THE IMPACT ON MARKET OUTCOMES OF
THE PORTFOLIO SELECTION OF LARGE
EQUITY INVESTORS

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The Impact on Market Outcomes of the Portfolio Selection of Large Equity Investors*

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Abstract

We study a setting in which several large investors select their portfolios of equity of the firms competing in a symmetric duopoly considering the impact of their interests on the managerial incentives. Assuming that investors objective is to maximize the value of their portfolios, we show that equilibrium portfolios will be symmetric, contributing to enhance the anticompetitive impact of the presence of large investors on price mark ups and profits.

Keywords: Market Power, Common Ownership, Minority Equity, Portfolio Selection.

JEL Classification: L13, L2, L4, L5.

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1 Introduction

The literature on antitrust and regulatory policy argues that common minority ownership of competing firms may have anticompetitive effects leading to an increase of price mark ups – e.g., O’Brien and Salop (2000). Indeed, the presence of large investors with significant ownership interests in firms competing in key economic sectors such as airlines, technology, banking, or pharmaceutical is prevalent, and seems to be on the rise. Azar et al. (2018), for example, documents that the portfolios of each of the Big Three investors (BlackRock, Vanguard, and State Street) contain significant and similar fractions of the shares of the main airlines, as well as of Apple and Microsoft. In this note we consider a simple setting that allows us to identify the equilibrium portfolios arising in competitive equity markets when large investors maximize the value of their portfolio considering the influence of their interests on managerial incentives.

In our setting, there are several large investors each of whom owns a fraction of the stock of shares of the two symmetric firms competing in an industry, and the remaining shares are owned by negligibly small investors. In line with the literature, e.g., O’Brien and Salop (2000), we assume that the weight of the interests of a large investor on the managerial objective of a firm is proportional to the fraction of the stock of shares she owns. In a stationary equilibrium firms’ strategic decisions must form an equilibrium of the industry, and the portfolio of large investors must be optimal given the equity prices (i.e., the firms’ profits).

We show that under standard assumptions there is a class of stationary equilibria in which large investors choose symmetric portfolios. In these equilibria, the anticompetitive effect of the presence of large investors is maximal, i.e., larger than when they hold asymmetric portfolios. Moreover, the anticompetitive effect is stronger the larger and more concentrated is the fraction of firms’ equity owned by large investors.

We proceed to calculate the equilibria in a linear industry, both under quantity competition and under price competition, in the presence of one and two large investors. We show that in either case all stationary equilibria involve the large investors holding symmetric portfolios. Numerical exercises suggest that the competitive effects of large investors decreases rapidly when the firms’ property is distributed evenly among a larger number of investors. We also study via numerical exercises how the results change when one of the firms has a cost advantage, and observe that equilibrium portfolios tend to involve larger positions on the inefficient firm, which induces a shift of production activity in favor of the efficient firm.

There is a growing literature studying the economic implications of market power and common ownership in alternative settings: partial equilibrium, e.g., Reynolds and Snapp (1986), Farrell and Shapiro (1990), general equilibrium, e.g., Azar and Vives (2021), and macroeconomics, e.g., De Loecker et al. (2020), Eeckhout (2021). To our knowledge this literature does not address the important issue of portfolio selection.
2 The Model

Consider a symmetric duopoly in which a firm’s business strategy is completely described by a non-negative real number (e.g., output, price). For \((r, s) \in \mathbb{R}_+^2\), we denote by \(\pi(r, s)\) the \textit{operating profits} of a firm whose strategy is \(r\) when the rival’s strategy is \(s\).

There are competitive equity markets in which the firms’ shares are traded. In these markets there are \(n\) large investors each with a portfolio \((x_i, y_i)\) containing a non-negligible fraction of the total stocks of the firms’ shares, but not so large as to grant managerial control of either firm, and a positive measure of small investors each with a negligible number of shares.

A firm’s managerial objective is to maximize a weighted sum of the value of its shareholders’ portfolios, where the weights are determined by the shareholders’ fraction of the firm’s total stock of shares. Thus, if the portfolios of large investors contain fractions \(x = (x_1, ..., x_n)\) of a firm’s stock of shares and fractions \(y = (y_1, ..., y_n)\) of the stock of shares of its rival, respectively, then the firm’s manager chooses its business strategy \(r \in \mathbb{R}_+\) to maximize

\[
\pi(r, s) + m(x, y)\pi(s, r),
\]

where \(m(x, y)\) is the weight of the rival’s profit, \(\pi(s, r)\), in the firm’s managerial objective function. Naturally, the function \(m\), which captures the influence on the managerial objectives of the minority interests (i.e., those of the large investors), depends on the legal framework.

We assume that given the portfolios of large investors \((x, y)\), the duopoly equilibrium is interior and unique. This assumption and the symmetry of our setting implies that there is a well defined function identifying the firms’ equilibrium strategies, \(r(x, y)\) and \(s(x, y) = r(y, x)\).

Hence, the function

\[
\Pi(x, y) := \pi(r(x, y), r(y, x)),
\]

identifying a firms’ equilibrium operating profits, as well as the functions

\[
U_i(x, y) := x_i\Pi(x, y) + y_i\Pi(y, x),
\]

identifying the value of the portfolio of each large investor \(i \in \{1, ..., n\}\), are well defined. Also, given \((x, y)\), firms’ shares are traded at prices \(\Pi(x, y)\) and \(\Pi(y, x)\), respectively. Therefore, the set of portfolios that large investor \(i\) can afford is

\[
B_i(x, y) = \{(x'_i, y'_i) \in \mathbb{R}_+^2 \mid \Pi(x, y)(x'_i - x_i) + \Pi(y, x)(y'_i - y_i) = 0\}.
\]

Definition. A \textit{stationary equilibrium} is a profile \((\bar{r}, \bar{s}, \bar{x}, \bar{y})\) such that (i) the firms’ strategies form an equilibrium, i.e., \((\bar{r}, \bar{s}) = (r(\bar{x}, \bar{y}), r(\bar{y}, \bar{x}))\), and (ii) the portfolio composition of each
large investor $i \in \{1, \ldots, n\}$ is optimal, i.e., $(x_i, y_i)$ solves the problem

$$\max_{(x, y)} U_i((x, y), (x_i, y_i)).$$

In a stationary equilibrium there are no incentives for further equity trading. Proposition 1 identifies an interesting class of stationary equilibria.

**Proposition 1.** Let $(\bar{x}, \bar{y})$ be a profile of portfolios of the large investors such that $x = y$, and assume that $\Pi$ is differentiable and increasing, and that for each $i \in \{1, \ldots, n\}$, $U_i((x_i, \bar{x}_{-i}), (y_i, \bar{y}_{-i}))$ is quasi-concave in $(x_i, y_i)$. Then $(r(\bar{x}, \bar{y}), r(\bar{y}, \bar{x}), x, y)$ is a stationary equilibrium.

**Proof.** Let $(\bar{x}, \bar{y})$ be such that $x = y$. We show that the marginal rate of substitution of every large investor $i \in \{1, \ldots, n\}$, given by

$$MRS_i(x, y) = \frac{\partial U_i(x, y)/\partial x_i}{\partial U_i(x, y)/\partial y_i},$$

evaluated at $(\bar{x}, \bar{y})$ is equal to the price ratio, which implies that her portfolio $(\bar{x}_i, \bar{y}_i)$ is optimal. For $k \in \{1, \ldots, 2n\}$ denote by $\partial_k \Pi$ the partial derivative of $\Pi$ with respect to the $k$th coordinate. Then $(\bar{x}, \bar{y}) = (\bar{y}, \bar{x})$ implies $\Pi(\bar{x}, \bar{y}) = \Pi(\bar{y}, \bar{x})$ and $\partial_k \Pi(\bar{x}, \bar{y}) = \partial_k \Pi(\bar{y}, \bar{x})$ for all $k$. Thus,

$$MRS_i(\bar{x}, \bar{y}) = \frac{\Pi(\bar{x}, \bar{y}) + \bar{x}_i \partial_i \Pi(\bar{x}, \bar{y}) + \bar{y}_i \partial_{n+i} \Pi(\bar{y}, \bar{x})}{\bar{x}_i \partial_{n+i} \Pi(\bar{x}, \bar{y}) + \Pi(\bar{y}, \bar{x}) + \bar{y}_i \partial_i \Pi(\bar{y}, \bar{x})} = 1,$$

and

$$\frac{\Pi(\bar{x}, \bar{y})}{\Pi(\bar{y}, \bar{x})} = 1. \square$$

It is interesting to observe that the presence of large investors benefits small investors who care exclusively about the value of their portfolios since $\Pi(x, x) > \Pi(0, 0)$ for all $x \in \mathbb{R}^n_+ \setminus \{0\}$. Moreover, when $x = y$ the portfolio of each large investor maximizes the industry profits, given for all $(x, y)$ by

$$\Pi(x, y) := \Pi(x, y) + \Pi(y, x),$$

since

$$\frac{\partial_i \Pi(x, y)}{\partial_{n+i} \Pi(x, y)} = \frac{\partial_i \Pi(x, y) + \partial_{n+i} \Pi(x, y)}{\partial_{n+i} \Pi(x, y) + \partial_i \Pi(y, x)} = 1.$$

**Remark 1.** Let the assumptions of Proposition 1 hold and assume that $\Pi$ is quasi-concave. Then in a stationary equilibrium such that $x = y$, the portfolio of every large investor $i \in \{1, \ldots, n\}$ maximizes the industry’s profits on $B_i(\bar{x}, \bar{y})$.

Naturally, the properties of stationary equilibria depend crucially on the function $m(x, y)$, which identifies the influence of the minority interests on the managerial objectives. In line with the literature, e.g., O’Brien and Salop (2000), Brito et al. (2018), in the exercises below we assume that the effect of each shareholder’s interests in shaping managerial incentives is
proportional to her stakes at the firm, i.e., that a firm’s managerial objective is to maximize

\[
(1 - \sum_{i=1}^{n} x_i) \pi(r, s) + \sum_{i=1}^{n} x_i (x_i \pi(r, s) + y_i \pi(s, r)),
\]

which amounts to assuming that the function \( m \) is given by

\[
m(x, y) = \frac{\sum_{i=1}^{n} x_i y_i}{1 - \sum_{i=1}^{n} x_i (1 - x_i)}.
\]

For the stationary equilibria \((r(x, x), r(x, x), x, x)\) identified in Proposition 1 the value of \( m \) is easily calculated as

\[
M(x) := m(x, x) = \frac{\sum_{i=1}^{n} x_i^2}{1 - \sum_{i=1}^{n} x_i (1 - x_i)} = \frac{\mu(x)^2 + \sigma^2(x)}{n^{-1} - \mu(x) + \mu(x)^2 + \sigma^2(x)},
\]

where \( \mu \) is the mean and \( \sigma^2 \) is the variance of the vector \( x \) identifying the fractions of the firms’ shares owned by the large investors. Note that \( M \) is increasing in both \( \mu \) and \( \sigma^2 \). Hence the lowest and largest values of \( M \) are reached when the shares are distributed evenly among the large investors, and when they are concentrated in a single large investor, respectively.

If there is a single large investor, for example, then \( \mu(x) = x \) and \( \sigma^2(x) = 0 \), and hence \( M(x) = x^2 / (1 - x + x^2) \) is increasing with the size of the investor’s portfolio, reaching the value of 1/3 when \( x = 1/2 \). If there are \( n > 1 \) large investors, then given the total fraction of the firms’ shares owned by the large investors, \( n \mu(x) \), the value of \( M \) ranges from \( \mu(x)^2 / \left( n^{-1} - \mu(x) + \mu(x)^2 \right) \) when the shares are evenly distributed among the large investors, i.e., \( x_i = \mu(x) \) for all \( i \), to \( \mu(x)^2 / \left( n^{-2} - \mu(x)n^{-1} + \mu(x)^2 \right) \) when they are owned by a single investor. If \( n \mu(x) = 1/2 \), for example, \( M \) ranges from \( 1 / (2n + 1) \) to \( 1/3 \). Thus, \( M \) is larger the larger and more concentrated are the stocks of shares of the large investors.

3 A Linear Industry

We study an industry in which the demand is linear and the firms produce the good with constant returns to scale. We consider in turn the cases of quantity and price competition.

Quantity Competition

Consider a symmetric homogeneous good duopoly. The inverse demand is \( P(Q) = \max\{A - bQ, 0\} \), where \( A, b \in \mathbb{R}_+ \), and \( Q \) is industry output. The firms’ strategic variable is output, which they produce with marginal cost \( c < A \). To simplify the analysis, with some loss of generality we normalize the parameters to \( A - c = b = 1 \), so that a firm’s operating profits are given for \((r, s) \in \mathbb{R}_+^2 \) by

\[
\pi_C(r, s) = \max\{(1 - r - s) r, 0\}.
\]
In this setting, we easily calculate the firms’ equilibrium output and operating profits as functions of the fractions of firms’ shares owned by the large investors, \((x, y)\), as

\[
r_C(x, y) = \frac{1 - m(x, y)}{3 - m(x, y) - m(y, x) - m(x, y)m(y, x)},
\]

and

\[
\Pi_C(x, y) = \frac{(1 - m(x, y))(1 - m(x, y)m(y, x))}{(3 - m(x, y) - m(y, x) - m(x, y)m(y, x))^2}.
\]

Calculating the set of stationary equilibria is challenging even in this simple setting, as the functions \(\Pi\) and \(U_i\) are complex high order polynomials. We verify with the help of Mathematica that \(\Pi\) is increasing; also, when there is a single large investor, that \(U_1\) is quasi-concave and that

\[
MRS_1(x, y) = \frac{\Pi(x, y)}{\Pi(y, x)} \Leftrightarrow y \geq x.
\]

Therefore, when \(n = 1\) in every stationary equilibrium \(x = y\). To identify the stationary equilibria when there are two large investors, we verify that \(U_i((x_i, \bar{x}_{-i}), (y_i, \bar{y}_{-i}))\) is quasi-concave with respect to \((x_i, y_i)\), and proceed to study the portfolio profiles in the Edgeworth box given their total fraction of firms’ shares, \((x_1 + x_2, y_1 + y_2)\). We verify that

\[
MRS_1(x, y) = MRS_2(x, y) \Leftrightarrow y_1 + y_2 \geq x_1 + x_2.
\]

Moreover, if \(x_1 + x_2 = y_1 + y_2\), then

\[
MRS_1(x, y) = MRS_2(x, y) \Leftrightarrow (y_1, y_2) = (x_1, x_2).
\]

Hence all the stationary equilibria satisfy \(x = y\) as well when \(n = 2\).

While these results may not be expected in general, we conjecture that in this linear setting they hold for \(n > 2\) as well. Assuming that this is the case, in a stationary equilibrium the output of each firm is

\[
r_C(x) = \frac{1 - M(x)}{3 - 2M(x) - M(x)^2},
\]

and the industry profits and consumer surplus are

\[
\Pi_C(x) = \frac{2(1 - M(x))(1 - M(x)^2)}{(3 - 2M(x) - M(x)^2)^2},
\]

\[
CS_C(x) = \frac{1}{2} \left( \frac{2(1 - M(x))}{3 - 2M(x) - M(x)^2} \right)^2.
\]

The curves in Figures 1a and 1b show the industry profits and consumer surplus, respectively, as a function of \(n\mu(x)\), the total fractions of firms’ equity held by \(n\) identical large investors (i.e., \(\sigma^2(x) = 0\)) for \(n = 1\) (the fuchsia curve), for \(n = 2\) (the brown curve), and for \(n = 3\).
(the green curve). The red and blue curves show these values at the monopoly and Cournot equilibrium, respectively. The graphs illustrate that as $n$ increases the industry profits decrease and the consumer surplus increases, reverting to their values at the Cournot equilibrium as $n$ grows large.

![Graphs showing industry profits and consumer surplus](#)

**Figures 1a and 1b**

In an industry in which firms are initially owned by small investors, a large investor entering the equity market, aware of its impact on the managerial incentives, will take equal positions in both firms. Thus, the impact of the entry of a large investor on the industry equilibrium output will be maximal given the size of its investment; that is, among the portfolios that are feasible to the investor, the equilibrium portfolio leads to the largest (smallest) industry profits (output). Hence the presence of a large investor in competitive equity markets fosters the anti-competitive effect of market power. Obviously, this effect is identified by the size of the portfolio of the large investor, $\mu(x) = x$. With a portfolio of maximum size, $x = 1/2$, the industry profits increase by 8%, and the consumer and total surpluses decrease by 19% and 5.5%, respectively, as compared to the Cournot equilibrium. Nonetheless, industry profits (consumer and total surplus) remain below (above) their value at the monopoly equilibrium, which is 12.5% larger (43.75% and 15.625% smaller, respectively) than at the Cournot equilibrium.

When there are two large investors who jointly own half of the firms’ shares, i.e., $x_1 + x_2 = 1/2$, the corresponding increase (decrease) of industry profits (total and consumer surplus) relative to the Cournot equilibrium ranges from the values given above when firms’ equity is entirely owned by one of the large investors, to the values 5.47% (12.11% and 3.32%, respectively) when it is evenly distributed among the two investors (i.e., $x_1 = x_2 = 1/4$). Thus, the impact of two large investors of equal size is considerably smaller than that resulting from a single large investor.

**Price Competition**

Consider a symmetric differentiated goods duopoly in which a firm’s strategic variable is the price of the good that produces with constant marginal cost, which we normalize to zero. The
operating profit of a firm that sells its good at the price $r$ when its rival’s price is $s$ is given by

$$\pi_B(r, s) = \max \left\{ \frac{(1 - \gamma - r + \gamma s) r}{1 - \gamma^2}, 0 \right\},$$

where $\gamma \in [0, 1]$ is the product differentiation parameter.

The analysis of this setting is somewhat more complex than that of the quantity competition. We use Mathematica to compute the functions $MRS_i$ and $\Pi$, and consider a fine grid of values of $\gamma$, for the cases of a single large investor ($n = 1$) and two large investors ($n = 2$). The results are analogous to the quantity setting scenario: in a stationary equilibrium the portfolio of a large investor contains identical fractions of shares of both firms, i.e., $x = y$. Thus, in a stationary equilibrium the price is

$$r_B(x) = \frac{(1 - \gamma)(2 + \gamma(1 + M(x)))}{4 - \gamma^2 (1 + M(x))^2}$$

and the industry profits and consumer surplus are

$$\Pi_B(x) = \frac{2 (1 - \gamma)}{1 + \gamma} \left( \frac{1 - \gamma M(x)}{2 - \gamma(1 + M(x))} \right)^2$$

$$CS_B(x) = \frac{1}{1 + \gamma} \left( \frac{1 - \gamma M(x)}{2 - \gamma(1 + M(x))} \right)^2.$$

Figures 2a and 2b show the industry profits and consumer surplus, respectively, for $\gamma = 0.9$, as a function of the total fraction of firms’ equity held by $n$ identical large investors (i.e., $\sigma^2(x) = 0$), when $n = 1$ (the fuchsia curve), $n = 2$ (the brown curve) and $n = 3$ (the green curve), as well at the Bertrand equilibrium (the blue line). The values of profits and consumer surplus at the equilibrium of a multiproduct monopoly (equal to $5/19$ and $5/32$, respectively) are outside the range of values shown in the figures. As in the in the case of quantity competition, the larger and more concentrated is the fraction of the stock of shares owned by the large investors, the is larger $M$, and hence the larger are the industry profits and the smaller is the consumer surplus.

Figures 2a and 2b
The comparative static properties of stationary equilibria with respect to the differentiation parameter $\gamma$ depend on the trade-off between the impact of $\gamma$ on consumer’s utility for the differentiated goods, which is larger the smaller is $\gamma$, and its effect on firms’ competition, which is more aggressive the larger is $\gamma$. As a result, the consumer surplus increases with $\gamma$ under Bertrand competition, but decreases with $\gamma$ when both goods are produced by a monopoly. Since the presence of large investors softens competition, the sign of the impact of an increment of $\gamma$ is negative (positive) for small (large) values of $\gamma$. Figure 3 shows the consumer surplus as a function of $\gamma$ under monopoly (the red curve), Bertrand competition for $M = 0$ (the blue curve), $M = 1/3$ (the fuchsia curve), $M = 1/6$ (the brown curve), and $M = 1/12$ (the green curve).

![Figure 3](image)

**Asymmetries**

Our framework and definition of stationary equilibrium are readily modified to accommodate asymmetric industries. To understand how asymmetries alter our conclusions, we consider a variation of the quantity setting linear duopoly described above in which the profit of a firm $i \in \{1, 2\}$ when its output is $r$ and its rival’s output is $s$ is $\pi_i(r,s) = (a_i - r - s)r$, where $a_1 = 1 > a_2$. The inequality $a_1 > a_2$ may result from, e.g., firm 1 producing the good more efficiently (i.e., with a lower constant marginal cost) than firm 2.

Deriving results for the asymmetric case is quite challenging: crucial properties, such as the quasi-concavity of the value of a portfolio, no longer hold even in our linear setting. To explore the properties of stationary equilibria, we solve a number of numerical examples that involve a single large investor and values of $a_2$ that are not too far from $a_1$. In these examples, optimal portfolios of the large investor, i.e., the solutions to the equation

$$MRS_1(x,y) = \frac{\Pi_1(x,y)}{\Pi_2(x,y)},$$

contain a larger fraction of shares of the inefficient firm than those of the efficient firm, i.e., satisfy $y > x$, implying $m_2(x,y) > m_1(x,y)$. Thus, in a stationary equilibrium there is a shift of the production activity in favor of the efficient firm.
Interestingly, while the total output is smaller, and the profits of the efficient firm are larger, than at the Cournot equilibrium, the profit of the inefficient firm may be smaller. When this occurs, and in sharp contrast to the symmetric case, the presence of a large investor makes worse off (relative to the Cournot equilibrium) the small investors who own equity of the inefficient firm. For example, for $a_2 = 0.8$, there is stationary equilibrium in which the portfolio of the large investor is $(0.0625, 0.2379)$, and the firms’ outputs and profits are $(q_1, q_2) = (0.4003, 0.1962)$ and $(\Pi_1, \Pi_2) = (0.1615, 0.03992)$, whereas at the Cournot equilibrium these values are $(q_1^*, q_2^*) = (0.4, 0.2)$ and $(\Pi_1^*, \Pi_2^*) = (0.16, 0.04)$. Thus, in this stationary equilibrium the output and profits of the efficient (inefficient) firm are larger (smaller) than at the Cournot equilibrium.

Moreover, in contrast to the symmetric case, the profits of an asymmetric industry may not be maximized in the presence of a larger investor, as this would require a larger production shift towards the efficient firm. In the numerical example just described, if the large investor were to choose the portfolio that maximizes the industry profits, then the resulting outputs would be $(q_1^*, q_2^*) = (0.4006, 0.1951)$, involving a larger production shift towards the efficient firm, which would benefit the large investor too.

References


