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Worldwide annual optimum tilt angle model for solar collectors and photovoltaic systems in the absence of site meteorological data

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Abstract

This study provides several models that allow to accurately compute the annual optimum tilt angle for fixed solar photovoltaic arrays or solar collectors, in any location of the world. The optimum tilt angle that maximizes the annual yielded energy can be therefore easily calculated in the absence of meteorological data and simulation software tools. The proposed models are calculated using global horizontal radiation data collected from 2551 sites all over the world. In the process, well-established submodels have been selected to estimate the hourly irradiance on any possible inclined surface, and its corresponding annual energy yield. After selecting the optimum angle for each location, through a regression analysis, a mathematical model that calculates annual optimum angles as a function of latitude has been developed. Furthermore, regression techniques such as neural networks and decision trees have been compared with the polynomial models. Finally, the results are compared to those obtained from high-quality 1-min measured irradiance data obtained at 52 research-class stations from the World Radiation Monitoring Center–Baseline Surface Radiation Network, providing a remarkably high number of validation data points. The results are analyzed, validated, and compared with previous research proposals proving the good performance of the proposed models.

Highlights

- Optimum tilt angle model developed with data collected from more than 2500 places.
- Different alternatives are discussed for coherent transposition model selection.
- Optimum tilt angle defined as function of latitude for any site.

Keywords: Solar photovoltaic; Solar collector; Optimum tilt angle; Solar radiation; Irradiance model
Wordcount: 7335 words.

List of Abbreviations

- Solar collector (SC)

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- Photovoltaic array (PVA)
- Azimuth (γ)
- Optimum azimuth angle (γ_{opt})
- Tilt angles (β)
- Optimum tilt angle (β_{opt})
- Typical meteorological year (TMY)
- Building-integrated photovoltaic (BIPV)
- Global Horizontal Irradiance (GHI)
- Diffuse solar irradiance (DIF)
- Global tilted irradiance (GTI)
- Direct normal irradiance (DNI)
- Diffuse horizontal irradiance (DHI)
- Root mean squared error (RMSE)
- Latitude in absolute value (L)
- Neural network (NN)
- Decision tree (DT)

1. Introduction

A key objective when installing a solar collector (SC) or a photovoltaic array (PVA) is to maximise energy output over a defined time period. Adjustments of PV installation angles can reduce total electricity generation costs [1]. Therefore, the design or control goal is to determine the tilt (β) and azimuth (γ) angles that increase the performance. It is known that in both applications the optimum angle that maximize incident radiation is the same. [2].

Annual optimum surface inclination should be calculated using site-specific weather data (TMY data, or satellite data) and software tools. But due to the the lack of access to meteorological data in many locations around the world, specially in some Africa, some South America or Asia regions, it is convenient for designers or users of small solar collectors or photovoltaic (PV) systems, to have access to a mathematical model for their optimal orientation. Even if local TMY data exist, some times (specially in small and medium size PV plants design) they are not used by the plant designers, managers, or the NGO founders. These situations could clearly benefit from a simple and universal equation that accurately estimates the optimum tilt angle as a function of latitude.

An extensive number of articles have attempted to establish the optimum tilt angle of solar systems for a specific geographic location around the world using diverse applications. Examples include PV systems, solar water heaters, solar cookers, solar powered thermoacoustic engines, building-integrated photovoltaic (BIPV) systems [3] or solar updraft tower power plants. Since 1950, studies that define the tilt optimal angle as a function of latitude by means of linear equations, based on few locations, have been carried out [4]. More than a hundred optimum tilt studies are reviewed in [5], which focused mainly on specific locations or regions around the world; however, only a few are validated due to a lack of measurements for more than one or two tilt angles.

In most solar energy applications, solar collectors and PVAs are placed according to the optimum azimuth angle (γ_{opt}), which is considered to be oriented to the equator [6]. To calculate annual optimum tilt angle β_{opt}, two main optimization criteria can be considered: maximum intercepted solar radiation and maximum intercepted solar irradiance. Maximizing total incident solar energy on the surface, either in extraterrestrial or in atmospheric conditions, is the main applied criterion [7]. Other optimization techniques applied are Artificial Neural Networks [8], Genetic Algorithms [9], or Particle-Swarm Optimization techniques [10].In some cases it is intended to optimize the effective output energy [11].

Two main approaches can be considered to obtain optimum angles (γ_{opt}, β_{opt}) when maximizing total incident solar energy on the surface: the search-based approach and the direct approach [12]. The direct approach consists of a mathematical model, which generates a formula to calculate the optimum tilt angle [13]; owing the independence of the azimuth angle, the value of γ is usually set to 0 in the Northern and 180 for the Southern Hemisphere. The direct approach is the more common method since it is normally simpler, although the precision depends on the model used to calculate incident radiation on the tilted surface, i.e., the transposition model [14]. In the search-based approach, pairs of tilt and azimuth angles, differently organized, are used to calculate the incident solar irradiance with a desired step size. The pair of angles with the higher solar irradiance along the considered period, is introduced as optimum. The quality and origin of data, as well as the selected transposition model, have a significant influence on the results [15].

This article has developed research based on two database types. First, the Global Horizontal Irradiance (GHI) hourly data from 2,551 sites all over the world is used, with latitudes between 70° north and 70° south. These data contain the typical meteorological year (TMY, TMY2, TMY3) acquired from [16] in the EnergyPlus database for every location. For each of these 2,551 sites, the PVA inclination yielding the maximum incident irradiance over a year has been calculated. Additionally, global, direct, and diffuse irradiance measurements from 52 WRMC-BSRN Stations [17] are used to validate previous results.

A deep search was carried out to select the most suitable model to estimate the global tilted irradiance (GTI) on inclined surfaces; the selected transposition model for diffuse irradiance (DIF) calculations, takes into account the diffuse radiation in the circumsolar region and a component isotropically distributed for the rest of the sky dome. A code has been implemented to determine the optimal tilt angle of 2,551 sites worldwide. A regression model with the site's latitude and the optimal tilt angle as inputs will be conducted and analysed to develop an algorithm that offers optimum tilt angles as a function of latitude for any site. Some other studies have carried out this methodology, but they used a much smaller dataset, and they typically used isotropic models [18]. As several authors mentioned [19], developing a universal model with good amount of high quality data from climatically and geographically dispersed sites is also essential. Also regression techniques such as neural networks and decision trees have been compared with the polynomial models.

The paper is organized as follows. Section 2 presents the methodology used to obtain the incident solar radiation on an inclined surface as a function of the tilt and azimuth angles. Section 3 describes the data selection and discusses the selected anisotropic diffuse transposition models. Section 4 firstly presents and discusses several methods for the calculation, and results for annual optimum angles as a function of latitude, followed by a comparison between methods. Finally, a comparison with other authors and experimental results is presented. Section 5 contains the discussion and conclusions to the paper.

2. Methodology

The applied methodology is summarized in Figure 1. Firstly, from TMY GHI data, the direct normal irradiance (DNI) hourly TMY data and the diffuse horizontal irradiance (DHI) hourly TMY data are obtained for each site from 2551 locations of the EnergyPlus data base. Then, for each site and 0º (North) or 180º (South) azimuth, hourly incident radiation for a year has been calculated for tilts from 0 º to 90º using appropriated transposition models. Therefore, for each site and each inclination, annual incident radiation has been obtained. The annual optimal angle is the one which maximizes annual incident energy. A vector of (2551x2) containing latitude of the location and optimum tilt angle is obtained. Several regression techniques have been compared, polynomial, neural networks and decision trees.

Tilt yielding the annual maximum energy was calculated using both hourly TMY and also high-quality 1-min measured DNI data from BSRN database. The results obtained for the optimum tilt angle are detailed in section 4.1; section 4.2 and 4.3 those results are analyzed, validated, and compared with previous research proposals.

3. Data and separation-transposition models selection

This section presents irradiance EnergyPlus [16] and BSRN [17] public databases since they will both be used to develop the proposed optimum tilt angle model. Furthermore, the relevant transposition models are analyzed to justify a choice.

Solar radiation, considered as both DNI and DHI, is needed with high precision to be able to adequately simulate the energy yield. Irradiance parameters of real-world databases should ideally be used, because the variability of the solar resource in a location from one year to the next produces some variations on the optimum tilt angle. Therefore, it is necessary to have solar resource data from a significant number of years, which can statistically represent a typical year.

A TMY dataset provides a single year of solar radiation hourly data along with other meteorological data and represents characteristic conditions over a long timeframe (for instance 30 years). As TMYs

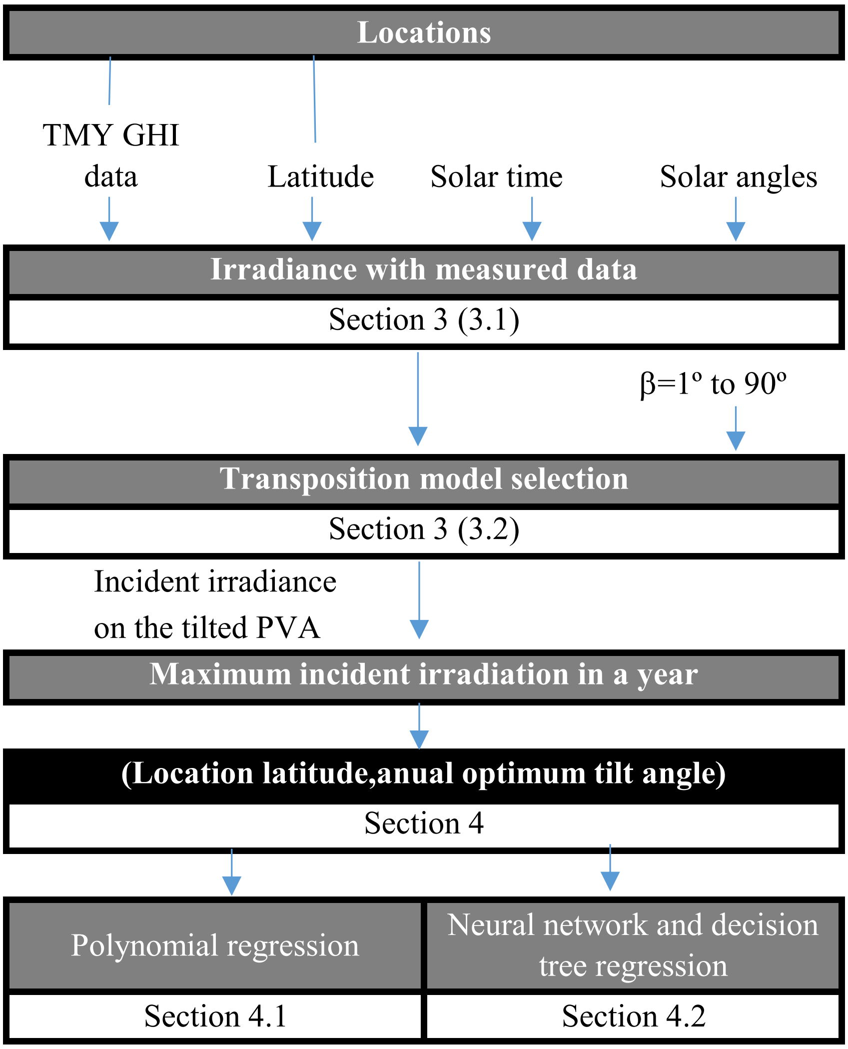

Figure 1: Flowchart representing methodology to calculate optimum angles.

represent typical non-extreme conditions, so they are not suitable for producing a worst-case weather scenarios designing systems and their components. However, these TMY datasets are certainly adequate for the general optimum angle calculation. Different versions of TMYs exist, (TMY,TMY2, TMY3) based on input data for different periods of time. These data are commonly provided by the local meteorological stations, estimated by satellite imagery or synthesized by statistical models. When measuring irradiance, DNI is not usually discernible from DIF scattered into the circumsolar region (by thin clouds, aerosols). Therefore, a decision on the angular size of corresponding solar disc band has to be made, and it will affect the calculation of the direct and diffuse contributions [20]. In cases where the measured GHI is available, a decomposition model is needed to estimate DNI and DIF components. For instance, when systematically producing DNI time series when GHI is obtained out of satellite imagery using the "cloud index" method [21].

Pyranometers are installed in some cases for the purpose of gathering suitable GHI or DNI data [22]. One difficulty is that, compared to GHI, DNI observations are not that common. Therefore, in most cases, with no measured data available, or if only a worst-case or an approximate analysis is desired, DNI is derived from modeled GHI by achieving a division into DNI and DIF. The global uncertainty is significantly contributed by the referred separation process [23]. Typically, the main source of error in GTI calculations is caused by the DNI/DIF separation process [24]. Notably, where there is a significant amount of solar resource, the DNI component is usually the dominant one, and therefore it is extremely important to adequately determine it. Deriving DNI from satellite-based estimates of hemispherical radiation instead of using measured GHI data contributes to bigger uncertainties, as might be expected [23].

Section 3.1 summarize the datasets used in this research and Section 3.2 describes the transposition model selection.

3.1. Meteorological and irradiance data

Two public meteorological databases are selected due to their quantity and quality of their data: EnergyPlus and BSRN. Regarding the EnergyPlus database [16], hourly meteorological data from 2,551 sites were selected across the world (see a map of locations in Fig. 2) of which 1,042 locations belongs to the USA, and more than 1,500 locations are found in many countries around the globe. All these correspond to those locations for which TMY data are publicly available. Weather data are arranged by region and country. Data owners are believed to have made every effort to ensure the quality of the data. Accuracy is variable, varying between 1.5 % and 5%.

A set of filters have been applied according to [25]. In addition, an irradiation tilted condition has been added in GTI calculations [26], which is greater than zero.

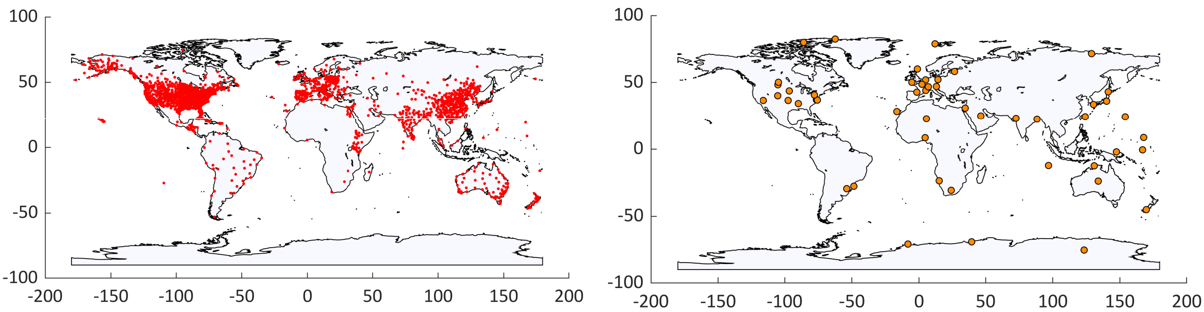

Figure 2: World map of meteorological sites EnergyPlus (left) and BSRN (right) used for model validation.

In addition, a high number of validation data points with 1-minute resolution were collected by 52 research-class BSRN stations in four different climate zones of the world, as shown in Fig. 2. Basic radiation measurements include GHI data in some cases and DNI and DHI in others. The stated accuracies for broadband solar measurements are 15 W/m2. The BSRN data archive is maintained by the WRMC and is updated regularly [17]. Data quality was performed at ETH Zurich.

According to [27], the ground-based decomposition models with the highest accuracy are the DIRINT model [28] and the BRL model, (also known as Ridley2 model) [29]. When the GHI is dominated by the DHI component, these models can have limitations. Nevertheless, the result is considered acceptable for the purposes of this research.

3.2. Transposition model selection

To calculate the diffuse incident irradiance from the DNI, DHI, or GHI data, a transposition isotropic or anisotropic model is used. A transposition model typically takes measured and/or modeled horizontal irradiance components as well as several geometrical parameters (i.e., sun elevation angle, inclination, incidence angle) to predict the irradiance components on the inclined surface. A usual assumption is that diffuse irradiance reflected from the ground into the solar surface is isotropic. In section 3.2.1, different models for the transposition of the diffuse component will be analyzed. The reflectance of the ground, named albedo, has a value between 0 and 1 (typically 0.2); which is discussed in section 3.2.2. Section 3.2.3 presents a comparison and selection of transposition models.

3.2.1. Transposition of the diffuse component

A transposition model was used to calculate the DIF component. The first transposition model [30] emerged in the 1960s. As an isotropic model, it assumes that the DIF is uniformly distributed over the sky dome.

Since the sky radiation has been proven as anisotropic for a long time, a large number of elaborated models have been proposed treating the diffuse component of the sky while taking this characteristic into account [31]. In most cases, the diffuse irradiance collected by a tilted panel is modeled according to a geometric frame of two or three parts. Third generation models treat the sky-diffuse component as anisotropic. Under this category, most of the models break down non-overcast irradiance as the addition of two elements (circumsolar and background sky diffuse). The Perez3 model [32], which consists of three components, is an exception. Early studies compared the the tilted irradiance models performance at daily or longer time frames while current studies work with minute or sub-minutal resolutions.

Many studies have tried to validate and compare transposition models [19]. Most of them tested the models for a maximum of two tilt angle and just one orientation. The RMSE and MBE error metrics are the ones chosen by most authors to compute the accuracy of the models. In some cases, the comparisons between models are more exhaustive [26].

In [19] anisotropic and isotropic models are compared and concluded that an anisotropic model of diffuse sky performs a better estimation of the available solar resource on a sloped surface compared to the diffuse isotropic sky model due to the strong dispersion effect of the aerosols. In [33], after comparing isotropic and several anisotropic sky diffuse models, it is recommended the use of the use of anisotropic models. Also, [34] implemented a similar analysis for daily radiation, concluding that the Gueymard model from 1987 performed the best.

According to Muneer et al.'s (2004) classification of transposition models [35], the validation developed by the researchers of these models, is usually done with data collected in diverse locations, geographically dispersed all around the world. They compared 10–20 models from all three generations. Sometimes, errors are found to be significant due to the approximations used in simplified models [36]. [37] ranked 10 models used to estimate the DIF on an inclined surface placed in Valladolid (Spain) through the RMSE and MBE errors. The result showed the Muneer model [38] achieved first, followed by the Reindl model [33], for daily and hourly values. In [39], 15 models (including the models of Perez [32], Gueymard

[19], Hay [40], and Skarveit [41]) were used. The model's performance, through the MBE and RMSE errors, was evaluated using data from two tilted surfaces measured in Ajaccio (France). The Perez model achieved the best performance among the tested models.

In [42], seven radiation models were checked on tilted surfaces and a simulation was implemented for building energy production. Hay [40] and Perez [32] were among the models tested.

Gueymard [24] compared 10 transposition models that were evaluated against the GTI measured in planes of fixed inclination, oriented to the south and a 2-axis tracker at NREL's Solar Radiation Research Lab. in Golden, CO. Among the selected models were the Hay [40], Skartveit [41], Perez [32], and Gueymard [19] models. The models were tested with irradiance data of one minute. The authors [24] found that under clear skies and for optimal inputs, the Gueymard and Perez models gave the best GTI estimation.

Among the cited studies, only a few developed a detailed study of the performance of the models for different sky conditions and identified [43], the Ma–Iqbal [44], and Perez models as the best ones in "all sky conditions." Different types of datasets were used to compare the estimated irradiance values derived from the various models in [45]. GTI is measured in three inclined surfaces; analyzing the results for clear day by six different models; for bright overcast days the best model's performance is achieved by Muneer [38], Reindl [33], Hay [40], Perez [32],and Gueymard [19] models; for dark overcast days the best results are shown in the Muneer[38], Gueymard [19] and Skartveit [41] models. Since the clouds act as an homogeneous layer giving an isotropic diffusion, in the case of overcast skies, the assumption of isotropy should be more adequate than other study cases. However, for clear or partially clouded skies, this assumption should not be made [46]. The solar resource received by different tilt planes is usually underestimated by isotropic models.

A performance study of 14 transposition models was presented in [47] using 10 min diffuse solar irradiation from the horizontal to inclined surface. For validation purposes, radiation data from eight months were used. The performance of the individual models were evaluated using statistical methods comparing the measured against the calculated global solar radiation on a south-facing surface inclined at a latitude angle. Given overcast conditions, the Perez model achieved the best performance.

The authors of [48] assessed and compared the performances of the four models, for diffuse solar radiation estimation in inclined planes located at Reunion Island (southern hemisphere). The research uses a detailed experimental configuration since there are 14 inclined planes available for testing, more than previous research works. In general, Pérez's model exhibits the best performance.

In [49] four common diffuse radiation models are used to analyse tilted diffuse irradiance of 27 Brazilian capital cities, being the Perez model more suited for the lower latitudes of the Brazilian North and Northeast regions.

Twelve models were tested against recorded south- and west-facing slope irradiances at Karaj (Iran), being Perez1 the model showing the best agreement with the measured tilted data [50].

Yang [26], evaluated 26 transposition models from four different locations (Eugene,Golden ,Oldenburg,Singapore), using the solar irradiance measurements from one year and a total of 18 case studies. According to the performance results of the linear classification in nRMSE, the first four families of models in the ranking were Perez (Perez1 [51], Perez2 [52], Perez3 [32], Perez4 [53]), Muneer [38], Hay [54, 55], and Gueymard [19]. While some models were recommended, a universal model was not found. [56] recommended the Iwaga model, which is based on continuous distribution functions and all-weather conditions to estimate DIF on building facades in Beijing. [12] concluded that, after comparing the results of the of the Perez, Hay, Gueymard and S&O [41], no unique and accurate model exists for all cities in Iran and that each model is suited to a specific location.

Of all the models available in the literature, the Perez family of models has been demonstrated to have a better performance in several sites such as Golden, CO [19], Duebendorf, Switzerland [42], Hong Kong [57], Egypt [58], Karaj, Iran [50], Belgium [47], Ajaccio [39], Isla Reunion [48] or Singapore [59].

The Perez family of models makes a three zone division in the sky hemisphere: the isotropic background, the horizon band, and the circumsolar disc. Perez0 is a more complex model but it had not been validated for different environments [51]. In the Perez1 model, energy from the horizon side is considered to come from an infinitesimally thin region. This simplification of the circumsolar model achieved better results than the original Perez0 model. Perez2 [53] is a Perez1 simplification which assumes that all circumsolar energy emerges from a point source. The coefficients used in Perez1 and Perez2 are fitted using hourly irradiation resolution data (from Trappes and Carpentras, France). Model performance may increase, in specific locations, by using locally-fitted coefficients [60], but for other locations these coefficients may not perform optimally. Hence, the Perez3 model was actualized with other coefficients using more data (from 3 European and 10 American sites), with a different sky's clearness partition. Perez4 has the Perez1 formulation but uses the binning and coefficients used in Perez3.

As a conclusion, among the models analyzed in the literature, and despite the absence of a universal model, the Perez model is chosen by most of the studies, as the most successful model; Perez3 [32] is the most widely used and accepted version of the Perez family. It is also particularly successful in low latitudes [61], but also for cloudy locations [62] or overcast conditions [47] and hence it is the model selected for the diffuse irradiance transposition calculations in this research.

3.2.2. Transposition of the ground-reflected component

A usual hypothesis is for the diffuse irradiance reflected from the ground into the solar surface to be assumed as isotropic, being created from the ground with a view factor of $(1 - \cos\beta)/2$. Models typically consider an albedo default value of 0.2 across sites, based on [63]; this value is generally used all throughout the literature [39].

Moreover, [64] used different albedo coefficients and concluded that there was no improvement in the performance of their estimations when compared to the use of a fixed 0.2 albedo value, for the moderately tilted surfaces. In their work, 12 sky diffuse sub-models were used with four albedo sub-models to estimate the GTI from GHI data. The ground reflected irradiance can be calculated using a simple isotropic model, with low influence on the final energy production. The isotropic model equation used is presented in [65], and it performs in one step the computation of the ground reflected irradiance and its transposition to the plane of array.

3.2.3. Transposition models comparison

Despite the the fact the Perez model has been selected as the diffuse transposition model through an extensive literature review, in this section a sensibility analysis is performed. For each of the 2,551 locations [16], the surface orientation producing the maximum annual energy in a year period was calculated using TMY. GTI was determined for each solar surface configuration with an inclination between 0° and 90° for surfaces oriented to the equator.

Fig. 3 shows the annual optimum tilt angle results by applying three different transposition models, the Perez3 model, selected in section 3.3.1; the Hay model, a simpler anisotropic model that show to perform well in the literature; and the isotropic model of Liu and Jordan. Furthermore, for a better comparison a polynomial regression curve is shown for each model. Perez gives higher inclination values than those obtained by the other two models in accordance with the expected results [12]. The Perez family of models divides the sky hemisphere into three zones and the Hay model assumes diffuse radiation to be composed of circumsolar and uniform background sky-diffuse components. The mean bias differs in line with the values derived from both types of anisotropic model, which increase as the inclination angle of the 250 planes increases. The reason for this increase is that the angle of inclination in the surface increases, the irradiance of the horizon band represents a larger part of the total irradiance. Only three zone models, such as Perez3, account for this region.

When the global radiation is predominantly diffuse, the main radiation is supposed to be received by the horizontal. This approach is reflected in [66]. The optimum tilt angle drops off along the clearness index. The Perez3 model allows to incorporate the effect of frequently overcast skies on the choice of optimum tilt angle [62].

This analysis shows the importance on the transposition model choice and its impact on the optimum tilt angle calculation.

4. Tilt angle model calculation for optimal annual energy yield

To select the solar surface tilt so that the incident energy can be maximized, there are two main approaches: search-based and a direct approach. This paper applies a search-based approach. With this approach, the GTI is calculated for different inclination angles with the desired step size, thus obtaining the optimum tilt angle when solar irradiance along the year is maximized.

As the selected transposition model influences the results [7], section 3 presents a detailed justification. For each of the 2,551 places from EnergyPlus irradiance database, the solar surface orientation and inclination yielding maximum annual energy over a year was obtained. With the aim of generating a simple and general expression for the world, the albedo is set to 0.2 and the surfaces are equator oriented. Equatorial direction provides the best azimuthal alignment in the absence of shading, horizon obstruction, or very different weather conditions during the day; is consistent with the results of previous researches [67]. Latitude, longitude, altitude, and hourly DHI and DNI data of each site are considered. Every solar surface configuration with angles varying in the tilt interval of 0° and 90° was used to calculate the incident irradiance.

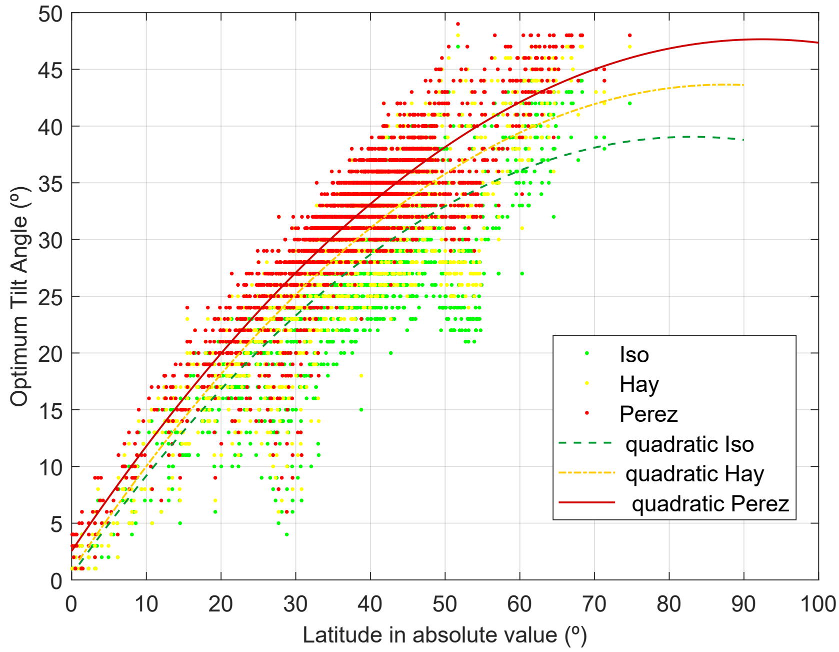

Figure 3: Annual optimum tilt angle as a function of latitude based on the calculation of three transposition models applied to 2,551 sites hourly irradiance data and their corresponding regression curves.

The results obtained for the optimum tilt angle are detailed in section 4.1; section 4.2; section 4.3 and 4.4 the results are validated and analyzed with previous research proposals.

4.1. Tilt angle model for optimal annual energy yield

It has been performed a least squares polynomial fit to the EnergyPlus database and three polynomial orders are shown in Fig. 4 with the corresponding coefficients in table 1, where L refers to latitude in absolute value. In all cases, polynomial models were fitted with the 85% of the data. Error results are calculated with the test set, composed by the 15 % of the remaining data. The convenience of distinguishing the northern and southern hemisphere latitudes has been examined, concluding that with the available data, it is not necessary as has been done up to now, and hence the optimal tilt angle can be expressed as a function of the absolute value of the latitude.

Fig. 4 shows the result of performing a least squares polynomial fit (linear, quadratic, and cubic) applied to the results of the optimum angle as a function of the latitude, calculated using the Perez3 transposition model. It can be concluded that the difference between quadratic and cubic models is bounded in the range of ±0.9° for latitudes 0°-70° and within -1° to +5° for all latitudes. No model can be clearly recommended because residuals for both options are very similar, and the main difference is for extreme latitudes where almost no data are available. Residuals are slightly better for the cubic polynomial: linear (RMSE=3.5565, R^2=0.8050), quadratic (RMSE=3.3982, R^2=0.8295), and cubic (RMSE=3.3934, R^2=0.8298).

Results using the BSRN sites for which enough DNI data were available have also been included. All three polynomial models are shown in Fig. 5 with the corresponding coefficients shown in table 1.

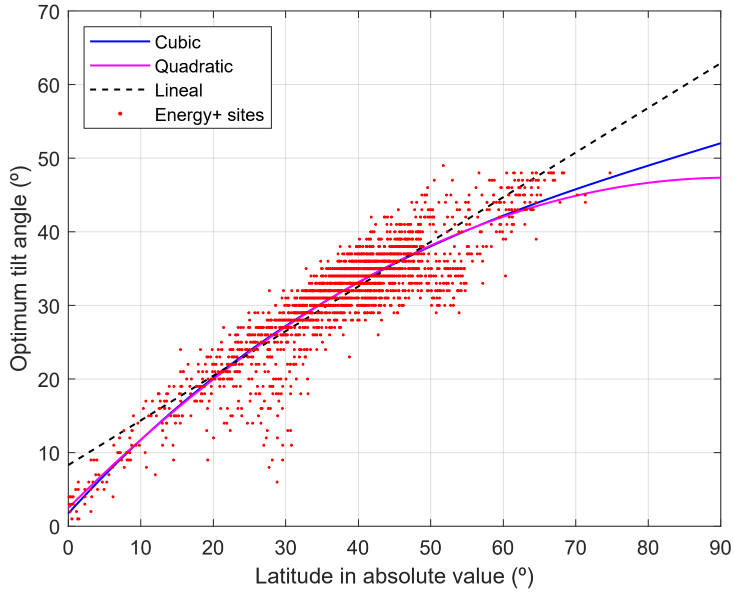

Figure 4: Optimum tilt angle models applying cubic, quadratic, and linear regression polynomials using the 2,551 sites EnergyPlus database.

Table 1: Regression coefficients for the polynomial: $\beta_{opt} = P_0 \cdot L^3 + P_1 \cdot L^2 + P_2 \cdot L + P_3$

Polynomial Degree	Cubic			Quadratic			Linear		
Database	EnergyPlus	BSRN	All-Data	EnergyPlus	BSRN	All-Data	EnergyPlus	BSRN	All-Data
P_0	3.308E-05	4.259E-05	2.904E-05	0	0	0	0	0	0
P_1	-0.008806	-0.01034	-0.008437	-0.005366	-0.005052	-0.0053	0	0	0
P_2	1.083	1.141	1.074	0.9815	0.9667	0.9772	0.6065	0.5557	0.602
P_3	1.738	2.156	1.819	2.479	3.421	2.555	8.311	9.972	8.471

In this case, it can be concluded that the difference between the cubic and quadratic linear regression polynomials is within ±3° for all latitudes.

Furthermore, to obtain some extra data for the extreme latitudes within 70°-90°, a last dataset was created and named All-Data, containing both the EnergyPlus and BSRN databases. The resulting regression coefficients are also shown in table 1. The difference between EnergyPlus and All-Data sets is bounded within ±0.2° for latitudes within 0°-70° and ±0.5° until 90° (see Fig. 6). Residuals for both are very similar and are only slightly better for the latter.

It has been found with the BSRN database that the interannual variability for the optimal tilt angle may vary as much as ±5% when periods of almost 10 years are considered, showing the convenience of using TMY or large time periods for these type of studies.

To facilitate the comparison of the different models and databases, Fig. 7 summarizes the effect of energy loss due to the tilt deviation from its local optimal. All 2,551 sites from the EnergyPlus database was analyzed at any possible tilt angle in 1 degree steps, computing the corresponding energy yield. To compare all of them, the results have been referred to each optimum angle (becoming the 0 in the x-axis) and to each maximum energy yield (becoming 100%). All curves have been overlapped and percentiles at

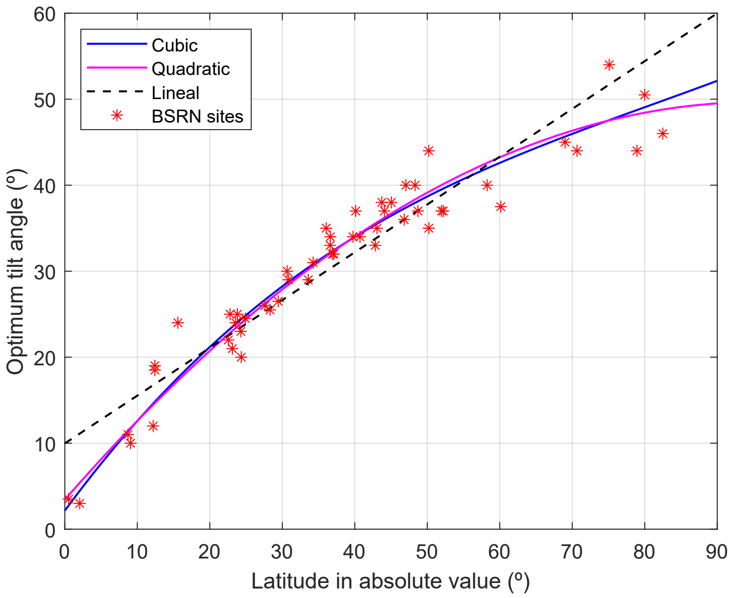

Figure 5: Optimum tilt angle models applying cubic, quadratic, and linear regression polynomials using the 52 sites in the BSRN database.

each tilt angle deviation calculated. In Fig. 7, some of those percentiles have been selected, showing fairly tight results between percentiles 5 to 95, and thus allowing the approximate energy loss to be estimated when deviating from. For angles deviations lower than 3° with respect to the optimum, the energy loss is not significant. The results are consistent with those of other studies indicated in other studies done for specific mid-latitude zone locations [14]. To facilitate quantitative estimations, the polynomial regression for the percentile 50 is shown in Eq. 1 with the equation parameters in table 2.

$$f(x) = p_1 \cdot x^4 + p_2 \cdot x^3 + p_3 \cdot x^2 + p_4 \cdot x + p_5 \tag{1}$$

4.2. Tilt angle model for optimal annual energy yield with neural networks and regression trees

In this subsection, an assessment of whether adding more complexity to the regression models or not is explained and shown. Regression techniques such as neural networks (NN) and decision trees (DT) have been compared with the polynomial models from the previous section.

Similarly to the polynomial models, both the NN and DT have been trained with each database individually as well as with both of them all together. In all cases, a 15% of the data have been kept from training and used only as a test dataset. The other 85% of the data have been used for training and validation (70% and 15% respectively). The validation dataset has been used to determine the characteristics of the regression models. For instance, in the NN, the validation set was used to

Table 2: Energy relative loss equation parameters due to tilt angle deviation for percentile 50.

Parameters	p_1	p_2	p_3	p_4	p_5
Value	7.373E-07	-1.611E-05	-0.01215	0.01304	100.2

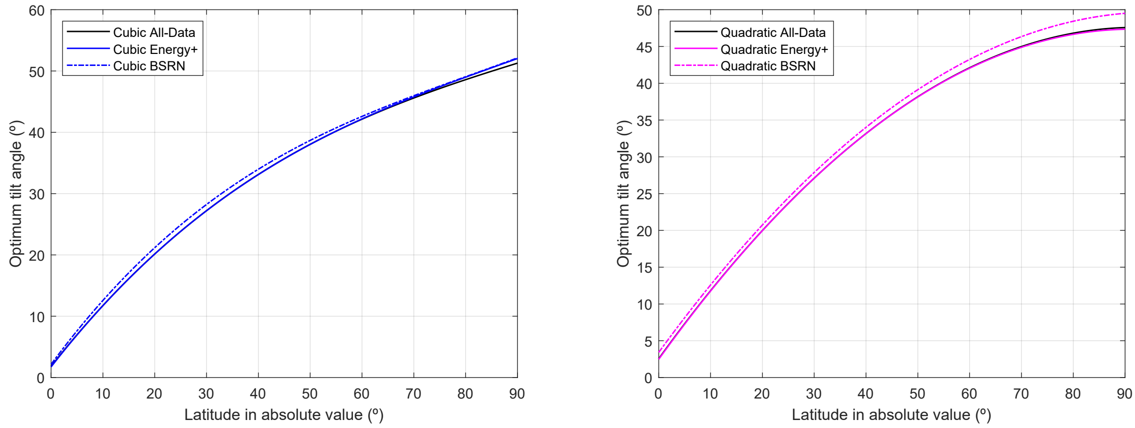

Figure 6: Comparison of optimum tilt angle regression models applying cubic and quadratic polynomials using the three data sets: EnergyPlus, BSRN, and All-data.

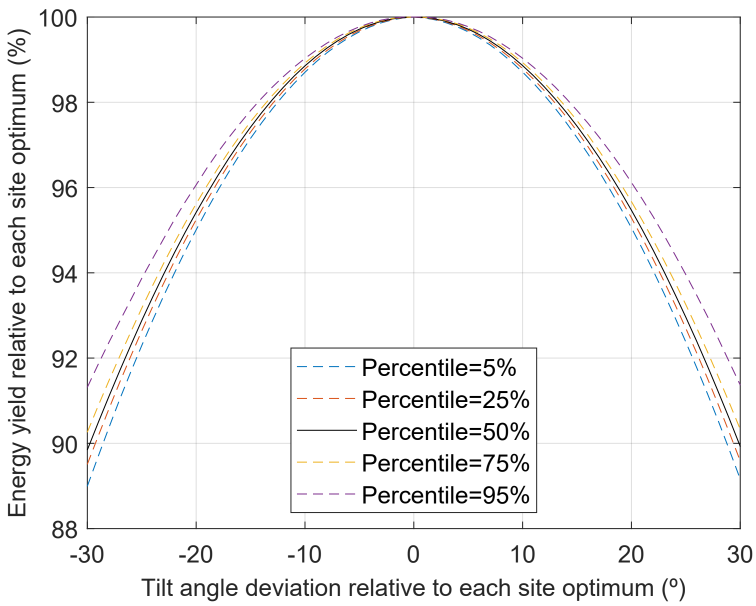

Figure 7: Energy relative percentiles loss due to the tilt angle deviation from a local optimal using the 2,551 EnergyPlus database.

determine the number of neurons in the hidden layer whereas in the DT it was used to determine the number of nodes. The data split has been randomly performed.

The structure of both models keeps the same arrangement than the polynomial algorithms. These provide easier performance comparison between them. Hence, the latitude in absolute value remains as the only input while the output of the models is the optimum tilt. All NN proposed have only one hidden layer with variable neurons depending on the database the model has been trained on. Variable learning gradient descent was the optimization algorithm selected to train the NN. All neurons in the hidden layer use an hyperbolic tangent sigmoid activation function while the output neuron use a linear function. There are always two bias neurons, one in the inputs and another in the hidden layer. The number of neurons is established in table 3. Regarding the DT, there is a variable number of nodes depending on the training database. This number can be seen on table 3. Nevertheless, all DT has the mean squared error as splitting criteria.

Table 3: Variable parameters of the NN and DT regression models for each training database.

Databases:	EnergyPlus	BSRN	All-Data
Number of neurons of the NN hidden layer	4	7	14
Number of nodes of the DT	151	15	111

All the regressions models have been tested with the test dataset, which is composed by locations not trained in the model. Hence, the results are more likely to be similar to those obtained by new locations. The performance metrics used, which can be seen in table 4, are: the mean bias, the mean absolute error (MAE) and the root mean squared error (RMSE). In Fig. 8, the NN and the DT optimum tilt angle models are shown along the polynomial models using both the EnergyPlus and BSRN databases. The behaviour is similar among all models, which is reflected in the error metrics from table 4.

Table 4: Performance metrics comparison among all regression models trained with all databases.

Regression models	EnergyPlus			BSRN			All-Data		
Error	bias	MAE	RMSE	bias	MAE	RMSE	bias	MAE	RMSE
Linear	0.0127	2.7176	3.5565	-1.454	4.1824	4.887	0.0376	2.7286	3.5654
Quadratic	-0.0102	2.5294	3.3982	-0.3846	2.6925	3.3037	-0.0064	2.5448	3.4126
Cubic	-0.0126	2.5223	3.3934	-0.4808	2.5943	3.2467	-0.0054	2.5383	3.4094
NN	0.0072	2.5164	3.3023	-1.1927	2.6518	3.6664	-0.198	2.4665	3.2812
DT	0.0545	2.4115	3.2747	-0.9051	2.9297	3.2267	0.0525	2.4145	3.2888

The performance of NN is highly correlated with the amount of data use for training as can be seen in table 4. Both the NN and DT slightly outperform the more simple polynomial models in almost all situations. While the complexity of these models is greater, the error differences between models are very small. Hence, for the sake of simplicity, the polynomial models are selected from now on to make the comparisons with other optimum tilt models. Nevertheless, the NN and DT can be found in a Github repository.

4.3. Comparison of tilt angle models for optimal annual energy yield

The proposed model (table 1) has been compared with another eight published models as shown in Fig. 9. For this task, the cubic polynomial using all 2,603 sites data from EnergyPlus and BSRN, namely the All-data set, is used. The commonly used rule of thumb, in which the yearly optimum tilt angle is about latitude in absolute value (L) ($\beta_{opt} = L$), [68] is also represented.

In [69], the optimal fixed installation angle is obtained using a direct approach for different time periods and latitudes in the northern hemisphere to maximize global radiation, using the isotropic model developed by [30]; for latitudes below 65°, the yearly optimal angles are positive and about 0.76 multiplied

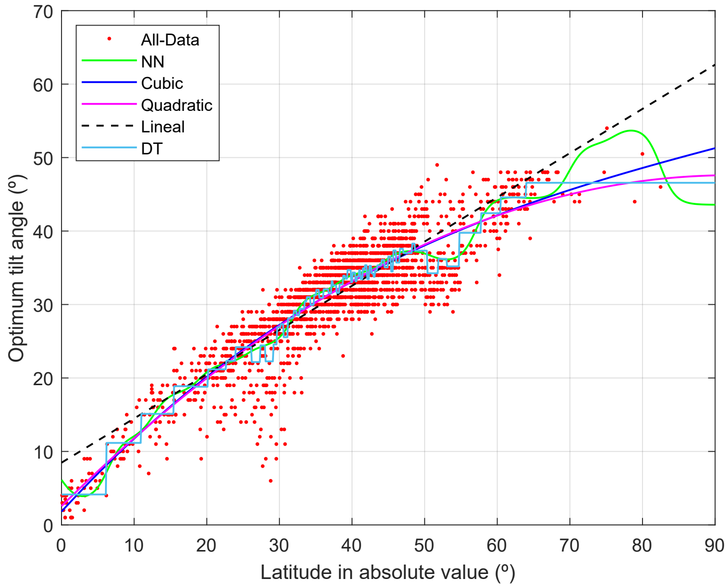

Figure 8: Optimum tilt angle models applying cubic, quadratic, linear, NN and DT regression techniques applied to All-Data.

by the latitude. Above this, the curve is flatter, $0.224 \cdot L + 33.65$. For high latitudes, the standard error of the mean values for irradiation estimation increases significantly.

In [13], the study was performed using global solar radiation data from the European Solar Radiation Atlas. The application of some empirical diffuse solar radiation models suggests that at high latitudes unreliable values can be produced.

[70] used four years data of daily global solar radiation on a horizontal surface to calculate the annual optimal tilt angle in 35 sites in different countries in the Mediterranean region. There is a test representing the relationship between annual optimum tilt and the latitude angles in which the quadratic model performed better, as shown in Eq. 2. the Reindl anisotropic diffuse transposition model is used to calculate GTI.

$$\beta_{opt} = 1.25351 \cdot L - 0.00728944 \cdot L2 \tag{2}$$

A complete photovoltaic generation model is developed (model SoL2015) in [71], and a quadratic relationship is established between latitude and optimum tilt angle in two cases: for clear sky conditions (see Eq. 3) and by TMY irradiance measurements taken in many locations across the world (see Eq. 4).

$$\beta_{opt} = 1.13 \cdot L - 0.004 \cdot L2 \tag{3}$$

$$\beta_{opt} = 2 + 0.92 \cdot L - 0.004 \cdot L2 \tag{4}$$

In [14], the analysis of β_{opt} results by direct approach is made in the mid-latitudes zone of North hemisphere; an isotropic model for diffuse and ground radiation components is used (see Eq. 5).

$$\beta_{opt} = 0.916 \cdot L + 1.171 \tag{5}$$

[72] used data from nine measurement stations in the Southern African Universities Radiometric Network, which was equipped with pyrheliometers and pyranometers to estimate the annual solar insolation in fixed panels with different angles. A linear model is shown in Eq. 6.

$$\beta_{opt} = L \pm 2.6 \tag{6}$$

More than 200 sites are used in [73] to estimate the energy production of grid-connected photovoltaic energy systems throughout the world. NREL's PVWatts simple calculator [74] is used to estimate optimal tilt angles for solar panels. A cubic polynomial fit to the optimal tilt angles as a function of latitude was developed from data for the Northern and Southern Hemispheres.

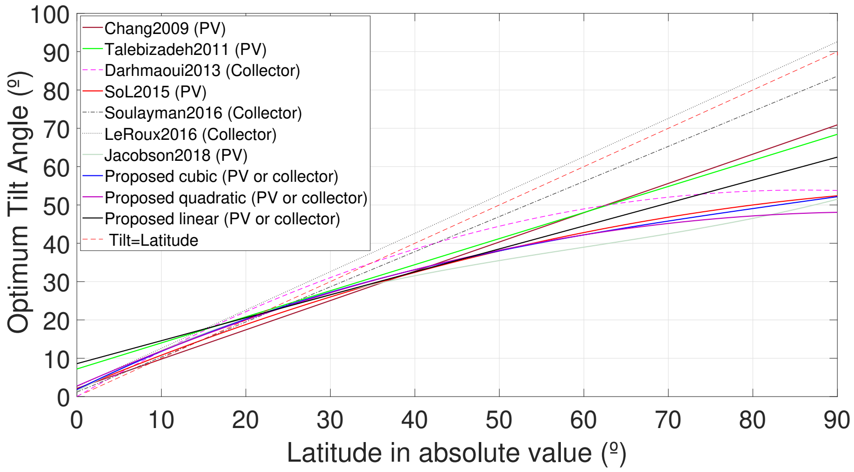

Figure 9: Tilt angle models for optimal annual energy yield as a function of latitude for different models.

In order to compare existing optimum tilt angle models with the proposed ones, both, SC or PV applications have been analysed. As shown in Fig. 9, there are significant differences but as stated previously these differences are only due to the data and the chosen mathematical approach, not related to the application. Among all, there is a group of authors using linear polynomials, Chang2009 [69],Talebizadeh2011 [75],soulayman2016 [14],Le Roux2016 [72]; those models and the proposed linear model in this paper is for comparison purposes only. It is clear the limitations of these kind of models in terms of residuals although some linear models can estimate decently the optimal tilt angle for latitudes within 10°-50°, except for [14] and [72] that indicate too large values close to a clear sky model approach. Among the quadratic polynomial approach, SoL2015, in [71], gets closer results to the proposed models due to the number of sites used in their regression. [70] shows a noticeable difference from [71] with an optimal tilt angle of up to 7° for some latitudes because of the small amount of sites (35) used. Among the only two cubic polynomials, in [73] and that proposed in this paper, the results are similar, although it is believed that the 2,603 sites used in this research must provide a better accuracy in the model as difference reach up to 3.5° for some latitudes.

Table 5: Comparison optimal tilt angle of proposed method against results from PVGIS and MODIS tools.

	BSRN Station									
	TAM	IZA	GOB	CAR	SON	PAL	CAM	CAB	TOR	LIER
Latitude (º)	22.79ºN	28.3093ºN	28.4249ºS	44.083ºN	47.054ºN	48.713ºN	50.2167ºN	51.9711ºN	58.254ºN	60.1389ºN
Proposed Model Cubic All-Data	22.28	26.12	26.19	35.21	36.65	37.43	38.11	38.89	41.50	42.24
PVGIS, TMY Data (CMSAF-SARAH)	24	24	26	38	37	36	39	39	39	40
MODIS (2013 data)	26	28	26	38	38	37.5	38	38	46	42

4.4. Validation of the proposed tilt angle model for optimal annual energy yield

With the aim of validating the proposed model, it has been compared it with the calculations of the annual optimum tilt angle using other software tools as well as published optimum tilt angles that have been validated.

First, the proposed model is compared with results from the PVGIS 5 tool [76] and MODIS method [6], as shown in table 5.

PVGIS is a validated model that obtains solar irradiance data from the Meteosat satellite cloud product; PVGIS calculates the GTI using the Muneer model [38]. Calculations are based on a time series of hourly values over more than 10 years, depending on the database (CMSAF-SARAH).). PVGIS also takes into account spectral, temperature, and wind effects.

The MODIS method uses a radiative transfer approach (MODIS) to calculate DHI, GTI and β_{opt} in all-sky conditions using cloud, ozone, water vapor, and aerosol input data. For this validation, published results in [6] of the MODIS method for the year 2013 and several BSRN sites were used.

Table 5 shows the results for this first validation. The main outcome is that the proposed model applied to randomly selected sites, shows very similar results to the PVGIS (for TMY) and MODIS (for the selected year) models, with an average difference of around 1.5º between the proposed model and both types of software.

Other researches that made annual optimum tilt angle calculations with just one site have been compared; annual optimum tilt angles from 95 sites published by multiple researchers have been gathered and compared with the all dataset chosen to obtain the proposed model [ref]. table A.8 in Appendix A summarizes the following research: [77, 62, 61, 78, 79, 12, 70, 80, 81, 82, 83, 84, 85, 86, 87, 88, 89, 90, 91, 92, 93, 94, 95, 96, 97, 98, 99, 100, 101, 102, 103, 75].

Fig. 10 presents the main findings. Most of the studies use isotropic or horizontal irradiance measurements along one year, which may explain differences between the results in sites of similar latitude. When comparing these values, the maximum difference between them was found to be within the range L $\pm$ 7°, which implies less than 1% energy deviation relative to the optimum. These results do not serve as a faithful contrast, since in no case do they use experimental measures at different inclinations and they usually use different transposition models. In some cases, an isotropic transposition model is used, which may explain why most of them are located on the upper bond of the reference set. Nevertheless, these results are considered positive and consistent with the proposed model.

Next, we analyzed, which fully justifies the differences found between the proposed model and the analysis of other researchers for a single site with short time irradiance measurements.

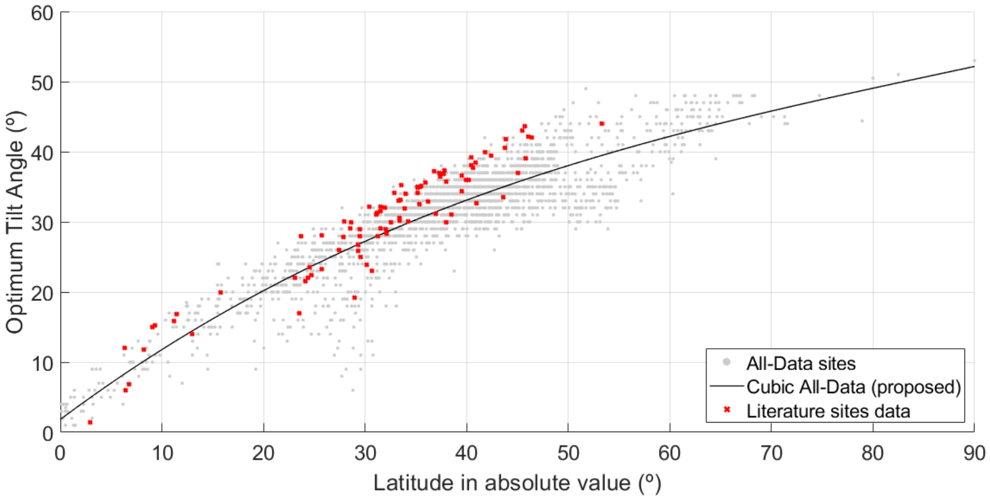

Figure 10: Tilt angle for optimal annual energy yield as a function of latitude for All-Data set and literature review sites.

An eight-year calculation (period 2010-2017) for several BSRN measuring stations is included as an example of the influence of the interannual variation in the calculations of the optimum inclination angle (table 6). Measurement data of GHI, DHI, and DNI have been used; for the same location, possible variations of the annual optimum tilt angle of up to 8% are detected with respect to the average value of the series of data analyzed. The importance of diffuse irradiance in the variation of the optimum interannual inclination angle is clear.

Table 6: Influence of the diffuse irradiance inter-annual variation on the optimal tilt angle calculation.

Year	Opt. Annual Titlt Angle (º)				
	CAB BSRN Station	CNR BSRN Station	NYA BSRN Station	E13 BSRN Station	SAP BSRN Station
2010	34	33	46	32	x
2011	37	34	42	34	36
2012	37	34	44	34	35
2013	37	33	46	33	33
2014	37	32	43	33	35
2015	37	33	44	34	35
2016	39	32	44	34	34
2017	36	35	44	x	34
Median	37	33	44	34	35
Med-Max (%)	5.41	3.03	4.55	0.00	2.86
Med-Min (%)	8.11	0.00	5.55	5.88	5.71

Owing to a lack of long term high-quality irradiance measurements with multiple tilt angles, it is not possible to validate the proposed optimum models using tilted irradiance measurements [6]. Most of the papers which develop or evaluate different transposition models include validation against measured GTI but few are based on experimental measurements at different tilted surfaces such as ([104] and

[105]) to validate the value of the annual optimum tilt angle. The experimental results are summarized and compared with the results obtained in this research (table 7) showing the good performance of the proposed models.

Table 7: Theoretical and experimental comparison for yearly optimal tilt angle.

	[Benghanem 2011 [104]] Latitude 24.55º	Dif(º)	[Dhimish2018 [105]] Latitude 53.3º	Dif(º)
Proposed All-Data cubic	23.54	-0.04	39.46	-0.46
Chang 2009 [69]	23.46	0.04	39.78	-0.78
Darhmaoui 2013 [70]	20.90	2.60	42.86	-3.86
Soulayman 2016 [14]	26.38	-2.88	46.10	-7.10
Le Roux 2016 [72]	23.66	-0.16	49.99	-10.99
Experimental opt tilt (º)	23.50		39.00	

5. Conclusions

The purpose of the study is obtaining the annual optimum tilt angle for PVAs and solar collectors for any latitude worldwide, with more accuracy than the previous proposals using recent large data sets of GHI global data. Two public databases, namely EnergyPlus and BSRN, with data from 2,603 sites across the world have been used to develop and validate the proposed optimum tilt angle model. It has been shown that although results are consistent when considering independent datasets, it is more convenient to treat all data as a whole.

It has been proposed a simple, but precise expression of the optimum tilt angle to be used with solar collectors or photovoltaic arrays, to capture the maximum annual incident solar irradiance at any latitude. The proposed model can be a quadratic or cubic polynomial, although the latter shows a slightly better performance. Adding more complexity to the regression models with neural networks or decission trees only improves the performance by decimals. Hence, for the sake of an easy implementation, the authors suggest the use of either the quadratic or cubic polynomial models.

The proposed polynomial model has been compared with other published regression models, showing large differences with those who have used a small number of sites, and closer results for models with a large number of sites. As the proposed model uses the largest set of quality data, it is believed to be the most accurate model to date. As stated by other researchers, no significant differences have been found in these models between PV or solar collector applications.

Finally, the proposed model has been validated against results by other sources such as PVGIS and MODIS, for a limited number of examples, as well as using some literature with experimental tilt calculations, showing similar and consistent results in all cases.

Appendix A. Annual optimum tilt angle in previous researches

Table A.8: Annual optimum tilt angle in previous researches.

Country	City	Latitude	Tilt Opt Offered by References	Reference
Malasya	Bangi	2.92	1.4	[70]
Nigeria	Benin City	6.33	12	[88]
Nigeria	Enugu	6.46	6	[97]
Turkey	Onitsha	6.78	6.8	[61]
Turkey	Wamba	8.18	11.8	[61]
Nigeria	Abuja	9.058	15	[88]
Turkey	Bauchi	9.29	15.3	[61]
Turkey	Potiskum	11.16	15.9	[61]
Turkey	Bursari	11.46	16.8	[61]
Nigeria	Katsina	12.99	14	[88]
Brasil	Brasilia	15.78	20	[71]
China	Guangzhou	23.13	22	[96]
Brasil	São Paulo	23.547	17	[71]
Bangladesh	Dhaka	23.73	28	[80]
Taiwan	Taichung	24.15	21.5	[85]
United Arab Emirates	Abbu Dhabi	24.4	22	[83]
Saudi Arabia	Madinah	24.55	23.5	[104]
Bangladesh	Mymensingh	24.75	22.4	[87]
Egypt	Luxor	25.69	28.1	[70]
Bangladesh	Rangpur	25.74	23.3	[87]
Australia	Brisbane	27.4	26	[102]
Algeria	Adrar	27.88	27.8	[78]
Egypt	Sharm Sheikkh	27.91	30.1	[70]
USA	Orlando	28.538	29.1	[84]
India	New Delhi	28.61	30	[77]
China	Changsha	29	19.22	[101]
Iran	Zahedan	29.28	26.7	[75]
Iran	Shiraz	29.32	25.88	[75]
Iran	Zahedan	29.49	28	[103]
Siria	Aqaba	29.5	29	[92]
Jordan	Aqaba	29.53	25	[93]
Iran	Kerman	30.15	23.95	[75]
Morocco	Agadir	30.43	32.2	[70]

China	Chengdu	30.67	23	[96]
Iraq	Nasiriyah	31.05	31	[95]
Libya	Syrte	31.19	31.3	[70]
Egypt	Alexandria	31.2	31.3	[70]
China	Shanghai	31.23	28	[96]
Palestine	Gaza	31.5	32.2	[70]
Pakistan	Lahore	31.5	31.5	[98]
Iran	Yazd	31.54	29.05	[75]
Jordan	Amman	31.96	32	[93]
Siria	Amman	32	29	[92]
Morocco	Smara	32.15	28.3	[70]
Iran	Birjand	32.52	29.93	[75]
Libya	Tripoli	32.89	34.1	[70]
Iraq	Baghdad	33.31	33	[95]
Iran	Tabas	33.36	30.16	[75]
Syria	Damascus	33.4	30.56	[94]
Syria	Damascus	33.51	33.1	[70]
Algeria	Mecheria	33.54	35.3	[70]
Lebanon	Beirut	33.89	31.9	[70]
Morocco	Fez	34.02	34	[82]
China	Xi‘an	34.26	30.1	[96]
Morocco	Larache	35.17	34.9	[70]
Cyprus	Nicosia	35.19	35	[86]
Cyprus	Nicosia	35.19	34.1	[70]
Greece	Keraklion	35.34	32.5	[70]
Iraq	Kirkuk	35.47	35	[95]
Syria	Latakia	35.54	35.1	[70]
Malta	Valetta	35.9	35.6	[70]
Syria	Aleppo	36.2	32.95	[94]
Tunisia	Tunis	36.8	37.2	[70]
Turkey	Adana	37	31.2	[79]
Tunisia	Bizerte	37.27	37	[70]
Spain	Seville	37.39	36.5	[70]
Turkey	Isparta	37.76	36.9	[70]
Italy	Marsala	37.8	37.4	[70]
Greece	Athens	37.98	35.7	[70]
Greece	Athens	38	30	[90]
Spain	Valencia	38.46	31	[46]
Turkey	Izmir	39.5	36.6	[81]

Turkey	Erzurum	39.54	34.4	[79]
China	Beijing	39.9	36	[100]
Turkey	Bursa	40.19	36	[70]
Spain	Madrid	40.42	39.2	[70]
Albania	Vlorë	40.47	38.1	[70]
Greece	Thessaloniki	40.64	37.7	[70]
Italy	Naples	40.85	38.5	[70]
Turkey	Istanbul	41	32.6	[79]
China	Shenyang	41.81	40	[100]
Montenegro	Podgorica	42.43	39.5	[70]
Canada	Toronto	43.6	33.5	[91]
Italy	Florence	43.77	40.5	[70]
Bosnia	Sarajevo	43.86	41.8	[70]
Canada	Ottawa	45	37	[91]
Italy	Milan	45.47	43	[70]
France	Lyon	45.76	43.7	[70]
Croatia	Zagreb	45.82	39.1	[70]
Slovenia	Ljubljana	46.06	42.1	[70]
Slovenia	Kredarica	46.38	42	[89]
Ireland	Galway	53.27	44	[62]

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