Are monetary-policy reaction functions asymmetric?: The role of nonlinearity in the Phillips curve

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Accepted February 2003

Abstract

This paper investigates the implications of a nonlinear Phillips curve for the derivation of optimal monetary policy rules. Combined with a quadratic loss function, the optimal policy is also nonlinear, with the policy-maker increasing interest rates by a larger amount when inflation or output are above target than the amount it will reduce them when they are below target. Specifically, the main prediction of our model is that such a source of nonlinearity leads to the inclusion of the interaction between expected inflation and the output gap in an otherwise linear Taylor rule. We find empirical support for this type of asymmetries in the interest rate-setting behaviour of four European central banks but none for the US Fed.

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\textit{JEL classification:} E52; E58

\textit{Keywords:} Taylor rules; Nonlinearities; Phillips curve

1. Introduction

For the most part, derivations of optimal rules for the conduct of monetary policy have taken place in a linear–quadratic (L-Q) framework, stemming from the combination of a quadratic objective function for the policymaker and a linear dynamic system
describing the economy; cf., inter alia, Taylor (1993, 1999), Svensson (1997) and Clarida et al. (1998, 2000). When the policy instrument is a short-term interest rate, this combination leads to a linear reaction function (Taylor rule) whereby central banks adjust nominal interest rates proportionally to inflation and output deviations from their targets.

There are, however, at least three good motives to challenge the L-Q paradigm underlying linear Taylor rules. First, it has been recognised for some time that the short-run inflation–output trade-off may be nonlinear. For instance, convexity may arise under the traditional Keynesian assumption that nominal wages are flexible upwards but rigid downwards, giving rise to a quasi-convex AS schedule; cf. Baily (1978). More recently, Akerlof et al. (1996) have further elaborated on this argument claiming that even a downward-sloping Phillips curve (in the inflation–unemployment space) might hold in the long run at very low rates of inflation due to the existence of money illusion on the part of the workers when there is a price stability. Conversely, Stiglitz (1997) argues in favour of a concave relationship when the output gap is negative on the grounds that firms operating under monopolistic competition may exhibit increasingly greater willingness to reduce prices under weak demand to avoid being undercut by rival firms. Orphanides and Wieland (2000) is, to our knowledge, the first paper to consider this type of nonlinearity in the derivation of optimal reaction functions. In particular, they allow for a zone-linear Phillips curve where inflation is essentially stable for a range of output gaps and changes outside this range, providing in this way a good theoretical rationale for inflation-zone as opposed to inflation point-targeting behaviour by central banks. From an empirical viewpoint, Latxon et al. (1995, 1999), Álvarez-Lois (2000), Gerlach (2000) and others have presented evidence in favour of a convex shape for several European countries and the US whereby the inflationary tendencies of capacity constraints on prices imply a considerably steeper Phillips curve when the output gap is positive than when it is negative. In every case, the derived implication is an asymmetric response of inflation with respect to the output gap.

Secondly, there is a growing body of research that departs from the standard assumption of a quadratic loss function by acknowledging the possibility that central banks may have asymmetric preferences with respect to inflation and/or output gaps. For example, given that some central bankers are supposed to be accountable to elected political officials, Cukierman (1999) points out that they may have greater aversion to recessions than to expansions. Under these asymmetric preferences, an inflation bias à la Barro–Gordon emerges even when the policy-maker targets the natural output level rather than a larger level. By contrast, Mishkin and Posen (1997) argue that a deflation bias might be a more likely outcome, since independent central banks often tend to deny the possibility that an expansionary monetary policy stance can reduce cyclical unemployment, and report some favourable evidence to this viewpoint for the Bank of Canada and the Bank of England. Clarida and Gertler (1997), in turn, have tested formally for the null hypothesis of symmetry and found evidence against it for the Bundesbank. More recently, Orphanides and Wieland (2000), Ruge-Murcia (2002), Dolado et al. (2002), Surico (2002) and Cukierman and Muscatelli (2002) have analysed the implications for the derivation of interest-rate reaction functions of assuming asymmetric preferences with respect to inflation and/or output by the policy-maker.
In particular, Dolado et al. (2002) find that, in the absence of certainty equivalence, when the central banker associates a larger loss to positive than to negative inflation deviations, uncertainty induces a prudent behaviour by the monetary authorities which is reflected by the inclusion of the conditional variance of inflation as an additional argument in the Taylor rule. Allowing as well for asymmetric preferences as regards the output gap, Cukierman and Muscatelli (2002) provide evidence showing that central banks in some G7 economies develop a precautionary demand for expansions and for low inflation once credibility building and disinflation have been achieved.

Lastly, there is a third source of nonlinearity which stems from uncertainty regarding the NAIRU or the trend growth rate of productivity. As Meyer et al. (2001) have shown, in periods of heightened uncertainty about the NAIRU (like the second half of the 1990s in the US following the IT-induced productivity acceleration), an optimal updating rule of the NAIRU leads to a nonlinear interest-rate policy according to which policy-makers are more cautious about adjusting interest rates in response to small output gaps than in a standard linear Taylor rule but more aggressive when they reach a certain threshold.

In view of these arguments, our goal of this paper is to extend the available evidence on the presence of asymmetric features in monetary policy rules. Specifically, our focus is restricted to the first source of nonlinearity. To this end, we re-examine the analytical implications of assuming a nonlinear short-term inflation–output trade-off in the derivation of such rules and provide some empirical evidence consistent with this nonlinearity. By assuming a quadratic functional form for the effects of the output gap in an accelerationist Phillips curve we obtain a modified Taylor rule which only differs from the conventional linear specification in that it includes an interaction between expected inflation and the output gap as an additional term in the Euler equation. This simple device allows us to capture the asymmetric response of the interest rate to inflation and output gaps which turns out to be optimal in this framework. Our results echo those recently derived by Schaling (1999) in a more restricted set-up than ours. Deriving this modified policy rule for the specific model considered here, together a cross-country empirical analysis supporting the usefulness of the proposed approach, is the contribution of the paper to the literature.

Our empirical approach relies upon testing for the statistical significance of the interaction term in the estimation of two types of models. First, we consider an Euler equation specification, in line with the influential approach by Clarida et al. (1998) to capture the performance of a policy rule in describing the evolution of a continuously adjusted short-term interest rate, like (say) an overnight interest rate. Second, we consider an ordered probit model which, as pointed out by Dolado and Maria-Dolores (2002), is a useful modelling strategy to analyse the determinants of decisions concerning adjustments in interest rates which only take place irregularly and in discrete increments, as is the case of discount rates. The proposed methodologies are applied to estimate the interest rate-setting behaviour of three European central banks (Banque

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1 Kim et al. (2002) have recently investigated the nature of nonlinearities in monetary policy rules using Hamilton’s (2001) flexible nonlinear methodology and conclude that our proposed interaction term is the one that does best in characterising nonlinear policy rules in the US Fed up to 1979.
de France, Bundesbank and Banco de España), the US Federal Reserve and the (surrogate) European Central Bank (ECB) over different sample periods.

The rest of the paper is organised as follows. Section 2 reviews the basic theory behind derivation of the optimal interest-rate reaction function under a nonlinear Phillips curve in a simple model along the lines of Svensson (1997). With this illustrative model in mind, we derive the main features of the nonlinear policy rule which serves as a benchmark for the empirical section. Section 3 presents the empirical results obtained from applying the two econometric methodologies described above to five central banks. Finally, Section 4 concludes.

2. Policy rules with a nonlinear Phillips curve

2.1. Basic principles

To make the basic point of the paper without introducing unnecessary complications, we modify a minimalist backward-looking model of the economy proposed by Svensson (1997) by allowing for a nonlinear Phillips curve. In this set-up, inflation is determined by an accelerationist Phillips curve, output follows a simple autoregressive process and the monetary policy instrument is a short-term nominal interest rate. Since the model is not based on explicit microfoundations, as in Clarida et al. (1998, 2000) or Woodford (1999), issues related to credibility and reputation are left aside, in exchange for expositional simplicity.

In every period, the policy-maker sets the nominal interest rate, \( i \), with the aim of maintaining inflation deviations from a target, \( \pi - \pi^* \), and the output gap, \( \tilde{y} \), close to zero. Assuming a quadratic per-period loss function in inflation and output performance, \( L(\tilde{\pi}_t, \tilde{y}_t) = \frac{1}{2}[\tilde{\pi}_t^2 + \lambda \tilde{y}_t^2] \), and a fixed discount rate, \( \delta \), the policy-makers’s objective in period \( t \) is to minimise the expected present discounted value of the per-period losses:

\[
E_t \sum_{s=0}^{\infty} \delta^s L(\tilde{\pi}_{t+s}, \tilde{y}_{t+s})
\]  \( (1) \)

subject to the following two equations describing the evolution of the economy:

\[
\pi_{t+1} = \pi_t + \alpha f(\tilde{y}_t) + u_{\pi,t+1}
\]  \( (2) \)

with

\[
f(\tilde{y}_t) = \tilde{y}_t + \phi \tilde{y}_t^2,
\]  \( \tilde{y}_t > -1/2\phi \)

and

\[
\tilde{y}_{t+1} = \beta \tilde{y}_t + \eta x_t - \xi r_t + u_{\tilde{y},t+1},
\]  \( (4) \)

where \( E_t \) is the conditional expectations operator, \( \delta \) and \( \beta \in [0, 1) \), and \( u_{\pi,t+1} \) and \( u_{\tilde{y},t+1} \) are zero-mean normally distributed shocks.
Eq. (2) represents an accelerationist Phillips curve (or AS schedule) where the output gap enters in a nonlinear way, as defined in Eq. (3). Linearity in (3) is recovered when $\phi = 0$ and the function is convex (concave) if $\phi > 0$ ($< 0$).\(^2\) We assume the function to be increasing ($1 + 2\phi \tilde{y} > 0$) since this is likely to be the case for realistic values of $\phi$ and $\tilde{y}$. Eq. (4), in turn, is an IS schedule where the output gap exhibits sluggish adjustment, and depends on the real interest rate ($r_t = i_t - E_t \pi_{t+1}$), and on a predetermined/exogenous variable, $x_t$, possibly capturing other determinants of interest-rate setting in open economies (see, e.g. Ball, 1998). The real interest rate affects output with one-period lag and, therefore, affects inflation with a two-period lag. This timing convention is in line with the extensive literature on the transmission mechanism of monetary policy which establishes that an innovation in monetary policy leads to a change in output in the short run with inflation only changing slowly later on (see e.g., Christiano et al., 1999).

Totally differentiating (1) with respect to $i_t$, subject to (2)–(4), yields the following Euler equation:

$$\lambda E_t \tilde{y}_{t+1} + \lambda \delta \beta E_t \tilde{y}_{t+2} + \delta \alpha E_t \tilde{\pi}_{t+2} (1 + 2\phi \tilde{y}_{t+1}) = 0.$$

(5)

Using (4) to replace $E_t \tilde{y}_{t+2}$ in terms of $E_t \tilde{y}_{t+1}$, $E_t x_{t+1}$ and $E_t r_{t+1}$, and solving for $i_t$ in period $t$ implies the following Taylor rule:

$$i_t = c_1 E_{t-1} \tilde{\pi}_{t+1} + c_2 E_{t-1} \tilde{y}_{t} + c_3 E_{t-1} x_t + c_4 E_{t-1} (\tilde{\pi}_{t+1} \tilde{y}_{t}),$$

(6)

where the $c_i$’s coefficients are functions from the set of structural parameters ($\delta, \lambda, \alpha, \mu, \phi, \zeta$ and $\beta$) so that $c_1 = 1 + \zeta/\lambda \zeta \beta$, $c_2 = (1 + \delta \beta^2)/\delta \zeta \beta$, $c_3 = \eta/\zeta$ and $c_4 = 2 \phi \alpha/\lambda \zeta \beta$.

The modified Taylor rule in (6) resembles a linear one except for the last term, namely, the interaction of expected inflation and the output gap. The intuition for the presence of this interaction term in the Euler equation is simple. If, for example, inflation is expected to be above its target at period $t + 1$, the real interest rate will be below its equilibrium value at period $t$ which, in turn, causes a higher output gap at $t + 1$ and higher inflationary pressure at $t + 2$. In the linear case, the policy-maker increases the interest rate by $c_1 E_{t-1} \tilde{\pi}_{t+1}$. However, if the Phillips curve is convex ($\phi > 0$), then the future inflationary pressure caused by the higher output gap will turn out to be larger than in the linear case. The policy-maker, anticipating this higher pressure captured by the interaction term, will react more forcefully, since in this case $c_4 > 0$. Conversely, if the Phillips curve is concave ($\phi < 0$), future inflationary pressure will be lower than in the linear case and the increase in the interest rate will be smaller ($c_4 < 0$). A similar intuition can be used to interpret an asymmetric response with respect to the output gap. If output is above trend at $t$, then the output gap at $t + 1$ will be positive as well, given the serial correlation in (4), leading to

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\(^2\)The quadratic functional form could be interpreted as a second-order approximation around $\tilde{y} \approx 0$ to other more flexible functional forms like the function $g(\tilde{y})$ discussed below (see Schaling, 1999) or the Linex function used by Nobay and Peel (2000). However, when such a more general shapes are considered as the primitive functional forms, the disturbance in Eq. (8) below will contain the approximation error which depends on forecasts and therefore is not uncorrelated with the instrumental variables. For this reason, we have taken the quadratic function as the primitive functional form.
Table 1
Estimated nonlinear Phillips curves

<table>
<thead>
<tr>
<th>Country</th>
<th>$\hat{\alpha}$</th>
<th>$\hat{\phi}$</th>
</tr>
</thead>
<tbody>
<tr>
<td>Germany</td>
<td>0.17**</td>
<td>0.41**</td>
</tr>
<tr>
<td></td>
<td>(0.02)</td>
<td>(0.16)</td>
</tr>
<tr>
<td>France</td>
<td>0.12**</td>
<td>0.33**</td>
</tr>
<tr>
<td></td>
<td>(0.03)</td>
<td>(0.14)</td>
</tr>
<tr>
<td>Spain</td>
<td>0.10**</td>
<td>0.38**</td>
</tr>
<tr>
<td></td>
<td>(0.02)</td>
<td>(0.17)</td>
</tr>
<tr>
<td>US</td>
<td>0.10**</td>
<td>0.09</td>
</tr>
<tr>
<td></td>
<td>(0.01)</td>
<td>(0.05)</td>
</tr>
<tr>
<td>Euro area</td>
<td>0.19**</td>
<td>0.29*</td>
</tr>
<tr>
<td></td>
<td>(0.03)</td>
<td>(0.15)</td>
</tr>
</tbody>
</table>

Note: White’s standard errors in parentheses; * and ** denotes statistical significance at the 10% and 5% significance levels, respectively.

a higher inflationary pressure at $t + 2$ than in the linear case because of the convex Phillips curve (see Table 1).

Although the previous argument has been derived in an Euler-equation context, it is interesting to know if a closed-form solution can be obtained. Unfortunately, since our model deviates from the L-Q framework (quadratic objective but nonlinear economic structure), the value function in the Bellman equation associated to (1) is not quadratic. Thus, the well-known arguments used by Svensson (1997, Appendix B) in order to derive an analytical solution of the optimal interest-rate reaction function in this backward-looking model cannot be applied. Instead, one should rely upon numerical dynamic programming algorithms, as the ones used by Orphanides and Wieland (2000), to obtain approximate solutions. However, as Schaling (1999) has shown, if the policy-maker is a pure point-inflation targetter ($\lambda = 0$) and, for algebraic purposes, a slightly modified function $g(\tilde{y}_t) = \tilde{y}_t/(1 - \phi \tilde{y}_t)$ is used instead of the quadratic $f(\tilde{y}_t)$ adopted in (2), then a simple closed-form solution exists.3 In effect, due to the recursive dynamic structure of (2) and (4), the interest rate in period $t$ should be set to achieve $E_t(\tilde{\pi}_{t+2}) = 0$. Assuming a deterministic problem (i.e., that the variances of the shocks in (2) and (4), $u_\pi$ and $u_y$, are zero) yields the following closed-form solution for the nominal interest rate:

$$
    i_t = \frac{(1/\xi \alpha)[\tilde{\pi}_t + \alpha f(\tilde{y}_t)]}{1 - (\phi/\alpha)[\tilde{\pi}_t + \alpha f(\tilde{y}_t)]} + \pi_t + f(\tilde{y}_t) + \beta \tilde{y}_t + \eta x_t,
$$

where $i_t$ is a nonlinear function of the inflation and output gaps which, when $\phi$ tends to zero, collapses to the conventional linear Taylor rule in this type of backward-looking

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3 Note that a second-order Taylor expansion of $g(\tilde{y}_t) = \tilde{y}_t/(1 - \phi \tilde{y}_t)$ around $\tilde{y}_t = 0$ yields $g(\tilde{y}_t) \approx \tilde{y}_t + \phi \tilde{y}_t^2$, that is exactly the quadratic function, $f(\tilde{y}_t)$, considered in (3). For a realistic range of values of $\tilde{y}_t$, like $[-0.04, 0.04]$ and $\phi = 0.3$ (see the estimates reported in Table 1), the two functions behave very closely.
model (see Svensson, 1997). The intuition for the presence of the nonlinear term in (7) is the same as the one provided for the interaction term in the Euler equation. For example, considering, for illustrative purposes, that \( \alpha = 0.5, \xi = 1, \) and \( \phi = 0.3, \) the appropriate interest rate changes stemming from \( \pm 0.5\% \) inflation gaps are \( \pm 1.5\% \) in the linear case whereas they are \( 1.93\% \) and \( -1.27\%, \) respectively, in the nonlinear case. Hence, when the Phillips curve is convex, the policy-maker reacts more forcefully (in absolute value) to positive inflation gaps than to negative gaps, and the opposite will happen under concavity (\( \phi < 0 \)).

Finally, a brief comment is offered about the implications of having a nonlinear Phillips curve on the inflation bias which typically occurs in the well-known Barro-Gordon’s analysis of discretionary monetary policy. Let us consider, for simplicity, a static optimisation problem where the loss function is \( L(\pi, y) = \frac{1}{2}[\pi^2 + \lambda(y - ky^*)^2] \), with \( k > 1 \) capturing the existence of labour-market rigidities or distorting taxes. Assume that the structure of the economy is given by the Phillips curve \( \pi = \pi^e + g(\tilde{y}) \) and \( \tilde{y} = p + u_y, \) where \( \pi^e \) represents the agents’ rational expectation of \( \pi; \tilde{y} \) is considered to be the control variable which depends linearly on a deterministic policy instrument, \( p, \) and \( u_y \) is an innovation. Then, taking expectations (E) in the first-order condition of the optimal discretionary policy yields:

\[
\pi^e = \lambda E \frac{k - 1}{[1 + \phi z]^2} y^* - \lambda E \frac{z}{[1 + \phi z]^3},
\]

where \( z = \pi - \pi^e. \) Note that with \( \phi = 0, \) the standard inflation bias, \( \lambda(k - 1)y^*, \) is recovered. Defining \( h(z) = z/(1 + \phi z)^3, \) we know from Jensen’s inequality that \( Eh(z) < h(Ez) \) if \( h(z) \) is concave. It can be easily checked that sign \( h''(z) = -\text{sign}(\phi). \) Thus, convexity of the Phillips curve, \( \phi > 0, \) implies concavity of \( h(z). \) Moreover, since in equilibrium (when \( z = 0 \)) \( h(Ez) = 0, \) we get \( Eh(z) < 0. \) Thereby, even when \( k = 1, \) convexity of the Phillips curve implies, on average, a positive inflation bias as long as there is output stabilisation (\( \lambda > 0 \)). Likewise, it is easy to check that, since \( \text{E}g(\tilde{y}) = 0 \) when \( z = 0, \) then convexity leads to \( \text{E}\tilde{y} < 0, \) namely, the expected level of output is lower than the natural level. The intuition for this deflation bias in expected output stems from the asymmetric interest rate-setting behaviour under a convex Phillips curve whereby policy-makers, in achieving a given inflation target, have a greater incentive to avoid periods of excess demand, as these require longer and/or more severe recessions to undo the inflation generated when output is above target.\(^6\)

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\(^4\) Allowing for uncertainty in the form of shocks to the output gap \( (u_y; t+1) \) in (4), a closed-form solution no longer exists but it can be proved that the interest-rate response to inflation gaps will be larger than in the model without uncertainty (see Schaling, 1999).

\(^5\) Without loss of generality, we set \( \pi^* = 0. \)

\(^6\) Nobay and Peel (2000) obtain similar results, except that the sign of the inflation bias is ambiguous, using a Linex functional form, \( \ell(\tilde{y}) = \gamma^{-1}(e^{\gamma\tilde{y}} - 1). \)
2.2. Econometric specifications

To assess the empirical support of the departure of the L-Q framework considered here, we rely upon two alternative econometric strategies which are described in turn.

2.2.1. Euler equation approach

First, we test for the statistical significance of the interaction term directly in the Euler equation derived in (6). For that, we replace the expectations by realised values in (6), yielding the following policy rule in $t$:

$$i_t = \text{cnst} + c_1 \tilde{\pi}_{t+k} + c_2 \tilde{y}_t + c_3 x_t + c_4 (\tilde{\pi}_{t+k} \tilde{y}_t) + \rho_1 i_{t-1} + v_t,$$

(8)

where, for estimation purposes, we have introduced two slight modifications in Eq. (8). First, in accord with most of the empirical literature, we take $k = 12$, instead of $k = 1$, to be the horizon used by central banks in forecasting inflation when data has monthly frequency as it is in our case. And, secondly, as is also conventional, we allow for a lagged dependent variable to capture interest-rate smoothing for which there are several motivations in the literature. While it is not possible to recover all structural parameters from the estimated coefficients in (8), what really matters from the viewpoint of this paper is that $c_4$ is the only coefficient which embodies information about the nonlinear Phillips curve, so that the restriction $\phi = 0$ implies $c_4 = 0$. Indeed, it is straightforward to check that the ratio $c_4/2(c_1 - 1)$ yields a direct estimate of $\phi$. Hence testing $H_0: \phi = 0$ is equivalent to testing $H_0: c_4 = 0$ as long as $c_1$ is different from unity. Since (8) is linear in the coefficients, the key advantage of directly testing $H_0: c_4 = 0$ is that it does not require estimating a nonlinear model in the parameters. As for the error term in (8) it is defined as

$$v_t = -[c_1(\tilde{\pi}_{t+k} - E_{t-1} \tilde{\pi}_{t+k}) + c_2(\tilde{y}_t - E_{t-1} \tilde{y}_k) + c_3(x_t - E_{t-1} x_t) + c_4(\tilde{\pi}_{t+k} \tilde{y}_t) - E_{t-1}(\tilde{\pi}_{t+k} \tilde{y}_t)],$$

where the term in brackets is a linear combination of forecast errors and therefore orthogonal to any variable in the information set available at $t - 1$. As is conventional in models involving rational expectations, the estimation method relies upon the choice of a set of instruments, $Z_t$, from the set of variables within the central bank’s information set, such as lagged variables that help forecast inflation and output or any other contemporaneous variables that are uncorrelated with the policy rule shock, $v_t$. Then, the generalized method of moments (GMM) can be used to estimate the parameter vector in (8) by exploiting the set of orthogonality conditions $E(v_t/Z_t) = 0$. Since the composite disturbance has an MA($k$) representation, due to the overlapping nature of the forecast errors, the Hansen–Hodrick weighting var–cov matrix, which puts a weight of 1 on the first $k$ covariances and 0 on all others, is used to implement GMM. Finally, Hansen’s $J$ test is used to test the over-identification restrictions.

2.2.2. Ordered probit approach

Secondly, to check how relevant is the role the interaction term in other types of interest-rate reaction functions, we estimate an ordered probit model which is the
appropriate econometric framework to analyse the determinants of decisions about the discrete and infrequent adjustments that characterise the behaviour of discount rates (see Dolado and Maria-Dolores, 2002). Under this approach, the underlying assumption is that the policy-maker takes a decision every period about adjusting a discount rate in terms of the following changes: Large or small reductions, no change, and large and small increases. Specifically, we will assume that these changes are discrete and use a breakdown of adjustments into the following five categories, captured by a set of dummy variables, which are ordered in steps of 25 basis points each:

(large decrease) \( c_t = 1 \iff \Delta i_t < -0.25 \),

(small decrease) \( c_t = 2 \iff \Delta i_t \in [-0.25, 0) \),

(no change) \( c_t = 3 \iff \Delta i_t = 0 \),

(small increase) \( c_t = 4 \iff \Delta i_t \in (0, 0.25] \),

(large increase) \( c_t = 5 \iff \Delta i_t > 0.25 \).

The observed dummy variable, \( c_t \), depends on a latent index, \( c_t^* \), according to the following rule:

\[
c_t = \begin{cases} 
1 & \text{if } c_t^* \leq \alpha_1, \\
2 & \text{if } \alpha_1 < c_t^* \leq \alpha_2, \\
3 & \text{if } \alpha_2 < c_t^* \leq \alpha_3, \\
4 & \text{if } \alpha_3 < c_t^* \leq \alpha_4, \\
5 & \text{if } c_t^* > \alpha_4, 
\end{cases}
\]

where \( c_t^* \) is taken to be a latent continuous random variable triggering adjustments and the \( \alpha \)'s are the thresholds that the latent variable must cross to change the value of \( c_t \). The underlying index is assumed to depend linearly on a set of covariates, \( x_t \), such that

\[
c_t^* = \beta'x_t + \epsilon_t = \beta_1 E_t(\tilde{\pi}_{t+k}) + \beta_2 \tilde{y}_t + \beta_3 E_t(\tilde{\pi}_{t+k} \tilde{y}_t) + \beta_4 s_t + \epsilon_t. \tag{9}
\]

The first three regressors in (9) mimic those considered in the derivation of the Euler equation while the nature of the remaining regressors, \( s_t \), will be discussed in Section 3. Although this specification has not been derived from a theoretical model on the adjustment decisions, it is bound to capture the same sort of considerations as in (6), this time in the discrete change framework which characterises the evolution of discount rates. Hence, we loosely interpret a test of \( H_0: \beta_3 = 0 \) as indirect evidence for \( H_0: \phi = 0 \). Assuming that \( \epsilon_t \) follows a n.i.d. \((0, \sigma^2_\epsilon)\), estimates of the parameter vector \((\alpha, \beta)\) are then obtained by maximising the following likelihood function
(see Maddala, 1983):

\[ l(z, \beta) = \sum_{t \in y = 1} \log \Phi(z_1 - \beta' x_t) + \sum_{j=2}^{4} \sum_{t \in y = j} (\log(\Phi(z_j - \beta' x_t)) - \Phi(z_{j-1} - \beta' x_t)) + \sum_{t \in y = 5} \log(1 - \Phi(z_4 - \beta' x_t)), \]

where \( \Phi(.) \) is the cumulative Gaussian distribution function.

3. Estimation results

3.1. Data

To estimate Eq. (8), we have used monthly data for three European countries (Germany, France and Spain) and the US. The sample periods are 1980(8)–1997(12) for Germany, 1988(7)–1997(12) for France, 1989(5)–1997(12) for Spain and 1984(1)–2001(9) for US. They correspond to recent spells where there was a virtually autonomous control over domestic monetary policy in each case. We have also considered quarterly data for the Euro area over the period 1984(Q1) to 2001(Q3) which has been constructed by OECD using weighted averages of the individual countries with GDP-weights measured in units of PPP at 1995 prices. In this case, \( k = 4 \) has been chosen. The idea is to study how a “surrogate” ECB would have behaved had it exerted monetary control over the Euro area during a period comprising the pre-EMU period (before 1999(Q1)) and afterwards.\(^7\)

The short-term intervention interest rates in (8) are: (i) overnight interest rates in Germany and France; (ii) the marginal intervention rate of auctions of “Certificados del Banco de España” in Spain; (iii) the Fed-Fund rate in the US; and (iv) a GDP-weighted average of short-term intervention interest rates for the EMU countries. For inflation we use the annual \((t/t - 12\) basis\) percentage rate in the CPI. For output we use (logged) Industrial Production Index and (logged) GDP for the Euro area (all variables are seasonally adjusted). To measure the output gap, we detrend (logged) output using the HP filter with a coefficient of 14.800 for the four individual countries and 1600 for the Euro area.\(^8\)

As regards the inflation target, \( \pi^* \), we consider a departure from the usual assumption that it is constant, as in Clarida et al. (1998), since in some countries inflation has slowly converged from above to its target value implying that a constant target seems to be less plausible than a gradually moving one. Instead we adopt a time-varying inflation target, \( \pi^*_t \), according to the following considerations: (i) for Germany, we take the inflation target to be the one established by the Bundesbank in its annual reports; (ii) for France, the German target inflation rate, given the close links between

\(^7\) As a referee has pointed out, this exercise implicitly assumes that the ECB has the same preferences as national central banks before EMU. Note, however, that Alesina et al. (2001) dispute this assumption.

\(^8\) The residuals from adjusting a cubic trend to logged output led to similar results.
both economies within EMU; for Spain, the official inflation rate in the budget laws up to 1995 and the target inflation rate reported by the Bank of Spain since 1996; for US, the target inflation rates in the reports of the Council of Economic Advisors; and finally, for the Euro-zone, the German target inflation rate again since German monetary policy served as an anchor to most of the other Euro area countries over the sample. Annual inflation target rates have been interpolated to a monthly frequency for the individual countries, and to a quarterly frequency for the Euro area. For the sake of completeness, however, results obtained with a constant inflation target are also reported below. As for the \( x_t \) variable, the German interest rate has been used for France and Spain, the US interest rate for Germany and the Euro area, and the growth rates of borrowed and total reserves for the US. The list of instruments is: a constant term, two lags of the interest rate, six lags of the inflation gap, six lags of the output gap, four lags of the interaction of inflation and output lags, two lags of (logged) raw materials price index. Further, in the case of the three European countries, two lags of the German interest rate (for France and Spain) and of the US interest rate (for Germany), and two lags of the (logged) effective real exchange rate have been included.

To estimate Eq. (9), we have used the repo rate for France, the marginal target rate in the interbank reserves market for Spain, the discount rate for Germany, and the target Funds rate for the US. Although these series have higher frequency, the changes have been aggregated to a fixed interval of a month, since this is the frequency at which information on inflation, output and some on the other determinants of interest-rate setting arrives. The sample periods are the same as above. Given that among the \( s_t \) variables we allow for duration effects, i.e., the time elapsed since the last change, the “surrogate” ECB has not been included in this econometric exercise, since a weighted aggregation of the durations of discount rates in each individual central bank would be meaningless.

3.2. Preliminary analysis: A nonlinear Phillips curve?

To get some preliminary evidence on the key channel for a nonlinear policy rule highlighted in this paper, Table 1 reports the results from estimating the nonlinear specification chosen for the Phillips curve in (2). For that, the change in inflation at \( t, \Delta \pi_t \), has been regressed on \( f(y_{t-1}) \) to estimate the parameters \( z \) and \( \phi \). A positive and statistically significant value of \( \phi \) implies a convex Phillips curve. The basic finding is that there is evidence favourable to a convex Phillips curve in all cases except in the US. For illustrative purposes, Fig. 1 depicts scatter plots of lagged output

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9 The inflation target in this case is constant since the German target rate did not change from 2% during the sample period considered for France.
10 In a previous version of this paper (see Dolado et al., 2000) we estimated directly the response of interest rates to positive and negative, and large and small, inflation and output gaps. To have observations of different sign in inflation gaps, the use of time-varying inflation targets was needed.
11 The data on the time-varying inflation target rates are available upon request.
12 White’s robust standard errors have been used to compute the \( t \)-ratios of \( z \) and \( \phi \) since, as pointed out by Dolado et al. (2002), there is strong evidence of a GARCH process in the residuals of such equations.
gap (horizontal axis) against change in inflation (vertical axis), together with the fitted quadratic function, for the US and the Euro area. As can be observed, the fitted curve is clearly convex in the Euro zone whereas it cannot be distinguished from a linear one in the US. One possible explanation for these contrasting results could be that European labour markets are known to suffer from higher real wage rigidity than the US labour market, giving rise to a steeper short-run inflation–output trade-off when output is above the natural level than when it is below it. For example, Nickell (1997) reports conclusive evidence about higher downwards than upwards wage rigidity in Europe, whereas such is not the case in the more flexible US labour market. Thus, this preliminary evidence seemingly supports the existence of a convex Phillips curve, at least in the three European countries and in the Euro area.

13 Alvarez-Lois (2000) finds a nonlinear relation between the change in inflation and a capacity utilization for the US. However, he uses quarterly data, whose lower frequency facilitates finding a nonlinear relationship, and his sample period is much longer, from 1960(Q1) to 2000(Q1).
3.3. Nonlinear Taylor rules

Table 2 displays the results obtained from estimating Eq. (8) by the GMM method described above. For each country, three specifications are presented in panels A, B and C. Results in panel A correspond to the specification excluding the \( x_t \) variables and allowing for time-varying inflation target, whereas those in panel B include \( x_t \). For the US and the Euro area, the coefficients on \( x_t \) for the different proxies that we tried were never statistically significant and therefore are not reported. Finally, panel C displays the results obtained with a constant inflation rate and allowing for \( x_t \). As for the coefficients on the lagged dependent variables, denoted in the table by \( \rho_i (i = 1, 2) \), we found that only one lag was significant in the case of the three European countries whereas two lags were needed for the US and the Euro area. The existence of an AR (2) specification for the US agrees with the findings of Clarida et al. (1998, 2000).

In general, it is worth noticing that the \( p \)-values of the \( J \)-test (denoted as \( p-J \) in Table 2) do not reject the over-identifying restrictions. Further, the \( p \)-values of the \( F \)-test about the joint significance of the coefficients obtained in the regression of \( \tilde{\pi}_{t+k}, \tilde{y}_t \) and \( (\tilde{\pi}_{t+k}, \tilde{y}_t) \) on the set of instruments is also reported since a poor fit in the first stage of the GMM procedure may raise concern about lack of identification (see Arellano et al., 1999).

The basic result to highlight is that the estimated coefficient \( c_4 \) on the interaction term is always highly significant for the three European countries and the Euro area. By contrast, it is not significant for the US, in accord with the evidence in the previous subsection and with the findings of Dolado et al. (2002) for the US using a longer sample period. Since the use of \( c_4 \) to test indirectly for the significance of \( \phi \) relies on \( c_1 \) being above unity it is worth discussing the results in this respect. In all cases, except in France (panel B) and Spain, the point estimates of \( c_1 \) are above unity, in line with an inflation-stabilising policy rule as explained by Clarida et al. (1998, 2000). Nonetheless, as those authors have pointed out, when the relevant \( x_t \) variable is a foreign interest rate, as in the three European countries, the correct interpretation is that the policy rule is a weighted average of the German interest rate for France and Spain, and the US interest rate for Germany (with a weight of \( c_3 \)) and a baseline policy rule (with a weight of \( 1 - c_3 \)). Thus the coefficient on expected inflation for these countries should be computed as \( c_1/(1 - c_3) \) yielding estimates of 1.25, 1.30 and 1.10 in Germany, France and Spain, respectively, according to the estimates of \( c_3 \) reported in panel B. The remaining coefficients are significant and correctly signed in all specifications. Finally, the results obtained for a constant inflation target are fairly similar to the ones with a time-varying target, despite some noticeable changes in the size of the constant term and \( c_2 \). In any case, the estimated coefficient on the interaction term remains significant, in agreement with the previous results.

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14 For France panel C is excluded since the German inflation target is constant for the chosen sample period.
15 Recall that \( c_1 = 1 + x/\lambda \xi \beta \) in (6), with all the parameters being positive.
Table 2
Estimated nonlinear reaction functions

<table>
<thead>
<tr>
<th>Country</th>
<th>Term</th>
<th>$\rho_1$</th>
<th>$\rho_2$</th>
<th>$\text{cnst}$</th>
<th>$c_1$</th>
<th>$c_2$</th>
<th>$c_3$</th>
<th>$c_4$</th>
<th>$\sigma_2$</th>
<th>$p$-value</th>
<th>$p$-value</th>
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<tbody>
<tr>
<td>Germany</td>
<td>$\pi^*$ variable $(A)$</td>
<td>$k = 12$</td>
<td>0.91</td>
<td>—</td>
<td>5.87</td>
<td>1.11</td>
<td>0.46</td>
<td>—</td>
<td>0.11</td>
<td>0.33</td>
<td>0.086</td>
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<tr>
<td></td>
<td>$k = 12$</td>
<td>+ US interest rate</td>
<td>0.91</td>
<td>—</td>
<td>4.80</td>
<td>1.05</td>
<td>0.39</td>
<td>0.16</td>
<td>0.17</td>
<td>0.31</td>
<td>0.085</td>
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<tr>
<td>France</td>
<td>$\pi^*$ variable $(A)$</td>
<td>$k = 12$</td>
<td>0.92</td>
<td>—</td>
<td>6.42</td>
<td>1.23</td>
<td>0.87</td>
<td>—</td>
<td>0.30</td>
<td>0.57</td>
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<tr>
<td></td>
<td>$k = 12$</td>
<td>+ German interest rate</td>
<td>0.68</td>
<td>—</td>
<td>3.66</td>
<td>0.47</td>
<td>0.34</td>
<td>0.64</td>
<td>0.14</td>
<td>0.53</td>
<td>0.085</td>
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<tr>
<td>Spain</td>
<td>$\pi^*$ variable $(A)$</td>
<td>$k = 12$</td>
<td>0.95</td>
<td>—</td>
<td>8.08</td>
<td>0.88</td>
<td>0.16</td>
<td>—</td>
<td>0.10</td>
<td>0.51</td>
<td>0.102</td>
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<tr>
<td></td>
<td>$k = 12$</td>
<td>+ German interest rate</td>
<td>0.93</td>
<td>—</td>
<td>7.36</td>
<td>0.78</td>
<td>0.14</td>
<td>0.29</td>
<td>0.10</td>
<td>0.49</td>
<td>0.097</td>
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<tr>
<td>US</td>
<td>$\pi^*$ variable $(A)$</td>
<td>$k = 12$</td>
<td>1.21</td>
<td>—</td>
<td>4.06</td>
<td>1.92</td>
<td>0.25</td>
<td>—</td>
<td>0.31</td>
<td>0.25</td>
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<tr>
<td></td>
<td>$k = 12$</td>
<td>+ German interest rate</td>
<td>1.19</td>
<td>—</td>
<td>2.93</td>
<td>2.16</td>
<td>0.26</td>
<td>—</td>
<td>0.45</td>
<td>0.33</td>
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<tr>
<td>Euro area</td>
<td>$\pi^*$ variable $(A)$</td>
<td>$k = 4$</td>
<td>1.30</td>
<td>—</td>
<td>2.86</td>
<td>1.46</td>
<td>0.82</td>
<td>—</td>
<td>0.40</td>
<td>0.45</td>
<td>0.035</td>
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Sample period:
- Germany: 1980:08 - 1997:12
- Euro area: 1984:01 - 2001:09
Table 2 (continued)

<table>
<thead>
<tr>
<th></th>
<th>$\rho_1$</th>
<th>$\rho_2$</th>
<th>const</th>
<th>$c_1$</th>
<th>$c_2$</th>
<th>$c_3$</th>
<th>$c_4$</th>
<th>$\sigma_\varepsilon$</th>
<th>p-J</th>
<th>p-F</th>
</tr>
</thead>
<tbody>
<tr>
<td>(C) $\pi^*$ constant</td>
<td></td>
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<tr>
<td>k = 4</td>
<td>1.36</td>
<td>-0.48</td>
<td>5.84</td>
<td>1.53</td>
<td>0.46</td>
<td>—</td>
<td>0.11</td>
<td>0.41</td>
<td>0.066</td>
<td>0.00/0.00/0.02</td>
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<td></td>
<td>(0.11)</td>
<td>(0.11)</td>
<td>(0.45)</td>
<td>(0.43)</td>
<td>(0.12)</td>
<td>—</td>
<td>(0.05)</td>
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Note: Standard errors in parentheses; p-J is the p-value of the J-test of over-identifying restrictions; p-F is the p-value of the F-test of the joint significance of the coefficients of $\tilde{\pi}_{t+k}, \tilde{y}_t$ and $(\tilde{\pi}_{t+k} \tilde{y}_t)$, respectively, on the instruments.

$^a$ $\pi^*$ in this case corresponds to a constant 2% German inflation target.

Next, as discussed in Section 2, an estimate of $\phi$ can be directly obtained from the estimates of $c_1$ and $c_4$. Since for the three European countries the correct interpretation of the coefficient on expected inflation seems to be $c_1/(1 - c_3)$, denoted as $\tilde{c}_1$, we take that value to compute the ratio given by $c_4/2(\tilde{c}_1 - 1)$ using the estimated coefficients in panel B for the three European countries and those in panel A for the US and the Euro area. That yields the following estimates of $\phi$: 0.34 (Germany), 0.23 (France), 0.50 (Spain), 0.17 (US) and 0.33 (Euro zone). Note that these values are fairly similar to those reported in Table 1. Indeed, using the delta method to compute 95% confidence intervals of $\phi$, we cannot reject that the differences between both sets of coefficients are statistically insignificant. Further, the only country for which the t-ratio of $\hat{\phi}$ is not significant is the US.

Finally, in order to ascertain the forecasting advantages of using the nonlinear Taylor rules estimated above to track the evolution of the short-term interest rates in the various cases under study, we computed the dynamically simulated fitted values of the linear and nonlinear models. The root-mean-squared error (RMSE) of the linear models are 0.83 (Germany), 0.71 (France), 1.92 (Spain), 1.38 (Euro area) and 0.76 (US) whilst the corresponding RMSE for the nonlinear rules are 0.70, 0.55, 1.48, 1.21 and 0.73, respectively. To test for whether these RMSEs are statistically significant, we have implemented the test statistic for predictive accuracy proposed by Diebold and Mariano (1995) which relies upon testing the null hypothesis $H_0$: $E(d_t) = 0$ where $d_t$ is the difference of the squared dynamically simulated residuals of two alternative models. As shown by those authors, $T^{1/2} \tilde{d}/\sigma$ is asymptotically distributed as $N(0,1)$, where $\tilde{d}$ is the sample average of $d_t$ and $\sigma$ is a nonparametric estimate of the long-run variance of $d_t$. The $p$-values of the corresponding test are: 0.0001 (Germany), 0.0002 (France), 0.0018 (Spain), 0.421 (US) and 0.0012 (Euro area). Thus, it appears that there are substantial advantages in using the nonlinear specification to predict the evolution of the short-term interest rates in all cases except in the US, in agreement with the evidence that a linear Phillips curve cannot be rejected for the latter country. As an illustration of this improvement, Fig. 2 depicts, together with the interest rate, the within-sample predictions in the Euro zone obtained with from a conventional linear policy rule (dotted line), stemming from the estimation of (8) without the interaction term, and from the modified rule (thin solid line), allowing for it.
3.4. Ordered probit model

As discussed above, the specific latent index of adjustments in (9) contains expected inflation (with \( k = 12 \)), the output gap and the interaction between both variables plus an additional set of controls, \( s_i \). Among those variables, we have considered both the change in the discount rate in the previous month, \( \Delta i_{t-1} \), and the number of months elapsed since the last intervention, \( D_t \), to capture persistence in interventions. Likewise, with the same motivation as the \( x_t \) variables in (6), changes in the FF/DM real exchange rate (\( \Delta rer_t \)) for France, and lagged changes of a foreign interest rate (\( \Delta i^{*}_{t-1} \)) have also been included.\(^{16}\) Since the regressors in the probit model are assumed to be uncorrelated with the error term, the procedure of replacing expectations of future variables by their realized value becomes invalid in this case. Thus, rather than using the previous approach, our strategy is based on constructing inflation forecasts from OLS regressions where the regressors are the instrumental variables used in the GMM approach.

Table 3 shows the results of the above exercise. Most relevant from our viewpoint is the finding that the coefficient on the interaction term, \( \beta_3 \), is estimated to be positive and significant for the three European countries and insignificant for the US, in broad agreement with the result obtained earlier. Thus, the results seem to be fairly robust to the use of this alternative methodology. Of independent interest are the findings that there are “duration” effects, in the sense that the probability of an adjustment depends positively on the time elapsed since the last intervention, and that the probability increases when the real exchange rate depreciates or when foreign discount rates rise.

\(^{16}\) The German interest rate is used for France and Spain, and the US interest rate for Germany. Other variables turned out to be not significant.
Table 3
Estimated ordered probit model

<table>
<thead>
<tr>
<th>Variables</th>
<th>Germany</th>
<th>France</th>
<th>Spain</th>
<th>USA</th>
</tr>
</thead>
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<tr>
<td>$\tilde{\pi}_{t+12}/\tilde{\pi}_t$</td>
<td>0.86</td>
<td>0.12</td>
<td>0.27</td>
<td>0.30</td>
</tr>
<tr>
<td></td>
<td>(0.49)</td>
<td>(0.04)</td>
<td>(0.09)</td>
<td>(0.12)</td>
</tr>
<tr>
<td>$\tilde{y}_t$</td>
<td>0.29</td>
<td>0.02</td>
<td>0.09</td>
<td>0.31</td>
</tr>
<tr>
<td></td>
<td>(0.21)</td>
<td>(0.05)</td>
<td>(0.05)</td>
<td>(0.08)</td>
</tr>
<tr>
<td>$\tilde{\pi}_{t+12}/\tilde{\pi}_t * \tilde{y}_t$</td>
<td>0.26</td>
<td>0.14</td>
<td>0.16</td>
<td>0.21</td>
</tr>
<tr>
<td></td>
<td>(0.12)</td>
<td>(0.07)</td>
<td>(0.06)</td>
<td>(0.29)</td>
</tr>
<tr>
<td>$\Delta \text{rer}_t$</td>
<td>—</td>
<td>0.70</td>
<td>—</td>
<td>—</td>
</tr>
<tr>
<td></td>
<td>—</td>
<td>(0.17)</td>
<td>—</td>
<td>—</td>
</tr>
<tr>
<td>$\Delta \text{i}_{t-1}$</td>
<td>—</td>
<td>1.94</td>
<td>0.93</td>
<td>—</td>
</tr>
<tr>
<td></td>
<td>—</td>
<td>(0.53)</td>
<td>(0.57)</td>
<td>—</td>
</tr>
<tr>
<td>$\Delta \text{i}_{t-1}$</td>
<td>0.63</td>
<td>0.55</td>
<td>1.31</td>
<td>1.57</td>
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<tr>
<td></td>
<td>(0.24)</td>
<td>(0.26)</td>
<td>(0.48)</td>
<td>(0.32)</td>
</tr>
<tr>
<td>$D_t$</td>
<td>0.18</td>
<td>0.12</td>
<td>0.05</td>
<td>0.18</td>
</tr>
<tr>
<td></td>
<td>(0.12)</td>
<td>(0.04)</td>
<td>(0.03)</td>
<td>(0.08)</td>
</tr>
<tr>
<td>$x_1$</td>
<td>-1.06</td>
<td>-0.92</td>
<td>-1.41</td>
<td>-1.14</td>
</tr>
<tr>
<td></td>
<td>(0.11)</td>
<td>(0.15)</td>
<td>(0.20)</td>
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<tr>
<td>$x_2$</td>
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<td>(0.13)</td>
<td>(0.16)</td>
<td>(0.02)</td>
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<tr>
<td>$x_3$</td>
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<td>0.21</td>
<td>0.97</td>
<td>0.32</td>
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<td>(0.19)</td>
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<td>$x_4$</td>
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<td>1.53</td>
<td>1.48</td>
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<td>(0.17)</td>
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<td>(0.16)</td>
</tr>
<tr>
<td>Log-likelihood</td>
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<td>-135.89</td>
<td>-127.41</td>
<td>-251.30</td>
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</table>

4. Concluding remarks

In this paper we search for asymmetries in the policy responses of five central banks to inflation and output gaps. We have argued that such responses can arise when the Phillips curve underlying the derivation of the optimal policy rule is nonlinear. To test for the existence of such asymmetric features we use two empirical approaches. The first one is based on the estimation of a Euler equation which allows for the interaction between expected inflation and the output gap while the second relies on the estimation of an ordered probit model to capture the discrete nature of changes in discount rates, allowing again for the interaction term.

We find significant evidence of nonlinearity in the policy rules of four European central banks after the 1980s, in the sense that they have tended to intervene with more virulence when inflation and output move above their target than what a linear Taylor rule would predict. However, that is not the case for the Fed, where a linear Phillips curve cannot be rejected. These contrasting results between European countries and the US can be interpreted by the fact that the convexity of the Phillips curve relies upon the existence of labour market rigidities and that those are much more severe in the former than in the latter.

In sum, the results in this paper seem to confirm the hypothesis that there are nonlinearities in the operating procedures of central banks when setting a short-term
interest rate to control monetary policy. Taking them into consideration may turn out to be helpful for financial market analysts when they forecast the evolution of interest rates on the basis of the already very popular usage of Taylor rules.

Acknowledgements

The current version has greatly benefited from suggestions made by an Editor and two anonymous referees. We also thank P. Álvarez-Lois, S. Gerlach, L. Svensson, F. Ruge-Murcia and seminar participants at CEMFI (Madrid), ECB (Frankfurt), Universidad de Salamanca and ESEM 2002 (Venice) for helpful comments. The usual disclaimer applies.

References