This is a postprint version of the following published document:


DOI: [10.1628/093245616x14798149327649](http://dx.doi.org/10.1628/093245616x14798149327649)

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The Governance of Perpetual Financial Intermediaries

by

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We reexamine the risk-sharing potential of intergenerational financial intermediaries, taking into account their governance structure. We argue that asset buffers of perpetual institutions are limited by the temptation of the living stakeholders to renegotiate contributions and distributions. We characterize the renegotiation constraint and show that it severely limits intergenerational risk sharing. Without renegotiation frictions, intermediaries cannot provide higher welfare than a market. The existence of (self-imposed) renegotiation costs relaxes the constraint. By forming a single monopolist intermediary, agents can further improve welfare. (JEL: G21, D91)

I Introduction

Providing insurance against liquidity risk is one of the main tasks of financial intermediaries. Seminal papers of Edgeworth (1888), Bryant (1980), and Diamond and Dybvig (1983) show how intermediaries can share risk in economies where production comes with gestation lags and depositors face stochastic liquidity needs. Diamond and Dybvig (1983) show that if agents face the risk of having to consume before a long-term production technology pays off, a financial intermediary is able to offer a consumption schedule in which early consumers are ex post subsidized by late consumers, and make all ex ante better off. Jacklin (1987) and Bhattacharya

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and Gale (1987) show that the presence of a market constrains the risk-sharing ability of intermediaries. An extensive literature now exists on bank risk sharing.\(^1\)

The object of our study is a Diamond (1965) overlapping generations (OLG) economy, where agents of every cohort face liquidity risk.\(^2\) We investigate whether institutions that collect endowments, hold a capital buffer that offers a perpetual income stream, and offer their beneficiaries contingent claims can improve welfare vis-à-vis the market mechanism.\(^3\) Unlike many related papers in the literature, we do not a priori rule out type verifiability, and thus also include insurance companies and pension plans as the object of our analysis.

We show that perpetual intergenerational intermediaries cannot be overfunded and, in our baseline analysis, cannot do better than finite-lived intragenerational intermediaries, because, in the absence of altruism, the size of the intermediaries’ asset buffers is constrained by the governance of the intermediaries by the living generation. The constraint on asset buffers arises because a generation that controls an intermediary with a large asset buffer is tempted to distribute assets to the detriment of future generations. Aside from altruism, purposefully introduced transaction costs and renegotiation frictions present a possible way to improve a perpetual institution’s capacity for buffer holding and risk sharing. Our model thus explains why widely held institutions such as defined pension plans or sovereign countries are rarely overfunded and why closely held institutions such as university colleges, churches, and wealthy families inflict disbandment hurdles upon themselves, by investing in difficult-to-liquidate assets, such as art, land, and real estate. In addition, perpetual institutions often have procedures and codes drawn up to prevent the contemporaneous stewards of the institutions’ wealth from delving into capital.\(^4\)

Our results arise from the introduction of the governance of financial institutions into a standard Diamond (1965) OLG economy. We do this by assuming that the operating rules (on investment and dividend distribution) are controlled by the gen-

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1 Haubrich and King (1990), Hellwig (1994), and von Thadden (1997, 1998) provide additional critique on Diamond–Dybvig risk sharing. Wallace (1988), Gorton and Pennacchi (1990), and Diamond (1997) suggest conditions under which banks can offer superior risk sharing.

2 Samuelson (1958) investigates an OLG economy where agents are born with a consumption deficit and need to borrow from the old.

3 Our paper concerns all perpetual institutions that carry a buffer of productive assets, or claims thereon, across generations that are subject to liquidity shocks. Banks are but one example of such institutions. Other intermediaries that fit our analysis are not only mutual funds, pension schemes, and insurance mutuals, but also countries, municipalities, university colleges, foundations, and families. These perpetual institutions have in common that they hold asset buffers that could potentially benefit a perpetual stream of beneficiary generations, but are controlled only by the living.

4 Goetzmann and Oster (2012) document that university endowment portfolios are highly slanted towards illiquid assets such as private equity and alternative investments. Hansmann (1990) finds that overfunded universities in the U.S. hold most of their wealth in physical assets, and identifies costs of excessive endowments in the form of diversion of funds to excessive facilities or esoteric research by faculty. Hansmann’s survey hints that some U.S. colleges may have reached their maximum capital accumulation.
erations that are alive, and decisions on changing these rules are made in periodic meetings where members decide whether to maintain the status quo distribution and investment regime, or to disband and offer all living members an alternative payoff schedule. We assume that if there exists a feasible disbandment proposal that improves the welfare of all living members, then it will be accepted and executed. This governance generates a fundamental lack of commitment within the institution, which leads us to study the set of investment buffers that are renegotiation-proof. We are able to characterize the renegotiation constraint: the frontier of allocations that do not lead to disbandment. Not surprisingly, the threat of disbandment limits the perpetual payoff schedule that an intermediary can offer to its members. In particular, we find that without renegotiation frictions, intermediaries cannot be overfunded vis-à-vis their obligations to the living.

The effect of the renegotiation constraint depends on whether financial institutions compete for depositors, or whether there is a single monopolistic financial intermediaries. We show that both under competition and under monopoly, the market allocation just satisfies the renegotiation constraint. Furthermore, the market allocation corresponds to the constrained optimal allocation for bank intermediaries, which are defined as institutions that cannot verify agents’ types and thus can only issue demandable debt securities. Nonbank intermediaries, such as insurance mutuals, defined-benefit pension plans, and government social security schemes, can redistribute consumption between early and late consumers in a way that is ex ante desirable. We find however that due to the renegotiation constraint, they cannot offer better schedules than finite-lived intragenerational intermediaries, and are only able to increase welfare vis-à-vis an incomplete market without securities written on individual liquidity needs.

We also study the monopoly case, which considers institutions that dominate the economy, such as country governments. In our model, these institutions can reduce the threat of renegotiation and may therefore be able improve welfare vis-à-vis a market economy when types are verifiable. The reason is that in the monopoly case, no active market will exist where the potentially disbanding stakeholders can sell the institution’s assets. There will however still be a shadow market where the ostracized younger generations are the potential buyers of these assets. We find that a monopolist configuration changes the renegotiation constraint with respect to the

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5 There is a large literature on the effects of lack of commitment, and the ensuing renegotiations, in contracts, beginning with Dewatripont (1988, 1989). A good overview can be found in Laffont and Martimort (2002).

6 The type of ex post redistribution depends on the depositors’ utility functions. We believe a desirable transfer from late to early consumers, as in Diamond and Dybvig (1983), to be most realistic.

7 This is consistent with reality, as it is well documented that very few pension funds are, or have ever been, overfunded (see, among others, Armstrong and Selody, 2005, or Andonov, Bauer, and Cremers, 2016). In practice an overfunding situation gives rise to mutually agreed-upon benefit increases, contribution holidays, or lump-sum distributions. Similarly, most countries have public debt, rather than large sovereign wealth funds, while clearly the latter would be beneficial for all future generations of citizens.
competitive case but does not represent a significant welfare change with respect to the case without type verifiability.

The conflict between current and future unborn generations in perpetual intermediaries has, to our knowledge, not been studied in the micro-economy literature. Intragenerational conflicts in the OLG literature typically are those between different living stakeholders. An example is the paper of Prescott and Rios-Rull (2000), who analyze a model where the young are tempted to ostracize old generations and restart the risk-sharing arrangement. In our study, parent coalitions are in control, and the danger looms that future generations will be abandoned. By specifically giving control to the living generations, our model offers insights into the asset allocation and governance mechanisms of perpetual intermediaries and institutions.

Another contribution of our paper is that it addresses an unrealistic tacit assumption in the banking literature: Most OLG banking papers implicitly assume that contracts between generations can be enforced by an invisible third party. An example is Allen and Gale (1997), who show how a banking system can achieve intergenerational income smoothing by holding an asset buffer, which is depleted whenever the risky dividends fall short of expectation, and replenished before and after. While they point out that a buffer-holding financial system is fragile because agents will abandon it when (through bad luck) the buffer is depleted, they do not investigate how the controlling generations would deal with an income windfall.

Also, the pension literature typically assumes perpetually fixed operating rules. Krueger and Kubler (2006), Gollier (2008), Bohn (2009), and others argue that defined-benefit pension schemes are superior to defined-contribution schemes in that they enable intergenerational transfers. The problem with these models is that they rule out renegotiation by the living generations, e.g., in response to increasing funding ratios.

Earlier papers that consider risk sharing in similar OLG models include Qi (1994), Bhattacharya and Padilla (1996), and Fulghieri and Rovelli (1998). Qi presents a model where a perpetual Diamond–Dybvig bank carries an asset buffer over time so as to take maximum advantage of available investment opportunities. Qi and the subsequent literature assume that the market mechanism does not achieve the golden rule investment level, where the marginal return equals the marginal utility (Phelps, 1961). Also, the present paper assumes that welfare can be improved by increasing the stationary investment level and increasing the size of the asset buffer that is passed from generation to generation.

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8 They use a result of Schechtman (1976) to show that such a system can offer its stakeholders the expected value of the stochastic output in all except a negligible number of periods.

9 See also Gottardi and Kubler (2006), Cui, De Jong, and Ponds (2011), and Beetsma and Bovenberg (2009) for analyses of intergenerational social security and defined-benefit pension schemes.

10 Other papers using this model include those of Bhattacharya, Fulghieri, and Rovelli (1998), who investigate the starting up of a bank economy; Qian, John, and John (2004), who analyze how transaction costs affect the equilibria; and Bencivenga and Smith (1991), who are concerned with economic growth.
Bhattacharya and Padilla (1996) point out that Qi’s allocation is not robust. In Qi’s allocation banks offer two-period returns that are higher than the return on the productive asset. They argue that this situation is not sustainable, because banks will open deposits with each other instead of in the real sector. Bhattacharya and Padilla then show that a government can improve on the market economy by using tax and subsidy schemes. Fulghieri and Rovelli (1998) show that intermediaries that can identify depositors’ ages or types can attain the same level of welfare. We find that the allocations suggested by Qi (1994), Bhattacharya and Padilla (1996), and Fulghieri and Rovelli (1998) are not renegotiation-proof: in all suggested equilibria, living depositors can make themselves uniformly better off by liquidating the intermediaries’ asset buffers.

Also related to this paper is the model of Gordon and Varian (1988), which considers an OLG economy where wages are stochastic and liquidity needs are fixed. The authors show that a government can improve welfare vis-à-vis a market economy by overcoming adverse selection and imposing wealth transfers from the lucky to the unlucky agents.

In the next section we describe the economy and the operating mode of perpetual intermediaries. In section 3 we describe how these institutions are governed, and model potential disbandment. Section 4 presents our main result, the renegotiation constraint, and discusses the relevance of the constraint to the banking literature. In section 5 we consider the renegotiation constraint of a monopolist intermediary, such as a government. We find that the renegotiation constraint is slightly less severe because monopolist intermediaries will suffer a price impact upon disbandment and selling secondary assets. Section 6 summarizes and concludes.

2 The Economy and Operating Rule of Perpetual Intermediaries

The object of our study is an infinite-horizon OLG model. On every date $t \in \mathbb{Z}$, a generation of infinitely many agents is born, each with an equal endowment of a homogeneous good that can be used for consumption or as input into riskless production. The economy features a market where risk-free assets accrue value at a per-period rate $r$. There is no uncertainty regarding this interest rate, and for the time being we assume that a single bank’s clientele does not affect $r$. In section 5 we relax this assumption and will consider monopolist intermediaries with a market impact.

Agents who enter the economy at date $t$ can be of two types: with probability $\varepsilon$ agents are impatient and live for one period only; with probability $1 - \varepsilon$ they are patient and live for two periods. Impatient agents consume only on their first birthday; patient agents, only on their second birthday. Agents born on date $t$ learn their type some time before their first birthday. An agent’s allocation is defined as a pair $(C_1, C_2)$.

Financial intermediaries cater to stable nongrowing clienteles that are large enough for there to be no uncertainty about the aggregate distribution of agent
types.\(^{11}\) We normalize the size of each generation of an intermediary’s clientele to unity so that, at any date \(t\), every intermediary has \(3 - \varepsilon\) clients in different stages of their life: \(1 - \varepsilon\) two-year-old, \(1 - \varepsilon\) one-year-old (\(1 - \varepsilon\) patient, \(\varepsilon\) impatient), and \(1\) newborn. In between dates the intermediary has \(2 - \varepsilon\) depositors: \(1\) young, and \(1 - \varepsilon\) old.

Clearly, if agents only depend on the market to share risk, their allocation is \(\{C_1, C_2\} = \{(1 + r) \cdot (1 + r)^2\}\). The key question of this paper is whether better allocations are attainable if agents organize to form buffer-carrying financial intermediaries that offer demandable debt securities in exchange for their endowments.

These securities are earmarked to give impatient depositors the right to withdraw \(R_1\) units of consumption good on their first birthday, and offer patient consumers \(R_2\) units on their second birthday. We limit our attention to intermediaries of constant size with stationary payout schedules and use \(Y\) to denote the corresponding stationary investment in risk-free assets. We define an intermediary’s operating rule as \(\{Y, R_1, R_2\}\).

Naturally, intermediaries are subject to an internal budget constraint that requires that withdrawals and investments be financed with new deposits and returns on earlier investments:

\[
\varepsilon R_1 + (1 - \varepsilon) R_2 \leq 1 + Y r.
\]

It can be easily seen that the periodic outflows to impatient and patient members is increasing in the intermediary’s buffer \(Y\). Indeed, an intermediary wishing to maximize the welfare of its members will try to reach the highest level of investment possible. Without exogenously specifying a maximum golden-rule investment level, there is no limit on the intermediary’s perpetual buffer.\(^{12}\) In the next section we will show that \(Y\) is endogenously limited by its governance: if only the living depositors determine the intermediary’s operating rule, the obtainable allocations are severely limited.

### 3 The Governance of Perpetual Institutions

To model the governance of the perpetual intermediary, we assume that its living depositors decide on the operating rule during a periodic general meeting. Without loss of generality, this meeting takes place between dates, after all agents learn their type, so that \(1\) young and \(1 - \varepsilon\) old members attend.\(^{13}\) In these meetings depositors vote to either (a) support the current operating rule, \(\{Y, R_1, R_2\}\), or (b) disband the

\(^{11}\) This is usually justified in terms of the law of large numbers. Duffie and Sun (2007) provide a formulation of independent random matching so that the aggregate distribution of individually random types is certain.

\(^{12}\) Notice that we do not specify a starting or an ending date. Bhattacharya, Fulghieri, and Rovelli (1998) show how intermediaries can reach stationarity from a starting date.

\(^{13}\) It can be shown that our key results hold if stakeholder meetings are held before types are known, or on dates, when \(3 - \varepsilon\) depositors, of three different cohorts, are present.
institution, liquidate all assets, and distribute the proceeds. We require unanimous support for a motion to be accepted.\textsuperscript{14}

We characterize a proposal to disband by the payout vector \(\{R_2^{\text{pat}}, R_1^{\text{imp}}, R_1^{\text{pat}}\}\), representing the distributions to the intermediary’s members at the date following the meeting: \(R_2^{\text{pat}}\) denotes the amount offered to the old, and \(R_1^{\text{imp}}\) and \(R_1^{\text{pat}}\) denote the payoffs to young impatient and patient respectively.

We further assume that there is a dead-weight disbandment cost \(kY\), where \(k \in [0,1]\).\textsuperscript{15} The disbandment costs \(k\) can be interpreted in different ways. They may be the costs of overcoming charters of constitution, or pure transaction costs that come with selling assets, or they may constitute search costs of finding the disbandment allocation. Finally, we can interpret \(k\) as feelings of remorse vis-à-vis all unborn future generations, who due to disbandment will not have access to the current allocation \(\{R_1, R_2\}\).

A disbandment proposal will only be voted on if it is feasible. A proposal is feasible if the liquidation value of the assets on the next date is sufficient to finance the disbandment schedule. That is,

\[
(1 - \varepsilon)(R_2^{\text{pat}} + R_1^{\text{pat}}) + \varepsilon R_1^{\text{imp}} \leq Y(1 + r)(1 - k).
\]

This expression (2) gives, on the left-hand side, the disbanding intermediary’s payouts to its members on the date following the governance meeting. On the right-hand side we find the net proceeds of the liquidation on that date.

\section{The Renegotiation Constraint}

For a disbandment motion to be successful it has to improve the payments received by both cohorts: for the young to support disbandment we need \(R_1^{\text{imp}} \geq R_1\) and \((1 + r)R_1^{\text{pat}} \geq R_2\); for the old to support it we need \(R_2^{\text{pat}} \geq R_2\).

Combining these inequalities with the feasibility constraint, we find the following renegotiation constraint:

\[
(1 - \varepsilon)R_2(1 + (1 + r)^{-1}) + \varepsilon R_1 \geq Y(1 + r)(1 - k).
\]

For any policy that does not satisfy (3) there is a feasible disbandment proposal that will receive unanimous support: The intuition behind this constraint is straightforward: the left-hand side represents the present value of the intermediary’s liabilities. On the right-hand side we find the liquidation value of the intermediary’s assets. When the value of the assets is greater than the present value of the liabilities,

\textsuperscript{14} Restricting renegotiation to full disbandment is without loss of generality. Providing for increased temporary payouts and partial asset sales generates the same renegotiation constraint. We require consensus to avoid agents being expropriated. Clearly, if a majority can ex post expropriate a minority, the intermediary will not attract depositors ex ante.

\textsuperscript{15} Also, the linearity assumption is without loss of generality. Making the disbandment cost nonlinear in \(Y\) would complicate the analysis without adding additional insights.
we have an overfunded intermediary. If the funding ratio is greater than \((1 - k)^{-1}\), the living can liquidate the asset buffer and pay themselves more than what the current operating rule offers them.

Equation (3) can be used to characterize the relationship between \(R_1\) and \(R_2\) by substituting into it the internal budget constraint. We find:

**Proposition 1 (renegotiation constraint with markets)** The withdrawal rights of perpetual intermediaries that earn a periodic return of \(r\) on their investments and face a disbandment cost \(k\) are limited by the following renegotiation constraint:

\[
(1 - \varepsilon) \frac{1}{(1 + r)^2} R_2 - k + \varepsilon R_1 \left( \frac{1}{1 + r} - k \right) \leq 1 - k.
\]

This renegotiation constraint is similar to a budget constraint in that it limits the allocation that perpetual intermediaries can offer. As the disbandment cost enters directly into the renegotiation through equation (3), it is immediate that a higher \(k\) will soften the renegotiation constraint and allow greater allocations.

Notice that for the special case \(k = 0\), we have the budget constraint that holds for intragenerational Diamond–Dybvig (1983) banks that pool their clients’ assets and offers withdrawal rights of either \(R_1\) or \(R_2\) so as to maximize ex ante expected utility.

Indeed, the key finding of this paper is that, in the absence of renegotiation costs, perpetual intergenerational intermediaries cannot accumulate more assets than the present value of their liabilities, and cannot improve on the allocation of intragenerational intermediaries:

**Corollary 1 (renegotiation constraint with costless disbandment)** In the absence of disbandment costs, the renegotiation constraint of intergenerational intermediaries that operate in a market economy is given by

\[
\frac{\varepsilon R_1}{1 + r} + \frac{(1 - \varepsilon) R_2}{(1 + r)^2} \leq 1.
\]

This result – which implies that, in the absence of frictions, intergenerational intermediaries cannot increase welfare relative to intragenerational intermediaries – is important, as it is not yet recognized in the literature. Instead, articles of Qi (1994), Bhattacharya and Padilla (1996), Allen and Gale (1997), Fulghieri and Rovelli (1998), and others argue that intergenerational intermediaries (banks, governments, or financial systems) are able to increase welfare over and above intragenerational intermediaries by carrying overfunded asset buffers. Our analysis shows that, because intermediaries are governed only by the living, none of these intergenerational intermediaries will survive unless significant disbandment costs exist. The present literature by and large ignores generational incentive compatibility and mostly postulates an infinite-lived welfare maximizer.

Despite the stringent renegotiation constraint, it may still be possible that intermediaries can improve on the market allocation. Whether such improvements are
possible depends on the nature of the institutions. Intermediaries that offer simple demandable debt securities, such as banks, need to prevent their patient clients from rolling over their deposits, and their impatient clients from selling their (securitized) deposits (in the market or to newborns). This means that for unconditional demandable debt securities, the following bank constraint needs to hold:

\[ R_2^1 = R_2. \]

If we have \( R_2^1 > R_2 \), patient depositors can increase their welfare by playing “withdraw and redeposit.” If \( R_2^1 < R_2 \), impatient depositors could increase their utility by selling their deposits to newborns as in Jacklin (1987). From (5) and (6) we find that, without disbandment costs, \( R_1 \leq 1 + r \) and \( R_2 \leq (1 + r)^2 \), implying that bank intermediaries cannot improve on the market allocation \( \{C_1, C_2\} = \{1 + r, (1 + r)^2\} \).

Unlike previous papers in the literature, we also consider intermediaries that can condition payouts on types, identities, or age. For example, insurance mutuals and social security schemes can condition payouts on the identity and type of depositors, and pension funds condition payouts on age. Such intermediaries are not constrained by (6).

We also recognize that in the economy’s population different client classes (with different impatient risks and different preferences) exist, so that intermediaries can offer a variety of \( \{R_1, R_2\} \) contracts. Still, because nonbank intermediaries are governed by the living generations only, their payout schedules have to abide by the renegotiation constraint (4).

In order to get an idea of the role of the different parameters, we calibrate the model so as to depict the renegotiation constraint for our simple model in a graph (see Figure 1). The main parameters are \( \varepsilon, k, \) and \( r \). The parameter \( \varepsilon \) captures the likelihood of being exposed to the liquidity shock in any given period. A set of values to consider for \( \varepsilon \) might range from the one assumed by Gertler and Kiyotaki (2013), who use a value for \( \varepsilon \) (which they denote by \( \pi \)) equal to 0.03, to the one assumed by Ennis and Keister (2003), who instead use 0.25. In our calibrations we find that \( \varepsilon \) has a relatively small effect, so we set it equal to 1/3, for clearer exposition, and because in our model a period is much longer than a year, as is often silently assumed in the existing literature. The parameter \( r \) represents the one-period return to the illiquid investment, which we set at 100%. If we assume active lives that last 60 years, and a potential illiquidity event that occurs after the first 30 years, our \( r \) of 100% corresponds to an annual real interest rate of 2.33%, which is close to the rates used in similar models by, amongst others, Mattana and Panetti (2014) and Cooley and Prescott (1995).\(^{16}\)

More important for our analysis is the illiquidity friction \( k \), as this parameter determines the welfare that can be achieved by our perpetual institutions. To calibrate \( k \) we can assume that institutions invest in public security markets, and consider

\(^{16}\) Mattana and Panetti (2014) use 2%, based on the average long-term growth rate of the U.S. GDP in real terms. At a time of higher growth and interest rates, Cooley and Prescott (1995) used 5%.
Figure 1
Institutions and Constraints

Notes: This figure depicts the various constraints and allocations for the case where $r = 100\%$ and $\varepsilon = 1/3$. Line $a$ is the budget constraint of the intragenerational Diamond–Dybvig intermediary, and the renegotiation constraint of the perpetual intermediary with no disbandment costs. Line $b$ is the renegotiation constraint with disbandment costs $k = 5\%$. Line $c$ is the golden-rule constraint suggested by Qi (1994) and others, $d$ is the bank constraint due to the rollover and side-trade threat, and $e$ is the no-interbank-deposits constraint suggested by Bhattacharya and Padilla (1996). $M$ is the market allocation; $Q$ is the allocation suggested by Qi (1994). $A$ is the constrained optimal allocation for an intragenerational nonbank intermediary, $B$ is the optimal allocation for the perpetual intermediary with disbandment costs, and $C$ is the optimal allocation if only the golden rule binds and agents have equal preferences in the impatient and in the patient state. The dashed lines show how the renegotiation constraint moves outward as $k$ increases.

Bid–ask spreads, direct transaction costs, or the excess returns of illiquid stocks compared to the most liquid ones. In any of these cases, the costs of liquidating assets in the stock market will lie between 1% and 5%.\textsuperscript{17} Alternative institutions can invest in less liquid assets, such as companies, real estate, or art. In this case, liquidation costs are likely to be much higher, perhaps as much as 20%.\textsuperscript{18} As a compromise, we use $k = 5\%$ as our base case, and conduct a sensitivity analysis.

In Figure 1 we depict the renegotiation constraint for a payout schedule targeted at clients with $\varepsilon = 1/3$, in an economy with $r = 100\%$. Line $a$ gives the renegotiation constraint in the absence of disbandment cost. The market allocation $M$ is on this line. $M$ is unlikely to be the optimal allocation on $a$ for all agents. It may well

\textsuperscript{17} See Stoll and Whaley (1983), Amihud and Mendelson (1986), and others.

\textsuperscript{18} Bernanke, Gertler, and Gilchrist (1999) use liquidation costs of 12% in an investigation of business cycles with equilibrium asset liquidation. In a similar study, Carlstrom and Fuerst (1997) use 20%.
be that some agents have higher utility for consumption in the impatient state than in the patient state, as is implied by the indifference curves in the figure. In such a case a nonbank intermediary that offers allocation $A$ may form.

Theoretically a pure exchange mechanism could also achieve allocations on line $a$. In such a market we would need the same number of contingent claim securities as there are agents, each agent would need to hold a sufficiently large number of these contingent claims to be sufficiently diversified, and each agent would need to monitor the resulting claims. Whether obtained by a market or by an intermediary, equilibrium allocations are constrained by line $a$, the renegotiation constraint of Proposition 1.

Line $b$ gives the renegotiation constraint of the perpetual intermediary if its disbandment costs are $k = 5\%$. As can be seen from the figure, the existence of disbandment costs enlarges the set of feasible allocations that perpetual intermediaries can offer. The relative improvement increases in the ratio $R_2/R_1$, because with a higher $R_2/R_1$ comes a higher $Y$, and a larger frictional loss in case of disbandment.

Line $c$ depicts $\varepsilon R_1 + (1 - \varepsilon) R_2 = (1 + r)^2$, the golden rule of Qi (1994), Bhattacharya and Padilla (1996), Fulghieri and Rovelli (1998), and later papers. These papers exogenously assume an optimal periodic investment of one unit that is invested for two periods. Bhattacharya and Padilla (1996) point out that a golden-rule investment level of unity is arbitrary but interesting in that it shows that the market equilibrium may not achieve the golden rule. Qi (1994) suggests that a bank or coalition can obtain the golden-rule investment and provide allocation $Q$ to its members. $Q$ is the only point on $c$ that is individually incentive-compatible, because agents could roll over claims (if $R_1^2 > R_2$) or engage in side trade (if $R_1^2 < R_2$), so that the bank constraint (equation (6)), depicted by line $d$, binds.

Bhattacharya and Padilla (1996) point out that $Q$ is not feasible because it tempts competing banks to open accounts with each other instead of investing in the production technology. They argue that an “interbank deposit constraint,” depicted by line $e$, needs to hold to avoid such interbank arbitrage. They then suggest ways by which a government, through a tax-and-subsidy scheme, could potentially attain the Pareto-optimal allocation $C$. Fulghieri and Rovelli (1998) point out that $C$ can also be obtained if agents’ ages are verifiable. Our analysis suggests that neither of these allocations is renegotiation-proof, as they both lie above the renegotiation constraint without frictions (represented by line $a$).

Our analysis shows that disbandment costs $k$ enable the institution to obtain increased stationary investment and higher allocations. The relationship between $k$ and the renegotiation constraint is illustrated by dotted lines (labeled $k = 5\%$, $k = 10\%$, etc.).

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19 To reduce the number of marketable contracts, Malinvaud (1972, 1973) and Penalva Zuasti (2008) suggest that insurance companies rather than markets deal with individual risks.

20 All these cited papers assume equal preferences for the consumption in the impatient and patient states, so that the Pareto-optimal allocation would be given by point $C$. 

Critical values of $k$ are $(1+r)^{-2}$ and $(1+r)^{-1}$ — in our example, 25% and 50%. When the first critical value is reached, there is no limit on $R_2$. To see this, notice that as $R_2$ becomes large, the periodic investment approaches $Y = (1-\varepsilon)R_2r^{-1}$, so that every period $(1-\varepsilon)R_2$ can be paid out. With $k > (1+r)^{-2}$, the first term in the negotiation constraint becomes negative, indicating that a higher $R_2$ can sustain a higher $R_1$, which takes us into welfare-improving territory. Thus, the renegotiation constraint if $k > (1+r)^{-2}$ is unlikely to bind before the (exogenous) golden-rule constraint does. Still, for the institution to be completely unconstrained by renegotiation threats, we need $k > (1+r)^{-1}$. In this case both terms on (4)’s left-hand side become negative, implying that all (positive) withdrawal schedules $\{R_1, R_2\}$ can be sustained in equilibrium.

5 The Monopolist Intermediary

In this section we consider a closed economy with a single collectively governed intermediary that collects deposits from all agents in the population. The governance structure and the basic idea of the renegotiation constraint are the same: if, in a candidate equilibrium, a disbandment allocation exists where all living agents increase their welfare by liquidating the asset buffer, then disbandment will obtain, thus rendering the candidate equilibrium invalid.

The presence of a monopolist complicates our analysis, as there no longer exists a market where the intermediary can sell its assets, and hence no unequivocal source with which to pin down the liquidation value of the asset buffer. There exists, however, a shadow market, because newborns will be ready to purchase assets in case they are not invited to join the intermediary. To properly understand the workings of this shadow market and identify the resulting shadow prices, it is important to model the available production technologies that represent the alternative investment opportunities for the newborns.

It can be easily seen that if the basic primary production technology has a gestation lag of one period, all assets will be priced at a yield of $r$, so that the analysis is the same as that of the previous section. It is however unrealistic that the periodic yield that we assumed in the previous sections is generated by one-period production only.

To model long-term production, we follow the previous literature and assume that the periodic interest rate $r$ is generated by a stationary sequence of two-period technologies, henceforth called projects. A project pays $R \equiv (1+r)^2$ two periods after the investment of one unit of consumption good.\footnote{To see that a stationary sequence of such projects generates a market interest rate of $r$, denote by $p_t$ the price at which seasoned (one-year-left-to-maturity) projects are bought and sold. To avoid arbitrage, we need $p_t = R/p_{t-1}$ for all $t$. The stationary equilibrium thus has $p = R/p$, for an interest rate of $r$.}

Now consider a monopolist intermediary that offers its members a schedule $\{R_1^*, R_2^*\}$, and periodically invests $Y^*$ in new projects. If the operating rule is sta-
tionary, the consequent internal budget constraint will be

\[
(7) \quad \varepsilon R_1^* + (1 - \varepsilon) R_2^* + Y^* \leq 1 + Y^* R.
\]

On the left-hand side we see the periodic outflows to depositors and new investments. On the right-hand side we have the inflows: the new deposits and the project payoffs. On every potential disbandment date the intermediary holds \( Y^* \) maturing projects and \( Y^* \) projects with one year left to maturity.

To preclude a feasible disbandment proposal from being successful, we again need the liquidation value of the assets to be lower than the present value of the liabilities to the living, or

\[
(8) \quad (1 - \varepsilon)R_2^* \left( 1 + \frac{p^*}{R} \right) + \varepsilon R_1^* \geq (Y^* p^* + RY^*)(1 - k),
\]

where \( p^* \) is the (unobservable) shadow price of intermediate projects paying \( R \) in one period. On the left-hand side we have the minimum of goods that the depositors require: the old require \( R_2^* \), the patient young require \( R_2^* p^* R^{-1} \) (so that next period they receive \( R_2^* \)), and the impatient young want at least \( R_1^* \). Here \( p^* \) is the project price on the disbandment date, when there are \( 2 - \varepsilon \) agents that need to carry consumption goods through time: \( 1 - \varepsilon \) patient one-year-olds and a measure one of newborns.

We will now study two cases: first we will consider the case where the newborns form a coalition too, and \textit{hold out} the intermediary controlled by their parents and grandparents. In this case the newborns threaten not to buy any seasoned projects if they are not invited to join the intermediary on the original terms. In section 5.2 we will consider the case where the newborns act competitively and bid up the price of seasoned projects to their marginal utility.

### 5.1 Newborn Generations Can Hold out the Monopolist Intermediary

If newborn generations can credibly threaten not to buy any assets sold by the incumbent monopolist, the shadow price of seasoned projects will be zero. In this case the intermediary could still disband and make all its living depositors better off, if its asset buffer is large enough. The chairperson can offer the patient young seasoned projects, but needs to offer the impatient young and the patient old consumption goods that are generated by its maturing projects. Since, by the internal budget constraint, we always have \( RY^* > (1 - \varepsilon) R_2^* \), the scarce resource will be goods, not projects. A feasible disbandment proposal thus exists if we have

\[
(9) \quad RY^*(1 - k) > (1 - \varepsilon) R_2^* + \varepsilon R_1^*.
\]

On the left-hand side we have the goods generated by the maturing projects; on the right-hand side we have the consumption needs of the old and the impatient young. Notice that if (9) just binds, the intermediary can offer the patient young all its \( Y^* \) projects, which will more than satisfy them.
To find the set of feasible \( \{R_1, R_2\} \) allocations we only need to take (9) and substitute \( Y^* \) using the internal budget constraint (7). After some algebra we find:

**Proposition 2 (Monopolist Intermediary Catering to Colluding Newborns)**

The withdrawal rights of a perpetual intermediary that invests in two-year projects earning a per-period return of \( r \), faces a disbandment cost \( k \), and caters to overlapping generations that can credibly threaten not to purchase any assets of a liquidating institution, are limited by the following condition:

\[
(1 - \varepsilon)R_2^* + \varepsilon R_1^* \leq \frac{R(1 - k)}{1 - Rk} \quad \text{if } k < \frac{1}{R}.
\]

For \( k = 0 \), we obtain \((1 - \varepsilon)R_2^* + \varepsilon R_1^* \leq R\), which implies \( Y^* \leq 1 \). This happens to be the golden-rule constraint considered by Qi (1994).\(^{22}\) The intuition of this analysis is that if a perpetual intermediary invests \( Y^* > 1 \), it can always make its living members better off by not inviting new depositors but instead distributing the payoffs from the maturing assets, \( Y^* R \), to its old and patient young members, and those of the \( Y^* \) projects to its patient members. In contrast, a perpetual intermediary with \( Y^* < 1 \) critically depends on cash inflows from new members to pay its consuming members and thus cannot disband, even if there are no associated costs.\(^{23}\)

If the disbandment costs are high enough (i.e., if \( k \geq R^{-1} \)), there is no renegotiation bound on the consumption schedules that can be offered by perpetual intermediaries. Only an exogenous golden rule would be able to limit the intermediary’s asset buffer.

The assumption that newborns – who are certain to live an additional period – collectively threaten not to buy (at any price) seasoned projects that pay \( R \) after one period may not be very realistic.\(^{24}\) The next subsection investigates the more natural case that assumes that newborns would act competitively, and bid up the price of seasoned assets to their competitive value in case the intermediary decides to disband.

### 5.2 The Monopolist Intermediary that Faces Competitive Newborns

The case where newborns act competitively introduces the additional complication that the price per project depends on the number of projects that the disbanding intermediary offers for sale. Since the patient young can be given projects rather

\(^{22}\) However, Qi (1994) does not give this argument for his assumption of a maximum investment limit \( Y^* \leq 1 \).

\(^{23}\) To see this, define \( D = \varepsilon R_1^* + (1 - \varepsilon) R_2^* = 1 + Y^* (R - 1) \) as the periodic distributions of perpetual intermediaries (the latter equality follows from the internal budget constraint (7)). We immediately see that \( Y^* > (\cdot) \iff D < (\cdot) Y^* R \), implying that intermediaries with \( Y^* > 1 \) can, and those with \( Y^* < 1 \) cannot, pay \( D \) out of their project payoffs.

\(^{24}\) Indeed, the disbandment subgame is not intuitive in the sense of Cho and Kreps (1987).
than goods, the number of projects that a disbanding intermediary needs to sell is
\[ X = Y^* - (1 - \varepsilon) R^* R^{-1}. \]

The final term of the left-hand side gives the minimum number of projects that
the disbanding intermediary needs to offer its patient young depositors to get their
vote.

Because the potentially disbanding monopolist is the only supplier of projects,
the competitive project price will depend on the number of projects it offers for sale
to the only potential buyers, the newborns. The following lemma gives the equilib-
rium price that ostracized newborns would pay if they were offered \( X \) projects in a
competitive auction:

**Lemma** If \( X \) projects are sold to competitive newborns, the clearing price per
project will be
\[ p = \frac{\varepsilon R}{X(R-1) + \varepsilon}. \]

The derivation of (12) follows from an analysis of the equilibrium that obtains
when newborns start a new market economy.\(^{25}\) In the appendix we show that if
a generation restarts without seasoned projects, the market return during the first
period will be zero, while the second and consecutive even periods will see a re-
turn of \( R \). Or, if no projects are offered, the ostracized first and consecutive odd
generations consume \( \{1, R\} \), while even generations consume \( \{R, R\} \).\(^{26}\) Hence, if
\( X \approx 0 \) projects are offered for sale, the equilibrium price will be \( p^* = R \), for a zero
yield. We show that if the economy is restarted with an auction of \( X > 0 \) seasoned
projects, the cyclicality will be progressively dampened and then reversed.\(^{27}\)

Plugging (12) into (11) and the binding budget constraint (7) into (8) gives
the renegotiation constraint for the monopolist case. It is a quadratic equation in
\( R^* (1 - \varepsilon) \), of which the positive root is given by the following proposition:

**Proposition 3 (Monopolist Intermediary Facing Competitive Newborns)**
The withdrawal rights of a perpetual intermediary that invests in projects paying \( R \) goods two periods after investment, faces a disbandment cost \( k \), and caters

\(^{25}\) Also, if ostracized newborns decide to start a new intermediate economy, the first
generations will receive allocations that are not better than those obtained in the market
economy. See Bhattacharya, Fulghieri, and Rovelli (1998) for details on how an interme-
diated economy can be started up.

\(^{26}\) This cyclical equilibrium is first mentioned in Bhattacharya, Fulghieri, and Rovelli

\(^{27}\) Notice that if more than \( \varepsilon \) projects are sold, they will fetch less than one good per
project, so that the restarting and consecutive odd generations consume more on their first
birthday than on their second birthday. This curious allocation is due to the assumption
that there is no storage technology. With a storage technology the price function would
be a step function that would unnecessarily complicate the derivation of the renegotiation
constraint. In the appendix we derive the renegotiation constraint for the case where there
is a storage technology.
to overlapping generations of competitive agents are limited by the following condition:

\[
R^*_{2} \leq \frac{R}{1 - Rk} - \frac{\varepsilon R^*_1 (R + 1)}{2(1 - \varepsilon)} + \frac{\sqrt{D - Rk(1 + R - 2\varepsilon R)}}{2(1 - Rk)(1 - \varepsilon)},
\]

where

\[
D = \varepsilon^2 R^*_1 (R - 1)^2 (1 - Rk)^2 + 4\varepsilon^2 R^2 \\
+ Rk(R - 1 - 2\varepsilon R)(2\varepsilon R^*_1 (R - Rk)(R - 1) + R(k - 1 - 2\varepsilon R) + 4\varepsilon)).
\]

It can be verified that for \( k = 0 \), the market allocation \( \{\sqrt{R}, R\} = \{1 + r, (1 + r)^2\} \) is on the monopolist’s renegotiation constraint, which shows that in the absence of disbandment costs, a monopolist bank, that needs to offer \( R_2 = R_1^2 \) to avoid rolling over of deposits and side trading, cannot offer a better allocation than a market economy or an economy with competing banks.

A nonbank monopolist that can make their payoffs contingent on type or age can do better than competing institutions, even in the absence of transaction costs. The following corollary formalizes these conclusions and gives the renegotiation constraint for the zero-disbandment case.

**Corollary 2 (costless disbandment)** In the absence of disbandment costs \((k = 0)\), the renegotiation constraint of a monopolist intermediary is given by

\[
R^*_2 \leq R + \varepsilon \frac{\sqrt{4R^2 + R^*_1 (R - 1)^2 - R^*_1 (R + 1)}}{2(1 - \varepsilon)}.
\]

The schedule \( \{\sqrt{R}, R\} \) just satisfies this condition. For all \( R, \varepsilon \), there exist:

(i) Allocations that meet the renegotiation constraint (14) for the disbandment-cost-free monopolist and do not meet the renegotiation constraint (5) for the disbandment-cost-free intermediary in competition. These schedules have \( R_1^2 > R_2 \).

(ii) Allocations that meet the renegotiation constraint for disbandment-cost-free competing intermediaries (5) and do not meet the renegotiation constraint for the disbandment-cost-free monopolist intermediaries (14). These schedules have \( R_1^2 < R_2 \).

We thus find that in the absence of disbandment costs \((k = 0)\), bank intermediaries cannot improve on the market equilibrium, whether there is only one intermediary acting as a monopolist or there are many competing banks. Nonbank intermediaries on the other hand can increase welfare, by shifting consumption between types. A monopolist nonbank intermediary can further increase welfare if agents place more value on early consumption than on late consumption. The reason for this is that a monopolist can hold a larger asset buffer without being subject to renegotiation.

Agents who favor a high \( R_2 \) and low \( R_1 \) may be better off when there are competing institutions. The reason for this is that intermediaries that offer high \( R_2/R_1 \)
Notes: We depict the monopolist intermediary’s renegotiation constraints, for $r = 100\%$ and $\varepsilon = 1/3$. Lines $a$ and $b$ give the renegotiation constraints in a competitive economy, for $k = 0$ and $5\%$ respectively; see Figure 1. Line $a^*$ is the renegotiation constraint if the intermediary has no disbandment costs and faces newborns who can threaten not to buy any seasoned projects. It coincides with line $c$ (Qi’s golden-rule constraint) of Figure 1. Line $a^{**}$ is the renegotiation constraint if the monopolist faces competitive newborns. The market allocation $M = \{2, 4\}$ is on this line. Lines $b^*$ and $b^{**}$ give the renegotiation constraints for monopolist intermediaries with disbandment costs $k = 5\%$, who face newborns threatening not to purchase any liquidated assets and competitively behaving newborns, respectively.

Figure 2 illustrates the renegotiation constraint for the monopoly case. Lines $a$ and $b$ give the renegotiation constraints if the intermediaries compete and disbandment costs are $k = 0$ and $k = 5\%$, respectively. Lines $a^*$ and $b^*$ are the renegotiation constraints if newborns can threaten not to purchase any assets from the monopolist intermediary controlled by their parent and surviving grandparent generations. Lines $a^{**}$ and $b^{**}$ give the renegotiation constraints if the monopolist intermediary faces competitive newborns who bid up the price of liquidated assets to their marginal utility. We clearly see that a monopolist nonbank intermediary who sets $R_1 > R_2$ can significantly improve on the allocation obtained in an economy with competing intermediaries.
In this article we have reexamined risk sharing in overlapping-generations economies. Ideally, wealth is redistributed as consumption while maximizing the periodic investment and output. Current models show how perpetual financial intermediaries can improve on the market economy by accumulating buffers of productive assets to smooth consumption and exploit production technologies.

In this paper we model the governance of such intermediaries and ask what happens if they are governed by the living generations only. We argue that investment and distribution schedules may be renegotiated if this benefits the living depositors at the cost of future generations. We find that the threat of renegotiation represents a major limitation on asset buffers of intergenerational institutions.

We characterize a renegotiation constraint, and find that it is so demanding that perpetual *inter*generational intermediaries cannot improve welfare beyond that obtained by finite-lived *intra*generational intermediaries. In particular, we find that perpetual institutions cannot be overfunded, because this leads to renegotiation.

We find that the renegotiation constraint is relaxed by incorporating frictions. We show that intermediaries are able to provide higher perpetual allocations if they commit to higher disbandment costs. Our intermediaries can be interpreted as banks, insurance companies, defined-benefit pension plans, sovereign states, or perpetual endowments of churches, colleges, or museums, which voluntarily invest in illiquid assets and make disbandments costly so as to safeguard their multigenerational existence. Open-end mutual funds also include liquidation frictions (in the form of redemption fees of back-end loads), but this is due to rebalancing costs that come from the inherent mismatch in liquidity of assets and liabilities (see Chordia, 1996). Institutions that are more closely related to the ones we analyze are closed-end mutual funds trading at a discount. Such funds could be interpreted as being overfunded financial intermediaries. Bradley et al. (2010) show that, as the discount at which closed-end funds trade increases, arbitrageurs take positions and disband or open-end the fund, and that statutory frictions can avoid such behavior.

Because countries are also perpetual institutions, we also analyze an economy with a single monopolist intermediary. We find that a mutually owned monopoly can improve welfare compared to a market economy if agents value consumption in the bad (early consumption) state high enough. The reason is that a monopolist intermediary faces a serious market impact if it wants to ostracize the future generations by liquidating its assets to the benefit of the living. The disbandment hurdle makes the out-of-equilibrium renegotiation subgame less attractive so that a larger asset buffer can be maintained.

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28 These costs are particularly severe for money market mutual funds, since they face the additional constraint of keeping their net asset value (total net assets / number of shares) at least equal to $1 at all times, because breaking the buck will likely cause mass withdrawals and contagion to other funds. See Schmid, Timmermann, and Wermers (2015).
For simplicity, and without loss of generality, our results are obtained assuming that renegotiation only occurs in the form of an outright shutdown of the scheme to new generations and a distribution of the assets to its existing members. In practice, renegotiation may take place in the form of benefit increases, contribution holidays, or lump-sum distributions. Such renegotiations, which occurred regularly in defined-benefit pension plans during the 1990s, are seen as key causes for the fact that most pension funds are currently underfunded. See Armstrong and Selody (2005) or Andonov, Bauer, and Cremers (2016).

We focus on a governance mechanism that requires complete agreement by all living participants to approve a disbandment, as we are interested in the effect of the existence of a governance mechanism in which one does not allow one group of living members to expropriate another. A more detailed study of how the details and political economy of the governance mechanism affect different institutions is an interesting topic for future research.

Another simplifying assumption in our model is the lack of uncertainty. In reality returns on assets and liabilities are stochastic, making it impossible to maintain a funding ratio of exactly 100%. Renegotiations in real life are expected, and ex ante contracted. Most insurance mutuals have statutory profit-sharing schemes that effectively cap their surpluses (Hansmann, 1985). Cooperatives and other endowment-carrying institutions renegotiate in the form of increased salaries or other benefits for their living beneficiaries, or equity redemptions of exiting members (Hansmann, 1999). Our claim that, in the absence of frictions, intermediaries cannot be overfunded is thus consistent with empirical observations.

Appendix

A.1 Proof of Proposition 1

From (1) we have \( Y \geq (\varepsilon R_1 + (1-\varepsilon)R_2 - 1)/r \). Substituting this into (3) gives

\[
(1-\varepsilon)R_2 \left(1 + \frac{1}{1+r}\right) + \varepsilon R_1 \geq (\varepsilon R_1 + (1-\varepsilon)R_2 - 1)(1+r)(1-k).
\]

Collecting terms gives

\[
(A1) \quad (1-\varepsilon)R_2 \left(\frac{r}{1+r} - 1 + k(1+r)\right) + \varepsilon R_1 (-1 + k(1+r)) \geq -(1+r)(1-k).
\]

29 Hansmann (1985) argues that the competitive advantage of mutuals in the life insurance business lies in the moral-hazard problem that life insurance sellers have vis-à-vis locked-in policyholders. Since in mutuals this shareholder–policyholder conflict is resolved, mutuals are particularly dominant in the life insurance business. See also Smith and Stutzer (1995).

30 Also related to our renegotiation constraint are the horizon problem, which causes underinvestment in LT projects by cooperatives (Olesen, 2007), and the access problem, which refers to limiting access to new generations and disbandment (Rey and Tirole, 2007).
and then (4):
\[(1 - \varepsilon)R_2 \left(\frac{1}{(1+r)^2} - k\right) + \varepsilon R_1 \left(\frac{1}{1+r} - k\right) \leq 1 - k.\]

_Q.E.D._

A.2 Proof of Proposition 2

Similarly, from (7) we find
\[(\text{A2}) \quad Y^* \geq \frac{\varepsilon R_1^* + (1 - \varepsilon)R_2^* - 1}{R - 1}.\]

Substituting this into (9) gives (10).

_Q.E.D._

A.3 Proof of the Lemma

Let \(\tau\) denote the date of the competitive sale of \(X\) assets by a disbanding intermediary. Potential buyers are newborns and the patient one-year olds who have been given \(R_1^\text{pat}\) goods to vote in favor of disbandment. Since these survivors have no consumption needs, they can buy (i) primary assets that cost unity and pay \(R\) on date \(\tau + 2\) or (ii) secondary assets that pay \(R\) on date \(\tau + 1\). We are looking for the equilibrium price of these assets, \(p_\tau\). Because there is no uncertainty, they also correctly anticipate all future prices \(p_{t>\tau}\). In competitive equilibrium we need \(R/p_\tau = p_{\tau+1}\) for all \(t\), to rule out arbitrage.

The market-clearing condition for time \(\tau\) is
\[(A3) \quad p_\tau = \frac{1 - Y_\tau}{X}.\]

where \(Y_\tau\) denotes the amount that the survivors invest in the production technology.

The market-clearing condition for all \(t > \tau\) is
\[(A4) \quad p_t = \frac{1 - Y_t + (1 - \varepsilon)(1 - Y_{t-1})R/p_{t-1}}{\varepsilon Y_{t-1}}.\]

The denominator gives the supply of projects. They come from the impatient who invested in the two-year production technology. The numerator gives the goods that purchase the projects. It is the aggregate endowment less the investment, plus the goods paid to the agents who in the previous period bought projects and remained patient. The expression (A3) shows that both \(Y_\tau\) and \(p_\tau\) are two-period. After some algebra using \(p_{t-1} = p_{t+1}, Y_{t+1} = Y_{t-1},\) and \(p_t = p_{t-1}/R,\) we find
\[(A5) \quad Y_t = 1 - \varepsilon \frac{R - p_t}{R - 1} \quad \text{for all} \ t.\]

Substituting (A4) in (A2) gives (12) of the Lemma.

_Q.E.D._
Notice that for \( X > \varepsilon \) we would have \( p_i < 1 \), which cannot be the case if a storage technology exists. If, in such a case, more than \( \varepsilon \) projects are offered, the price will be unity, and the first and consecutive odd generations will consume \( \{ R, R \} \) while even generations will consume \( \{ 1, R \} \).

A.4 Proof of Proposition 3

Substituting the binding internal budget constraint (A1) into (11) and rewriting gives

\[
X = \frac{\varepsilon R^*_1 + (1-\varepsilon)R^*_2 - 1}{R-1} - \frac{(1-\varepsilon)R^*_3}{R}.
\]

(A6)

Substituting (A5) into (12) gives

\[
p^* = \frac{\varepsilon R^2}{\varepsilon R R^*_1 + (1-\varepsilon)R^*_2 - (1-\varepsilon)R}.
\]

Substituting (A1) (binding) and (A6) into (8) gives

\[
(1-\varepsilon)R^*_2 \left( 1 + \frac{\varepsilon R}{\varepsilon RR^*_1 + (1-\varepsilon)R^*_2 - (1-\varepsilon)R} \right) + \varepsilon R^*_3 \\
\geq \frac{(\varepsilon R^*_1 + (1-\varepsilon)R^*_2 - 1)\varepsilon R^2(1-k)}{(\varepsilon RR^*_1 + (1-\varepsilon)R^*_2 - (1-\varepsilon)R)(R-1)} + \frac{(\varepsilon R^*_1 + (1-\varepsilon)R^*_2 - 1)R(1-k)}{R-1}.
\]

Multiplying both sides by \( (\varepsilon RR^*_1 + (1-\varepsilon)R^*_2 - (1-\varepsilon)R)(R-1) \) gives

\[
(1-\varepsilon)R^*_2 ((\varepsilon RR^*_1 + (1-\varepsilon)R^*_2 - (1-2\varepsilon)R)(R-1) \\
+ \varepsilon R^*_1 (\varepsilon RR^*_1 + (1-\varepsilon)R^*_2 - (1-\varepsilon)R)(R-1)) \\
\geq (\varepsilon R^*_1 + (1-\varepsilon)R^*_2 - 1)\varepsilon R^2(1-k) \\
+ (\varepsilon R^*_1 + (1-\varepsilon)R^*_2 - 1)\varepsilon RR^*_1 + (1-\varepsilon)R^*_2 - (1-\varepsilon)R)(R(1-k)).
\]

Collecting terms, changing sign, and simplifying gives

\[
(R^*_2(1-\varepsilon))^2(1-Rk) \\
+ R^*_2(1-\varepsilon)(R^*_1 \varepsilon(R + 1)(1-Rk) - 2R(1-\varepsilon) + Rk(1 + R - 2\varepsilon R)) \\
+ R^*_2 \varepsilon R^2(1-Rk) - R^*_1 \varepsilon R(1-\varepsilon + R - \varepsilon R - 2Rk(1-\varepsilon)) + (1-2\varepsilon)R^2(1-k) \leq 0.
\]

The discriminant of this quadratic equation is

\[
D = R^*_2 \varepsilon R^2(R-1)^2(1-Rk)^2 + 4R^2 \varepsilon^2 \\
+ Rk(1-2\varepsilon R)(2R^*_1(1-Rk)\varepsilon(R-1) + R(k(R-1-2\varepsilon R) + 4\varepsilon)),
\]

of which the positive root is

\[
R^*_2(1-\varepsilon) = \frac{2R(1-\varepsilon) - R^*_1 \varepsilon(R + 1)(1-Rk) - Rk(1 + R - 2\varepsilon R) + \sqrt{D}}{2(1-Rk)}.
\]
so that
\[
R_2^* = \frac{2R(1 - \varepsilon) - R_1^* \varepsilon (R + 1)(1 - Rk) - Rk(1 + R - 2\varepsilon R) + \sqrt{D}}{2(1 - Rk)(1 - \varepsilon)},
\]
\[
R_2^* = \frac{R}{(1 - Rk)} - \frac{\varepsilon R_1^*(R + 1)}{2(1 - \varepsilon)} + \frac{\sqrt{D} - Rk(1 + R - 2\varepsilon R)}{2(1 - Rk)(1 - \varepsilon)},
\]
which gives the renegotiation constraint (13).

Q.E.D.

In case there is also a storage technology in the economy, the relevant shadow price is unity if \(X > \varepsilon\). Substituting (A1) (binding) and \(p^* = 1\) into (8), we find

\[
(A7) \quad (1 - \varepsilon)R_2^*\left(1 + \frac{1}{R}\right) + \varepsilon R_1^* \leq \frac{R + 1}{R - 1}(\varepsilon R_1^* + (1 - \varepsilon)R_2^* - 1)(1 - k),
\]

which after some algebra evaluates to

\[
R_2^* \geq \frac{R}{(1 - \varepsilon)(1 - Rk)}\left(1 - k - \varepsilon R_1^*\left(\frac{2}{R + 1} - k\right)\right).
\]

In the presence of a storage technology the renegotiation constraint given by both (13) and (A7) must hold. Figure 2 would change as shown in Figure A1. Due to the storage technology, the renegotiation constraints \(a^{**}\) and \(b^{**}\) are more strict than in Figure 2, because a disbanding intermediary can always sell its assets for unity.

\[\text{Figure A1}\]

Renegotiation with Storage
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