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Hybrid Multi-objective Orbit-Raising Optimization with Operational Constraints

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Abstract

The optimal design of orbit raising trajectories is formulated within a Multi-Objective Hybrid Optimal Control framework. The spacecraft can be equipped with chemical, electric or combined chemical-electric propulsion systems. The model incorporates realistic effects of the space environment and complex operational constraints. An automated solution strategy, based on two sequential steps, is proposed for this problem. In the first step, operational constraints are not enforced and the control law of the electric engine is parameterized by a Lyapunov function. A heuristic global search algorithm selects the propulsion system and optimizes the guidance law. Approximate Pareto-optimal solutions are obtained trading off propellant mass, time of flight and solar-cell degradation. In the second step, candidate solutions are deemed as initial guesses to solve the Nonlinear Programming Problem resulting from direct transcription of the operationally constrained problem. The proposed approach is applied to two transfer scenarios to the Geostationary orbit. Results show the effectiveness of the methodology to generate not only rapid performance estimates for preliminary trade studies, yet also accurate calculations for the detailed design. Additionally, it is identified that the application of operational restrictions causes minor penalties in the objective function.
1. Introduction

Geostationary Equatorial Orbit (GEO) is known to be the most commercially profitable orbit [1]. Satellites can be placed there directly by the launch vehicle or transferred from a parking orbit by their on-board propulsion. The most classical strategy relies on Chemical Propulsion (CP) due to its maturity and reliability. Additionally, it guarantees a very short transfer time, often ranging from hours to few days starting from GTO (Geostationary Transfer Orbit). However, there has been much interest in GEO satellites equipped with Electric Propulsion (EP) as a means for saving costs. This is due to the fact that the required propellant mass is reduced when compared to CP. Thus, larger payloads or lower-wet-mass spacecraft are achievable. In March 2015, the first ever all-electric satellites incorporating the Boeing 702-SP architecture were launched [2]. Two years later, the platform Eurostar E3000EOR by Airbus reached an operative status. Moreover, the use of electric propulsion for orbit raising is included in the next generation of satellite platforms, such as the Airbus’ Eurostar Neo, expected to be launched in 2021.

Nevertheless, the significant mass savings attained by EP come at the cost of a higher transfer time. The low thrust levels require the engine to fire almost continuously for weeks or months. Additionally, a major concern associated with all-electric transfers is the long transit time through the Van Allen radiation belts [3]. The impacting energized particles may be hazardous to on-board electronics and cause degradation of solar panels. A viable alternative to obtain intermediate performances may be represented by a combination of both CP and EP into a Combined Chemical-Electric Propulsion (CCEP) system. Customers may be provided with more payload mass than Fully Chemical Transfers (FCT), yet maintaining an acceptable on-orbit delivery time and reducing the radiation exposure of Fully Electric Transfers (FET). The mission sequence would consist on a chemical phase followed by an electric phase.
Consequently, the design of orbit raising missions is modeled in this work as a multi-objective optimization problem. Flight time, propellant mass and radiation damage are considered as competing objectives. The spacecraft may be equipped with on-board CP, EP and CCE. In the case of CCE propulsion, the mission sequence consists on a chemical phase followed by an electric phase. The resulting trajectory is determined by a sequence of discrete events, defining the chemical firings and the electric engine on and off switchings; and by continuous dynamics, describing the motion during the thrusting and coasting phases. Note that neither the sequence of discrete events nor the propulsive configuration are known a priori and they have to be obtained as part of the optimal solution. As a result, the optimization problem is formally posed as a Multi-Objective Hybrid Optimal Control Problem (MO-HOCP). General frameworks for the description of HOCPs and its corresponding mathematical formalism are presented, e.g. by Branicky [4] and Buss [5]. Particular frameworks for space mission planning are introduced by Chilan et al. [6] and Ross et al. [7].

The goal of this paper is to efficiently determine the set of Pareto-Optimal solutions for the aforementioned HOCP. Therefore, a solution methodology able to concurrently optimize and explore the design space of FCT, FET and Combined-Chemical-Electric Transfers (CCET) is desired. The optimization of each transfer trajectory is qualitatively different from each other and it has been traditionally treated separately.

The optimization of FCT is a well-known problem and has been profusely studied in the literature; [8, 9, 10, 11, 12] provide a partial, representative list of such prior works. Early approaches [8, 9, 10] assumed either coplanar transfers, circular orbits or two-impulse strategies. Algorithms for \(n\)-impulsive transfers between non-coplanar elliptical orbits have been developed by Jezewski [11] and Abdelkhalik et al. [12]. The former developed an iterative method to calculate local minima solutions. The latter employed a genetic algorithms to find global optimal solutions over the design space.

Conversely, it becomes much more challenging when dealing with FET due to the nonlinearity and non-convexity of the continuous dynamics and constraints.
Additionally, realistic effects of the space environment such as Earth oblateness, eclipse or radiation damage should be included to increase the fidelity of the model. Furthermore, the avoidance of crossings with the GEO torus is desirable due to the likelihood of impact with operational satellites. Other operational constraints such as slew rate limitations on the thrust pointing angle or phasing to a certain orbital slot may have to be imposed in the solution. Last but not least, achievable and acceptable results while using reasonable amount of computational time and memory are highly desirable.

Historically, in order to reduce the complexity of FET, either the control law or the trajectory (even both of them) has been assumed to be known “a priori”, i.e., predefined. This strategy allowed some authors to obtain analytical solutions (e.g., [13][14]) for certain special cases. Other researchers combined it with averaging techniques (e.g., [15][16][17][18][19]) or asymptotic analysis (e.g., [20][21]) to effectively estimate the dynamics of the system. A different approach utilizes closed form feedback control laws derived from predetermined Lyapunov functions [22][23][24]. Several works [23][25] employed a multi-objective evolutionary algorithm to optimize the parameters of the Q-law Lyapunov function proposed by Petropoulos [23]. This technique was proven to permit a rapid evaluation of the trade-off space and to provide reasonable estimates of the optimal performances. However, this approach does not allow to target a desired true longitude. To overcome this limitation, Lantukh, et al. [27] presented the EQlaw, an extension of the Q-law able tackle rendezvous problems. This methodology was combined with a multi-objective heuristic algorithm in [28] to explore the trade-space of electric orbit raising transfers with respect to transfer time and propellant mass. Lorenz et al. [29] further included the radiation dose absorbed by the spacecraft as the third optimization objective.

Solutions from preceding methodologies for FET are suboptimal and may not fulfill the constraints. However, they can serve as initial guesses for more sophisticated optimizers including high fidelity models for the dynamics and constraints. As an illustration, Betts [30] and Schäff [31] applied a direct transcription method along with a Non-Linear Programming (NLP) solver. The
former included slew rate restrictions, whereas the latter further considered phasing, GEO torus avoidance and radiation constraints, although only a brief description of the models employed were provided. Other authors, such as Singh et al. [32], have coupled heuristic algorithms with gradient-based NLP solvers to identify globally optimal solutions in a multi-objective design space.

Fewer researches have investigated the use of CCEP systems for orbit raising trajectories to GEO. Spitzer [33] proposed a suboptimal operational strategy where the CP establishes a geo-synchronous elliptical orbit with zero inclination. A final EP phase with fixed inertial attitude reduces the eccentricity to zero. Oleson et al. [34] demonstrated the propellant mass savings attainable by replacing a portion of the chemical transfer with an EP stage. Mailhe and Heister [35] considered a vehicle design for a coplanar transfer assuming constant tangential thrust for the EP phase. Oh et al. [36] proposed a combined graphical-analytical method for estimating performances. They incorporated the software SEPSPOT (Solar Electric Propulsion Trajectory Optimization Program) [37] to obtain minimum-time EP transfers. Jenkin [38, 39] performed mission trade studies by applying HYTOP (Hybrid Trajectory Optimization Program) to determine the EP transfer that maximizes delivered mass. A chemical engine was allowed to fire at apogee. Kluever [40] contributed with a comprehensive analysis based on a chemical impulse at apogee and on the parametrization of the low-thrust segment by curve fits of stored optimal solutions obtained offline.

Furthermore, Kluever [41] provided a purely analytical design tool to rapidly perform trade studies. Ceccherini et al. [42] derived a preliminary design procedure to evaluate the usefulness of CCEP for a given payload. In their work, the EP trajectory is optimized with the package LT2O (Low-Thrust Trajectory Optimization) following the minimum time philosophy. Their work was later extended in [43] to perform system-trajectory optimization to GEO orbit. Last but not least, Macdonald et al. [44] compute propellant optimal trajectories while imposing limits to the maximum radiation absorbed.

Despite the excellent insight into CCET problem that has been provided by [33, 34, 35, 36, 38, 39, 40, 41, 42, 43, 44], they have neglected important
characteristics of the problem. For instance, many of them did not account for space environmental effects such as [33, 34, 35, 36, 37, 38, 39, 40, 41], who ignored the Earth-shadow eclipse effect during the EP transfer to GEO or [35, 42, 43, 40, 44], who did not account for Earth-oblateness. Refs. [33, 35, 38, 39, 40] did not quantify the radiation effects caused by transit through the Van Allen radiation belts. Other studies were not flexible enough. [33, 34, 38, 39, 40] assumed a fixed or constrained starting orbit for the EP phase and [42, 43, 44] restricted the use of the chemical engine to planar maneuvers. Last but not least, [40, 41, 42] performed an offline optimization procedure of the EP phase, yet they were not able to provide optimized guidance laws.

In this paper, we contribute to the field of research on methodologies for the optimal design of orbit raising trajectories by providing a general framework and a solution approach that overcomes the limits of the previous approaches. The contribution lies on the development of a fast, robust and flexible optimization algorithm to cast the aforementioned MIO-HOCP including eclipse, radiation damage and earth-oblateness effects. The algorithm does not impose any constraint on the initial or final orbit for the electric phase, nor on the direction of the chemical maneuvers. Our approach concurrently optimizes the guidance laws and performances for CCET, FCT and FET, which has not previously been addressed in the literature. Additionally, the modeling, implementation and evaluation of the complex operational constraint associated to the avoidance of the GEO torus within a direct transcription scheme is a novel contribution.

For such purpose we propose a sequential two-step algorithm termed MOLTO-OR (Multi-Objective Low-Thrust Optimizer for Orbit Raising). In Step 1 we combine the contributions made by Abdelkhalik et. al [12] for FCT and the predefined Q-law from Petropoulos [23] for FET to create a new single algorithm that is additionally able to account for CCET. A heuristic algorithm is in charge of selecting the propulsion system among CP, EP or CCEP and optimizing the trajectory. A whole family of Pareto quasi-optimal solutions in terms of propel-lant mass, time of flight and solar-cell degradation is obtained. Results comprise the number, magnitude, direction and location of the chemical impulses as well
as the low-thrust steering law. Although at this stage operational constraints are not applied, users can greatly benefit from quick preliminary tradeoffs assessments. In Step 2, provided the solution from Step 1 as initial guess, the NLP problem resulting from application of a direct transcription technique is solved. The guidance for the low-thrust control is open-loop and no longer predefined. Operational constraints such as slew rate limitations, phasing and GEO torus avoidance can be imposed in this step.

The paper is organized as follows: In Section 2 the orbit raising optimization problem is formally stated as a MO-HOCP. The selected dynamical models for both the low-thrust trajectory and the chemical maneuvers are presented in Section 3. The modeling of the propulsion systems, the space environment and the operational constraints are described in Section 4. In Section 5 we explain the proposed solution algorithm MOLTO-OR. Later, in Section 6 the algorithm capabilities are tested. Firstly, an analysis of optimality and robustness of the Lyapunov control law is performed. Secondly, the capabilities of the Step 1 algorithm will be tested on an orbit raising trajectory from Low-Earth-Orbit (LEO) to GEO. Thirdly, the capabilities of the Step 2 algorithm to incorporate operational constraints are explored for an orbit raising trajectory from Geostationary Transfer Orbit (GTO) to GEO. Finally, some conclusions and perspectives are discussed in Section 7.

2. Mathematical Framework

The trajectory of a spacecraft equipped with either a low-thrust engine, chemical propulsion or a combination of both can be modeled as a hybrid dynamical system (HDS), that is, a system with interacting continuous and discrete dynamics. The continuous dynamics determines the geocentric trajectory during thrusting or coasting phases. Each phase represents a different working condition and consequently a different dynamical description. The discrete dynamics characterizes the discontinuous behavior of the system such as the on/off switchings of the low-thrust engine or the instantaneous firings of the chemical
engine. We consider that the EP engine can be switched off for propellant saving reasons or if there is not enough power available from the batteries during eclipse. The multi-objective optimization of the control vector of a spacecraft modeled as a HDS is formulated as MO-HOCP. Hereafter, we present the general mathematical framework for HDS and for MO-HOCP based on the one proposed by Buss [5].

The application of this framework to optimization of low-thrust multigravity-assisted trajectories have been presented in Ref. [45].

2.1. Hybrid Dynamical System

The state of a HDS is determined by the continuous state vector $x(t) \in \mathcal{X} \subset \mathbb{R}^n$, which is constrained to be in the set $\mathcal{X}$ and the discrete state vector $q(t) \in \mathcal{Q} \subset \mathbb{N}^n$, which is constrained to be in the set $\mathcal{Q}$. The system can be controlled by a continuous control vector $u(t) \in \mathcal{U} \subset \mathbb{R}^m$, which belongs to the set $\mathcal{U}$, and by a discrete control vector $v(t) \in \mathcal{V} \subset \mathbb{N}^n$, which belongs to the set $\mathcal{V}$. Therefore, the time-evolution of a HDS is given by:

$$\dot{x} = f(x, q, u, v, t) \text{ if } s_j(x, q, u, v, t) \neq 0, \quad j = 1, \ldots, n_s$$

(1)

$$x(t_{j+1}^+) = \phi_j(x, q, u, v, t_j) \text{ if } s_j(x, q, u, v, t_j) = 0, \quad j = 1, \ldots, n_s$$

(2)

The continuous behavior of the HDS is described by the set of differentiable equations $f : \mathcal{X} \times \mathcal{Q} \times \mathcal{U} \times \mathcal{V} \rightarrow \mathbb{R}^n$, whereas the discontinuous behavior is characterized by the $n_s$ discontinuity surfaces $s_j : \mathcal{X} \times \mathcal{Q} \times \mathcal{U} \times \mathcal{V} \rightarrow \mathbb{R}$ and transition maps $\phi_j : \mathcal{X} \times \mathcal{Q} \times \mathcal{U} \times \mathcal{V} \rightarrow \mathcal{X}$ for $j = 1, \ldots, n_s$. Discontinuous behavior is caused by events occurring when the hybrid state intersects discontinuity surfaces [5]. The times $t_j$ at which these events occur, are called event transition times. The states $x(t_j^+)$ and $q(t_j^+)$ just after a discrete event are determined by the transition maps. As long as all discontinuity surfaces $s_j(x, q, u, v, t) \neq 0$, the system evolves continuously according to Eq. [1].

2.2. Multi-Objective Hybrid Optimal Control Problem

The MO-HOCP is to find the set of continuous $u(t)$ and discrete $v(t)$ control vectors that minimize the multi-objective function $J(u, v, t)$. Solutions must
obey both the continuous and discrete dynamics (1)-(2) and are subject to:
\begin{align}
  u(t) &\in \mathcal{U} \subset \mathbb{R}^{n_u}, \quad v(t) \in \mathcal{V} \subset \mathbb{Z}^{n_v}, \quad \forall t \in [t_0,t_f], \\
  x(t) &\in \mathcal{X} \subset \mathbb{R}^{n_x}, \quad q(t) \in \mathcal{Q} \subset \mathbb{N}^{n_q}, \quad \forall t \in [t_0,t_f], \\
  0 &\leq g(x, u, q, v, t), \quad t \in [t_0,t_f], \\
  x(t_0) &= x_0(x, q, u, v, t_0), \quad q(t_0) = q_0(x, q, u, v, t_0) \\
  x(t_f) &= x_f(x, q, u, v, t_f), \quad q(t_f) = q_f(x, q, u, v, t_f)
\end{align}

Here, \( t_0 \) and \( t_N = t_f \) are the beginning and final times. The minimization of the multi-objective function is subject to initial and terminal conditions on the state vector (6)-(7), admissible values for the continuous and discrete control and state variables (3)-(4) and further inequality constraints (5) given by the function \( g : \mathcal{X} \times \mathcal{Q} \times \mathcal{U} \times \mathcal{V} \times \mathbb{R} \rightarrow \mathbb{R}^{n_r} \).

3. Dynamical Modelling

In this section we present the mathematical description of the set of continuous differential equations and discrete event functions for a spacecraft equipped with CCEP system. The model also includes FCT and FET as particular cases. The former does not allow thrusting phases whereas the latter does not permit chemical firings during the transfer.

3.1. Continuous and Discrete State Variables

The spacecraft’s continuous state vector can be determined by the set of modified equinoctial elements \( \{ p, f, g, h, k, L \} \). Given that low-thrust transfers are characterized by very long transfer times, it is convenient to use the true longitude \( L \) as the independent variable, instead of time \( t \). Additionally, the evolution of the spacecraft’s mass \( m \) is needed to fully describe its dynamics. The set of dependent variables results in the following continuous state vector \( x = [p, f, g, h, k, t, m] \). The working condition of the electric engine is determined by the discrete state variable \( q \in \{ 0, 1 \} \), where ‘0’ designates the coasting mode and ‘1’ indicates the thrusting mode. The switching between both modes of operation is governed by the discrete dynamics and controls.
3.2. Continuous and Discrete Control Variables

The EP system produces a continuous thrust force of magnitude $T$ on the vehicle when switched on. The steering law is determined by the continuous control vector $u_e = [\alpha, \beta]$. Here, $\alpha$ is the azimuth angle measured in the orbit plane from the circumferential direction and positive away from the gravitational centre, whereas $\beta$ is the declination angle measured out of the orbital plane and positive along the angular momentum. The on/off switchings during sunlight are managed by the binary control vector $v_e = [0, 1]$. The coasting state is required when '0', while a burning phase is demanded for '1'. The CP provides $n$-instantaneous velocity changes on the spacecraft. They are determined by the set of controls $u_{c;j} = [\theta_j, \Delta V_j, \alpha_j, \beta_j]$ for $j = 1, \ldots, n$, where $\theta_j$ is the true longitude at which the $j^{th}$ impulse of magnitude $\Delta V_j$ and direction $\alpha_j$ and $\beta_j$ is performed. Here $\alpha_j$ is the in-plane angle measured from the tangential direction and positive away from the gravitational centre and $\beta_j$ is the declination angle.

3.3. Continuous Dynamics

The continuous evolution of the continuous state vector is given as:

$$\frac{dp}{dL} = \frac{1}{\sigma} \sqrt{\frac{2p}{\mu}} \omega$$

$$\frac{df}{dL} = \frac{1}{\sigma} \sqrt{\frac{2p}{\mu}} \left( \sin La_o + \frac{1}{w} \left( (w + 1) \cos L + f \right) a_o - \frac{g}{w} \left( h \sin L - k \cos L \right) a_o \right)$$

$$\frac{dg}{dL} = \frac{1}{\sigma} \sqrt{\frac{2p}{\mu}} \left( -\cos La_o + \frac{1}{w} \left( (w + 1) \sin L + g \right) a_o + \frac{1}{w} \left( h \sin L - k \cos L \right) a_o \right)$$

$$\frac{dh}{dL} = \frac{1}{\sigma} \sqrt{\frac{2p}{\mu}} \omega_k$$

$$\frac{dk}{dL} = \frac{1}{\sigma} \sqrt{\frac{2p}{\mu}} \sin L$$

$$\frac{dL}{dL} = \frac{1}{\sigma} \omega$$

$$\frac{d\mu}{dL} = -\frac{2}{\sigma} \omega$$

(8)
where

\[
\sigma = \sqrt{\frac{1}{\mu w}} (b \sin L - k \cos L) \omega + \sqrt{\frac{w}{\mu}} \left(\frac{w}{\mu}\right)^2
\]  

(9)

\[
w = 1 + f \cos L + g \sin L
\]  

(10)

\[
l^2 = 1 + h^2 + k^2
\]  

(11)

Here, \( \dot{m} \) is the fuel consumption rate of the electric engine and \( \mu \) is the gravitational constant of the Earth. The perturbing acceleration \( \mathbf{a} = (a_r, a_\theta, a_h) \) is projected onto the rotating radial frame. For the purpose of this paper, the only non-two-body accelerations are due to Earth \( J_2 \) and thrust. Adding different perturbing accelerations is as simple as converting them to the rotating radial frame and including them in \( \mathbf{a} \). The Earth oblateness is accounted in a similar manner to [30]. The acceleration due to thrust \( \mathbf{a}_T \) is given as:

\[
\mathbf{a}_T = \frac{T}{m} \mathbf{d}
\]  

(12)

where \( \mathbf{d} = (\cos \beta \cos \alpha, \cos \beta \sin \alpha, \sin \beta) \) is the unitary vector in the direction of the thrust. The thrust magnitude \( T \) of the electric engine and its fuel consumption rate \( \dot{m} \) depends on the specific propulsion system, whose models will be presented in the next section.

3.4. Discrete Dynamics

The discrete dynamics of the system are governed by controlled events, i.e., the discrete velocity impulses of the CP and the discrete on/off switchings of the EP system, and by autonomous events, i.e., the electric engine shut down during eclipse. The discrete event functions associated to them, including discontinuity surfaces and transition map functions, will be formally defined hereafter.

3.4.1. Chemical Propulsion Burns

The effect of a chemical engine translates into discrete changes of the continuous state vector that are given by the transition map functions \( \phi_c \). The discontinuity surfaces \( s_c \) constrain the firings to occur when the spacecraft reaches the true longitude selected by the control parameter \( \theta \) as long as the EP is
switched off. Note that, the discrete state is not affected by this maneuver, i.e., \( q(L_i^s) = q(L_i^c) \). Therefore, the discrete dynamics is expressed as follows:

\[
\begin{align*}
\text{Burn: } & s_{o,i} : L_i^c = \theta_f, \quad j = 1, \ldots, n \\
& \phi_{o,i} : q(L_i^c) = q(L_i^c), \quad x(L_i^c) = \mathcal{C}(x(L_i^c), u_{s,j}), \quad j = 1, \ldots, n
\end{align*}
\]

where \( n \) is the number of allowed impulses and \( \mathcal{C} : \mathcal{X} \times \mathcal{U} \times \mathbb{R} \rightarrow \mathcal{X} \) is the procedure to compute the successor continuous state \( x(L_i^c) \) of the spacecraft after the \( j \)th chemical maneuver as a function of the predecessor continuous state \( x(L_i^c) \) and the control parameters \( u_{s,j} \).

3.4.2. Electric Engine on/off switching

The EP engine switchings are controlled by the discrete control \( v_e \) as long as the spacecraft is in sunlight conditions. They are modeled by the discontinuity surfaces \( s_{\text{on}} \) and \( s_{\text{off}} \) and by the transition maps \( \phi_{\text{on}} \) and \( \phi_{\text{off}} \). Additionally, the spacecraft shutdown during eclipse is represented by the discrete event function \( s_{\text{off}2} \) and transition map \( \phi_{\text{off}} \). Therefore, the following equations are included for a complete description of the system:

\[
\begin{align*}
\text{Switching on: } & s_{\text{on}} : q(L_i^c) = 0, \quad v_e(L_i^c) = 1, \quad \delta(x, L_i^c) = 1 \\
& \phi_{\text{on}} : q(L_i^c) = 1, \quad x(L_i^c) = x(L_i^c) \\
\text{Switching off: } & s_{\text{off1}} : q(L_i^c) = 1, \quad v_e(L_i^c) = 0 \\
& s_{\text{off2}} : q(L_i^c) = 1, \quad \delta(x, L_i^c) = 0 \\
& \phi_{\text{off}} : q(L_i^c) = 0, \quad x(L_i^c) = x(L_i^c)
\end{align*}
\]

In previous Eqs. (14)-(15) the effect of the eclipse is included via the shadow function \( \delta(x, L_i^c) : \mathcal{X} \times \mathbb{R} \rightarrow \mathbb{I} \). It represents a binary-valued function that takes the value ‘1’ when the spacecraft is in sunlight and ‘0’ when it is in umbra conditions. Note that, an explicit relation between the discrete state, the discrete control vector and the shadow function can be deduced:

\[
q(v_e, x, L) = v_e \delta(x, L)
\]

3.5. Transfer Sequence

Throughout this work, it is assumed that the chemical firings are executed fully before the electric phase begins. This constraint arises because the high
amount of power required by the electric propulsion system to operate may
require deployed solar array panels [47], which would be incompatible with the
high forces applied to the spacecraft by the chemical propulsion.

4. Propulsion System, Space Environment and Constraints Modeling

In this section we present the models for the propulsion systems, the space
environment effects, such as the Earth shadow, the Van Allen belts radiation
and the solar-cell damage. Additionally, models for slew rate limits, slot-phasing
and GEO torus avoidance constraints will be introduced.

4.1. Electric Engine

Solar electric power system generates power based on its distance to the Sun
and the angle between the array plane and the Sun vector. In this analysis the
Sun is assumed to be 1 A.U. from the spacecraft at all times and the arrays are
always perfectly pointed at the Sun. If the power available to the propulsion
system is given by \( P \), then the maximum thrust that can be generated is:

\[
T = \frac{2g_0 I_{sp,e}}{I_{sp,e}} \quad (17)
\]

and the rate of propellant expenditure due to the engine firing is derived as:

\[
\dot{m} = \frac{2g_0 P}{g_0 I_{sp,e} I_{sp,e}} \quad (18)
\]

where \( g_0 \) is the Earth gravitational acceleration at sea level, \( I_{sp,e} \) is the specific
impulse and \( \eta \) is the efficiency factor of the electric engine respectively.

4.2. Chemical Engine

The mass of the spacecraft after a chemical maneuver \( m(L_{i}^+) \) is computed
as a function of the mass before the maneuver \( m(L_{i}^-) \) according to:

\[
m(L_{i}^+) = m(L_{i}^-) e^{-\Delta\nu/(I_{sp,c} g_0)} \quad (19)
\]

where \( I_{sp,c} \) is the specific impulse of the chemical thruster.
4.3. Earth Shadow

It is reasonable to assume that both the Sun and Earth are spherical bodies as suggested in Ref. [30]. We further assume that the Sun is infinitely far away from the Earth resulting in a cylindrical shadow. The radius of the cylinder is therefore equal to the radius of the earth $R_e$ and its axis is determined by the unitary vector from the Earth to the Sun $\mathbf{s}$. Let us define $r_\perp$ as the distance of the spacecraft to the axis of the cylinder and it can be obtained as:

$$r_\perp = \sqrt{r^2 - r^T \mathbf{s}}$$

(20)

where $r = \|\mathbf{r}\|$. The condition for the spacecraft to be inside the cylinder can be simply expressed as $r_\perp \leq R_e$. In addition to this, in order to be inside the shadow region of the cylinder, the constraint $r^T \mathbf{s} \leq 0$ must be fulfilled. Combining both conditions, a necessary and sufficient condition for an eclipse to happen is derived as:

$$r^T \mathbf{s} + \sqrt{r^2 - R_e^2} \leq 0$$

(21)

Thus, the binary shadow function $\delta$ has the following form:

$$\delta = \begin{cases} 0 & \text{if } r^T \mathbf{s} + \sqrt{r^2 - R_e^2} \leq 0 \\ 1 & \text{otherwise} \end{cases}$$

(22)

4.4. Radiation Environment

Following the same approach as in Ref. [48], the geomagnetic field is modeled by considering a magnetic dipole with an axis parallel to the Earth’s magnetic axis passing through its center of mass. The omnidirectional radiation flux, defined as the flux of all particles averaged over all directions at any location owing to protons and electrons, can be computed as a function of the McIlwain’s coordinate $\mathcal{L}$ and the latitude $\lambda$. Furthermore, Ref. [48] shows that the radiation impact from electrons during the orbit-raising trajectory represents only a small fraction of what the satellite undergoes in GEO over a period of 10-15 years. Hence, in this work, only the radiation damage caused by protons is considered.
The omnidirectional flux of protons of energy equal or greater than $E$ can be expressed by the following analytical form \cite{48}:

$$\Phi \left( L, \lambda, E \right) = a \left( L, E \right) e^{-b \left( L, E \right) \lambda}$$ \hspace{1cm} (23)

where $a \left( L, E \right)$ and $b \left( L, E \right)$ are given by:

$$a \left( L, E \right) = a_0 e^{a_1 E + a_2 \left( a_3 + E \right) \lambda^2}$$ \hspace{1cm} (24)

$$b \left( L, E \right) = b_0 + b_1 E + b_2 L + b_3 E L + b_4 E^2 + b_5 L^3$$ \hspace{1cm} (25)

Radiation models like AP-8 or AP-9 can be used to find the values of the fitting parameters $a_j$ and $b_j$. In our approach we will use the fitting parameters obtained for the AP-8 model that can be found in \cite{48}. Given that the coordinates $(L, \lambda)$ depends uniquely on the position of the spacecraft, the radiation flux can be expressed in terms of the continuous state vector $x$ and the true longitude $L$, so that $\Phi \left( x, L, E \right) = \Phi \left( L, \lambda, E \right)$. Therefore, the total fluence of protons of all energy levels greater than or equal to $E$ encountered along the orbit-raising trajectory until a certain time $t$ is given by:

$$\Phi \left( x, L, E \right) = \int_{t_0}^{t} \Phi \left( x, L, E \right) dt$$ \hspace{1cm} (26)

4.5. Radiation Damage

Solar-cell degradation causes power loss. The amount of degradation is a function of the type of solar cells, amount and material of the shielding provided, the energies of radiation encountered along the path and the number of particles for each energy level. We utilize the parametric relationships from Ref. \cite{49} in terms of the displacement damage dose and non-ionizing energy loss in order to compute the power $P$ available to the spacecraft:

$$\frac{P}{P_0} = 1 - C \log \left( 1 + \frac{D_d}{D_s} \right)$$ \hspace{1cm} (27)

where $D_d$ is the displacement dose of protons for given solar cells, $D_s$ and $C$ are constants specific to the type of solar cells. The displacement damage dose is defined as:

$$D_d = \int_{t_0}^{t_s} \int_{E_s}^{E_i} \frac{\partial \Phi \left( x, L, E \right)}{\partial E} S_p \left( E \right) dE dt$$ \hspace{1cm} (28)
where $S_p(E)$ is the non-ionizing energy loss. The rate of change of the radiation flux with respect to the energy levels can be obtained analytically from Eq. (23):

$$\frac{\partial \Phi_p(x, L, E)}{\partial L} = [a_1 - \lambda^2(b_1 + b_2 L)] \Phi_p(x, L, E)$$ (29)

4.6. Slew Rate Limits

Limits are often imposed on the angular rate of change of the thrust direction vector as a requirement from the attitude subsystem. For a small true longitude interval $\delta L$, the angle between two unit direction vectors is defined by:

$$d^T(L) d(L + \delta L) = \cos \theta$$ (30)

It then follows that an approximate rotation rate, assuming a linear variation, is given by:

$$\dot{\theta} \approx \frac{1}{t(L + \delta L) - t(L)} \cos^{-1} \left[ d^T(L) d(t + \delta L) \right] \leq \dot{\theta}_{\text{lim}}$$ (31)

where $\dot{\theta}_{\text{lim}}$ is the maximum allowed rotational velocity. The value of $\delta L$ is typically chosen to coincide with the integration step of the dynamical equations.

4.7. GEO Torus Avoidance

The GEO torus is the volume in space where most of the satellites in GEO are located. The region is also populated by uncontrolled objects that are subject to variations in altitude due to orbit perturbations. During the transfer, any crossing of the torus poses a certain collision risk with high value assets. In our study, we define the torus as a toroid around Earth with a rectangular cross-section centered at GEO (see Figure 1). The rectangular cross-section is termed GEO box and its width and height are $2l_r$ and $2l_z$ respectively. The inner $r_{GEO-}$ and outer $r_{GEO+}$ radius of the toroid are defined as:

$$r_{GEO-} = r_{GEO} - l_r$$ (32)

$$r_{GEO+} = r_{GEO} + l_r$$ (33)

where $r_{GEO}$ is the radius of the GEO orbit. Additionally, we define the North and South boundaries as the planes parallel to Equatorial plane but located at
a distance \( l_z \) above and below respectively. The intersection of the spacecraft’s trajectory with such planes determines \( r_N \) and \( r_S \), where the subscript \( N \) and \( S \) refers to the North and South boundary respectively and the symbol \( r_{NS} \) is used to refer to both indistinctly.

The conditions to avoid crossing the GEO torus can be evaluated as a function of the projections onto the Equatorial plane of the radiuses at the north/south boundary of the GEO arc:

\[
\begin{align*}
    r_{NS} \cos \phi_{SN} & \leq r_{GEO} - r_N \cos \phi_N \quad \text{or} \quad r_{NS} \cos \phi_{SN} \geq r_{GEO} + r_S \cos \phi_S \quad (34)
\end{align*}
\]

where \( \phi_{SN} \) is the elevation angle, measured out of the Equatorial plane, \( r_{NS} \) and it is computed as \( \phi_{NS} = \pm \sin^{-1} l_z/r_{NS} \), where we select the + sign for \( r_N \) and the − sign for \( r_S \). The previous complementary Eqs. (34) can be reformulated as a set of nonlinear inequality constraints as follows:

\[
\begin{align*}
    (r_{GEO} - r_N \cos \phi_N)(r_{GEO} - r_S \cos \phi_S) & \geq 0 \quad (35) \\
    (r_{GEO} - r_N \cos \phi_N)(r_{GEO} + r_S \cos \phi_S) & \geq 0 \quad (36) \\
    (r_{GEO} - r_{NS} \cos \phi_{SN})^2 - (2l_z)^2 & \geq 0 \quad (37)
\end{align*}
\]

Note that the previous expression are not valid for small inclinations, where no crossings of the trajectory with the north/south boundaries occurs. In such case, the value of \( l_z \) can be reduced and the constraints could be applied until...
the inclination reaches zero. Also, at least the last orbit must be unconstrained in order to allow targeting of the final orbit.

### 4.8. Phasing constraints

In case the satellite is to be transferred to GEO, it will have to occupy a single slot within this orbit. Phasing is the operation to target the certain longitude of the assigned slot. This constrained is applied to the final longitude:

\[
\cos L_f = \cos L_T(t_f), \quad \sin L_f = \sin L_T(t_f)
\]  \hspace{1cm} (38)

where \(L_T\) is the target final true longitude, which is a function of the arrival time, as the Earth rotates with respect to the inertial reference frame. Note that, the previous constraint is expressed as a trigonometric relation in order to make it independent of the number of revolutions.

### 5. Solution Approach

MOLTO-OR is a two-step solution approach for the MO-HOCP problem under consideration. The algorithm is schematically depicted in Figure 2. In MOLTO-OR Step 1 we incorporate a parametric model of the low-thrust control law based on the Lyapunov function Q-law [23], pursuing the goal of developing a flexible and robust algorithm able to rapidly find solutions, which would approximate the optimal performances as well as the mission design variables, needing minimum information from the user. In MOLTO-OR Step 2 we include the complete model of the trajectory seeking for accuracy and robustness in addition to the possibility of including complex constraints with ease.

#### 5.1. MOLTO-OR Step 1: Multi-objective Global Heuristic Search

In this step we convert the multi-objective HOCP into an unconstrained multi-objective mixed-integer parameter optimization problem with a small set of design variables. Due to that fact and to the requisite of evaluating many different scenarios simultaneously we have chosen a population-based heuristic algorithms as the most adequate technique to solve it. This step requires the
rapid evaluation of FCT, FET and CCET. In the following lines we describe the particular design variables and evaluation procedure separately for each transfer case. Thereafter, we present the complete optimization algorithm.

5.1.1. Fully Chemical Transfer (FCT)

A series of \( n \)-chemical maneuvers are performed sequentially. Solar-cell degradation is not considered for this transfer as the crossing of the radiation belts is very fast. The location, magnitude and direction of the \( j \)th impulse is defined by \( u_{c;j} = [\theta_j, \Delta V_j, \alpha_j, \beta_j] \) for \( j = 1, \ldots, n - 2 \). The trajectory between the \((j-1)\)th and the \(j\)th impulses is assumed to follow a Keplerian arc and is computed analytically between \( \theta_{j-1} \) and \( \theta_j \). A Lambert’s arc is assumed for the last two impulses in order to guarantee that the satellite is inserted into the final orbit exactly. Three additional variables are included to define the last two impulses: the parameter \( \theta_{n-1} \) represents the true longitude on the orbit where the first Lambert’s impulse occurs; the parameter \( t_c \) represents the time of flight on the transfer arc; and \( \theta_n \) is the true longitude on the final orbit at the time of the satellite arrival. The set of parameters \( \mathcal{C} \) defining the FCT is:

\[
\mathcal{C} = (u_{c;1}, \ldots, u_{c;n-2}, \theta_{n-1}, t_c, \theta_n)
\]  

Note that the number of impulses \( n \) is not a design variable. Instead, it is assumed to be equal to the maximum number of allowable impulses \( n_{max} \).
Table 1: MOLTO-OR Step 1 VS MOLTO-OR Step 2 main features

<table>
<thead>
<tr>
<th></th>
<th>MOLTO-OR Step 1</th>
<th>MOLTO-OR Step 2</th>
</tr>
</thead>
<tbody>
<tr>
<td>Solution Approach</td>
<td>Genetic Algorithm</td>
<td>Hermite-Simpson +</td>
</tr>
<tr>
<td></td>
<td>NLP solver</td>
<td></td>
</tr>
<tr>
<td>Dynamical Model</td>
<td>$3D + J_2 + \text{shadow} + \text{radiation}$</td>
<td>$3D + J_2 + \text{shadow} + \text{radiation}$</td>
</tr>
<tr>
<td>Control Model</td>
<td>Lyapunov Q-law</td>
<td>Optimal control</td>
</tr>
<tr>
<td>Propulsive system</td>
<td>Free</td>
<td>Fixed</td>
</tr>
<tr>
<td>Transfer Type</td>
<td>Free</td>
<td>Fixed</td>
</tr>
<tr>
<td>Thrust Model</td>
<td>Constant</td>
<td>Constant</td>
</tr>
<tr>
<td>$I_{sp}$ Model</td>
<td>Constant</td>
<td>Constant</td>
</tr>
<tr>
<td>Ephemerides</td>
<td>Analytic approximation</td>
<td>Analytic approximation</td>
</tr>
<tr>
<td>Programming Language</td>
<td>C++</td>
<td>AMPL</td>
</tr>
<tr>
<td>Heuristic solver</td>
<td>NSGA-II</td>
<td>N/A</td>
</tr>
<tr>
<td>Gradient-based solver</td>
<td>N/A</td>
<td>Ipopt</td>
</tr>
</tbody>
</table>

defined by the user. The number of impulses is then optimized by driving to zero the magnitude of the unnecessary $\Delta V$'s.

5.1.2. Fully Electric Transfer (FET)

In the case of a FET, the control vector is is predefined and determined by the Q-law Lyapunov function. This guidance scheme was originally proposed by Petropoulos [23] using orbital elements, yet in this work we use the formulation presented by Gávor I. V. in [26] with modified equinoctial elements. The Q-law is based on a proximity quotient, $Q$, which quantifies the proximity of the osculating orbit to the target orbit. During the transfer at each instant the Q-law method chooses the thrust angles that reduce the $Q$ value the most quickly. The set of parameters $E$ defining the low-thrust guidance scheme is defined as:

$$E = (W_p, W_f, W_o, W_i, \eta_a, \eta_r, m, n, r)$$  \hspace{1cm} (40)

Here, $(W_p, W_f, W_o, W_i, W_k)$ are the weighting factors associated to each equinoctial element, $(\eta_a, \eta_r)$ are the absolute and relative effectiveness coefficients, which allow introducing coasting arcs and finally $(m, n, r)$ are the scaling factors. Let us define the $Q_{min}$ function as the procedure that computes the spacecraft
thruster orientation as a function of $E$, the actual state $x$ and the desired final state $x_f$ using the Q-law as guidance:

$$[u_e, v_e, Q] = Q_{law}(E, x, x_f, L) \quad (41)$$

The dynamical system in Eq. (8) is integrated with a Runge-Kutta-Fehlberg 7(8) until the proximity quotient $Q$ reaches a threshold $Q_{th}$, which means that the spacecraft has targeted the orbit. Note that the transfer time is not a design variable and is obtained after the integration of the trajectory.

During the integration of the equations of motion, the displacement damage dose at every instant has to be computed, updating the power available to the spacecraft and the magnitude of the thrust when necessary. In order to compute the radiation damage caused to the solar arrays of a satellite, consider a discrete set of energy levels $E = (E_1, E_2, \ldots, E_m)$, such that $E_1 = E_l$ and $E_m = E_u$ with $E_l$ and $E_u$ being respectively the lower and the upper bounds of the set of $m$ energy levels under consideration. This is reasonable as the non-ionizing energy loss $S_P(E)$ is typically provided as tabular data. Based on this discretization the energy integral in Eq. (28) can be approximately obtained as:

$$\sum_{i=1}^{m-1} \frac{1}{2} \left( \frac{\partial \Phi_p(x, L, E_i)}{\partial E} S_P(E_i) + \frac{\partial \Phi_p(x, L, E_{i+1})}{\partial E} S_P(E_{i+1}) \right) (E_{i+1} - E_i) \quad (42)$$

5.1.3. Combined-Chemical-Electric Transfer (CCET)

The CCET consists on a chemical segment followed by an electric phase. The chemical phase is obtained following the same procedure than for the FCT, except for the last Lambert arc. In this case the last two impulses are applied in a similar manner that the $n^{th}$ 2 previous firings. The final state after the $n^{th}$ impulse is used to define the initial orbit for the electric phase. The same methodology than for FET is then applied to target the desired orbit. Thus, the set $\mathcal{H}$ of parameters that determines a CCET is:

$$\mathcal{H} = (u_{c,1}, \ldots, u_{c,n}, W_p, W_f, W_b, W_c, \eta_p, \eta_f, m, n, r) \quad (43)$$
5.1.4. Optimization Algorithm

The complete algorithm has to select the optimal propulsive system as well as to optimize the corresponding trajectory. The propulsive system is selected by the integer variable \( z \). Here the CP, EP and CCEP are represented by \( z = 1, 2, 3 \) respectively. Let us define \( Z = \{ z \} \) as the set of propulsive options. The set \( P \) of design variables for the complete algorithm is:

\[
P = \mathcal{C} \cup \mathcal{E} \cup \mathcal{H} \cup Z
\]

The complete optimization problem is thus defined as finding the set of parameters \( P \) such that the following multi-objective fitness function is minimized:

\[
J = (m_f, t_f, P_f)
\]

where \( m_f, t_f, P_f \) are respectively the mass, time and power of the solar arrays at the end of the transfer. A summary of the optimization variables can be found in Table 2 along with their lower bounds (Lb) and upper bounds (Ub). The objective function is computed following the flow chart depicted in Fig.3. The user only needs to provide information about the initial and final orbit along with the mass, propulsive and power generation system characteristics. The user-defined parameters and summarized in Table 3. Note that, the algorithm used to compute the lambert arc at the end of a FCT may not converge. In this case, a penalty vector is associated to lead the heuristic solver to discard them for the next generations. Similarly, the convergence of the proximity quotient \( Q < Q_{th} \) is not always assured. Thus, a maximum number of integration steps is imposed to avoid the integration scheme from running indefinitely. If this value is exceeded, a penalty is added to the fitness function.

5.2. MOLTO-OR Step 2: Local Deterministic Optimization

In Step 2, our goal is to reduce the multi-objective HOCP to a single-objective Large-scale Nonlinear parameter optimization problem and solve it with robust classical gradient-based solvers. For such purpose, we assume that the propulsive system and the number of chemical firings is known and provided
Figure 3: MOLTO-OR Step1: Flow chart of the fitness function

by Step 1. However, we do not impose the thrust/coast sequence of the EP. It will be optimized by the algorithm instead. As shown in Eq. (46) the sequence is determined by the binary control $v_e$, and the binary shadow function $\delta$. However, gradient-based solvers only accept continuous variables and continuous and differentiable functions within the search domain. The binary control $v_e$ is relaxed so that it can continuously vary within the $[0, 1]$ interval. Additionally, the binary shadow function $\delta$ is approximated by a smoothing function, where $\epsilon$ is the smoothing parameter, as:

$$\delta = \frac{1}{1 + e^{-(\epsilon (\tau + \sqrt{2m_1})}}$$  \quad (46)
Table 2: MOLTO-OR Step 1 Variables

<table>
<thead>
<tr>
<th>Variable</th>
<th>Meaning</th>
<th>Lb</th>
<th>Ub</th>
</tr>
</thead>
<tbody>
<tr>
<td>$\theta_i$</td>
<td>True Longitude for $i^\text{th}$ CP maneuver</td>
<td>0</td>
<td>$2\pi$</td>
</tr>
<tr>
<td>$\Delta V_i$</td>
<td>Magnitude of for $i^\text{th}$ CP maneuver</td>
<td>0</td>
<td>$\Delta V_{\text{max}}$</td>
</tr>
<tr>
<td>$a_i$</td>
<td>In-Plane angle of the $i^\text{th}$ CP maneuver</td>
<td>$\pi$</td>
<td>$\pi$</td>
</tr>
<tr>
<td>$\beta_i$</td>
<td>Out-of-Plane angle of the $i^\text{th}$ CP maneuver</td>
<td>$\pi/2$</td>
<td>$\pi/2$</td>
</tr>
<tr>
<td>$t_c$</td>
<td>Transfer time to last CP maneuver</td>
<td>$t_{c,\text{max}}$</td>
<td>$t_{c,\text{min}}$</td>
</tr>
<tr>
<td>$W_{f,x,y,z}$</td>
<td>Q-law weighting factors</td>
<td>1</td>
<td>100</td>
</tr>
<tr>
<td>$\eta_{x,y,z}$</td>
<td>Q-law effectiveness coefficients</td>
<td>0</td>
<td>0.98</td>
</tr>
<tr>
<td>$m_0, n, r$</td>
<td>Q-law scaling factors</td>
<td>0.01</td>
<td>10</td>
</tr>
<tr>
<td>$\xi$</td>
<td>Propulsion subsystem option</td>
<td>0</td>
<td>2</td>
</tr>
</tbody>
</table>

Table 3: User-Defined parameters

<table>
<thead>
<tr>
<th>Variable</th>
<th>Meaning</th>
</tr>
</thead>
<tbody>
<tr>
<td>$t_0$</td>
<td>Initial Launch date</td>
</tr>
<tr>
<td>$a_0, e_0, i_0, \omega_0, \Omega_0$</td>
<td>Initial Orbital elements</td>
</tr>
<tr>
<td>$a_f, e_f, i_f, \omega_f, \Omega_f$</td>
<td>Final Orbital elements</td>
</tr>
<tr>
<td>$n_{\text{max}}$</td>
<td>Max number of chemical impulses</td>
</tr>
<tr>
<td>$t_{c,\text{max}}, t_{c,\text{min}}$</td>
<td>Max/Min transfer time to last CP maneuver</td>
</tr>
<tr>
<td>$m_0$</td>
<td>Initial Spacecraft mass</td>
</tr>
<tr>
<td>$I_{E,p,e}$</td>
<td>Electric Engine Parameters</td>
</tr>
<tr>
<td>$I_{C,t}, \Delta V_{\text{max}}$</td>
<td>Chemical Engine Parameters</td>
</tr>
<tr>
<td>$P_0, C, D_0, SP(E)$</td>
<td>Solar Array characterization</td>
</tr>
</tbody>
</table>

As a result, the discrete state $q$ of the spacecraft is no longer a binary function. We discretize the states $x_j = x(L_j)$ and controls $u_j = u(L_j)$ and $v_j = v(L_j)$ on a selected uniform grid $L_j$, for $j = 1, \ldots, n$, where $n$ is number of grid points. The dynamical equations are imposed as defect constraints based on the Hermite-Simpson collocation scheme \cite{50}. The energy integral in Eq. \eqref{eq:28} is evaluated with the same scheme shown in Eq. \eqref{eq:42}, whereas the time integral uses the Hermite-Simpson discretization.

Slew rates limits are imposed as path constraints at each node as formulated in Eq. \eqref{eq:31}. The value $\delta L = L_j - L_{j-1}$ is chosen as the spacing between grid-points. The phasing constraint (see Eq. \eqref{eq:33}) is imposed as a terminal
constraint. The application of the GEO torus avoidance constraints in Eqs. (35)-(37) requires the determination of the radius at the North/south boundary of the GEO box. Thus, interior point constraints have to applied at $L_k$, for $k = 1, \ldots, m$: $r_z(L_k) = \pm l_z$. Here, $r_z$ is the vertical projection of the position vector $r$ onto the cartesian reference frame. Then, it holds $r_{NS} = r(L_k)$. In MOLTO-OR Step 2, the user has to select to optimize time of flight, propellant mass, final power available or a weighted sum of them.

6. Test cases

In this section we employ MOLTO-OR to solve two different orbit raising problems to deploy a satellite into GEO. We consider the two most common injection orbits, GTO and LEO, as departure orbits. The classical orbital elements for both the departure and arrival orbits are presented in Table 5. Also the spacecraft characteristics are summarized in Table 6. In the GTO-GEO case the spacecraft is assumed to provide always the nominal power to the spacecraft, whereas in the LEO-GEO case it decreases with the solar array degradation.
Table 5: Orbit Parameters

<table>
<thead>
<tr>
<th>Orbits</th>
<th>a/Rₐ</th>
<th>e</th>
<th>i (deg)</th>
<th>Ω (deg)</th>
<th>ω (deg)</th>
</tr>
</thead>
<tbody>
<tr>
<td>LEO 1</td>
<td>0.984</td>
<td>0</td>
<td>28.5</td>
<td>0</td>
<td>0</td>
</tr>
<tr>
<td>GTO 3</td>
<td>3.820</td>
<td>0.731</td>
<td>27</td>
<td>99</td>
<td>0</td>
</tr>
<tr>
<td>GEO 6</td>
<td>6.4607</td>
<td>0</td>
<td>0</td>
<td>0</td>
<td>0</td>
</tr>
</tbody>
</table>

Table 6: Spacecraft parameters and transfer cases

<table>
<thead>
<tr>
<th>Transfer</th>
<th>m₀ (kg)</th>
<th>tₑₐₙₑ (s)</th>
<th>η</th>
<th>Pₖ (kW)</th>
<th>tₑₐₙₑ (s)</th>
<th>ΔVₑₐₙₑ (km/s)</th>
</tr>
</thead>
<tbody>
<tr>
<td>GTO-GEO</td>
<td>450</td>
<td>3300</td>
<td>0.65</td>
<td>5</td>
<td>330</td>
<td>3</td>
</tr>
<tr>
<td>LEO-GEO</td>
<td>1200</td>
<td>3300</td>
<td>0.65</td>
<td>10</td>
<td>330</td>
<td>3</td>
</tr>
</tbody>
</table>

We also consider that the satellite solar arrays are comprised by GaAs solar cells for which C = 0.2904 and Dx = 1.10E+9 [51]. The proton non-ionizing energy loss $S_p(E)$ values for different energy levels used in the simulations correspond to Gallium Arsenide (GaAs) solar cells and are determined from Ref. [52]. Furthermore, we consider that the satellite solar array shielding is capable of stopping protons of all energies less than or equal to 5 MeVs. Hence, $E_l = 5$ MeV for the numerical simulations. The contribution of energies greater than 1000 MeV is neglected, that is $E_u = 1000$ MeV.

MOLTO-OR Step 1 is fully coded in C++. The Non-dominated Sorting Genetic Algorithm NSGA-II [53] was selected as heuristic solver for the small-scale NLP problem and the set of genetic parameters is shown in Table 1. In MOLTO-OR Step 2, we use the Interior Point solver Ipopt [54] to solve the resulting Large-scale NLP and the AMPL [55] programming language as interface, which employs automatic differentiation to compute gradients of the objective and constraint functions of the pertinent optimization problem. The initial date is set for 1 January 2000 for all the simulations, which are performed using a Intel Core i7 (2.5GHz) computing system. The optimality and feasibility tolerances in Ipopt were set to $10^{-8}$ and 20 grid nodes per orbital revolution were chosen for the Hermite-Simpson transcription model. The ephemerides for the Sun vector, used for the eclipse calculations are approximated by analytical ephemerides with an accuracy of 36 arcsec between 1950 and 2050.
Table 7: Genetic algorithm parameters

<table>
<thead>
<tr>
<th>Population</th>
<th>Generation</th>
<th>Crossover</th>
<th>Mutation Rate</th>
<th>Mutation Range</th>
</tr>
</thead>
<tbody>
<tr>
<td>150</td>
<td>100</td>
<td>0.8</td>
<td>0.2</td>
<td>0.2</td>
</tr>
</tbody>
</table>

The proposed scenarios will be used to carry out three main analysis. Firstly, we perform an optimality analysis for the GTO-GEO case to demonstrate the importance of selecting a proper initial guess for MOLTO-OR Step 2. The Q-law will be evaluated and compared to other promising near-optimal low-thrust control law. In particular, we use the approach developed by Gao [18], which consists on a predefined control law combined with analytical orbital averaging including Earth $J_2$, coasting mechanism and eclipse constraints. In the second example, the full capabilities of MOLTO-OR will be tested for a LEO-GEO transfer case, comparing the solutions obtained for CP, EP and CCEP as a function of time of flight, propellant mass and solar-cell degradation. Finally, the penalties in the objective function associated to the imposition of operational constraints will be evaluated for the GTO-GEO case using MOLTO-OR Step 2.

6.1. MOLTO-OR Optimality Analysis

In this first case, the GTO-GEO transfer case is solved with MOLTO-OR Step 1 considering only FET and regarding propellant mass and time of flight. Several simulations were carried out. On average, the entire population was feasible by completing the 3rd generation. At the 10th generation, 75% of the population lied along a distinct non-dominated front, whereas from generation 10 to 20 the front was progressively shifted towards lower times of flight and propellant masses. From generation 20 to 30, solutions spread along the Pareto front to generate a more uniform set. After 40 generations were completed, members of the population were uniformly distributed along the front and no later improvement, in terms of non-domination, was observed. Average computational times are summarized in Table 8.

The set of solutions corresponding to a mission duration of 75, 100, 120, 150, 200 days was selected for comparison. Note that MOLTO-OR Step1 does not
allow to impose constraints to match a specific time of flight. However, due to the well-spread Pareto front computed, solutions were found in the neighborhood of these times within a margin of the order of hours. Then, the parametric control law proposed by Gao [18] was implemented and optimized for the given transfer times by means of propellant reduction. Figure 4 displays the performances obtained from both methods. It shows that MOLTO-OR Step 1 is able to obtain a lower propellant mass for the same transfer time than Gao’s approach.

MOLTO-OR Step 2 algorithm was fed by the previous Step 1 solutions in order to obtain the minimum-propellant trajectory for each of the mission times under consideration. Computed propellant masses are shown in Figure 4. It can be seen that results are different when using the initial guess from the Q-law and from Gao. It was found that the NLP-solver did not vary the number of revolutions provided by the initial guess. Therefore, a parametric search over the number of orbital revolutions for each mission time was carried out. For such purpose, an additional constraint was imposed on the final true longitude, forcing the solver to deviate to a solution with a different number of revolutions.

Results obtained are plotted in Figure 5. The particular performances obtained for the number of revolutions provided by the Q-law and Gao are highlighted. It can be deduced that, for fixed-time minimum propellant mass EP transfers, there exists a local optimal solution per each feasible number of revolutions. Furthermore, there is an optimal number of revolutions for which the propellant mass is globally minimal. The number of revolutions of MOLTO-OR Step 1 and Gao’s approach are compared in Table 9 with the globally optimal value found in parametric search. In general, trajectories from the Q-law are closer to that optimum, differing less than a 0.5% in propellant mass.

The existence of multiple locally minima was previously addressed by Betts [30] and Graham [56]. It can be attributed both to the periodic nature of the trajectories as well as to the complicated interaction of the nonlinear and nonconvex constraints. In particular, gradient-based solvers will not vary the number of revolutions provided by the initial guess. Therefore, the selection of a proper initial guess for a direct transcription scheme is crucial.
Table 8: MOLTO-OR Step 1: GTO-GEO Computational Time

<table>
<thead>
<tr>
<th>Variable</th>
<th>Avg. CPU Time</th>
</tr>
</thead>
<tbody>
<tr>
<td>Population</td>
<td>0.41 s</td>
</tr>
<tr>
<td>Generation</td>
<td>10.12 s</td>
</tr>
<tr>
<td>Total</td>
<td>8.54 min</td>
</tr>
</tbody>
</table>

Figure 4: Propellant Mass vs Time of Flight for the first and second step

6.2. LEO-GEO Transfer Case

In this second example, we use MOLTO-OR Step 1 to obtain the Pareto front for the LEO-GEO transfer case, exploiting its capability to concurrently optimize FC, FET and CCET. The objectives under consideration are the time of flight, propellant mass consumed and radiation damage. The electric engine is assumed to use all the power available to the spacecraft. Thus, the thrust dismisses with the radiation displacement dose. Results of the 3D Pareto Front are shown in Figures 6-8. Computational times are summarized in Table 10.

Table 9: Number of revolutions

<table>
<thead>
<tr>
<th>Case</th>
<th>75 days</th>
<th>100 days</th>
<th>120 days</th>
<th>150 days</th>
<th>200 days</th>
</tr>
</thead>
<tbody>
<tr>
<td>MOLTO-OR Step 1</td>
<td>101</td>
<td>156</td>
<td>184</td>
<td>229</td>
<td>279</td>
</tr>
<tr>
<td>Gao [18]</td>
<td>101</td>
<td>164</td>
<td>197</td>
<td>240</td>
<td>288</td>
</tr>
<tr>
<td>Global Optimum</td>
<td>105</td>
<td>156</td>
<td>183</td>
<td>218</td>
<td>277</td>
</tr>
</tbody>
</table>
Figure 5: Locally optimal solutions for a fixed number of revolutions

FCT are optimal for short transfer times (less than 1 day), avoiding the Van-Allen radiation Belts at the cost of consuming propellant mass fractions higher than 73%. Therefore, the power of the solar arrays is not degraded. FET are optimal for transfer times longer than 250 days for minimum fuel trajectories whereas for minimum radiation damage they are not optimal until 300 days. Propellant mass fractions are within the range of 18-20% and they decrease at a rate of 0.006% per day. The solar array power losses are between 18-24% and increase at a rate of 0.12% per day. Intermediate performances are obtained with CCET consisting on two chemical impulses followed by a low-thrust arc for trajectories shorter than one month, and on one chemical impulse before the electric phase for longer transfers. It can be seen that the propellant mass consumed for CCET decreases at a constant rate of 0.18% per day of transfer time increased. However, regarding the radiation damage two different regimes are identified: until 125 days the power loss increases at a rate of 0.17% per day, whereas for longer transfers it occurs at a rate of 0.03% per day.
Table 10: MOLTO-OR Step 1: LEO-GEO Computational Time

<table>
<thead>
<tr>
<th>Variable</th>
<th>avg. CPU Time</th>
</tr>
</thead>
<tbody>
<tr>
<td>Population</td>
<td>0.1920 s</td>
</tr>
<tr>
<td>Generation</td>
<td>9.8 s</td>
</tr>
<tr>
<td>Total</td>
<td>16.8 min</td>
</tr>
</tbody>
</table>

The different total \(\Delta V\)'s for each propulsion system are represented in Figure 6 as a function of time for the minimum radiation damage trajectories and for the minimum propellant mass solutions. It can be seen that, for the same time of flight, dedicating more \(\Delta V\), i.e., more fuel for the chemical engine, and less to the EP is beneficial in terms of radiation damage. The maximum \(\Delta V\) for the CP is 5500 m/s whereas for the electric engine is 6700 m/s.

In Figure 7, we represent the minimum fuel-time trajectories obtained...
from MOLTO-OR Step 1 and we compare them with the ones outputted from MOLTO-OR Step 2. In MOLTO-OR Step 2 different fixed-time minimum fuel problems were solved, using the trajectories from MOLTO-OR Step 1. No operational constraints were considered. A detailed view of the fully chemical transfer is shown, where the point at 0.2 days corresponds to the Hohmann transfer and the solution at 0.05 days correspond to the minimum time solutions with a maximum allowable $\Delta V$ of 3 km/s. The average gain in terms of fuel by re-optimization with Step 2 is 3%. It can be noted that results provided by MOLTO-OR Step 1 are close to the optimal ones and makes the nonlinear programming solver converge within less than 1000 iterations and less than 30 min of computing time. Regarding the computational time, it increases as the number of revolutions of the trajectory increases, ranging from a couple of min-
utes for CCET up to one hour for the FET with 350 days transfer, due to the number of variables that results from the discretization scheme.

In Figures 11(a)-11(f), a set of representative CCET are displayed. The red color represents the shadow region, the clear blue the thrusting arcs, and the dark blue the coasting arcs. Note that, a minimum-fuel hybrid transfer with a fixed flight time incorporates coasting arcs during the electric orbit raising. Therefore, as many authors previously did, it is not optimal to assume a constant thrust after the chemical phase. Trajectories 11(a)-11(b) includes two-chemical impulses. The former raises the apogee higher than GEO and decreases the inclination, while the second one decreases the eccentricity of such orbit performing a burn at the apogee. In trajectory 11(c)-11(d) a chemical firing raises the apogee to a lower value than GEO, and the electric engine then raises it above GEO. Note that, in this case the CP system is not used for reducing the inclination. The configuration of trajectories in Figs. 11(e)-11(f) implies a small initial burn followed by an electric phase with a reduced number of coasting arcs. The duration of these coast arcs will be shortened as the magnitude of the chemical firing is reduced. In the limit case, when the magnitude of the chemical firing is zero, the electric engine will be continuously firing. Such case corresponds to the minimum time case of the FET.
Figure 11: LEO-GEO combined chemical-electric transfers trajectories

(a) ToF = 20 days
(b) ToF = 20 days
(c) ToF = 60 days
(d) ToF = 60 days
(e) ToF = 110 days
(f) ToF = 110 days
6.3. Analysis of Operational Constraints

We perform an analysis on how the application of constraints in MOLTOR Step 2 affects the objective function. The operational restrictions under consideration are: complete avoidance of the geostationary ring, a slew rate limitation of 25 deg/h and phasing at a longitude of 90 deg. Each of them will be evaluated independently for the GTO-GEO transfer scenario for a series of fixed-time minimum fuel cases. In particular, the same mission times from the first test case will be used. All simulations converged to a local optimal solution and the performances obtained are summarized in Table 11. The number of GEO torus crossings, the maximum angular velocity during the transfer, the final longitude in the GEO orbit and the propellant mass penalty with respect to the unconstrained or nominal case are shown.

Regarding the number of GEO torus crossings, it can be seen that the 150
days transfer case has the maximum number. The optimal avoidance of the 104 crossings can be accomplished by carrying less than 1% more fuel. It can be highlighted that avoiding crossings also increments the maximum angular velocity and it changes the arrival longitude. Illustrations of the trajectories nearby the GEO torus for the unconstrained and constrained cases are represented in Figures 12(a)-12(f) for the 100, 120 and 150 days transfers. The trajectory is projected onto a rotating radial frame. It can be seen how the algorithm is able to eliminate all the crossings, except for the last one that is required to reach GEO. Trajectories cross the torus at the beginning, mid and end of the transfer. The combination of increasing the argument of perigee while reducing inclination makes the radius of the ascending node increase and decrease continuously. This region tends to be above GEO as the time of flight increases due to the change of inclination starting at a higher apogee.

Imposing slew limits constraints has a more clear effect in the case of higher angular velocity, i.e., the minimum time case. For that case, the penalty is 1.31%, whereas for the others is three orders of magnitude less. As an illustration, Figure 13 compares the angular velocity of the thrust vector for the nominal case with the constrained one for the 100 days transfer. As one may expect, the angular velocity is higher at the of the beginning the transfer, as the orbital periods are smaller and maneuvers have to be performed faster. In the constrained case, it can be seen how the optimal solution consists on a saturated profile. Note that, the limitation in the angular velocity for the cases under consideration did not increase the number of GEO crossings. The phasing constraint was found to have a larger penalty for the 120 days case. This can be explained as we are forcing the spacecraft to arrive almost at a complimentary slot, i.e., differing 180 degrees, to the locally optimal one.

Finally, all previous constraints are imposed simultaneously. The 150 days transfer was found to be the most penalized. Note that the combined effect of all the constraints, has a greater impact in propellant mass than just adding the contributions of each of them separately. Furthermore, computational times for the constrained case double the ones for the unconstrained scenario.
(a) 100 days: Crossings

(b) 100 days: No Crossings

(c) 120 days: Crossings

(d) 120 days: No Crossings

(e) 150 days: Crossings

(f) 150 days: No Crossings

Figure 12: GEO torus detail

(a) Slow rate limit off

(b) Slow rate limit on

Figure 13: Slow rate comparisons for 100 days case
7. Conclusions

The concurrent optimization of chemical, electric and combined chemical-electric orbit raising trajectories is solved as a multi-objective Hybrid Optimal Control Problem. We describe our tool MOLTO-OR (Multi-Objective Low-Thrust Optimizer for Orbit Raising), based on a two-step procedure that includes realistic effects on the space environment such as Earth oblateness, Earth eclipse and solar array degradation. It incorporates models for complex operational constraints: slew rate limits, GEO torus avoidance and phasing restrictions. The first step, namely MOLTO-OR Step 1, allows for preliminary trade studies as it provides the user in one run a wide set of candidate solutions that closely estimates the optimal performances and mission design parameters. The second step, namely MOLTO-OR Step 2, allows for a refinement of the trajectory, improving optimality and imposing operational constraints.

MOLTO-OR has been successfully tested on two transfer missions to GEO: from GTO and LEO. It has been shown that the NLP resulting from direct transcription of the fixed-time minimum-fuel problem has a local optimum for each number of revolutions. For the GTO-GEO case, using the Q-law as initial guess for MOLTO-OR Step 2 allow to obtain solutions with a propellant mass fraction 0.5% worse than the global optimal. Our analysis on MOLTO-OR Step 1 for a LEO-GEO case with solar cell degradation effects, has shown that Combined Chemical Electric Propulsion Systems fill the gap between Chemical and Electric Propulsion. In particular, it has been revealed that the transfer time interval where each propulsion system is optimal differs for the minimum fuel case and for the minimum solar-cell degradation case. Additionally, for a mission with a given transfer time, increasing the propellant dedicated to the chemical maneuvers will reduce the total radiation damage. Also, the solar-cell degradation increases at a lower rate with respect to transfer time for Combined Chemical-Electric propulsion than for fully electric transfers. Finally, the imposition of operational constraints for the GTO-GEO has been shown to penalize the design with less than 6.37% propellant mass.
Acknowledgments

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