Who Quits Next?
Firm Growth in Growing Economies

Supplementary Material

Julieta Caunedo* Emircan Yurdagul†
Cornell University Universidad Carlos III

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A Appendix: Data

Our data source for plant-level information is the World Bank Enterprise Surveys. The survey is conducted by private contractors with samples of varying size depending on the size of the economy under analysis, targeting formal firms with at least 5 employees. It is constructed via stratified random sampling. The strata are firm size, business sector, and geographic region within a country. The manufacturing and services sectors are the primary business sectors of interest. Firms with 100% government/state ownership are not eligible to participate in the survey. Where available, we use the standardized data that includes samples from 2006 to 2014. Countries for which information is not available in the 2006-2014 dataset, we use the older sample that covers 2002-2005. For countries for which we observed multiple cross-sections in the newer dataset, we treat each country-year pair as one economy to build the measures presented in Figure 1. Taking country-specific averages to form one measure per country gives similar results.

The variables we use from the ES are: current employment ("Permanent, full-time employees end of last complete fiscal year", *l1*), past employment ("Permanent, full-time employees three fiscal years ago", *l2*), current sales ("Last fiscal year's total sales", *d2*), past sales ("Total annual sales three years ago", *n3*) and the year the establishment started its operations ("Year establishment began operations", *b5*). The last variable gives the age of

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*452 Uris Hall, Ithaca, NY. 14853. Phone: +1 607 255 0248. E-mail: julieta.caunedo@cornell.edu.

†(Corresponding author) Calle Madrid, 126. Getafe, Spain 28903. Phone: +34 91 624 95 53. E-mail: eyurdagu@eco.uc3m.es.

an establishment.[1] We adjust for country-specific weights (wt). Sales information is in local currency units. We adjust sales by the Consumer Price Index of each country as reported in the World Development Indicators (WDI), also provided by the World Bank. Finally, we use GDP per capita (constant 2005 US$) from the WDI to measure income per capita. For each country-year sample, we take the average growth rate (log difference) of GDP per capita in the last 20 years to form the growth rates of income per capita. We omit country-year observations with negative value for this average growth rate. Overall, we hold a harmonized dataset with 138 country-year level observations.

For each country-year pair, we obtain Pareto tail indices from the employment size distribution. We first form 15 employment categories, where category i corresponds to 3^{i-1} integers. We drop the categories with less frequency than the one with higher order. Then we regress the logarithm of the number of firms on the logarithm of the median point of each category. The coefficient of the latter is our tail index. A Pareto distribution captures the tails well. Specifically, the average R-squared statistic in the aforementioned regressions is 0.98. The standard deviation of estimated tail parameters is 0.20. We give the key summary statistics for our sample in tables A1 and A2.

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[1] This construction of the age of the establishment allows is robust to acquisitions of the establishment by another company, or the mergers of the firm with another one.

Table A1: Summary statistics for the sample countries

Country	Year	GDP pc ($)	GDP pc growth (%)	q	Tail index	Average plant age
Angola	2006	1838	0.059	0.476	2.54	10.93
Angola	2010	2374	0.060	0.605	1.97	9.75
Armenia	2009	1952	0.078	0.550	1.94	12.47
Armenia	2013	2298	0.070	0.452	1.68	14.74
Azerbaijan	2009	3018	0.093	0.552	1.82	15.46
Azerbaijan	2013	3253	0.077	0.364	1.99	13.75
Bahamas, The	2010	21109	0.004	0.355	1.98	22.97
Bangladesh	2007	541	0.033	0.409	1.80	16.14
Bangladesh	2013	713	0.037	0.418	1.61	18.48
Belarus	2008	4174	0.065	0.574	1.61	16.29
Belarus	2013	4922	0.056	0.835	1.86	13.30
Belize	2010	3935	0.014	0.374	2.35	17.31
Benin	2009	552	0.012	0.321	1.96	14.90
Bhutan	2009	1629	0.052	0.450	2.03	15.05
Bolivia	2006	1078	0.016	0.393	2.30	19.68
Bolivia	2010	1205	0.019	0.376	2.06	26.08
Botswana	2006	5656	0.031	0.330	1.92	12.02
Botswana	2010	6061	0.027	0.527	1.78	16.10
Brazil	2009	5239	0.015	0.515	1.70	23.55
Bulgaria	2007	4420	0.043	0.486	1.97	11.35
Bulgaria	2009	4500	0.039	0.596	2.14	14.73
Bulgaria	2013	4808	0.034	0.264	1.98	15.94
Burkina Faso	2009	447	0.031	0.291	2.02	13.07
Cameroon	2009	924	0.011	0.411	1.92	16.98
Central African Republic	2011	357	0.005	0.441	1.89	13.27
Chile	2010	8678	0.030	0.462	1.72	24.36
China	2012	3378	0.086	0.372	1.96	11.39
Colombia	2006	3567	0.015	0.630	2.21	15.94
Colombia	2010	3984	0.018	0.392	2.23	15.70
Congo, Rep.	2009	1828	0.009	0.485	1.84	19.24
Costa Rica	2010	5504	0.026	0.271	1.95	21.16
Croatia	2007	11280	0.045	0.413	2.07	15.84
Croatia	2013	10556	0.026	0.256	1.99	16.78
Czech Republic	2009	14354	0.029	0.484	1.96	14.05
Czech Republic	2013	14648	0.024	0.291	2.18	17.32
Dominican Republic	2010	4572	0.039	0.318	1.93	19.54
Ecuador	2006	3103	0.011	0.522	1.89	19.29
Ecuador	2010	3283	0.012	0.431	1.94	17.81
El Salvador	2006	2977	0.024	0.395	2.05	19.14
El Salvador	2010	3037	0.019	0.375	2.14	18.35
Estonia	2009	10091	0.050	0.631	2.16	13.71
Estonia	2013	12056	0.049	0.519	2.56	16.25
Gambia, The	2006	424	0.002	0.493	1.90	11.33
Georgia	2008	1839	0.071	0.417	1.70	10.05
Georgia	2013	2160	0.061	0.519	2.29	10.85
Guatemala	2006	2126	0.013	0.293	2.15	18.52
Guatemala	2010	2210	0.012	0.367	1.69	25.46
Honduras	2006	1471	0.019	0.415	1.82	19.39
Honduras	2010	1539	0.017	0.328	1.80	23.56
Hungary	2009	10997	0.026	0.486	1.99	14.70
Hungary	2013	11435	0.022	0.436	1.90	15.23
India	2014	1235	0.051	0.456	2.27	17.42
Indonesia	2009	1492	0.024	0.373	2.00	15.20
Israel	2013	24342	0.026	0.356	2.08	22.98
Jordan	2013	2855	0.023	0.224	2.10	16.67
Kazakhstan	2009	4473	0.051	0.489	1.76	10.62
Kazakhstan	2013	5425	0.050	0.417	2.03	11.49
Kenya	2007	572	0.009	0.494	1.82	14.33
Kenya	2013	642	0.012	0.165	1.93	21.70
Kyrgyz Republic	2009	570	0.030	0.460	1.76	17.20
Kyrgyz Republic	2013	627	0.029	0.507	1.77	13.36
Lao PDR	2009	602	0.048	0.363	2.10	12.63
Lao PDR	2012	725	0.050	0.425	1.85	12.86
Latvia	2009	8126	0.058	0.542	2.11	12.43
Latvia	2013	9636	0.055	0.334	2.14	13.58
Lebanon	2013	7199	0.018	0.247	2.18	22.30
Lesotho	2009	821	0.024	0.401	1.76	15.66
Liberia	2009	189	0.063	0.703	2.41	9.40

Note: GDP per capita growth is the average in the last 20 years. Tail index is the estimated tail parameter of a Pareto distribution from the calculations described in Appendix A. q denotes the estimated probability of success for each country. For more details, see Appendix A.

Table A2: Summary statistics for the sample countries, continued

Country	Year	GDP pc ($)	GDP pc growth (%)	q	Tail index	Average plant age
Lithuania	2009	8616	0.023	0.574	2.35	13.16
Lithuania	2013	10653	0.038	0.455	2.39	14.86
Macedonia, FYR	2009	3576	0.023	0.392	1.97	15.88
Macedonia, FYR	2013	3840	0.022	0.267	2.13	14.87
Madagascar	2009	282	0.000	0.366	1.80	18.31
Malawi	2009	254	0.022	0.604	1.78	15.07
Malawi	2014	274	0.020	0.306	1.75	18.80
Mali	2007	437	0.028	0.525	2.14	12.07
Mexico	2006	8167	0.011	0.545	2.25	17.69
Mexico	2010	8035	0.007	0.392	1.72	20.45
Moldova	2009	917	0.022	0.475	2.03	12.40
Moldova	2013	1138	0.029	0.529	1.93	12.88
Mongolia	2009	1216	0.040	0.535	2.13	12.16
Mongolia	2013	1777	0.052	0.478	2.29	12.16
Montenegro	2009	4424	0.024	0.469	2.34	13.98
Montenegro	2013	4690	0.022	0.250	2.26	15.45
Morocco	2013	2499	0.027	0.377	1.81	21.06
Mozambique	2007	335	0.043	0.466	1.82	16.23
Namibia	2006	3785	0.023	0.545	2.31	13.28
Namibia	2014	4571	0.023	0.285	2.31	8.40
Nepal	2009	362	0.024	0.354	2.29	11.71
Nepal	2013	408	0.025	0.448	2.59	14.20
Nicaragua	2006	1208	0.026	0.471	2.02	17.87
Nigeria	2007	882	0.037	0.550	2.31	10.83
Oman	2003	12553	0.012	0.343	1.78	13.90
Pakistan	2013	794	0.016	0.376	1.67	21.84
Panama	2006	4968	0.025	0.297	1.95	24.94
Panama	2010	6243	0.033	0.454	2.24	14.99
Paraguay	2010	1795	0.009	0.388	1.89	21.58
Peru	2006	2883	0.026	0.470	1.96	19.30
Peru	2010	3561	0.033	0.410	1.91	16.88
Philippines	2009	1330	0.021	0.369	2.08	17.86
Romania	2009	5655	0.037	0.489	2.33	13.48
Russian Federation	2009	6094	0.034	0.619	1.65	15.42
Russian Federation	2012	6846	0.034	0.392	2.10	11.85
Rwanda	2006	305	0.067	0.472	1.87	12.63
Rwanda	2011	393	0.062	0.235	1.80	9.66
Senegal	2007	787	0.016	0.413	2.00	13.67
Senegal	2014	802	0.012	0.345	2.04	17.84
Serbia	2009	4066	0.038	0.606	2.12	18.94
Serbia	2013	4303	0.033	0.558	2.17	15.15
Sierra Leone	2009	357	0.008	0.532	2.19	16.54
Slovak Republic	2009	13924	0.043	0.465	1.96	12.65
Slovak Republic	2013	15371	0.039	0.346	1.72	15.81
Slovenia	2009	19178	0.031	0.542	1.89	19.18
Slovenia	2013	18640	0.023	0.408	2.03	20.62
South Africa	2007	5894	0.017	0.569	1.93	17.80
Sri Lanka	2011	1725	0.043	0.510	2.08	24.03
Sudan	2014	973	0.036	0.615	2.27	13.49
Swaziland	2006	2387	0.013	0.530	1.99	13.69
Sweden	2014	46061	0.019	0.347	2.01	29.29
Tajikistan	2008	397	0.022	0.512	1.68	18.19
Tajikistan	2013	486	0.027	0.481	1.89	14.29
Tanzania	2006	457	0.027	0.582	2.28	12.73
Tanzania	2013	579	0.029	0.116	2.03	14.08
Thailand	2004	2578	0.021	0.415	1.79	15.45
Timor-Leste	2009	635	0.037	0.540	2.01	7.01
Togo	2009	383	0.004	0.357	2.16	10.41
Turkey	2008	7733	0.030	0.481	1.93	17.44
Turkey	2013	8720	0.028	0.614	2.07	15.98
Uganda	2006	344	0.037	0.419	1.99	12.81
Uganda	2013	429	0.035	0.106	2.14	11.16
Ukraine	2008	2206	0.030	0.503	1.94	16.16
Ukraine	2013	2099	0.020	0.344	2.16	15.67
Uruguay	2006	5426	0.011	0.583	2.12	26.71
Uruguay	2010	6873	0.023	0.487	2.04	22.36
Vanuatu	2009	2147	0.008	0.530	1.70	19.60
Venezuela, RB	2010	6020	0.006	0.591	1.96	21.59
Vietnam	2009	855	0.054	0.510	1.91	11.77
Zambia	2007	765	0.024	0.618	1.80	17.35

Note: GDP per capita growth is the average in the last 20 years. Tail index is the estimated tail parameter of a Pareto distribution from the calculations described in Appendix A. q denotes the estimated probability of success for each country. For more details, see Appendix A.

Robustness checks for the empirical evidence of Section II

This subsection includes three parts, each addressing the robustness of the empirical results in the main body of the paper to one particular additional consideration. First, we attempt to clean the aggregate and plant-level growth rates from capital accumulation. Second, we take into consideration the role of sectoral composition in these growth rates. Finally, we clean our sample from country-year observations that with large and statistically significant time trends in aggregate growth in the recent past.

Capital. The production function in our model has two components, labor and productivity. Accordingly, the capital accumulation is included in the latter. We opted for focusing on a labor productivity measure that includes the capital, since the ES does not provide information on the past levels of capital costs (nor on other input costs) which would leave impossible to construct the growth rates of productivity at the plant level.

Here, we try to address the concerns regarding the capital accumulation by cleaning the aggregate growth in the last 20 years from that of the capital. In particular, we compute the growth rate of capital per capita in the last 20 years for each country-year pair in our sample, multiply by 0.33 and subtract it from the growth rate of income per capita in the last 20 years. The source for capital per capita is the Penn World Table 8.0 (PWT). We use this "net" income growth in the aggregate growth measures. The choice of 0.33 as the capital share is consistent with our choice for the labor share in the paper of 0.66. Row I of Table A3 replicates the main statistics highlighted in our benchmark estimates for this exercise. Panel (a) shows that the correlation between the prevalence of positive idiosyncratic productivity growth and the growth rate of income per capita (net of capital) is 0.30, larger and more significant than our benchmark results (in spite of a smaller sample due to limited information on capital in the PWT). Panels (b) and (c) show that the employment growth does not significantly correlate with aggregate growth, as is the case in the benchmark exercise. The relationship between average firm age and aggregate growth is negative and significant, with a correlation coefficient of -0.28, milder than the corresponding coefficient of -0.42 in our benchmark results.

Table A3: Robustness of results in Section II

		Panels				Sample size
		(a)	(b)	(c)	(d)	
		Benchmark				
	Correlation	0.18	0.11	0.12	-0.46	139
	(t-statistic)	(2.17)	(1.28)	(1.44)	(-6.00)	
		Capital				
I.	Net agg. growth	0.30	-0.01	0.08	-0.28	50
		(3.38)	(-0.05)	(0.87)	(-3.11)	
II.	Net agg. and plant growth	0.34	-0.01	0.08	-0.28	50
		(3.91)	(-0.05)	(0.87)	(-3.11)	
		Sectoral composition				
III.	Controlling agg.	0.19	0.14	0.15	-0.40	139
		(2.26)	(1.67)	(1.78)	(-5.12)	
IV.	Controlling plant level	0.20	0.11	0.05	-0.46	139
		(2.34)	(1.36)	(0.58)	(-6.00)	
V.	Controlling agg. and plant level	0.22	0.14	0.07	-0.40	139
		(2.68)	(1.65)	(0.80)	(-5.12)	
		Aggregate volatility and transitions				
VI.	Residual	0.19	0.05	0.09	-0.44	115
		(2.03)	(0.49)	(0.91)	(-5.27)	
VII.	Low volatility	0.19	0.11	0.11	-0.45	125
		(2.15)	(1.15)	(1.27)	(-6.00)	
VIII.	Weak time trend	0.24	0.15	0.15	-0.50	104
		(2.52)	(1.59)	(1.55)	(-6.00)	

Note: The table reports results of replicating the exercises in Section II under alternative approaches to the data. In paranthesis we report the t-statistics. Row I uses the growth rate of income per capita after subtracting 0.33 times the growth rate of capital per capita in the last 20 years. Row II complements this by defining firm productivity growth as increasing productivity by more than 0.33 times the growth rate of capital per capita. Row III uses the growth rate of output per capita after controlling for the sectoral composition of plants. Row IV uses the unconditional aggregate growth rates, but defines the plant-level growth measures as increments relative to the world-wide sector-specific medians. Row V combines exercises in rows III and IV. Row VI uses growth rates of income per capita conditional on the levels of income per capita in year 2000. Row VII drops from the sample the country-year observations with the highest decile of the standard deviation in the income per capita growth rate. Row VIII drops the observations with the largest and significant time trends in the growth rate of income per capita. See the text for further details.

In an additional effort to control for the dynamics led by capital accumulation, we extend the previous exercise as follows. We define plant-level productivity growth as an episode in which productivity grows more than 0.33 times the growth rate of the capital per capita in the country in the last 20 years.[2] The results are documented in row II. Panel (a) shows that the correlation between this measure of the prevalence of productivity growth and aggregate growth is more positive and significant than the previous results, and also more positive than our benchmark results depicted in the Section II of the paper. Panel (c) illustrates that the correlation of the fraction of firms with employment growth among the firms that

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[2] If the plant-level productivity (as defined in Section II of the paper) grows by x percent, and the capital per capita in the country grew by y percent on average in the last 20 years, we count this as a positive (net) productivity growth if and only if $x \geq 0.33 \times y$. (For our benchmark exercises, we were counting the cases with $x \geq 0$.)

experienced this net productivity growth, and aggregate growth (net of capital accumulation) is very similar to the benchmark results and the previous set, with a correlation that is almost identical.[3]

Sectoral composition. The analysis in our paper abstracts from the role of the variations in the sectoral composition across countries on shaping firm-level and aggregate growth. The firms in our model produce homogenous goods, and we do not distinguish in the data according to firms' sectors. We follow this approach for simplicity.

In this section, our objective is to show that the empirical findings of our paper are robust to controlling for the sectoral composition of the economies under focus. For this purpose, we use the information in the ES about the sector (out of 37 categories) that each firm belongs to.

In the first set of exercises, we regress the growth rate of income per capita in the last 20 years, on the current fraction of plants from each of the 37 sectors. The residual growth rate for each country-year sample (once readjusted to preserve the mean of the raw growth rate of income per capita) is our net aggregate growth measure. We follow the same procedure to control for the sectoral composition in the average age of the plant. Row III of Table A3 shows that our qualitative results are similar to the benchmark estimates. Panel (a) documents that the fraction of firms with positive productivity growth correlates significantly with this residual aggregate growth rate of income per capita (0.19, similar to 0.18 in the benchmark exercise). The employment growth patterns correlate less strongly with the aggregate growth as it is the case for our results in Section II; though significant at 10 percent, as panels (c) and (d) show. Finally, average age across firms is negatively correlated with this measure of aggregate growth, with a coefficient that is only slightly less negative than the benchmark estimates (-0.40 as opposed to -0.46).

The exercises we report in row III do not modify the definition of the idiosyncratic growth of plants that we use in the main body of the manuscript. Nevertheless, one can also use the information on the sector of each firm to determine its productivity growth in the last years relative to that of the rest of the firms in the same sector in our cross-country

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[3] Notice that the other correlations relating the relationship between employment growth frequency and aggregate growth, and the average firm age and aggregate growth, are the same as in row I by construction.

sample. Accordingly, we complement the first set of exercises by defining the indicator of the productivity and employment growth relative to the median plant in the same broad sector in our sample. In row IV, we use these idiosyncratic measures of plant-level growth with the raw aggregate growth rates also used in Section II. We find that our results in Section II are also robust to these additional modifications in the empirical strategy. In particular, Panel (a) shows that the fraction of firms growing in productivity more than the world-wide sector-specific median correlates positively with the aggregate growth with a coefficient of 0.20 that is significant at the 5 percent level. Panel (b) shows that the fraction of firms growing in employment more so than the world-wide sector-specific median lacks significant correlation with the aggregate growth. Panel (c) validates this for the subsample of firms that have grown in their relative productivity.

Finally, Row V combines the exercises in rows III and IV, by using aggregate growth rate and average age conditional on current composition, and plant-level growth rates relative to the sector-specific median. Our qualitative results also go through in this exercise.[4]

Growth volatility and transitions. The theoretical model in our paper is built on the assumption that the countries under study are on a balanced growth path (BGP). This assumption is a compromise from reality in order to assure tractability in our model solution, and isolate the mechanisms of focus in our study.

There are at least two aspects that can be concerning about the assumption of BGP. First, our dataset spans countries at different stages of development which can play a role in shaping their growth rates. Second, there can be short run fluctuations in the growth rates that we might be interpreting as long run growth rates.

First, we try to see if the facts we highlight in Section II are robust to controlling for the level of countries' income. In order to do so, we compute residual growth after we control for the income per capita group of a country and the interaction between the income per capita group and the level of income per capita in 2000. In particular, we estimate

$$\bar{\gamma}_{it} = \alpha + \delta_g + \beta_g \log(y_{i2000}) + \epsilon_{it}$$

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[4] Repeating the robustness checks here by allocating plants into two broad sectors as Manufacturing and Services (instead of 37 more narrow sectors) leads to similar results.

Then, we shift the residuals (the error term) by the average raw growth rate, so that we maintain the same average in the residuals as the original growth rates. Row VI of Table A3 shows that the correlation patterns we highlight in Section II are robust to controlling for the initial income levels. Panel (a) shows that the correlation between the aggregate growth and the fraction of firms with positive productivity growth is similar to the ones in the benchmark exercise, and significant at 5 percent level. There is no significant correlation between employment growth patterns of plants and aggregate growth rates, as panels (b) and (c) document. Finally, the average plant age exhibits a significant correlation of -0.44 with the aggregate growth rate conditioned on the level of income, similar to the results we obtained with row aggregate growth rates.

Next, we include two more sets of robustness checks to clear our data from the countries with large idiosyncratic variation in the aggregate growth during the horizon of analysis. In particular, in our second exercise, we consider a subsample of country-year observations with relatively stable growth rates over the 20 years prior to the survey year. We call the growth rates stable if the standard deviation over the sample period is below the 90th percentile of the growth rate standard deviation distribution.[5] This filters country-year observations with a standard deviation of growth rate above 0.072.[6]

Row VII Panel (a) of Table A3 shows that the fraction of firms with productivity growth has a significant positive correlation of 0.19 with aggregate growth in income per capita, similar to the corresponding correlation of 0.18 that we have with the overall sample. Panel (b) documents that the fraction of firms with employment growth does not have a significant correlation with the aggregate growth, as is the case for our benchmark results. Panel (c) shows that this is also true if we only look at firms which experienced productivity growth. Finally, the last panel shows that the negative relationship between average firm age and the growth rate of income per capita is also present in this selected sample with lower variation in aggregate growth rates, with a corresponding correlation of -0.45.

In the second exercise to make our sample more consistent with the assumption of BGP, we omit country-year observations with relatively strong time trends in the aggregate growth

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[5] Doing the same exercise with top 5 percentile gives similar results.

[6] In the overall sample, the 5th, 25th, 50th percentile of the standard deviation in recent growth rates are 0.012, 0.023 and 0.033, respectively.

rates in the last 20 years. In particular, we regress observed growth rates on a time variable and a constant for each country. Among the cases with trend significantly different from zero at 5 percent level, we exclude economies that have a time trend in the top decile. This corresponds to a coefficient that is larger than 1.5 percentage points in absolute value. Row VIII Panel (a) of Table A3 shows that the highlighted empirical facts of Section II are robust to this selection of country-year observations. The correlation between the fraction of plants with employment growth and the growth rate of income per capita is slightly larger than the benchmark results with a coefficient of 0.24 that is significant at 5 percent level. The corresponding correlation for employment growth, not only for all the firms but also for those experiencing productivity growth, is also insignificant with this exercise (Panels (b) and (c)). Panel (d) shows that the average firm age has a correlation of -0.50, slightly stronger than the corresponding correlation of -0.46 that we document for the original sample.[7]

Overall, these robustness checks assure that our empirical results of Section II of the main body of the paper are not driven by a few cases with rapid transitions in their income levels, or large fluctuations in the growth rates.

B Appendix: Proofs

Proposition 1 (Proof). Using the flow of profits for non-innovative for T periods, which by definition accounts for the number of firms left to operate, we can get:

$$\begin{aligned} V_N(z,w) &= \sum_{t=0}^{T-1} \left(\frac{1-\delta}{R}\right)^t (1-\theta)\theta^{\frac{\theta}{1-\theta}} \frac{z}{(w\gamma^t)^{\frac{\theta}{1-\theta}}} - \sum_{t=0}^{T-1} f_N \left(\frac{1-\delta}{R}\right)^t w\gamma^t \\ &= \underbrace{(1-\theta)\theta^{\frac{\theta}{1-\theta}} \frac{1-\left(\frac{1-\delta}{R\gamma^{\frac{\theta}{1-\theta}}}\right)^T}{1-\frac{1-\delta}{R\gamma^{\frac{\theta}{1-\theta}}}}}_{B_{NT}} \frac{z}{w^{\frac{\theta}{1-\theta}}} - \underbrace{f_N \frac{1-\left(\frac{1-\delta}{R}\gamma\right)^T}{1-\frac{1-\delta}{R}\gamma}}_{D_{NT}} w, \end{aligned}$$

Hence, the value function can be represented as $V_N(z,w) = B_{NT}\frac{z}{w^{\frac{\theta}{1-\theta}}} - D_{NT}w$.

Differently from the value of a non-innovative firm, the one of an innovative firm exhibits

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[7] Doing the same exercise with top 5 and 25 percentiles gives similar results.

an infinite horizon. The value function satisfies:

$$V_I(z,w) = \max_{\phi\geq 1}(1-\theta)\theta^{\frac{\theta}{1-\theta}}\frac{z}{w^{\frac{\theta}{1-\theta}}} - c\frac{(\phi^\tau - 1)z}{w^{\frac{\theta}{1-\theta}}} - f_I w + \frac{1-\delta}{R}[qV_I(\phi z, w') + (1-q)V_I(z,w')].$$

Using the guess that $V_I(z,w) = B_I\frac{z}{w^{\frac{\theta}{1-\theta}}} - D_I w$, and the FOC for investment, we get

$$B_I = (1-\theta)\theta^{\frac{\theta}{1-\theta}} - c(1-\tau)\left[\frac{\frac{1-\delta}{R}qB_I}{c\tau\gamma^{\frac{\theta}{1-\theta}}}\right]^{\frac{\tau}{\tau-1}} + c + \frac{\frac{1-\delta}{R}(1-q)B_I}{\gamma^{\frac{\theta}{1-\theta}}}.$$

Since the value function solves an infinite horizon problem, the fixed cost component of the value function satisfies:

$$D_I w = \sum_{t=0}^{\infty}\left(\frac{1-\delta}{R}\right)^t f_I w = \frac{f_I}{1-\frac{1-\delta}{R}\gamma}w,$$

which verifies that $D_I = \frac{f_I}{1-\frac{1-\delta}{R}\gamma}$. ■

Proposition 2 (Proof). We first argue that $B_I > B_N$. The coefficient B_I satisfies:

$$B_I\frac{z}{w^{\frac{\theta}{1-\theta}}} = \max_{\phi\geq 1}(1-\theta)\theta^{\frac{\theta}{1-\theta}}\frac{z}{w^{\frac{\theta}{1-\theta}}} - c\frac{(\phi^\tau - 1)z}{w^{\frac{\theta}{1-\theta}}} + \frac{1-\delta}{R}[qB_I\frac{z\phi}{w^{\frac{\theta}{1-\theta}}\gamma^{\frac{\theta}{1-\theta}}} + (1-q)B_I\frac{z}{w^{\frac{\theta}{1-\theta}}\gamma^{\frac{\theta}{1-\theta}}}].$$

This implies that, if the optimal investment rule is interior (i.e. $\phi > 1$) we have:

$$B_I\frac{z}{w^{\frac{\theta}{1-\theta}}} > (1-\theta)\theta^{\frac{\theta}{1-\theta}}\frac{z}{w^{\frac{\theta}{1-\theta}}} + \frac{1-\delta}{R}B_I\frac{z}{w^{\frac{\theta}{1-\theta}}\gamma^{\frac{\theta}{1-\theta}}}.$$

$$B_I > \frac{(1-\theta)\theta^{\frac{\theta}{1-\theta}}}{1-\frac{1-\delta}{R\gamma^{\frac{\theta}{1-\theta}}}} = B_N.$$

We also know that $D_I > D_N$. Hence, there exists $\hat{z} > 0$ such that:

$$V_I(\hat{z},w) = B_I\frac{\hat{z}}{w^{\frac{\theta}{1-\theta}}} - D_I w = B_N\frac{\hat{z}}{w^{\frac{\theta}{1-\theta}}} - D_N w = V_N(\hat{z},w).$$

Define the function:

$$s(z,w) \equiv -V_I(z,w) + V_N(z,w) = -(B_I - B_N)\frac{z}{w^{\frac{\theta}{1-\theta}}} + (D_I - D_N)w$$

to denote the surplus obtained from a possible transaction between an exiting innovative firm and an entrant non-innovative firm. Notice that $s(\hat{z},w) = 0$. Moreover, since $B_I - B_N > 0$ and $D_I > D_N$, for any $z > \hat{z}$:

$$s(z,w) = -(B_I - B_N)\frac{z}{w^{\frac{\theta}{1-\theta}}} + (D_I - D_N)w < -(B_I - B_N)\frac{\hat{z}}{w^{\frac{\theta}{1-\theta}}} + (D_I - D_N)w = 0,$$

and for any $z < \hat{z}$:

$$s(z,w) = -(B_I - B_N)\frac{z}{w^{\frac{\theta}{1-\theta}}} + (D_I - D_N)w > -(B_I - B_N)\frac{\hat{z}}{w^{\frac{\theta}{1-\theta}}} + (D_I - D_N)w = 0.$$

Finally, $s(z,w)$ is strictly increasing in w for any $z > 0$. In particular, $\lim\limits_{w\to\infty} s(z,w) > 0$, and $\lim\limits_{w\to 0} s(z,w) = \infty$. Hence, for any $z > \hat{z}$, there exists $\bar{t} \in (1,\infty)$ such that, $s(z, w\gamma^{\bar{t}}) = 0$. Thus, any innovative firm will exit in finitely many periods. ■

Proposition 3 (Proof). Define: $X_I \equiv E_I(z)$, $X_N \equiv E_N(z)$. Also define the cumulative distribution function for the group I as:

$$F_I(z) = \begin{cases} 1 - \left(\frac{\hat{z}}{z}\right)^{\lambda}, & \text{if } z \geq \hat{z}; \\ 0, & \text{o/w.} \end{cases}$$

Moreover, let M_I and M_N be the number of firms in each group, which will be constant in the BGP, let $\alpha \equiv \frac{M_I}{M_I + M_N}$. To show that $\phi = \mu$ we first show that $\mu \leq \phi$ along the BGP. Define $\bar{z} \equiv \max\{\hat{z}, \frac{\mu}{\phi}\hat{z}\}$, and $a \equiv \frac{\hat{z}}{\bar{z}}$. Then

$$X_I' = (1-\delta)q\int_{\bar{z}}^{\infty}(\phi z)\frac{\lambda\hat{z}^{\lambda}}{z^{\lambda+1}}dz + (1-\delta)(1-q)\int_{\hat{z}'}^{\infty} z\frac{\lambda\hat{z}^{\lambda}}{z^{\lambda+1}}dz$$

$$+[\delta + (1-\delta)qF_I(\bar{z}) + (1-\delta)(1-q)F_I(\hat{z}')]X_I'$$

$$= \frac{q\int_{\bar{z}}^{\infty}(\phi z)\frac{\lambda\hat{z}^{\lambda}}{z^{\lambda+1}}dz + (1-q)\int_{\hat{z}'}^{\infty} z\frac{\lambda\hat{z}^{\lambda}}{z^{\lambda+1}}dz}{1 - qF_I(\bar{z}) - (1-q)F_I(\hat{z}')} = \frac{q\left(\frac{\phi}{\mu}\right)a^{\lambda-1} + (1-q)\mu^{-\lambda}}{qa^{\lambda} + (1-q)\mu^{-\lambda}}\mu\frac{\lambda}{\lambda-1}\hat{z}$$

For non-innovative firms we need to keep track of all the endogenous exits from the innovative group for the last $\hat{t}$ periods. Define:

$$k(\check{z}) \equiv q\int_{\check{z}}^{\check{z}/a}(\phi z)\frac{\lambda \check{z}^{\lambda}}{z^{\lambda+1}}dz + (1-q)\int_{\check{z}}^{\mu\check{z}} z\frac{\lambda\check{z}^{\lambda}}{z^{\lambda+1}}dz = q\frac{\lambda}{\lambda-1}\phi[1-a^{\lambda-1}]\check{z} + (1-q)\frac{\lambda}{\lambda-1}[1-\mu^{1-\lambda}]\check{z}$$

Then:

$$\begin{aligned} X_N &= (1-\delta)^{\hat{t}}k(\hat{z}\mu^{-\hat{t}}) + (1-\delta)^{\hat{t}-1}k(\hat{z}\mu^{-\hat{t}+1}) + .. + (1-\delta)k(\frac{\hat{z}}{\mu}) \\ &= \left(q\frac{\lambda}{\lambda-1}\phi[1-a^{\lambda-1}] + (1-q)\frac{\lambda}{\lambda-1}[1-\mu^{1-\lambda}]\right)(1-\delta)\mu^{-1}\frac{1-(1-\delta)^{\hat{t}}\mu^{-\hat{t}}}{1-(1-\delta)\mu^{-1}}\hat{z} \end{aligned}$$

Moreover:

$$\gamma = \frac{w'}{w} = \left(\frac{\alpha X_I' + (1-\alpha)X_N'}{\alpha X_I + (1-\alpha)X_N}\right)^{1-\theta} =$$

$$= \left(\frac{\alpha\frac{q\left(\frac{\phi}{\mu}\right)a^{\lambda-1}+(1-q)\mu^{-\lambda}}{qa^{\lambda}+(1-q)\mu^{-\lambda}}\mu^{\hat{t}} + (1-\alpha)\left(q\phi[1-a^{\lambda-1}] + (1-q)[1-\mu^{1-\lambda}]\right)\left(\sum_{t=0}^{\hat{t}-1}(1-\delta)^{t+1}\mu^{-t}\right)}{\alpha\mu^{\hat{t}} + (1-\alpha)\left(q\phi[1-a^{\lambda-1}] + (1-q)[1-\mu^{1-\lambda}]\right)\left(\sum_{t=0}^{\hat{t}-1}(1-\delta)^{t+1}\mu^{-t}\right)}\mu\right)^{1-\theta}$$

Since $\gamma = \mu^{1-\theta}$:

$$q\left(\frac{\phi}{\mu}\right)a^{\lambda-1} + (1-q)\mu^{-\lambda} = qa^{\lambda} + (1-q)\mu^{-\lambda}$$

Which gives

$$\frac{\phi}{\mu} = a \leq 1 \Rightarrow \phi \leq \mu$$

We prove $\mu \leq \phi$ by contradiction. Suppose that $\mu > \phi$, i.e., threshold productivity grows at a faster rate than the productivity growth of the successful innovative firms. Then, a measure $F_I(\hat{z}\frac{\mu}{\phi};t) - F_I(\hat{z};t)$ innovative firms will not be able to stay in the market even if they successfully increase their productivity. This implies that these firms would not invest in productivity growth, since the increments for the exiting firms are nullified by assumption. Then the value of a firm with productivity $z \in [\hat{z}, \hat{z}\frac{\mu}{\phi}]$ is:

$$V^I(z,w) = (1-\theta)\theta^{\frac{\theta}{1-\theta}}\frac{z}{w^{\frac{\theta}{1-\theta}}} - f_I w + \frac{1-\delta}{R}V^I(z,w\gamma)$$

Since the firm endogenously exits next period, we have: $V^I(z,w\gamma) \leq V^N(z,w\gamma)$.

Then, using also $f_I > f_N$, we get:

$$\begin{aligned} V^N(z,w) &= (1-\theta)\theta^{\frac{\theta}{1-\theta}}\frac{z}{w^{\frac{\theta}{1-\theta}}} - f_N w + \frac{1-\delta}{R}V^N(z,w\gamma) \\ &> (1-\theta)\theta^{\frac{\theta}{1-\theta}}\frac{z}{w^{\frac{\theta}{1-\theta}}} - f_I w + \frac{1-\delta}{R}V^I(z,w\gamma) = V^I(z,w) \end{aligned}$$

This contradicts $\hat{z} < z$ being the threshold productivity level for innovating firms; hence, shows that $\phi = \mu$. To prove the second part, notice that the distribution next period, given the current distribution is:

$$\begin{aligned} F_I'(z) &= (1-\delta)\,q\left[F_I(\frac{z}{\phi}) - F_I(\hat{z}\frac{\mu}{\phi})\right] + (1-\delta)\,(1-q)\,[F_I(z) - F_I(\hat{z}\mu)] + \\ & G(z)[\delta + (1-\delta)(qF_I(\hat{z}\frac{\mu}{\phi}) + (1-q)\,F_I(\hat{z}\mu))] \end{aligned}$$

Because entrants are drawn from the equilibrium distribution, $G(z) = F_I'(z)$ and the initial distribution is assumed to be Pareto:

$$\begin{aligned} F_I'(z) &= (1-\delta)\,q\left[\left(1-\left(\phi\frac{\hat{z}}{z}\right)^{\lambda}\right) - F_I(\hat{z})\right] + (1-\delta)\,(1-q)\left[1-\left(\frac{\hat{z}}{z}\right)^{\lambda} - F_I(\mu\hat{z})\right] + \\ & \left(1-\left(\mu\frac{\hat{z}}{z}\right)^{\lambda}\right)[\delta + (1-\delta)(qF_I(\hat{z}) + (1-q)\,F_I(\mu\hat{z})] \end{aligned}$$

Since $\phi = \mu$ and $F_I(\hat{z}) = 0$, we get: $F_I'(z) = 1 - \left(\mu\frac{\hat{z}}{z}\right)^{\lambda}$ which shows that the distribution next period is Pareto with threshold $\hat{z}\mu$. ■

Proposition 4 (Proof). Define $m(\check{z}) \equiv (1-q)\int_{\check{z}}^{\mu\check{z}} \frac{\lambda\check{z}^{\lambda}}{z^{\lambda+1}}dz M_I = (1-q)[1-\mu^{-\lambda}]M_I$ then

$$M_N = (1-\delta)^{\hat{t}}m(\hat{z}\mu^{-\hat{t}}) + (1-\delta)^{\hat{t}-1}m(\hat{z}\mu^{-\hat{t}+1}) + .. + (1-\delta)m(\frac{\hat{z}}{\mu})$$

$$M_N = (1-q)[1-\mu^{-\lambda}]\left(\sum_{t=0}^{\hat{t}-1}(1-\delta)^{t+1}\right)M_I = (1-q)[1-\mu^{-\lambda}]\frac{1-\delta}{\delta}[1-(1-\delta)^{\hat{t}}]M_I$$

$$(1-\alpha) = \alpha(1-q)[1-\mu^{-\lambda}]\frac{1-\delta}{\delta}[1-(1-\delta)^{\hat{t}}],$$

which proves the claim since $\hat{t}$ is constant. In order to show that the total measure of firms

(M) is constant, notice that Proposition 3 implies:

$$X_I = \frac{\lambda}{\lambda - 1}\hat{z}, \qquad X_N = (1-q)\frac{\lambda}{\lambda-1}[1-\mu^{1-\lambda}](1-\delta)\mu^{-1}\frac{1-(1-\delta)^{\hat{t}}\mu^{-\hat{t}}}{1-(1-\delta)\mu^{-1}}\hat{z}$$

From the exit condition of group N, we know that: $(1-\theta)\theta^{\frac{\theta}{1-\theta}}\tilde{z} = f_N w^{\frac{1}{1-\theta}}$, where

$$w = \frac{\theta}{(1-(\alpha f_I + (1-\alpha) f_N)M)^{1-\theta}} \left(\alpha M X_I + (1-\alpha) M X_N\right)^{1-\theta}$$

Hence,

$$(1-\theta)\theta^{\frac{\theta}{1-\theta}} = \tfrac{\theta^{\frac{1}{1-\theta}} f_N M}{(1-(\alpha f_I+(1-\alpha)f_N)M)} \tfrac{\lambda}{\lambda-1}\left(\alpha + (1-\alpha)(1-q)[1-\mu^{1-\lambda}](1-\delta)\mu^{-1}\tfrac{1-(1-\delta)^{\hat{t}}\mu^{-\hat{t}}}{1-(1-\delta)\mu^{-1}}\right)\mu^{-\hat{t}}$$

This gives us the equation for M, given two constant endogenous objects α and $\hat{t}$. Hence, M is constant along the BGP. ■

C Appendix: Robustness checks for the quantitative implications of the model

C.A Curvature in the profit function

We calibrate two alternative economies in which the parameter θ is estimated from the data. To do it, we construct country-specific estimates which are then averaged across countries and follow two alternative approaches.

First, we estimate the labor share computing the ratio of labor payments relative to sales, $\frac{w(l+f_I)}{y}$. Because overhead costs differ for innovative and non-innovative firms, we consider only labor payments across the top 10th percentile of the employment distribution (as we do when we identify the probability of success) under the assumption that these correspond to innovative firms. Unfortunately, we don't have direct measures of overhead costs in the harmonized ES dataset. Hence, we use a common overhead cost across countries corresponding to the calibrated parameter in our benchmark economy.[8] This yields an

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[8] To be specific, for each firm, we estimate a share of productive labor as $\frac{\text{Employment-1}}{\text{Employment}}\frac{\text{Labor payments}}{\text{Sales}}$.

average estimate across countries of $\theta = 0.786$.

Second, we allow for the technology of production for firms in the innovative sector to display decreasing returns. If returns to scale are summarized by a span of control as in Lucas (1978), ζ, and the production technology is $y = z^{\varsigma} l^{\theta} = (z l^{\frac{\theta}{\varsigma}})^{\varsigma}$, the labor share in the economy corresponds to $\frac{\theta}{\zeta}$. Therefore, the parameter θ can be inferred from the product of the labor share and the estimate of the span of control, $\frac{w(l+f_I)+rk}{y} = \varsigma$ where rk is the cost of capital. Note that while in the model firms only use labor inputs for production, in the data both labor and capital are observable inputs into firms' value added. We can then identify both the labor share and the span-of-control by computing ratios of factor payments and sales. In particular, $\frac{w(l+f_I)}{y} = \theta$ and $\frac{w(l+f_I)+rk}{y} = \varsigma$. Our estimate for the parameter of interest is below the share used in the benchmark economy ($\theta = 0.57$ in the data).

Table A4 shows that the predictions of the model in terms of the variation in growth rates and the average age of firms across countries are robust to these changes. Our benchmark economy explains 67% of the variation in growth rates, in between the measure explained by the economy with constant returns to scale and estimated labor share (72%) and that with decreasing returns to scale (64%).

Table A4: Robustness, parameter choices

	Benchmark	θ Estimates		Cost function	
		DRS	CRS	$\tau = 1.2$	$\tau = 5$
sample size*	126	94	111	116	111
		Parameters			
θ	0.66	0.57	0.79	0.66	0.66
f_N	4	4.3	3.5	4	4
c	0.17	0.28	0.06	0.31	0.05
δ	5.09%	5.11%	4.56%	5.17%	4.80%
		Moments			
mean growth	3.09%	3.04%	3.03%	3.09%	3.07%
mean age	16.09	16.14	16.40	16.32	16.37
		Goodness of Fit			
S^2_{γ}	0.67	0.64	0.72	0.67	0.73
corr$(\gamma, \gamma_{\text{model}})$	0.23	0.24	0.14	0.24	0.20
corr$(age, age_{\text{model}})$	0.19	0.19	0.21	0.17	0.19

Note: We compare the benchmark economy to four alternative ones. In columns two and three, we estimate the curvature of the profit function of innovative firms, as determined by θ from the ES data. In the second column, we report estimates for the labor share, $\theta = 0.786$. In the third column we report estimates assuming decreasing returns to scale, so that $\theta = 0.56$ combines the labor share and the span of control. In the fourth and fifth columns we use alternative specifications for the curvature in the innovation cost function. *sample size corresponds to the number of countries for which we are able to solve for the equilibrium allocation.

C.B Curvature in the cost function

In the model, the main source of variation in growth rates across countries is firms' incentive to innovate. These incentives depend on the level of uncertainty that firms face, but also on the costs of innovation. In our benchmark calibration, the cost function was assumed quadratic. Here, we assume that the cost function is either close to linear, $\tau = 1.2$ or more convex than in our benchmark calibration, $\tau = 5$.

Table A4 shows that the predictions of the model in terms of the variation in growth rates and the average age of firms across countries are robust to these changes. Our benchmark economy explains 67.3% of the variation in growth rates, in between the measure explained by the economies with a linear cost function (66.9%) and that with higher cost curvature (73%).

C.C Aggregate growth rates

At the end of Appendix A, we discuss the robustness of our empirical results of Section II to alternative exercises that (i) control for the role of income levels in shaping aggregate growth, (ii) cleans the data from country-year observations with large variations and trends in the recent growth rates. Here, we show the robustness of our model performance to these alternative approaches to the data.

Table A5 reports our findings. Importantly, the calibrated parameters and the model fit to the data, are similar to the benchmark results highlighted in Section IV. Especially the calibrations in which we consider the residual growth rates and low volatility in growth rates, resemble closely the results from the benchmark exercise. This suggests that our results are not simple artifacts of the sample of countries under analysis, nor of their stages in the development path.[9]

When we focus on a set of country-year observations without strong time-trends, we find that the goodness of fit of the model to the data improves under every measure. In that sense, we view our results as conservative.

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[9] Our results from the extension with the intensive margin shocks to investment returns (Section IV.D) are also robust to using these alternative filtering methods for the growth rates. We omit the numerical results from these computations for the sake of brevity.

Table A5: Robustness, growth rate estimations

	Benchmark	Growth Estimates		
		Income controls VI	Low volatility VII	No-time trends VIII
sample size*	126	105	113	92
		Parameters		
c	0.17	0.17	0.18	0.17
δ	5.09%	5.03%	5.02%	5.07%
		Moments		
mean growth	3.09%	3.06%	2.93%	3.17%
mean age	16.09	16.35	16.40	16.16
		Goodness of Fit		
S_γ^2	0.67	0.69	0.65	0.71
corr($\gamma, \gamma_{\text{model}}$)	0.23	0.25	0.20	0.33
corr(age, age_{model})	0.19	0.19	0.16	0.26

Note: We compare the benchmark economy to three alternative ones. In the second column we use the residual growth rate after regressing the growth rates used in the benchmark on income per capita group dummies and the interaction of these with the level of the income per capita in 2000. In the third column, we exclude country-year observations whose growth rate variation in the last 20 years exceed top 10 percent in the sample. In the fourth column, we exclude countries with a significant time trend in growth rates in the last 20 years. *sample size corresponds to the number of countries for which we are able to solve for the equilibrium allocation.

C.D Capital accumulation and sectoral composition

Our robustness checks in Appendix A for the empirical results of Section II included accounting for the role of capital accumulation in aggregate growth, and for the role sectoral composition in our firm-level measures and aggregate growth. Here, we show that using the corresponding measures we obtained in the Appendix Section A in these two sets of exercises to quantify the model performance gives results similar to the ones highlighted for our benchmark calibration.

For capital, we followed two alternative approaches to the data. In the first one, we computed "net" measures of aggregate income growth by taking out the growth rate of capital (multiplied by the capital share) in the last 20 years. In the second one, we also adjusted the firm-level growth productivity measure by subtracting the country-specific capital growth rate (also multiplied by the capital share) from the firms' productivity growth. Columns I and II of Table A6 shows that feeding these measures of aggregate and firm-level growth into our quantitative model does not affect the model performance in matching the observed aggregate growth rates in the data.

Table A6: Robustness, capital and sectors

	Benchmark	Capital		Sectoral Composition		
		I.	II.	III.	IV.	V.
Sample Size	126	47	47	124	110	110
				Parameters		
c	0.17	0.12	0.12	0.17	0.13	0.13
δ	5.09%	5.08%	5.04%	5.10%	5.39%	5.32%
				Moments		
mean growth	3.09%	6.27%	6.37%	3.13%	3.09%	3.18%
mean age	16.1	14.27	14.27	16.0	15.9	15.9
				Goodness of Fit		
S^2	0.67	0.66	0.67	0.71	0.66	0.66
corr γ	0.23	0.28	0.41	0.25	0.30	0.32
corr age	0.19	-0.03	-0.03	0.25	0.16	0.33

Note: We compare the benchmark numbers for the model fit to those in five alternative approaches to the data. Column I uses (for the data) the growth rate of income per capita after subtracting 0.33 times the growth rate of capital per capita in the time horizon of focus. Column II complements this analysis by defining firm productivity growth as increasing productivity by more than 0.33 times the growth rate of capital per worker. Column III uses the growth rate of output per capita after controlling for the sectoral composition of plants in our ES samples. Column IV uses the unconditional aggregate growth rates, but defines the plant-level growth measures as increments relative to the world-wide sector-specific medians. Column V combines the exercises in rows III and IV. See the text for further details.

In columns III, IV and V, we use the variables computed in Section A to control for the role of sectoral composition. Our results of model performance prove robust to using aggregate growth rates conditional on the sectoral composition (Column III), measures of firm growth relative to the world-wide sector-specific median (Column IV), and using the latter two computations simultaneously (Column V).

C.E Introducing country-specific innovation costs

In our benchmark calibration, we assume that innovation costs (c) are the same across economies. The uniform cost of investing in technology is a simplification that allows us to better highlight the role of the probability success in shaping aggregate growth. We can complement our benchmark results by computing the implied innovation costs that would rationalize each country's growth rate given the estimated probability of success (q) and the remaining benchmark calibrated parameters. As shown in Figure A1, we find a strong negative correlation between the implicit costs of adoption and the observed growth rates, with a correlation coefficient of -0.67. These results are consistent with previous work by Parente and Prescott (1994), who argue that institutional arrangements and barriers to

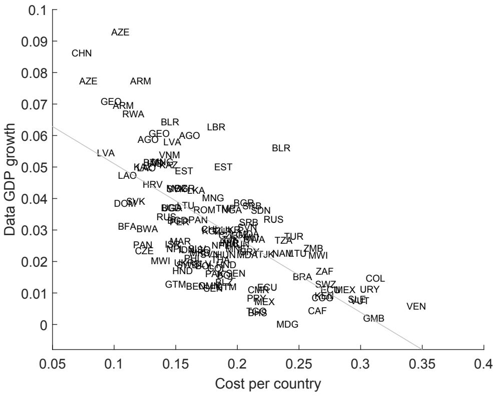

Figure A1: Aggregate growth and implied innovation costs

Note: Implied innovation costs that rationalize observed growth rates across countries given the probability of success q estimated from the data.

adoption may account for a substantial portion of the difference in levels of income per capita.

D Possible factors behind the probability of success

This paper studies the probability of success in productivity investment as a structural parameter. However, this probability is likely to be determined by technological or institutional factors. In this section, we discuss a few characteristics that may yield disparate probabilities of success across countries and time. Specifically, we run Probit regressions on the indicator of increased plant-level labor productivity as a dependent variable, and a number of candidate aggregate indicators of economic infrastructure and business friendliness of institutions as explanatory variables – separately and jointly.[10]

Technological infrastructure. One way to increase firm-specific labor productivity is innovation, and in economies with poor technological infrastructure the likelihood of success in innovation can be lower. To capture the quality of the infrastructure of an economy we use three alternative measures. First, we use the value lost due to electrical shortages as a percentage of sales to measure the reliability of access to electricity. Second, we use the fraction of the labor force with tertiary education to capture the human capital in a country. Notice that better educated workers might imply faster on-the-job learning, giving a higher

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[10] The equation we use to this estimation is $P(z_{ijt+1} > z_{ijt}|X_{it}) = \Phi(X_{it}'\beta)$, where i denotes a plant, j denotes a country and t is a time index. Notice that the explanatory variables are country-specific.

probability of increases in workers' productivity. Finally, we control for the level of R&D-expenditure in a country (as a percent of the GDP). The results of these estimations are given in Table A7 of Appendix D.[11] Columns (1) to (3) show that the measures that we use to capture the quality of infrastructure can have a significant impact on the probability of success. The marginal effects indicate that 1 percentage point increase in the average losses due to electric outages (relative to sales, %) decreases the probability of success by 0.45 percentage points. One percentage point increase in the share of the labor force with tertiary education is associated to 0.4 percentage point increase in the probability of success. Column (7) documents that the effects of these two variables remain significant when all the indicators in this section are included in the regression. As Column (8) shows, this is robust to including the available plant level characteristics in the regressions, specifically the gender of the owner, the experience of the manager and the age of the firm. R&D expenditure as a percentage of GDP has a significant effect in the individual regression reported in Column (3) but not once we filter out the effect of the other characteristics.

Institutions. Firms' rents may easily be ceased in countries with an ineffective judicial system, undeveloped protection of investors' rights, or with prevalent corruption. We use three variables to capture the business-friendliness of the institutional environment. In the ES dataset, respondents are asked the extent to which courts and corruption present an obstacle for their business. We use the country-year specific average of these responses to capture the relationship of the quality of the judicial system and corruption. We also use *Investor protection index*, available in the Doing Business database by the World Bank. This index measures how well-protected investors are against the misuse of their funds by others, including firm employees.[12]

We find that in countries where courts are more often categorized as an obstacle, the firms' probability of success is lower. To be specific, for a one level (out of four) increase

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[11] Value loss due to outages is from the ES database. We average responses across establishments to generate country-year specific measures. Data on the fraction of the labor force with tertiary education, and on the R&D expenditure are provided by the World Development Indicators. It is likely that high probability of success would imply more resources being allocated to R&D. Nevertheless, the aggregate measures that we use are not likely to be affected by a firm-specific innovative activity.

[12] Obstacle measures in the ES take values 0 to 4 for each plant, 4 meaning a severe obstacle. *Investor protection index* taxes values from 0 to 10, 10 being the best outcome.

in the severity of this problem, the probability of success decreases by 6 percentage points. Better investor protection relates positively to probability of success. The implied effect of a 1 point increase in the investor protection index is a 2 percentage points increase in the probability of success. Differences in investor protection significantly explain disparities in the probability of success even when all factors are considered together (columns (7) and (8)). In contrast, we find that the marginal effect of corruption on the probability of success is not significant (columns (3), (7) and (8)).

References

Lucas, R. E. (1978, Autumn). On the Size Distribution of Business Firms. *Bell Journal of Economics 9*(2), 508–523.

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Table A7: Probability of success and aggregate characteristics

	(1)	(2)	(3)	(4)	(5)	(6)	(7)	(8)
Value lost due to outages (% of sales)	-0.00450**						-0.0138***	-0.0136***
	(-2.476)						(-3.028)	(-3.102)
Labor force with tertiary edu (%)		0.00437***					0.00265*	0.00268*
		(2.865)					(1.866)	(1.889)
R&D expenditure (% of GDP)			0.0986***				0.0547	0.0559
			(3.333)				(1.454)	(1.505)
Courts obstacle				-0.0578**			-0.0111	-0.0122
				(-2.427)			(-0.306)	(-0.339)
Investor protection index					0.0237***		0.0213**	0.0199**
					(3.350)		(2.041)	(2.006)
Corruption obstacle						-0.0193	0.0596	0.0625
						(-1.287)	(1.535)	(1.627)
Individual characteristics	No	No	No	No	No	No	No	Yes
Log pseudolikelihood	-82.2	-37.7	-30.7	-82.9	-79.4	-83.6	-19.5	-19.2
Observations	57,674	22,485	27,956	57,813	49,769	59,454	14,909	14,647

Robust z-statistics in parentheses
*** p<0.01, ** p<0.05, * p<0.1

Note: The table reports the marginal effects estimated from a Probit model. Marginal effects are computed at the mean value of each explanatory variable. It also shows the McFadden's Pseudo R-squared statistic. Value loss due to outages (as a percentage of sales) is collected at the plant level and we average across establishments to construct a country-year specific value. Obstacle measures are constructed by averaging within a country-year the responses of establishments regarding how big of an obstacle (0 to 4, 4 meaning very severe) an issue is for their operations. *Investor protection index* is a qualitative index taking values between 0 to 10, 10 being the best outcome. Only the last column include characteristics at the individual plant level, specifically the gender of the owner, manager's experience and the age of the firm. In that estimation only the dummy for having a female owner is significant, with a coefficient of -0.04 and a z-statistic of -2.3.