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Location, inventory and testing decisions in closed-loop supply chains: a multimedia company

Abstract

Our partnering firm is a Chinese manufacturer of multimedia products that needs guidance developing its imminent closed-loop supply chain (CLSC). To study this problem, we take into account location, inventory, and testing decisions in a CLSC setting with stochastic demands of new and time-sensitive returned products. Our analysis pays particular attention to the different roles assigned to the reverse distribution centers (DCs) and how each option affects the optimal CLSC design. The roles considered are collection and consolidation, additional testing tasks, and direct shipments with no reverse DCs. The problem concerning our partnering firm is formulated as a scenario-based chance-constrained mixed-integer program and it is reformulated to a conic quadratic mixed-integer program that can be solved efficiently via commercial optimization packages. The completeness of the model proposed allows us to develop a decision support tool for the firm and to offer several useful managerial insights. These insights are inferred from our computational experiments using data from the Chinese firm and a second data set based on the U.S. geography. Particularly interesting insights are related to how changes in the reverse flows can impact the forward supply chain and the inventory dynamics concerning the joint DCs. Supplementary materials are available for this article. Go to the publishers online edition of IISE Transaction, datasets, additional tables, detailed proofs, etc.

Keywords: closed-loop supply chain, conic quadratic mixed-integer program, role of reverse DCs, Chinese manufacturer

1. Introduction

Sichuan Changhong Electric Co., Ltd. (a.k.a Changhong) is a Chinese company leader in TV sales in the national market that has been in business for around 60 years. In recent years, the company has expanded to more industries, such as the multimedia, air conditioner, refrigerator, IT, communication, and the eco-friendly battery industries. Alongside its accelerated development, the major challenges of running the company are related to sustaining the logistics service level to maintain customer satisfaction, while at the same time administering the increment of logistics costs that take up a significant proportion of the total cost. Moreover, the industries where this company operates suffer from environmental pressure in the form of new legislations associated
with electronic products such as the “Regulations on Administration of Collection and Disposal of Waste Electrical Appliances and Electronic Products”, which is similar to the Waste Electrical and Electronic Equipment Directive (WEEE) in Europe, and financial allowances for collecting specific electronic products. All these factors impel the company to update its logistics network, starting with analyzing the best closed-loop supply chain (CLSC) of the newly launched line of multimedia products that including liquid crystal televisions (LCTVs), computers, and mobile phones.

Time has a central role in reverse supply chain design and management (Guide et al., 2006; Janzen & Rosier, 2008) and this is reflected in our modeling approach. Although Changhong’s multimedia products can be used for about ten years, their real life-cycle is only a few years because new products are launched frequently. For example, the life-cycle of intelligent TV sets in China has reduced to 3~4 years in recent years (Lenovo, 2012). PCs are another type of consumer electronic product that can lose value at rates in excess of 1% per week (Blackburn et al., 2004). In particular, for a real example of a consumer electronics firm, for $1000 worth of product returns nearly half the product value (>45%) is lost in the return process.

This paper responds to different calls for more research on generalized CLSC network design models with practical impact and methodology development (Akçalı et al., 2009; Aras et al., 2010; Blackburn et al., 2004; Ferguson, 2010; Guide & Van Wassenhove, 2009; Souza, 2013). The structure of the supply chain studied is a good representation of the multimedia products’ CLSC of our Chinese manufacturer and of a great number of other firms (Akçalı et al., 2009; Fleischmann et al., 2001; Souza, 2013). It consists of a three-tiered closed-loop supply chain network with capacitated forward and reverse manufacturing facilities (MFs), capacitated forward and reverse distribution centers (DCs), and retailers (Figure 1). Further details of the CLSC studied are described in Section 3.

Figure 1: Structure of the supply chain
The three-tiered structure of the CLSC can generate a discussion about the possible roles that might be taken by the reverse DCs (Figure 2). We consider three options. The first one assumes that reverse DCs are collection and consolidation centers (our base scenario). For example, Cummins exemplifies this case, where new engines are shipped to the dealer (customer) via a DC and the used products go from the dealer to a used products depot where products get stocked and credit is issued to customers for the returns. From this depot all returns are shipped to one of two remanufacturing plants where testing is done (Souza, 2013). The second option assumes that these DCs could also be testing and separating between remanufacturing and disposable products. Testing at the DCs is typically more expensive than at the reverse MFs due to a reduction on economies of scale and increased labor costs, but less volume needs to be shipped to the reverse MFs (Sections 3.6 and 6.2). This option is illustrated by Changhong and by Ford, that collects cores from 4,000 dealers to a facility to evaluate them for quality. Bad quality cores are disposed of and good ones are kept in a warehouse for future remanufacturing (Souza, 2012). The third option (Section 6.3) is to directly ship the returns from the retailers to the remanufacturing plants. This option does not have reverse DCs, it is generally the most expensive in terms of shipping costs, but it is the fastest one (Abdallah et al., 2012; Drake et al., 2008).

Figure 2: Three options of reverse flow structure.

To sum up, in this paper we seek to answer the following main research questions: How do different roles assigned to the reverse DCs impact the CLSC? How do different degrees of a product’s marginal value of time and different rates of returned product influence the location and inventory decisions of a CLSC?

This paper contributes to the CLSC literature by providing novel research in three directions. First, it models a novel integrated three-tiered closed-loop supply chain network problem and it provides an efficient associated solution approach. Our solution approach is general, flexible, and
it allows to model desirable features of the CLSC problem such as risk pooling and consolidation activities at the DCs and uncertain forward and reverse demand flows with nonlinear safety stocks. Second, it provides a detailed description of the CLSC challenges of our partnering firm, Changhong, that is planning to start its large-scale remanufacturing operations for multimedia products. Among different features, the model is scenario-based and it is being utilized by the firm as a decision support tool. Third, it provides new industry-wide managerial insights related to location of DCs and MFs, inventory management at the DCs, and testing location decisions.

The rest of the paper is organized as follows. Section 2 presents a comprehensive literature review. The problem for Changhong is stated and a nonlinear mixed-integer program with chance constraints is developed in Section 3. In Section 4, the program is transformed into a conic quadratic mixed-integer program and a class of valid inequalities is introduced to strengthen the formulations. The case study about Changhong is developed in Section 5. Managerial insights are explored in Section 6. In Section 7, the conclusions are drawn and further research directions are outlined.

2. Literature review

The literature associated with closed-loop supply chains has increasingly emerged in recent years. For comprehensive reviews of this literature we refer the reader to the work of Akçali et al. (2009), Dekker et al. (2004), Ferguson (2010), Guide & Van Wassenhove (2009), and Souza (2013). Before we dive into reviewing different types of CLSC network design problems, we briefly cite relevant work on forward and reverse supply chain design literatures that have served as an inspiration to subsequent work in CLSCs. The forward supply chain design literature is very extensive and we refer to two books (Graves & de Kok, 2003; Snyder & Shen, 2011) that cover the most relevant aspects. The research in reverse supply chain design is more recent. Some of the most representative papers are Alumur et al. (2012) that proposed a profit maximization modeling framework for a deterministic multi-period reverse logistics network design problem and Tancrez et al. (2012) that studied a deterministic location-inventory problem in a three-level supply chain. We note that in most of the related reverse supply chain design literature, simplistic inventory costs were added to the objective function without considering the demand uncertainty or the risk pooling impact as we do in our approach.

Next, we review literature that integrates both forward and reverse flows into a unique CLSC
model. First of all, we observe that the integrated research on location and inventory decisions for CLSC models, which our paper models because it lead to lots of cost-saving opportunities, is very limited. [Abdallah et al. (2012) and Diabat et al. (2015)] proposed the uncapacitated closed-loop location-inventory model which integrates the inventory decision into the location allocation decisions in a closed-loop supply chain. Both flows in their studies contain only two levels: the DCs distribute a single product to different retailers in forward flows while remanufacturing centers collect the returns from the retailers. [Zhang et al. (2015)] studied a three-tiered supply chain with stochastic forward demand and time-sensitive returned products. They integrated location and inventory decisions at the DC level as our current paper but they assumed a unique uncapacitated manufacturer. This assumption simplifies the study of the network and does not allow for a complete study of testing decisions. Unlike Zhang et al. (2015), our study further models stochasticity to represent the company’s current uncertainties related to capacities and volume of returned product.

Continuing with modeling stochasticity in a CLSC problem, this uncertain behavior can be modeled using scenarios that describe a possible future state attached to a known probability distribution. [Keyvanshokooh et al. (2016)] proposed a profit maximizing CLSC design problem that defined stochastic scenarios for transportation costs. These transportation cost scenarios were generated using a Latin Hypercube Sampling method and an accelerated stochastic Benders decomposition algorithm is proposed for solving their model. [Üster & Hwang (2017)] used a scenario-based technique to model uncertainty of new product demand and product returns in a CLSC design problem. A Benders’ decomposition approach was proposed to solve the suggested two-stage stochastic mixed integer linear programming model. [Chen et al. (2015)] studied a profit maximizing CLSC with uncertain market size, return quantity and return quality that are defined in a set of scenarios. They studied the choice between remanufacturing and recycling and claimed that this choice depends upon market characteristics and cost structure.

Other research on CLSC network design close to our work is cited in the following lines. These models do not consider inventory decisions or time value of money that can generate nonlinearities as it is the case of our paper. Thus, these papers can be modeled as mixed-integer linear problems (MILP) and can use standard solution techniques such as Benders’ decomposition or tabu search. [Sahyouni et al. (2007)] developed three generic facility location models for the integrated distribution and collection of products in closed-loop supply chains. These models quantified the value of
integrated decision making in the design of forward and reverse logistics networks throughout
different stages of a product’s lifecycle. Üster et al. (2007) considered a multi-product closed-loop
supply chain network design problem where collection centers and remanufacturing facilities were
located while coordinating the forward and reverse flows in the network so as to minimize the
processing, transportation, and fixed location costs. Easwaran & Üster (2009, 2010) proposed
MILPs to consider multi-product capacitated closed-loop logistic network design problems, which
locate network facilities such as manufacturing/ remanufacturing facilities and DCs and determine
the material flows. An extension to these two papers is presented in Üster & Hwang (2017) which
considered uncertainty in new product demand and product return values. Özceylan & Paksoy
(2013) developed an MILP to investigate a multi-period and multi-part CLSC with deterministic
and time-varying demands. Soleimani et al. (2016) proposed a scenario-based multi-period, multi-
product CLSC network with stochastic demand and price in a MILP programming structure.

3. Problem statement and formulation

Figure 1 illustrates the three-tiered structure of the network, which is based on the real application
of Changhong. The first tier consists of two kinds of capacitated manufacturing facility
(MF) candidates: manufacturing facilities that process the new products (forward MFs) and re-
manufacturing facilities (reverse MFs) that refurbish returned products or disassemble them and
ship back to the manufacturing facilities the available components that can be used for producing
secondary market products. We assume that the reverse MFs can only be opened near forward MFs
to minimize unnecessary transportation and leverage resources and expertise. This is a common
setting in real applications and an assumption extensively considered in closed-loop supply chains
(Abdallah et al., 2012; Diabat et al., 2015). The second tier consists of three types of capacitated
DC candidates: forward DCs that store the new products shipped from the MFs, reverse DCs that
only store the returned products collected from the retailers, and joint DCs that store both new and
returned products. For the base model, it is assumed that reverse and joint DCs collect returned
products but do not inspect them. The third tier is represented by the retailers who not only sell
the new products but also collect the returned products.

There are two product flows in the network. One is the forward flow, which is represented by
new products that are shipped from the forward MF to the retailers via forward or joint DCs to

6
satisfy the stochastic customers’ demand. The other flow is the reverse flow, which is represented
by returned products that are shipped from the retailers to the reverse MF via reverse or joint DCs
to be reprocessed at the reverse MFs. Changhong is particularly interested in collecting end-of-use
multimedia products because they still have significant value left. For example, they are interested
in recovering the chips from used multimedia phones. Nonetheless, the reverse flow can include
three types of returned products: consumer returns that follow return policies and are mostly new,
end-of-use returns, and end-of-life returns. Indeed, consolidation of different types of returns is a
recommended practice that enables sharing services (Janzen & Rosier, 2008). In this paper, we
do not differentiate between different types of returned products, but this can certainly be done
by assuming different rates of returned product, marginal values of time, and fractions of disposed
returned product. For our base model, we assume that all returns are shipped to reverse or joint
DCs and then to remanufacturing facilities, where testing and other processing operations take
place depending on the state of the product. If there are suitable components, these will be re-used
to obtain new products that will be sold to secondary markets. We compare the base model with
a second model that expands the role of the reverse and joint DCs to implementing testing and
disposition activities (Section 3.6) and a third model that skips the reverse DCs all together.

Single-sourcing strategies in both retailer-to-DC and DC-to-MF assignments are considered,
where each retailer gets the new products from a single DC and each DC replenishes its inventory
from a single MF in the forward flow, while the returned products are shipped to a single MF via
a single DC. Although a single-sourcing strategy reduces the span of sourcing options that multi-
sourcing can bring, single sourcing is used in practice because it helps to ease control of the inventory
decisions throughout the supply chain and reduces the internal friction of a company. Furthermore,
this strategy is also preferred at the DC and retailer levels because it simplifies operational decisions
that do not require the use of advanced information technology to coordinate and track shipments
and deliveries (Easwaran & Uster, 2009). This is corroborated by our partnering firm, which prefers
a single sourcing strategy given that the company is starting its CLSC implementation.

3.1. Management of inventories

In our model we integrate inventory management decisions with location decisions. When man-
aging inventories in a supply chain, there are two critical decisions. The first one is to determine
the number of stocking locations or distribution centers to have. Then, the amount of inventory maintained at each of the centers has to be determined. Separate decision-making on the location and inventory tasks results in a degree of sub-optimization \cite{Daskin2002}. Therefore, integrated inventory and location decisions when studying the optimal design of a supply chain has been extensively studied and has been shown to achieve risk-pooling benefits, inventory cost reductions, and possibly line-haul shipping benefits \cite{Daskin2002, Shen2003}. In the context of CLSC, this is still very relevant because some locations can share forward and returned flows, as it is the case of our joint DCs. In this context, modeling inventory levels and accounting for capacities are critical factors \cite{Diabat2015, Zhang2015}.

To obtain the benefit of a risk pooling strategy, inventories are kept at the DCs where safety stock is retained. So, in this problem, inventory management decisions are only modeled at the DC level. Additional modeling of inventories at the MF level would require the design of a multi-echelon inventory system that would add an extra layer of complexity to the problem without providing any novel insights related to distribution and location decisions. Forward and joint DCs should manage to satisfy the retailer demand at a specific level of service. In particular, an approximation to the \((Q,r)\) model with Type-I service (or in-stock probability) is used for managing the stock of new products, where \(Q_j^F\) is the reorder quantity at DC \(j\) obtained from solving a capacitated EOQ model and \(r_j\) is the reorder point at DC \(j\) \cite{Ozsen2008}. This reorder point is defined as the sum of safety stock and inventory that is equal to the demand during lead time. The inventory policy followed by joint or reverse DCs is as follows: once the quantity of returned products at a DC attains to a predetermined value, all of it will be shipped to the corresponding remanufacturing facility. This quantity is the solution of a capacitated EOQ model. Note that the system is capacitated at the MF and DC levels.

### 3.2. The model

The problem is to determine which MFs should be opened to process new products or handle the returned products, which DCs candidates should be opened to store the new and returned products, and how to allocate the open DCs to the open MFs and allocate the retailers to the open DCs. The objective is to minimize the fixed charges of locating the MFs and DCs, and the operational costs associated with the forward and reverse flows.
Before proposing the model, the following additional assumptions are made: (1) Volumes of new and returned products at each retailer are i.i.d. and follow normal distributions; (2) Different normal distributions of volumes of returned product are represented in different scenarios with associated scenario probabilities; (3) Transportation capacity is sufficient; (4) No transshipment between DCs is allowed; (5) Lead times at the retailers are neglected (it does not affect our analysis).

To model the problem, Tables 1, 2, and 3 define the parameters and decision variables used throughout this paper.

| $I$ | Set of retailers indexed by $i$ |
| $J$ | Set of candidate DC sites indexed by $j$ |
| $K$ | Set of candidate manufacturing facilities indexed by $k$ |
| $S$ | Set of scenarios for different rates of returned product indexed by $s$ |

Table 1: Sets.

In summary, the problem is modeled as follows:

$$\begin{align*}
\min_{X,Y} \quad & Z = \sum_{k \in K} p_k Z^F_k + \sum_{j \in J} f_j^F X^F_j + \sum_{k \in K} p_k Z^R_k + \sum_{j \in J} f_j^R X^R_j - \sum_{j \in J} \delta^C_j X^C_j \\
& \quad + \sum_{s \in S} \omega_s \Gamma_s + \lambda \sum_{s \in S} \omega_s \left| \Gamma_s - \sum_{s' \in S} \omega_{s'} \Gamma_{s'} \right|,
\end{align*}$$

(1)

s.t. \quad \sum_{j \in J} Y^F_{ij} = 1, \sum_{j \in J} Y^R_{ij} = 1, \forall i \in I,

(2)

$$Y^F_{ij} \leq X^F_j, Y^R_{ij} \leq X^R_j, \forall i \in I, \forall j \in J,$$

(3)

$$X^C_j \leq X^F_j, X^C_j \leq X^R_j, \forall j \in J,$$

(4)

$$\sum_{k \in K} V^F_{jk} = X^F_j, \sum_{k \in K} V^R_{jk} = X^R_j, \forall j \in J,$$

(5)

$$V^F_{jk} \leq Z^F_k, V^R_{jk} \leq Z^R_k, \forall j \in J, \forall k \in K,$$

(6)

$$Z^R_k \leq \sum_{j \in J} V^F_{jk}, \forall k \in K,$$

(7)

$X^F_j, X^R_j, X^C_j$ 1, if candidate location $j$ is selected as a forward/reverse/joint DC, and 0 otherwise

$Y^F_{ij}, Y^R_{ij}$ 1, if new/returned products of retailer $i$ is served/collected by DC $j$, and 0 otherwise

$Z^F_k, Z^R_k$ 1, if MF $k$ is selected to be a manufacturer/ remanufacturer, and 0 otherwise

$V^F_{jk}, V^R_{jk}$ 1, if DC $j$ is served by manufacturer/ remanufacturer $k$, and 0 otherwise

$\Gamma_s$ Operational cost under scenario $s$

$Q^F_s, Q^R_s$ Shipment quantity of new and returned products at DC $j$ under scenario $s$

$Y^F_j, Y^R_j = (Y^F_{1j}, ..., Y^F_{Ij})^T = (Y^R_{1j}, ..., Y^R_{Ij})^T$

Table 2: Decision variables.
\begin{equation}
\Gamma_s \geq \sum_{j \in J} \left\{ \sum_{i \in I} \beta \chi_d i j \mu^F_{i j} \gamma^F_{i j} + \tilde{W}_i \gamma^F_{i j} (D^F_{j i}, Q^F_{j i}) + \theta h \sum_{s \in S} S^F_{j i} (Y^F_{j i}, L^F_{j i}) \right\} + \sum_{j \in J} \left\{ \sum_{i \in I} \beta \chi_d i j \mu^R_{i j} \gamma^R_{i j} + \tilde{W}_i \gamma^R_{i j} (D^R_{j i}, Q^R_{j i}) + W \cdot R (Y^R_{j i}, Q^R_{j i}) \right\}, \forall s \in S,
\end{equation}
\begin{equation}
\Pr \left\{ Q^F_{j i} + Q^R_{j i} + r_j - \tilde{D}^F_{j i} \geq C_{j i} \right\} \leq \tilde{\rho}, \forall j \in J, s \in S,
\end{equation}
\begin{equation}
\Pr \left\{ \sum_{j \in J} \tilde{D}^F_{j i} V^F_{j k} \geq C^F_{j i} \right\} \leq \rho^F, \forall k \in K,
\end{equation}
\begin{equation}
\Pr \left\{ \sum_{j \in J} \tilde{D}^R_{j i} V^R_{j k} \geq C^R_{j i} \right\} \leq \rho^R, \forall k \in K, s \in S,
\end{equation}
\begin{equation}
\sum_{j \in J} D^F_{j i} V^F_{j k} \geq \sum_{j \in J} D^R_{j i} V^R_{j k}, \forall k \in K, s \in S.
\end{equation}
\begin{equation}
Q^F_{j i}, Q^R_{j i}, \Gamma_s \geq 0, \forall j \in J, s \in S,
\end{equation}
\begin{equation}
X^F_{i j}, X^R_{i j}, X^C_{i j}, Y^F_{i j}, Y^R_{i j}, Z^F_{i k}, Z^R_{i k}, V^F_{j k}, V^R_{j k} \in \{0, 1\}, \forall i \in I, j \in J, \forall k \in K,
\end{equation}
where, $\overline{WI}_j^F$ and $\overline{WI}_j^R$ are working inventory costs associated with new and returned products, respectively. They are defined as follows.

$$\overline{WI}_j^F (D_j^F, Q_{js}^F) = \begin{cases} 
F_j^F \frac{D_j^F}{Q_{js}^F} + \beta (g_j^F + a_j^F Q_{js}^F) \frac{D_j^F}{Q_{js}^F} + \frac{\theta h}{2} Q_{js}^F, & \forall j \in J, Q_{js}^F > 0, \\
0, & Q_{js}^F = 0,
\end{cases}$$

(15)

$$\overline{WI}_j^R (D_j^R, Q_{js}^R) = \begin{cases} 
F_j^R \frac{D_j^R}{Q_{js}^R} + \beta (g_j^R + a_j^R Q_{js}^R) \frac{D_j^R}{Q_{js}^R} + \frac{\theta h}{2} Q_{js}^R, & \forall j \in J, Q_{js}^R > 0, \\
0, & Q_{js}^R = 0,
\end{cases}$$

(16)

and, $D_j^F = \chi \sum_{i \in I} Y_{ij}^F, a_j^F = \sum_{k \in K} \bar{a}_{jk}^F V_{jk}^F, g_j^F = \sum_{k \in K} \bar{g}_{jk}^F V_{jk}^F, L_j = \sum_{k \in K} \bar{L}_{jk}^F V_{jk}^F$. These equations are defined as the sum of fixed ordering cost, MF-to-DC shipping costs, and the average order inventory holding costs per year. The detailed information about the definition of the working inventory costs can be found in Shen et al. (2003). $\overline{SS}_j (Y_j^F, L_j)$ is the safety stock, which is formulated below.

$$\overline{SS}_j (Y_j^F, L_j) = z_\alpha \sqrt{L_j \sum_{i \in I} (\sigma_i^F)^2 Y_{ij}^F}.$$  

(17)

And, $R (Y_j^R, Q_{js}^R)$ is the total average value loss per year at DC j, which is defined as

$$R (Y_j^R, Q_{js}^R) = R_{inv} (Q_{js}^R) + R_{tr} (Y_j^R) = \gamma U \left( \frac{\chi Q_{js}^R}{2} + \sum_{i \in I} \left( \frac{d_{ij} + a_j^R}{\kappa} \right) \chi \mu_{is}^R Y_{ij}^R \right).$$  

(18)

The average value loss is associated with inventory time and transportation time of returned products. A detailed explanation is provided in Section 3.5.

The objective function consists of six components. The first five components are associated with location costs, which are independent of the scenarios related to different rates of returned products. The first two components are the location costs of the forward MFs and forward DCs, respectively. The third and fourth components are the location costs of the reverse MFs and reverse DCs, respectively. The fifth component is the cost savings from co-location of forward and reverse DCs, where the benefit is obtained from exploiting economies of scale. Similarly to Sahyouni
et al. (2007), fixed cost savings occur when assuming $s_j^C \leq \min\{f_j^F, f_j^R\}$. The sixth component is associated with the operational costs, which are modeled as a weighted sum of mean and deviation of operational costs based on a mean absolute deviation framework. A detailed explanation of the sixth component is provided in Section 3.3.

Constraints (2) state that a retailer is served by exactly one DC due to single sourcing policy. Constraints (3) ensure that a retailer is served only by an open DC. Constraints (4) give the definition of joint DCs, i.e., a joint DC acts as not only a forward DC but a reverse DC as well. Constraints (5) ensure that an open DC is assigned to exactly one MF due to the single sourcing policy. Constraints (6) stipulate that a DC is served by an open MF. Constraints (7) state that a reverse MF can only be open if the corresponding forward MF is opened. Constraints (8) state the expected total operational costs under each scenario. The expected operational cost associated with the forward flows consists of shipment cost, working inventory cost, and safety stock cost. The expected operational cost associated with the reverse flows is similar except that safety stock is substituted with the weight value loss. Chance constraints (9) represent that during the replenishment lead time of new products, the probability that the inventory accumulation at DC exceeds the capacity of that DC is less than a predetermined value, $\bar{\rho}$. Chance constraints (10) and (11) state that the probabilities that the quantities of new and returned products processed at a MF per year exceed their maximum processing capacities are less than predetermined values, $\rho^F$ and $\rho^R$, respectively. A detailed explanation of constraints (9), (10) and (11) is provided in Section 3.3. Constraints (12) ensure the balance between the volume of new and returned product. That is, the quantity of new products produced at every manufacturing facility should be larger than the quantity of returned products collected at the corresponding remanufacturing facility in a year. Constraints (13) and (14) are nonnegative and standard integrality constraints, respectively.

3.3. Mean absolute deviation framework associated with the operational costs

Several modeling tools such as stochastic programming and robust optimization are extensively used to tackle uncertainty. In particular, deviation risk measures such as mean absolute deviation (MAD), mean variance (MV), and probability risk measures such as Conditional Value at Risk (CVaR) and Value at Risk (VaR) have shown to be advantageous in analyzing stochastic supply chain management problems over the past decades (Agrawal & Seshadri, 2000; Hung et al., 2013;
These techniques have also been used in CLSC management (Soleimani et al., 2014). Some particular reasons encourage us to utilize the mean absolute deviation (MAD) measure as the measure of risk to model uncertain portions of our problem: (1) MAD is intuitive and easily interpretative in practice; (2) the MAD is a very versatile and robust measure for exploring supply chain related risk issues. The most uncertain portion of our problem is the one related to operational costs, which are less strategic and require more short term details. In practice, firms, including our partnering firm, might feel comfortable assigning probabilities to a set of feasible operational scenarios. For this reason, a scenario-based robust approach, which falls in the stochastic programming domain, is employed to model the operational costs part of the problem. In particular, for our problem, each scenario represents a specific volume of returned product. We define the set of scenarios as $S$, the probability of scenario $s \in S$ as $\omega_s$, and the expected operational costs of scenario $s$ as $\Gamma_s$. The expectation is defined as $\sum_{s \in S} \omega_s \Gamma_s$ and the risk measure is characterized by the mean absolute deviation (MAD) of the operational costs, which is formulated as $\sum_{s \in S} \omega_s |\Gamma_s - \sum_{s' \in S} \omega_{s'} \Gamma_{s'}|$.

3.4. Capacity constraints

Under scenario $s \in S$, the capacities of both the reverse MFs and DCs are considered in the model. For each forward (reverse) MF, the yearly processing capacity restricts the volumes of new (returned) products processed, which equals to the total quantity of demand of new products (shipment of returned products) of the retailers who are linked with that MF. For stochastic demands of new products and collection quantity of returned products, we use chance constraints to formulate the capacity constraints associated with each MF, which are modelled as

$$Pr \left\{ \sum_{j \in J} \tilde{D}_j^F V_{jk}^F \geq C_k^F \right\} \leq \rho^F \quad \text{and} \quad Pr \left\{ \sum_{j \in J} \tilde{D}_j^R V_{jk}^R \geq C_k^R \right\} \leq \rho^R, \forall k \in K, \forall s \in S .$$

At the DC level, we use a chance constraint to formulate the capacitated constraint associated with each DC as follows $Pr \left\{ Q_{js}^F + Q_{js}^R + r_j - \tilde{D}_{j,k}^F \geq C_j \right\} \leq \bar{\rho}, \forall j \in J, \forall s \in S$, where we model the inventory accumulation at a DC during replenishment lead time. This equals the sum of shipment quantity of new and returned products and reorder point of new products minus demand of new products during that lead time. Note that this constraint is describing a worst-case scenario in which both full quantities $Q_{js}^F$ and $Q_{js}^R$ coincide at the same time at the DC.
3.5. Value loss

Time and cost trade-offs have attained considerable attention in supply chain management. Compared to the benefit of shortening the new products flow time, it is much more beneficial for returned products to shorten their flow time because these products are characterized by more time-varying prices and shorter residual life-cycle, especially for time-sensitive and short life-cycle products. Moreover, processing of new products is often a priority in real applications, which further leads to the delay of processing of returned products in an integrated supply chain. For the above reasons, in this research we consider the time value of returned products. As modeled in Zhang et al. (2015), $R(\mathbf{Y}_j^R, Q_{j,s}^R)$ represents the average value loss of returned product per year, which is caused by the delay of the storage time and transportation time of returned products. The daily marginal value of time ($\gamma$) represents the decay rate of returned products (Blackburn et al., 2004), which is illustrated by the slope of the line in Figure 3. Since the average returned products inventory on hand is $\chi Q_{j,s}^R/2$ in a DC, the average value loss of returned product per year, denoted by $R_{\text{inv}}(Q_{j,s}^R)$, is defined as $R_{\text{inv}}(Q_{j,s}^R) = \frac{\gamma U \chi}{2} Q_{j,s}^R$, where $U$ is the initial price of returned products.

Value loss caused by transportation time, denoted by $R_{\text{tr}}(\mathbf{Y}_j^R)$, is defined as follows. Let $\kappa$ be the daily transportation cost per unit returned product and so $\frac{d_{ij} + a_{ij}^R}{\kappa}$ represents the time spent by a returned product in transportation from retailer $i$ to DC $j$ and from DC $j$ back to the assigned MF. Thus, the average amount of returned products in transportation per year at DC $j$ is $\sum_{i \in I} \left( \frac{d_{ij} + a_{ij}^R}{\kappa} \right) \chi \mu_i^R Y_{ij}^R$. In total, the total average value loss per year at DC $j$ is defined as $R(\mathbf{Y}_j^R, Q_{j,s}^R) = R_{\text{inv}}(Q_{j,s}^R) + R_{\text{tr}}(\mathbf{Y}_j^R) = \gamma U \left( \frac{\chi Q_{j,s}^R}{2} + \sum_{i \in I} \left( \frac{d_{ij} + a_{ij}^R}{\kappa} \right) \chi \mu_i^R Y_{ij}^R \right)$. This definition is modified when testing is done at the reverse DCs (Section 3.6), due to products being discarded.

![Figure 3: Time value of product returns.](image-url)
3.6. Testing and disposition decisions at the DC level

If testing and disposition is done at the reverse and joint DCs instead of at the remanufacturing site, less returned products will be shipped for remanufacturing to the reverse MFs. At the same time, testing and disposition decisions are typically more expensive at the DCs. In particular, only a fraction $\tau_i$ of returned product is acceptable for remanufacturing and the rest, $1 - \tau_i$, will be disposed of as scrap or recyclable material. For notation convenience, we split the per unit cost $a_j^R$ into cost of shipment $a_j^{ship,R}$ and cost of testing $a_j^{test,R}$. For the sake of simplicity, the cost of testing is assumed to be the same regardless of the DC ($a_j^{test,R} = a_j^{test,R}$). This scenario affects the model in the following ways:

- Less returned products will be shipped from DCs to MFs. In fact, equations (16) change to
  \[
  F_j^R = \chi \sum_{i \in I} T_i R_j Y_{ij}^R + \beta (g_j^R + a_j^{ship,R} Q_{js}^R) + \beta a_j^{test,R} \chi \sum_{i \in I} \mu_{is} R_j Y_{ij} + \frac{\theta h}{2} Q_{js}^R.
  \]

- Less returned products will be processed at the remanufacturing MFs. In particular, equations (10), (11) and (12) should only include a proportion $\tau_i$ of returned products.

- The average value loss will be affected and it will depend on the values of $a_j^{ship,R}$ and $a_j^{test,R}$.

On the one hand, less quantity will be shipped from DC to reverse MF and, on the other hand, testing at the DCs needs to be done for all units. The value loss changes as follows:

\[
\gamma U \left( \frac{\chi Q_{js}^R}{2} + \sum_{i \in I} \left( \frac{d_{ij} + a_j^{test,R}}{\kappa} \chi \mu_{is} Y_{ij}^R + \frac{a_j^{ship,R}}{\kappa} \chi T_i R_j Y_{ij} \right) \right).
\]

In Section 6, we study the trade-off between the savings that imply shipping less returned product to the reverse MFs and the increased cost of testing at the DCs.

4. Model reformulation

The proposed model is formulated as a scenario-based chance-constrained mixed-integer program that is normally hard to solve to optimality in a reasonable amount of time. We employ the methodology suggested by Atamtürk et al. (2012) for joint location-inventory models to reformulate ($\mathcal{P}$) as a conic quadratic mixed-integer program (CQMIP) that will be directly solved via commercial optimization packages.
Definition 1. A conic quadratic mixed-integer program (CQMIP) is an optimization problem of the form:

\[
\begin{align*}
\min & \quad c^T x \\
\text{s.t.} & \quad \|A_i x + b_i\|_2 \leq d_i^T x + e_i, \quad i = 1, \ldots, p,
\end{align*}
\]

where \(x \in \mathbb{Z}^n \times \mathbb{R}^m\), \(c \in \mathbb{R}^{(n+m)}\), \(A_i \in \mathbb{R}^{n_i \times (n+m)}\), \(b_i \in \mathbb{R}^{n_i}\), \(d_i \in \mathbb{R}^{(n+m)}\), \(e_i \in \mathbb{R}\), \(\| \cdot \|_2\) is the Euclidean norm, and all parameters are rational.

This reformulation to a CQMIP is detailed in Appendix A and it is beneficial in several ways: 1) it reduces the burden of developing special purpose algorithms such as column generation and Lagrangian relaxation based methods, 2) it allows to consider novel generalized modeling aspects (for example it does not require to assume a certain proportion between the retailer mean and variance demand) and 3) it leads to efficient computational performance.

Proposition 1. Problem (\(\mathcal{P}\)) is equivalent to a CQMIP.

PROOF. See online supplement. □

Finally, we note that the set functions in constraints (A.9), (A.12), and (A.14) are submodular. Therefore, we can add extremal extended polymatroid inequalities [Atamtürk & Narayanan, 2008] to strengthen the formulations and improve the computational results.

Proposition 2. The extremal extended polymatroid inequality \(\sum_{i \in I} \pi_i Y_{ij} \leq \Omega_j\) is valid for \(\mathcal{Q}_f\), which represents the lower convex envelope of the sets of solutions which satisfy constraints (A.9), i.e. \(\mathcal{Q}_f = \text{conv} \left\{ (Y_j^F, \Omega_j) \in \{0, 1\}^I \times R : \Omega_j \geq f(S) = \sqrt{\sum_{k \in K} \sum_{i \in S} \bar{L}_{jk}(\sigma_i^F)^2}, \forall S \subseteq I \right\}\), where \(\pi_i = \sqrt{\sum_{k \in K} \sum_{i \in S(i)} L_{jk}(\sigma_i^F)^2} - \sqrt{\sum_{k \in K} \sum_{i \in S(i-1)} L_{jk}(\sigma_i^F)^2} \in \mathcal{E}P_f, S = \{i|Y_{ij} = 1\}\), and \(S(i) = \{(1), (2), \cdots, (i)\}, 1 \leq i \leq |i|\) for some permutation.

PROOF. See Zhang et al. (2015). □

A greedy algorithm proposed by Edmonds (1970) is used to efficiently find these valid inequalities.

An alternative reformulation can also be done based on a sample average approximation (SAA) approach [Luedtke & Ahmed, 2008; Pagnoncelli et al., 2009] with the use of efficient solution algorithms for constrained programs [Luedtke, 2014; Ahmed et al., 2017; Xie & Ahmed, 2017]. In the online supplement, we are suggesting an SAA approach to solve our model with more general settings related to the random variables.
5. Case study: CLSC for multimedia products at Sichuan Changhong Electric

As initially described in the introduction, we study the design of a Chinese electronic’s company’s CLSC network for its newly launched multimedia products (e.g., liquid crystal TVs, multimedia phones, etc). Note that, as convened with the firm, this study is based on all volume of multimedia products without differentiating between individual products. In this section, we describe the current firm’s supply chain structure and the main results after using our decision support tool. Details related to data and validation of our approach are included in the online supplement.

5.1. The company’s CLSC structure

The forward flow supply chain is three-layered with three manufacturing locations (MFs), a second layer of six central distribution centers (CDCs), and a third layer composed by 51 regional distribution centers (RDCs). When a RDC is located in one of the six regions where a CDC is located, the facility acts as both CDC and RDC. The products from the RDCs are dispatched to the actual final retailers, however we do not model this part of the supply chain. In effect, we are assuming that RDCs act as the retailers of our model. This assumption is valid for the following reasons: 1) the RDC-to-retailer shipment cost is paid by the retailers and 2) inventory and location costs at the RDCs are not incurred by the company because these facilities and its operations are outsourced to a third-party logistics company.

The remanufacturing strategy currently adopted by the company collects a very small volume of returned multimedia products, refurbishes and puts them into secondary markets. Alternatively, disassembles them and reuses the available electronic components (some IC modules) or recycles valued material in Printed Circuit Boards (PCBs). The products are collected by retailers in the regions and shipped back to collection points (small reverse DCs), where the collector inspects the returned products for integrality. The current reverse strategy tests at the reverse DCs, but the firm would like to explore the idea of testing at the reverse MFs due to cheaper labor costs. The direct shipment option is discarded by the managers because direct shipping cost from retailers to reverse MFs could be significantly higher than the cost of indirect shipments, specially for large volume products such as LCTVs.

The company currently deals with a small volume of returned products, but this is not a representative scenario of how they should plan for its reverse flows strategy on the long run. Thus,
Table 4: Scenarios of rates of returned product considered in the robust closed-loop supply chain network

<table>
<thead>
<tr>
<th>Scenario</th>
<th>1</th>
<th>2</th>
<th>3</th>
<th>4</th>
</tr>
</thead>
<tbody>
<tr>
<td>Rate of returned product ($\phi_s$)</td>
<td>0.1</td>
<td>0.3</td>
<td>0.5</td>
<td>0.7</td>
</tr>
<tr>
<td>Probability</td>
<td>0.3</td>
<td>0.4</td>
<td>0.2</td>
<td>0.1</td>
</tr>
</tbody>
</table>

the quantity of returned products is defined to be the product of the new multimedia products demand with a rate. Moreover, the remanufacturing capacity of the company is set to be 0.3 times the production capacity associated with the new multimedia products. At this early stage, it is challenging for Changhong to provide an accurate estimate of the rate of returned products. In its absence, a discrete probability table to forecast future rates was developed with the managers (Table 4). Then, the quantity of returned products is uncertain and dependent on each scenario of rate of returned product and on the characterization of demand of new products. Under scenario $s$, the mean ($\mu_{is}^R$) and variance ($\sigma_{is}^2$) of returned products at retailer $i$ is $\phi_s \mu_{is}^F$ and ($\phi_s \sigma_{is}^F$)$^2$, respectively, where $\phi_s$ is the return rate under scenario $s$.

5.2. Main results

We first find the optimal CLSC solutions using our modeling approach with return rates of 0.1, 0.3, 0.5 and 0.7 with the same remanufacturability fraction ($\tau = 0.5$) assumed in all four cases. All results are substantially different in terms of total costs, assignments, and opened locations (see Figure S1, online supplement). Next, we run the scenario-based model which (robust) solution is represented in Figure 4. The solution has the same location of facilities as the optimal network structure under 0.3 rate of returned product. However, several assignments between retailers and reverse DCs (RDC) are different. The total cost of the robust solution is CNY440862124, which is 1.1% more expensive than the optimal solution for a 0.1 rate of returned product and 0.3% cheaper for a 0.7 rate.

Next, for different remanufacturability fractions, we compare the integrated robust solution (integrated solution because it refers to the problem with forward and reverse flows together) with the robust solution obtained by fixing the current forward supply chain (sequential solution) (Table S1 online supplement). Our approach predicts total cost savings in the range of 40% if the integrated solution is implemented. The large majority of savings come from the management of a less costly forward supply chain. In fact, it turns out that the current forward supply chain has too many forward MFs and DCs making it notably expensive when operating a full functioning reverse supply
Figure 4: Robust closed-loop supply chain network obtained by the integrated approach. MF = manufacturing location, RMF = remanufacturing location, FDC = forward DC, JDC = joint DC, and RDC = reverse DC. Total cost = CNY44,086,2124, the numbers of MF, RMF, FDC, RDC, and JDC are 2, 2, 3, 7, 3, respectively.

In turn, this large number of opened forward MFs favors the cost of the reverse supply chain that, for some instances, is slightly cheaper than the reverse supply chain recommended by the integrated robust solution.

Another relevant analysis for Changhong is the exploration of the major differences between testing at the reverse DCs (current practice) and at the reverse MFs. The firm is currently outsourcing testing to the collector company, but if testing was done at the MFs it would most likely be done in-house and it could potentially be cheaper than it currently is. As noted in Section 3.6, the key aspects in this analysis are the testing costs difference and the remanufacturability level. In our experiments for the company, a fixed incremental testing cost is set and, from this, a threshold remanufacturability fraction can be identified. In our experiments (Table S2 of the online supplement) this value is set between 0.8 and 0.9 and we can observe that for larger $\tau$ values than the threshold it is cheaper to test at the MFs and for values below the threshold it is more economical to test at the reverse DCs. Also, we note that the optimal CLSC when testing at the reverse MFs has less reverse DCs than with testing at the reverse DCs.

6. Managerial insights

The goal of this section is to use our CLSC model to infer useful managerial insights that address our research questions. To observe the system’s behaviors more clearly, we use a single scenario model that is derived from the robust model (Section 3 of online supplement). A second set of
data sets based on the U.S. geography provided in Daskin (1995) is used to generate experimental data and discuss the insights of this section. However, the same experiments associated with the Changhong data set and with multiple scenarios (robust model) with the Daskin (1995) data set are reported in Sections 6 and 7 of the online supplement, respectively. All three sets of experiments demonstrate analogous managerial insights, which implies the generalization of the observations to supply chain cases based on the U.S. and Chinese geographies and to multiple rates of returned product. Nevertheless, we note that our numerical observations are based on some assumptions such as the single-sourcing policy, the (Q, r) inventory policy and certain specific parameter values. These are all primarily drawn from the multimedia industry and may not be representative of other industries.

The features studied include the impact on costs, location, and inventory decisions of: testing and disposition decisions at the DC level (Section 6.2), direct returned products shipments (Section 6.3), the marginal value of time of returned product (Section 6.4), and the rate of returned products (Section 6.5). The most novel and intriguing insights are highlighted as Observations and the results that coincide with past literature or observed practice are acknowledged to establish validity.

6.1. Experimental data

We employ a well-known array of data sets that have been extensively used in the joint location-inventory and CLSC literatures (Daskin et al., 2002; Shen et al., 2003; Diabat et al., 2015). The assumptions related to demand and cost characterizations provided are of standard practice. The data associated with the MFs, DCs and the retailers is generated from the data sets given in Daskin (1995) about the 1990 U.S. Census. We use three data sets, the 49-city, 60-city, and 88-city demand nodes, where the demands of new and returned products are assumed to be proportional to the population of the U.S. states, capitals and largest cities as explained in Table B.10 in Appendix B, respectively. We note that these assignments designate a proportionally larger new products demand than returned product flows as it is also true in practice (Easwaran & Üster, 2009, 2010; Üster & Hwang, 2017). The data associated with the location of the candidate DCs and the retailers is provided in the form of longitude and latitude, where the distances between each node are calculated using the great circle distance. For these experiments each candidate DC is assumed to be located at a demand node site (retailer). Moreover, there are 5 candidate MF
locations positioned at the same longitude and latitude as the largest 5 demand nodes (retailers). The specific locations of the candidate MFs are not significant. In fact, the same experiments with the candidate locations at the smallest retailer sites trigger the same major managerial insights.

All parameter values and extra notation related to the experiments are listed in Table B.10. The experiments are run on an HP 380 G7 server running CentOS 7.0 operating system and the algorithm is coded in C++. All computation times exclude input times. We used the MIQCP solver of CPLEX 12.5, which solves CQMIP relaxations at the nodes of the branch-and-bound tree, with CPLEX heuristics turned off. The online supplement contains a study of the computational performance of the modeling approach (Tables S3 and S4), where we use the 49-city and 88-city data sets. We can state that our approach is efficient and the usage of valid cuts improves performance significantly with randomized parameter values. To run the experiments in this section and infer managerial insights we use the 60-city data set. The default values of the parameters used in this section are set to: $|K| = 5$, $\gamma = 0.10\%$, $\xi = 0.4$, $\text{cap}_{60} = 800$, $\theta = 0.1$, $\beta = 0.005$, $W = 1$, $\kappa = 100$.

6.2. The impact of testing and disposition decisions at the DC level

Some firms, including Changhong, prefer to diversify earlier (at the reverse and joint DC sites) than later (at the remanufacturing sites). This strategy directly implies a reduction in shipping costs between reverse DCs and reverse MFs because some of the returned products can be disposed of. However, testing and disposition at DCs can be more expensive due to higher set up costs at the DCs and reduced economies of scale related to labor and other types of costs. According to the setting and notation in Section 3.6 testing costs are considered at DCs and are initially set to be one (i.e., $a_j^{\text{test},R} = 1$), while we assume that testing at the MFs costs zero and per-unit shipping costs from DCs to reverse MFs are valued the same regardless of whether testing is done at DC or reverse MFs (i.e. $a_j^{\text{ship},R} = a_j^R$). These assumptions can be interpreted as setting $a_j^{\text{test},R}$ as the incremental cost of testing at the DCs compared to testing at the reverse MFs.

The results with different remanufacturability fractions ($\tau_i$) are reported in Table 5. The total costs ($\text{Cost}_T$), reverse flow costs ($\text{Cost}_R$), value loss ($\text{Cost}_L$), and number of reverse DCs tend to increase when increasing $\tau_i$ because the amount of ”remanufacturable” returned products is larger. Also, when $\tau_i$ increases the number of reverse and joint DCs show a decreasing trend to offset with the increase in the number of remanufacturing sites. This effect is consistent with the observations
<table>
<thead>
<tr>
<th>( \tau_i )</th>
<th>Cost components</th>
<th>MFs</th>
<th>DCs</th>
</tr>
</thead>
<tbody>
<tr>
<td></td>
<td>( \text{Cost}_T )</td>
<td>( \text{Cost}_F )</td>
<td>( \text{Cost}_R )</td>
</tr>
<tr>
<td>0.10</td>
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<td>87831</td>
<td>42969</td>
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<td>0.20</td>
<td>137338</td>
<td>87850</td>
<td>46370</td>
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<td>0.30</td>
<td>141525</td>
<td>87867</td>
<td>50971</td>
</tr>
<tr>
<td>0.40</td>
<td>144261</td>
<td>87886</td>
<td>54847</td>
</tr>
<tr>
<td>0.50</td>
<td>146975</td>
<td>87926</td>
<td>58687</td>
</tr>
<tr>
<td>0.60</td>
<td>149591</td>
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<tr>
<td>0.70</td>
<td>152137</td>
<td>87919</td>
<td>67867</td>
</tr>
<tr>
<td>0.80</td>
<td>158253</td>
<td>87936</td>
<td>64697</td>
</tr>
<tr>
<td>0.90</td>
<td>160150</td>
<td>87921</td>
<td>65581</td>
</tr>
<tr>
<td>1.00</td>
<td>161859</td>
<td>87930</td>
<td>66615</td>
</tr>
<tr>
<td>-</td>
<td>145421</td>
<td>87930</td>
<td>50194</td>
</tr>
</tbody>
</table>

Table 5: Costs and location results under different \( \tau \) when testing and disposition decisions at the DC level \( (a_j^{\text{test, } R} = 1) \). The solution with testing at the MFs is represented by \( a_j^{\text{test, } R} = 0, \tau_i = 1 \). Note: check Table B.10 for notation.

obtained in Üster & Hwang (2017). Further, in these experiments we observe that the impact of varying \( \tau_i \) on the forward flow supply chain is insignificant.

If we compare these experiments that represent different scenarios of testing at the DCs with the solution of testing at the MFs (last row), we highlight that:

**Observation 1.** Testing downstream (at the reverse DCs) leads to solutions with larger number of opened reverse DCs. This behavior is more significant as remanufacturability of the returned products decreases.

The observation is explained by noting that if testing is done closer to the retailers (reverse DCs) there are shipping cost savings of discarded products. This is connected to the fact that testing at the reverse MFs is a better option for products with high remanufacturability levels. The latter can also be observed in Figure 5 that shows the threshold line where the colored area represents the instances at which testing at the DCs is cheaper and the white area represents the \( (a_j^{\text{test, } R}, \tau) \) values at which testing at the MF level is cheaper. The threshold line is evidently decreasing and, for the case of our example, an additional cost of testing at the DCs of more than \$2 per unit should not be accepted.

### 6.3. The impact of direct shipments at the reverse chain

This option is considered by firms that have a high time-sensitive product and want to minimize the time spent at the reverse supply chain by skipping the reverse DCs. This simplifies the model because variables related to reverse or joint DCs and the respective constraints are no longer needed.
Figure 5: The colored area represents specific combinations of \((a_j^{test,R}, \tau)\) where testing and disposition decisions at the DC level are cheaper. Note: the specific \(\tau\) values in the threshold line have been identified via linear interpolation between a lower and upper bound with 0.05 precision (or higher precision).

To observe all major effects of direct shipment reverse flows in the CLSC we run different combinations of per-unit shipment costs. These are represented in the first column of Table 6, that is the ratio of per-unit shipment cost between DCs and MFs by the shipment cost between DCs and retailers. From practice, it is assumed that shipments from MF to DC are proportionally cheaper. So, according to our definition, the larger the ratio is the more expensive the per-unit shipment cost between MFs and DCs becomes. Note that for the rest of experiments, we are assuming this ratio to be 0.4. For direct shipments, the per-unit shipment cost between reverse MFs and retailers is set to be the cost between DCs and retailers in indirect shipments. For each ratio, the results associated with indirect and direct shipment of returned products are listed in the first and second rows, respectively.

The effects observed in Table 6 related to the direct shipments reverse flows option are the expected. Compared with the indirect shipments option, lower value loss and an increased number of MFs to compensate for the lack of reverse DCs are observed. When shipment costs between MFs and DCs increase (i.e. increased ratios) the total costs and forward flow costs increase significantly for both types of shipment (indirect and direct) of returned products. For indirect shipments, the loss value has a decreasing trend. Also for both shipment options, the number of open MFs increases and the number of DCs decreases.

Moreover, we observe (particularly for ratios 0.3, 0.4, and 0.5 in Table 6) that direct shipments of returned products can affect the optimal design of the forward supply chain as follows:
Observation 2. **Direct shipments for returned products prompt optimal forward supply chains with more forward MFs and less forward DCs than the indirect shipments option.**

Finally, by comparing total costs of both options we infer the following dynamic:

Observation 3. **The direct shipment option is specially interesting for instances where transportation costs between MFs and DCs tend to be as expensive as between DCs and retailers.**

6.4. **The impact of marginal value of time of returned products**

In our approach, returned products’ marginal value of time ($\gamma$), which refers to the degree of time sensitivity of the product’s price, impacts the optimal design of the CLSC. Table 7 shows the trade-off between costs and time where:

Observation 4. **For highly time-sensitive returned products (large $\gamma$) total and reverse flow costs become larger and more MFs are opened than for less time-sensitive returned products.**

This result is in line with Blackburn et al. (2004) that states that low marginal value of time products should have centralized supply chain structures and high marginal value of time products decentralized ones. By definition, the value loss of returned products increases for larger $\gamma$, while the impact on the forward flow costs and cost savings of co-location is not significant.

Table 6: Costs and location results under different ratios when indirect/direct shipments of returned products

<table>
<thead>
<tr>
<th>Shipment</th>
<th>Ratio</th>
<th>Cost components</th>
<th>MFs</th>
<th>DCs</th>
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<tbody>
<tr>
<td></td>
<td></td>
<td>Cost$_r$</td>
<td>Cost$_{fr}$</td>
<td>Cost$_{dr}$</td>
</tr>
<tr>
<td>indirect</td>
<td>0.2</td>
<td>119614</td>
<td>68277</td>
<td>44350</td>
</tr>
<tr>
<td>direct</td>
<td>0.2</td>
<td>135449</td>
<td>88141</td>
<td>59191</td>
</tr>
<tr>
<td>indirect</td>
<td>0.3</td>
<td>132836</td>
<td>75277</td>
<td>47698</td>
</tr>
<tr>
<td>direct</td>
<td>0.3</td>
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<td>direct</td>
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</tr>
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<td>96535</td>
<td>62007</td>
</tr>
<tr>
<td>indirect</td>
<td>0.9</td>
<td>166438</td>
<td>97238</td>
<td>61377</td>
</tr>
<tr>
<td>direct</td>
<td>0.9</td>
<td>162527</td>
<td>96730</td>
<td>62007</td>
</tr>
<tr>
<td>indirect</td>
<td>1.0</td>
<td>167137</td>
<td>97284</td>
<td>62085</td>
</tr>
<tr>
<td>direct</td>
<td>1.0</td>
<td>162528</td>
<td>96732</td>
<td>62007</td>
</tr>
</tbody>
</table>
If we study the effect to shipment quantities at joint DCs, we can observe that in our approach the theoretical EOQ value of forward products at any DC is not affected by $\gamma$. On the contrary, the theoretical EOQ value of returned products decreases when increasing $\gamma$ so as to reduce the loss value of returned products. For capacitated DCs, the impact of $\gamma$ is exacerbated and for larger $\gamma$ the shipment quantities of new products ($Q^F_j$) are generally larger due to reduced $Q^R_j$ quantities. This dynamic is shown in the last two columns of Table 7. Figure 6 illustrates these quantities for the specific joint DC 6. Thus, this is another evidence that changes in the reverse supply chain can affect forward supply chain flows:

**Observation 5.** In capacitated joint DCs, higher time-sensitive returned products tend to have larger optimal forward shipment sizes than less time-sensitive returned products.

### 6.5. The impact of the rate of returned products

The quantity of returned products is generated by multiplying the quantity of new products with a return rate that represents the return percentage of new products. As expected, the total cost and number of opened facilities become larger when the rate of returned product increases. Note that the forward supply chain is also affected by this increase of the rate of returned products (as also seen in Fleischmann et al. 2001). In Table 8 one extra MF is required to be opened and
<table>
<thead>
<tr>
<th>Rate</th>
<th>MFs</th>
<th>DCs</th>
<th>Cost components</th>
<th>F</th>
<th>R</th>
<th>#F</th>
<th>#R</th>
<th>#C</th>
</tr>
</thead>
<tbody>
<tr>
<td>0.2</td>
<td>128893</td>
<td>87887</td>
<td>CostT</td>
<td>37423</td>
<td>805</td>
<td>4357</td>
<td>2, 3, 4</td>
<td>18</td>
</tr>
<tr>
<td>0.4</td>
<td>147125</td>
<td>87937</td>
<td>CostT</td>
<td>51577</td>
<td>1198</td>
<td>8809</td>
<td>2, 3, 4</td>
<td>18</td>
</tr>
<tr>
<td>0.6</td>
<td>163905</td>
<td>88109</td>
<td>CostT</td>
<td>64166</td>
<td>1759</td>
<td>13389</td>
<td>2, 3, 4</td>
<td>19</td>
</tr>
<tr>
<td>0.8</td>
<td>176015</td>
<td>88407</td>
<td>CostT</td>
<td>81535</td>
<td>2169</td>
<td>8241</td>
<td>1, 2, 3, 4</td>
<td>18</td>
</tr>
<tr>
<td>1.0</td>
<td>184488</td>
<td>88584</td>
<td>CostT</td>
<td>88394</td>
<td>2794</td>
<td>10304</td>
<td>1, 2, 3, 4</td>
<td>21</td>
</tr>
</tbody>
</table>

Table 8: Costs and location results under different rates of returned products. Note: \( \mu^R_i = \text{return rate} \times \mu^F_i \), \( (\sigma^R_i)^2 = (\text{return rate})^2 \times (\sigma^F_i)^2 \) and \( C^R_i = 1.0 \times \chi \sum_{i \in I} \mu^F_i \times 0.4 \)

some additional forward and joint DCs are opened. This is because the increasing flow of returned product reduces the capacity of the joint DCs.

As generally observed in practice and as manifested by managers at Changhong, rates of returned products can vary across the lifetime of a product and are a challenging parameter to estimate a priori. Thus, using our 60-city data set, we study how robust are the optimal solutions to changes in this rate. For each specific rate of returned product in Table 9, the first row lists the optimal solution (with this specific return rate, see model 4 of online supplement) and the second row reports a greedy solution. This greedy solution fixes the opened MFs/DCs obtained from the optimal solution of the second row of its immediately smaller rate experiment. Extra facilities are opened (column “O”) if deemed cost beneficial and other facilities might be closed (column “C”) if they are not being used despite incurring fixed location costs. We compare both the optimal (first row) and the greedy solutions (second row) per each rate and observe that the second one has a total cost in a range between 0.01% and 0.40% higher than the optimal solution. We have also used this greedy approach for Changhong’s data and the heuristic solutions only range 0.03-0.05% higher than the optimal solution. Thus, we can state that:

<table>
<thead>
<tr>
<th>Rate</th>
<th>Total cost</th>
<th>Manuf.</th>
<th>Remanuf.</th>
<th>Forward DC</th>
<th>Reverse DC</th>
</tr>
</thead>
<tbody>
<tr>
<td></td>
<td>#</td>
<td>O</td>
<td>C</td>
<td>#</td>
<td>O</td>
</tr>
<tr>
<td>0.2</td>
<td>3</td>
<td>2, 3, 4</td>
<td>-</td>
<td>3</td>
<td>2, 3, 4</td>
</tr>
</tbody>
</table>

Table 9: Costs and location results under different return rates. '#': the number of the facilities. 'O': the opened facilities. 'C': the closed facilities. '-': that there are no MFs/DCs opened or closed. *: 0, 2, 3, 4, 6, 8, 9, 11, 12, 14, 17, 21, 27, 28, 29, 31, 45, 55. **: 2, 3, 4, 8, 29, 45
Observation 6. *Greedy algorithms that myopically add MFs and DCs to the optimal solution with lower rate of returned products can be effective in finding solutions relatively close to the optimal solution for larger rates of returned product.*

Even though this heuristic is simple and provides solutions relatively close to the optimal, Changhong’s managers prefer the scenario-based solution, which is an a priori solution that is going to be implemented once. The heuristic suggested assumes that facilities will be opened and closed if there are changes of the rate of returned product to avoid situations such as having an overcapacitated network if the rate decreases significantly.

7. Conclusions

During the past decade several review articles in CLSC highlight the lack of research that thoroughly studies managerial insights in CLSC design. To address this demand and to serve the interest of our partnering firm (Sichuan Changhong Electric Co. Ltd.) that wishes to study its CLSC network for newly launched multimedia products, we build a CLSC model and run extensive computational experiments. In particular, we look at the effects on the CLSC of the different roles assigned to reverse DCs, different marginal value of time of returned products, and changes in the rate of returned products. Our experiments are run using our partnering firm’s data based on the Chinese geography and a second data set based on the U.S. census and geography.

We propose a novel model that accommodates several relevant features such as joint location and inventory decisions, uncertain demand of forward and returned flows, finite capacities at the MFs and DCs, EOQ-approximation inventory policies with risk pooling, loss in value over time of returned products and savings in co-location. The proposed model is a scenario-based nonlinear program with chance constraints that is reformulated as a conic quadratic mixed-integer program and is solved efficiently by a commercial optimization package. The completeness of the suggested model allows practitioners to use a decision support tool that is very close to the real operations of the supply chain. Nonetheless, this is at the expense of being able to directly derive analytical results which would required a drastically different and more stylized model. For this reason, the derivation of our managerial insights is based on our extensive computational experiments.

Some of the most relevant insights directed to supply chain managers in the multimedia industry
are related to testing decisions, where we observe that testing upstream (at the reverse MFs) is more favorable for products with high remanufacturability fractions, while testing downstream (at the reverse DCs) is preferable for low remanufacturable products. We also observe that testing downstream leads to solutions with a larger number of opened reverse DCs. Furthermore, the closed-loop design of the system brings some interesting insights related to how changes in the reverse supply chain affect the forward supply chain. For example, direct shipments of returned products prompt optimal forward supply chains with more open MFs and less open forward DCs. Also, higher time-sensitive returned products tend to have larger open MFs and also tend to have larger optimal forward shipment quantities in its joint DCs. Finally, in our paper we show that scenario-based robust and myopic heuristics approaches are effective tools to consider when managers are uncertain about key parameter values, such as the rate of returned product, at the strategic phase of the CLSC design.

At last, we outline some future research directions. For example, using a multi-sourcing strategy in both retailer-to-DC and DC-to-MF assignments decisions and considering relocation and reassignment costs of the MFs and DCs between periods. Secondly, developing efficient solution algorithms with the sample average approximation (SAA) approach is an interesting research direction to jointly manage all random variables of our problem. Thirdly, another natural extension of this problem is to consider other mean-risk frameworks such as mean-CVaR and mean-VaR and make a comparison with the mean absolute deviation model. Fourthly, it is interesting to compare and differentiate CLSC strategies in different market context when considering different national economic policies such as government subsidy policies. Finally, the objective of Sichuan Changhong Electric is to implement a fully integrated and operational CLSC for multimedia products using our decision support tool. One aspect to explore is the operational problem concerning their pricing policy.

Acknowledgement

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Appendix A. Equivalent CQMIP formulation

\[
\begin{align*}
\min_{X,Y} Z &= \sum_{k \in K} \mu_k^F Z_k^F + \sum_{j \in J} f_j^F X_j^F + \sum_{k \in K} \mu_k^R Z_k^R + \sum_{j \in J} f_j^R X_j^R - \sum_{j \in J} \sigma_j^C X_j^C \\
+ \sum_{s \in S} \omega_s \Gamma_s + \lambda \sum_{s \in S} \omega_s \left[ \Gamma_s - \sum_{s' \in S} \omega_{s'} \Gamma_{s'} + 2A_s \right],
\end{align*}
\]

\[s.t. \quad (3) \sim (7).\]

\[
\begin{align*}
\sum_{j \in J} \sum_{k \in K} V_{ijk}^F &= 1, \forall i \in I, \\
V_{ijk}^F \leq V_{ijk}^R, V_{ijk}^F \leq V_{ijk}^R, \forall i, j \in J, k \in K, \\
V_{ijk}^R &\leq Y_{ijk}^R, V_{ijk}^R \leq Y_{ijk}^R, \forall i, j \in J, k \in K, \\
\Gamma_s &\geq 2 \left( \sum_{i \in I} \beta \chi d_{ij} \mu_k^F \chi_{ij}^F + \theta h z_n \Omega_j + \beta \sum_{k \in K} \sum_{i \in I} \theta_{jk} \chi_{i} \chi_{jk} \chi_{ijk} F_{ij}^j + \frac{\theta h}{2} \Delta_{js}^F \right) \\
+ \sum_{j \in J} \left( \sum_{i \in I} \beta \chi d_{ij} \mu_k^R \chi_{ij}^R + \beta \sum_{k \in K} \sum_{i \in I} \theta_{jk} \chi_{i} \chi_{jk} \chi_{ijk} F_{ij}^j + \frac{\theta h}{2} \Delta_{js}^R \right) \\
+ W \Gamma_U \left( \frac{1}{K} \sum_{k \in K} \sum_{i \in I} \mu_k^F \chi_{ij}^F + \frac{1}{K} \sum_{k \in K} \sum_{i \in I} \mu_k^R \chi_{ij}^R + \theta h + W \gamma U \chi \Omega_s^R \right), \forall i \in I, j \in J, k \in K, s \in S, \\
\Omega_s^2 \geq \sum_{k \in K} \sum_{i \in I} \left( \sigma_i^2 \right)^2 \left( \Gamma_s^F \right)^2, \forall j \in J, \\
Q_{js}^F + (z_n - z_j) \Omega_j + Q_{js}^R \leq C_j, \forall i \in I, j \in J, k \in K, \\
\sum_{j \in J} \sum_{i \in I} \chi_{i} \chi_{ijk} F_{ij}^j + z_{i} \chi_{ijk} F_{ij}^j \leq C_j^F, \forall k \in K, \\
\left( \Omega_s^F \right)^2 - \sum_{j \in J} \sum_{i \in I} \chi_{i} \left( \sigma_i^F \right)^2 \left( \Gamma_s^F \right)^2 \geq 0, \forall k \in K, \\
\sum_{j \in J} \sum_{i \in I} \chi_{i} \chi_{ijk} F_{ij}^j + z_{i} \chi_{ijk} F_{ij}^j \leq C_j^R, \forall k \in K, s \in S, \\
\left( \Omega_s^R \right)^2 - \sum_{j \in J} \sum_{i \in I} \chi_{i} \left( \sigma_i^R \right)^2 \left( \Gamma_s^R \right)^2 \geq 0, \forall k \in K, s \in S, \\
\sum_{j \in J} \sum_{i \in I} \chi_{i} \chi_{ijk} F_{ij}^j \geq \sum_{j \in J} \sum_{i \in I} \chi_{i} \chi_{ijk} F_{ij}^j, \forall k \in K, s \in S, \\
\Delta_{js}, \Delta_{js}^F, \Omega_j, \Omega_k^F, \Omega_k^R, Q_{js}^F, Q_{js}^R, \Gamma_s, A_s \geq 0, \forall j \in J, k \in K, s \in S,
\end{align*}
\]
\( X^F_i, X^R_i, X^C_i, Y^F_j, Y^R_j, Z^F_k, Z^R_k, V^F_{j,k}, V^R_{j,k}, \tilde{V}^F_{i,j,k}, \tilde{V}^R_{i,j,k} \in \{0,1\}, \forall i \in I, \forall j \in J, \forall k \in K. \) \hspace{1cm} (A.17)

### Appendix B. Parameter values

<table>
<thead>
<tr>
<th>Parameter</th>
<th>Value</th>
</tr>
</thead>
<tbody>
<tr>
<td>( C_i^{F(R)} )</td>
<td>( 1.0 \times \chi \sum_{i \in I} \mu_i^{F(R)} \times \xi, \xi \in [0.3, 0.4, 0.5, 0.6, 0.7] )</td>
</tr>
<tr>
<td>( \xi )</td>
<td>Weight factor associated with MF processing capacities</td>
</tr>
<tr>
<td>( d_{ij} )</td>
<td>Per-unit shipment costs associated with great circle distance</td>
</tr>
<tr>
<td>( \tilde{a}<em>{ij}, \tilde{a}</em>{ik}^F )</td>
<td>0.4 ( \times d_{ik} ), testing at MFs is valued as 0</td>
</tr>
<tr>
<td>( f^F_j, f^R_j )</td>
<td>Fixed location cost (Daskin, 1995) divided by 100</td>
</tr>
<tr>
<td>( \mu^F_j, (\sigma^F_j)^2 )</td>
<td>Demand 1 (Daskin, 1995) divided by 1000, ( \mu )</td>
</tr>
<tr>
<td>( \mu^C_j, (\sigma^C_j)^2 )</td>
<td>Demand 2 (Daskin, 1995) divided by 1000, ( \mu )</td>
</tr>
<tr>
<td>( \hat{s}_j )</td>
<td>( 0.2 \min {f^F_j, f^R_j} ) (Sahvouni et al., 2007)</td>
</tr>
<tr>
<td>'cap49', 'cap60'</td>
<td>Capacity of DCs in 49-city and 60-city data sets</td>
</tr>
<tr>
<td>'F', 'R', 'C'</td>
<td>Represent specific forward, reverse, and joint MFs or DCs</td>
</tr>
<tr>
<td>'#F', '#R', '#C'</td>
<td>Represent total number of each of these facilities that is open</td>
</tr>
<tr>
<td>'Cost_F', 'Cost_R'</td>
<td>Forward and reverse costs (include MF and DC fixed location, transportation and inventory costs)</td>
</tr>
</tbody>
</table>

<table>
<thead>
<tr>
<th>Parameter</th>
<th>Value</th>
</tr>
</thead>
<tbody>
<tr>
<td>( F^F_j, F^R_j, \tilde{g}^F_{j,k}, \tilde{g}^R_{j,k} )</td>
<td>10</td>
</tr>
<tr>
<td>( \bar{\gamma} )</td>
<td>0.10%</td>
</tr>
<tr>
<td>( z_\alpha, \alpha )</td>
<td>1.96, 97.5%</td>
</tr>
<tr>
<td>( \bar{\rho}, \rho^F, \rho^R )</td>
<td>2.5%</td>
</tr>
<tr>
<td>( U, \kappa )</td>
<td>100</td>
</tr>
<tr>
<td>( p^F_k, p^R_k )</td>
<td>10 ( \times f^F_j, 10 \times f^R_j )</td>
</tr>
<tr>
<td>( h, \chi, L_{j,k} )</td>
<td>1</td>
</tr>
<tr>
<td>'Cost_T'</td>
<td>Total cost</td>
</tr>
<tr>
<td>'Cost_g'</td>
<td>Cost savings</td>
</tr>
<tr>
<td>'Cost_L'</td>
<td>Value loss</td>
</tr>
</tbody>
</table>

Table B.10: The parameter values and extra notation related to the experiments. Note: Fixed location costs for both 49-city and 60-city data sets represent the median home value in that city, Demand 1 from the 49-city data set represents the population of each U.S. state, Demand 2 from the 49-city data set represents the population of the first 60 cities of a larger list of all state capitals and the 50 largest U.S. cities, and Demand 2 from the 60-city data set represents the number of households for all 60 nodes.


