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A fast reconstruction algorithm for time-resolved X-ray tomography in bubbling fluidized beds

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Abstract.

A new tomographic reconstruction algorithm is proposed for fast image reconstruction. The results are based on a high speed X-ray tomography system, consisting of 3 X-ray sources and 32 detectors for each source. The proposed algorithm combines void measurements of each X-ray beam into a triangular mesh, which is formed by the intersection points of all the beams. Simulations and real fluidized bed data are utilized to assess the quality of the proposed algorithm compared to the Simultaneous Algebraic Reconstruction Technique (SART). The influence of the number, position and diameter of the phantoms on the proposed reconstruction method is studied. The new method provides images with similar quality to SART reconstructions, although obtaining smaller bubbles sizes. The low computing time needed to reconstruct each image with the new method, which is more than 5000 times faster than SART for a 40x40 mesh, encourages the use of the new method for the online image reconstruction of X-ray measurements.

Keywords:

X-ray tomography; SART; MIRR; fluidized bed

Nomenclature

a detector index

b detector index

A_{cal} fitted coefficient

B_{cal} fitted coefficient

C_{cal} fitted coefficient

i source index

I radiation intensity

I_0 photon intensity in the absence of material

$\overline{I_{ref}}$ mean intensity of the reference state

j detector

M total number of rays

N the total number of pixels

n number of bubbles

p_i solids fraction on the line

S_t value of the triangle

U gas velocity

U_{mf} minimum fluidization velocity

W_{ij} weighing factor for pixel j

x distance traveled by the beam

Z_{ab} value of the intersection point

Greek symbols

α_j^n pixel value of j after the n^{th} SART step

β weight controller of the OSL

$\mu_{material}$ absorption coefficient of the material

Abbreviations

AGA Adaptive Genetic Algorithms

ART Algebraic Reconstruction Technique

CT Computed Tomography

ECT Electrical Capacitance Tomography

FBP Filtered Back Projection

GA Genetic Algorithms

MIR Mean Intensity Ratio

MIRR Mean Intensity Ratio Reconstruction

OSL one-step-late algorithm

SART Simultaneous Algebraic Reconstruction Technique

SSIM Structural Similarity index

1. Introduction.

The excellent heat and mass transfer of fluidized beds have encouraged its use in a wide variety of applications, such as gasification, combustion, particle coating and granulation. Fluidized beds also stand out due to the great range of particle sizes, which covers from several millimeters to nanometers, which can be employed as a solid phase [1, 2]. In these reactors, the gas-solid interactions determine the behavior and performance of the process. However, the impossibility of direct observation of the gas-solid phase complicates the full understanding of fluidized beds. Many experimental techniques have been proposed to describe the properties of the gas-solid motion [3]. Tomographic techniques are a good solution for the noninvasive visualization of the internal flow in a fluidized bed [4]. X-ray Computed Tomography (CT) and Electrical Capacitance Tomography (ECT) have been used to obtain the cross-sectional view of the bed, distinguishing between gas and solid phase, and thus, giving an image of the internal motion of the bed [5 - 7]. Although X-ray CT is more expensive and more complex to apply than ECT, its straight field lines give superior spatial resolution [8]. For further discussion and details regarding the comparison between X-ray CT and ECT, the reader is referred to Rautenbach et al. [7].

X-ray CT makes use of an X-ray beam with high energy that passes through the bed. Opposite to the X-ray-source, a detector measures the attenuation caused by the interaction of the X-ray with the solids. In this way, the changes in the density of the gas-solid phase, which can be produced by a passing bubble or by the increase of the bed expansion, are recorded. Time-averaged attenuation measurements of an X-ray CT system were used to study the gas holdup and solid distribution inside a fluidized bed [9 - 11]. However, these time-averaged results do not reflect the dynamics of the system. To solve that, fast X-ray CT has been developed [12, 13]. Using such a high speed X-ray system it is possible to reconstruct bubble velocities and bubble volumes at high superficial velocities [14 - 16]. The high temporal resolution of this system, 2500 frames per second, encourages its use for the continuous monitoring and control of the experiments. However, the transformation from the X-ray attenuation measurements to a tomography image is still a slow and difficult process.

Several approaches have been applied to reconstruct the X-ray data [16]. The algorithms have to deal with a relatively low number of data points, which is restricted by the space needed to place the 3x32 detectors around the bed. A very frequently used algorithm for image reconstruction is the Filtered Back Projection (FBP) [17]. However, this method is unstable with noisy data. Other methods, such as Simultaneous Algebraic Reconstruction Technique (SART), Genetic Algorithms (GA) and Adaptive Genetic Algorithms (AGA) have been used to obtain accurate reconstructions [18 - 21]. Although these methods can reconstruct the voidage distribution (i.e., bubble pattern) in fluidized beds rather well, the high computational times make online reconstruction impossible .This lack of information during real experiments makes it difficult to detect malfunction of the equipment. For instance, consider the study of a pressurized fluidized bed. Using the current reconstruction methods, the results are obtained off line, consuming more time to analyze the data and with a lower safety level during the tests since there is no online feedback of the process. To solve that, a faster tomography reconstruction algorithm is developed.

In this paper, an algorithm is proposed for the fast image reconstruction of the data measured using a high speed X-ray tomography system. The algorithm combines the void measurements of each X-ray beam into a triangular mesh, which is formed by the intersection points of all the beams. This approach improves the algorithm velocity, although it limits the grid resolution by the geometry of the X-ray system. The accuracy and computation time of the method is compared to the SART algorithm using both data from simulations and from fluidization experiments.

2. Experimental setup.

The experiments are conducted in a Perspex tube, wall thickness 5 mm, with an inner diameter of 0.14 m and a height of 1.4 m. The air distributor consists of a porous plate, sintered bronze, with pore sizes ranging from 30 to 70 μm and a thickness of 7 mm. As bed material we use polystyrene Geldart B [1] particles with a density of 1100 kg/m^3 and a mean diameter of 0.56 mm. Air at ambient temperature is used to fluidize the bed. The compressed airflow is controlled using a mass-flow controller. The minimum fluidization velocity is measured at U_{mf} = 0.12 m/s. The static bed height is 0.6 m.

The X-ray sources are placed at 120º around the column. Each source generates two fan beams giving information in two measurements planes. 32 detectors are placed opposite to the sources for each plane and source (Fig. 1). The location of the 96 detectors per plane around the bed ensures the simultaneous measurement of the X-ray attenuation from different positions. All the detectors record at a rate of 2500 Hz. This measurement setup was developed by Mudde et al. [12]; further details can be found in Mudde [22].

Figure 1. Schematic diagram of the X-ray scanner. (a) Side view showing upper and lower detector array. (b) Top view.

3. Tomographic reconstruction.

3.1. Measuring principle.

Each detector measures the attenuation caused when an X-ray beam passes through a certain amount of material. For a monochromatic radiation through a homogeneous slab of material, the attenuation of the radiation is described by the Lambert-Beer law:

$$I(x) = I_0 \cdot \exp\left(-\mu_{material} \cdot x\right) \qquad (1)$$

where $I(x)$ is the radiation intensity measured by the detector after passing a material of thickness x, I_0 is the photon intensity in the absence of material and $\mu_{material}$ is the absorption coefficient of the material in the slab. The generated X-ray beam produces a wide spectrum of photon energies. Low energy photons are more easily absorbed by the material, causing the energy distribution of the beam to shift to a higher mean energy, which is known as beam hardening [23]. In order to deal with this, all detectors are individually calibrated by placing different amounts of packed bed material (seven in total) in between a source and its detectors. The seven point calibration, ranging from a completely empty to a completely filled bed, allows the fitting to Eq. (2):

$$I(x) = A_{cal} + B_{cal} \cdot \exp\left(-x/C_{cal}\right) \quad (2)$$

with x the distance traveled by the beam through the powder phase, and A_{cal}, B_{cal} and C_{cal} the fitted coefficients. In this way, the measured intensity can be transformed to a measured amount of bed material.

3.2. Simultaneous Algebraic Reconstruction Technique (SART).

The Simultaneous Algebraic Reconstruction Technique (SART) is an iterative method that minimizes the error generated between the reconstructed image and the measured attenuation map. In this method, the cross-section of the bed is divided into a square pixel array; see Fig. 2. To reduce the salt and pepper noise usually present in the Algebraic Reconstruction Techniques (ART), it simultaneously applies to a pixel the average of the corrections generated by all rays. However, this quality improvement increases the computation time. The SART algorithm is described by:

$$\alpha_j^{n+1} = \alpha_j^n + \frac{1}{W_{+,j}} \sum_{i=1}^{N} \left(\frac{p_i - \tilde{p}_i}{W_{i,+}} W_{ij} \right) \quad (3)$$

where $W_{+,j} = \sum_{i=1}^{N} W_{ij}$, $W_{i,+} = \sum_{j=1}^{M} W_{ij}$, M is the total number of rays and N is the total number of pixels.

Figure 2. Definition of the grid and W_{ij} matrix.

The so-called one-step-late (OSL) algorithm is implemented to further reduce the pepper and salt noise, improving the contrast of the reconstructed image [24]. This extra step is developed adding a median root function to Eq. (3).

$$\alpha_{OSL,j}^{n+1} = \frac{1}{1+\beta\frac{\alpha_{OSL,j}^{n} - Med\left(\alpha_{OSL,j}^{n}\right)}{Med\left(\alpha_{OSL,j}^{n}\right)}} \quad (4)$$

where α_j^n is the value of pixel *j* after the n^{th} SART step and $\alpha_{OSL,j}^{n}$ is the same after the n^{th} one-step-late correction. The parameter to control the weight of the correction is set to $\beta = 0.1$ since it is found to improve the reconstruction process [22]. The median filter $Med(\alpha_j)$ uses a 3x3 neighborhood to replace the value of the j^{th} pixel by the averaged value of the j^{th} neighborhood.

Furthermore, the algorithm averages the data in steps of 10 samples for each detector in order to reduce the noise in the signal, which also reduces the time resolution to 250 frames per second. In this work a 40x40 grid was employed for the SART image reconstruction, which ensures a spatial resolution of 0.35 cm [21]. Table 1 summarizes the properties used for the SART reconstruction.

Table 1. Parameters for SART.

3.3. Mean Intensity Ratio Reconstruction (MIRR) method.

The novel, fast reconstruction approach we propose in this paper is based on the mean intensity ratio (MIR). This ratio compares the mean intensity of the X-ray measurement to the mean intensity of a reference state for each detector. This ratio is expressed as follows:

$$MIR_{ij} = \frac{I_{ij}}{\overline{I_{ref}}_{ij}} \quad (5)$$

where *i* identifies the source, *j* is the detector, *I* is the beam intensity of the test under analysis, and $\overline{I_{ref}}$ the mean intensity measured at the reference state. Similarly to the SART algorithm, the intensity data (*I*) is averaged in steps of 10 samples in order to reduce the noise of the signal, obtaining a time resolution of 250 frames per second.

The intensity of the reference state ($\overline{I_{ref}}$) is measured for a packed bed during 30 seconds. Thus, this ratio gives information about the void fraction of each X-ray beam. Figure 3 illustrates the influence of the bubbles on this ratio. This simulated test presents two empty and circular phantoms on a homogeneous dense phase (Fig. 3-a). In this figure, the black color identifies the empty space while the white color points to the solid powder. The position and number of each beam is sketched in Fig. 3-a using different colors for the sources. The MIR values of the detectors shown in Fig.3-b/c/d are obtained dividing the attenuation produced due to the phantoms by the reference intensity measured for a packed bed. As can be seen for the detectors of each source (Fig. 3-b/c/d), when an X-ray beam passes through a phantom, the ratio is greater than 1, varying also its values with the increase of the empty part of the path. Otherwise, the ratio is equal to 1 since there is no difference between the solid phase of the reference state and the phantom test. Also, when the phantoms are placed together, as they are seen by source 2 (Fig. 3-c), it is difficult to elucidate the boundaries of each phantom. In order to increase the difference between both phases during fluidization, a moving threshold is employed. The value of this threshold should be placed as (i) a safety margin since fluctuations around 10% are usually shown for MIR values in the absence of bubbles, which could produce the appearance of false bubbles, and (ii) to reduce the effect of the bed expansion during fluidization, which would increase the MIR limit between dense and void above 1. It is placed 25% above the minimum MIR value of each reconstructed image. Such a value is set for the bubbling regime studied in this work, and might change with the increase of the bed expansion.

Figure 3. MIR values for the 2 phantoms test. (a) Position of the simulated phantoms and X-ray beams. (b) MIR values of detectors array 1. (c) MIR values of detectors array 2. (d) MIR values of detectors array 3.

The detectors measure the attenuation of the X-ray beams over a straight line. Therefore, it is necessary to combine the measurements from all the detectors to transform these linear measurements to an image. The SART algorithm uses a pixel representation of the cross-section of the fluidized bed to combine the X-ray measurements. As was pointed out before, this process increases the computation time. In contrast, MIRR triangulates the cross-section domain using the intersection points between the X-ray beams as vertex.

Each triangle is generated using the intersections between the sources 1-2, 1-3 and 2-3 (Fig. 4).To combine the information of the beams that pass through a intersection point, the product of its MIR values is employed. This process gives a triangular mesh of points with a level value associated to each one. A triangular pattern can be seen connecting the intersection points of all the sources (Fig. 4-a). In order to transform these points into a mesh, a method to combine the information is needed. The connection follows the Delaunay triangulation [25]. This method maximizes the minimum angle of all the angles of the triangles in the triangulation ensuring that the circumcircle associated with each triangle (i.e., the circle through all three vertices) contains no other point in its interior. It provides the vertex of each triangle, which values would be combined to obtain the surface value of the triangle. In this way, each triangle contains the attenuation measurements from the three sources. As the objective is to discriminate between solid and void phases, the multiplication is also used to compute the surface values. A detail of this mesh shows the different triangle sizes (Fig. 4-b). As can be seen comparing Fig. 4-a and Fig. 4-b, the spatial resolution varies locally, showing higher spatial resolution close to the vessel walls. This accuracy change is produced since more X-rays cross this zone [7]. Thus, as Delaunay triangulation is based on the intersection points between X-beams, the lowest spatial resolution is found near to the center of the bed, which is approximately 0.5 cm.

The proposed methodology is outlined in Fig. 5, and can be summarized as follows:

Figure 4. Definition of the triangular grid (a) bed domain and (b) detail of the mesh.

Figure 5. Flowchart of the MIRR algorithm.

- Computation of the intersection points between all the lines and transformation to a triangular mesh using the Delaunay triangulation.
- Definition of the reference data computing the mean intensity of each detector. The reference state is measured during 30 seconds after gently filling the bed.
- Thresholding of the MIR values in order to increase the difference between the powder and the empty phases. A moving threshold is proposed: when the bed is fluidized, the powder is expanded even in the absence of bubbles. For the detectors of each source (*i*) the threshold is fixed to:

$$Threshold_i = 1.25 \cdot \min\left(MIR_{ij}\right) \qquad (6)$$

  and the MIR values will be set to:

$$\text{if} \quad Threshold_i > MIR_{ij} \rightarrow MIR_{ij} = 0 \quad (7)$$

  This threshold reduces the diameter of the phantoms in the reconstructed images, but it facilitates the reconstruction process.
- Identification of the number of bubbles (*n*). The direct combination of the MIR values for all the detectors would add shadows to the reconstructed picture. To solve that, each bubble is reconstructed individually. The identification of bubbles is carried out by: i) filtering the MIR results using a moving average filter with 3 points to reduce the noise caused by the signal, and ii) identifying the position of the maximum and minimum intensity of the detectors of each source in order to fix the boundaries of each bubble.
- Creation of the input signal used in the reconstruction. As the strategy consists of the individual reconstruction of each bubble, it is necessary to introduce separately each bubble into the reconstruction algorithm. Thus, *n* signals are created to reconstruct each bubble. The boundary limits of each bubble, which were obtained in the previous step, are employed. However, the number of bubbles shown by MIR values can vary depending on the source, as was shown in Fig. 3 where source 2 only detects 1 big bubble, while sources 1 and

3 detects 2 bubbles. To solve that, the source that detects the greater number of bubbles is used to create the signals. Figure 6 shows the signals created for the simulated test of Fig. 3. Each input signal (Fig. 6-b1/b2/b3 and Fig. 6-c1/c2/c3) consists of the measured MIR after the thresholding and the bubble identification. In this example, both sources 1 and 3 could be used to create the signals of the phantoms. Finally, source 3 is used since the phantoms are identified by a greater number of detectors.

- Reconstruction of each bubble. As each bubble is reconstructed individually, and then it is added to the image, this step is repeated for the *n* bubbles identified. The value associate to each intersection point (*Z*) is defined as:

$$(Z_{ab})_{i=1-i=2} = MIR_{i=1,j} \cdot MIR_{i=2,j} \quad (8)$$

being i = 1 and i = 2 sources 1 and 2, and a and b indicate the 32 beams per source. This expression is also applied for sources 2-3 and 1-3. This product is computed for all the detectors and all the sources. Similarly, the surface value of each triangle (*S*) is computed as:

$$S_t = (Z_{ab})_{1-2} \cdot (Z_{ab})_{2-3} \cdot (Z_{ab})_{1-3} \quad (9)$$

where $(Z_{ab})_{1\text{-}2}$, $(Z_{ab})_{2\text{-}3}$ and $(Z_{ab})_{1\text{-}3}$ denote the vertex of the t triangle.

Figure 6. Example of signal creation for a simulated 2 phantom test.

4. Experimental procedure.

To study the feasibility of the proposed reconstruction algorithm, two types of analysis were conducted: (i) using artificial data, and (ii) using real X-ray data. In (i) the accuracy of the proposed reconstruction method is evaluated simulating the experimental setup. In these tests, circular phantoms are simulated in a bed with an inner diameter of 0.14 m. These circular objects represent the cross-sectional shape of a bubble when it passes through the measurement plane. In these simulations, it is assumed that the phantoms are completely empty. The empty path that should be measured by the X-ray detector of one measurement plane is simulated. This path length is introduced into the SART algorithm in order to obtain the reconstruction image of the phantom test. To use the proposed reconstruction approach, this simulated path length is transformed to the mean intensity ratio using the calibration function of the bed material. The SART reconstruction results are used to check the reliability of the proposed approach. To do that, the Structural Similarity (SSIM) index is employed [26]. The SSIM index is a quality measure of one of the images being compared, provided the other image is regarded as of perfect quality. SSIM varies between 0, for completely different images, to 1 for the same pictures.

The practical application of the proposed reconstruction method is studied using real X-ray measurements in the bed with 0.14 m inner diameter. Similarly to the phantom tests, two empty cylinders are placed into a packed bed. The cylinders have an inner diameter of 36 and 58 mm and a 2 mm Perspex wall. The measurement planes pass through these circular voids in a similar way as shown in Fig. 3-a. The attenuation measurement of the X-ray beams recorded at the bottom plane is used to reconstruct the tomographic images with SART and MIRR algorithms. Furthermore, fluidization experiments at an air velocity ratio of U/U_{mf} = 2 were also analyzed. In this test, the X-ray measurements were carried out at 0.17 m above to the distributor plate, where vigorous bubbling was observed.

5. Results and discussion.

The results show the performance of the proposed reconstruction algorithm compared to the SART results. Both methods were programmed in Matlab and the image reconstructions were carried out with a Intel Core 2.4 GHz processor. Simulated phantom tests are analyzed to check the accuracy of the MIRR approach. The computation time is also presented for these simulated data. Later, image reconstructions of real experiments with two phantoms and with a fluidized state are studied.

5.1. Simulated data: Phantom tests.

Figure 7 presents the reconstruction results of SART and MIRR for two, three, four and five simulated phantoms. The exact position and diameter of each phantom is plotted in pink. In these gray scale images, the black color indicates empty space, while the white color represents the solid phase. The threshold results are also presented in this figure. For SART, a 0.6 threshold gives the optimal contrast to differentiate between the dense phase and gas voids [21]. The threshold effect was also analyzed using the SSIM index for the MIRR reconstructions. A 0.3 threshold was found to be the best value for improving the MIRR images. Furthermore, the reconstructed images presented in Fig. 7 are also analyzed using the SSIM index. The comparison between the simulated test and each image is presented in Fig. 8.

Focusing on the two phantom test (Fig. 7-a1), the original SART image shows both bubbles with good accuracy in size and position. Only a few shadows are shown around the boundary of the biggest phantom, which disappear after the image threshold (Fig. 7-a2). The original MIRR image shows a rather poor contrast between solid and voids (Fig. 7-a3). This effect is produced by the values measured with the mean intensity ratio when a phantom is present. As was shown before (Fig. 3-b/c/d), a local maximum appears on the MIR values of each source when the X-rays passed through a phantom. The combination of all the values causes the appearance of a maximum near to the center of the phantom, complicating the determination of the phantom boundary. The threshold of such an image (Fig. 7-a4) improves the recognition of both phantoms. Nevertheless, the threshold image of MIR shows shadows around the big phantom, obtaining a result similar to the original SART reconstruction (Fig. 7-a1).

Three big phantoms are simulated in order to study the effect of increasing the area occupied by voids on the MIRR reconstruction results. The quality of the SART reconstruction decreases with the presence of these big phantoms, showing some shadows especially on the right side of the picture (Fig. 7-b1). Without prior knowledge, it is difficult to know the exact number and size of the phantoms. Nevertheless, the threshold eliminates the shadows on the image giving an accurate result of the phantom shapes (Fig. 7-b2). Compared with the original SART, more shadows appear using MIRR (Fig. 7-b3). As the area occupied by the voids is increased, a lower number of X-ray beams pass between the phantoms, complicating the identification of the phantom boundaries. As the MIRR algorithm does not include an error-correcting step to minimize the error generated between the reconstructed image and the measured attenuation map, the shadows are also shown after the threshold of the image (Fig .7-b4). In such a figure, only the biggest phantom could be recognized, while some shadows appear between the other phantoms.

To analyze the influence of the position on the MIRR reconstructed images, four phantoms are symmetrically placed on the simulated bed. All the phantoms can be recognized for the original and threshold results of SART (Fig. 7-c1/c2, respectively). In the MIRR results (Fig. 7-c3) the left phantom cannot be identified while the right phantom shows an elliptical shape. In contrast, the upper and lower phantoms are represented with good accuracy. Such a different reconstruction result is produced by the relative position of the phantoms with respect to the sources. The detectors of source 1 can only identify two phantoms with similar shape and another bigger empty space which is the presence of the right and left phantoms on the same X-ray lines. A similar effect appears for the detectors of the sources 2 and 3. Thus, the triangulation of these values produces the reconstruction of shadows near the center of the bed and around the right phantom (Fig. 7-c4).

A more complex phantom test is simulated to check the limits of the proposed algorithm. The five phantoms can be identified in the original results of SART, Fig. 7-d1, in spite of the presence of small shadows. Similarly to the other tests, the threshold of SART improves the quality of the reconstruction (Fig. 7-d2). MIRR (Fig. 7-d3) initially obtained a similar reconstruction to the original SART (Fig. 7-d1) even in the position of some shadows. In the same way, the MIRR threshold (Fig. 7-d4) improves the reconstruction accuracy to values around to the original SART result.

Figure 7. Original and thresholded reconstruction results for 2, 3, 4 and 5 phantoms with SART and MIRR. The pink circles indicate the position and shape of the phantoms.

The SSIM analysis, Fig. 8, confirms the results visually shown in Fig. 7. For all the phantom tests, the threshold improves the quality of the reconstructions. In the 2, 4 and 5 phantoms test, MIRR with threshold obtains accurate images near to the SART quality. Only when the area occupied by voids is high, such as in the 3 phantoms test, both the original and threshold MIRR images are far from the SART quality.

Figure 8. SSIM values for the 2, 3, 4 and 5 phantoms reconstructions of the original and threshold results of SART and MIRR.

Regarding to the computing time needed, Table 2 shows the CPU times in seconds for one image reconstruction of one plane. It is clear that SART consumes much more time than MIRR independently of the number of phantoms. The low computational time of MIRR is caused by: i) the use of simple mathematical operations, such as the multiplication, to perform the reconstruction algorithm, and ii) the absence of an iterative error-correcting step to minimize the error generated between the reconstructed image and the measured attenuation map, which reduces the accuracy of the images but avoids any convergence process.

Table 2. **Computing times for the reconstruction of one frame. Both reconstruction techniques are implemented in Matlab.**

5.2. Experimental data: Phantom tests.

To assess the reliability of the proposed reconstruction approach, real X-ray measurements are carried out through a packed bed with two empty cylinders. The cylinders have an inner diameter of 36 and 58 mm, respectively, with 2 mm Perspex wall thickness. The position of these phantoms is indicated in red color in the reconstructed images presented in Fig. 9. The figure shows the original and threshold image reconstructions obtained with SART and MIRR algorithms. Some noisy pixels appear around the phantoms in SART reconstruction (Fig. 9-a). Nevertheless, the shape and the position of the phantoms are rather good. The shadows are removed by the threshold of the picture, showing a good match in the position of both bubbles (Fig. 9-b). Similar results are obtained using MIRR, even for the position of most of the shadows (Fig. 9-c). In these pictures (Fig. 9-c/d) the reconstructed phantoms show lower sizes, especially for the smallest phantom. This is a result of the thresholding step of the mean intensity ratio values (Eq. (7)). Furthermore, the triangulation of the domain deforms the spherical form of these phantoms, as can be clearly seen for the smallest phantom.

Figure 9. Reconstruction results for experimental data with 2 phantoms. (a) Original result of SART. (b) SART with threshold. (c) Original result of MIRR. (d) MIRR with threshold. The red color points to the position and shape of the phantoms and the column.

SSIM values between the ideal position of the phantoms and the reconstructed pictures with SART and MIRR are shown in Fig. 10. Similar results are obtained comparing the original and threshold reconstructions of both methods. The corrector step of the SART method combined with the threshold performed the best SSIM result.

Figure 10. SSIM values for experimental data with 2 phantoms.

5.3. Experimental data: Fluidized bed test

In this section the performance of the proposed algorithm is studied for real fluidized bed data. The measurements are carried out at 17 cm above to the distributor plate at U/U_{mf} = 2. These conditions ensure a bubbling state at the upper and lower measurements planes. A signal of 24 seconds is analyzed using SART and MIRR approaches with threshold. The image reconstructions for both methods, 5750 per plane, are combined in a video (see supplementary material).

The frames of both videos are analyzed using the SSIM index comparing SART to MIRR images. In this way, the performance of the MIRR algorithm is studied. According to SSIM results, both methods are more similar if SSIM values are closer to 1. Figure 11 shows the evolution in time of SSIM values for the lower plane during 1 second of measurements. Furthermore, Fig. 11 points to the snapshots (*a*-*f*) that are presented in Figs. 12 and 13 for the visual analysis. These pictures show the upward movement of two bubbles at different times.

Figure 11. SSIM ratio comparing SART with MIRR reconstruction for the fluidized test.

As expected, SSIM index shows no difference between SART and MIRR when no bubbles pass through the measurement plane (Fig. 11). When a bubble crosses the lower plane, some differences appear during the rising and disappearing of the bubble. Such variations are typically caused by the smaller bubble size obtained with MIRR. Comparing SART and MIRR images with 1 bubble (Fig. 12-a/b and Fig. 13-a/b, respectively), both methods show similar position and shape of the incoming bubble. The appearance of a second bubble reduces the similarity between both methods, Fig. 11. As the size of both bubbles increase, SSIM index ratio is reduced to a minimum value at time "*e*". During the bubbles rising, the position of the second bubble is also reconstructed with good accuracy by MIRR methods, although it presents white pixels inside the second bubble (Fig. 13-c). A similar size reduction in the MIRR results is shown comparing Fig. 12-d/e to Fig. 13-d/e. Only the last frame of MIRR (Fig. 13-f) shows slight differences compared to SART when the bubbles leave the measurement plane (Fig. 12-f). This difference is caused by an error in the identification of the number of bubbles in the MIRR method. During the bubble collapse, the mean intensity ratio of the detectors present some peaks. If these peaks are big enough, they can be identified as individual bubbles by the MIRR algorithm. Thus, some of them can be reconstructed as separated bubbles, producing the appearance of small shadows on the upper zone of the bed (Fig. 13-f).

Figure 12. SART reconstruction snapshots of the analyzed video at different times. The red color points to the bed column.

Figure 13. MIRR reconstruction snapshots of the analyzed video at different times. The red color points to the bed column.

Once the quality of MIRR reconstruction was compared to SART images using SSIM, the bubble size distribution was estimated using both methods. The signal of 23 seconds of the real fluidized bed data is analyzed. The reconstructed images of both methods were stacked for the lower X-ray plane obtaining a pseudo 3D representation of bubbles/voids, as can be seen in Figure 14. The z-axis represents the time at which a bubble pass the detector plane, which coincide with the horizontal axis of Figure 15. This makes it possible to detect the bubbles and their properties, such as the location in the bed, the crossing time of the measurement plane and their pseudo 3D volume. Figure 14 shows the results of this analysis for 2 s of data (from 5 to 7 s in Fig. 15), which are analyzed using both MIRR and SART reconstructions. Similar results are obtained for the pseudo 3D results of both methods, as can be seen comparing Fig. 14-a and Fig. 14-b. Also, a detail of the bubbles detected from 5.08 to 5.40 s is presented in Fig. 14-c, for SART, and Fig.14-d for MIRR.

Figure 14. Pseudo 3D reconstruction of the void shapes. Lower measurement plane at 0.17 m and U/U_{mf} = 2. The *y*- and *x*-axis represent the length dimensions while *z*-axis shows time. Note that *z* values are shown in reverse time. (a) SART result over 2 s. (b) MIRR result over 2 s. (c) SART detail. (d) MIRR detail.

The void volume of each bubble using both methods is presented in Figure 15. The vertical axis represents the void volume, calculated considering a 40 x 40 pixel grid, while the horizontal axis shows the time at which the center of the pseudo 3D bubble passed through the measurement plane. MIRR reconstructions, which were estimated using a triangular mesh, were transformed into a 40 x 40 rectangular grid. Comparing SART and MIRR, similar void volumes are obtained for most of the bubbles. However, a few bubbles present lower volumes for MIRR. For instance consider the bubbles detected at *t* = 5.24 s. At this time, SART detects one bubble while MIRR results show two bubbles, each one with roughly half volume of SART bubble. Due to that, 49 bubbles were detected using SART, while 62 bubbles were shown using MIRR. Such an effect is caused by an error in MIRR algorithm during the identification of the number of bubbles, which divided a bigger bubble into smaller bubbles. Thus, the relative error in the identification of the number of bubbles was 20% when using MIRR, while the number in the total bubble volume is 2%.

Figure 15. Void volume of each bubble obtained using SART and MIRR methods.

5. Conclusions.

We have proposed the Mean Intensity Ratio Reconstruction (MIRR) as an alternative image reconstruction technique for X-ray tomography measurements in fluidized beds. It uses the mean intensity ratio in a triangulated mesh together with the individual reconstruction of each bubble. Although the reconstructing results are typically not as good as those obtained with the Simultaneous Algebraic Reconstruction Technique (SART), MIRR is more than 5000 times faster than SART, which will allow online image reconstruction and thus monitoring of the fluidization.

The accuracy of the MIRR method depends on the size and position of the bubbles. With the MIRR method, bubble sizes are underestimated due to the threshold step. As the area occupied by some voids in one measurement plane is increased, the quality of the MIRR results is reduced due to the appearance of shadows. Furthermore, the position of the bubbles could also reduce the quality of the MIRR result. Nevertheless, the analysis of the bubble size distribution and SSIM values of MIRR and SART suggest that MIRR images with threshold are in most cases close to the original SART result.

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Figure 15. Void volume of each bubble obtained using SART and MIRR methods.

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