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Campaign Contests*

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Abstract

I develop a formal model of political campaigns in which candidates choose how to distribute their resources over two different policy issues. I assume that campaigning on an issue has two simultaneous effects, both rooted in social and cognitive psychology: It increases the perceived quality of the advertising candidate in that issue (persuasion) and it makes the issue more salient (priming), thereby increasing the issue’s perceived importance to the voters. I show that, unlike in the extant literature, interior pure strategy equilibria, in which every candidate campaigns on all issues, exist, if persuasion is sufficiently effective. However, candidates “specialize ” by spending more than their contender on the issue, in which they hold a comparative advantage. Further, I show that an issue receives more aggregate spending, if it becomes more important or if voters’ opinions on candidates’ qualities in the issue become weaker. A candidate increases his vote share during the campaign contest, if he has a comparative advantage on the issue that receives more aggregate spending. The contest may therefore be biased in one candidate’s favor and an a priori less popular candidate might be the actual odds on favorite.

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1 Introduction

There is an election between two candidates upcoming and Candidate 1 is currently more popular in the polls than Candidate 2, leading with 5 percentage points over his opponent. Both candidates have identical campaign budgets and employ their funds with equal efficiency. There are no shocks to voters’ preferences and there is no randomness in the campaign. Can we then conclude that Candidate 1 is going to win the election? In this paper I show that the answer to that question is “no.” The reason for this is that campaign contests are often biased in one candidate’s favor; they benefit the candidate who has a comparative advantage on the issue that receives the greatest amount of campaign spending. If a candidate trails his opponent by not too great a margin at the outset of the campaign contest, he thus could be likely to come out ahead on Election Day.

Political campaigns in many western democracies are often best described as contests in which candidates and political parties spend significant amounts of time and money—or effort—in an attempt to influence voters’ decisions at the ballot. The prime example is the US, where spending during these campaign contests by the two main candidates during presidential elections has experienced an average growth rate of 24.6 per cent between campaigns over the period from 1984 to 2016 and has reached a maximum of more than $1.2 billion in 2012. Candidates allocate these funds over multiple policy issues and how issues are strategically targeted matters for electoral outcomes. Nevertheless, the vast majority of the literature studying campaign contests focusses on one-dimensional campaigns in which campaign spending creates valence. Another important strand of literature studies multi-dimensional campaign contests where candidates compete through issue strategic selection. In this class of models, the function of campaigning is to prime issues, i.e., to strategically manipulate which issues the voters consider important on Election Day. Both of these approaches have led to a series of interesting results and have deepened our understanding of how campaign contests are fought and what consequences they have. But, taken in isolation, they also have significant shortcomings.

In the current paper I combine features of these two approaches to political campaigns to further our understanding of their workings and consequences. There are two candidates competing in a campaign contest and who need to decide how to allocate their campaign resources to the different policy issues. Campaigning on an issue has two simultaneous effects: it persuades the voters of the issue specific quality of the advertising candidate, and it primes the issue, thereby manipulating voters’ issue importance ranking. Persuasion is similar to creating issue specific valence and hence relates to the literature on endogenous valence, while priming relates to the literature studying strategic issue selection. Voters are heterogeneous in their candidate evaluations and may also differ in their issue importance weights. Candidates choose an allocation of their budgets that maximizes their respective vote shares.

1Calculated using data from https://www.fec.gov/data/. Recent studies point to the importance of campaign contests, see for example Erikson and Palfrey (1998, 2000) or Franz and Ridout (2007, 2010). Kang et al. (2018), Arbour (2014), Belanger and Meguid (2008), or Sigelman and Buell (2004) show that spending is distributed over a whole range of important issues in the US, while Wagner and Meyer (2014), Meyer and Wagner (2016), or Dolezal et al. (2014) show the same for campaigns in Europe.


Combining the two existing approaches generates a set of novel testable predictions:

- The campaign contest has a unique interior pure strategy Nash equilibrium. Thus, both candidates campaign on both issues.

- A candidate campaigns with greater intensity on an issue than his contender, if he has a comparative advantage on that issue.

- Spending on an issue increases when the issue becomes more important or when voters’ conviction regarding candidates’ qualities on the issue decreases.

- A candidate increases his electoral support during the campaign contest, if he has a comparative advantage on the issue on which aggregate spending is greater. Campaign contests tend to structurally benefit one candidate over the other.

The first main result states that interior pure strategy equilibria, in which both candidates campaign on both issues, generally may exist. This is in contrast to the extant theoretical literature studying issue selection, where candidates never campaign on the same issues and hence “diverge.” The result can hence theoretically rationalize what most of the empirical literature on issue selection finds. The second main result shows that results from Amorós and Puy (2013) and Aragonès et al. (2015) remain valid even though the framework was changed significantly by allowing for persuasion. However, the current model predicts a more nuanced version of their results, because candidates do not spend all their resources on the issue of their comparative advantage. The third result follows from a contest theoretical logic, and while I am not aware of a similar result in the literature, the underlying intuitions are familiar from other contexts. The last of the main results shows that an intuition by Bartels (1992), who surmised that campaigning will not help any candidate unlike there are significant differences in campaign budgets or campaigning technologies, does not necessarily apply. If campaigning both persuades voters and primes issues, the contest tends to benefit one of the candidates even if both have similar means or use similar campaigning technologies.

The paper is organized as follows. The remainder of this section places the paper in the context of the relevant literature. The next section discusses how campaign contests are likely to affect voters’ attitudes towards candidates. Section 3 introduces a model of campaign contests, and Section 4 studies equilibrium campaigning. Section 5 derives implications of campaign contests for candidate selection. Section 6 concludes. All proofs are contained in the appendix.

Related Literature. The paper contributes to the extensive literature studying competitive vote buying or endogenous valence, for example Snyder (1989), Klumpp and Polborn (2006), Herrera et al. (2008), Meirowitz (2008), Ashworth and Bueno de Mesquita (2009), Serra (2010), Denter and Sisak (2015), Iaryczower and Mattozzi (2012), Boyer et al. (2017), Balart et al. (2018), or Casas (2018). In contrast to these papers, I study the allocation of a campaign budget over different issues.

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5The following quote due to Sigelman and Buell (2004) nicely summarizes the gap between theoretical predictions and empirical evidence in the issue selection literature: “There is no shortage of explanations for why issue convergence is such a rare commodity in American campaigns. Perhaps surprisingly, though, there is a shortage of convincing evidence that issue convergence really is a rare commodity.”

6See for example Konrad (2009) for an overview.
Because campaigning changes how voters evaluate candidates on an issue, the model is a version of a Blotto game. A Blotto game is a situation in which players allocate resources to a number of different contests (here issues), and typically the player spending most on a certain contest wins it for sure. Papers contributing to this literature are the classical treatise of [Borel (1953), Shubik and Weber (1981), Roberson (2006), Chowdhury et al. (2013), Kovenock and Roberson (2011), or Hortala-Vallve and Llorente-Saguer (2012)]. This kind of models has been used to model electoral competition before, for example by [Myerson (1993), Laslier and Picard (2002), or Boyer et al. (2017)]. The current paper differs because the value of a battlefield is determined endogenously, since an issue’s importance increases in the amount of spending directed to it. Moreover, unlike in these papers, in the current paper success on individual battlefields—or issues—is not modelled as all-pay auctions but as a smooth function of campaign spending. This implies that, unlike in this literature, pure strategy Nash equilibria typically exists.

The paper also contributes to the literature on strategic issue selection with endogenous issue weights, examples include [Petrocik (1996), Riker (1996), Amorós and Puy (2013), Aragonés et al. (2015), or Dragu and Fan (2016)]. The current paper innovates by introducing persuasion: the way voters evaluate candidates’ issue specific competence or quality is determined endogenously during the campaign contest. In this literature, candidates never campaign on the same set of issues in equilibrium. Empirical research, however, refutes this conclusion, and finds that some form of “convergence” on the issues is the norm rather than the exception, see [Sigelman and Buell (2004), Damore (2005), Kaplan et al. (2006), or Green and Hobolt (2008)]. The model presented in the current paper allows for imperfect convergence and is thus a step forward in reconciling theory and data.

While the above literature studied non-informative campaigns, some papers focus on candidates’ incentives to provide information during campaign contests. [Gul and Pesendorfer (2012)] study how parties release information regarding a payoff relevant state variable over time and [Alonso and Cámara (2016) or Denter et al. (2020)] study how a biased information provider may influence voters’ decision at the ballot using Bayesian Persuasion. [Polborn and Yi (2006)] study informative positive and negative campaigning. In all these papers the policy space is one-dimensional and thus, unlike in the current paper, issue selection as well as issue priming cannot play a role. [Egorov (2015) and Basu and Knowles (2018)] study informative campaigning with two-dimensional policy spaces. [Egorov (2015)] studies the incentives of an incumbent and a challenger to campaign either on the first or on the second issue in a model in which campaigning directly reveals information about a candidate’s competence and when issue selection also signals information regarding one’s privately known competence on the issue one campaign on. In contrast to his paper, in [Basu and Knowles (2018)] candidates can campaign on both issues at once, but they drop the assumption that voters may draw inference about a candidate’s competence also from issue selection. As in the current paper, both [Egorov (2015) and Basu and Knowles (2018)] show that candidates may choose to campaign on the same issues in equilibrium. However, unlike in the current paper, in their papers campaigning does not prime policy issues, which creates the strict incentive found in the literature to campaign on different issues in the first place. Moreover, in contrast to these papers, in the current paper neither candidates nor voters are restricted in the sense that they can only campaign on or observe campaign spending on one policy issue. Finally,

\[7\] An exception is Hortala-Vallve and Llorente-Saguer (2012).
none of these papers studies the political consequences of the campaign contest for candidate selection on Election Day.

2 Effects of Campaigning: Priming and Persuasion

Given the multi-dimensionality of campaign contests, there are two distinct ways in which they may affect voters. First, campaigns could have an across-issues dimension and may change how voters view the different issues. In particular, campaigns may change how voters prioritize issues or which weights voters attach to the different issues. Second, there may be a within-issue dimension as well. That is, candidates’ campaigning on a given issue could change the way voters see candidates’ qualities on that issue. This distinction between across- and within-issue effects is similar to the distinction used by Bartels (2006), who differentiates between priming and persuasion. We will follow Bartels by referring to the different effects of campaigning in the same way. In the remainder of this section I discuss how persuasion and priming are likely to change voters’ attitudes.

Priming. The fact that priming an issue can raise this issue’s importance relative to other issues is well known in political science, see for example Bartels (2006) or Aragonès et al. (2015) and the respective references provided therein. This effect has its roots in cognitive psychology. Priming is a cognitive process that activates accessible categories in the mind of a person. Exposure to a stimulus makes the related categories of the stimulus easier accessible and the categories become more important in the mind of individuals. Smith and Mackie (2007) put it like this: “[…] anything that brings an idea to mind—even coincidental, irrelevant events—can make it accessible and influence our interpretation of behavior” (p. 67). In the specific example of a political campaign, priming makes an issue more salient and thus individuals evaluate the issue as more relevant for making decisions (see Iyengar and Kinder 1987 or Weaver 2007). Priming is hence closely related to the theory of agenda setting (see for example the discussion in Willnat 1997). When evaluating two issues, the primed issue is still in the memory and becomes more important. Priming can therefore “alter the standards by which people evaluate election candidates” (Severin and Tankard 1997). In the sequel, I will assume that campaigning on an issue increases this issue’s relative importance and decreases the importance of the other issues.

Persuasion. There are many reasons to suspect that campaigning changes how voters view candidates conditionally on an issue. One simple reason could be that campaigning provides information regarding policy platforms. If voters are risk averse, this will on average increase the advertising candidate’s issue specific evaluation, as uncertainty is reduced. Similarly, issue specific political persuasion tends to improve how voters esteem a certain candidate on that issue. Persuasion could take the form of providing costly evidence of a candidate’s issue specific valence, for example by highlighting a candidate’s professional background as a business leader or veteran. Skaperdas and Vaidya (2012) study this kind of persuasion. Similarly, persuasion may take the form of Bayesian persuasion as pioneered by Kamenica and Gentzkow (2011). An application of this model to political persuasion is for example Alonso and Camara (2016). In both cases, campaigning on an issue will at least in expectation improve a candidate’s assessment on that issue.
Finally, there is reason to expect a positive persuasion effect of campaigning based on what psychologists call the *mere-exposure effect*. According to this effect “repeated exposure to an object results in greater attraction to that object” (Hogg and Vaughan 2008, p. 170), because it creates familiarity. The effect was first systematically described by Zajonc (1968) and there is ample evidence of its importance for human attitudes, see for example Bornstein (1989), Tom et al. (2007), Moon et al. (2009), or Fang et al. (2007). The importance of the mere-exposure effect for advertising is well-established, see for example Yoo (2008) or Lee et al. (2015). Moreover, Moorthy and Hawkins (2005) show that mere repetition of an ad increases a product’s perceived quality.

There are sufficient reasons to believe that persuasion effects are relevant and that voters esteem a candidate more if the candidate campaigns on the issue. In the following I will assume that by campaigning on an issue a candidate indeed increases his perceived quality.

### 3 Campaign Contests

In this section I introduce a theoretical model of campaign contests. Two politicians $j \in \{D, R\}$ compete in a campaign for a political office by exerting effort. While effort could mean many different things, for specificity I stick to the interpretation of buying TV advertising. There is a measure-one continuum of voters, indexed by $v$. Voters care about two policy issues $i \in \{1, 2\}$. They assign to each candidate a relative quality belief $\theta^i_{v,j} \in (0, 1)$, where relative quality is defined in a way such that $\theta^i_{v,D} + \theta^i_{v,R} = 1$. It is useful to define $\theta^i_{v,D} \equiv 1 - \theta^i_{v,R}$ and work with this convention in the following. To assess the overall relative quality of a politician, voters assign a weight $\varphi_v \in (0, 1)$ to issue 1 and $1 - \varphi_v$ to issue 2. Voters’ assessments of candidates’ relative quality on issue $i$ are distributed on $\Theta^i = [\theta^i, \bar{\theta}^i] \subset (0, 1)$. Similarly, issue importance assessments are distributed on $\Omega = [\varphi, \bar{\varphi}] \subset (0, 1)$. Every voter $v$ is hence completely described by $s_v \in S \equiv \Theta^1 \times \Theta^2 \times \Omega$. Voters’ assessments of candidates and issues are uncorrelated with respective marginal distributions $I(\varphi_v)$ and $C^i(\theta^i_v)$.

Following the literature, voters are assumed to have *weighted issue preferences* (e.g. Krasa and Polborn 2010) and evaluate candidates as follows:

\[
\begin{align*}
  u_D(x, s_v) &= c(x^1, \theta^1_v)w(x, \varphi_v) + c(x^2, \theta^2_v)(1 - w(x, \varphi_v)), \\
  u_R(x, s_v) &= (1 - c(x^1, \theta^1_v))w(x, \varphi_v) + (1 - c(x^2, \theta^2_v))(1 - w(x, \varphi_v)),
\end{align*}
\]

where $c(x^i, \theta^i_v) \in [0, 1]$ is voter $v$’s evaluation of Candidate $D$’s relative quality on issue $i$, taking into account campaign spending $x^i \equiv (x^i_D, x^i_R)$ and the initial evaluation $\theta^i_v$. Similarly, $w(x, \varphi_v) \in [0, 1]$ is the voter’s evaluation of the importance of issue 1, depending on campaign spending $x \equiv (x^1, x^2)$ and the initial evaluation $\varphi_v$. It follows that when $u_D(x, s_v) > \frac{1}{2}$, voter $v$ prefers $D$ over $R$, and vice versa if $u_D(x, s_v) < \frac{1}{2}$. A voter is indifferent between $D$ and $R$ when $u_D(x, s_v) = \frac{1}{2}$. Moreover, $u_R(x, s_v) = 1 - u_D(x, s_v)$.

As mentioned before, I follow Bartels (2006) and assume that campaigning has the two simultaneous effects described above, *persuasion* and *priming*. Persuasion implies that the assessment of a candidate’s

\[8\] Interestingly, the mere-exposure effect also has a significant impact on how researchers value the quality of different academic journals (Serenko and Bontis 2011).
quality is improving in the number of published TV ads, \( x^i_j \). Priming leads to a reassessment of issues’ relative importance. I assume the following priming technology:

**Assumption 1.** Campaigning changes a voter’s beliefs about issues’ relative importance in the following way:

\[
w(x, \varphi_v) = \max \{ \min \{ \varphi_v + \eta \left( g \left( x^1_D + x^1_R \right) - g \left( x^2_D + x^2_R \right) \right), 1 \}, 0 \}
\]

for \( g(0) = 0 \), \( g'(x) > 0 \), \( g''(x) \leq 0 \), and \( \eta \geq 0 \).

Spending on an issue increases that issue’s relative importance and decreases the importance of all other issues. \( \eta \) is a parameter that measures the overall effectiveness of priming and will be useful for comparative statics.

Voter \( v \)’s after-campaigning assessment of candidates’ relative quality on issue \( i \) is \( c(x^i, \theta^i_v) \). I assume the following persuasion technology:

**Assumption 2.** \( c(x^i, \theta^i_v) \in [0, 1] \) is \( C^2 \) in all arguments and has the following properties:

1. **Concavity:** \( c(x^i, \theta^i_v) \) is strictly concave and increasing in \( x^i_D \) and strictly convex and decreasing in \( x^i_R \).

2. **Symmetry:** \( c(x, y, \theta^i_v) = 1 - c(y, x, 1 - \theta^i_v) \).

3. **Neutrality:** \( c(x, x, \theta^i_v) = \theta^i_v \).

Assumption 2 is very similar to what Dixit (1987) or Hoffmann and Rota-Graziosi (2012) impose on contest success functions. Each player’s effort has a positive but diminishing marginal effect on his issue specific relative evaluation. \( c(x^i, \theta^i) \) is symmetric in the sense that if we exchange candidates’ efforts and initial evaluations, we also exchange their post campaigning evaluation. Moreover, if both choose the same level of effort on issue \( i \), their relative evaluation is unchanged; efforts neutralize each other. Throughout most of the analysis, that is with the exception of Section 4.1 and some examples, I will impose another assumption, namely that the first unit of persuasive effort is very effective, \( \lim_{x^i_j \to 0} \frac{\partial c(x^i, \theta^i_v)}{\partial x^i_j} = +\infty \). This assumption precludes the existence of corner equilibria, which are not the focus of this paper, and makes the analysis slightly more convenient.

Lemma in Appendix A.3 shows that Assumption 2 implies that given a symmetric spending profile on issue \( i \), \( x^i_D = x^i_R \), the marginal impact of campaign spending on candidates’ relative evaluation depends on voter \( v \)’s initial assessment through \( \theta^i_v(1 - \theta^i_v) \), which one can interpret as a measure of the voter’s undecidedness on issue \( i \). Throughout the analysis I will assume that in such a situation it is weakly easier to influence a voter who is undecided than a voter who has a clear favorite on issue \( i \):

**Assumption 3.** The marginal impact of campaign spending on a voter’s relative evaluation of candidates on issue \( i \) is weakly increasing in a voter’s undecidedness \( \theta^i_v(1 - \theta^i_v) \). Formally, \( \frac{\partial^2 c(x^i, \theta^i_v)}{\partial x^i_D \partial \theta^i_v} \bigg|_{x^i_D=x^i_R} \geq 0 \) if \( \theta^i_v \leq \frac{1}{2} \), and \( \frac{\partial^2 c(x^i, \theta^i_v)}{\partial x^i_D \partial \theta^i_v} \bigg|_{x^i_D=x^i_R} \leq 0 \) else.

Starting at \( \theta^i_v = \frac{1}{2} \), when \( \theta^i_v \) gets closer to zero or one, the effectiveness of campaigning does not increase. Thus, the assumption formalizes the words of Festinger et al. (1956): “A man with a conviction
is a hard man to change.” A similar assumption is made explicit in a recent paper by Balart et al. (2018), where the effectiveness of campaign spending depends on and decreases in platform polarization. Similarly, Ashworth and Bueno de Mesquita (2009) show that if voters have concave policy utility, the marginal effectiveness of campaign spending decreases in policy divergence. $\theta_v(1-\theta_v)$ can be interpreted as a measure of policy divergence, where $\theta_v(1-\theta_v) = \frac{1}{4}$ implies candidates choose the same platforms on issue $i$ and therefore candidates converge completely. When for a certain voter $\theta_v(1-\theta_v)$ gets closer to zero, this voter’s conviction increases.

We need one final assumption to facilitate the analysis. Given Assumptions 2 and 3, it is not guaranteed that a unique pure strategy equilibrium exists even when $\eta = 0$, that is when the game becomes a standard contest with exogenous prizes or weights. The next assumption constrains the magnitude of the cross derivatives of $c^i$ and is sufficient to guarantee this:

**Assumption 4.** For all $x^i \in [0, B]^2$ and for all $\theta^i \in (0, 1)$,

$$
\frac{\partial^2 c(x^i, \theta^i_v)}{\partial (x^i_D)^2} \frac{\partial c(x^i, \theta^i_v)}{\partial x^i_R} > \frac{\partial^2 c(x^i, \theta^i_v)}{\partial x^i_D \partial x^i_R} > 
\frac{\partial^2 c(x^i, \theta^i_v)}{\partial x^i_R^2} \frac{\partial c(x^i, \theta^i_v)}{\partial x^i_D}.
$$

Assumptions 2 to 4 define something that in other contexts is often called a contest success function, see for example Skaperdas (1996). Most standard contest success functions are special cases of $c(x^i, \theta^i_v)$, for example the generalized logit, or Tullock, contest success function studied by Snyder (1989), Skaperdas and Grofman (1995), Klumpp and Polborn (2006), Balart et al. (2018), or Bouton et al. (2018), or the tournament model with potential head starts studied by Lazear and Rosen (1981), Herrera et al. (2008), or Denter and Sisak (2015).

Voting is probabilistic and the probability that a voter casts her ballot for candidate $j$ is $u_j(x, s_v) \in (0, 1)$. This implies that voter $v$ is more likely to cast his ballot for $D$ if $u_j(x, s_v) > \frac{1}{2}$, for $R$ if $u_j(x, s_v) < \frac{1}{2}$, and chooses both politicians with equal probabilities when she is indifferent, i.e., when $u_j(x, s_v) = \frac{1}{2}$. Campaign spending has no immediate marginal costs but candidates are endowed with a use-it-or-lose-it budget $B > 0$ that they can distribute over the different issues. Since it is always beneficial to increase spending on one of the two issues, in equilibrium the budget constraint needs to hold with equality, and hence $x^2_j = B - x^1_j$. Each candidate allocates his campaign budget to the different issues to maximize his vote share $\pi_j(x) = E[u_j(x, s_v)]$, where $E[\cdot]$ is the expectation operator. The equilibrium concept is Nash equilibrium.

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9 For example, assume $\theta^i_v = \frac{1}{2} (1 - (b^i_v - p^i_D)^2 + (b^i_v - p^i_R)^2) \in [0, 1]$, where the policy space is $[0, 1]$, $b^i_v \in [0, 1]$ is voter $v$’s ideal point on issue $i$ and $p^i_D = p^i + \epsilon$ and $p^i_R = p^i - \epsilon$ for appropriately chosen $\epsilon$, which can be interpreted as a measure of platform divergence and when $\epsilon = 0$ candidates choose the same platforms and thus converge perfectly. Then $\theta^i_v(1-\theta^i_v)$ decreases in platform polarization $\epsilon$.

10 In tournament models Assumption 4 holds only when the variance of the additive noise variable is sufficiently large, as otherwise the marginal impact of campaign spending becomes zero at one point.
4 Equilibrium Campaigning

We now study equilibrium behavior by both candidates in the campaign contest. To derive intuitions for how the two main effects of campaigning influence candidates' incentives, I begin with an analysis of the two effects in isolation. That is, I first study behavior in a campaign contest when campaigning only persuades but leaves issues' relative importance unchanged, and then I continue by studying the other polar case, i.e., a campaign contest that only primes issues but does not persuade voters. As we will see, the two effects have very different consequences for equilibrium campaigning, and these differences relate to comparative advantages, which I define as follows:

**Definition 1 (Comparative Advantage).** Let

\[
\sigma_i \equiv E[\theta^i_v] - \bar{\theta},
\]

where \( \bar{\theta} \equiv \frac{1}{2} E[\theta^1_v + \theta^2_v] \). Candidate D has a comparative advantage on issue i if \( \sigma_i > 0 \). If \( \sigma_i < 0 \), Candidate R has a comparative advantage in i and if \( \sigma_i = 0 \), no candidate has a comparative advantage on that issue.

By the nature of comparative advantages, it is not possible that one candidate has a comparative advantage on all issues. This follows directly from \( \sigma^1 + \sigma^2 = 0 \). In particular, either every candidate has exactly one comparative advantage or no candidate has a comparative advantage. For example, in the simplest case with a single voter comparative advantage boils down to a comparison of \( \theta^1 \) and \( \theta^2 \). For example, if \( \theta^1 = 0.8 \) and \( \theta^2 = 0.6 \), D has an absolute advantage on all issues and a comparative advantage on issue 1.

4.1 Two Polar Benchmarks

To motivate the study of an integrated model with both priming and persuasion, I now study both effects in isolation. Hence, I now study a campaign contest that either only primes or only persuades. This will deliver two important benchmarks and shows why an integrated model with both effects is necessary to generate the results of this paper:

**Proposition 1.**

1. Persuasion: Let \( \eta = 0 \). The campaign contest has a unique interior Nash equilibrium in pure strategies in which candidates converge completely, i.e., they spend the same amount on all issues \( i \), \( x^i_D = x^i_R = x^i \in (0, B) \).

2. Priming: Let \( \eta > 0 \) and \( c(x^i, \theta^i_v) = \theta^i_v \) for all \( x^i \in [0, B]^2 \). If candidates have comparative advantages, the campaign contest has a unique Nash equilibrium in which each candidate spends all of his budget on the issue where he has the comparative advantage. Therefore, candidates diverge completely, i.e., they spend never campaign on the same issue \( i \), \( x^i_D \cdot x^i_R = 0 \). When candidates have no comparative advantage, any spending profile \((x^1_D, x^1_R) \in [0, B]^2\) is an equilibrium.

Persuasion leads candidates to adopt identical strategies and they will converge completely in any equilibrium. Priming has the opposite effect and taken in isolation leads candidates to diverge perfectly. Both are extreme predictions and fail to explain the empirical evidence that was mentioned in the
introduction. In particular, [Sigelman and Buell (2004)] showed that imperfect convergence is the best description of observed campaigning behavior and that one candidate tends to spend more on a certain subset of issues and less on the remaining ones. This implies equilibrium should be interior, i.e., both candidates campaign on both issues, but with differing intensities. As we will see in the sequel, an integrated model with both priming and persuasion can rationalize such behavior. Furthermore, as we will see in Section 5, the interaction of both effects leads to interesting novel results.

4.2 An Integrated Model

In this section I now turn to study the integrated model that allows both effects to be present in a campaign contest. The following proposition establishes that an interior pure strategy Nash equilibrium exists:

**Proposition 2.** The game has an interior pure strategy Nash equilibrium. Moreover, for sufficiently small but positive \( \eta \) and \( \frac{\partial^2 c(x^i, \theta)}{\partial x_D \partial x_R} \), the game has a unique Nash equilibrium.

The proposition shows that interior pure strategy equilibria may well exist in a campaign contest. Thus, adding persuasion to a standard priming model qualitatively changes the campaign contest’s equilibrium in an important way: Both candidates spend positive amounts on both issues. The assumption of an infinite marginal product of the first unit of persuasion is not necessary to arrive at this result, and it suffices that the first unit of persuasion is sufficiently effective. Moreover, a sufficient condition for uniqueness of Nash equilibrium is that \( \eta \) and \( \frac{\partial^2 c(x^i, \theta)}{\partial x_D \partial x_R} \) are sufficiently small. The reason is that in this case both candidates’ best response functions are nowhere “too steep.” Condition (A.3) in Appendix [A.2](#) provides the details for this to be the case. For the remainder of the paper I assume that this condition holds.

The following example shows that \( \eta \) need not be really close to zero to guarantee uniqueness and that priming can be quite effective:

**Example 1:** Assume that \( B = 1 \) and that

\[
w(x, \varphi) = \max \left\{ \min \left\{ \varphi_v + \eta \left( x_D^1 + x_R^1 - x_D^2 - x_R^2 \right), 1 \right\}, 0 \right\}
\]

as well as

\[
c(x_i, \theta^i) = \max \left\{ \min \left\{ \theta_v^i + \kappa \left( \left( x_D^i - \frac{1}{2} (x_D^i)^2 \right) - \left( x_R^i - \frac{1}{2} (x_R^i)^2 \right) \right), 1 \right\}, 0 \right\}
\]

for some \( \kappa > 0 \) and \( \eta > 0 \). Moreover, let \( \theta^i + \frac{\zeta}{2} < 1, \theta^i - \frac{\zeta}{2} > 0, \varphi + 2\eta < 1, \varphi - 2\eta > 0 \). The individual decision problems of the candidates are strictly concave and the campaign contest has a unique Nash equilibrium in pure strategies. For large enough \( \kappa \), the equilibrium is interior, i.e., \( x^i_j \in (0, 1) \).

The proofs to all the examples in the paper can be found in Appendix [B](#). Note that the assumptions in the example define an upper boundary \( \bar{\eta} = \min \left\{ \frac{1 - \varphi}{2}, \frac{\varphi}{2} \right\} \leq \frac{1}{3} \) for \( \eta \). Whenever, \( \eta < \bar{\eta} \), the campaign contest has a unique pure strategy Nash equilibrium.

---

\[ \text{[A.2]} \] These assumptions guarantee that \( c(x^i, \theta^i) \in (0, 1) \) and \( w(x, \varphi_v) \in (0, 1) \) for all \( x \).
4.2.1 Issue Selection: Convergence or Divergence?

I now begin with the analysis of equilibrium campaigning. In a first step I focus on strategic issue selection, that is with which intensity the two candidates campaign on an issue and which candidate puts greater emphasis on a given issue. As we will see in Section [5], strategic issue selection is an important determinant of candidate selection on Election Day. Therefore, it is important to understand what determines strategic issue selection by the candidates.

In an interior pure strategy Nash equilibrium behavior follows from the set of first order conditions (henceforth FOCs). If one rearranges the FOCs appropriately, they reveal the most important intuitions for candidates’ relative issue emphasis in equilibrium:

\[
\frac{\partial \pi_D(x)}{\partial x_D} = E \left[ \frac{\partial c(x^1, \theta^1)}{\partial x_D} w(x, \varphi_v) - \frac{\partial c(x^2, \theta^2)}{\partial x_D} (1 - w(x, \varphi_v)) \right] + \left[ (c(x^1, \theta^1) - c(x^2, \theta^2)) \frac{\partial w(x, \varphi_v)}{\partial x_D} \right]
\]

\[
\frac{\partial \pi_R(x)}{\partial x_R} = E \left[ -\frac{\partial c(x^1, \theta^1)}{\partial x_R} w(x, \varphi_v) + \frac{\partial c(x^2, \theta^2)}{\partial x_R} (1 - w(x, \varphi_v)) \right] - \left[ (c(x^1, \theta^1) - c(x^2, \theta^2)) \frac{\partial w(x, \varphi_v)}{\partial x_R} \right]
\]

The respective first two terms in both FOCs, highlighted by the expectation operators, relate to the marginal effect of persuasion, and they enter both FOCs in a similar way. They reveal that both candidates have an incentive to spend more on an issue that is of greater importance, which comes as no surprise. The third term in each FOC, again highlighted by an expectation operator, is the marginal effect of priming an issue. As we can see, priming affects candidates’ utilities in opposite ways, because the two terms either have opposite signs or are both zero, which is the case if there are no comparative advantages. When a candidate has a comparative advantage on issue \(i\), highlighting this issue has two beneficial effects: it persuades voters and it primes the issue in which the candidate is relatively strong. The contender, who has a comparative disadvantage on \(i\), also benefits from persuasion, but suffers from priming the issue. Hence, priming creates incentives to emphasize issues differently, if candidates have comparative advantages. But because campaigning also persuades voters, divergence is not too extreme, and candidates have an incentive to also campaign on the issues of their respective comparative disadvantage.

Proposition 3 formalizes this intuition:

**Proposition 3 (Issue Selection and Comparative Advantage).** A candidate spends more on issue \(i\) than his opponent if and only if he has a comparative advantage on \(i\). Both candidates spend the same on issue \(i\) if and only if nobody has a comparative advantage on \(i\). Formally, \(\text{Sign}[x^i_D - x^i_R] = \text{Sign}[\sigma^i]\), \(i = 1, 2\).

The proposition highlights the possibility of perfect convergence in the absence of comparative advantages, something that can be sustained also in pure priming campaigns (see Proposition 1). More importantly, Proposition 3 allows for *imperfect convergence* in interior equilibrium, where candidates

\[12\]
have different focus in their campaign strategies. This may help explain previous empirical findings. As we saw earlier, a campaign that only primes or only persuades will never generate such a result. An integrated model, however, is able to achieve this and may thus be valuable to foster our understanding of campaign contests.

In the setting of Example 1 this looks as follows:

**Example 1 (Continued):** Consider the campaign contest described in Example 1 above. Equilibrium spending on issue 1 is

\[
\begin{align*}
    x_D^1 &= \frac{E[\varphi_1] - 2\eta}{1 - 4\eta} + \frac{2\eta(E[\theta_1^1] - E[\theta_1^2])}{\kappa(1 - 4\eta)}, \\
    x_R^1 &= \frac{E[\varphi_2] - 2\eta}{1 - 4\eta} - \frac{2\eta(E[\theta_1^2] - E[\theta_2^2])}{\kappa(1 - 4\eta)},
\end{align*}
\]

and spending on issue 2 follows from \( x_j^2 = 1 - x_j^1 \), \( j = D, R \). If \( E[\theta_1^1] = E[\theta_1^2] \), and hence if no candidate has a comparative advantage, \( x_D^1 = x_R^1 \), and thus candidates converge perfectly. Otherwise \( \text{Sign}(x_D^1 - x_R^1) = \text{Sign}(\sigma^i) \) with imperfect convergence/divergence. If we measure issue convergence following Sigelman and Buell (2004) by \( C(x) = 1 - \frac{1}{2} \left( \text{Abs}[x_D^1 - x_R^1] + \text{Abs}[x_D^2 - x_R^2] \right) = 1 - \text{Abs}[x_D^1 - x_R^1] = 1 - \text{Abs} \left[ \frac{4\eta(E[\theta_1^2] - E[\theta_1^2])}{\kappa(1 - 4\eta)} \right] \), the level of issue convergence decreases in \( \text{Abs} \left[ E[\theta_1^1 - \theta_1^2] \right] \).

What are the main takeaways of this section? We see that in a quite general setting it is comparative advantage that determines campaign agendas. This means that whether candidates converge on an issue or not is not related to issue ownership, as hypothesized by Petrocik (1996) and others. Puglisi and Snyder (2015) write that “a/according to the issue ownership hypothesis, introduced by Petrocik (1996), an issue is said to be Democratic (or “owned” by the Democratic party) if the majority of citizens stably believe that Democratic politicians are better at handling the main problems related to that issue than Republican politicians.” Therefore, an issue is owned if a candidate has an absolute advantage over his contender. To the contrary, comparative advantage implies a relative advantage, and is hence a quite different concept. Moreover, we saw that candidates may well campaign on all important issues. The model may hence help to explain why empirical research finds no evidence for (i) candidates to be diverging significantly in their strategies (see for example Sigelman and Buell 2004) and for (ii) owned issues to be decisive for whether candidates diverge in the first place (see for example Kaplan et al. 2006).

### 4.2.2 Campaign Agendas

Next I turn to candidates’ “aggregate incentives” to address an issue and study which issue dominates the campaign contest in terms of aggregate campaign spending\(^{13}\). This question is not only interesting in itself, but has important implications for candidate selection on Election Day (see Section 5).

Define aggregate campaign spending on issue \( i \) as \( X_i^j = x_D^i + x_R^i \). To get closer to an understanding of what determines campaign agendas we take another look at the first order conditions. From there it follows that in any interior equilibrium the following must hold:

\[
E \left[ \left( \frac{\partial c(x_1^1, \theta_1^1)}{\partial x_D^1} - \frac{\partial c(x_1^2, \theta_2^2)}{\partial x_R^1} \right) w(x, \varphi_v) \right] = E \left[ \left( \frac{\partial c(x_2^2, \theta_2^2)}{\partial x_D^2} - \frac{\partial c(x_2^2, \theta_2^2)}{\partial x_R^1} \right) \left( 1 - w(x, \varphi_v) \right) \right].
\]

\(^{13}\)All results in this section should readily extend to any number of issues \( n \geq 2 \).
This is independent of priming and comparative advantages and hence, loosely speaking, in an interior equilibrium aggregate incentives to address an issue are mostly driven by the candidates’ desire to persuade. How strong persuasion incentives are depends on the issues’ importance through $\varphi_v$ and potentially on the degree of voters’ conviction through $\theta_v^i(1 - \theta_v^i)$ (see Assumption 3). Ceteris paribus, an issue should receive a greater share of total campaign spending when its relative importance increases, when voters become more and more undecided as to the candidates’ relative competence on the issue (when $\theta_v^i(1 - \theta_v^i)$ increases), and when voters’ become more and more decided as to the candidates’ relative competence on the other issue (when $\theta_v^j(1 - \theta_v^j)$ decreases). Of course, if for a single voter $\theta_v^i$ or $\varphi_v$ changes, this has no effect at all on spending, because this voter has zero mass. The above reasoning and the results below apply to situations where voter $v$ has positive mass and changes his candidate assessment $\theta_v^i$ and issue assessments $\varphi_v$.

Our first formal result of this section states the intuitive result that an issue receives indeed greater aggregate spending when it’s importance increases:

**Proposition 4.** Aggregate spending on an issue increases in the issue’s importance.

With respect to $\varphi_v$, the comparative statics are clear: As an issue’s importance increases, aggregate spending on it also increases. If, for example, an economic crisis shifts attention to the issue of job security, or if an epidemic shifts attention to the issue of health care, we should expect candidates to increasingly campaign on these issues.

Note that the proposition suggests that if an issue is sufficiently important, both candidates may campaign hardest on the same issue. This also means that one of them campaigns with the greatest intensity on his weakest issue, the issue of the comparative disadvantage. The reason for this is, as before, that when an issue is important, persuasion is also important, and so independent of comparative advantages it may be worthwhile to campaign intensely on an issue in which one is perceived weak:

**Example 1** (Continued): $D$ spends more on issue 2 than on issue 1 if

$$E[\varphi_v] < \frac{1}{2} - \frac{2\eta (E[\theta_v^1] - E[\theta_v^2])}{\kappa}.$$  

For example, if $\eta = \frac{1}{10}, \kappa = \frac{1}{4}, E[\theta_v^1] = \frac{55}{100}, E[\theta_v^2] = \frac{45}{100}$, and $E[\varphi_v] = \frac{2}{5}$, then $x_D^1 = \frac{7}{15} < \frac{8}{15} = x_D^2$, and hence $D$ spends most of his budget on his weakest issue.

Although aggregate incentives depend mostly on the incentives to persuade, the comparative statics with respect to voters’ strength of opinion are not as clear cut. The reason is that changing $\theta_v^i$ not only changes how easy it is to persuade a voter, but it also changes comparative advantages, and hence relative issue emphasis. In the general model comparative statics with respect to $\theta_v^i$ are hard to get, because also comparative advantages determine a candidate’s marginal product of persuasion in equilibrium. However, to get an idea in which direction comparative statics go we can study them when $\eta$ is sufficiently small. Then, it can be shown that comparative statics are indeed as intuition suggests:

**Proposition 5.** In a neighborhood of $\eta = 0$, aggregate spending on issue $i$ increases in $\theta_v^i(1 - \theta_v^i)$ and decreases in $\theta_v^{-i}(1 - \theta_v^{-i})$. 


Figure 1: Aggregate campaign spending as a function of relative effectiveness of persuasion \( \nu \) and \( E[\varphi_v] \).

The proposition confirms the intuition developed above. Moreover, note that \( \eta \) being small is not necessary for the result to hold, but it helps to derive the result formally. The following example shows that it may well generalize:

**Example 2:** Consider the campaign contest defined in Example 1, but let

\[
c(x^i, \theta^i_v) = \max \left\{ \min \left\{ \theta^i_v + \kappa^i \left( \left( x^i_D - \frac{1}{2} x^i_D^2 \right) - \left( x^i_R - \frac{1}{2} x^i_R^2 \right) \right), 1 \right\}, 0 \right\}
\]

for some \( \kappa^i > 0 \). Moreover, let \( \tilde{\theta}^i + \frac{\kappa^i}{2} < 1 \) and \( \theta^i_v - \frac{\kappa^i}{2} > 0 \). Then

\[
\text{Sign}[X^1 - X^2] = \text{Sign} \left[ E[\varphi_v] - \frac{\kappa^2}{\kappa^1 + \kappa^2} \right] = \text{Sign} \left[ E[\varphi_v] - \frac{1}{1 + \nu} \right].
\]

The example shows how an issue’s importance and the issue-specific marginal effectiveness of persuasion together shape aggregate incentives to address an issue. Following our previous discussion, the potential differences in \( \kappa^i \) could be rationalized as stemming from voters’ convictions on the issues, \( \kappa^i = E[\kappa^i_v] = E[\kappa(\theta^i_v)] \), where \( \kappa \) is a symmetric function with maximum at \( \theta^i_v = \frac{1}{2} \) and which is increasing on \( \theta^i_v \in [0, \frac{1}{2}) \) and decreasing on \( \theta^i_v \in (\frac{1}{2}, 1] \).

Figure 1 shows how \( \nu \) and \( E[\varphi_v] \) together influence aggregate spending on the issues. The figure also summarizes our results from Propositions 4 and 5: Aggregate spending on an issue \( i \) increases in its ex-ante importance and \( \theta^i_v(1 - \theta^i_v) \) and decreases in \( \theta^i_v(1 - \theta^i) \).

The results we just derived help us explain something that we can observe frequently in real campaigns. Voters often feel that candidates do not listen to their needs and talk about things that they deem of secondary importance. A recent example is the 2016 presidential campaign between Donald Trump and Hillary Clinton. During the course of the year, the number of surveyed Americans stating that candidates actually talked about issues they really cared about hovered between 62 and 56 percent, and just a month before the election took place, in October, this number dropped to 48 percent.\(^{14}\)

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\(^{14}\)The data comes from Gallup, see https://news.gallup.com/poll/196607/sharp-drop-views-candidates-talk-key-issues.aspx?g_source=POLITICS&g_medium=topic&g_campaign=tiles (last retrieved: June 5, 2019).
Hence, a significant fraction of the electorate perceived the public discourse to be errant. Moreover, this perception that candidates focus on the “wrong” issues was similar for Democrats and Republicans. Another example of candidates not focusing on voters’ priority issues can be found in the US 2008 presidential campaign. Both John McCain and Barrack Obama spent heavily on the issue Taxes, making it the most important issue in terms of aggregate campaign spending of this campaign. But Taxes was not among the five most important issues at the time. Hence, it appears that campaign contests often lead candidates to focus on issues that are not really important to voters.

While Proposition 4 shows that more important issues tend to receive greater attention by the candidates, Proposition 5 as well as Example 2 revealed that an issue’s importance alone is not sufficient to tell whether it receives greater attention than another issue. Figure 1 reveals that an issue, in which voters are undecided, may well receive greater spending than a more important issue. Thus, our model can help to shed light on why candidates shape campaigns in a way such that issues of secondary importance receive the bulk of attention. The next proposition formalizes this observation as a possibility result:

**Proposition 6.** Candidates may campaign hardest on any issue, even the least important. If voters have strong convictions on important issues and are undecided on an unimportant issue, the latter may dominate the campaign in terms of aggregate spending.

If voters are easily swayed on an issue, both candidates may have strong incentives to campaign on it, even if there are other and more important issues. Of course, this may lead candidates to campaign on the most important issues with the greatest intensity, but also less intuitive equilibria with inverse agendas are possible.

## 5 Implications for Candidate Selection

So far the focus was on understanding candidate behavior in campaign contests, but not on the consequences campaign contests have for political outcomes. In this section I will study how campaign contests influence candidates’ chances at the ballot.

To find an answer to this question we need to study how campaign contests influence candidates’ equilibrium vote shares. Bartels (1992) hypothesized that campaign contests are likely to have no consequences at all, and he provides the following intuition:

“In a world where most campaigners make reasonably effective use of reasonably similar resources and technologies most of the time, much of their effort will necessarily be without visible impact, simply because every campaigner’s efforts are balanced against more or less equally effective efforts to produce the opposite effect.”

(Bartels 1992, p. 267)

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16 For sources of this data, see [https://news.gallup.com/poll/108331/obama-has-edge-key-election-issues.aspx](https://news.gallup.com/poll/108331/obama-has-edge-key-election-issues.aspx) (last retrieved: June 5, 2019).
In fact, many studies have derived this result formally, see for example Proposition 2 in Meirowitz (2008), Proposition 2 in Iaryczower and Mattozzi (2013), or Propositions 1 and 2 in Denter and Sisak (2015). And in fact this is also what Proposition 1 suggests: whenever campaigning only persuades or only primes, equilibrium campaigning leaves candidate’s vote shares unchanged. The reason is, as Bartels correctly anticipated, that candidates’ efforts balance and offset each other. In the next Proposition I challenge this conclusion and show that campaign contests tend to structurally benefit one of the candidates:

**Proposition 7.** In a neighborhood of \( \eta = 0 \), Candidate D benefits during the campaign contest if

\[
\Psi \equiv (E[\theta_1^1] - E[\theta_2^2])(X^1 - X^2) > 0,
\]

Candidate R benefits if \( \Psi < 0 \), and no candidate benefits if \( \Psi = 0 \).

Absent comparative advantages, no candidate benefits and the campaign contests remains neutral. This result makes intuitive sense, because in this case perceived competence does not change, and since it is equal in both issues, shifting issue weights has no consequence for candidate selection. Similarly, if both issues receive the same amount of spending, \( X^1 = X^2 \), relative issue importance does not change, and hence changes in issue specific candidate evaluation balance each other out. However, when candidates have comparative advantages, agenda setting starts to become important. Having a comparative advantage on an issue means one further improves one’s standing with the voters on that issue, but loses on the other. Whether or not this is beneficial depends on whether or not the issue of the comparative advantage becomes more important during the campaign contest. Therefore, the intuitions derived in Sections 4.2.1 for relative issue emphasis and 4.2.2 for campaign agendas together determine which candidate can use the campaign contest to the own advantage. For example, if a candidate has a comparative advantage in the issue that is significantly more important, he is likely to improve his vote share. To the contrary, if both issues are of similar importance to voters, the candidate with the comparative advantage in the issue, in which voters have weaker opinions about candidates’ relative quality, is likely to improve his standings with the voters.

Proposition 7 shows who benefits from the campaign contest, but the formal analysis focussed on the case where \( \eta \) is small. Even in this special case it is only possible to make clear statements about which issue receives greater emphasis when both the issues’ relative importance and voters’ convictions point in the same direction (see Propositions 4 and 5). When these two are in conflict, however, it is unclear which issue receives greater spending, unless we specify the details of the campaign contest. If we define the contest as in the above example, we get a clear prediction:

**Example 2** (Continued): Candidate D benefits during the campaign if

\[
\Psi \equiv (E[\theta_1^1] - E[\theta_2^2])(X^1 - X^2) > 0 \Leftrightarrow (E[\theta_1^1] - E[\theta_2^2]) \left( E[\varphi_v] - \frac{\kappa^2}{\kappa_1 + \kappa_2} \right) > 0,
\]

Candidate R benefits if \( \Psi < 0 \), and no candidate benefits when \( \Psi = 0 \).

The example and Proposition 7 show that the intuitions derived by Bartels (1992) are valid only in certain environments. His intuition is, in a sense, based on the argument that a candidate can simply
Figure 2: A campaign with $\eta = \frac{1}{10}$, $\kappa = \frac{1}{2}$, and $E[\theta^2_v] = \frac{45}{100}$. In the upper, blue-shaded area $D$ loses the majority during the campaign contest, while in the lower, brown shaded area $D$ gains a majority.

“copy” what his contender is doing to avoid to be on the losing side of the contest. The integrated model with both priming and persuasion reveals that this is often impossible. Choosing exactly the same effort allocation as one’s contender implies that campaigning does not change how voters view the candidates’ qualities, but it may change issues’ relative importance. If candidates have comparative advantages, such a spending profile cannot be optimal for both. Similarly, choosing an effort allocation that keeps issues’ relative importance constant implies that each candidate improves on one issue and loses on another. Unless both issues have the same importance, this cannot be good for both candidate, either. Therefore, campaign contests tend to benefit one of the candidates.

Note that $\Psi$ is independent of a candidate’s current popularity with the voters, and thus the identity of the candidate, who benefits from the contest, is independent of candidates’ initial popularity. An interesting question is whether the contest can change vote shares enough to make a candidate, who trails his contender in the polls at the campaign outset, the actual favorite on Election Day. Other authors have asked this question before. Prato and Wolton (2018) also show that under certain conditions the ex-ante less popular candidate might structurally benefit during a campaign contest. In their paper, which has a unique policy dimension, this is the case if the race is imbalanced due to partisan preferences. They show that the a priori less popular candidate remains less popular in equilibrium, even though the gap between candidates may become smaller. The following example shows that this is different in the current model, and that upsets on Election Day might predictably occur:

**Example 1** (Continued): Assume $E[\theta^1_v] > E[\theta^2_v]$, $E[\varphi_v] > \frac{1}{2}$, and $u^0 < \frac{1}{2}$, i.e., that $D$ is the less popular candidate at the campaign outset. Candidate $D$ is able to turn this initial disadvantage into an
advantage during the campaign contest if
\[
\eta > \frac{1}{2} - E \left[ \theta_1 \varphi + \theta_2 (1 - \varphi) \right] \\
2(1 - E [\theta_1 + \theta_2])
\]

The example shows that indeed the a priori weaker candidate might have better chances to win the election than his seemingly stronger contender. This leads us back to the question posed in the first paragraph of the introduction: Is it possible that a candidate who initially trails his contender by 5 percentage points should be considered the stronger candidate? Consider Example 1 and let \( \theta_1 = \frac{1}{4} \), \( \theta_2 = \frac{59}{100} \), \( \varphi = \frac{1}{3} \), \( \eta = \frac{85}{1000} \), and \( \kappa = \frac{1}{2} \). Then the campaign contest has a unique interior pure strategy equilibrium and \( D \)'s vote share increases from \( \theta_1 \varphi + \theta_2 (1 - \varphi) = 0.475 \) to 0.505. Hence, \( D \) turns a deficit of 5 percentage points into a narrow victory with a 1 percentage point lead. Figure 2 shows combinations of parameter values for which the identity of the stronger candidate changes endogenously during the campaign contest of Example 1.

6 Conclusion

I have developed a model of campaign contests in which candidates compete for electoral success by spending time or money on different policy issues. The novelty is that I allow for simultaneous issue priming and persuasion. This allows me to derive a set of new testable predictions about candidate behavior, which have important consequences for candidate selection on Election Day.

The paper motivates different lines of future research. First and foremost, it will be interesting to take the model’s predictions to the data. To be able to formulate clear predictions, one needs to adjust parts of the model, because candidates campaign usually on more than two issues. However, as discussed in Footnote 12, the model’s results should carry over to any number of policy issues \( n \geq 2 \), when comparative advantage is redefined appropriately. The model’s main testable predictions regarding campaigns spending and its consequences are then the following:

- Both candidates campaign on all important issues, but specialize according to their comparative advantages.
- Ceteris paribus, the amount of aggregate spending directed to an issue increases in the issue’s perceived importance.
- Ceteris paribus, the amount of aggregate spending directed to an issue decreases in voters’ conviction regarding candidates’ qualities.
- A candidate tends to increase his vote share during the campaign contest if he has a comparative advantage on an issue that dominates the campaign in terms of aggregate spending.

Second, it will be interesting to extend the model by allowing for more than two candidates. In its current form, the model studies campaign contests between two candidates, which is the norm in majoritarian political systems like the one in the US. In countries with a proportional representation (PR) electoral system, for example Germany or Spain, we usually observe more than two parties
competing. Some authors have already shown that the electoral system has a significant impact on candidates’ incentives to campaign, see for example [Iaryczower and Mattozzi (2013)] or [Denter and Sisak (2015)]. Thus, it would be interesting to extend the model to allow for multiple candidates/parties to compete. Some authors have already shown that the electoral system has a significant impact on the degree of candidates’ specialization on the issues, and hence are likely to have consequences also on Election Day.

A Mathematical Appendix

A.1 Proof of Proposition 1

A.1.1 Proof of Proposition 1

A campaign that only persuades. Candidates’ strategy spaces are convex and compact, as \( x^j_i \in [0, B] \). Also note that individual payoff functions are continuous in all variables by Assumptions 1 and 2. To ensure existence of a pure strategy Nash equilibrium, it hence suffices to show that payoff functions are strictly quasi-concave in \( x^j_i \), because this allows us to apply standard results (see for example Theorem 1.2 in [Fudenberg and Tirole, 1991]).

When \( \eta = 0 \), the first derivative of \( j \)’s payoff function is

\[
\frac{\partial \pi_D(x)}{\partial x_D} \bigg|_{\eta=0} = E \left[ \frac{\partial c(x^j_i, \theta^j_i)}{\partial x_D} \varphi_v + \frac{\partial c(x^j_i, \theta^j_i)}{\partial x_D} \frac{dx^j_i}{dx_D} (1 - \varphi_v) \right]
\]

\[
= E \left[ -\frac{\partial c(x^j_i, \theta^j_i)}{\partial x_D} \varphi_v + \frac{\partial c(x^j_i, \theta^j_i)}{\partial x_D} \frac{dx^j_i}{dx_D} (1 - \varphi_v) \right]
\]

and

\[
\frac{\partial^2 \pi_D(x)}{\partial (x_D)^2} \bigg|_{\eta=0} = E \left[ \frac{\partial^2 c(x^j_i, \theta^j_i)}{\partial (x_D)^2} \varphi_v + \frac{\partial c(x^j_i, \theta^j_i)}{\partial (x_D)^2} \frac{dx^j_i}{dx_D} (1 - \varphi_v) \right]
\]

\[
= E \left[ -\frac{\partial^2 c(x^j_i, \theta^j_i)}{\partial (x_D)^2} \varphi_v + \frac{\partial c(x^j_i, \theta^j_i)}{\partial (x_D)^2} \frac{dx^j_i}{dx_D} (1 - \varphi_v) \right] < 0,
\]

implying individual payoffs are strictly concave and thus also quasi-concave. Hence, equilibrium exists.

Moreover, because \( \frac{\partial c(x^j_i, \theta^j_i)}{\partial x^j_i} \bigg|_{x^j_i=\infty} = \infty \), equilibrium must be interior.

To prove symmetry, the following lemma is useful:

Lemma 1.

\[
\left. \frac{\partial c(x^i, \theta^i)}{\partial x_D} \right|_{x_D=x^i_D} = - \left. \frac{\partial c(x^i, \theta^i)}{\partial x_D} \right|_{x_D=x^i_R} = \left. \frac{\partial c(x^i, 1 - \theta^i)}{\partial x_D} \right|_{x_D=x^i_D} = - \left. \frac{\partial c(x^i, 1 - \theta^i)}{\partial x_D} \right|_{x_D=x^i_R}
\]

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Proof. By Assumption \(2 \), \( c(x, x, \theta_v^i) = \theta_v^i \). Thus,
\[
\frac{\partial c(x_D^i, x_R^i, \theta_v^i)}{\partial x_D^i} \bigg|_{x_D^i=x_R^i} + \frac{\partial c(x_D^i, x_R^i, \theta_v^i)}{\partial x_R^i} \bigg|_{x_D^i=x_R^i} = 0 \iff \frac{\partial c(x_D^i, x_R^i, \theta_v^i)}{\partial x_D^i} \bigg|_{x_D^i=x_R^i} = -\frac{\partial c(x_D^i, x_R^i, \theta_v^i)}{\partial x_R^i} \bigg|_{x_D^i=x_R^i}
\]
and of course as well
\[
\frac{\partial c(x_D^i, x_R^i, 1-\theta_v^i)}{\partial x_D^i} \bigg|_{x_D^i=x_R^i} = -\frac{\partial c(x_D^i, x_R^i, 1-\theta_v^i)}{\partial x_R^i} \bigg|_{x_D^i=x_R^i}.
\]
Moreover, also by Assumption \(2 \),
\( c(x, y, \theta_v^i) = 1 - c(y, x, 1-\theta_v^i) \) and thus \( \frac{\partial c(x, y, \theta_v^i)}{\partial x} = \frac{\partial c(x, y, 1-\theta_v^i)}{\partial x} \) and of course as well
\[
\frac{\partial c(x, y, \theta_v^i)}{\partial x} \bigg|_{x=y} = -\frac{\partial c(y, x, 1-\theta_v^i)}{\partial x} \bigg|_{x=y}.
\]
Therefore,
\[
-\frac{\partial c(x, \theta_v^i)}{\partial x} \bigg|_{x=x_R^i} = \frac{\partial c(x^i, \theta_v^i)}{\partial x_D^i} \bigg|_{x_D^i=x_R^i} = -\frac{\partial c(x^i, 1-\theta_v^i)}{\partial x_D^i} \bigg|_{x_D^i=x_R^i} = \frac{\partial c(x^i, 1-\theta_v^i)}{\partial x_D^i} \bigg|_{x_D^i=x_R^i},
\]
which proves the lemma.

When candidates choose identical spending profiles, they have the same marginal utility of campaigning on the different issues. To proceed we need one more lemma:

**Lemma 2.** If \( c(x^i, \theta_v^i) \in (0, 1) \),
\[
-\frac{\partial c(x^i, \theta_v^i)}{\partial x_D^i} \bigg|_{x_D^i=x_R^i} = \frac{\partial c(x^i, \theta_v^i)}{\partial x_D^i} \bigg|_{x_D^i=x_R^i} = \frac{\partial c(x^i, 1-\theta_v^i)}{\partial x_D^i} \bigg|_{x_D^i=x_R^i} = \frac{\partial c(x^i, 1-\theta_v^i)}{\partial x_D^i} \bigg|_{x_D^i=x_R^i} \equiv 1.
\]

Proof. First note that by Lemma \(1 \),
\[
\frac{\partial c(x^i, \theta_v^i)}{\partial x_D^i} \bigg|_{x_D^i=x_R^i} = \frac{\partial c(x^i, \theta_v^i)}{\partial x_D^i} \bigg|_{x_D^i=x_R^i} = 0 \iff \rho(x^i, \theta_v^i) \equiv -\frac{\partial c(x^i, \theta_v^i)}{\partial x_D^i} \bigg|_{x_D^i=x_R^i} = 1.
\]
Now take the derivative of \( \rho(x^i, \theta_v^i) \) with respect to \( x_D^i \):
\[
\frac{\partial \rho(x^i, \theta_v^i)}{\partial x_D^i} = -\frac{\partial^2 c(x^i, \theta_v^i)}{\partial x_D^2} \bigg( \frac{\partial c(x^i, \theta_v^i)}{\partial x_D^i} \bigg)^2 - \frac{\partial c(x^i, \theta_v^i)}{\partial x_D^i} \bigg( \frac{\partial c(x^i, \theta_v^i)}{\partial x_D^i} \bigg)^2.
\]
This is negative if and only if

\[ - \left( \frac{\partial^2 c(x^i, \theta^i_v)}{\partial (x_D^i)^2} \frac{\partial c(x^i, \theta^i_v)}{\partial x_R^i} - \frac{\partial c(x^i, \theta^i_v)}{\partial x_D^i} \frac{\partial^2 c(x^i, \theta^i_v)}{\partial x_R^i \partial x_D^i} \right) < 0 \]

\[ \Leftrightarrow \frac{\partial^2 c(x^i, \theta^i_v)}{\partial (x_D^i)^2} \frac{\partial c(x^i, \theta^i_v)}{\partial x_R^i} > \frac{\partial c(x^i, \theta^i_v)}{\partial x_D^i} \frac{\partial^2 c(x^i, \theta^i_v)}{\partial x_R^i \partial x_D^i} > 0 \]

This is the case by Assumption 4 and hence \( \rho^i(x^i, \theta^i_v) \) is monotone decreasing in \( x_D^i \). In a similar way we can prove that \( \rho^i(x^i, \theta^i_v) \) monotonically increases in \( x_R^i \). Thus,

\[
\rho^i(x^i, \theta^i_v) \begin{cases} > 0 & \text{if } x_D^i - x_R^i < 0, \\ < 0 & \text{if } x_D^i - x_R^i > 0, \end{cases}
\]

which proves the lemma.

Note that the lemma implies that when \( x_D^i > x_R^i \), \( \frac{\partial c(x^i, \theta^i_v)}{\partial x_D^i} < \frac{\partial c(x^i, \theta^i_v)}{\partial x_R^i} \) and vice versa:

**Corollary 1.**

\[
\frac{\partial c(x^i, \theta^i_v)}{\partial x_D^i} + \frac{\partial c(x^i, \theta^i_v)}{\partial x_R^i} \begin{cases} > 0 & \text{if } x_D^i - x_R^i < 0, \\ < 0 & \text{if } x_D^i - x_R^i > 0, \end{cases}
\]

The marginal product of persuasion of Candidate D relative to Candidate R depends on who spends more on the issue so far, independent of \( \theta^i_v \). Next consider the first order conditions. In an interior equilibrium, both FOCs need to hold simultaneously. Thus,

\[ E \left[ \frac{\partial c(x^i, \theta^i_v)}{\partial x_D^i} \varphi_v - \frac{\partial c(x^i, \theta^i_v)}{\partial x_R^i} (1 - \varphi_v) \right] = 0 = E \left[ \frac{\partial c(x^i, \theta^i_v)}{\partial x_D^i} \varphi_v + \frac{\partial c(x^i, \theta^i_v)}{\partial x_R^i} (1 - \varphi_v) \right] \]  

(A.1)

By Lemma 4, (A.1) holds if \( x_D^1 = x_R^1 \), as both the LHS and RHS are zero. Now assume \( x_D^1 \neq x_R^1 \), for example \( x_D^1 > x_R^1 \). Then Corollary 4 tells us that the LHS of (A.1) is negative, while the RHS is positive, because \( x_D^1 > x_R^1 \Leftrightarrow x_D^2 < x_R^2 \). Hence, this cannot be the case in interior equilibrium. Similarly, \( x_D^1 < x_R^1 \) is not possible, either. Hence, in any interior equilibrium, both candidates campaign with identical intensity on all issues, \( x^i = x^i_R \) for \( i = 1, 2 \), and thus they converge completely.

To prove uniqueness, evaluate the first order condition given \( x_D^1 = x_R^1 = x^1 \) and reorganize:

\[
E \left[ \left. \frac{\partial c(x^i, \theta^i_v)}{\partial x_D^i} \right|_{x_D^1=x^1} \right] = E \left[ \left. \frac{1 - \varphi_v}{\varphi_v} \right|_{x_D^1=x^1} \right].
\]
If we can show that this is monotone in $x^1$ equilibrium must be unique. Taking the derivative with respect to $x^1$ yields

$$
E \left[ \frac{\partial c(x^1, \theta^1_{x^1})}{\partial x^1_{D}} \right]_{x^1_{j} = x^1} + \left( \frac{\partial^2 c(x^1, \theta^1_{x^1})}{\partial x^1_{D} \partial x^1_{R}} \right)_{x^1_{j} = x^1} \right)
$$

where we already used that $dx^2_j/dx^1_j = -1$. If this is strictly positive or strictly negative equilibrium is unique. To determine the sign of this we need a last lemma:

**Lemma 3.**

$$
\frac{\partial^2 c(x^i, \theta^i_{x^i})}{\partial x^i_{D} \partial x^i_{R}} \bigg|_{x^i_{j} = x^i} = -\frac{1}{2} \left( \frac{\partial^2 c(x^i, \theta^i_{x^i})}{\partial (x^i_{D})^2} \bigg|_{x^i_{j} = x^i} + \frac{\partial^2 c(x^i, \theta^i_{x^i})}{\partial (x^i_{R})^2} \bigg|_{x^i_{j} = x^i} \right).
$$

**Proof.** We know from Lemma [ ] that $\frac{\partial c(x^i, \theta^i_{x^i})}{\partial x^i_{D}} \bigg|_{x^i_{D} = x^i_{R} = x^i} + \frac{\partial c(x^i, \theta^i_{x^i})}{\partial x^i_{R}} \bigg|_{x^i_{D} = x^i_{R} = x^i} = 0$. Totally differentiating with respect to $x^i$ yields

$$
\frac{\partial^2 c(x^i, \theta^i_{x^i})}{\partial x^i_{D} \partial x^i_{R}} \bigg|_{x^i_{D} = x^i_{R} = x^i} = -\frac{1}{2} \left( \frac{\partial^2 c(x^i, \theta^i_{x^i})}{\partial (x^i_{D})^2} \bigg|_{x^i_{D} = x^i_{R} = x^i} + \frac{\partial^2 c(x^i, \theta^i_{x^i})}{\partial (x^i_{R})^2} \bigg|_{x^i_{D} = x^i_{R} = x^i} \right),
$$

which is the condition stated in the lemma.

Using the lemma condition [A.2] simplifies to

$$
E \left[ \frac{\partial c(x^1, \theta^1_{x^1})}{\partial x^1_{D}} \bigg|_{x^1_{j} = x^1} \left( \frac{\partial^2 c(x^2, \theta^2_{x^2})}{\partial (x^2_{D})^2} \bigg|_{x^2_{j} = x^2} + \frac{\partial^2 c(x^2, \theta^2_{x^2})}{\partial (x^2_{R})^2} \bigg|_{x^2_{j} = x^2} \right) \bigg] \left( \frac{\partial^2 c(x^1, \theta^1_{x^1})}{\partial (x^1_{D})^2} \bigg|_{x^1_{j} = x^1} + \frac{\partial^2 c(x^1, \theta^1_{x^1})}{\partial (x^1_{R})^2} \bigg|_{x^1_{j} = x^1} \right) \right] < 0.
$$

In a similar fashion we can establish the result using the first order condition of candidate $R$. Thus, equilibrium is unique, which proves the proposition.
A campaign that only primes. If campaigning only primes issues, the first derivative of $j$’s payoff function is

$$
\frac{\partial \pi_D(x)}{\partial x_D} \big|_{(x', \theta'_v) = \theta'_i, v} = E \left[ \left( \theta^1_v - \theta^2_v \right) \left( \frac{\partial w(x', \varphi_v)}{\partial x_D} + \frac{\partial w(x', \varphi_v)}{\partial x_D} \frac{dx_D^2}{dx_D} \right) \right]
$$

$$
= E \left[ \left( \theta^1_v - \theta^2_v \right) \left( \frac{\partial w(x', \varphi_v)}{\partial x_D} - \frac{\partial w(x', \varphi_v)}{\partial x_D} \right) \right]
$$

$$
= -E \left[ \left( \theta^1_v - \theta^2_v \right) \left( \frac{\partial w(x', \varphi_v)}{\partial x_D} + \frac{\partial w(x', \varphi_v)}{\partial x_D} \right) \right]
$$

Note that $\text{Sign} \left[ E \left[ \left( \theta^1_v - \theta^2_v \right) \right] \right] = \text{Sign}[\sigma^1]$, as

$$
\text{Sign} \left[ \sigma^1 \right] = \text{Sign} \left[ E[\theta^1_v] - \frac{1}{2} (\theta^1_v + \theta^2_v) \right] = \text{Sign} \left[ E \left[ \theta^1_v - \theta^2_v \right] \right].
$$

Therefore, if $\sigma^1 = \sigma^2 = 0$, any combination of strategies is a Nash equilibrium, as campaigning has no effect whatsoever. Otherwise, i.e., when $\sigma^1 > 0$ and $\sigma^2 < 0$, $D$ spends all of his budget on issue 1 and $R$ spends all of his budget on issue 2, because $E \left[ \left( \frac{\partial w(x', \varphi_v)}{\partial x_D} - \frac{\partial w(x', \varphi_v)}{\partial x_D} \right) \right] > 0$, and the opposite is true when $\sigma^1 < 0$ and $\sigma^2 > 0$. In other words, candidates never campaign on the same issue and thus they diverge completely.

### A.2 Proof of Proposition 2

When $\eta$ is small the proof follows by a continuity argument from the proof of Proposition [1](#) and implies also uniqueness. However, for larger $\eta$ it is not clear that utilities are quasi-concave. Baye et al. (1993) provide sufficient conditions for existence of a pure strategy Nash equilibrium when payoffs may fail to be quasi-concave. In particular, (i) strategy spaces need to be convex and compact and the aggregator function $U^{agg}(x) \equiv \pi_D(x) + \pi_R(x)$ needs to be (ii) diagonally transfer continuous and (iii) diagonally transfer quasi-concave. (i) clearly holds by assumption. (ii) follows from Proposition 2 of Baye et al. (1993), as both $\pi_D$ and $\pi_R$ are continuous, which is a weaker condition than diagonal transfer continuity. Finally, $U^{agg}$ is constant (= 1) and thus quasi-concave, which by Proposition 1 of Baye et al. (1993) is sufficient for diagonal transfer quasi-concavity. Thus, a pure strategy Nash equilibrium exists. Moreover, a corner equilibrium cannot exists as the marginal product of $x^j_i$ would be either $+\infty$ ($x^j_i = 0$) or $-\infty$ ($x^j_i = B$). We can conclude that all equilibria must be interior. Moreover, each payoff function has an interior maximum and at this maximum the payoff functions must be concave. Hence, Nash equilibria are determined by the respective first-order conditions.

A sufficient condition for uniqueness of equilibrium is that the slope of the best response functions is smaller than 1 in absolute value for all $x$. Given the FOCs, we can use the implicit function theorem to calculate this slope. Let $c^1_v \equiv c(x, \theta^1_v)$, $c^2_v \equiv c(x, \theta^2_v)$, and $w_v \equiv w(x, \varphi_v)$. Totally differentiating $D$’s
but not necessary for uniqueness of Nash equilibrium. It generally holds when both strategy equilibrium nevertheless, as our previous analysis has revealed. (A.3) is thus indeed sufficient when faced voter yields

\[
\frac{\partial^2 u_D(x, s_v)}{\partial x_D^2} = -\frac{\partial w_D}{\partial x_D} \left( \frac{\partial c_v}{\partial x_D} + \frac{\partial^2 c_v}{\partial x_D^2} \right) + \frac{\partial^2 \rho_{w_D}}{\partial (x_D)^2} (c_v^1 - c_v^2) + w_v \frac{\partial^2 c_v}{\partial x_D^2} (1 - w_v) \frac{\partial^2 c_v}{\partial x_D^2}.
\]

The slope of D’s best response is then

\[
\frac{\partial x_D}{\partial x_R} = E \left[ -\frac{\partial^2 u_D(x, s_v)}{\partial x_D^2} \frac{\partial^2 u_D(x, s_v)}{\partial x_D^2} \right].
\]

Using the same steps we can calculate

\[
\frac{\partial x_R}{\partial x_D} = E \left[ -\frac{\partial^2 u_R(x, s_v)}{\partial x_D^2} \frac{\partial^2 u_R(x, s_v)}{\partial x_D^2} \right].
\]

A sufficient condition for uniqueness of equilibrium is then the following:

\[
1 > E \left[ -\frac{\partial^2 u_D(x, s_v)}{\partial x_D^2} \frac{\partial^2 u_D(x, s_v)}{\partial x_D^2} \right] > -1 \forall x \in [0, B]^2 \text{ and } 1 > E \left[ -\frac{\partial^2 u_R(x, s_v)}{\partial x_D^2} \frac{\partial^2 u_R(x, s_v)}{\partial x_D^2} \right] > -1 \forall x \in [0, B]^2 \quad (A.3)
\]

Note that when \( \eta = 0 \) this condition is not necessarily fulfilled, but there is nevertheless a unique pure strategy equilibrium nevertheless, as our previous analysis has revealed. (A.3) is thus indeed sufficient but not necessary for uniqueness of Nash equilibrium. It generally holds when both \( \eta \) and \( \left| \frac{\partial^2 c_v}{\partial x_D^2} \right| \) are sufficiently small, because then

\[
\left| \frac{\partial^2 u_D(x, s_v)}{\partial x_D^2} \frac{\partial^2 u_D(x, s_v)}{\partial x_D^2} \right|_{\eta=0} = -\frac{\varphi_v \frac{\partial^2 c_v}{\partial x_D^2} (1 - \varphi_v) \frac{\partial^2 c_v}{\partial x_D^2}}{\varphi_v \frac{\partial^2 c_v}{\partial x_D^2} (1 - \varphi_v) \frac{\partial^2 c_v}{\partial x_D^2}} \in (-1, 1).
\]

### A.3 Proof of Proposition 3

The candidates’ respective objective is to maximize

\[
\pi_D(x) = E \left[ c(x^1, \theta^1) w(x, \varphi_v) + c(x^2, \theta^2) (1 - w(x, \varphi_v)) \right] s.t. \ x_D^1 + x_D^2 = B
\]

\[
\pi_R(x) = E \left[ (1 - c(x^1, \theta^1)) w(x, \varphi_v) + (1 - c(x^2, \theta^2)) (1 - w(x, \varphi_v)) \right] s.t. \ x_R^1 + x_R^2 = B \quad (A.4)
\]
We know that equilibrium is interior, and hence determined by the system of first order conditions:

\[
\frac{\partial \pi_D(x)}{\partial x_D} = E \left[ \frac{\partial c(x^1, \theta^1_v)}{\partial x_D} w(x, \varphi^1_v) + \frac{\partial c(x^2, \theta^2_v)}{\partial x_D} dx_D^2 (1 - w(x, \varphi^1_v)) \right] \\
+ E \left[ (c(x^1, \theta^1_v) - c(x^2, \theta^2_v)) \left( \frac{\partial w(x, \varphi_v)}{\partial x^1_D} + \frac{\partial w(x, \varphi_v)}{\partial x^2_D} \right) \right] \\
- E \left[ (c(x^1, \theta^1_v) - c(x^2, \theta^2_v)) \left( \frac{\partial w(x, \varphi_v)}{\partial x^1_D} - \frac{\partial w(x, \varphi_v)}{\partial x^2_D} \right) \right]
\]

\[
\frac{\partial \pi_R(x)}{\partial x_R} = E \left[ \frac{\partial c(x^1, \theta^1_v)}{\partial x_R} w(x, \varphi^1_v) - \frac{\partial c(x^2, \theta^2_v)}{\partial x_R} dx_R^2 (1 - w(x, \varphi^1_v)) \right] \\
- E \left[ (c(x^1, \theta^1_v) - c(x^2, \theta^2_v)) \left( \frac{\partial w(x, \varphi_v)}{\partial x^1_R} + \frac{\partial w(x, \varphi_v)}{\partial x^2_R} \right) \right] \\
+ E \left[ (c(x^1, \theta^1_v) - c(x^2, \theta^2_v)) \left( \frac{\partial w(x, \varphi_v)}{\partial x^1_R} - \frac{\partial w(x, \varphi_v)}{\partial x^2_R} \right) \right]
\]

(A.5)

Note that \( \frac{\partial w(x, \varphi_v)}{\partial x_D} = \frac{\partial w(x, \varphi_v)}{\partial x_R} = \frac{\partial w(x, \varphi_v)}{\partial x} \). In any interior equilibrium it needs to hold that:

\[
E \left[ \left( \frac{\partial c(x^1, \theta^1_v)}{\partial x_D} + \frac{\partial c(x^1, \theta^1_v)}{\partial x_R} \right) w(x, \varphi^1_v) - \left( \frac{\partial c(x^2, \theta^2_v)}{\partial x_D} + \frac{\partial c(x^2, \theta^2_v)}{\partial x_R} \right) (1 - w(x, \varphi^1_v)) \right]
= -2E \left[ (c(x^1, \theta^1_v) - c(x^2, \theta^2_v)) \left( \frac{\partial w(x, \varphi_v)}{\partial x^1} - \frac{\partial w(x, \varphi_v)}{\partial x^2} \right) \right]
\]

Note that when \( x^1_D > x^1_R \) we also have \( x^2_D < x^2_R \). By Corollary 1 this means that

\[
\text{Sign} \left[ E \left[ \left( \frac{\partial c(x^1, \theta^1_v)}{\partial x_D} + \frac{\partial c(x^1, \theta^1_v)}{\partial x_R} \right) w(x, \varphi^1_v) - \left( \frac{\partial c(x^2, \theta^2_v)}{\partial x_D} + \frac{\partial c(x^2, \theta^2_v)}{\partial x_R} \right) (1 - w(x, \varphi^1_v)) \right] \right] = -\text{Sign}[x^1_D - x^1_R].
\]

Consequently, in any interior equilibrium

\[
-\text{Sign}[x^1_D - x^1_R] = \text{Sign} \left[ -2E \left[ (c(x^1, \theta^1_v) - c(x^2, \theta^2_v)) \left( \frac{\partial w(x, \varphi_v)}{\partial x^1} - \frac{\partial w(x, \varphi_v)}{\partial x^2} \right) \right] \right]
\leftrightarrow \text{Sign}[x^1_D - x^1_R] = \text{Sign} \left[ E \left[ (c(x^1, \theta^1_v) - c(x^2, \theta^2_v)) \right] \right].
\]

To see that this also implies that \( \text{Sign}[x^1_D - x^1_R] = \text{Sign} \left[ E \left[ \theta^1_v - \theta^2_v \right] \right] \), we prove the following lemma:

**Lemma 4.**

\[
\text{Sign} \left[ \frac{\partial x_D}{\partial \eta} \bigg|_{\eta=0} - \frac{\partial x_R}{\partial \eta} \bigg|_{\eta=0} \right] = \text{Sign} \left[ E \left[ \theta^1_v - \theta^2_v \right] \right].
\]

**Proof.** To derive comparative statics of spending with respect to \( \eta \) we totally differentiate the system of first-order conditions and evaluate the result at \( \eta = 0 \). Totally differentiating the FOCs and evaluating
the result at \( \eta = 0 \) yields
\[
\begin{align*}
\frac{\partial^2 \pi_D(x)}{\partial(x_D)^2} \bigg|_{\eta=0} &= E[\varphi_v] E \left[ \frac{\partial^2 c(x^1, \theta_1^1)}{\partial(x_D)^2} \right] + (1 - E[\varphi_v]) E \left[ \frac{\partial^2 c(x^2, \theta_2^2)}{\partial(x_D)^2} \right], \\
\frac{\partial^2 \pi_D(x)}{\partial x_D \partial x_R} \bigg|_{\eta=0} &= E[\varphi_v] E \left[ \frac{\partial^2 c(x^1, \theta_1^1)}{\partial x_D \partial x_R} \right] + (1 - E[\varphi_v]) E \left[ \frac{\partial^2 c(x^2, \theta_2^2)}{\partial x_D \partial x_R} \right], \\
\frac{\partial^2 \pi_R(x)}{\partial x_D \partial x_R} \bigg|_{\eta=0} &= -2(g(X^1) - g(X^2)) E \left[ \frac{\partial c(x^1, \theta_1^1)}{\partial x_D} \right] - E \left[ \theta_1^1 - \theta_2^2 \right] \left( g'(X^1) + g'(X^2) \right), \\
\frac{\partial^2 \pi_R(x)}{\partial x_R^2} \bigg|_{\eta=0} &= -E[\varphi_v] E \left[ \frac{\partial^2 c(x^1, \theta_1^1)}{\partial x_R^2} \right] - (1 - E[\varphi_v]) E \left[ \frac{\partial^2 c(x^2, \theta_2^2)}{\partial x_R^2} \right], \\
\frac{\partial^2 \pi_R(x)}{\partial x_D \partial x_R} \bigg|_{\eta=0} &= -E[\varphi_v] E \left[ \frac{\partial^2 c(x^1, \theta_1^1)}{\partial x_D \partial x_R} \right] - (1 - E[\varphi_v]) E \left[ \frac{\partial^2 c(x^2, \theta_2^2)}{\partial x_D \partial x_R} \right], \\
\frac{\partial \varphi}{\partial x_D} \bigg|_{\eta=0} &= 2(g(X^1) - g(X^2)) E \left[ \frac{\partial c(x^1, \theta_1^1)}{\partial x_D} \right] + E \left[ \theta_1^1 - \theta_2^2 \right] \left( g'(X^1) + g'(X^2) \right).
\end{align*}
\]

Define
\[
M = \left( \begin{array}{cc}
\frac{\partial^2 \pi_D}{\partial(x_D)^2} & \frac{\partial^2 \pi_D}{\partial x_D \partial x_R} \\
-\frac{\partial^2 \pi_D}{\partial x_D \partial x_R} & \frac{\partial^2 \pi_D}{\partial x_R^2}
\end{array} \right), \quad M_D = \left( \begin{array}{cc}
-\frac{\partial^2 \pi_D}{\partial x_D \partial x_R} & \frac{\partial^2 \pi_D}{\partial x_R^2} \\
\frac{\partial^2 \pi_D}{\partial x_D \partial x_R} & -\frac{\partial^2 \pi_D}{\partial x_D \partial x_R}
\end{array} \right), \quad M_R = \left( \begin{array}{cc}
\frac{\partial^2 \pi_D}{\partial x_D \partial x_R} & -\frac{\partial^2 \pi_D}{\partial x_R^2} \\
-\frac{\partial^2 \pi_D}{\partial x_D \partial x_R} & \frac{\partial^2 \pi_D}{\partial x_D \partial x_R}
\end{array} \right),
\]

Equilibrium comparative statics are \( \frac{\partial x_D^1}{\partial \eta} = \frac{|M_D|}{|M|} \) and \( \frac{\partial x_R^1}{\partial \eta} = \frac{|M_R|}{|M|} \). Using Lemmas 1 and 3 together with \( \eta = 0 \) and \( x_R^i = x_D^i \) yields
\[
\left. \frac{\partial x_D^1}{\partial \eta} \right|_{\eta=0} - \left. \frac{\partial x_R^1}{\partial \eta} \right|_{\eta=0} = 4E[\theta_1^1 - \theta_2^2] \left( g'(X^1) + g'(X^2) \right)
\]
\[
E \left[ \varphi_v \left( \frac{\partial^2 c(x^1, \theta_1^1)}{\partial x_D^2} - \frac{\partial^2 c(x^1, \theta_1^1)}{\partial x_R^2} \right) + (1 - \varphi_v) \left( \frac{\partial^2 c(x^2, \theta_2^2)}{\partial x_D^2} - \frac{\partial^2 c(x^2, \theta_2^2)}{\partial x_R^2} \right) \right].
\]

As the sign of the denominator is clearly positive, the sign of the whole expression depends on the sign of the numerator, and hence on \( E[\theta_1^1 - \theta_2^2] \).

Hence, when \( \eta \) small but positive, the candidate with the a priori comparative advantage spends more on an issue. Say \( D \) has comparative advantage on 1. Could \( R \) spend more on this issue when \( \eta \) increases further? Note that spending is a continuous function of all parameters. For \( R \) to spend more on 1, continuity implies that first there must be \( \eta \) such that both spend the same. But that is not possible, as when both spend the same comparative advantage remains unchanged, and thus \( D \) needs to spend more on it. Moreover, could it be true that one candidate spends more than another on an issue absent comparative advantages? The answer is no again. To see this note that if \( E[\theta_1^1] = E[\theta_2^2] + \epsilon \) for any \( \epsilon > 0, x_D^1 > x_R^1 \). Moreover, for any \( \epsilon < 0, x_D^1 < x_R^1 \). By continuity (along this sequence of equilibria), when \( \epsilon = 0, x_D^1 = x_R^1 \). This implies that \( \text{Sign} \left[ x_D^1 - x_R^1 \right] = \text{Sign} \left[ E[\theta_1^1 - \theta_2^2] \right] \) is generally true.

A.4 Proof of Proposition 4

As mentioned in the text, the proof assumes that a positive mass of voters change their issue weights \( \varphi_v \). Otherwise, such a change has no effect on spending.

To prove the proposition we totally differentiate the system of FOCs and use Cramer’s rule to calculate comparative statics. When the FOCs are, respectively, \( \frac{\partial \pi_D(x)}{\partial x_D} = 0 \) and \( \frac{\partial \pi_R(x)}{\partial x_R} = 0 \), for
comparative statics we need $\frac{\partial^2 \pi_j(x)}{\partial (x_j^1)^2}$, $\frac{\partial^2 \pi_j(x)}{\partial x_j^1 \partial x_j^2}$, and $\frac{\partial^2 \pi_j(x)}{\partial x_j^1 \partial \varphi_v}$. Define

$$M = \left( \begin{array}{cc} \frac{\partial^2 \pi_D}{\partial (x_D^1)^2} & \frac{\partial^2 \pi_D}{\partial x_D^1 \partial x_R} \\ \frac{\partial^2 \pi_R}{\partial x_D^1 \partial x_R} & \frac{\partial^2 \pi_R}{\partial (x_R^1)^2} \end{array} \right), \quad M_D = \left( \begin{array}{cc} -\frac{\partial^2 \pi_D}{\partial x_D^1 \partial \varphi_v} & \frac{\partial^2 \pi_D}{\partial x_D^1 \partial \varphi_v} \\ \frac{\partial^2 \pi_R}{\partial x_D^1 \partial \varphi_v} & -\frac{\partial^2 \pi_R}{\partial (x_R^1)^2} \end{array} \right), \quad \text{and} \quad M_R = \left( \begin{array}{cc} -\frac{\partial^2 \pi_D}{\partial (x_D^1)^2} & -\frac{\partial^2 \pi_D}{\partial x_D^1 \partial \varphi_v} \\ -\frac{\partial^2 \pi_R}{\partial x_D^1 \partial \varphi_v} & -\frac{\partial^2 \pi_R}{\partial (x_R^1)^2} \end{array} \right).$$

Then

$$\frac{\partial x_D^1}{\partial \varphi_v} = \frac{|M_D|}{|M|} = -\frac{\frac{\partial^2 \pi_D}{\partial x_D^1 \partial \varphi_v} - \frac{\partial^2 \pi_D}{\partial x_R \partial \varphi_v}}{\frac{\partial^2 \pi_R}{\partial (x_D^1)^2} - \frac{\partial^2 \pi_R}{\partial x_D^1 \partial x_R}}$$

and

$$\frac{\partial x_R^1}{\partial \varphi_v} = \frac{|M_R|}{|M|} = -\frac{\frac{\partial^2 \pi_D}{\partial (x_D^1)^2} - \frac{\partial^2 \pi_D}{\partial x_D^1 \partial \varphi_v}}{\frac{\partial^2 \pi_R}{\partial x_D^1 \partial \varphi_v} - \frac{\partial^2 \pi_R}{\partial (x_R^1)^2}}.$$

Therefore,

$$\frac{\partial X^1}{\partial \varphi_v} = \frac{\partial x_D^1}{\partial \varphi_v} + \frac{\partial x_R^1}{\partial \varphi_v} = \frac{\frac{\partial^2 \pi_D}{\partial x_D^1 \partial \varphi_v} \left( \frac{\partial^2 \pi_D}{\partial (x_D^1)^2} - \frac{\partial^2 \pi_D}{\partial x_R \partial x_D^1} \right) + \frac{\partial^2 \pi_D}{\partial x_D^1 \partial \varphi_v} \left( \frac{\partial^2 \pi_R}{\partial (x_D^1)^2} - \frac{\partial^2 \pi_R}{\partial x_D^1 \partial x_R} \right)}{\frac{\partial^2 \pi_D}{\partial (x_D^1)^2} - \frac{\partial^2 \pi_D}{\partial x_D^1 \partial x_R}}.$$

We know that in an interior equilibrium $\frac{\partial^2 \pi_j(x)}{\partial (x_j^1)^2} < 0$. Moreover, from the condition for uniqueness $A.3$, we know that $1 > \frac{\partial^2 \pi_j(x)}{\partial (x_j^1)^2} > 1$. Thus, if $\frac{\partial^2 \pi_j}{\partial x_j^1 \partial \varphi_v} > 0$, $j = D, R$, then $\frac{\partial X^1}{\partial \varphi_v} > 0$ and thus aggregate effort increases in $\varphi_v$. Differentiating the respective FOCs with respect to $\varphi_v$ yields

$$\frac{\partial^2 \pi_D}{\partial x_D^1 \partial \varphi_v} = \frac{\partial c(x, \theta_D^1)}{\partial x_D^1} + \frac{\partial c(x, \theta_D^1)}{\partial \theta_D^1} > 0$$

and

$$\frac{\partial^2 \pi_R}{\partial x_R^1 \partial \varphi_v} = -\frac{\partial c(x, \theta_R^1)}{\partial x_R^1} - \frac{\partial c(x, \theta_R^1)}{\partial \theta_R^1} > 0.$$

Therefore, we can conclude that aggregate spending on issue 1 is increasing in $\varphi_v$, which proves the proposition. Thus, if for a positive mass of voters $\varphi_v$ increases, aggregate spending on issue 1 also increases.

A.5 Proof of Proposition 5

As mentioned in the text, the proof assumes that a positive mass of voters change their candidates’ relative competence assessments $\theta_v^2$. Otherwise, such a change has no effect on spending.

To prove the proposition we assume $\eta = 0$ and totally differentiate the system of FOCs. As before, we then use Cramer’s rule to calculate comparative statics. When the FOCs are, respectively, $\frac{\partial \pi_D(x)}{\partial x_D} = 0$ and $\frac{\partial \pi_R(x)}{\partial x_R} = 0$, for comparative statics we need $\frac{\partial^2 \pi_j(x)}{\partial (x_j^1)^2}$, $\frac{\partial^2 \pi_j(x)}{\partial x_j^1 \partial x_j^2}$, and $\frac{\partial^2 \pi_j(x)}{\partial x_j^1 \partial \varphi_v}$. Comparative statics with respect to $\theta_v^2$ are analogous. Define

$$M = \left( \begin{array}{cc} \frac{\partial^2 \pi_D}{\partial (x_D^1)^2} & \frac{\partial^2 \pi_D}{\partial x_D^1 \partial x_R} \\ \frac{\partial^2 \pi_R}{\partial x_D^1 \partial x_R} & \frac{\partial^2 \pi_R}{\partial (x_R^1)^2} \end{array} \right), \quad M_D = \left( \begin{array}{cc} -\frac{\partial^2 \pi_D}{\partial x_D^1 \partial \varphi_v} & \frac{\partial^2 \pi_D}{\partial x_D^1 \partial \varphi_v} \\ \frac{\partial^2 \pi_R}{\partial x_D^1 \partial \varphi_v} & -\frac{\partial^2 \pi_R}{\partial (x_R^1)^2} \end{array} \right), \quad \text{and} \quad M_R = \left( \begin{array}{cc} -\frac{\partial^2 \pi_D}{\partial (x_D^1)^2} & -\frac{\partial^2 \pi_D}{\partial x_D^1 \partial \varphi_v} \\ -\frac{\partial^2 \pi_R}{\partial x_D^1 \partial \varphi_v} & -\frac{\partial^2 \pi_R}{\partial (x_R^1)^2} \end{array} \right).$$

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Then
\[ \frac{\partial x_D^1}{\partial \theta_v^1} = |M_D| = \frac{\partial^2 \pi_{uD}}{\partial x_1^2} \frac{\partial^2 \pi_{uR}}{\partial (x_1^R)^2} + \frac{\partial^2 \pi_{uR}}{\partial x_1^R \partial x_1^R} \frac{\partial^2 \pi_{uD}}{\partial x_1^R \partial x_1^R} \]
and
\[ \frac{\partial x_R^1}{\partial \theta_v^1} = |M_R| = \frac{\partial^2 \pi_{uD}}{\partial (x_1^R)^2} \frac{\partial^2 \pi_{uR}}{\partial x_1^R \partial x_1^R} - \frac{\partial^2 \pi_{uR}}{\partial x_1^R \partial x_1^R} \frac{\partial^2 \pi_{uD}}{\partial x_1^R \partial x_1^R} \]
Therefore,
\[ \frac{\partial X^1}{\partial \theta_v^1} = \frac{\partial x_D^1}{\partial \theta_v^1} + \frac{\partial x_R^1}{\partial \theta_v^1} = \frac{\partial^2 \pi_{uD}}{\partial x_1^2} \frac{\partial^2 \pi_{uR}}{\partial (x_1^R)^2} + \frac{\partial^2 \pi_{uR}}{\partial x_1^R \partial x_1^R} \left( \frac{\partial^2 \pi_{uD}}{\partial x_1^R \partial x_1^R} - \frac{\partial^2 \pi_{uD}}{\partial x_1^R \partial x_1^R} \right). \]

We know that in an interior equilibrium \( \frac{\partial^2 \pi_{uj}}{\partial x_j^2} < 0 \). Moreover, from the condition for uniqueness \( \text{(A.3)} \), we know that \( 1 > \frac{\partial^2 \pi_{uj}}{\partial x_j^2} > -1 \). Thus, if \( \frac{\partial^2 \pi_{uj}}{\partial x_j^2} > 0 \), \( j = D, R \), then \( \frac{\partial X^1}{\partial \theta_v^1} > 0 \) and thus aggregate effort increases in \( \theta_v^1 \). To the contrary, if \( \frac{\partial^2 \pi_{uj}}{\partial x_j^2} < 0 \), \( j = D, R \), then \( \frac{\partial X^1}{\partial \theta_v^1} < 0 \) and thus aggregate effort decreases in \( \theta_v^1 \). Differentiating the respective FOCs with respect to \( \theta_v^1 \) yields
\[ \frac{\partial^2 \pi_{uD}}{\partial x_1^2} \frac{\partial^2 \pi_{uR}}{\partial (x_1^R)^2} = \varphi_v \frac{\partial^2 c(x_1, \theta_v^1)}{\partial x_1^2} \frac{\partial^2 c(x_1, \theta_v^1)}{\partial x_1^2}, \quad \frac{\partial^2 \pi_{uR}}{\partial x_1^R \partial x_1^R} \frac{\partial^2 \pi_{uD}}{\partial x_1^R \partial x_1^R} = -\varphi_v \frac{\partial^2 c(x_1, \theta_v^1)}{\partial x_1^R \partial x_1^R}. \]

Both are weakly positive when \( \theta_v^1 < \frac{1}{2} \) and weakly negative when \( \theta_v^1 > \frac{1}{2} \) by Assumption \( \text{(3)} \). In other words, both weakly increase in \( \theta_v^1(1-\theta_v^1) \). When \( \theta_v^1 = 0 \) both expressions are zero. Therefore, aggregate spending on issue 1 weakly increases in \( \theta_v^1(1-\theta_v^1) \). When \( x^1 = 0 \) both expressions are zero. Therefore, aggregate spending on issue 2 weakly increases in \( (1-\theta_v^1)^2 \). Moreover, because \( X^1 + X^2 = B \), \( \frac{\partial X^1}{\partial \theta_v^1} = -\frac{\partial X^2}{\partial \theta_v^1} \), which proves that spending on issue \( i \) weakly decreases in \( \theta_v^1(1-\theta_v^1) \).

### A.6 Proof of Proposition \([\text{7}]\)

To prove the proposition, I study equilibrium when \( \eta = 0 \) and then totally differentiate the system of FOCs at the symmetric equilibrium:
\[ E \left[ \frac{\partial c(x^1, \theta_v^1)}{\partial x^1} \bigg|_{x_1^D = x_1^R = x^1} \right] = E \left[ \frac{1 - \varphi_v}{E[\varphi_v]} \frac{\partial c(x^2, \theta_v^2)}{\partial x^2} \bigg|_{x_2^D = x_2^R = B - x^1} \right]. \]  

This is the FOC for both candidates at the symmetric equilibrium.

The vote share of \( D \) as a function of \( \eta \) is
\[ \Upsilon(\eta) = E \left[ c(x^1(\eta), \theta_v^1) \left( \varphi_v + \eta \left( g(x_1^D(\eta) + x_1^R(\eta)) - g(2B - x_1^D(\eta) - x_1^R(\eta)) \right) \right) \right. \]
\[ + \left. c(x^2(\eta), \theta_v^2) \left( 1 - \left( \varphi_v + \eta \left( g(x_1^D(\eta) + x_1^R(\eta)) - g(2B - x_1^D(\eta) - x_1^R(\eta)) \right) \right) \right] \]

Note that \( \Upsilon(0) = E[\theta_v^1 \varphi_v + \theta_v^2 (1 - \varphi_v)] \). When \( \eta \) now increases, the vote shares changes in the following...
Thus, way:

\[
Y'(\eta)\big|_{\eta=0} = E[\varphi_v] \left( \frac{\partial c(x_1(\eta), \theta_1^v)}{\partial x_D} \bigg|_{\eta=0} \right)
- (1 - E[\varphi_v]) \left( \frac{\partial c(x_1(\eta), \theta_1^v)}{\partial x_D} \bigg|_{\eta=0} \right)
+ (E[\theta_1^v] - E[\theta_2^v]) (g(X^1) - g(X^2))
\]

It follows from Lemma \[1\] that

\[
\frac{\partial c(x_1(\eta), \theta_1^v)}{\partial x_D} \bigg|_{\eta=0} = \frac{\partial c(x_1(\eta), \theta_1^v)}{\partial x_D} \bigg|_{\eta=0} \quad \text{and} \quad \frac{\partial c(x_2(\eta), \theta_2^v)}{\partial x_D} \bigg|_{\eta=0} = \frac{\partial c(x_2(\eta), \theta_2^v)}{\partial x_D} \bigg|_{\eta=0}.
\]

Thus,

\[
Y'(\eta)\big|_{\eta=0} = E[\varphi_v] \left( \frac{\partial c(x_1(\eta), \theta_1^v)}{\partial x_D} \bigg|_{\eta=0} \right)
- (1 - E[\varphi_v]) \left( \frac{\partial c(x_1(\eta), \theta_1^v)}{\partial x_D} \bigg|_{\eta=0} \right)
+ (E[\theta_1^v] - E[\theta_2^v]) (g(X^1) - g(X^2))
\]

Finally, using \[A.6\] it becomes apparent that

\[
\left( E[\varphi_v] E \left[ \frac{\partial c(x_1(\eta), \theta_1^v)}{\partial x_D} \bigg|_{\eta=0} \right] - (1 - E[\varphi_v]) E \left[ \frac{\partial c(x_1(\eta), \theta_1^v)}{\partial x_D} \bigg|_{\eta=0} \right] \right) = 0,
\]

and hence the condition boils down to

\[
Y'(\eta)\big|_{\eta=0} = (E[\theta_1^v] - E[\theta_2^v]) (g(X^1) - g(X^2)).
\]

When \(Y'(0) > 0\), for sufficiently small but positive \(\eta\), \(D\) benefits from the campaign contest, while \(R\) can use the contest to his advantage if \(Y'(0) < 0\). If \(Y'(0) = 0\) the candidates’ vote shares remain constant. As \(g'(x) > 0\), \(\text{Sign}[g(X^1) - g(X^2)] = \text{Sign}[X^1 - X^2]\), and thus I can rephrase the condition for \(D\) to benefit as

\[
\Psi = (E[\theta_1^v] - E[\theta_2^v]) (X^1 - X^2) > 0.
\]

This proves the proposition.
B The Examples

B.1 Example 1:

Given the assumptions, \( c(x^i, \theta_v^i) \in (0, 1) \) and \( w(x, \varphi_v) \in (0, 1) \) for all \( [x_D^1, x_R^1] \in [0, B]^2 \). The FOCs are

\[
\frac{\partial \pi_D}{\partial x_D^1} = 2\eta(E[\theta_v^1] - E[\theta_v^2] - \kappa + 2\kappa \cdot x_D^1) + \kappa(E[\varphi_v] - x_D^1) = 0,
\]

\[
\frac{\partial \pi_R}{\partial x_R^1} = 2\eta(E[\theta_v^2] - E[\theta_v^1] - \kappa + 2\kappa \cdot x_R^1) + \kappa(E[\varphi_v] - x_R^1) = 0,
\]

while the SOCs are

\[
\frac{\partial^2 \pi_j}{\partial (x_j^1)^2} = \kappa(4\eta - 1) < 0 \iff \eta < \frac{1}{4}.
\]

The SOCs hold whenever \( \eta < \frac{1}{4} \), which is the case by assumption, as \( \varphi + 2\eta < 1 \) and \( \varphi - 2\eta > 0 \). Hence, the candidates’ decision problems are concave and strategy spaces are compact and convex, implying that a pure strategy Nash equilibrium exists. Moreover, the FOCs are linear and independent of the other candidate’s effort choice. Thus, there is a unique pure strategy Nash equilibrium, and if \( \kappa \) is sufficiently large, this equilibrium must be interior. In this case we find

\[
x_D^1 = \frac{E[\varphi_v]}{1 - 4\eta} + \frac{2\eta (E[\theta_v^1] - E[\theta_v^2] - \kappa)}{\kappa(1 - 4\eta)} = \frac{E[\varphi_v]}{1 - 4\eta} + \frac{2\eta (E[\theta_v^1] - E[\theta_v^2])}{\kappa(1 - 4\eta)} - \frac{2\eta}{1 - 4\eta},
\]

\[
x_R^1 = \frac{E[\varphi_v]}{1 - 4\eta} + \frac{2\eta (E[\theta_v^2] - E[\theta_v^1] - \kappa)}{\kappa(1 - 4\eta)} = \frac{E[\varphi_v]}{1 - 4\eta} + \frac{2\eta (E[\theta_v^2] - E[\theta_v^1])}{\kappa(1 - 4\eta)} - \frac{2\eta}{1 - 4\eta}.
\]  

This implies

\[
X^1 = \frac{2(E[\varphi_v] - 2\eta)}{1 - 4\eta} \quad \text{and} \quad X^2 = \frac{2 \left((1 - E[\varphi_v]) - 2\eta\right)}{1 - 4\eta}.
\]

Given (B.1), equilibrium vote shares are

\[
S_D = \frac{E[\theta_v^1 \varphi_v + \theta_v^2 (1 - \varphi_v)] - 2\eta (E[\theta_v^1 + \theta_v^2])}{1 - 4\eta},
\]

\[
S_R = 1 - S_D.
\]  

(B.2)

Given \( (B.1) \), the candidate having the comparative advantage in the more important issue benefits during the campaign contest if and only if

\[
S_D > S_D^0 \iff S_D^{0} = \frac{E[\theta_v^1 \varphi_v + \theta_v^2 (1 - \varphi_v)] - 2\eta (E[\theta_v^1 + \theta_v^2])}{1 - 4\eta} > S_D^0.
\]

If \( E[\varphi_v] > \frac{1}{2} \), this is the case when \( E[\theta_v^1] > E[\theta_v^2] \), otherwise if \( E[\theta_v^1] < E[\theta_v^2] \). Thus, the candidate having the comparative advantage in the more important issue benefits during the campaign contest.
If $S^0_D < \frac{1}{2}$, Candidate $D$ is a priori weaker than Candidate $R$. If $S^0_D > \frac{1}{2}$, Candidate $D$ is a posteriori stronger than Candidate $R$. If both are true at the same time, $D$ is able to turn an initial disadvantage into an advantage. Simple manipulations show that $S^0_D > \frac{1}{2} \iff S^0_D > \frac{1}{2} + 2\eta (E[\theta^1_0 + \theta^2_0] - 1)$. Note that when $E[\varphi_v] > \frac{1}{2}$, $S^0_D < \frac{1}{2}$ implies that $E[\theta^1_0 + \theta^2_0] < 1$. Thus, we can further reformulate to get

$$\eta > \frac{\frac{1}{2} - S^0_D}{2(1 - E[\theta^1_0 + \theta^2_0])} = \frac{1}{2} - E[\theta^1_0 \varphi_v + \theta^2_0 (1 - \varphi_v)]$$

which is the condition from the example.

## B.2 Example 2:

Given the assumptions, $c(x^i, \theta^i_v) \in (0, 1)$ and $w(x, \varphi_v) \in (0, 1)$ for all $[x^i_D, x^i_R] \in [0, 1]^2$. A proof analogous to the one in Example 1 establishes existence of a unique pure strategy Nash equilibrium.

Take the derivatives of candidates’ payoff functions with respect to their respective effort in issue 1:

$$\frac{\partial \pi_D}{\partial x_D} = \kappa^1 E[\varphi_v](1 - x_D^1) - \kappa^2 (1 - E[\varphi_v])x_D^1 + 2\eta (E[\theta^1_0] - E[\theta^2_0])$$

$$\frac{\partial \pi_R}{\partial x_R} = \kappa^1 E[\varphi_v](1 - x_R^1) - \kappa^2 (1 - E[\varphi_v])x_R^1 - 2\eta (E[\theta^1_0] - E[\theta^2_0])$$

This system of equations has no simple closed-form solution, but we can use the fact that $\frac{\partial \pi_D}{\partial x_D} + \frac{\partial \pi_R}{\partial x_R} = 0$ in any interior equilibrium. Summing the two FOCs up, we get

$$\frac{\partial \pi_D}{\partial x_D} + \frac{\partial \pi_R}{\partial x_R} = 0$$

$$\iff 22\kappa^1 \varphi_v - 2\eta(x_D^1 + x_R^1 - 1)(\kappa^1(x_D^1 + x_R^1 - 2) - \kappa^2(x_D^1 + x_R^1)) + (x_D^1 + x_R^1)(\kappa^2(\varphi_v - 1) - \kappa^1 \varphi_v) = 0$$

$$\iff 22\kappa^1 \varphi_v - 2\eta(\kappa^1(x_D^1 + x_R^1 - 2) - \kappa^2(\varphi_v - 1) - \kappa^1 \varphi_v) = 0,$$

where, as before, $X^1 = x_D^1 + x_R^1$. Defining $\nu \equiv \frac{\kappa^1}{\kappa^2}$, assuming $\kappa^1 \neq \kappa^2 \iff \nu \neq 1$, and solving for $X^1$, we obtain

$$X^1 = \sqrt{\frac{4\eta ((\nu^2 - 1) \varphi - 3\nu + 1) + 4\eta^2 (\nu + 1)^2 + ((\nu - 1) \varphi + 1)^2 + \eta (6\nu - 2) - \varphi (\nu - 1) - 1}{4\eta \nu - 1}}.$$

We already know from Proposition 4 that $X^1$ increases in $\varphi_v$ (the formula also reveals this). Hence, there is potentially a threshold value $\tilde{\varphi}$ that solves $X^1|_{E[\varphi_v] = \tilde{\varphi}} = 1$, and where $X^1 > 1$ if $E[\varphi_v] > \tilde{\varphi}$ and $X^1 < 1$ if $E[\varphi_v] < \tilde{\varphi}$. This value, if it exists, is defined by

$$X^1|_{E[\varphi_v] = \tilde{\varphi}} = 1$$

$$\iff \sqrt{4\eta ((\nu^2 - 1) \varphi - 3\nu + 1) + 4\eta^2 (\nu + 1)^2 + ((\nu - 1) \varphi + 1)^2 + \eta (6\nu - 2) - \varphi (\nu - 1) - 1} = 4\eta (\nu - 1)$$

$$\iff \tilde{\varphi} = \frac{1}{1 + \nu}.$$

Note that $\frac{1}{1 + \nu} \in (0, 1)$, and a meaningful threshold $\tilde{\varphi}$ exists. Therefore, $\text{Sign} \left[ X^1 - X^2 \right] = \text{Sign} \left[ E[\varphi_v] - \frac{1}{1 + \nu} \right]$.  

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If $\kappa^1 = \kappa^2$, $\nu = 1$. Aggregate spending on issue 1 is then (see also the proof of Example 1)

$$X^1 = \frac{2(2\eta - E[\varphi_v])}{4\eta - 1}.$$

Thus, also in this case $\text{Sign } [X^1 - X^2] = \text{Sign } [E[\varphi_v] - \frac{1}{1+\nu}]$. \hfill \Box

References


