

Simulations without data updates using analytical attitude propagator GSAM for spin stabilized satellites

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Abstract. The objective of this work is to validate the GSAM propagator using new data provided by the National Institute for Space Research (INPE) from SCD1 and SCD2 data collection satellites, with emphasis on long interval simulations without daily data updates. Originally, only 40 days of data were available to test the program, constraining any attempts to measure its precision more accurately. Recently, over two decades of data regarding both satellites' orbital and attitude parameters were provided, allowing further studies and validation of the program. The rotational motion equations are composed by the gravity gradient torque, aerodynamic torque, solar radiation pressure torque, residual and eddy current magnetic torques, the latter using a dipole geomagnetic model. The results are considered fitting when the mean deviation between the calculated variables and the real satellite data stay within 0.5° for the right ascension and declination angles and 0.5 rpm for the spin velocity. Intervals that meet the required precision were found for all years, from three to up to 15 days of simulation without data update. The consistent detection of such intervals further corroborate the use of the propagator to estimate the orientation of the satellites studied in their missions.

1. Introduction

It is unarguable that satellites are vital to tackle several tasks of modern life, influencing society from weather forecasting to intelligence and defense. Across most of the range of uses for these devices, knowing their orientation at all times is fundamental. This work is focused on satellites with low eccentricity orbits and spin stabilization, characterized by rotation around the axis of greatest principal moment of inertia.

When in space, satellites are subject to multiple external forces, affecting their rotational motion by inducing different torques. The main causes of perturbation for these objects [1] are the gravity gradient torque (GGT), derived from the Earth's gravity force attracting the satellite's non-uniformly distributed mass; the solar radiation torque (SRT), consequence of the variation of momentum of incident photons hitting the surface of the satellite; the aerodynamic torque (AT), caused by collision between molecules from the high atmosphere with the satellite's surface; the residual magnetic torque (RMT), result of the interaction between the magnetic moment over the rotation axis of the satellite and the geomagnetic field; and, lastly, the eddy



currents torque (ECT), originated from the interaction between small induced currents on metal parts of the satellite and Earth's magnetic field.

Mathematical models for torques acting over the satellite were presented in Yu [2], Thomas and Cappelari [3], Wertz [4], Venkataraman and Carrara [5], and Carrara [6]. However, the models for external torques used in this paper were presented in Zanardi et al [1] and Zanardi et al [7].

1.1. Rotational motion equations and analytical solution

The equations that describe the rotational motion of spin stabilized satellites, as showed in [8], depend on the external torque components acting on the satellite's fixed system and on the greatest moment of inertia around the axis of rotation.

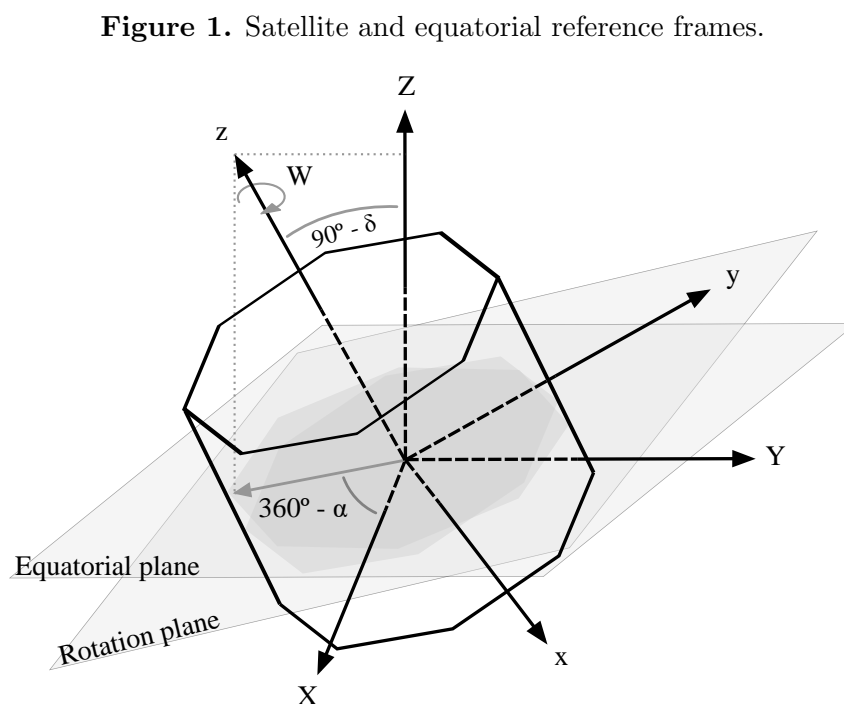
The equations are given in terms of the right ascension angle, the rotation axis declination angle and the absolute value of the spin velocity [8, 4], as follows:

$$\frac{d\alpha}{dt} = \frac{N_x}{I_z W \cos(\delta)} \quad (1)$$

$$\frac{d\delta}{dt} = \frac{N_y}{I_z W} \quad (2)$$

$$\frac{dW}{dt} = \frac{N_z}{I_z} \quad (3)$$

where I_z is the axis of greatest principal moment of inertia around the spin axis and N_x , N_y , and N_z are the net torque components on all three axes of the satellite coordinate system. Figure 1 shows how the main variables are defined, based on the satellite and equatorial reference frames.



As shown in [4], integrating the equations aforementioned results in the following analytical solutions, valid for one orbital period:

$$W = \left(W_0 + \frac{N_{GGz}}{N_{ECz}} \right) e^{\frac{N_{ECz}t}{I_z}} - \frac{N_{GGz}}{N_{ECz}} \quad (4)$$

$$\delta = \frac{t}{I_z} \left(N_{ECy} - \frac{N_{ys}N_{ECz}}{N_{GGz}} \right) + \frac{N_{ys}}{N_{GGz}} \ln \left(\frac{W}{W_0} \right) + \delta_0 \quad (5)$$

$$\alpha = \frac{t}{I_z \cos(\bar{\delta})} \left(N_{ECx} - \frac{N_{xs}N_{ECz}}{N_{GGz}} \right) + \frac{N_{xs}}{N_{GGz} \cos(\bar{\delta})} \ln \left(\frac{W}{W_0} \right) + \alpha_0 \quad (6)$$

where, in equation (4), W is the spin velocity in *rad/s*, W_0 is the initial value for the spin velocity, N_{GGz} and N_{ECz} are the components in the z -axis for the gravity gradient and the eddy currents torques, respectively. In equation (5), N_{ECy} and N_{ys} are the components in y for the eddy currents torque and the sum of components of torques in the y -axis that do not multiply by W , respectively, while δ_0 is the initial value for the rotation axis declination. Lastly, in equation (6), $\bar{\delta}$ is the average between the calculated declination and its initial value, N_{xs} is the sum of components of torques in the x -axis that do not multiply by W , while α_0 is the initial value for the rotation axis declination.

1.2. Objectives

Motta [9] has created a propagator based on the previously shown solutions and named it GSAM, after the first letters of the torques used in the analytical solutions: **G**avity gradient torque, **S**olar radiation torque, **A**erodynamic torque and **M**agnetic torques. In his work, the program successfully predicts the motion of data collection satellites SCD1 and SCD2, staying within the required precision of 0.5° for the declination angle and 0.5 *rpm* for the spin velocity, and exceeding it for the right ascension angle. At the time, only 40 days of data from both satellites were available to test the propagator.

Thus, the objective of this work is to use the new data provided by the National Institute for Space Research (INPE), spanning over two decades for both satellites, to validate the propagator. More specifically, the program will be used without daily data updates throughout the simulations, expecting to find intervals in which the spatial orientation of the satellites can be effectively calculated.

2. Methodology

The propagator was programmed to be used in two different simulation modes. In case it is daily updated, the satellite attitude will be propagated for 24 hours, stored then replaced by data from the following day to be propagated for another 24 hours, continuing this logic until the 20-day interval is fully simulated. In case it is decided to not update the data daily, the program will use its own results after the first day and continue onto the next one, carrying on the errors and consequently increasing imprecision of the results.

This work is focused on the simulation mode without daily data updates. Although it provides less precise results when compared to daily updated simulations, it is a better way to test the program's accuracy predicting the satellites' orientations from a single set of original data. Since the propagator relies on its own results to carry out the calculations, slight deviations are enough to completely throw off the predictions, making intervals as short as three days already considered a positive result.

Once a three-day interval was found for a given year, simulations incrementing the interval a day at a time were carried out, in an attempt to find the longest streak of days for which the

required precision was met for all variables. Even though a year of data could provide multiple valid intervals, only one per year is displayed in the results.

3. Results

The following tables compile the mean deviations for the three variables studied – the right ascension angle, the declination angle and the spin velocity. Table 1 displays the results for satellite SCD1 and Table 2 for satellite SCD2. Every year available in the data had at least one three-day interval found that complied with the required precision of 0.5° for the angles and 0.5 rpm for the spin velocity, with up to 15 days in which the results were satisfactory.

Table 1. Intervals simulated for the SCD1 satellite without daily data updates.

Intervals	$\Delta\alpha(^{\circ})$	$\Delta\delta(^{\circ})$	$\Delta W(rpm)$
04/18/1994 – 04/22/1994 [5 days]	0.17790	-0.45656	0.11101
10/28/1995 – 11/01/1995 [5 days]	0.07738	-0.41353	-0.35120
01/02/1996 – 01/16/1996 [15 days]	-0.29075	-0.43014	0.45433
03/01/1997 – 03/03/1997 [3 days]	-0.37972	-0.26662	-0.01055
10/07/1998 – 10/11/1998 [5 days]	-0.49706	0.29712	0.04867
04/08/1999 – 04/10/1999 [3 days]	-0.45026	-0.13577	-0.09739
07/18/2000 – 07/20/2000 [3 days]	-0.38526	0.19135	-0.11972
11/16/2001 – 11/20/2001 [5 days]	0.16665	0.33697	-0.48203
11/15/2002 – 11/21/2002 [7 days]	-0.48920	0.37255	0.10286
04/19/2003 – 04/21/2003 [3 days]	-0.22399	-0.16082	-0.15237
03/27/2004 – 03/29/2004 [3 days]	0.02606	-0.36459	0.22003
04/20/2005 – 04/26/2005 [7 days]	0.11788	-0.40805	0.19131
03/08/2006 – 03/12/2006 [5 days]	-0.26800	-0.22240	0.18244
04/24/2007 – 04/26/2007 [3 days]	-0.19683	-0.40298	0.04629
11/05/2008 – 11/09/2008 [5 days]	-0.43800	-0.32397	0.21187
01/02/2009 – 01/04/2009 [3 days]	-0.14575	-0.34140	-0.42109
07/04/2010 – 07/06/2010 [3 days]	-0.32233	0.12538	0.17492
03/30/2011 – 04/01/2011 [3 days]	-0.45187	-0.43051	0.04165
06/02/2012 – 06/06/2012 [5 days]	-0.29404	-0.02209	0.37730
03/29/2013 – 03/31/2013 [3 days]	0.21022	-0.36115	-0.03581

Table 2 displays the SCD2 satellite results and is noticeably shorter, starting from 1999 instead of 1994, due to different launch dates. Nonetheless, satisfying simulated intervals were found for every year of data available.

Another point to be made is the fact that the SCD2 satellite has an attitude control system aboard, capable of correcting its orientation. These changes in attitude cannot be predicted or modeled by the propagator used in this work, resulting in zero deviation for the days in which active control is used. Subsequently, intervals that came across any of these active control days were not considered a valid result, since it would interfere with the natural motion of the spacecraft. Consequently, only intervals up to 7 days were found for the SCD2 satellite, not necessarily because the propagator failed to calculate its orientation but rather for running into days that did not fit into the model being used.

Table 2. Intervals simulated for the SCD2 satellite without daily data updates.

Intervals	$\Delta\alpha(^{\circ})$	$\Delta\delta(^{\circ})$	$\Delta W(rpm)$
01/22/1999 – 01/24/1999 [3 days]	0.30556	0.10094	-0.29867
07/18/2000 – 07/22/2000 [3 days]	-0.05006	0.29679	0.08889
10/27/2001 – 10/29/2001 [3 days]	0.12642	-0.01798	-0.01820
01/20/2002 – 01/23/2002 [4 days]	0.48542	-0.28938	0.04586
03/11/2003 – 03/14/2003 [4 days]	0.18118	-0.48589	-0.29120
10/26/2004 – 11/01/2004 [7 days]	-0.19840	-0.47573	-0.09672
04/10/2005 – 04/14/2005 [5 days]	0.11257	0.17204	-0.27467
01/25/2006 – 01/29/2006 [5 days]	0.03288	-0.10873	-0.44078
01/12/2007 – 01/14/2007 [3 days]	0.08399	-0.49943	0.25652
09/28/2008 – 09/30/2008 [3 days]	0.34838	0.44847	0.08860
01/10/2009 – 01/12/2009 [3 days]	0.01145	0.33512	-0.02276
03/31/2010 – 04/02/2010 [3 days]	-0.49924	0.00634	0.01061
11/17/2011 – 11/21/2011 [5 days]	0.06169	-0.20135	0.17831
01/10/2012 – 01/16/2012 [7 days]	0.39958	-0.10756	-0.30012
01/10/2013 – 01/12/2013 [3 days]	0.27716	0.18341	-0.09206

Through analysis of Tables 1 and 2, it is possible to see that the deviation for the spin velocity, in the majority of the simulations, tends not to be the highest among the rest of the variables, rarely being the value that surpasses the precision limits and characterizes the simulation unsuccessful. This happens due to the fact that the spin velocity is calculated rather independently when compared to the angles, both of which require W to be found beforehand [4]. Consequently, in order for the angles to have low deviation values, it is implicit that the same happens for the spin velocity.

The following figures showcase the longest interval found during this work, being 5 times longer than what is considered a successful simulation. The graphs contain the deviations for the right ascension angle, the declination angle and the spin velocity, respectively, for the SCD1 interval going from 01/02/1996 to 01/16/1996.

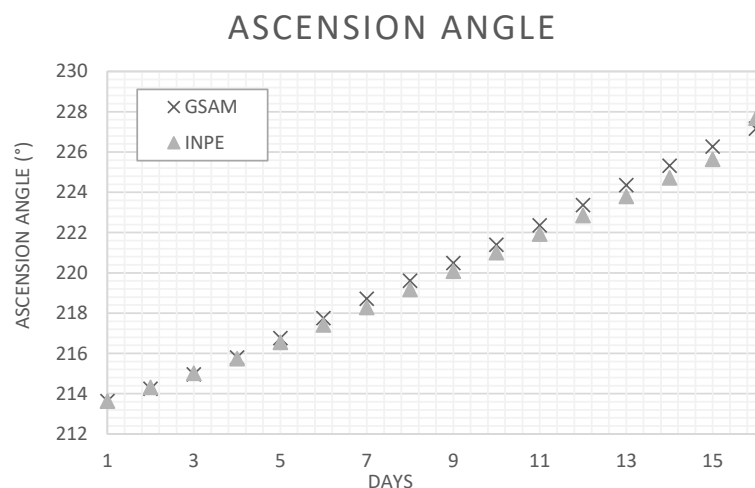
Figure 2. Right ascension angle for the SCD1 satellite without data updates, from 01/02/1996 to 01/16/1996.

Figure 3. Declination angle for the SCD1 satellite without data updates, from 01/02/1996 to 01/16/1996.

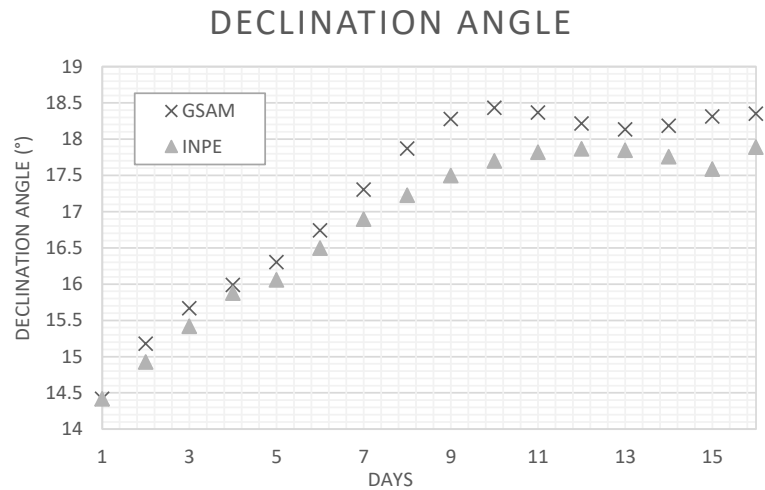
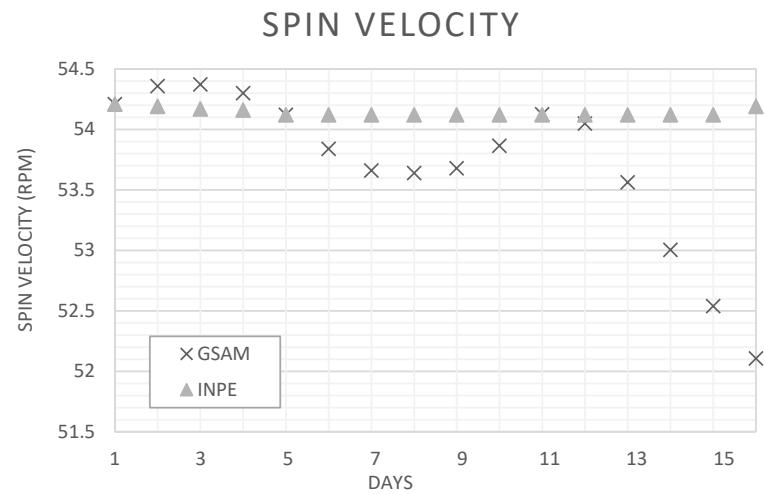


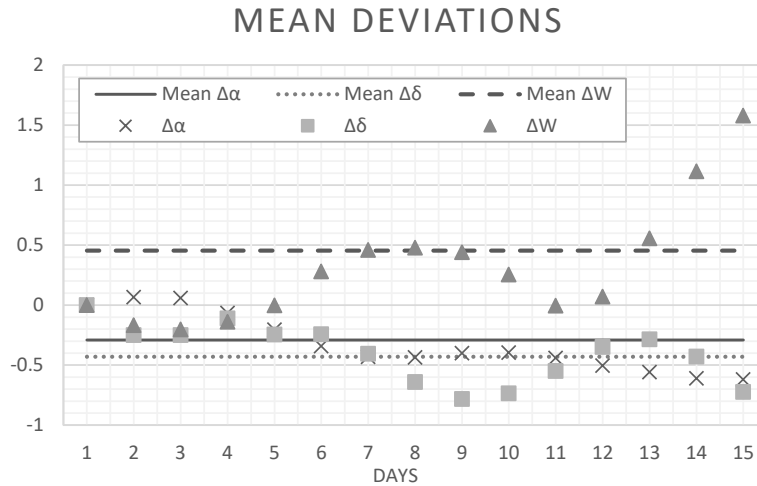
Figure 4. Spin velocity for the SCD1 satellite without data updates, from 01/02/1996 to 01/16/1996.



As shown in Table 1, the mean deviations for the right ascension angle, declination angle and spin velocity displayed in Figure 5 are -0.29075 , -0.43014 and 0.45433 , respectively, and are represented by the different lines in the following figure.

Each deviation is calculated based on the absolute difference between the real data for a given variable and its calculated value, making it possible for deviations to be negative, as the graph shows.

Figure 5. Mean deviations for the SCD1 satellite without data updates, from 01/02/1996 to 01/16/1996.



As previously mentioned, the absence of an attitude control system aboard the SCD1 satellite allows the automation of 3-day simulations without daily data updates. Since SCD2 requires manual inputs to register active control days in the program, it was not possible to perform the same analysis for the second satellite. Table 3 shows to number of successful 3-day intervals found for every year, along with the yearly percentage it represents.

Table 3. Number of 3-day intervals found for the SCD1 without daily data updates.

Year	Intervals	Yearly percentage
1994	47	13.62%
1995	66	19.13%
1996	74	21.45%
1997	51	14.78%
1998	47	13.62%
1999	39	11.30%
2000	30	8.696%
2001	46	13.33%
2002	46	13.33%
2003	45	13.04%
2004	45	13.04%
2005	51	14.78%
2006	36	10.43%
2007	39	11.30%
2008	40	11.59%
2009	40	11.59%
2010	28	8.116%
2011	21	6.087%
2012	9	2.609%
2013	10	2.898%

Several successful intervals were found for every year in the available data, gradually decreasing in number over time as the satellite aged and consequentially drifted from its original orbital parameters. Nonetheless, these intervals show that the propagator can consistently calculate the satellite's attitude over the course of its mission, something that had not been shown before due to the data available at the time of previous works. [9]

4. Conclusions

The results displayed in Tables 1 and 2 illustrate the fact that valid intervals were found in every year of data available. This does not imply that these are the longest streak of days for each year but merely demonstrate that they exist. In order to find the longest interval for each year in which the propagator calculates the parameters within the required precision, it would be necessary to simulate every possible three-day interval for every year and then increase the successful intervals by one day at a time until they no longer stay within the required precision.

Furthermore, the results also show that there is a slight negative correlation between the mean deviations for the right ascension and declination angles, what is even more evident when analyzing the results that surpassed the required precision and did not make it into tables. The correlation between these variables arise from the fact that the torques' influence vary independently over time and affect each variable differently, as the analytical solutions show [4]. This corroborates the proposition that the propagator heavily relies on the data provided and does not have an internal bias toward specific parameters, considering the mean deviation for the right ascension angle in previous works. [7, 9]

Finally, combining the results of this work with previous validations of this propagator [10], it is possible to assume that this program could be used for future attitude calculations for the SCD1 and SCD2 satellites, either combining both data update modes or using the simulations without daily data updates for short periods.

Acknowledgments

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