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Quote Inefficiency in Options Markets

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Abstract

In an arbitrage-free economy with non-zero bid-ask spreads the existence of payoffs whose price is lower than the price of a dominated payoff cannot be discarded in general. However, when the former price corresponds to trivial portfolios which involve buying or selling one unit of the basis assets, its presence is a severe market anomaly which we refer to as an inefficient quote. In an empirical study, we report evidence that indicates that in options markets both the frequency and the magnitude of these anomalies are substantial and we document puzzling patterns in their behavior.

Keywords and phrases: Inefficient Quotes, Bid-Ask Spread, Law of One Price, Index Options

JEL Classifications: C61, C63, D4, G1

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I Introduction

In contrast with other fields of asset pricing, the simple two-period model has gone a long way in developing testable predictions for European options and futures prices. In addition, many of the advances in this simple framework have been accomplished by resorting to fairly weak economic assumptions, namely, the law of one price, absence of arbitrage and absence of second-order stochastic dominance.

The first one and weakest of these assumptions simply precludes different prices for identical payoffs. Most tests of the law of one price are designed to check the validity of one of its specific pricing predictions. For example, Gould and Galai (1974), Klemkoski and Resnick (1980,1979) and Kamara and Miller (1995), to mention just a few, concentrate their efforts on the well-known put-call parity. Protopapadakis and Stoll (1983) and Yadav and Pope (1994) empirically examine the relationship between spot and futures prices whereas Bharadwaj and Wiggins (2001) look into potential misspricings of Box Spread combinations.

Absence of arbitrage is a slightly stronger departure point, which thus includes the law of one price as a special case. In a frictionless market it can be linked to the existence of a bounded solution to the two-period portfolio problem for at least one investor with strictly monotonic preferences, or equivalently, to the existence of a strictly positive stochastic discount factor (SDF). The additional pricing constraints that this assumption brings about are unfortunately less tight and must be formulated in terms of price bounds. Work in this direction was pioneered by Perrakis and Ryan (1984) and Ritchken (1985). Tests of no-arbitrage conditions in this simple framework can be found in Balbás, Longarela and Pardo (2000), Balbás, Longarela...

A third turn of the screw can be accomplished by resorting to the absence of second-order stochastic dominance. The focus is now in discarding a price which will prevent any investor with monotonic and concave preferences from taking a position in the corresponding asset. This approach has been proved fruitful and it has managed to deliver bounds which are tighter than their no-arbitrage counterparts. Important contributions in this area are Levy’s (1985) seminal work and more recently Constantinides and Perrakis (2002). On the empirical front, Constantinides et al. (2009) under quite general assumptions report widespread instances of second-order stochastic dominance.

Recent developments in option pricing seem to indicate that the above assumptions have done their job and that all pricing implications based on them have been exhausted. This is the implicit conclusion that one can draw from, for example, the price bounds derived by Cochrane and Saá-Requejo (2001), and Bernardo and Ledoit (2000).

As it turns out, we can go one step backward rather than forward in our list of assumptions and find, as our arguments will show, unexploited meaningful implications for European options and futures prices which do not even require the concavity assumption on preferences associated to the lack of second-order stochastic dominance.

The focus of analysis in this paper may be illustrated with a real life example. Table I reports a subset of best bid and ask quotes that were observed on October 10, 2006 at 13:36 for End-Of-the-Month (EOM) European options on the CME E-mini
S&P 500 Futures Index with maturity November 29, 2006. Prices are expressed in index points and the current price of the Index is 1355.5.

Arbitrage is not possible at these prices; however, one can still wonder if it is possible to superreplicate the payoff resulting from buying (selling) any of this individual options at a price that is lower (higher) than their associated best ask (bid) quote by using a combination of the available assets.\(^1\) As it turns out, examining this possibility is remarkably simple.

### Table I

This table displays a subset of best bid and ask quotes that were observed on October 10, 2006 at 13:36 for EOM European options on the CME E-mini S&P 500 Future Index with maturity November 29, 2006. The first two columns indicate the asset, the third and fourth columns show the bid and ask prices and their associated volumes are displayed in columns six and seven. The strike of the options is listed in column eight. The current price of the Index is 1355.5. All prices are expressed in index points. The bond quotes refer to the price of one index point to be delivered at maturity. The last two columns present the optimal superreplicating portfolio associated with the inefficient bid quote of asset 4.

<table>
<thead>
<tr>
<th>Asset</th>
<th>Type</th>
<th>Bid</th>
<th>Ask</th>
<th>Vol. Bid</th>
<th>Vol. Ask</th>
<th>Strike</th>
<th>(w^b)</th>
<th>(w^a)</th>
</tr>
</thead>
<tbody>
<tr>
<td>1</td>
<td>Bond</td>
<td>0.992869</td>
<td>0.992869</td>
<td>-</td>
<td>-</td>
<td>150</td>
<td>0</td>
<td>0</td>
</tr>
<tr>
<td>2</td>
<td>Call</td>
<td>18.50</td>
<td>18.75</td>
<td>81</td>
<td>20</td>
<td>1360</td>
<td>0</td>
<td>1</td>
</tr>
<tr>
<td>3</td>
<td>Put</td>
<td>23.25</td>
<td>24.00</td>
<td>61</td>
<td>213</td>
<td>1360</td>
<td>1</td>
<td>0</td>
</tr>
<tr>
<td>4</td>
<td>Put</td>
<td>150.75</td>
<td>155.75</td>
<td>60</td>
<td>60</td>
<td>1510</td>
<td>0</td>
<td>0</td>
</tr>
</tbody>
</table>

Consider, for example, a short position on Asset 4 which at this point would give its holder 150.75 index points and whose payoff is

\[
- \max (1510 - S_T, 0)
\]  

\(^1\)At this minute there were quotes available for the future contract and a much larger number of options. However, for the sake of simplicity and for illustrative purposes, we do not consider them.
where $S_T$ denotes the uncertain value of the underlying index at maturity. Any alternative portfolio of the available assets would now reward its owner with an amount equal to

$$0.993w^b_1 - 0.993w^a_1 + 18.5w^b_2 - 18.75w^a_2 +$$

$$+ 23.25w^b_3 - 24w^a_3 + 150.75w^b_4 - 155.75w^a_4$$

and it will deliver a payoff at maturity given by

$$w^a_1 - w^b_1 + \left( w^a_2 - w^b_2 \right) \max (S_T - 1360) +$$

$$+ \left( w^a_3 - w^b_3 \right) \max (1360 - S_T) + \left( w^a_4 - w^b_4 \right) \max (1510 - S_T, 0) .$$

where $w^a_i$ and $w^b_i$ denote the nonnegative weights of the long and short positions on asset $i$, respectively.

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**Figure 1:** Payoffs in our example

The goal is thus to determine whether there exist weights for which the maximum
value of (2) exceeds 150.75 and whose associated payoff (3) is greater or equal than (1) for all possible values of $S_T$. This latter condition may be found difficult to examine since it involves in principle an infinite number of constraints. Fortunately, almost all of them are redundant. First, note that although $S_T$ can take any nonnegative value, it is fairly safe to expect that it will never exceed, for example, 10 times the current value of the underlying, so that it can be assumed to lie in the interval $(0, 13555)$. Second, the payoff of the portfolio is a piecewise linear function of $S_T$ with kinks at the strike values of the options included. In this particular example, this payoff will at most have kinks at 1360 and 1510. As a result, it is easy to see that (3) will be greater than (1) for all values of $S_T$ in $(0, 13555)$ if and only if it satisfies such condition for all values of $S_T$ in $\{0, 1360, 1510, 13555\}$. The feasible set of super-replicating portfolios is thus defined by these five inequalities together with the nonnegativity constraints on the portfolio weights. Hence, our task of maximizing (2) subject to the super-replicating condition is reduced to solving a simple linear program with eight decision variables and twelve constraints. In our example, the optimal value of its objective function is equal to 153.43 and the corresponding vector of optimal weights is given in Table I. A graphical description of the payoffs involved is presented in Figure 1.\footnote{In order to improve the visibility of the graph, we only plot the payoffs for values of the underlying up to 5000 instead of 13555.}

Clearly, this situation is at odds with a competitive price setting process for two reasons. Firstly, no trader will ever accept the best bid quote for asset 4 (at least a rational investor who is not exposed to exorbitant transaction fees) since the alternative portfolio is clearly superior. This renders the bid quote as uninformative. Secondly,
any market maker can offer a better bid price within the interval $(150.750, 153.43)$, for example, $152$. If this position is taken and therefore she is required to purchase the inefficient put, she only needs to implement the super-replicating portfolio. Her combined position will give a payoff at maturity identical to the one resulting from buying a call option with strike price $1510$, a payoff which is obviously greater or equal than zero regardless of what the final value of the underlying is. Thus, she can pocket the difference $153.43 - 152$ without bearing any risk at all. Furthermore, if the underlying at maturity happens to be above $1510$, an additional profit of $S_T - 1510$ index points may be obtained. This is not an arbitrage opportunity but it is indeed the opportunity of an arbitrage opportunity.

In formalizing this new concept, our theoretical and empirical considerations accommodate the presence of volume constraints and trading fees. Furthermore, in order to go beyond an analysis of the efficiency of individual quotes, we introduce a measure that quantifies the overall degree of quote inefficiency of the market. Our analysis has the virtue of simplicity since, as it has been illustrated above, the definition of these objects and its computation involves simple linear programming.

With these theoretical tools in our hands, we empirically examine the quality of the price-setting process of options market participants. Our evidence indicates a clear presence of these anomalies for a large sample of EOM European options on CME E-mini S&P 500 futures. Furthermore, a deeper analysis of our results shows some puzzling patterns in the behavior of these misspricings.\(^3\) Specifically, we find

\(^3\)The term misspricing is perhaps abusive in as much as these inefficiencies are not outright violations but rather opportunities available to sophisticated traders to achieve certain payoffs via a cheaper superreplicating strategy. However, as exemplified here, they also open the possibility to implement an arbitrage strategy whenever an inefficient quote is accepted.
that quotes tend to be on average inefficient when the associated option, be it a call or a put, is in the money, whereas they are highly efficient when the options lie in the out-of-the-money region. Our findings are largely robust since they do not seem to be explained away by execution risk, margin costs or the unavailability of borrowing and they persist even if we impose that the super-replicating portfolios must contain whole numbers. In addition, by further examining a shorter sample of European option quotes on the Dow Jones EURO STOXX 50 Index, the Dax Index and the FTSE Index, we confirm that our results are not a temporary feature of a particular options market.

The remainder of the paper is organized as follows. The next section presents our notation and preliminaries. Section 3 explains the details of the implementation of our empirical application and its results are presented and discussed. Section 4 concludes.

II Notation and preliminaries

In a two-period model, let \( z \) be an \( N \)-dimensional random vector representing the payoffs of \( N \) securities indexed by \( i = 1, \ldots, N \). These assets can be purchased (sold) at prices \( p^a \in \mathbb{R}^N \) (\( p^b \in \mathbb{R}^N \)) where it is assumed that \( p^b \leq p^a \). Portfolios of these basis assets, represented by vectors \( w \) in \( \mathbb{R}^N \), can also be traded whenever they satisfy certain volume constraints.\(^4\) The set of admissible portfolios is thus defined

\(^4\)Our goal is to model the inside quotes of the limit order book (also known as the best bid and ask quotes). The volume constraints of our model represent the order size of these quotes. We stick to the term "volume constraints" or "attached volume" in our empirical application since we prefer to save the term "size" to refer to the magnitude of the inefficiency of a quote.
as

\[ W \equiv \{ w \in \mathbb{R}^N : -\overline{w}^b \leq w \leq \overline{w}^a \} \]

where \( \overline{w}^a \) (\( \overline{w}^b \)) is a strictly positive \( N \)-vector giving the corresponding maximum values of the long (short) position. To simplify our exposition, we focus on the finite-dimensional case with \( S \) states of nature indexed by \( s = 1, \ldots, S \) which occur with strictly positive probability.\(^5\) The set of traded payoffs is given by\(^6\)

\[ X \equiv \{ x : x = z'w, w \in W \} \]

which is a convex set in \( \mathbb{R}^S \). Furthermore, the function \( \varphi : W \rightarrow \mathbb{R} \) gives the price of any portfolio as

\[ \varphi (w) \equiv p^a w^+ - p^b w^- \]

where \( w^+ \equiv [\max (0, w_1), \ldots, \max (0, w_N)]' \) and \( w^- \equiv (-w)^+ \). Also, it is convenient for our purposes to define the function \( \pi : X \rightarrow \mathbb{R} \)

\[ \pi (x) \equiv \inf_w \{ \varphi (w) : w \in W, w'z \geq x \} . \]  \( (4) \)

which delivers the lowest price at which agents can purchase a payoff that superreplicates \( x \). Note that the optimization defining the above function is a simple linear program.

\(^5\) In the context of our empirical application this assumption is clearly realistic since the relationships between random variables used in this section need only be evaluated at a finite number of states of nature as we will see later.

\(^6\) We skip the label "almost sure" when establishing equalities and inequalities involving random variables. Also, in order to avoid complicating our notation, the same expression may be used to denote a random variable and the vector in \( \mathbb{R}^S \) containing its possible values.
Absence of arbitrage will be assumed from the start which in our context is defined as follows.\textsuperscript{7}

**Definition 1** A portfolio \( w \in W \) is an arbitrage if \( \varphi(w) < 0 \) and \( w'z \geq 0 \).

As it happens, in an arbitrage-free economy where the bid-ask spread is present, a payoff may be generated with different portfolios that also have a different price. More generally, a payoff whose price is lower than the price of another payoff where the former dominates the latter may exist. Obviously, one should expect that the \textit{expensive} payoff will always be avoided by any traders with reasonable preferences. Although the concept of inefficiency underlying this argument can be defined in a broader way, an analysis over general payoffs is less relevant since the corresponding potential inefficiencies may be entirely due to the existence of non-zero bid-ask spreads.\textsuperscript{8} However, this is not the case when the focus is on trivial portfolios that involve buying or selling one unit of the basis assets. An ask (bid) quote for a basis asset which makes its purchase (sell) inefficient is a nuisance since no investor with reasonable preferences will ever accept such a price. If the market maker has incentives to induce as many trades as possible, one should not observe this particular inefficiencies. Formally, one should not observe the following:

**Definition 2** The quote \( p_i^a \) (\( p_i^b \)) is inefficient if \( \pi(x_i) < p_i^a \) (\( \pi(-x_i) < -p_i^b \)).

\textsuperscript{7}This definition is the equivalent in our context of what Ingersoll (1987) labels as arbitrage of the second type.

\textsuperscript{8}In an earlier version of this paper the authors provide further details about this point. Their analysis includes a higher degree of formalization with characterizations and some additional theoretical results which transcend the object of the paper’s empirical analysis.
In the sequel, we will refer to $\pi(x_i)$ as the optimal ask (bid) quote of asset $i$. Thus, whenever the optimal ask quote is not equal to $p^a_i$ and hence the difference $p^a_i - \pi(x_i)$ is strictly positive, $p^a_i$ is defined as inefficient. The value of these differences gives the size of the price improvement that can be implemented in each case. Any market participant can always offer at no risk an ask price $\overline{p}_i$ in the interval $[p^a_i, \pi(x_i)]$ and, if this position is taken, she will profit from this transaction by an amount equal to $\overline{p}_i - \pi(x_i)$. The prospect of this profit, together with competition among investors and those incentives offered by the exchange to induce transactions, should drive the price $p^a_i$ down and make it equal to $\pi(x_i)$ thereby removing these inefficiencies. An identical argument applies when we consider bid quotes and thus one should not observe $\pi(-x_i) < -p^b_i$. Establishing the size of these discrepancies for each bid and ask price in financial markets is by itself a relevant empirical exercise.

It is also interesting to quantify the overall degree of inefficiency of the market. Although one may consider different alternatives, a definition of a measure for such purpose is simple in the absence of volume constraints. However, their presence requires that such definition is done in a more careful way since, even though all the quote improvements associated with inefficient quotes are obviously possible individually, it might not be feasible due to the volume constraints to implement them all together.

Define the objects

$$W^a = \{ w \in W : w \in \mathbb{R}^N_+, w \leq \overline{w}^a \}$$

$$W^b = \{ w \in W : -w \in \mathbb{R}^N_+, -\overline{w}^b \leq w \},$$
that is, the sets of admissible portfolios which contain only long and short positions, respectively. In addition, for any \( w^a \in W^a \) and \( w^b \in W^b \), let \( X (w^a, w^b) \) be the set of payoffs in \( X \) such that
\[
x = \sum_{i=1}^{N} x_i^a + x_i^b
\]
where \( x_1^a, \ldots, x_N^a, x_1^b, \ldots, x_N^b \in X \) and
\[
\begin{align*}
x_i^a & \geq w_i^a z_i, & \forall i, \quad (5) \\
x_i^b & \geq -w_i^b z_i, & \forall i.
\end{align*}
\]
Hence, \( X (w^a, w^b) \) contains all those elements of \( X \) which can be decomposed into the sum of \( 2N \) payoffs that are also in \( X \) and which superreplicate the long positions given by \( w^a \) and the short positions given by \( w^b \) as in (5). Now, consider the function \( f : W^a \times W^b \longrightarrow \mathbb{R} \)
\[
f (w^a, w^b) \equiv \min_{x \in X (w^a, w^b)} \pi (x)
\]
This mapping gives the lowest price at which one can attain the individual long and short positions in \( w^a \) and \( w^b \), respectively.

Note that
\[
w^a p^a - w^b p^b \geq f (w^a, w^b) \geq \sum_{i=1}^{N} \pi (w_i^a z_i) + \pi (-w_i^b z_i) \geq \pi \left( (w^a - w^b)^T z \right)
\]
The first inequality is strict if there is any price inefficiencies associated with any of the positions taken in \( w^a \) and \( w^b \). The second inequality becomes an equality when \( W = \mathbb{R}^N \) (no volume constraints) and it can be strict otherwise. Finally, the
last inequality follows from the fact that any \( x \) satisfying \( x = (w^a - w^b)'z \) is not necessarily in \( X(w^a, w^b) \). Note that when \( W = \mathbb{R}^N \) (no volume constraints), we have that

\[
 f(w^a, w^b) = \sum_{i=1}^{N} \pi(w^a_i z_i) + \pi(-w^b_i z_i),
\]

that is, in the absence of volume constraints all quote improvements are possible individually.

Thus, the value

\[
 w^a^* p^a - w^b^* p^b - f(w^a, w^b)
\]

gives the difference between the corresponding total price of the payoff \( (w^a - w^b)'z \) when the transaction is implemented at the prevailing bid and ask prices of each one of the individual positions assumed in \( w^a \) and \( w^b \), and the minimum price at which this can be done by exploiting any existing price inefficiencies. Thus, it is natural to define a measure of (absolute) overall price inefficiency as

\[
 A(p^a, p^b, w^a, w^b) \equiv \max_{w^a \in W^a, w^b \in W^b} \left( w^{a^*} p^a - w^{b^*} p^b - f(w^a, w^b) \right). \tag{7}
\]

This value can be interpreted as the maximum profit that could be obtained if all inefficiencies were exploited simultaneously as much as allowed by the volume constraints. Also, since \( A \) will depend on the total liquidity of the market, it is interesting to consider also a relative measure given by

\[
 a(p^a, p^b, w^a, w^b) \equiv \frac{A(p^a, p^b, w^a, w^b)}{w^{a^*} p^a + w^{b^*} p^b} \tag{8}
\]
so that one obtains the value of the maximum discrepancy defined in (7) as a fraction of the total value of the transactions available. This ratio satisfies what it is expected from a proper relative measure, that is,

\[ 0 \leq a(p^a, p^b, \overline{w}^a, \overline{w}^b) \leq 1, \]

which follows from the fact that both numerator and denominator in (8) are non-negative and that, since we assume absence of arbitrage, the function \( f(w^a, w^b) \) is non-negative which implies that

\[ w^a p^a - w^b p^b - f(w^a, w^b) \leq w^a p^a - w^b p^b \leq w^a p^a + w^b p^b \leq \overline{w}^a p^a + \overline{w}^b p^b. \]

It is important to note that the computation of optimal quotes and the measure above is an easy task. Indeed, as we have already mentioned the function \( \pi(\cdot) \) is given by the solution of a linear program. It is straightforward to see that this fact also implies that the value in (7) inherits this simplicity. Accommodating integer constraints on the decision variables of these programs is also simple. In this case, a mixed-integer linear program needs to be solved instead.

The goal of the remaining sections is to investigate the presence of quote inefficiencies in option markets and to measure their magnitude. Also, an analysis of any potential patterns in their behavior is implemented.
III  Empirical analysis

A  Data and implementation

Our study relies on a sample containing synchronized minute-by-minute data on all available best bid and ask quotes, together with their attached volume, for End-of-the-Month European options on the CME E-mini S&P 500 Futures Index and for their underlying futures contract over the period May 12, 2006 to December 29, 2006. All these derivatives are traded electronically at the Chicago Mercantile Exchange. The futures contract is one-fifth the size of the standard CME S&P 500 futures contract and it closely tracks the price movements of the S&P 500 index. The options are contracts with expirations available every month whereas the futures contracts have quarterly maturities. In our sample we have options associated with the nine different expiration dates taking place from June 2006 and February 2007. A description of our option data is presented in Table II.

Table II

This table describes the bulk of our main data. Columns 1 and 2 give the number of days and maturities examined, respectively. The total number of quotes is indicated in column 3 and the next two columns indicate in percentage the fraction over this total of call and put options. Finally, the last three columns display the minimum tick, the minimum transaction fees and the monetary value of an index point.

<table>
<thead>
<tr>
<th>#Maturities</th>
<th>#Days</th>
<th>#Minutes</th>
<th>Quotes</th>
<th>Min</th>
<th>Minimum</th>
<th>I.Point</th>
</tr>
</thead>
<tbody>
<tr>
<td></td>
<td></td>
<td></td>
<td>Total</td>
<td>Call (%)</td>
<td>Put (%)</td>
<td>Tick</td>
</tr>
<tr>
<td>9</td>
<td>149</td>
<td>177,898</td>
<td>13,088,864</td>
<td>43.9</td>
<td>56.1</td>
<td>12.5 $</td>
</tr>
</tbody>
</table>

In addition, our proxy for the riskless bond price is computed from the daily BBA LIBOR rate, specified in the appropriate currency, whose maturity is closest
to the expiration date of our derivatives. This value is the primary benchmark for short term interest rates globally and it is used as the basis for settlement of interest rate contracts on many of the world’s major futures and options exchanges (including LIFFE, Deutsche Term Börse, Chicago Mercantile Exchange, Chicago Board of Trade, SIMEX and TIFFE) as well as most Over the Counter (OTC) and lending transactions.\(^9\)

Hence, at every minute we consider all bid and ask quotes offered together with their attached volume for call options, put options and the futures contract with the same expiration date. We filter out those observations which lack either the bid quote or the ask quote. This improves the reliability of our data and guarantees that the bid-ask spread can always be computed. Unconstrained borrowing and lending through the riskless bond is always allowed. Based on this market structure, the value \(\pi(\cdot)\) corresponding to the payoff at maturity resulting from (selling) purchasing one unit of each available call and put option is computed and compared with its (bid) ask quote. A quote inefficiency will be declared every time the appropriate difference between the two numbers exceeds the minimum tick. These objects and their computation involve simple linear programming. Finally, our measure of overall quote inefficiency will be computed at every point in time. Our results will be then examined in several dimensions, namely, time to maturity, interval of the trading session and moneyness.

It should be emphasized that our theoretical framework is realistic in this context. In particular, our two-period model is justified and so is the assumption of a finite

\(^9\)These rates are available at http://www.bbalibor.com.
number of states of nature. To see this latter point, note that even though the constraints of our programs involve random variables which can not be assumed to be discrete, they can be easily proved to hold if and only if a discrete version of them holds. Indeed, if one assumes that the underlying’s value at maturity, is bounded below and above, the typical constraint

\[ w'z \geq x \]

holds if and only if it holds for \( S \) states of nature. \( S - 2 \) of these outcomes correspond to the underlying taken value equal to each available strike price. The remaining two states of nature occur when the underlying takes its maximum and minimum values. Thus, the current payoff structure can be expressed through a payoff matrix whose dimension is equal to \( N \times (K + 2) \) where \( K \) is the number of available strike prices at each point in time. For the purpose of our computations, the minimum and maximum value of the underlying is assumed to be equal to zero and 10 times its prevailing value, respectively. This particular point has also been explicitly illustrated in the example presented in the Introduction.

It is very difficult to establish the exact amount of transactions fees that investors are effectively bearing. Even though those fees charged by the exchange can be observed, clearing fees are usually kept confidential by the clearing firm. We get around this problem in the following way. Rather than speculate about what the value of the fees is, they are disregarded in our calculations of optimal quotes and instead we report the shadow transaction fees, that is, the minimum value of this cost that would eliminate the associated quote inefficiency. In order to compute this
amount, it will be assumed that these unobservable total transaction fees are the same for futures and options and that no fees are charged when trading the riskless bond. This shadow value can then be compared with the observable part of the total fee, that is, the fee charged to market makers by the exchange. This latter quantity which we will refer to as the minimum transaction fees is in our case equal to 0.0069 index points (0.345$). Additional benchmark fees are hard to come by. The most comparable transaction fees reported in the literature are associated with options on the S&P 500. Kamara and Miller (1995) and Ackert and Tian (2001) use values of 2$ of 4$ per option whereas Constantinides et al. (2009) set these fees to values ranging from 19 to 75 cents. We will thus use 4$ as the maximum transaction fees.

In addition, we also wish to quantify the potential tightening of the bid-ask spread that can be accomplished by setting the inefficient quote equal to its optimal value. We thus report the actual size of this inefficiency as a percentage of the spread and we refer to this value as the relative spread improvement.

Most of our theoretical constructions and notation can be illustrated by referring again to our example presented in Table I. In this case $N = 4, K = 2$ and $S = K + 2 = 4$. Specifically, at that point the price of the Index is equal to 1355.5 and hence the minimum and maximum assumed for the value of the index at maturity is 0 and 13555. These values together with the two different strikes, 1360 and 1510 give the 4 states of nature for the price of the underlying at maturity that need to be considered in these case. Thus, checking for inefficiencies entails solving a linear program with $2 \times N = 8$ decision variables (long and short positions in each asset), and $S = 4$ constraints (plus the nonnegativity constraints on the decision variables).
At these prices only the bid quote 150.75, attached to the call option whose strike price is 1510, is inefficient. Indeed, the optimal bid quote of this option can be proved to be equal to 153.43 which is the value of the function in (4), multiplied by $-1$, evaluated at the payoff resulting from selling one unit of this call option. The optimization that defines this function is solved for the portfolio indicated in the last column of the table, that is,

$$-\pi (-x_2) = -\varphi (w^*) = 153.43$$

where

$$w^* = (-150, 1, -1, 0)' .$$

The discrepancy between the bid and optimal quote is thus equal to 2.68 index points (134\$) which is larger than the minimum tick of 0.25 index points (12.5\$) for these options. This means that this bid quote is considered inefficient in our study. Furthermore, the shadow transaction fee is obtained as

$$\frac{2.68 - 0.25}{2} = 1.215 \text{ (60.75\$)}$$

which is equal to the size of the inefficiency minus the minimum tick, divided by the number of options and futures contracts involved in $w^*$. This value is approximately 176 times larger than our minimum transaction fees (the ones paid by market makers to the exchange) and around 15 times larger than our maximum transaction fees (the largest ones reported in Kamara and Miller (1995)).
Also, the relative spread improvement is computed by dividing the size of the inefficiency with the bid-ask spread, that is,

\[
\frac{2.68 - 0.25}{155.75 - 150.75}
\]

an amount representing a potential improvement of the spread as high as 48.6%.

Although there are 60 contracts offered at the inefficient bid quote, the number of units available of its associated super-replicating portfolio is only 20 (the volume constraint on purchases of the call option becomes binding at this point; see Table I). Hence, this is the maximum number of contracts out the total of 60 that can be offered at the improved price. We will refer to this quantity as the implied volume of its associated quote inefficiency.

The measure \( A \), whose details are given in Section II, quantifies the economic value of overall inefficiency at each point in time. Its value can be interpreted as the maximum profit that could be obtained if all the inefficiencies detected were exploited simultaneously as much as allowed by the volumes constraints. Given the available set of assets in this example, the bid quote of asset 4 is the only inefficient one. Furthermore, there is no other way to improve this quote than by using the indicated super-replicating portfolio. Hence, the value of the measure of absolute overall quote inefficiency, \( A \), can be computed in this case by multiplying the implied volume of 20 by the difference between the size of the inefficiency and the minimum tick, \( 2.68 - 0.25 \), giving a value of 48.6 (2430$). If more inefficiencies were present \( A \) could be obtained by summing over the products of the inefficiency sizes (minus the minimum tick) and their associated implied volumes as long as
their super-replicating portfolios do not overlap. However, if this overlapping is present, the value of $A$ cannot be reasoned in this simple way. Indeed, the attached super-replicating portfolios of any other inefficient quote could require part of the contracts already contained in $w^*$ and thus the two super-replicating portfolios could not be implemented at the same time. In the presence of this overlapping, the linear program that computes the value of $A$ prioritizes those inefficiencies which add a larger value to its objective function. Furthermore, it leaves out those ones attached to a super-replicating portfolio whose volume has been exploited by the former ones and which as a result cannot be implemented at the same time.

Given our focus on the prices of call and put options in our empirical analysis, we do not include the futures contracts or the bond in the objective function that once optimized gives the value of $A$, even though we allow this type of assets to be part of the super-replicating portfolios.\footnote{Even if we did, our results would not change given the assumed zero bid-ask spread for the riskless bond and the fact that the futures quotes are almost never inefficient since their bid-ask spread is rarely greater than the minimum tick.} Hence, the relative measure of overall quote inefficiency, $a$, will not include the future contracts and bonds to calculate the maximum discrepancy as the total value of transactions. In our example, $a$ is equal to 18 basis points, that is, the monetary value of the overall inefficiency is equal to a fraction of 0.18\% the total value of all volume available.\footnote{$a = \frac{81 \times 18.5 + 20 \times 18.75 + 61 \times 23.25 + 213 \times 24 + 60 \times 150.75 + 60 \times 155.75}{48.6} = 48.6$}

\section*{B Results}

In reporting our findings, we will often make use of both conditional and unconditional averages when it comes to values associated with the magnitude of these
anomalies (shadow transaction fees, size, relative spread improvement, absolute and relative overall price inefficiency). The former is computed as the average given that quote inefficiencies exist whereas the latter takes the average over all observations assigning value zero to those observations in which no inefficiencies appeared.

A summary of the general results is presented in Table III. As we can see in column 2, at least one inefficient quote is present in around 90% of the minutes that have been considered. Given the large number of available quotes per minute in our sample, this may sound unremarkable. However, things become more revealing when looking at the fraction of inefficient quotes over the total number of quotes; approximately one out of every seven quotes is declared uninformative. Overall, call quotes are slightly more inefficient than put quotes and so are ask quotes with respect to their bid counterparts.

### Table III

This table displays a summary of our main results broken down by quote category if applicable. The first column gives the category. The second column shows the percentage of minutes in which at least one inefficient quote was detected and the third one presents the fractions of uninformative quotes in each indicated category. All remaining columns report average values given that inefficient quotes were present and if applicable, unconditional values are shown in parenthesis. Size is listed in column four whereas shadow transaction fees and relative spread improvements are reported in columns five and six, respectively. The value of our absolute measure of overall inefficiency and its corresponding relative value are indicated in the next two columns. The last column contains the values of implied volume. B.P. stands for basis points.

<table>
<thead>
<tr>
<th>Inefficient Quotes</th>
<th>Min(%)</th>
<th>Size($)</th>
<th>S.T.F ($</th>
<th>RS$I(</th>
<th>A ($</th>
<th>a (B.P)</th>
<th>l. Vol</th>
</tr>
</thead>
<tbody>
<tr>
<td>Total</td>
<td>89.13</td>
<td>14.71</td>
<td>44.78</td>
<td>15.12</td>
<td>29.35</td>
<td>34,475.00</td>
<td>0.299</td>
</tr>
<tr>
<td>Call</td>
<td>15.46</td>
<td>(6.59)</td>
<td>(2.22)</td>
<td>(4.32)</td>
<td>(5,071.27)</td>
<td>(0.044)</td>
<td>110.88</td>
</tr>
<tr>
<td>Put</td>
<td>14.13</td>
<td>(6.15)</td>
<td>(2.23)</td>
<td>(4.06)</td>
<td>47.42</td>
<td>15.78</td>
<td>28.77</td>
</tr>
<tr>
<td>Bid</td>
<td>12.78</td>
<td>(6.06)</td>
<td>(2.11)</td>
<td>(3.72)</td>
<td>42.77</td>
<td>14.03</td>
<td>29.51</td>
</tr>
<tr>
<td>Ask</td>
<td>16.64</td>
<td>(7.12)</td>
<td>(2.33)</td>
<td>(4.91)</td>
<td></td>
<td></td>
<td></td>
</tr>
</tbody>
</table>

A summary of the general results is presented in Table III. As we can see in column 2, at least one inefficient quote is present in around 90% of the minutes that have been considered. Given the large number of available quotes per minute in our sample, this may sound unremarkable. However, things become more revealing when looking at the fraction of inefficient quotes over the total number of quotes; approximately one out of every seven quotes is declared uninformative. Overall, call quotes are slightly more inefficient than put quotes and so are ask quotes with respect to their bid counterparts.
As for the magnitude of these inefficiencies, size represents nearly four times the minimum tick on (conditional) average and their overall level, as measured by $A$, is also quite substantial. In addition, spreads associated with inefficiencies can be reduced by around 30% if optimal quotes are offered instead. Obviously, if one takes transaction fees into consideration, these numbers will be somewhat reduced. Nevertheless, given the reported values of the shadow transaction fees, the actual fees would have to be 44 and nearly 4 times larger than the minimum and maximum transaction fees, respectively.

The reported implied volumes confirm that the volume constraints on the super-replicating portfolios are not restricting potential improvements to a small fraction of the contracts offered at an inefficient price. These volumes are slightly larger for bid and put quotes.

A partial description of the behavior over time of our results is given in Figure 2 where averages over the nine different expirations are shown. As we can see in the upper-left plot, far from maturity the presence of inefficient quotes is prominent, although it evolves in a heavily fluctuating fashion. During the second last month the percentage of inefficiencies declines and becomes more stable. The series concludes with a trend upwards that begins thirty days from maturity. Remarkably, this pattern is almost exactly replicated by the unconditional average of the relative spread improvement whose series is displayed in the bottom-right graph. The unconditional value of our measure of relative overall inefficiency, $a$, also seems to evolve in a similar manner. This similarity is not clearly visible in the upper-right plot due to the extreme values recorded over the last two days before expiration. By
Figure 2: Daily averages

presenting in a smaller box the same values within two and sixty days to maturity, the scale of the graph allows us to visualize this common pattern.\textsuperscript{12} In any case, the evolution of all these series does not seem to comove with the number of available quotes as it can be concluded when they are compared with the trends shown in the bottom-left plot.\textsuperscript{13}

\textsuperscript{12}The correlation between the percentage of inefficiencies and $a$ is strongly significant and equal to 0.62.

\textsuperscript{13}It should be emphasized that this does not imply that the probability of finding an inefficient quote at a given instant is not affected by the number of available quotes.
Figure 3: Averages along the trading session
Figure 3 shows graphs for the same magnitudes as in the previous figure but in this case we report instead averages over the whole sample period for each 30-minute interval of the trading session. As we can see, if we exclude the first interval of the session, a U-shaped pattern seems to dominate all graphs and thus inefficiencies, both in frequency and magnitude, are more prominent during the first and last intervals of the session. In this case, we find a stronger connection between the evolution of inefficiencies and the number of available quotes.

The breakdowns reported in the summary results describe a picture which seems to be consistent along the two time dimensions examined above. As it is shown in Figure 4, ask and call quotes are uniformly more inefficient than their bid and put counterparts, respectively.

We turn now to examine our results with respect to the moneyness of the options, where the latter is defined as the strike price divided by the current value of the underlying. Figure 5 shows what is perhaps the most striking result of this empirical study. As we can see in the upper-left graph, quotes are very often inefficient when the associated option, call or put, is in the money. As soon as the out-of-the-money region is entered, the frequency of uninformative quotes drops dramatically and it becomes negligible. The unconditional averages of the relative spread improvement is shown in the upper-right plot. Quotes are on average inefficient when the option is in the money with potential relative spread improvements which mostly lie well-above 20% outside the at-the-money interval. It is important to emphasize that these are unconditional averages, so the values reported are computed over all options regardless of whether they are declared inefficient or not. In this light, the scope
Figure 4: Breakdowns over time to maturity and interval of the trading session
of the severity of these results becomes clearer since Figure 5 provides evidence suggesting that not only transaction fees are unlikely to explain away the presence of these inefficient quotes when they appear but also that it seems safe to conclude that quotes are inefficient on (unconditional) average when they correspond to an in-the-money option. In the bottom panels of Figure 5 we examine any potential disparities between bid and ask quotes. The higher frequency of inefficient ask quotes and their larger misspricings appear to prevail over almost all the spectrum of moneyness.

We have extended our analysis over the previous three dimensions by dividing each group of call and put quotes into their associated bid and ask prices and thus creating a breakdown into four categories. Our results indicate no major insights that could enrich our investigation and for the sake of brevity we choose not to report them. In any case, the patterns over moneyness for call and put quotes uncovered above persist after this additional breakdown.

To summarize this section, we have presented quite solid evidence documenting the prominence of inefficient quotes. In addition, we find that these anomalies are concentrated around the in-the-money region of the options. Our next step is to examine the robustness of these two conclusions.

C Robustness checks and discussion

C.1 Other markets, another time

Our documented anomalies may be temporary and/or market specific and not a stable feature of option markets. In order to address this concern, we have also examined option prices from another three indexes, namely, Dax, FTSE 100 and
Figure 5: Inefficiencies vs. moneyness
Dow Jones EURO STOXX 50. This sample, retrieved from Bloomberg, contains synchronized minute-by-minute data on all available best bid and ask quotes for European options and futures contracts on the three indexes, together with their attached volume, over the period November 24, 2005 to January 31, 2006. All these contracts have maturity on March 17, 2006. Once again, all observations which lack either the bid or the ask quote have been eliminated.

Our proxy for the riskless bond price is again computed from the daily BBA LIBOR rate specified in the appropriate currency. A description of this new option data is presented in Table IV.

Table IV

This table describes the bulk of our additional option data. The first column displays the corresponding underlying of the options. Columns 2 and 3 give the number of days and minutes in each one of the three markets, respectively. The total number of prices is indicated in column 4 and the remaining columns indicate in percentage the fraction over this total of call and put options observed. Finally, the last three columns display the minimum tick, the minimum transaction fees and the monetary value of an index point.

<table>
<thead>
<tr>
<th></th>
<th>#Days</th>
<th>#Minutes</th>
<th>Total</th>
<th>Call (%)</th>
<th>Put (%)</th>
<th>Min.</th>
<th>Minimum</th>
<th>I. Point</th>
</tr>
</thead>
<tbody>
<tr>
<td>Dax</td>
<td>48</td>
<td>21,553</td>
<td>1,124,449</td>
<td>46.00</td>
<td>54.00</td>
<td>0.5€</td>
<td>0.35€</td>
<td>5 €</td>
</tr>
<tr>
<td>Ftse</td>
<td>45</td>
<td>19,774</td>
<td>370,984</td>
<td>49.43</td>
<td>50.57</td>
<td>5£</td>
<td>0.25£</td>
<td>10£</td>
</tr>
<tr>
<td>E. Stoxx</td>
<td>44</td>
<td>19,553</td>
<td>585,788</td>
<td>46.69</td>
<td>53.31</td>
<td>1 €</td>
<td>0.30€</td>
<td>10 €</td>
</tr>
</tbody>
</table>

Although these options are not on the future contract, it is important to emphasize that the underlying indexes themselves are never part of the portfolios under consideration. We use instead their synthetic replica involving the future contract and the bond. A summary of the general results is presented in Table V. As we can see, the frequency of inefficient quotes and their associated magnitude are even more
substantial. Furthermore, both the conditional and unconditional shadow transaction fees clearly exceed the value of the minimum transaction fees (the fees paid by market makers). The relative spread improvement is around 5% larger than reported for the main sample. In addition, call options are more often inefficient than put options, specially for the Dax and Euro Stoxx markets, a feature that our E-mini options also displayed.

Table V

This table displays a summary of our results for options on the Dax, FTSE 100 and Dow Jones EURO STOXX 50 indexes, broken down by quote category if applicable. For each panel, the first column gives the corresponding category. The second column shows the percentage of minutes in which at least one inefficient quote was detected and the third one presents the fractions of uninformative quotes in each indicated category. All remaining columns report average values given that inefficient quotes were present and if applicable, unconditional values are shown in parenthesis. Size is listed in column four whereas shadow transaction fees and relative spread improvements are reported in columns five and six, respectively. The value of our absolute measure of overall inefficiency and its corresponding relative value are indicated in the next two columns. The last column contains the values of implied volume. B.P. stands for basis points.

<table>
<thead>
<tr>
<th>Panel A: DAX Options</th>
<th>Inefficient Quotes (%)</th>
<th>Size ($)</th>
<th>S.T.F. ($)</th>
<th>RSI (%)</th>
<th>A ($)</th>
<th>a (B.P.)</th>
<th>I. Vol.</th>
</tr>
</thead>
<tbody>
<tr>
<td>Total</td>
<td>92.14</td>
<td>25.14</td>
<td>10.93</td>
<td>3.57</td>
<td>34.86</td>
<td>-23.818</td>
<td>6.1785</td>
</tr>
<tr>
<td>Call</td>
<td>30.46</td>
<td>11.32</td>
<td>3.38</td>
<td>34.55</td>
<td>27.59</td>
<td>-23.418</td>
<td>608.00</td>
</tr>
<tr>
<td>Put</td>
<td>20.66</td>
<td>20.6</td>
<td>3.8</td>
<td>3.7</td>
<td>30.25</td>
<td>6.3025</td>
<td>638.25</td>
</tr>
<tr>
<td>Bid</td>
<td>24.18</td>
<td>2.15</td>
<td>0.78</td>
<td>7.26</td>
<td>26.91</td>
<td>3.6825</td>
<td>398.94</td>
</tr>
<tr>
<td>Ask</td>
<td>26.09</td>
<td>10.77</td>
<td>3.1</td>
<td>34.91</td>
<td>28.21</td>
<td>3.9625</td>
<td>416.54</td>
</tr>
</tbody>
</table>

<table>
<thead>
<tr>
<th>Panel B: FTSE Options</th>
<th>Inefficient Quotes (%)</th>
<th>Size ($)</th>
<th>S.T.F. ($)</th>
<th>RSI (%)</th>
<th>A ($)</th>
<th>a (B.P.)</th>
<th>I. Vol.</th>
</tr>
</thead>
<tbody>
<tr>
<td>Total</td>
<td>70.18</td>
<td>20.38</td>
<td>18.1</td>
<td>5.49</td>
<td>27.51</td>
<td>9.96610</td>
<td>1.034</td>
</tr>
<tr>
<td>Call</td>
<td>20.41</td>
<td>17.42</td>
<td>5.23</td>
<td>28.09</td>
<td>25.71</td>
<td>10.03</td>
<td>416.79</td>
</tr>
<tr>
<td>Put</td>
<td>20.34</td>
<td>18.76</td>
<td>5.74</td>
<td>26.91</td>
<td>25.81</td>
<td>435.58</td>
<td>436.72</td>
</tr>
<tr>
<td>Bid</td>
<td>19.66</td>
<td>17.4</td>
<td>5.06</td>
<td>26.76</td>
<td>28.18</td>
<td>436.72</td>
<td>416.54</td>
</tr>
</tbody>
</table>

31
Table V (Continued)

<table>
<thead>
<tr>
<th>Panel C: STOXX Options</th>
</tr>
</thead>
<tbody>
<tr>
<td>Min(%)</td>
</tr>
<tr>
<td>--------</td>
</tr>
<tr>
<td>Total</td>
</tr>
<tr>
<td>Call</td>
</tr>
<tr>
<td>Put</td>
</tr>
<tr>
<td>Bid</td>
</tr>
<tr>
<td>Ask</td>
</tr>
</tbody>
</table>

This distinctive evolution may arise as a result of the patterns over moneyness discussed earlier. Indeed, if we consider the availability of call and put option quotes in all four markets broken down into two moneyness intervals as displayed in Table VI, we can see that call option quotes are more often attached to strikes belonging to the in-the-money region than put option quotes.

Table VI

This table displays in its first column the corresponding underlying of the options. Column two reports the percentage of call quotes out total call quotes whose strike price moneyness is less than 1.02. Column three gives the percentage of put quotes out of total put quotes whose moneyness is greater than 0.98. m stands for moneyness defined as strike price divided by the current value of the underlying.

<table>
<thead>
<tr>
<th>Call quotes (%) with m &lt; 1.02</th>
<th>Put quotes (%) with m &gt; 0.98</th>
</tr>
</thead>
<tbody>
<tr>
<td>E-mini</td>
<td>66.87</td>
</tr>
<tr>
<td>Dax</td>
<td>65.69</td>
</tr>
<tr>
<td>Ftse</td>
<td>77.58</td>
</tr>
<tr>
<td>E. Stoxx</td>
<td>73.58</td>
</tr>
</tbody>
</table>
A quite consistent pattern across all markets is also found when considering bid and ask quotes. The inefficiency of the latter seems to be more common than that of the former. The fact that (inefficient) bid quotes are always bounded below by zero whereas (inefficient) ask quotes are in principle unbounded above may provide a simple explanation for this feature.

Finally, the behavior of quote inefficiencies with respect to the moneyness of the options is displayed in Figure 6. The patterns that these plots uncover are identical
to the ones reported in the previous section.

C.2 Execution risk

One possible source of concern in evaluating our results may be the presence of a high degree of execution risk. Indeed, when a market maker (or another type of investor) proceeds to improve an existing inefficient quote, his or her improved position is riskless as long as the super-replicating portfolio stays on the electronic book. If this portfolio becomes unavailable soon after the position has been improved, this may stop investors from taking advantage of these opportunities (of arbitrage opportunities). A first reply to this concern is that if this type of risk is prominent, it should also prevent investors from taking advantage of regular arbitrage. However, in our entire sample we have not detected any exploitable arbitrage violations. As a second reply, we have performed an attempt to establish the size of this risk by using as a proxy the length of the time interval that the optimal quote which could replace the inefficient quote (adjusted by the minimum tick) would remain riskless. The average length of this time interval in our sample is equal to 15.94 minutes. This number seems high enough to safely discard execution risk as a serious concern.

C.3 Dropping the perfect divisibility assumption

Another consideration might stem from the characteristics of the positions of the super-replicating portfolios. Even though the assumption of perfectly divisible assets is used in calculating these portfolios, in reality one can not purchase a fractional number of derivatives. In order to address this issue, we have repeated our calcula-
tions by imposing that optimal quotes must rely on portfolios whose positions, with the exception of the riskless asset, must all be multiples of 100 due to the fact that the options can only be traded in contracts of 100.\textsuperscript{14} Thus, we have replaced the standard linear program that gives optimal quotes in (4) by a linear mixed-integer optimization. Our general results can be found in Table VII and Figure 7. As we see, the effect of relaxing the assumption of perfect divisibility is almost negligible.

C.4 Constrained borrowing and margin requirements

The assumption that traders have access to unconstrained borrowing may be in some instances questionable. In addition, Frazzini and Pedersen (2012) emphasize the extreme embedded leverage of deep-in-the-money options as a source of important pricing effects. To remove any potential explanations based on any of these arguments, it is important to take a look at our results when this assumption is relaxed. Any attempt to fully introduce borrowing constraints must include three elements. First, borrowing should be excluded as a possibility. Second, option writing should not be allowed unless realistic margin requirements are considered. Finally, the total cash outlay of the super-replicating portfolio should not exceed the one associated with the option being replicated. Clearly, the latter is not relevant here since by definition the combined positions associated with an inefficient quote involve no losses. For this very same reason, margin costs are negligible for market makers since their accounts are always margined on their net positions.

\textsuperscript{14}In order to accomplish this, we made use of the ILOG CPLEX technology which offers fast algorithms to solve mixed-integer programs.
Table VII

This table summarizes our results for most of our robustness checks. The first row contains the benchmark case discussed in the main text. The second one analyses the case where positions are constrained to be integer. The case of constrained borrowing is presented in the third row. The next row displays the results corresponding to the case where margin costs are addressed by eliminating all option writing. The fifth row presents the case that combines the two previous cases. The final entry contains the results that discard those quotes whose associated implied volatility takes extreme values. For all these four cases, column two reports the percentage of minutes in which at least one inefficient quote was detected and the third one presents the fractions of uninformative quotes. The last three columns display the conditional average values of size, shadow transaction fees and relative spread improvements whereas their unconditional counterparts are shown in parenthesis.

<table>
<thead>
<tr>
<th></th>
<th>Min (%)</th>
<th>Inefficient Quotes (%)</th>
<th>Size($)</th>
<th>S.T.F ($)</th>
<th>RSI (%)</th>
</tr>
</thead>
<tbody>
<tr>
<td>Real</td>
<td>89.13</td>
<td>14.71</td>
<td>44.78</td>
<td>15.12</td>
<td>29.35</td>
</tr>
<tr>
<td>Integer</td>
<td>85.68</td>
<td>12.39</td>
<td>51.80</td>
<td>13.42</td>
<td>30.34</td>
</tr>
<tr>
<td>C.B</td>
<td>86.21</td>
<td>11.50</td>
<td>44.55</td>
<td>12.89</td>
<td>26.38</td>
</tr>
<tr>
<td>M.C</td>
<td>86.19</td>
<td>9.63</td>
<td>48.50</td>
<td>19.79</td>
<td>30.39</td>
</tr>
<tr>
<td>C.B &amp; M.C</td>
<td>80.29</td>
<td>6.25</td>
<td>42.36</td>
<td>20.34</td>
<td>26.41</td>
</tr>
<tr>
<td>I. Volatility</td>
<td>88.91</td>
<td>13.84</td>
<td>43.88</td>
<td>12.93</td>
<td>26.07</td>
</tr>
</tbody>
</table>

Hence, our first step is to look at our results with the added feature that only borrowing is forbidden. A quick inspection of Table VII and Figure 7, where we present the results of this whole section, shows that this additional constraint has no major impact in our findings.

As opposed to market makers, any other investor’s account is instead margined on its short positions. Santa-Clara and Saretto (2009) present very strong evidence suggesting that the cost associated with the writing of options can be very substantial. We follow a radical approach in order to eliminate margin costs as a potential issue.
We have produced our computations with the constraint that the super-replicating portfolio can contain no short positions on the options while borrowing is still allowed. This case is relevant when considering regular investors who have access to borrowing. Even after this extreme precaution, our main results largely persist.

Finally, we redo our calculations by removing both borrowing and option writing as a possibility. Although inefficient quotes appear with much less frequency in this case, their presence is nonnegligible and the moneyness patterns clearly survive. Note that we have wiped out half the positions available in the market at each point in time. We have imposed this severe constraint so that the concerns in this section are addressed in an unequivocal way leaving no room for the real size of the cost of option writing to be an issue. A less restrictive approach allowing for option writing with realistic margin requirements would deliver higher inefficiencies.

C.5 Filtering away quotes with extreme implied volatility values

There is an important fraction of quotes in our sample whose implied volatilities are unreasonable or undefined. As in Santa-Clara and Saretto (2009), we have filtered away all observations with an implied volatility larger than 200% or lower than 1%. Table VII and Figure 7 reports again the outcome. Our two main results clearly survive after the implementation of this filter.

This exercise also allows us to compare the volatility of efficient and inefficient options over the moneyness dimension, a comparison that we present in Figure 8. Inefficient quotes have a higher implied volatility over the full spectrum of moneyness regardless of the particular type of quote.
Figure 7: Percentage of inefficiencies vs. moneyness: robustness checks
Figure 8: Implied volatility of efficient and inefficient quotes vs. moneyness
C.6 Discussion

It seems difficult to explain away our results and in particular the striking patterns that we have documented when examining quote inefficiency as a function of moneyness. Running the risk of giving a somehow tautological explanation, we conjecture that the existence of redundant assets may be playing a crucial role in here. Consider the put-call parity. This relationship tells us that either calls or puts are actually an asset too many. Therefore, inefficient quotes can appear as the result of the market acknowledging this fact and rendering one of them useless. However, what we actually observe is a more sophisticated way of dealing with this redundancy since from the behavior of inefficient quotes the market seems to discard in-the-money calls and in-the-money puts. We have some preliminary results concerning the behavior of trades in the US market which seem to confirm this appreciation since puts and calls are traded in a large majority of cases when they are out of the money. Nevertheless, for this explanation to be totally convincing, it would be important to find out what force, supply or demand, is driving this pattern. One may speculate that there is only demand for options when they are out the money and this is why inefficient quotes appear since liquidity providers find no use in offering competitive quotes for the in-the-money options. However, it may also be that this lack of demand for in-the-money options is actually caused by the presence of inefficient quotes. Distinguishing between these two driving forces is something that trade and quote data alone cannot do. A complete picture will require further research and clearly model-based explanations. Along these lines, an interesting issue deserving special attention would be to examine the relationship between inefficiency and competition
at the exchange. A more monopolistic supply of options may be associated with a larger degree of misspriced quotes.

IV Conclusions

This paper introduces a methodology which aims at analyzing the quality of bid and ask quotes in financial markets. Its power is illustrated in an empirical application which involves high-frequency data on three different options markets. Our findings reveal that a high fraction of quotes in these markets is uninformative and they uncover certain puzzling patterns in the behavior of these inefficiencies. These results may qualify and/or complement previously reported improvements in liquidity and efficiency in options markets as a result of the introduction of computerized trading systems (Eldor et al., 2006).

Our analysis can be beneficial for financial exchanges, since it provides a set of tools that can help in designing the right set of incentives for liquidity providers in this type of markets. Furthermore, our documented regularities may already illuminate these institutions on how to reshape certain aspects of their regulatory framework.

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