

Microscopic theory for radiation-induced zero-resistance states in 2D electron systems: Franck-Condon blockade

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Microscopic theory for radiation-induced zero-resistance states in 2D electron systems: Franck-Condon blockade

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We present a microscopic model on radiation-induced zero resistance states according to a novel approach: Franck-Condon physics and blockade. Zero resistance states rise up from radiation-induced magnetoresistance oscillations when the light intensity is strong enough. The theory begins with the *radiation-driven electron orbit model* that proposes an interplay of the swinging nature of the radiation-driven Landau states and the presence of charged impurity scattering. When the intensity of radiation is high enough, the driven-Landau states (vibrational states) involved in the scattering process are spatially far from each other and the corresponding electron wave functions no longer overlap. As a result, a drastic suppression of the scattering probability takes place and current and magnetoresistance exponentially drop. Finally, zero resistance states rise up. This is an application to magnetotransport in two-dimensional electron systems of the Franck-Condon blockade, based on the Franck-Condon physics which in turn stems from molecular vibrational spectroscopy. *Published by AIP Publishing.*
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Radiation-induced magnetoresistance (R_{xx}) oscillations (RIRO)^{1,2} turn up in high mobility two-dimensional electron systems (2DESs) under illumination at low temperature ($T \sim 1$ K) and low magnetic fields (B) perpendicular to the 2DES. When increasing radiation power (P), maxima and minima oscillations increase, however the latter evolve into zero resistance states (ZRS).^{1,2} Many experiments^{3–22} and theoretical explanations^{23–39} have been proposed to understand these effects but no consensus among the people devoted to this field has yet been reached. Therefore, we have to admit that, to date, RIRO and ZRS are still open issues that remain in the cutting edge of condensed matter physics regarding the radiation-matter interaction. This is especially true in the case of ZRS, perhaps the most intriguing and challenging effect in this field. Despite the fact that plenty of theories have been developed for RIRO, when it comes to ZRS only a few theoretical models have been put forward.^{23,27,40,44,45} In general, they predict negative R_{xx} , while it was not experimentally confirmed. On the other hand, the most accepted theory on ZRS is based on the formation of current and electrical field domains;⁴⁰ the key is the existence of an inhomogeneous current flowing through the sample due to the presence of a domain structure. Yet, this is a macroscopical model that overlooks any microscopic approach on ZRS.

In this letter, we develop a microscopic theory for ZRS that is based on the *radiation-driven electron orbit model*. This model, in turn, is based on the exact solution of the electronic wave function in the presence of a static magnetic field interacting with radiation and a perturbation treatment for elastic scattering due to randomly distributed charged impurities. This scattering between Landau states, LS (vibrational states), is successfully completed when there is a net overlap between the initial and final wave functions (see Fig. 1). In this model, the LS semiclassically describe orbits driven by radiation,

“driven LS,” whose center positions oscillate according to the radiation frequency. This radiation-driven oscillation alters dramatically the scattering conditions. In some cases, the LS advance during the scattering jump and, on average, the

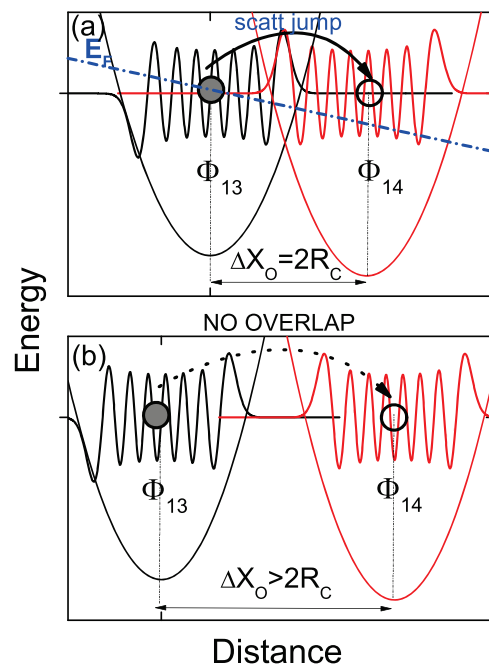


FIG. 1. Schematic diagrams of electron scattering between Landau states. In (a), there is an important overlap between Landau states. The case of Φ_{13} and Φ_{14} is shown as an example. Then, the charged impurity scattering is very likely to occur. For this to happen, it is essential that the distance between the guiding centers of the Landau states is around twice the cyclotron radius or less. In (b), we observe the opposite situation. Now the distance is bigger than twice the cyclotron radius and the overlap between the Landau states does not exist. Then, the scattering process is extremely unlikely to happen. The circles represent the guiding center of the Landau states.

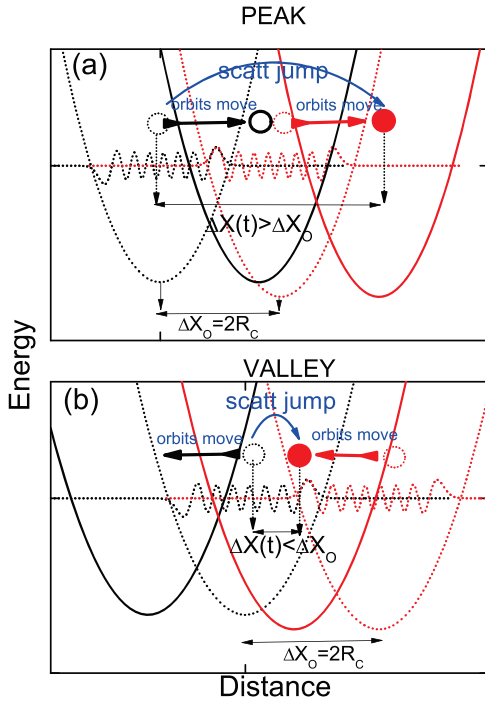


FIG. 2. Schematic diagram for scattering between Landau states in the presence of radiation. Under radiation, the Landau states are harmonically driven in a swinging motion with the radiation frequency. In (a), the Landau states move forward and on average the electrons advance further than the dark case (peaks). In (b), the Landau states move backward and on average the electrons advance less than in the dark case (valleys). For both panels, dotted parabolas represent the initial driven Landau states and the solid ones the final states after the scattering event. The circles represent the corresponding guiding center positions of the Landau states before (dotted) and after (solid) scattering.

advanced distance by electrons is bigger than in the dark giving rise to peaks in RIRO (see Fig. 2(a)). In others, the LS go backward during the jump and the net distance is smaller obtaining valleys (see Fig. 2(b)). But in all of them, there must be a net overlap of wave functions in order to have important and valuable contributions to R_{xx} .

This idea is similar to the one in Franck-Condon (FC) physics and extensively used in vibrational spectroscopy and molecular quantum mechanics.^{41,42} ZRS turn up when the radiation intensity is high enough. Then, it can happen that the final LS ends up behind the initial position of the scattering jump. Although this process corresponds to a good overlap between LS, the average advanced distance is equal to zero and does not contribute to R_{xx} . Then, we can consider other final LS much distant with respect to the scattering initial position. These LS could still end up ahead of that position during the scattering-driven process even at very high light intensities. Nevertheless, these LS do not significantly overlap and the corresponding contribution to R_{xx} exponentially drops (see Fig. 3). As a result, scattering rate, current, and R_{xx} are dramatically suppressed, electrons remain in their initial LS, and ZRS rise up. This effect is known as Franck-Condon blockade⁴³ and it is at the heart of the physical origin of ZRS.

In the *radiation-driven electron orbits model*, the electron time-dependent Schrödinger equation with a time-dependent force and magnetic field is exactly solved to study the magnetoresistance of a 2DES subjected to radiation at low B and temperature, T .^{23,46,47} Accordingly, the exact

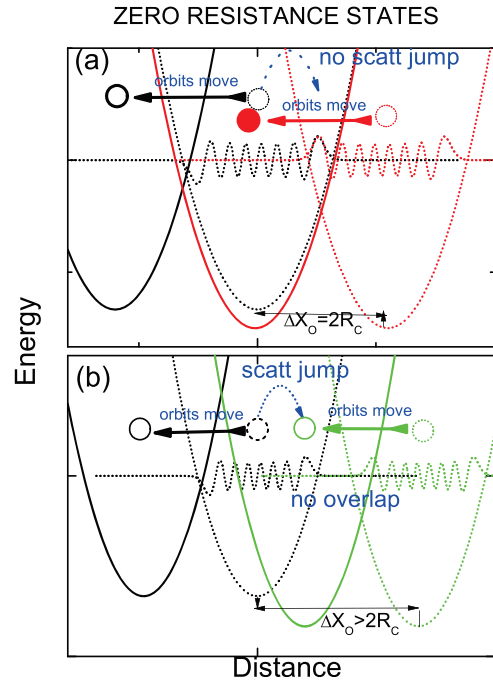


FIG. 3. Schematic diagram explaining the physical origin of ZRS. In (a), when the intensity of radiation is high enough and the Landau states move backwards it may happen that the final Landau state, initially at a distance of twice the cyclotron radius ($2R_c$), ends up behind the scattering initial position. Now and although the overlap is important, the average advanced distance is equal to zero. In (b), scattering processes to Landau states at more than $2R_c$. These Landau states end up still ahead of the initial jump position and there is a positive advanced distance. However, the overlap between these involved Landau states is negligible and the final magnetoresistance exponentially drops and ZRS show up.

expression of the obtained electronic wave function reads $\Psi_n(x, t) \propto \phi_n(x - X_0 - x_{cl}(t), t)$, where ϕ_n is the solution for the Schrödinger equation of the unforced quantum harmonic oscillator. Thus, the obtained wave function (Landau state or Landau orbit) is the same as the one of the standard quantum harmonic oscillators where the guiding center, X_0 , without radiation, is displaced by $x_{cl}(t)$. $x_{cl}(t)$ is the classical solution of a negatively charged, forced and damped, harmonic oscillator

$$\begin{aligned} x_{cl}(t) &= \frac{-eE_0}{m^* \sqrt{(w_c^2 - w^2)^2 + \gamma^4}} \cos(wt - \beta) \\ &= -A \cos(wt - \beta), \end{aligned} \quad (1)$$

where E_0 is the intensity of radiation, w is the radiation frequency, and w_c is the cyclotron frequency. γ is a phenomenologically introduced damping factor for the electronic interaction with acoustic phonons. β is the phase difference between the radiation-driven guiding center and the driving radiation itself and it is given by $\tan \beta = \frac{\gamma^2}{w_c^2 - w^2}$. Thus, the guiding center lags behind radiation a phase constant of β . When the damping parameter γ is important ($\gamma > w \Rightarrow \gamma^2 \gg w^2$), then $\tan \beta \rightarrow \infty$ and $\beta \rightarrow \frac{\pi}{2}$. Now, the time-dependent guiding center is $X = X_0 + x_{cl} = X_0 - A \sin wt$. This physically implies that the orbit guiding centers oscillate harmonically at the radiation frequency w , but radiation leads the guiding center displacement in $\frac{\pi}{2}$.

The longitudinal conductivity σ_{xx} in the 2DES is obtained applying the Boltzmann transport theory. With this theory and within the relaxation time approximation, σ_{xx} is given by the following equation:^{48–54}

$$\sigma_{xx} = e^2 \int_0^\infty dE \rho_i(E) (\Delta X)^2 W_I \left(-\frac{df(E)}{dE} \right), \quad (2)$$

with E being the energy and $\rho_i(E)$ the density of initial Landau states. W_I is the remote charged impurity scattering rate, given, according to the Fermi's Golden Rule, by $W_I = N_I \frac{2\pi}{\hbar} |\langle \phi_m | V_s | \phi_n \rangle|^2 \delta(E_m - E_n)$, where N_I is the impurity density and E_n and E_m are the energies of the initial and final LS, respectively. V_s is the scattering potential for charged impurities,⁵³ ΔX is the average distance advanced by the electron between orbits in every scattering jump in the x direction and is given by,⁵⁵ $\Delta X = \Delta X^0 - A \sin(2\pi \frac{w}{w_c})$. ΔX^0 is the distance between the guiding centers of the LS involved in the scattering event. Since all LS oscillate in phase, this distance remains constant during the driving motion and is the same with or without radiation.

After some algebra, we get to an expression for σ_{xx} ^{56,57}

$$\sigma_{xx} = \frac{e^2 m^*}{\pi \hbar^2} \left[\Delta X^0 - A \sin\left(2\pi \frac{w}{w_c}\right) \right]^2 \times W_I \left[1 + \frac{2X_s}{\sinh(X_s)} e^{-\frac{\pi\Gamma}{\hbar w_c}} \cos\left(\frac{2\pi E_F}{\hbar w_c}\right) \right], \quad (3)$$

where $X_s = \frac{2\pi^2 k_B T}{\hbar w_c}$, k_B is the Boltzmann constant, Γ is the Landau level width, and E_F is the Fermi energy. To find the expression of R_{xx} , we use the well-known tensorial relation $R_{xx} = \frac{\sigma_{xx}}{\sigma_{xx}^2 + \sigma_{yy}^2} \simeq \frac{\sigma_{xx}}{\sigma_{xy}^2}$, where $\sigma_{xy} \simeq \frac{n_e e}{B}$, n_e is the electron density, and $\sigma_{xx} \ll \sigma_{xy}$.

To apply the Franck-Condon physics to the problem of ZRS, we need to properly develop the matrix element inside the scattering rate W_I . This matrix element can be expressed as^{52–54}

$$|\langle \phi_m | V_s | \phi_n \rangle|^2 = \sum_q |V_q|^2 |I_{nm}|^2 \delta_{k'_y, k_y + q_y}, \quad (4)$$

where $V_q = \frac{e^2}{\epsilon(q+q_s)}$, ϵ is the dielectric constant, and q_s is the Thomas-Fermi screening constant.⁵³ The integral I_{nm} is given by

$$I_{nm} = \int_{-\infty}^{\infty} e^{iq_x x} \phi_m(x - X') \phi_n(x - X) dx, \quad (5)$$

where $X = X_0 - A \sin \omega t$ and $X' = X'_0 - A \sin \omega t$ are the guiding centers of ϕ_n and ϕ_m , respectively. Expanding the exponential in the integral in powers of $q_x x$: $e^{iq_x x} = 1 + iq_x x - \frac{1}{2} q_x^2 x^2 - \dots$. On the one hand and using a screened Coulomb potential, x is of the order of the Thomas-Fermi screening length $1/q_s$, $x \sim 1/q_s \simeq 5 \times 10^{-9}$ m for GaAs.⁵⁸ On the other hand, $q_x \sim q = 2k_F \sin \frac{\theta}{2}$ (Ref. 58) where θ is the scattering angle and k_F is the Fermi wave vector. For high mobility samples, the scattering is mainly described by long range, small angle (charged impurity) scattering. Then,

we assume that for the samples used in experiments this angle is small or very small.⁵⁹ We have taken an average scattering angle of $\theta \leq 10^\circ$ and for the Fermi wave vector $2k_F \simeq (3-1) \times 10^8 \text{ m}^{-1}$ for a 2DES with the experimental electron density.¹ This gives for $q_x \sim 10^6 - 10^7 \text{ m}^{-1}$ and then $q_x x \sim 10^{-3} - 10^{-2} \ll 1$. We therefore make a good approximation retaining only the first term in the above expansion: $e^{iq_x x} \rightarrow 1$. The final outcome is that the integral I_{nm} becomes an overlap integral of the LS involved in the scattering process: $I_{nm} = \int_{-\infty}^{\infty} \phi_m(x - X') \phi_n(x - X) dx$. This result implies that an important overlap between the initial and final LS will give, through the term $|I_{nm}|^2$, an intense scattering and in turn an intense R_{xx} . This principle is known in Franck-Condon physics and extensively used in molecular vibrational spectroscopy.^{41,42} We translate it now into magneto-transport in the 2DES and calculate the square of the vibrational overlap integral, $|I_{nm}|^2$, the *Franck-Condon factor*. The expression for the Franck-Condon factor (FC) reads⁶⁰

$$|I_{nm}|^2 = \frac{n!}{m!} \left[\frac{\Delta X_0^2}{2R^2} \right]^{m-n} e^{-\frac{\Delta X_0^2}{2R^2}} \left[L_n^{m-n} \left(\frac{\Delta X_0^2}{2R^2} \right) \right]^2, \quad (6)$$

where $m \geq n$, $R^2 = \frac{\hbar}{eB}$ is the square of the magnetic length and L_n^{m-n} is the associate Laguerre polynomials.

In Fig. 4, we show the calculated FC factor versus ΔX_0 in units of cyclotron radius (R_c) for three different B : $B = 0.1, 0.2$ and 0.3 T. For each case, we also present the Landau level index for the Fermi energy and the scattering process considered in the simulation. We observe, as expected, that the FC factor presents important values only when $\Delta X_0 \leq 2R_c$ (important overlap between LS) and exponentially drops when $\Delta X_0 > 2R_c$ (negligible overlap). As in vibrational transitions in infrared molecular spectroscopy

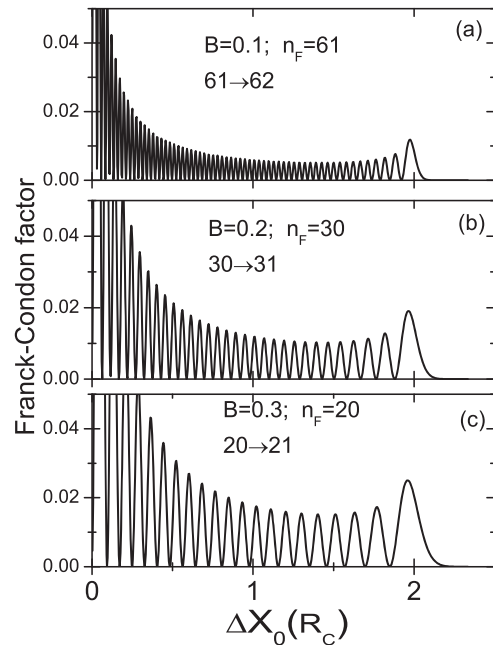


FIG. 4. Franck-Condon factor vs ΔX_0 in units of cyclotron radius R_c for three different B : $B = 0.1, 0.2$, and 0.3 T. In each panel, we present also the Landau level index for the Fermi energy and the Landau levels indexes for the scattering process considered in the simulation.

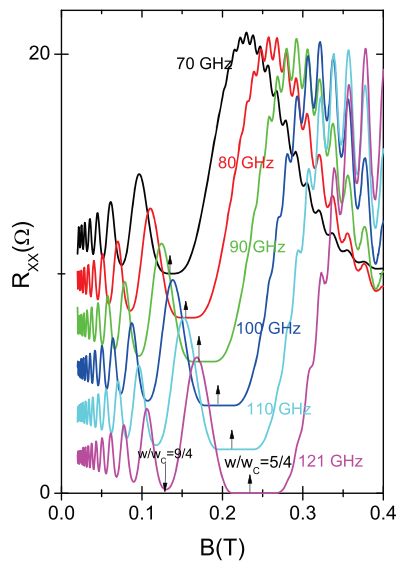


FIG. 5. Calculated R_{xx} vs B under illumination for different radiation frequencies to study the dependence of radiation-induced oscillations on the frequency.

with the spectroscopic lines, here the FC factor defines the intensity of the scattering. Thus, when the LS involved in the scattering event are at a distance of $2R_c$ or less than the FC factors (see Fig. 4), W_I gives important and non-negligible values. Now, with a not very intense radiation, the final driven LS always ends up ahead of the LS initial position of scattering giving rise to bigger or smaller ΔX : peaks or valleys, respectively, in R_{xx} . This is described in Fig. 2. We can get to a totally different scenario if we further increase P reaching a situation where the final LS ends up behind the initial scattering jump position and then, although with an important value for the FC factor, the average advance distance is zero. Nevertheless, we can consider further away LS at more distance than $2R_c$ so that they end up, even at high P , ahead of the initial scattering position giving a net advanced distance. Yet, there is no overlap now and the FC factor turns out to be negligible. This physical scenario corresponds to the rise up of ZRS. This situation is described in Fig. 3. Figure 5 shows calculated R_{xx} vs B for different radiation frequencies. ZRS positions move according to the change of radiation frequency ω , keeping the ratio, $\frac{\omega}{\omega_c} = j + \frac{1}{4}$.³⁷ Simulated ZRS are very clearly obtained for $j = 1$.

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