in the limit \(\Psi\) is decreasing in the precision of the newspaper’s signal. Thus, efficiency of the collective decision improves if the newspaper is less informed. Intuitively, the newspaper’s persuasive power is lower when it is less informed, which limits its ability to overcome the independent signals.

6 Conclusions

This paper developed a model of political persuasion in which voters observe a report about a state from a biased newspaper, as well as independent signals, and communicate this information to other voters on social media. The messages that each voter receives are thus correlated, because they are partly based on the newspaper’s report. Voters ignore this correlation and form posterior beliefs about the state, before voting based on these beliefs. As connectivity increases, the probability of an efficient voting outcome can increase or decrease, depending on the prior belief of the voters. When connectivity is large, each voter observes a large number of independent signals; nevertheless, the probability of an incorrect collective decision remains positive even in the limit.

These results have implications for contexts other than elections. Consider, for example, the question of the effect of social connectivity and communication technologies on the ability of citizens to mobilise for a protest against the government (Enikolopov et al., 2020).
Suppose that a government-run media outlet is attempting to ensure that protest participation does not exceed a certain threshold. Our results suggest that in this situation, increased social connectivity makes it harder for citizens to mobilise for a protest if the government is ex ante unpopular, while organising a protest against an ex ante popular government becomes easier.

Alternatively, consider a committee that consults a biased expert before making a decision. Should committee members be allowed to deliberate, and if yes, how much time should they be given? If deliberation involves information exchange, longer deliberation time allows more members to reveal their signals. This has a similar effect to an increase in connectivity in our model. Our results suggest that longer deliberation time is desirable if the prior probability of the expert’s preferred state is high, but not if it is low.

Future research can extend this analysis in a number of ways. One potential extension could be to endogenise the decision of whether to read the newspaper. Another way to extend the model would be to consider a dynamic setting in which voters observe messages from their social media friends before choosing which social media connections to maintain in the next stage. This would enable the model to account for endogenous formation of social networks based on common political views.
References


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Appendix

Proof of Proposition [1]. The proof consists of three steps. First, we prove the existence of a unique threshold $\bar{q}(\gamma)$ and characterise it. Then we prove an auxiliary lemma that shows a necessary condition for an editorial policy to be optimal. After that we characterise the optimal editorial policy when babbling is not optimal.

Let $V(\theta, r)$ be the share of voters that vote for the government in state $\theta$ when the newspaper sends report $r$. The newspaper receives a payoff of 1 if $V(\theta, r) \geq \frac{1}{2}$, and a payoff of 0 otherwise.

Recall that independent evidence $S$ is distributed normally with variance $n + 1$ and mean $(n + 1)\mu$ if $\theta = 1$, and mean $-(n + 1)\mu$ if $\theta = 0$. Therefore, the probability that [1] holds for a voter with $n$ connections is

$$1 - \Phi \left[ \frac{S_n(p_0, p_1) - \theta(n + 1)\mu + (1 - \theta)(n + 1)\mu}{\sqrt{n + 1}} \right]$$
Hence, if \( p_0 = 0 \), then \( V(\theta, 1) = 1, \forall \theta \in \{0, 1\} \). If \( p_0 > 0 \), we have

\[
V(\theta, 1) = \begin{cases} 
\sum_{n=0}^{N} \gamma_n \left( 1 - \Phi \left[ \frac{S_n(p_0, \mu) + (n+1)\mu}{\sqrt{n+1}} \right] \right) & \text{if } \theta = 0 \\
\sum_{n=0}^{N} \gamma_n \left( 1 - \Phi \left[ \frac{S_n(p_1, \mu) - (n+1)\mu}{\sqrt{n+1}} \right] \right) & \text{if } \theta = 1
\end{cases}
\]  

(A.1)

Similarly, the probability that \[2\] holds for a voter with \( n \) connections is

\[
1 - \Phi \left[ \frac{S_n(1 - p_0, 1 - p_1) - \theta (n+1)\mu + (1 - \theta)(n+1)\mu}{\sqrt{n+1}} \right]
\]

Hence, if \( p_1 = 1 \), then \( V(\theta, 0) = 0, \forall \theta \in \{0, 1\} \). If \( p_1 < 1 \), we have

\[
V(\theta, 0) = \begin{cases} 
\sum_{n=0}^{N} \gamma_n \left( 1 - \Phi \left[ \frac{S_n(1 - p_0, 1 - p_1) + (n+1)\mu}{\sqrt{n+1}} \right] \right) & \text{if } \theta = 0 \\
\sum_{n=0}^{N} \gamma_n \left( 1 - \Phi \left[ \frac{S_n(1 - p_0, 1 - p_1) - (n+1)\mu}{\sqrt{n+1}} \right] \right) & \text{if } \theta = 1
\end{cases}
\]  

(A.2)

Suppose that the newspaper babbles, that is, \( p_0 = p_1 \). Then, \( S_n(p_0, p_1) = S_n(1 - p_0, 1 - p_1) = \frac{1}{2\mu} \ln \frac{1-q}{q} \). Thus, we have

\[
V(0, 0) = V(0, 1) = \sum_{n=0}^{N} \gamma_n \left( 1 - \Phi \left[ \frac{\frac{1}{2\mu} \ln \frac{1-q}{q} + (n+1)\mu}{\sqrt{n+1}} \right] \right)
\]

which is strictly increasing in \( q \). When \( q = \frac{1}{2} \), \( V(0, r) = \sum_{n=0}^{N} \gamma_n (1 - \Phi [\sqrt{n+1}]) < \frac{1}{2} \).

When \( q \to 1 \), then \( V(0, r) \to 1 \). By the intermediate value theorem, this implies that there exists a unique \( \bar{q}(\gamma) \in (\frac{1}{2}, 1) \) such that \( V(0, r) = \frac{1}{2} \) if and only if \( q = \bar{q}(\gamma) \). If \( q \geq \bar{q}(\gamma) \), then \( V(0, r) \geq \frac{1}{2} \), and since \( V(1, r) > V(0, r) \), this implies that the newspaper can achieve its maximum payoff by babbling.
If \( q < \bar{q}(\gamma) \), then \( V(0, r) < \frac{1}{2} \). Then, if the newspaper babbles, the government can receive \( \frac{1}{2} \) of all votes in, at most, state 1. Hence, the payoff of the newspaper is at most \( q \). This is identical to the payoff that the newspaper receives by truthfully revealing the state, that is, setting \( p_0 = 0 \) and \( p_1 = 1 \). However, by setting \( p_1 = 1 \) and \( p_0 = \epsilon \) for some sufficiently small \( \epsilon > 0 \) the newspaper can achieve a payoff of \( q + (1 - q)\epsilon > q \). Hence, babbling is not optimal in this case. By the same reasoning, truthful revelation is not optimal, either.

When \( q < \bar{q}(\gamma) \), the newspaper selects an editorial policy such that \( p_0 < p_1 \). Then \( V(\theta, 1) > V(\theta, 0) \), \( \forall \theta \in \{0, 1\} \). Together with the fact that for each \( r \) more voters vote for the government in state 1, this implies that

\[
V(1, 1) > \max\{V(1, 0), V(0, 1)\} > \min\{V(1, 0), V(0, 1)\} > V(0, 0).
\]

The following auxiliary lemma proves another property of the optimal editorial policy:

**Lemma 3.** Suppose \( q < \bar{q}(\gamma) \). Under an optimal editorial policy \((p_0, p_1)\), \( \forall \theta \in \{0, 1\} \), we have \( V(1, 1) > V(0, 1) \geq \frac{1}{2} \) and \( V(0, 0) < V(1, 0) < \frac{1}{2} \).

**Proof.** We will show that a profitable deviation exists from any editorial policy that does not satisfy the condition in the lemma.

If \( V(1, 1) < \frac{1}{2} \), then the newspaper’s payoff is 0, so the newspaper can gain by deviating to editorial policy \((0, 1)\), which ensures it a payoff of \( q \).

If \( V(1, 1) \geq \frac{1}{2} \) and \( \max\{V(1, 0), V(0, 1)\} < \frac{1}{2} \), then the newspaper receives a payoff of 1 if and only if \( \theta = 1 \) and \( r = 1 \). Its expected utility is then \( qp_1 \leq q \). But we have shown that the newspaper can achieve a payoff strictly larger than \( q \).

If \( \min\{V(1, 0), V(0, 1)\} \geq \frac{1}{2} \) and \( V(0, 0) < \frac{1}{2} \), then the newspaper receives a payoff of
1 unless \( \theta = 0 \) and \( r = 0 \). The newspaper’s payoff then equals \( q + (1 - q)p_0 \). Furthermore, \( V(1, 0) \geq \frac{1}{2} \) implies that \( p_1 < 1 \), because otherwise \( V(1, 0) = 0 \). Now consider a deviation to an editorial policy \( (p'_0, p'_1) = \left( \frac{p_0}{p_1}, 1 \right) \). This deviation does not affect the value of \( S_n(p_0, p_1) \), and hence \( V(1, 1) \) and \( V(0, 1) \) remain unchanged. Furthermore, under this deviation in state 1 the newspaper always sends report 1. Hence, the newspaper always receives a payoff of 1 when \( \theta = 1 \), and when \( \theta = 0 \) it receives a payoff of 1 with probability \( p'_0 = \frac{p_0}{p_1} \). Its expected utility is thus \( q + (1 - q)\frac{p_0}{p_1} \), which is higher than the original utility since \( p_1 < 1 \).

Finally, \( V(0, 0) \geq \frac{1}{2} \) is impossible when \( q < q(\gamma) \).

Hence, at the equilibrium we must have \( V(1, 1) > \max \{V(1, 0), V(0, 1)\} \geq \frac{1}{2} > \min \{V(1, 0), V(0, 1)\} > V(0, 0) \), where the strict inequalities follow from the fact that \( V(1, r) > V(0, r) \), and \( V(\theta, 1) > V(\theta, 0) \). If \( V(1, 0) \geq \frac{1}{2} > V(0, 1) \), then the newspaper receives a payoff of 1 if and only if \( \theta = 1 \). Then its payoff is equal to a payoff from truthfully revealing the state which, as shown above, is suboptimal. Hence, at the equilibrium, \( V(1, 1) > V(0, 1) \geq \frac{1}{2} \), and \( V(0, 0) < V(1, 0) < \frac{1}{2} \).

Consider any editorial policy \((p_0, p_1)\) that satisfies the condition in Lemma 3. Suppose that \( p_1 < 1 \). Then the expected utility of the newspaper is \( qp_1 + (1 - q)p_0 \). Consider a deviation to an editorial policy \((p'_0, p'_1) = \left( \frac{p_0}{p_1}, 1 \right) \) (note that we have ruled out the case when \( p_1 = 0 \)). This deviation leaves \( S_n(p_0, p_1) \) unchanged, as it only depends on the ratio of \( p_0 \) and \( p_1 \). Hence \( V(1, 1) \) and \( V(0, 1) \) do not change. Thus, \((p'_0, p'_1)\) satisfies the condition in Lemma 3. The newspaper’s expected utility under \((p'_0, p'_1)\) then equals \( qp'_1 + (1 - q)p'_0 = q + (1 - q)\frac{p_0}{p_1} \), which is higher than the utility under \((p_0, p_1)\), as \( p_1 < 1 \). Hence, this deviation is profitable, so at the optimum we must have \( p_1 = 1 \).
The equilibrium expected utility of the newspaper then equals \( q + (1 - q) p_0 \). The newspaper chooses \( p_0 \) to maximise it, subject to satisfying the condition given in Lemma 3. The maximum is attained when \( V(0, 1) = \frac{1}{2} \), which, given \( p_1 = 1 \), is equivalent to the expression in the proposition. Existence is guaranteed because \( V(0, 1) \) approaches 1 as \( p_0 \to 0 \), and \( V(0, 1) < \frac{1}{2} \) when \( p_0 = 1 \) if \( q < \bar{q}(\gamma) \). Uniqueness follows from the fact that \( V(0, 1) \) is monotone in \( p_0 \).

Proof of Lemma 1. We have shown that \( \bar{q}(\gamma) \) is defined by (3). Note that

\[
\Phi \left[ \frac{1}{2} \ln \frac{1 - \frac{1}{\bar{q}(\gamma)}}{\bar{q}(\gamma)} + \frac{n + 1}{\sqrt{n + 1}} \right]
\]

is increasing in \( n \), because

\[
\frac{d}{dn} \left[ \frac{1}{2\mu} \ln \frac{1 - \frac{1}{\bar{q}(\gamma)}}{\bar{q}(\gamma)} + \frac{n + 1}{\sqrt{n + 1}} \right] = -\frac{1}{2} (n + 1)^{-\frac{3}{2}} \frac{1}{2\mu} \ln \frac{1 - \bar{q}(\gamma)}{\bar{q}(\gamma)} + \frac{1}{2} (n + 1)^{-\frac{1}{2}} \mu > 0
\]

where the inequality follows from the fact that \( \bar{q}(\gamma) > \frac{1}{2} \), which implies that \( \ln \frac{1 - \frac{1}{\bar{q}(\gamma)}}{\bar{q}(\gamma)} < 0 \).

Therefore, a shift from \( \gamma \) to \( \bar{\gamma} \) decreases the left-hand side of (3). Then to maintain the equality, \( \bar{q}(\gamma) \) must increase.

Proof of Proposition 2. Notice that it follows from (4) that \( \Psi \) decreases in \( p_0^* \). We next show the effect of connectivity on \( p_0^* \). We have

\[
S_n(p_0, 1) = \frac{1}{2\mu} \ln \frac{1 - q}{q} + \frac{1}{2\mu} \ln p_0.
\]

Rearranging the expression in Proposition 1 using the fact that \( \sum_{n=0}^{N} \gamma_n = 1 \), we can then write the expression that defines optimal bias as

\[
\sum_{n=0}^{N} \gamma_n \Phi \left[ \hat{h}(n) \right] = \frac{1}{2} \tag{A.3}
\]
where
\[ \hat{h}(n) \equiv \frac{1}{2\mu} \left( \ln \frac{1-q}{q} + \ln p_0 \right) (n+1)^{-\frac{1}{2}} + (n+1)^{\frac{1}{2}} \mu \]

Note that \( \hat{h}(n) \) is increasing in \( p_0 \). If \( \hat{h}(n) \) is increasing in \( n \), then a shift from \( \gamma \) to \( \tilde{\gamma} \) increases the left-hand side of (A.3), so \( p_0 \) has to decrease to restore equality. Hence, to prove the result, it is sufficient to show that \( \hat{h}(n) \) is increasing in \( n \). Approximating \( n \) by a continuous variable and differentiating yields
\[ \frac{d\hat{h}(n)}{dn} = -\frac{1}{4\mu} \left( \ln \frac{1-q}{q} + \ln p_0 \right) (n+1)^{-\frac{3}{2}} + \frac{1}{2} (n+1)^{-\frac{1}{2}} \mu \]

for this to be positive for all \( n \), it is sufficient to have \( \ln \frac{1-q}{q} + \ln p_0 \leq 0 \). Suppose the opposite is the case, i.e. \( \ln \frac{1-q}{q} + \ln p_0 > 0 \). Then \( \hat{h}(n) > 0 \) for all \( n \). Hence,
\[ \sum_{n=0}^{N} \gamma_n \Phi \left[ \hat{h}(n) \right] > \sum_{n=0}^{N} \gamma_n \Phi [0] = \frac{1}{2} \sum_{n=0}^{N} \gamma_n = \frac{1}{2} \]

which violates (A.3). Hence, \( \ln \frac{1-q}{q} + \ln p_0 \leq 0 \), implying that \( \frac{d\hat{h}(n)}{dn} > 0 \) for all \( n \), and hence \( \hat{h}(n) \) is increasing in \( n \).

**Proof of Proposition 3.** When \( \gamma_N = 1 \), the threshold prior is characterised by (3) as
\[ \Phi \left[ \frac{1}{2\mu} \ln \frac{1-q(\gamma)}{q(\gamma)} + (N+1) \mu \right] = \frac{1}{2} \]

Hence, \( \tilde{q}(\gamma) = \frac{1}{1+e^{-2\mu^2(N+1)}} \), so \( \lim_{N \to \infty} \tilde{q}(\gamma) = 1 \). Thus, in the limit babbling is not optimal...
for any $q \in (0, 1)$. Proposition 1 then implies that for sufficiently large $N$, $p_0$ solves

$$
\Phi \left[ \frac{1}{2\mu} \ln \frac{1-q}{q} + \frac{1}{2\mu} \ln p_0 + (N + 1) \mu \right] = \frac{1}{2}
$$

and hence

$$
p_0 = \frac{q}{1-q} e^{-2(N+1)\mu^2}
$$

which converges to 0 as $N \to \infty$. Thus, $\lim_{N \to \infty} \Psi = \lim_{N \to \infty} (1 - (1 - q) p_0) = 1$. 

Proof of Proposition 4. We know that $\Psi$ decreases in $p_0^*$. We next show the effect of connectivity on $p_0^*$. We have $S_n(p_0, 1) = \frac{1}{2\mu} \ln \frac{1-q}{q} + \frac{1}{2\mu} (n + 1) \ln p_0$. Rearranging the expression in Proposition 1, we can then write the expression that defines optimal bias as

$$
\sum_{n=0}^{N} \gamma_n \Phi [h(n)] = \frac{1}{2}, \tag{A.4}
$$

where

$$
h(n) \equiv \frac{1}{2\mu} (n + 1)^{-\frac{1}{2}} \ln \frac{1-q}{q} + \left( \frac{1}{2\mu} \ln p_0 + \mu \right) (n + 1)^{\frac{1}{2}}.
$$

Note that $h(n)$ is increasing in $p_0$. If $h(n)$ is increasing in $n$, then a shift from $\gamma$ to $\tilde{\gamma}$ increases the left-hand side of (A.4), so $p_0$ has to decrease to restore equality. If $h(n)$ is decreasing in $n$, then a shift from $\gamma$ to $\tilde{\gamma}$ decreases the left-hand side of (A.4), so $p_0$ has to increase to restore equality. If $h(n)$ is constant in $n$, then a shift from $\gamma$ to $\tilde{\gamma}$ leaves the left-hand side of (A.4) unchanged, so $p_0$ remains unchanged.

Thus, to prove the result, it is sufficient to show that for all $n$, $h(n)$ is (i) strictly increasing in $n$ if $q > \frac{1}{2}$; (ii) strictly decreasing in $n$ if $q < \frac{1}{2}$; and (iii) constant in $N$ if $q = \frac{1}{2}$. 

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Approximating $n$ by a continuous variable and differentiating yields

$$\frac{dh(n)}{dn} = -\frac{1}{4\mu} (n+1)^{-\frac{3}{2}} \ln \frac{1-q}{q} + \frac{1}{2} \left( \frac{1}{2\mu} \ln p_0 + \mu \right) (n+1)^{-\frac{1}{2}}$$

Note that $h(n)$ is increasing (decreasing) in $n$ if and only if the derivative is positive (negative) for all $n$.

To show (i), suppose that $q > \frac{1}{2}$. Then $\ln \frac{1-q}{q} < 0$. To show that $\frac{dh(n)}{dn} > 0$ for all $n$, it is then sufficient to show that $\frac{1}{2\mu} \ln p_0 + \mu \geq 0$. Suppose the opposite is the case, i.e. $\frac{1}{2\mu} \ln p_0 + \mu < 0$. Then $h(n) < 0$ for all $n$. Hence,

$$\sum_{n=0}^{N} \gamma_n \Phi [h(n)] < \sum_{n=0}^{N} \gamma_n \Phi [0] = \frac{1}{2} \sum_{n=0}^{N} \gamma_n = \frac{1}{2}$$

which violates [A.4]. Hence, $\frac{1}{2\mu} \ln p_0 + \mu > 0$, implying that $\frac{dh(n)}{dn} > 0$ for all $n$, and hence $h(n)$ is strictly increasing in $n$.

To show (ii), suppose that $q < \frac{1}{2}$. Then $\ln \frac{1-q}{q} > 0$. To show that $\frac{dh(n)}{dn} < 0$ for all $n$, it is then sufficient to show that $\frac{1}{2\mu} \ln p_0 + \mu \leq 0$. Suppose the opposite is the case, i.e. $\frac{1}{2\mu} \ln p_0 + \mu > 0$. Then $h(n) > 0$ for all $n$. Hence,

$$\sum_{n=0}^{N} \gamma_n \Phi [h(n)] > \sum_{n=0}^{N} \gamma_n \Phi [0] = \frac{1}{2} \sum_{n=0}^{N} \gamma_n = \frac{1}{2}$$

which violates [A.4]. Hence, $\frac{1}{2\mu} \ln p_0 + \mu \leq 0$, implying that $\frac{dh(n)}{dn} < 0$ for all $n$, and hence $h(n)$ is strictly decreasing in $n$.

To show (iii), suppose that $q = \frac{1}{2}$. Then $\ln \frac{1-q}{q} = 0$. To show that $\frac{dh(n)}{dn} = 0$ for all $n$, it is then sufficient to show that $\frac{1}{2\mu} \ln p_0 + \mu = 0$. Suppose the opposite is the case, i.e.
\[
\frac{1}{2\mu} \ln p_0 + \mu = B \text{ for some } B \neq 0. \text{ Then } h(n) = B (n + 1)^{\frac{1}{2}} \neq 0, \text{ and }
\]

\[
\sum_{n=0}^{N} \gamma_n \Phi[h(n)] > \sum_{n=0}^{N} \gamma_n \Phi\left[B(n + 1)^{\frac{1}{2}}\right] = \Phi\left[B(n + 1)^{\frac{1}{2}}\right] \neq \frac{1}{2}
\]

which violates (A.4). Hence, \( \frac{1}{2\mu} \ln p_0 + \mu = 0 \), implying that \( \frac{dh(n)}{dn} = 0 \) for all \( n \), and hence \( h(n) \) is constant in \( n \).

\[\Box\]

**Proof of Proposition 5.** We have shown in the proof of Proposition 3 that when \( \gamma_N = 1 \), babbling is not optimal for sufficiently large \( N \). Proposition 1 then implies that for sufficiently large \( N \), \( p_0 \) solves

\[
\Phi\left[ \frac{\frac{1}{2\mu} \ln \frac{1-q}{q} + \frac{1}{2\mu} (N + 1) \ln p_0 + (N + 1) \mu}{\sqrt{N + 1}} \right] = \frac{1}{2}
\]

Therefore,

\[
p_0 = \left( \frac{1 - q}{q} \right)^{-\frac{1}{N+1}} e^{-2\mu^2}
\]

which converges to \( e^{-2\mu^2} \) as \( N \to \infty \). This, together with the fact that \( \Psi = 1 - (1 - q) p_0 \), implies the result.

\[\Box\]

**Proof of Proposition 6.** We have shown in the proof of Proposition 3 that when \( \gamma_N = 1 \), babbling is not optimal for sufficiently large \( N \). Then, given Proposition 1 and our expression for \( S_n(p_0, p_1) \), the optimal value of \( p_0 \) solves

\[
\Phi\left[ \frac{\frac{1}{2\mu} \ln \frac{1-q}{q} + \frac{1}{2\mu} k (n + 1) \ln p_0 + (n + 1) \mu}{\sqrt{n + 1}} \right] = \frac{1}{2}
\]
and hence
\[
\ln p_0 = -\ln \frac{1 - q}{q} - (N + 1) 2\mu^2 \frac{1}{k(N + 1)}
\]

Thus,
\[
\lim_{N \to \infty} \ln p_0 = \lim_{N \to \infty} -2\mu^2 \frac{N}{k(N)}
\]

If \(\lim_{N \to \infty} \frac{k(N)}{N} = L\), then \(\lim_{N \to \infty} \ln p_0 = -2 \mu^2 \frac{N}{L}\), so \(\lim_{N \to \infty} p_0 = e^{-\frac{2\mu^2}{L}}\). If \(\lim_{N \to \infty} \frac{k(N)}{N} = 0\), then \(\lim_{N \to \infty} \ln p_0 = -\infty\), so \(\lim_{N \to \infty} p_0 = 0\), implying the result.

\[\square\]

**Proof of Proposition 7.** We know that \(\Psi\) decreases in \(p_0^*\). We next show the effect of connectivity on \(p_0^*\). When babbling is not an optimal policy, the same approach as the one used to prove Proposition 4 implies that the optimal editorial policy ensures that the government receives at least \(\tau\) votes if and only if \(r = 1\). Hence, at the optimum, the newspaper selects \(p_1 = 1\) and \(p_0\) such that

\[
\sum_{n=0}^{N} \gamma_n \left( 1 - \Phi \left[ \frac{\frac{1}{2\mu} \ln \frac{1 - q}{q} + \frac{1}{2\mu} (n + 1) \ln p_0 + (n + 1) \mu}{\sqrt{n + 1}} \right] \right) = \tau \quad (A.5)
\]

We can write (A.5) as
\[
\sum_{n=0}^{N} \gamma_n \Phi [h(n)] = 1 - \tau
\]

where \(h(n)\) is defined as in the proof of Proposition 4. Following the logic of that proof, it is sufficient to show that for all \(n\), \(h(n)\) is (i) strictly increasing in \(n\) if \(q > \frac{1}{2}\) and \(\tau \leq \frac{1}{2}\); and (ii) strictly decreasing in \(n\) if \(q < \frac{1}{2}\) and \(\tau \geq \frac{1}{2}\). Approximating \(n\) by a continuous variable...
and differentiating yields

\[
\frac{dh(n)}{dn} = -\frac{1}{4\mu} (n + 1)^{-\frac{3}{2}} \ln \frac{1-q}{q} + \frac{1}{2} \left( \frac{1}{2\mu} \ln p_0 + \mu \right) (n + 1)^{-\frac{1}{2}}
\]

Note that \( h(n) \) is increasing (decreasing) in \( n \) if and only if the derivative is positive (negative) for all \( n \).

To show (i), suppose that \( q > \frac{1}{2} \) and \( \tau \leq \frac{1}{2} \). Then \( \ln \frac{1-q}{q} < 0 \). To show that \( \frac{dh(n)}{dn} > 0 \) for all \( n \), it is then sufficient to show that \( \frac{1}{2\mu} \ln p_0 + \mu \geq 0 \). Suppose the opposite is the case, i.e. \( \frac{1}{2\mu} \ln p_0 + \mu < 0 \). Then \( h(n) < 0 \) for all \( n \). Hence,

\[
\sum_{n=0}^{N} \gamma_n \Phi[h(n)] < \sum_{n=0}^{N} \gamma_n \Phi[0] = \frac{1}{2} \sum_{n=0}^{N} \gamma_n = \frac{1}{2} \leq 1 - \tau
\]

which violates (A.4). Hence, \( \frac{1}{2\mu} \ln p_0 + \mu \geq 0 \), implying that \( \frac{dh(n)}{dn} > 0 \) for all \( n \), and hence \( h(n) \) is strictly increasing in \( n \).

To show (ii), suppose that \( q < \frac{1}{2} \) and \( \tau \leq \frac{1}{2} \). Then \( \ln \frac{1-q}{q} > 0 \). To show that \( \frac{dh(n)}{dn} < 0 \) for all \( n \), it is then sufficient to show that \( \frac{1}{2\mu} \ln p_0 + \mu \leq 0 \). Suppose the opposite is the case, i.e. \( \frac{1}{2\mu} \ln p_0 + \mu > 0 \). Then \( h(n) > 0 \) for all \( n \). Hence,

\[
\sum_{n=0}^{N} \gamma_n \Phi[h(n)] > \sum_{n=0}^{N} \gamma_n \Phi[0] = \frac{1}{2} \sum_{n=0}^{N} \gamma_n = \frac{1}{2} \geq 1 - \tau
\]

which violates (A.4). Hence, \( \frac{1}{2\mu} \ln p_0 + \mu \leq 0 \), implying that \( \frac{dh(n)}{dn} < 0 \) for all \( n \), and hence \( h(n) \) is strictly decreasing in \( n \).
Proof of Proposition 8. Analogously to our earlier reasoning, let \( \tilde{q}(\gamma, \tau) \) be the value of \( q \) such that babbling is optimal if and only if \( q \geq \tilde{q}(\gamma, \tau) \). Then when \( \gamma_N = 1 \), \( \tilde{q}(\gamma, \tau) \) is defined similarly to (3) as

\[
\Phi \left[ \frac{1}{2\mu} \ln \frac{1-\tilde{q}(\gamma, \tau)}{\tilde{q}(\gamma, \tau)} + \frac{N+1}{2} \mu \right] = 1 - \tau
\]

Hence,

\[
\tilde{q}(\gamma, \tau) = \frac{1}{1 + e^{-2\mu(\mu(N+1)+\sqrt{N+1}\Phi^{-1}(1-\tau))}},
\]

so \( \lim_{N \to \infty} \tilde{q}(\gamma, \tau) = 1 \). Thus, babbling is not optimal for any \( q \in (0, 1) \) when \( N \) is sufficiently large, and the optimal bias is defined by

\[
\Phi \left[ \frac{1}{2\mu} \ln \frac{1-q}{q} + \frac{1}{2\mu} (N+1) \ln p_0 + \frac{N+1}{2} \mu \right] = 1 - \tau.
\]

Therefore,

\[
p_0 = \left( \frac{1-q}{q} \right)^{-\frac{1}{N+1}} e^{-2\mu^2+2\mu \frac{1}{\sqrt{N+1}} \Phi^{-1}(1-\tau)},
\]

which converges to \( e^{-2\mu^2} \) as \( N \to \infty \), implying the result.

Proof of Lemma 2. Note that for any \( \mu > 0 \) and a given report sent by the newspaper \( r \), the share of voters voting for the government is greater if \( \theta = 1 \) than if \( \theta = 0 \). Hence, if it is possible to convince a majority of voters in state \( \theta = 0 \) it is also possible to do so in state \( \theta = 0 \). Hence, to prove the lemma it suffices to provide a condition such that persuasion is possible if \( \theta = 0 \).

The newspaper is maximally persuasive when it reports its signal truthfully, \( p_1 = 1 \) and
\( p_0 = 0 \). However, in this case it never sends report \( r = 1 \) in state \( \theta = 0 \). Hence, persuasion is generally possible if and only if there exists \( p_0 > 0 \) under which a majority of voters votes for the government in state \( \theta = 0 \).

When \( p_1 = 1 \) and \( p_0 = 0 \), the belief after receiving report \( r = 1 \) and independent evidence \( S \) is

\[
\pi = \frac{q}{q + (1 - q) \left( \frac{1 - \alpha}{\alpha} \right)^{n+1} e^{-2s\mu}}.
\]

Thus, a voter with \( n \) connections votes for the government if and only if the evidence she receives is at least \( S_n(1 - \alpha, \alpha) \), where \( S_n \) is defined as above. This implies that the share of voters voting for the government is

\[
\sum_{n=0}^{N} \gamma_n \left( 1 - \Phi \left[ \frac{S_n(1 - \alpha, \alpha) + (n+1)\mu}{\sqrt{n+1}} \right] \right).
\]

Using the definition of \( S_n(1 - \alpha, \alpha) \), this policy of truthful revelation persuades a majority to vote for the government if and only if

\[
\sum_{n=0}^{N} \gamma_n \left( 1 - \Phi \left[ \frac{S_n(1 - \alpha, \alpha) + (n+1)\mu}{\sqrt{n+1}} \right] \right) \geq \frac{1}{2}
\]

\[
\Leftrightarrow \sum_{n=0}^{N} \gamma_n \Phi \left[ \frac{S_n(1 - \alpha, \alpha) + (n+1)\mu}{\sqrt{n+1}} \right] \leq \frac{1}{2}
\]

(A.6)

Since \( p_0 \) must be positive to ever send \( r = 1 \) in state \( \theta = 0 \), the above inequality must be strict.

The lemma states that a necessary and sufficient condition for (A.6) to hold for any \( \gamma \) is

\[
\max \{ 1, \frac{1-q}{q} \frac{1-\alpha}{\alpha} \} < e^{-2\mu^2}.
\]

We prove both parts separately.
Sufficiency: Suppose the condition in the lemma holds. Observe that $\Phi$ is a strictly increasing function. If no voter is connected to another voter, $\gamma_0 = 1$, (A.6) holding as strict inequality simplifies to

$$\Phi \left[ \frac{1}{2\mu} \ln \frac{1-q}{q} + \frac{1}{2\mu} \ln \frac{1-\alpha}{\alpha} + \mu \right] < \frac{1}{2} \iff \frac{1}{2\mu} \ln \frac{1-q}{q} + \frac{1}{2\mu} \ln \frac{1-\alpha}{\alpha} + \mu < 0$$

$$\iff \ln \frac{1-q}{q} + \ln \frac{1-\alpha}{\alpha} < -2\mu^2 \iff \frac{1-q}{q} \frac{1-\alpha}{\alpha} < e^{-2\mu^2}$$

The condition in the lemma ensures that the last inequality holds. Hence, (A.6) holds when $\gamma_0 = 1$.

To show that (A.6) holds for any connectivity distribution, let

$$\kappa := \frac{1}{2\mu} \ln \frac{1-q}{q} + \frac{1}{2\mu} (n+1) \ln \frac{1-\alpha}{\alpha} + (n+1)\mu$$

If $\kappa$ weakly decreases in $n$, truthful revelation persuades a majority of voters for any number of connections $n$, and hence persuasion is generally possible. Approximate $n$ by a continuous variable and take the derivative of $\kappa$ with respect to $n$:

$$\frac{\partial \kappa}{\partial n} = \frac{1}{2\mu} \ln \frac{1-\alpha}{\alpha} + \mu$$

The condition in the lemma ensures that this is negative. Thus, if $\max\{1, \frac{1-q}{q}\} \frac{1-\alpha}{\alpha} < e^{-2\mu^2}$, persuasion is feasible for any $\gamma$, and hence it is generally possible. Note that when $\alpha = 1$, the inequality always holds as $e^{-2\mu^2} > 0$.

Necessity: Assume the condition is violated. If $\max\{1, \frac{1-q}{q}\} \frac{1-\alpha}{\alpha} > e^{-2\mu^2}$ and $\max\{1, \frac{1-q}{q}\} = 1$, $\kappa$ linearly increases in $n$ and converges to positive infinity. This means that for $\gamma_N = 1$
and sufficiently large $N$ persuasion is impossible, because (A.6) does not hold. On the other hand, if $\max\{1, \frac{1-q}{q} \frac{1-\alpha}{\alpha}\} > e^{-2\mu^2}$ and $\max\{1, \frac{1-q}{q}\} = \frac{1-q}{q}$, then for $\gamma_0 = 1$ persuasion is impossible. Finally, note that when $\max\{1, \frac{1-q}{q} \frac{1-\alpha}{\alpha}\} = e^{-2\mu^2}$, (A.6) holds with equality, which, as explained above, implies that persuasion is not generally possible. Hence, $\max\{1, \frac{1-q}{q} \frac{1-\alpha}{\alpha}\} < e^{-2\mu^2}$ is necessary for persuasion to be possible.

**Proof of Proposition 9.** Given that the state is $\theta = 1$, the newspaper sends report $r = 1$ with probability $z_1 = \alpha p_1 + (1 - \alpha)p_0$. In state $\theta = 0$ this probability is $z_0 = \alpha p_0 + (1 - \alpha)p_1$. Thus, the belief of a voter with $n$ connections and independent evidence $S$ after report $r$ are

$$\pi(z_0, z_1 \mid r = 1) = \frac{q}{q + (1 - q) \left(\frac{z_0}{z_1}\right)^{n+1}} e^{-2\mu S}$$

and

$$\pi(z_0, z_1 \mid r = 0) = \frac{q}{q + (1 - q) \left(\frac{1-z_0}{1-z_1}\right)^{n+1}} e^{-2\mu S}.$$ 

These beliefs are identical to those in Section 3 under full correlation neglect except that the newspaper now chooses $z_i$ instead of $p_i$. Recall that we assume without loss of generality that $p_1 \geq p_0$, which implies that $z_0 \leq z_1$.

Using similar logic as in the proof of Proposition 1 we can show that the unique optimal editorial policy must have $p_1 = 1$ and $p_0 \in (0, 1)$ such that $V(0, 1) = \frac{1}{2}$. Note that this also implies that Proposition 4 remains valid if persuasion is generally possible. The proof is identical, replacing $p_i$ by $z_i$.
Finally, if \( \gamma_N = 1 \), the optimal editorial policy follows from

\[
1 - \Phi \left[ \frac{S_N(z_0, 1) + \mu(N + 1)}{\sqrt{N + 1}} \right] = \frac{1}{2} \iff S_N(z_0, 1) = -\mu(N + 1).
\]

Solving for \( p_0 \) we find

\[
p_0 = \frac{\alpha \left( \frac{1-q}{q} \right)^{\frac{1}{N+1}} - e^{2\mu^2} + \alpha e^{2\mu^2}}{\alpha \left( \frac{1-q}{q} \right)^{\frac{1}{N+1}} - \left( \frac{1-q}{q} \right)^{\frac{1}{N+1}} + \alpha e^{2\mu^2}}
\]

In the limit, as \( N \to \infty \), we get

\[
\lim_{N \to \infty} p_0 = \frac{\alpha - e^{2\mu^2} + \alpha e^{2\mu^2}}{\alpha - 1 + \alpha e^{2\mu^2}}
\]

This is strictly between 0 and 1 if \( \frac{1-\alpha}{\alpha} < e^{-2\mu^2} \), which, by Lemma 2, holds when persuasion is generally possible. \( \square \)