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On Higher-Order Statistics of the Channel Model for UAV-to-Ground Communications

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Abstract—Unmanned-aerial-vehicle (UAV) communication systems are envisioned to play an important role in 5G and beyond 5G (B5G) systems. UAV-to-ground communications in urban cities are often characterized by highly dynamic propagation environments that can be described by composite fading channels. Most of the UAV-to-ground systems are based on first order (FO) performance evaluation, however, the models based on FO statistics are insufficient for characterization of time variant fading channels. We provide comprehensive mathematical framework for derivation of second order (SO) statistics over double-scattered, double-shadowed (DS-DS) fading channels, modeled as the product of double Nakagami-m (DN) and double inverse Gamma (DIG) random processes (RPs). In particular, we obtained exact mathematical expressions for average fade duration (AFD) and level crossing rate (LCR) of the proposed UAV-to-ground channel model. Moreover, the exact, integral form SO statistical expressions are approximated by Laplace Integration (LI) and exponential LI in order to provide closed form, easily computing mathematical expressions. Numerical results show that approximate and exact results are fitting well, especially for higher output threshold values. The impact of DS-DS fading severities on the SO statistics are well investigated. Furthermore, the proposed method is extended to analyze SO performances for the selection scenario of UAV with the highest signal level from among N-UAVs links.

Index Terms—B5G communications, 6G communications, Average fade duration, Level crossing rate, Unmanned aerial vehicle (UAV).

I. INTRODUCTION

In the recent years, we have seen increased research interest in unmanned aerial vehicles (UAV) based communications. UAVs are being used for parcel, medicine delivery, monitoring of social distancing, monitoring fleet, farming and others. Yet, there are many challenges needed to be addressed and among the most important is accurate channel modeling for UAV communications [1]-[2]. The urban topology is dense with buildings, features, and has huge densely populated areas. In such scenarios channel modeling and coverage due to building and structures is a major problem. UAV-to-ground communication mainly operates in line of sight (LOS) environments with increased mobility (IM) of communication nodes. In urban cities, besides LOS-IM communications, non-LOS IM communications is expected. The UAV-to-ground channel model capable of accounting for multipath and shadowing effects, and that is in accordance with experiments can be modeled as the double-scattered, double-shadowed (DS-DS) fading channel [3]-[4]. The paper [3] provides probability density function (PDF), cumulative distribution function (CDF), outage probability and channel capacity (C) of DS-DS fading channel modeled as the products of double Nakagami-m (DN) and double inverse Gamma (DIG) random processes (RPs). However, performance analysis of UAV communications obtained in [3]-[9] are mainly observed through first order (FO) statistical measures.

In addition to evaluation of the FO statistics, level crossing rate (LCR) and average fade duration (AFD), also known as the second order (SO) statistics can further broaden understanding of the time-variant fading channels (TVFC). Namely, the LCR addresses TVFC by determining time rate of change of the signal envelope, while AFD addresses TVFC by determining the mean time of the signal envelope being below a specified threshold. In particular, those measures can be useful for the channel coding, block interleave and overall system design. Furthermore, 5G and beyond 5G (B5G) expected requirements include ultra-reliable, low-latency communications (URLLCs) which requires the system performance analysis with respect to time [10]. UAV communication performances obtained in [11]-[15] are mainly observed through SO statistical measures. However, SO performances of DS-DS UAV fading channels have not been observed in open literature so far.

This paper provides the development of a non-GBSM framework for derivation of SO statistics. The main contributions of our study are:

- Derivation of mathematical expressions for LCR and
AFD of DS-DS fading channel, modeled as the product
of independent but not identically distributed (i.i.d) DN
and DIG RPs.
- Derivation of approximate closed-form SO statistics by
  using Laplace integration (LI) and exponential LI
  methods.
- Confirmation of obtained SO statistical results by Monte
  Carlo simulations.
- SO performance analysis for the selection scenario of
  UAV with the highest signal level from among N-UAV
  links.
- Analysis of the impact of DS-DS multipath and shadowing
  severity parameters on the observed performance measures.

II. SYSTEM MODEL

The composite fading channel for UAV communications
can be modeled as double-scattered (DS), double-adjacent
(DS) fading channel. Namely, DS-DS can be modeled as the product
of two Nakagami-m random variables (denoted as $X_{N1}$ and
$X_{N2}$) and two inverse Gamma random variables (denoted as
$X_{ds}$ and $X_{ds}$), given by [4, Eq. (3)]:

$$X_{out} = X_{N1}X_{ds}X_{N2}X_{ds} = X_{N1} \times \frac{1}{X_{N2}X_{ds}}$$

where inverse Gamma random variables can be expressed
as squared inverse Nakagami-m random variables [16, Eq.
(2.52)]. Namely, $X_{ds} = \frac{1}{Y_{ds}}$ and $X_{ds} = \frac{1}{Y_{ds}}$.
Nakagami-m PDFs are, respectively [16, Eq. (2.52)]:

$$p_{X_{N1}}(x_{N1}) = \frac{2}{\left(m_{1}/\bar{x}_{m1}\right)^{m_{1}}} [x_{N1}]^{m_{1}-1} e^{-\frac{m_{1}}{\bar{x}_{m1}} [x_{N1}]^{2}} , i = 1, 4$$

The PDF of the product of Nakagami-m and inverse Gamma
random variables, denoted as $Y_1$, can be expressed as:

$$p_{Y_1}(y_1) = \int_{0}^{\infty} \int_{0}^{\infty} \frac{4m_{1}^{m_{1}}m_{2}^{m_{2}}}{\Gamma(m_{1})\Gamma(m_{2})} \frac{m_{1}-1}{\bar{x}_{m1}^{m_{m1}}} [x_{N1}]^{m_{1}-1} \frac{m_{2}-1}{\bar{x}_{m2}^{m_{m2}}} [x_{N2}]^{m_{2}-1} dx_{N1} dx_{N2}$$

Similarly, using the same mathematical approach of PDF by
where $Y_2 = Y_1X_{N1}$ is:

$$p_{Y_2}(y_2) = \frac{8m_{1}^{m_{1}}m_{2}^{m_{2}}m_{3}^{m_{3}}}{\Gamma(m_{1})\Gamma(m_{2})\Gamma(m_{3})} \frac{m_{1}-1}{\bar{x}_{m1}^{m_{m1}}} [x_{N1}]^{m_{1}-1} \frac{m_{2}-1}{\bar{x}_{m2}^{m_{m2}}} [x_{N2}]^{m_{2}-1} \frac{m_{3}-1}{\bar{x}_{m3}^{m_{m3}}} [x_{N3}]^{m_{3}-1} dx_{N1} dx_{N2}$$

The PDF of the product of DN and DIG random variables,
which as $p_{X_{N1}}(x_{out})$, where $X_{out} = \frac{1}{X_{ds}}$ can be expressed as:

$$p_{X_{N1}}(x_{out}) = \int_{0}^{\infty} \int_{0}^{\infty} \frac{4m_{1}^{m_{1}}m_{2}^{m_{2}}}{\Gamma(m_{1})\Gamma(m_{2})} \frac{m_{1}-1}{\bar{x}_{m1}^{m_{m1}}} [x_{N1}]^{m_{1}-1} \frac{m_{2}-1}{\bar{x}_{m2}^{m_{m2}}} [x_{N2}]^{m_{2}-1} dx_{N1} dx_{N2}$$

where $\frac{1}{m_{1}}$ is already provided in (5).

The PDF of $p_{X_{ds}}(x_{ds})$ is:

$$p_{X_{ds}}(x_{ds}) = \frac{16}{\Gamma(m_{1})\Gamma(m_{2})\Gamma(m_{3})} \frac{m_{1}-1}{\bar{x}_{m1}^{m_{m1}}} [x_{N1}]^{m_{1}-1} \frac{m_{2}-1}{\bar{x}_{m2}^{m_{m2}}} [x_{N2}]^{m_{2}-1} \frac{m_{3}-1}{\bar{x}_{m3}^{m_{m3}}} [x_{N3}]^{m_{3}-1} dx_{N1} dx_{N2}$$

where $\frac{1}{m_{3}}$ is already provided in (5).
The $E_i$ in (9) can be calculated by exponential LI for the following set of functions: $\gamma N_i$, $f_i = 1$,

$$f_3 = \frac{m_1^3}{T^2_1} + \frac{m_2^3}{T^2_1} + \frac{m_3^3}{T^2_1} + \frac{m_4^3}{T^2_1},$$

$$- (2m_1 + 4 - 1)\ln x_N - (2m_2 - 2k - 1)\ln x_N$$

(13)

The LCR for a given threshold $x_{TH}$, denoted as $L_{x_{TH}}$, is given by:

$$L_{x_{TH}} = \int_0^\infty x_{out} P_{x_{out}}(x_{out}) dx_{out}$$

(14)

where $x_{out}$ is the first derivative of $x_{out}$. The $P_{x_{out}}(x_{out})$ can be derived from the joint PDF of independent RPVs $x_{out}$, $x_{out2}$, $x_{out3}$ and $x_{out4}$,

$$P_{x_{out}}(x_{out}) = \int_0^\infty dx_{N2} \int_0^\infty dx_{N3}$$

(15)

where $P_{x_{out}}(x_{out})$ can be transformed as:

$$P_{x_{out}}(x_{out}) = \int_0^\infty dx_{N2} \int_0^\infty dx_{N3}$$

(16)

After substitutions, (17) in (16), (16) in (15) and (15) in (14), respectively, the $L_{x_{TH}}$ becomes:

$$L_{x_{TH}} = \int_0^\infty x_{out} P_{x_{out}}(x_{out}) dx_{out}$$

(17)

where,

$$P_{x_{out}}(x_{out}) = \int_0^\infty dx_{N2} \int_0^\infty dx_{N3}$$

(18)

The integral $E_2$ can be solved by LI, already given by (10), (11) and (12) for the following $\gamma N_i$, $f_i = 1$,

$$f_2 = \frac{m_1^2}{T^2_1} + \frac{m_2^2}{T^2_1} + \frac{m_3^2}{T^2_1} + \frac{m_4^2}{T^2_1}$$

(19)

Finally, the $L_{x_{TH}}$ can be expressed as:

$$L_{x_{TH}} = 16\pi^2 \gamma N_i \int_0^\infty \frac{dx_{N2} dx_{N3} dx_{N4}}{\sqrt{x_{N2} x_{N3} x_{N4}}}$$

(20)

where $x_{N1}$, $x_{N2}$, $x_{N3}$ and $x_{N4}$ are the first derivatives of $x_{N1}$, $x_{N2}$, $x_{N3}$ and $x_{N4}$, respectively. Since the linear transformation of zero mean Gaussian RVs is a zero mean Gaussian RV, the variance of $x_{out}$ is also a zero mean Gaussian RV and $\sigma^2_{x_{out}}$ can be expressed through the variances of $x_{N1}$, $x_{N2}$, $x_{N3}$ and $x_{N4}$, respectively.
The presented results show that exact analytical expression (integral form expression) for $Afd_X(x_{TH})$ can be evaluated as:

$$Afd_X(x_{TH}) = \frac{F_X(x_{TH})}{Lev_X(x_{TH})}$$  \hspace{1cm} (26)

The system model from UAV-to-ground communications is extended to include selection of UAV with the highest signal envelope level from among $N$-UAV's over $N$ i.i.d ground-to-UAV links.

The CDF for extended UAV channel model over DS-DS fading, denoted as $F_X^E(x)$ can be expressed as:

$$F_X^E(x) = F_X(x)^N$$  \hspace{1cm} (27)

LCR with UAV selection over DS-DS fading can be obtained as:

$$Lev_X^{(S)}(x_{th}) = NLev_X(x_{TH})F_X(x_{TH})^{-1}$$  \hspace{1cm} (28)

Finally, the $Afd_X^{(S)}(x_{th})$ for DS-DS channel model with UAV selection is:

$$Afd_X^{(S)}(x_{th}) = \frac{F_X(x_{TH})}{NLev_X(x_{TH})}$$  \hspace{1cm} (29)

### III. NUMERICAL RESULTS

The SO statistics of UAV-to-ground communications over DS-DS channel fading model in terms of various fading DS-DS severities are efficiently evaluated and presented in numerical results. Moreover, the SO statistics of the extended model with UAV selection is then numerically evaluated and investigated in terms of different system model parameters. The presented results show that exact analytical expression (integral form expression) for $Lev_X(x_{TH})$ fits well with the approximation (closed form expressions approximated by L1). Moreover, the exact analytical expression (integral form expression) for $F_X(x)$ is approximated by exponential L1. The presented results for $Afd_X(x_{TH})$ (ratio of $Lev_X(x_{TH})$ and $F_X(x_{TH})$) between exact form solution and approximation fits well only for higher $x_{TH}$ values.

The DS-DS fading channel is modeled as the product of DN and DIG, where results are presented for various DS-DS multipath severity parameters ($m_n, m_3$) and various values of DS-DS shadowing severity parameters ($m_1, m_3, m_4$). The variances in (21) are expressed as $\sigma^2_{z_n} = \pi^2 f_n^2 \frac{m_n}{L_1}$, $i = 1, 4$ where the maximum Doppler frequencies are assumed to be the same [19, Eq (43)]. $f_n = f_{m_n} = \sqrt{\frac{f_{m_1}}{f_{m_n}}}$.

Fig. 1 provides the behavior of $Lev_X(x_{TH})$ normalized by $f_{m_n}$. It can be seen that by increasing all DS-DS severity parameters, $Lev_X(x_{TH})$ decreases in the whole observable $x_{TH}$ dB regime, which in turn can provide improvement in the system performances. The behavior of $Afd_X(x_{TH})$ multiplied by $f_{m_n}$ is provided in Fig. 2. The numerical results show that by increasing all DS-DS severity parameters, $Afd_X(x_{TH})$ slightly decreases in lower $x_{TH}$ threshold dB regime while $Afd_X(x_{TH})$ increases in higher $x_{TH}$ dB regime. Moreover, the impact of DS-DS fading severities on $Afd_X(x_{TH})$ is stronger for higher $x_{TH}$.

Fig. 3 provides normalized $Lev_X^{(S)}(x_{th})$. The increasing number of UAVs causes $Lev_X^{(S)}(x_{th})$ values to decrease for lower $x_{th}$ and $Lev_X^{(S)}(x_{th})$ values to increase for higher $x_{th}$. Moreover, it can be observed that the number of UAVs have stronger impact on $Lev_X^{(S)}(x_{th})$ for lower dB $x_{th}$ values. The $Afd_X^{(S)}(x_{th})$ multiplied by $f_{m_n}$ is presented in Fig. 4. It is evident that increasing number of UAVs, can improve system performances, since $Afd_X^{(S)}(x_{th})$ decreases in whole $x_{th}$ dB regime. Similarly, it can be noticed that number of UAVs have stronger impact on $Afd_X^{(S)}(x_{th})$ for lower $x_{th}$.
The LI approximation formulas for derivation of closed form analytical statistics of the product of DN and DIG RPs. The LI applied particular, we provide mathematical expressions for the S0 envelope level from among is extended to include UA V selection with the highest signal Afd Fig. 4.

Lcr

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IV. CONCLUSION

The SO statistics in urban UA V-to-ground communication systems over DS-DS fading channel is considered. The model is extended to include UA V selection with the highest signal envelope level from among N UA-Vs over N i.n.i.d links. In particular, we provide mathematical expressions for the SO statistics of the product of DN and DIG RPs. The LI approximation formulas for derivation of closed form analytical expressions as well as Monte Carlo simulations for verification of obtained results are efficiently applied. The increase in DS-DS multipath and shadowing fading severity parameters can provide system performance improvement for lower dB threshold values in relation to SO statistics. Moreover, the increasing number of UA-Vs can further provide system performance improvements.

REFERENCES