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C Omitted Proofs

C.1 Proof of Lemma A7

Proof. For implementability of $z_i(\cdot)$ through some $x_P(\theta_P, \theta_D) + x_D(\theta_D, \theta_P) \le X$ we invoke Theorem 3 in Border (2007). The conditions are as follows.

For every message $m \in \{1, K\}$, let $m^c := \{k \in \{1, K\} | k \neq m\}$ be its complement. Further, let $p(1) \equiv p$ and $p(K) \equiv (1 - p)$. Fix some γ and non-negative z_i for every i. Then there exists an ex-post feasible x_i (i.e. $x_i(\theta_i, \theta_{-i}) \in [0, X]$ and $x_P(\theta_P, \theta_D) + x_D(\theta_D, \theta_P) \le X$) that implements z_i if and only if the following constraints are satisfied:

- $\forall m, n \in \{1, K\}$:

$$p(m)z_i(m) + p(n)z_{-i}(n) \le \tag{EPI}$$

$$X(1 - Pr(L)) - X(1 - \gamma(m^c, n^c))p(m^c)p(n^c)$$

- $\forall m, i$:

$$z_i(m) \le X(1 - \gamma_i(m)). \tag{IF}$$

Plugging in the values at the optimum from Section 3 verifies the inequalities.

If condition (M) is violated, the equilibrium is no-longer monotonic. Instead, overlapping strategies may be possible: If, e.g., $b_P(1)K < b_P(K)$ the likelihood of meeting a low-cost type for $\theta_D = K$ is too high compared to that of $\theta_D = 1$. $\theta_D = K$ has strong incentives to *provide more evidence* than $\theta_D = 1$. Further, because belief systems are consistent, whenever $\theta_D = K$ faces a $\theta_P = 1$, that low-cost type (rationally) expects to face $\theta_D = K$ with large probability. This provides an incentive for $\theta_D = K$ to compete more aggressively and for $\theta_P = 1$ to compete softer than under condition (M). The equilibrium strategy support in the non-monotonic equilibrium is depicted in Figure 8. $\theta_D = 1$ and $\theta_D = K$ overlap on the middle interval but are otherwise “close to monotonic”. $\theta_P = K$’s support covers the whole interval, $\theta_P = 1$ only competes on the middle interval. In addition, a high-cost Defendant also has a mass point at 0.

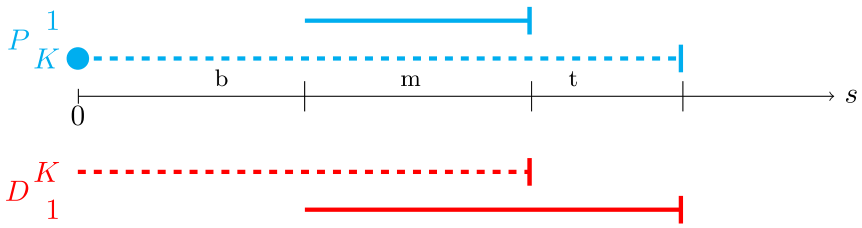

Figure 8: **Strategy support of P and D if monotonicity fails.**

Inside the space of non-monotonic equilibria there is no interior solutions for the same reasons as in Appendix A. The designer picks $b_P(1)$ equal to any discontinuity point or at the respective borders. That is, either $b_P(1) = 0$ or $b_P(1) = \max\{b_D(1), b_P(K)/K\}$. If $b_P(1) = b_D(1) = \rho_i$ under non-monotonicity, the first-order condition of the designer’s problem is monotone in ρ_i, requiring $\rho_i = 0$ which is never optimal. If $b_P(1) = b_P(K)/K$ utilities converge to their monotone counterparts and thus, the solution is no different than that for monotonicity. Finally, $b_P(1) = 0$ is never optimal as the objective is always decreasing at this point. $\square$

C.2 Proof of Lemma B1

Proof. The proof follows Siegel (2014). We omit proving uniqueness and the following properties: (i) the equilibrium is in mixed strategies, (ii) the equilibrium support of both disputants shares a common upper bound, and (iii) the equilibrium support is convex and at most one disputant has a mass point which is at 0. All arguments apply exactly as in Siegel (2014).

Each disputant θ_i holds belief $b_i(\theta_i)$, and maximizes

$$(1 - b_i(\theta_i))\, XF^K_{-i}(a) + b_i(\theta_i) XF^1_{-i}(a) - a\theta_i,$$

over a. Define the partitions $I_1 = (0, \overline{a}^K_D]$, $I_2 = (\overline{a}^K_D, \overline{a}^K_P]$ and $I_3 = (\overline{a}^K_P, \overline{a}^1_P]$. We define indicator functions $\mathbb{1}_{\in I_l}$ with value 1 if $a \in I_l$ and 0 otherwise. Similar the indicator function $\mathbb{1}_{>I_l}$ takes value 1 if $a > \max I_l$ and 0 otherwise. Disputant θ_i mixes such that the opponent's first-order condition holds on the joint support. The densities are

$$f^1_D(a) = \mathbb{1}_{\in I_2} \frac{K}{Xb_P(K)} + \mathbb{1}_{\in I_3} \frac{1}{Xb_P(1)}, \quad f^K_D(a) = \mathbb{1}_{\in I_1} \frac{K}{X(1 - b_P(K))},$$
$$f^1_P(a) = \mathbb{1}_{\in I_3} \frac{1}{Xb_D(1)}, \qquad f^K_P(a) = \mathbb{1}_{\in I_1} \frac{K}{X(1 - b_D(K))} + \mathbb{1}_{\in I_2} \frac{1}{X(1 - b_D(1))}.$$

This leads to the following cumulative distribution functions:

$$\begin{aligned}
F^1_D(a) &= \mathbb{1}_{\in I_2} a \frac{K}{Xb_P(K)} + \mathbb{1}_{\in I_3} \left(\frac{a}{Xb_P(1)} + F^1_D(\overline{a}^K_D) \right) + \mathbb{1}_{>I_3}, \\
F^K_D(a) &= \mathbb{1}_{\in I_1} a \frac{K}{X(1 - b_P(K))} + \mathbb{1}_{>I_1}, \\
F^1_P(a) &= \mathbb{1}_{\in I_3} \frac{a}{Xb_D(1)} + \mathbb{1}_{>I_3}, \\
F^K_P(a) &= \mathbb{1}_{\in I_1} \left(a \frac{K}{X(1 - b_D(K))} + F^K_P(0) \right) + \mathbb{1}_{\in I_2} \left(\frac{a}{X(1 - b_D(1))} + F^K_D(\overline{a}^K_D) \right) + \mathbb{1}_{>I_2}.
\end{aligned}$$

Disputants' Strategies: Interval Boundaries. The densities define the strategies up to the intervals' boundaries. These boundaries are determined as follows

1. $\overline{a}^K_D$ is determined using $F^K_D(\overline{a}^K_D) = 1$, i.e. $\overline{a}^K_D f^K_D(a) = 1$ for $a \in I_1$. Substituting yields
$$\overline{a}^K_D = \frac{X(1 - b_P(K))}{K}.$$
2. For any $\overline{a}^K_P$, $\overline{a}^1_P$ is determined using $F^1_P(\overline{a}^1_P) = 1$, i.e. $\left(\overline{a}^1_P - \overline{a}^K_P\right) f^1_P(a) = 1$ with $a \in I_3$. Substituting yields
$$\overline{a}^1_P = \overline{a}^K_P + Xb_D(1).$$
3. $\overline{a}^K_P$ is determined by $F^1_D(\overline{a}^K_P) = 1$. That is, $\left(\overline{a}^K_P - \overline{a}^K_D\right) f^1_D(a) + \left(\overline{a}^1_P - \overline{a}^K_P\right) f^1_D(a') = 1$ with $a \in I_2, a' \in I_3$. Substituting yields
$$\overline{a}^K_P = \overline{a}^K_D + \left(1 - \frac{b_D(1)}{b_P(1)}\right) \frac{Xb_P(K)}{K}.$$
4. $F^K_P(0)$ is determined by the condition $F^K_P(\overline{a}^K_P) = 1$, i.e. $F^K_P(0) = 1 - \overline{a}^K_D f^K_P(a) - \left(\overline{a}^K_P - \overline{a}^K_D\right) f^K_P(a')$ with $a \in I_1, a' \in I_2$. Substituting yields
$$F^K_P(0) = 1 - \frac{1 - b_P(K)}{1 - b_D(K)} - \left(1 - \frac{b_D(1)}{b_P(1)}\right) \frac{b_P(K)}{1 - b_D(1)} \frac{1}{K}. \quad \square$$

C.3 Proof of Lemma B2

Proof. A public signal implies a lottery over several (internally consistent) information structures.

Take the set $\{\rho_A, \rho_B, b_A(1)\}$ that maximizes A2 in Appendix A. Assume that it violates neither (IC^1) and is feasible. By the definition of an optimum this implies that no other information structure provides a higher value of A2. Thus, no lottery over information structures can improve upon that optimum either. Hence signals have no use. □

D Alternative Implementation for Mediation

In this section we show that the abstract optimal ADR mechanism can be implemented by a mediation mechanism in which a disputant can secure herself a settlement solution by claiming a moderate reservation value. More precisely, the game is as follows.

1. Both disputants claim a reservation value, $r_i \in \{w_i, s_i\}$, with $s_i > w_i$
2. The case settles with probability 1 if at least one disputant claimed reservation value w_i
3. If both disputants claimed reservation value s_i, the case goes to litigation with probability $\alpha = \gamma(1, 1)$.

Let $\tilde{m}_i \in \{w_i, s_i\}$. Suppose there is a settlement solution. Then, the mediator clears the shares as follows: Party i, who reported $\tilde{m}_i$, receives ex-post share $\tilde{x}_i(\tilde{m}_i, \tilde{m}_{-i})$.

Take the numerical example with $(X, K, p) = (1, 3, 1/5)$. This game has an equilibrium in which (i) the high type mixes between reporting w_i and s_i and (ii) the probability of settlement is the same as that under the optimal mechanism.

Suppose that K_i reports s_i with probability σ_i. Moreover, let $\sigma_P = \frac{p(1+p)}{(1-p)^2} = \frac{6}{16}$ and $\sigma_D = \frac{p}{1+p} = \frac{1}{6}$. Given this strategy, we have $Pr(L|\theta_i, \theta_{-i}) = \gamma(\theta_i, \theta_{-i})$ for all type combinations (θ_i, θ_{-i}).

Next, we construct the expected shares, $\tilde{z}_i(\tilde{m}_i)$ with $\tilde{m}_i \in \{w_i, s_i\}$, such that (i) reporting w_i yields to expected share $\tilde{z}_i(1)$ and K_i is indeed indifferent between reporting w_i and s_i. Then, it directly follows that 1_i strictly prefers to report s_i.

- 1_P receives expected share $\tilde{z}_P(s_i) = z_P(1) = \frac{76}{165}$
- K_P receives $U_P(K) = 0$ whenever there is litigation. Thus, she needs expected share $\tilde{z}_P(w_i) = \tilde{z}_P(s_i) = z_P(1)$
- 1_D receives expected share $\tilde{z}_i(s_i) = z_D(1) = \frac{14}{33}$
- K_D receives $U_D(K) = \frac{2}{15}$ whenever there is litigation. Thus, $\tilde{z}_D(K)$ must satisfy $z_D(1) + \gamma_D(K)U_D(K) = \tilde{z}_D(w_i)$ or $\tilde{z}_D(w_i) = z_D(1) + \gamma_D(1)U_D(K) = \frac{14}{33} + \frac{3}{11}\frac{2}{15} = \frac{76}{165}$.

The ex-post shares $\tilde{x}_i(\tilde{m}_i, \tilde{m}_{-i})$ that give rise to these expected shares solve the following system of equation.

$$\tilde{z}_P(s) = \tilde{x}_P(s, s)(1-\alpha)(p + (1-p)\sigma_D) + \tilde{x}_P(s, w)(1-p)(1-\sigma_D) \tag{15}$$

$$\tilde{z}_P(w) = \tilde{x}_P(w, s)(p + (1-p)\sigma_D) + \tilde{x}_P(w, s)(1-p)(1-\sigma_D) \tag{16}$$

$$\tilde{z}_D(s) = (1-\tilde{x}_P(s, s))(1-\alpha)(p + (1-p)\sigma_P) + (1-\tilde{x}_P(w, s))(1-p)(1-\sigma_P) \tag{17}$$

$$\tilde{z}_D(w) = (1-\tilde{x}_P(s, w))(p + (1-p)\sigma_P) + (1-\tilde{x}_P(w, w))(1-p)(1-\sigma_P) \tag{18}$$

Substituting $\alpha = \frac{6}{11}$, σ_D, σ_P, and $\tilde{z}_i(\tilde{m}_i)$ the system becomes

$$\frac{76}{165} = \tilde{x}_P(s, s)\frac{5}{33} + \tilde{x}_P(s, w)\frac{2}{3} \tag{19}$$

$$\frac{76}{165} = \tilde{x}_P(w,s)\frac{1}{3} + \tilde{x}_P(w,w)\frac{2}{3} \tag{20}$$

$$\frac{10}{33} = \tilde{x}_P(s,s)\frac{5}{22} + \tilde{x}_P(w,s)\frac{1}{2} \tag{21}$$

$$\frac{89}{165} = \tilde{x}_P(s,w)\frac{1}{2} + \tilde{x}_P(w,w)\frac{1}{2} \tag{22}$$

The following ex-post shares

$$\begin{pmatrix} \tilde{x}_P(s,s) \\ \tilde{x}_P(s,w) \\ \tilde{x}_P(w,s) \\ \tilde{x}_P(w,w) \end{pmatrix} = \begin{pmatrix} \frac{8}{15} \\ \frac{94}{165} \\ \frac{4}{11} \\ \frac{84}{165} \end{pmatrix}$$

with $\tilde{x}_D(\tilde{m}_D, \tilde{m}_P) = 1 - \tilde{x}_P(\tilde{m}_P, \tilde{m}_D)$ implement an equilibrium. Thus, the allocation (including the probability of settlement) is the same as that of the optimal abstract ADR mechanism.