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Vibration augmentation of the solids volume dragged by the wake of a bubble rising in a fluidized bed

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Abstract

In a gas fluidized bed, bubbles are mainly responsible for the displacement and mixing of the bed particles, with a direct impact on the bed heat and mass transfer. In this work, the motion of solids in the wake region of an isolated bubble rising in a vibrated fluidized bed is experimentally studied using combined digital image and bubble averaging techniques. The results show that the bubble wake behavior depends on the vibration phase and the bubble position in the bed. Particles penetrate inside the bubble volume during the deceleration of the bed vessel, provoking their characteristic wavy contour. The volume of particles dragged by the bubble wake is affected by the vibration phase due to the interaction of the bubble with the compression and expansion waves travelling inside the vibrated bed. Independently increasing the vibration amplitude or frequency augments the volume of particles dragged by the bubble wake.

Keywords: Vibrated fluidized bed, Bubble, Wake, Pseudo-2D, DIA, PIV

1. Introduction

In a gas fluidized bed, bubbles are mainly responsible for the displacement and mixing of the bed particles. While ascending in the bed, each bubble pushes the particles upwards in its dome and drags the particles in its wake. This has a great impact on the homogeneity of the bed species and temperature, which directly affects the bed heat and mass transfer. Fluidization enhancement techniques, such as the vibration of a gas fluidized bed, can be used to modulate the particle mixing produced by bubbles in bed reactors. Vibration of a conventional gas fluidized bed aids to break gas channeling and avoid agglomeration, facilitates fluidization and enhances mixing or segregation in the bed depending on the gas superficial velocity and the vibration conditions. Vibration is also a way of stabilizing the system and gain some control over the bed dynamics [1–5]. Despite its advantages, vibration introduces complexities in the dynamics of the bed that are far from being fully understood. These complexities range from the bubbling behavior of the bed to the behavior of the particles around the bubbles. In fact, as mentioned before, one of the main
virtues of conventional fluidized beds is the ability to mix particles due to bubble motion inside the bed. In particular, the particles dragged by the wake of rising bubbles are of paramount importance for the mixing and segregation phenomena in fluidized beds [6, 7]. Knowledge of the complex physical phenomena and the dragging ability of bubbles in Vibrated Fluidized Beds (VFB) as compared to conventional fluidized beds may be used to improve design and control of the existing VFBs and to gain insight into possible new applications of these non-conventional systems.

Among the different experimental techniques aimed at characterizing the bed and bubble behavior in VFBs, Digital Image Analysis (DIA) is one of the most used, not only in VFBs but also in conventional fluidized beds [5, 8–18]. Conventional DIA can be combined with Particle Image Velocimetry (PIV) techniques to characterize the motion of particles in the bed. This PIV technique has been extensively used in the past to characterize both conventional and vibrated fluidized beds [13, 14, 17, 19–23]. PIV can be useful to validate existing models of the solids motion inside conventional (non-vibrated) fluidized beds. For example, Müller et al. [19] studied the eruption of a bubble in a fluidized bed by means of PIV. They observed that, even after eruption, the potential flow theory was capable of predicting the velocity of the particles on the roof of the bubble. Hernández-Jiménez et al. [24] carried out two-fluid model simulations of the system used in [19]. The results of the simulations indicated that the potential flow theory provides a more accurate prediction for the particle velocities in erupting bubbles than other semi-empirical relations available in the literature. Very recently, Fullmer et al. [25] analyzed different velocimetry techniques employed to study granular systems: PIV, Particle Tracking Velocimetry (PTV) and Optical Flow Velocimetry (OFV). They concluded that none of them stands out over the others and PIV, in particular, was found to have a noticeable grid dependence. However, they also argued that future effort on the verification of velocimetry codes specifically for the dense granular suspensions frequently encountered in fluidization would be welcomed in future publications.

Existing experimental studies in VFBs are mainly focused on beds operated in bubbling regime. These works include studies on the effect that vibration has on the bed and bubble dynamics [1, 5], the air pressure and void fraction fluctuations induced by vibration [26–30] and the aggregation [28, 31] and segregation phenomena [3, 18, 32–34]. Some works have also investigated the effect of vibration on the bubble behavior in VFBs [8, 10, 11]. A change of the flow pattern in the vibrated bed with regard to a conventional bed was observed by Mawatari et al. [9] and Cano-Pleite et al. [12]. Cano-Pleite et al. [16] studied the oscillatory behavior of the bed bulk and bubbles. The results in [12, 16] show that both the mean and oscillatory behaviors of bubbles are different in the lower (\(y < 0.2\) m) and upper (\(y > 0.2\) m) halves of a freely bubbling bed.

The use of PIV has been successfully extended to VFBs [17, 23, 35]. The results presented in these works reveal the suitability of PIV for the characterization of the motion of particles in a VFB and show that the application of PIV in a system of reference that moves with the bed or the bubbles is a valuable
mean to evaluate the particle behavior in the bed. In particular, Cano-Pleite et al. [17] used PIV to analyze the oscillatory movement and the wave propagation phenomenon in a pseudo-2D operated at minimum fluidization conditions. They observed that the oscillatory motion of the bed vessel promoted the formation of compression and expansion solid waves that travel through the bed and divide the bed bulk into two clearly distinguishable regions with opposed downward and upward solids motion. Cano-Pleite et al. [23] demonstrated that the behavior of the particles around the bubble were strongly affected by these compression and expansion wave fronts travelling in the bed. The validity of the potential flow theory for isolated bubbles rising in a vibrated fluidized bed operated at minimum fluidization conditions could be questioned based on the usual incompressible and irrotational flow assumptions of the theory. Nevertheless, comparisons of the experimental results in [23] with the Davidson & Harrison potential flow model [36] revealed that this model could be still applied for the motion of particles around bubbles rising in a vertically vibrated fluidized bed. However, although Cano-Pleite et al. [23] showed that the behavior of the bubble wake is cyclically affected by the oscillation of the bed vessel and the bed bulk, no systematic characterization was done and further analyses are required to understand the behavior of the particles in the volume of the wake of a bubble rising in a VFB.

Several works investigating the importance of the wake effect on the bed performance can be found in the literature. For example Patil et al. [37] analyzed the wall-to-bed heat transfer in gas–solid bubbling fluidized beds by means of experiments and simulations. They observed an increase of the heat transfer coefficient in the wake of the bubbles due to the scouring action of the enhanced solids circulation near the wall. Recently, Medrano et al. [38] studied in detail the mass flux distributions in a freely bubbling fluidized bed. They found that the solids fluxes increase as a function of the vertical position and depend on the excess gas velocity, contrary to the well-known two-phase theory of fluidized beds, which assumes the vertical solids fluxes to be constant. Medrano et al. [38] extended a PIV/DIA technique to analyze the amount of solids carried by bubbles on their wakes. The importance of the bubble wake was highlighted in Medrano et al. [38] since it allows to solve the internal mass balance for the solids phase in the fluidized bed and it was shown to give a proper description of the solids flow rates, which is of paramount importance in the phenomenological modeling of fluidized beds. In particular, the constant solid flux model classically employed over-predicted approximately by the double the actual average solids flux experimentally obtained in [38], which implied different slopes in the species mole fractions when different models for the wake parameter were employed. However, despite the importance of a good understanding of the behavior of the bubble wake in a fluidized bed, there are no works in the literature, to the authors’ best knowledge, that specifically analyze the behavior of the particles in the bubble wake region in a vibrated fluidized bed system.

The aim of the present work is to experimentally study the motion of particles in the bubble wake region of an isolated bubble rising in a vibrated fluidized bed, with the ultimate objective of quantifying the
volume of particles transported by the wake of bubbles when the bed is subjected to vibration. The novel
results presented in this work make use of the averaging of bubbles method described in [23] to average
the bubble contours and the velocity of solids in the proximity of the bubbles using both DIA and PIV.
This methodology allows to obtain an average behavior of the particles in the region of the bubble wake
as a function of the vertical position of the bubble in the bed and the vibration phase. The results here
presented provide a better understanding on the complex behavior of the fluid dynamics in the wake region
of bubbles rising in VFBs and are of practical interest to analyze the ability of the bubble wake to drag
particles upwards in a vibrated fluidized bed as a function of the vibration phase and the bed vessel vibration
amplitude and frequency. Characterization of this dragging ability can help to gain a better understanding
of the mixing and the segregation phenomena, with implications to heat and mass transfer, in vibrated
fluidized bed systems.

2. Experimental setup

Figure 1 represents the experimental facility used in this work. The bed vessel consists of a pseudo-2D
fluidized bed of dimensions 0.3 x 0.6 x 0.01 m (width $W$, height $H$ and thickness $K$, respectively). The
bed was placed on a structure that is sinusoidally vibrated in the vertical direction with a frequency $f$ and
amplitude $A$ by means of two vibro-motors situated at both sides of the fluidized bed. The front and rear
walls of the bed were made out of glass and the rear wall was painted in black to increase contrast of the
dense and bubble phases for DIA and PIV purposes. More details about the experimental facility can be
found in [17, 23, 39].

The bed vessel was filled with ballotini glass beads of type B according to Geldart’s classification [40]
with a size range between 425 and 600 $\mu$m and a density of 2500 kg/m$^3$. Approximately a 5 % of the particles
were painted in black. Thus, the dense phase presents an speckle pattern whose displacement could be easily
detected in the cross-correlation of the bed images during the PIV processing. The minimum fluidization
velocity of the particles at static conditions was measured using the mean value of the signal of a pressure
probe attached to the rear wall of the bed at 2.5 cm above the distributor and resulted in $U_{mf} = 0.28$ m/s.

Similarly to Cano-Pleite et al. [23], the air supply was divided into two different lines: one path of the
supply line was connected to the plenum of the bed and was used to achieve minimum fluidization conditions
in the bed for each of the vibration conditions tested. The second path was used to inject bubbles in
the system through a hole drilled in the rear wall of the vessel at a distance of 5 cm to the distributor, as
schematized in Figure 1. The volume of the injected bubbles was regulated by controlling the mass and
pressure of a preloaded volume of gas confined between 'Valve 1' and 'Valve 2' in Figure 1. The front view
of the bed was illuminated by two spotlights and was recorded with a high speed camera (Redlake Motion pro X3) at a frame rate of 250 images per second. The instantaneous position and velocity of the bed vessel and, therefore, its vibration amplitude and frequency were calculated by tracking the displacement of five white squares of small sizes glued to the plenum wall.

Each of the experiments comprised around 14 s, which corresponded to the injection of, approximately, 10 isolated bubbles. The duration of the experiments and the number of injected bubbles could not be increased due to limitations of the acquisition system. For each of the experiments, the bed was fluidized and the vibro-motors were switched on more than 10 s before starting recording the bed to avoid the acquisition of start-up effects. Some test bubbles were also injected before triggering the high speed camera, to visually guarantee optimal experimental conditions before recording.

The experiments carried out in the present work investigate the separate effect of the vibration frequency \( f \) and the vibration amplitude \( A \). The experiments are listed in Table 1, together with the value of the vibration strength, \( \Lambda = A(2\pi f)^2/g \), for each of them. Experiment 1 was selected as the base case with \( A = 4.2 \) mm and \( f = 14.2 \) Hz. The effect of the vibration frequency and amplitude were independently investigated by increasing \( f \) at nearly fixed \( A \) (experiments 1, 4 and 5 in Table 1) and by increasing \( A \) at a fixed \( f \) (experiments 1, 2 and 3). The cases tested were selected to cover the widest range of frequencies and amplitudes that the experimental facility allows. It was verified that the vibration frequencies evaluated in this work were considerably far from the resonance frequency of the bed material, evaluated following Barletta et al. [41]. Finally, for the sake of comparison, bubbles were also injected in a non-vibrated system (experiment 6). The static bed height for all the experiments was \( H_0 = 0.375 \) m (\( H_0/W = 1.25 \)). This value of \( H_0 \) was considered large enough to have sufficient data to characterize the motion of particles around the isolated bubbles along the bed height. During the experimental campaign it was observed that larger static bed heights (e.g. \( H_0/W = 1.5 \)) increased the particle rain inside the bubbles, which complicated a precise determination of the bubble contour and the evaluation of the solids velocity on top of the injected bubbles. Smaller static bed heights were not considered, as they would prevent from a clear analysis of the behavior of the particles as a function of the vertical position of the bubble inside the bed, especially due to the presence of compression-expansion waves travelling through the bed [17].

3. Data processing

3.1. Particle Image Velocimetry

The solids velocity, \( \vec{v}_s \), in the bed bulk and the bubble contour were calculated by means of PIV using the code MATPIV 1.6.1. [42]. A DIA mask that discriminates between the dense phase and the bubble
phase is used for each image. Then, the velocity vectors are calculated exclusively in the regions identified as dense phase during the bubble detection procedure.

The velocity of solids around a bubble was also investigated as a function of the phase of the bed vessel by means of an averaging of cycles method [16, 17, 23, 39, 43]. As the bed manifests a cyclic behavior of period $T$, the cyclic oscillation of the bed vessel in each period can be divided into $M$ phase intervals $\left(\frac{2(k-1)\pi}{M} < \phi < \frac{2k\pi}{M}\right)$ $k = 1 \ldots M$, to describe the transient behavior of the particles in terms of the phase, $\phi \in [0, 2\pi]$ instead of the time. This allows to calculate the time average velocity of the solids at every phase interval. In this averaging of cycles method, the beginning of the cycle, phase $\phi_i = 0$ at $t = t_0$, corresponds to the bed vessel at its central position and moving upwards and the end of the cycle, $\phi_i = 2\pi$ at $t_0 + T$, to the bed vessel at its central position and moving again upwards after the bed has finished its descending stroke and it is in the middle of the ascending one.

3.1.1. Bubble averaging method

The averaging of bubbles method presented in Cano-Pleite et al. [23] was used to statistically characterize the particle velocity in the bubble wake region. This method consists in averaging all the bubbles identified along an experiment in order to obtain the average distribution of the solids velocity around an average bubble that statistically represents all the analyzed bubbles. The bubble averaging method is qualitatively described in the following lines and is schematized in Figure 2. More details about the bubbles averaging method can be found in Cano-Pleite et al. [23].

The instantaneous area-equivalent bubble diameter ($D_{b,i} = \sqrt{4A_{b,i}/\pi}$), velocity ($V_{bub,i}$), centroid coordinates, $((x,y)_{b,i}^{(C)})$ and perimeter $((x,y)_{b,i}^{(P)})$ are obtained, by means of DIA, for every image frame acquired with the high speed camera (numbered with $i$) in which an isolated bubble is present in the system. The contour of the bubble is then normalized and translated to a new system of coordinates, defined by a normalized pair of coordinates $(X,Y)$. The centroids of all the bubbles are placed at $X = 0$ and $Y = 0$ in this new coordinate system and their size is scaled so that their equivalent radius is equal to unity. This operation is repeated for all the bubble contours identified along an experiment. Then, the average contour $(X,Y)_{b,avg}^{(P)}$ is defined as the contour that averages all the analyzed bubbles, as schematized in Figure 2(b).

The same operation is carried out for the solids velocity vectors, $\vec{v}_s(x,y)_i$, around the bubble. The normalization of these velocity vectors is carried out dividing the horizontal and vertical components of the velocity vectors by the mean vertical velocity of the centroid of all the analyzed bubbles. This yields the normalized solids velocity of each bubble in the new system of normalized coordinates, $\vec{V}_s(X,Y)_i$. However, the position of the velocity vectors, in the new system of coordinates $(X,Y)$ may vary from one frame to another, since they are displaced during their normalization. Thus, the average velocity around the bubble
in the new coordinate system, $\vec{V}_{s}(X,Y)_{avg}$ is obtained by dividing the new system of coordinates in $N_{bins}$ averaging windows and linearly interpolating the velocity vectors within every window to its center. The size of the averaging windows is such that the distance between their centers is similar to the normalized distance between PIV vectors in the initial PIV image. Once all the bubble contours and the velocity vectors are averaged in the new coordinate system, the normalized mean solids velocity of the particles as a function of the angle around a bubble, $\theta$, could be determined, where $\theta$ is the angle between the position of the velocity vector and a vertical line passing through the bubble centroid. The interpolation of the solids velocity allows the determination of the fitting circle $(X,Y)^{(F)}$, of radius $R$, that better fits the average bubble contour $(X,Y)^{(P)}_{b,avg}$ in an angle range $\theta \in [-120^\circ, 120^\circ]$ and the new fitting circle $(X,Y)^{(F)}_{n}$, with radius $R^*$, which represents the radius at which the velocity of the solids in the average bubble is maximum at $\theta = 0^\circ$ (see [23] for more details). The values of the normalized solids velocity vectors are interpolated to this new fitting circle, which is used for the determination of the velocity of the bubble front and the solids velocity in a system of reference that moves with the bubble front [23].

3.1.2. Wake velocity and volume

The wake behavior was analyzed in this work by means of the velocity and normalized volume of the particles dragged by the wake as a function of the vibration phase. Previous works brought to light the different behavior of bubbles in a VFB depending on their vertical position along the bed height [16, 23]. Thus, the results were analyzed differentiating between bubbles situated in the lower ($y_b = 0.1 - 0.2$ m) and upper ($y_b = 0.2 - 0.3$ m) sections of the bed. These limits also ensure that the bubbles are fully developed along the heights investigated, avoiding analyzing bubbles in their early formation stages ($y_b < 0.1$ m) or bubbles perturbed by their eruption at the surface of the bed ($y_b > 0.3$ m).

The velocity of the bubble wake is calculated as the average vertical velocity of the solids inside a region delimited by a circular sector of angle $\theta_w = 20^\circ$ and a distance to the bubble center smaller than the radius, $R$, of the circle that better fits the equivalent bubble contour, $(X,Y)^{(F)}$, as indicated in Figure 2(c). Given the oscillatory nature of the system, the velocity of the particles in the bubble wake region can be studied in an absolute system of reference or in a moving system of reference. This relative velocity is referred to a system of coordinates that moves at the same vertical velocity as the bubble front, which is defined as the vertical velocity of the solids at the fitting circle $(X,Y)^{(F)}_{n}$ and averaged within an angle $\theta \in [-10^\circ, 10^\circ]$ at the dome of the average bubble (see $V_{front}$ in Figure 2). In previous works analyzing the particle behavior around a bubble rising in a VFB [23], the use of a relative velocity that refers to the bubble front maximum velocity resulted in a better definition of the potential flow model. This allowed to avoid the effect of particle rain on the calculation of the solids motion around the bubble. Given the fact the oscillation of the bubble velocity and the vibration of the bed vessel are directly linked [39], the use of this relative system of reference also serves to determine the solids velocity in a system of reference that is directly related to the bubble,
facilitating the interpretation of the results and enhancing their relevance. Vertically vibrated fluidized beds exhibit the particularity of the existence of regions in the vicinity of the bubble in which the solids velocity are largely affected, at certain phases, by the presence of compression-expansion waves travelling inside the bed \cite{16, 17}. Therefore, in view of the results and previous studies, the definition here presented for the evaluation of the relative system of reference is considered appropriate to provide a quantitative analysis of the evaluation of the velocity of the particles in the bubble wake region and the evaluation of the capability of the bubble wake to drag particles upwards in a vibrated fluidized bed.

The volume of particles dragged upwards by the bubble wake is arbitrarily calculated in Equation (1) as the volume of the dense phase under the bubble where the solids velocity is larger than the velocity of the bubble front, calculated as indicated in the previous paragraph. This novel evaluation was specifically conceived due to the complexity of evaluating the wake volume in a VFB and differs from other definitions found in the literature for conventional fluidized beds. For example, in \cite{44} the wake volume was calculated by setting a threshold that accounts for the particles having a velocity larger than 0.8 times the bubble velocity and in \cite{45–47} the evaluation of the bubble wake volume is based on geometric arguments.

The volume belonging to the bubble wake can be graphically seen in Figure 2(c) and is evaluated as the volume of particles below the bubble center (i.e. \( Y < 0 \)) with upwards vertical relative solids velocity, \( V_r \). It was observed at high vibration frequencies that, during the ascending stroke of the bed vessel, the wake volume was sufficiently large to interact with the compression wave front below it, making it difficult to differentiate between the bubble wake and the solids pushed upwards by the compression wave front. To prevent an overestimation of the bubble wake volume, the evaluation of the bubble wake volume was limited to \( Y > -3 \). The evaluation of the bubble wake volume is done by averaging the bubble contours and solids velocities in phase intervals of \( \Delta \phi = 0.25\pi \) along the experiments. To increase the resolution of the results in the definition of the bubble wake volume, a subgrid interpolation of the normalized relative velocity vectors of the solids, \( \vec{V}_r \), was carried out. Thus, the number of \( n \) windows in which the velocity vectors are averaged and the wake volume is evaluated was increased to \( N_w = 3N_{bins} \). Neglecting velocity gradients perpendicular to the PIV plane, the wake volume is therefore defined as:

\[
VOL_w = \sum_{n=1}^{N_w} X_w Y_w K \gamma_n
\]  

(1)

with

\[
\gamma_n = \begin{cases} 
1 & \text{if } V_{r,n} \geq 0 \text{ and } -3 < Y_n < 0 \\
0 & \text{otherwise}
\end{cases}
\]  

(2)

where \( K \) is the bed thickness and \( X_w \) and \( Y_w \) are, respectively, the horizontal and vertical dimensions of
the $N_w$ analysis windows in which the solids velocity is evaluated. It was observed that further increasing
the number of analysis windows did not yield an appreciable change on the obtained results. To obtain
results independent on the bubble diameter, the wake volume was normalized dividing it by the volume of
the average bubble in each phase interval of analysis:

$$VOL_w = \frac{VOL_w}{A_{b,avg}}$$  \hspace{1cm} (3)

where $A_{b,avg}$ is the visible area of the normalized average bubble. That is, the area enclosed by the average
bubble contour $(X,Y)_{b,avg}^{(P)}$.

4. Results and discussion

The results are divided in three sections. The first section qualitatively analyzes the motion of particles
in the wake region of an isolated bubble rising in the vibrated fluidized bed. This analysis serves to discern
the particle behavior in the wake of the bubble and to provide an explanation to the quantitative results
presented in the second section. The second section is therefore supported by the visual observations
presented in the first section and it is devoted to a quantitative analysis of the velocity and the volume of
particles dragged by the bubble. The third section analyzes the effect of the bed vessel vibration amplitude
and frequency on the volume of particles dragged by the bubble wake.

4.1. Particle behavior in the bubble wake region

Figure 3 shows five consecutive snapshots of a bubble rising in the studied vibrated fluidized bed at a bed
height $y_b \simeq 0.2$ m. The sine wave on top of each figure represents the bed vessel position and the phase at
which each of the snapshots was taken. Figure 3 allows to analyze the interaction of solids with the bubble
and to discern the effect of the bed bulk cyclic compression and expansion on the behavior of the particles
in the bubble wake region. The black arrows correspond to the velocity vectors calculated using PIV whereas
the red arrows are manually overimposed to highlight the main solids path.

[Figure 3 about here.]

For the bubbles and the conditions of the present work, it was observed that the wake generally penetrates
the bubble during the descending stroke of the bed vessel ($\phi = 0.5\pi - 1.5\pi$ rad). This penetration is identified
by the volume of solids that enter the bubble contour, cyclically modifying the expected spherical cap shape
of the bubbles. In particular, wake penetration in the bubble is detected when a wavy pattern appears at
the bubble bottom. This is shown in Figure 3(c), where the green circle specifically highlights the wake
region. As observed in previous works using a bed of similar characteristics [17], the bed vessel commences
to descend in a phase close to the one represented in Figure 3(a). In this phase, the particles move upwards
below the bubble because the bed bulk maintains the inertial motion of the upwards displacement of the bed vessel. The presence of the bubble perturbs this upward motion of particles, causing the regions of low solids velocities observable in Figure 3(a,e).

In the snapshots shown in Figure 3(b,c) the bed vessel is on its descending stroke, which implies an expansion of the bed bulk, according to [17]. During this descending stroke the bubble velocity decreases and a negative velocity of the solids of the bed bulk to the right and left sides of the bubble appears due to the expansion of the bed bulk. The particles in the bubble wake region are not affected by this downward solids velocity and maintain their inertial upward displacement. This causes the penetration of the bubble wake in the interior of the bubble contour, producing the characteristic wavy contour of bubbles in a vibrated fluidized bed, which was also observed in previous works [23, 39, 43]. Also below the bubble, in Figure 3(b,c), the volume of solids that is dragged by the bubble wake has positive vertical velocity and can be discriminated from the solids moving downwards outside the wake of the bubble. This leads to the appearance of an inflexion point (i.e. a point of null solids velocity) and a region of low solids velocities below the bubble.

As observed in previous works [17] the bed vessel impacts the bed bulk in a phase close to $\phi = 1.5\pi$ rad. This promotes an upward velocity of particles below the bubble in the subsequent phases, as reflected in Figure 3(d,e). Additionally, in Figure 3(d), the volume of particles that have penetrated the bubble in the phases $\phi = 0.6\pi - 1.5\pi$ rad decreases due to the increase of the bubble velocity and the collapse to the bubble contour of the wavy contour formed in the bubble wake in the previous phases of the cycle. In Figure 3(e), the volume of particles penetrating the bubble has disappeared and the velocity of the particles below the bubble is mainly vertical, as the particles are moving upwards due to the ascending stroke of the bed vessel, which causes the presence of a compression wave inside the bed [17].

4.2. Velocity and volume of particles in the bubble wake region

The wake behavior described in the previous section was analyzed by means of the normalized average velocity of the particles dragged by the bubble wake, as indicated in Section 3.1.2. Figure 4 shows the velocity of the solids in the wake region as a function of the vibration phase for bubbles situated in the upper and lower sections of the bed. Each point depicted in the figure averages bubble contours in phase intervals of $\Delta\phi = 0.25\pi$ rad and situated at a specific height interval, which corresponds to the averaging of more than 50 bubble contours. The normalized absolute velocity of particles in the wake region, $V_{w,abs}$, is maximum for a phase interval of the bed vessel close to $\phi = 0$ (see also Figure 3). In that phase, the bed vessel velocity is maximum and the bubble centroid follows the motion of the bed bulk, which presents the solids compression wave travelling in its interior [12, 23, 39, 43]. The normalized solids velocity in the wake region progressively decreases as both the bed bulk and the bubbles decelerate and commences to increase approximately when the bed vessel commences the ascending stroke ($\phi \sim 1.4\pi$ rad) due to the presence of
the waves at the bottom of the bubble that penetrate its contour (see Figure 3).

The velocity of the particles in the bubble wake can be more easily analyzed in terms of their normalized relative velocity, $V_{w,rel}$, as depicted in Figure 4. For a bubble situated in the lower section of the bed, $V_{w,rel}$ reaches a minimum value at a phase close to $\phi = 0.6\pi$ rad, which corresponds to the start of the descending stroke of the bed vessel (note that the bed vessel commences to descend at $\phi = \pi/2$). From that phase value on, and until a value of $\phi = 2\pi$ rad, the wake normalized relative velocity commences to increase and presents a velocity that is larger than that of the bubble front. These results confirm the qualitative observations made in Figure 3 in which the wake penetration is appreciable for a phase $\phi \approx 0.9\pi$ rad. The relative velocity of the wake increases for $\phi > 0.6\pi$ rad, indicating that the wake penetrates into the bubble. This behavior is delayed for a bubble situated in the upper section of the bed (i.e. $y = 0.2 \sim 0.3$ m). The reason is that both the bubble velocity and the compression and expansion waves affecting the bed bulk oscillation [17] present a larger phase delay (with respect to the bed vessel oscillation) as the distance of the bubble to the distributor increases. For both $V_{w,abs}$ and $V_{w,rel}$, the bubble wake presents a larger velocity when the bubble is situated in the lower section of the bed than when it is situated in the upper section of the bed. This may be attributed to the progressive damping of the compression and expansion solids velocity waves as they travel inside the bed, which have a direct impact on the solids velocity below the bubble. The standard deviation of the normalized solids velocity in the bubble wake can be evaluated considering the velocity vectors that lie within the wake analysis region ($V_{w}$ in Figure 2). The resulting value of this standard deviation is very small in most of the studied cases. For bubbles situated in the lower region of the bed, the average of the standard deviation of the normalized velocity within the bubble wake, calculated as the mean value of the standard deviations of all the phase intervals shown in Figure 4, is 0.016, with a maximum value of 0.02, whereas for bubbles situated in the upper region of the bed, this average normalized standard deviation is 0.04 with a maximum value of 0.12. The small values of the standard deviations in this evaluation are a good indicator of the reproducibility of the results, as they indicate that a narrow distribution of velocities can be expected for the average velocity vectors that are used for the evaluation of the velocity of the bubble wake, confirming the applicability of the averaging of bubbles method.

[Figure 4 about here.]

Quantitatively, the wake behavior described in the previous lines can be further analyzed by means of the volume of particles dragged upwards in the bubble wake. This volume of particles is calculated following the indications of Section 3.1.2. Figure 5 shows maps of the vertical normalized relative velocity magnitude, $V_r$, together with the volume of particles dragged by the bubble wake for four different phase intervals of size $\Delta \phi = 0.25\pi$ rad. Thus, each of the maps in Figure 5 is obtained by the averaging of more than 100 bubble contours, which ensures the robustness of the bubble averaging method. From Figure 5 it is clear that the volume of particles dragged by the wake is directly affected by the oscillatory motion of the bed bulk. The
compression wave traveling inside the bed can be identified by the large solids velocities in the regions below the bubble, as in Figure 5(a). This front directly affects the volume of the wake, as it collaborates with the bubble dragging ability, making the bubble to more easily drag the particles below it, which can be clearly observed in the phase interval $\phi = 0 - 0.25\pi$ rad in the figure. Equivalently, the larger downward velocity of solids in the vicinity of the bubble in the phase intervals $\phi = 0.5\pi - 0.75\pi$ rad and $\phi = \pi - 1.25\pi$ rad decreases the wake volume even if the particles in the region below the bubble still present a large solids velocity at the bottom caused by the compression wave in the previous phases, as shown in Figure 5. At the latter stages of the bed vessel vibration in $\phi = 1.5\pi - 1.75\pi$, the compression wave again commences to increase the volume of particles dragged by the bubble, this is noticeable by the increase of the velocity of the particles below the bubble between Figure 5(c) and Figure 5(d). This effect is also observable in Figure 7(c) and Figure 7(d).

[Figure 5 about here.]

Figure 6 shows the normalized volume of the bubble wake in Exp. 1 for different intervals of the bubble vertical position and 8 phase intervals. As a reference, Figure 6 also includes the volume of particles dragged by a bubble in the case the bed is not vibrated (Exp. 6) and for a height interval of $y = 0.1 - 0.2$ m. Due to the procedure employed to generate the isolated bubbles, the volume of gas injected to create a bubble in the non-vibrated bed was not enough to keep the bubble expected growth and bubbles tended to split in the upper section of the bed under no vibration conditions. Under vibration conditions, the isolated bubbles were more stable and less prone to split while rising in the bed remaining isolated until they reached the surface of the bed. Consequently, to make the comparison with the non-vibration conditions consistent, only the height interval of $y = 0.1 - 0.2$ m is employed here for the non-vibrated case. In Figure 6, the increase of the wake volume with an increase of $\phi$, as a consequence of the compression wave, can be clearly observed in the phase interval $\phi = 3\pi/2 - 2\pi$ rad (of the next cycle). Similarly to the observations done in Figure 5, during the phase interval $\phi = 0 - \pi/2$ rad, the volume of particles dragged upwards by the bubble decreases with $\phi$ due to the commencement of the bed expansion (see Figures 3 and 5). Also, in the phase range $\phi = \pi/2 - 3\pi/2$ rad, when the bubble is in the lower section of the bed, the volume of solids dragged by the wake of the bubble is nearly similar to that of the case in which the bed is not vibrated (using the same identification criteria as in the vibrated system). Noticeably, the phase range $\phi = \pi/2 - 3\pi/2$ rad corresponds to the phase interval in which the wake penetrates the bubble, as commented in Figure 3. In that phase range, in addition to the smaller volume of particles dragged below the bubble (Figure 5), the particles that penetrated the bubble contour are not affected by the bulk motion around the bubble and present an upward velocity that is smaller than the velocity of the bubble front. The same trend of the wake volume with the vibration phase described in Figure 6 is followed for all the vibration conditions tested (i.e. the wake volume is minimum in the range $\phi = \pi/2 - 3\pi/2$ rad).
4.3. Effect of vibration amplitude and frequency

The effects of the vibration amplitude and frequency on the normalized wake volume of bubbles rising in the bed are investigated in Figure 7. The figure presents the averaged value of the normalized volume evaluated along the whole bed height, i.e., \( y_b = 0.1 - 0.3 \) m. The non-vibrated case is not included in the figure due to the difficulty to obtain stable isolated bubbles in the \( y_b = 0.2 - 0.3 \) m interval when the bed was not vibrated. The normalized volume of the dragged particles increases when increasing both the vibration amplitude and frequency of the vibration of the bed vessel. When increasing \( f \) or \( A \), the vibration strength, \( \Lambda \), increases and a larger amount of momentum is introduced in the system, which might favor the dragging of a larger amount of particles by the bubble in the phase interval \( \phi = 3\pi/2 - \pi/2 \) rad (of the next cycle). Interestingly, vibration of the bed vessel at the lower amplitude (\( A = 2.8 \) mm) reduces the volume of the bubble wake to zero in the phase range \( \phi = \pi/2 - 3\pi/2 \) rad. Note that, in that phase interval, the bed vessel is performing its descending stroke, which corresponds to the expansion of the bed bulk. This null value of the bubble wake volume (as defined in this work) for \( A = 2.8 \) mm is attributed to the low energy of the compression wave travelling inside the bed. The bed bulk compression front that promotes the large volume of particles dragged in the bottom region of the bubble in phases \( \phi = 3\pi/2 - \pi/2 \) rad (of the next cycle) is not energetic enough to promote a large velocity in the bubble wake that could overcome the decrease of the velocity of the particles due to the expansion of the bed bulk in the phases \( \phi = \pi/2 - 3\pi/2 \). This situation is, therefore, similar to the one observed for bubbles situated in the upper section of the bed in Figure 6. Besides, as shown in Figure 7, increasing the vibration amplitude clearly increases the normalized wake volume to values above zero in the phase range \( \phi = \pi/2 - 3\pi/2 \) rad. For the highest vibration frequency (\( f = 17.1 \) Hz) and the phase interval \( \phi = 1.75\pi - 0.5\pi \) rad (of the next cycle) and for the frequency \( f = 15.6 \) Hz and the phase interval \( \phi = 0 - 0.25\pi \) rad, the wake volume is sufficiently large to interact with the compression wave front below it. This front also travels at a velocity larger than the velocity of the bubble front and that makes difficult to differentiate between the solids dragged by the bubble wake and the solids pushed upwards by the compression wave front.

Table 2 presents the average values along all the vibration phases of the normalized volume dragged for the bubbles in the lower section of the bed, \( y_b = 0.1 - 0.2 \) m, and for the different vibration conditions indicated in Table 1. To avoid the extraction of misleading conclusions due to the interaction of the compression wave below the bubble and the bubble wake, the results presented in the table are averaged in the phase interval \( \phi = \pi/2 - 3\pi/2 \) rad. Only values for the normalized wake volume in the lower part of the bed are extracted so that they can be compared with the non-vibration conditions. The constant average values
depicted in Table 2 can serve as a reference on how vibration is capable of enhancing the dragging ability of the bubble wake in a vibrated fluidized bed system and its implications. The results show that vibration clearly creates a larger volume of solids dragged by the bubble wake. Comparison of the values reported in the table indicates that the average volume of particles dragged by the bubble wake can be increased by a factor of 10 between different vibration conditions (including no vibration).

As stated in the literature [48], the shape of the bubbles varies as they grow in the bed, leading to an increase of the volume of particles dragged by the wake in comparison to the volume of the bubble, i.e. the bubble wake parameter. Medrano et al. [38] discussed in detail the importance of this wake parameter for the phenomenological modeling of fluidized bed reactors. In the particular case of methane reforming, they found deviations of around 10 % in the hydrogen mole fraction profile when different models for the wake parameter were employed, since it is proportional to the solids flux. Thus, taking [38] as a reference, it can be considered that the increase of the volume of particles dragged by the bubble wake achieved under vibrating conditions could significantly improve the performance of the gas-solid reactor. It is worth to mention that, despite the fact the definition of the bubble wake region of this work and the method employed by Medrano et al. [38] for the evaluation of the bubble wake parameter are not directly comparable, the relative difference among the vibrating conditions and between the non-vibrating and vibrating conditions here presented can be illustrative. Further detailed experiments and numerical simulations would be required, but the results here presented confirm the promising potential of a vibrated bed to enhance the performance of gas-solid reactions.

5. Conclusions

The behavior of the wake of a bubble rising in a pseudo-2D fluidized bed subjected to vertical vibration was experimentally studied in the present work by means of combined DIA and PIV techniques. The bed material was Geldart B particles, the bed vessel was vibrated at different frequencies and amplitudes and was operated at minimum fluidization conditions. Bubbles were sequentially injected in the system and the solids motion in the bubble wake region was analyzed by means of a bubble averaging method that averages the behavior of the particles to obtain statistically meaningful results.

The results shown in the present contribution provide an explanation to the cyclic penetration of particles in the bubble contour, promoting the characteristic wavy shape of bubbles in a vibrated fluidized bed. This cyclic particle penetration is caused by the compression and expansion of the bed bulk, which also has a direct effect on the velocity of particles in the bubble wake region. The PIV results of the average solids velocity in the wake region were also used to estimate the volume of particles dragged by the bubble wake.
According to the results, this volume of particles cyclically varies as a function of the vibration phase. The maximum volume of particles dragged by the bubble wake is reached during the ascending stroke of the bed vessel, as it is directly affected by the presence of the solids compression wave traveling through the bed bulk. The results also revealed that the increase of the bed vibration amplitude or frequency promotes a net augmentation of the volume of particles dragged by the bubble wake. However, this volume exhibits a cyclic behavior and presents phases in which, for a sufficiently low vibration strength, the volume of particles dragged by the bubble is inferior to the average volume dragged by a bubble in a non-vibrated fluidized bed in the lower part of the bed.

The above results can help to understand the interaction between the bed bulk oscillation and the bubble behavior in a vibrated fluidized bed. More importantly, these results indicate that vibration of the bed vessel can be used to gain some control over the volume of particles dragged by the bubbles, which directly affects the particle mixing and the heat and mass transfer in the bed, an aspect of great relevance in vibrated systems such as dryers and chemical conversion reactors.

**Nomenclature**

\( A \) = bed vibration amplitude (m)
\( A_b \) = bubble area (m²)
\((x, y)_{(C)}\) = bubble centroid coordinates (m)
\( D_b \) = bubble diameter (m)
\((X, Y)_{(P)}\) = fitting circle coordinates (-)
\((X, Y)_{(P)}^n\) = new fitting circle coordinates (-)
\( f \) = vibration frequency (Hz)
\( H \) = bed height (m)
\( H_0 \) = static bed height (m)
\( K \) = bed thickness (m)
\( M \) = number of phase intervals for averaging of cycles (-)
\( N_{bins} \) = number of windows for velocity averaging (-)
\((x, y)_{(P)}\) = bubble perimeter coordinates (m)
\( R \) = fitting circle radius (-)
\( R^* \) = radius of the new fitting circle (-)
\( T \) = period of vibration (s)
\( t \) = time (s)
\( U_{mf} \) = minimum fluidization velocity (m/s)
\( V \) = vertical velocity (m/s)
$V_b(t)$ = instantaneous vertical velocity of a bubble (m/s)
$V_{bub}$ = bubble velocity (m/s)
$V_r$ = normalized vertical velocity of solids relative to the bubble front velocity (-)
$\vec{V}_r$ = normalized solids velocity vector relative to the bubble front velocity (-)
$\vec{V}_s(X,Y)$ = normalized solids velocity vector (-)
$\vec{u}_s$ = solids velocity vector (m/s)
$v$ = vertical solids velocity (m/s)
$W$ = bed width (m)
$X$ = normalized horizontal coordinate (-)
$x$ = horizontal coordinate (m)
$Y$ = normalized vertical coordinate (-)
$y$ = vertical coordinate (m)

Greek letters
$\Delta \phi$ = phase interval (rad)
$\theta$ = angle from the vertical direction (°)
$\theta_b$ = bubble rising angle (°)
$\Lambda$ = vibration strength parameter (-)
$\phi$ = bed vessel vibration phase (rad)

Subscripts
$avg$ = average
$b$ = bubble
$bed$ = bed vessel
$r$ = relative
$w$ = wake

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<table>
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<th>Exp.</th>
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Table 2: Normalized volume of particles dragged by the bubble wake averaged over the phase interval $\phi = \pi/2 - 3\pi/2$ rad. $y_b = 0.1 - 0.2$ m

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