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Mechanically amplified milli-Newton thrust balance for direct thrust measurements of electric thrusters for space propulsion

Mick Wijnen MSc, dr. Jaume Navarro-Cavallé, dr. Pablo Fajardo

Abstract

Direct thrust measurements by means of a thrust balance are the golden standard for measuring thrust and concurrently the specific impulse in electric thrusters. To measure these properties in the novel class of electrode-less plasma thrusters a new thrust balance based on the VAHPER (Variable Amplitude Hanging Pendulum with Extended Range) concept has been developed.

The thrust balance has a mechanical amplification mechanism with an angular magnification of $31^\circ/\mu\text{N}$. Using Lagrangian mechanics we show the thrust balance loaded with a 5.2 kg thruster prototype to behave like a damped harmonic oscillator with a natural frequency of 0.37 Hz. A variable damping system provides damping with an optimal damping ratio of 0.78 which corresponds to a settling time of only 1.8 s. Both the model as well as the damping and calibration system have been validated.

To accommodate the particularities of medium power electrode-less plasma thrusters the thrust balance design includes the following features: an optical displacement sensor, water cooling, liquid metal connectors, dedicated vacuum-rated electronics for auto-leveling, remote (in-vacuum) calibration and temperature monitoring.

To test the thrust balance, measurements were performed on a 500W Helicon Plasma Thruster breadboard model. When loaded with this thruster the measured stiffness of the system was $12.67 \pm 0.01 \text{ mN/mm}$. For this stiffness the thrust range is 150 mN with a 0.1 mN resolution. The relative uncertainty on the thrust measurements is found to be on the order of 2%.

Index Terms

direct thrust measurement, thrust balance, force measurement, calibration, space propulsion, electric propulsion, helicon plasma thrusters

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I. INTRODUCTION

Space propulsion is rapidly electrifying, as well established technologies such as Hall-effect thrusters (HET) and gridded ion thrusters (GIT) are becoming the propulsive subsystem of choice for many new spacecraft, for both scientific [1], [2], [3] as well as for commercial [4], [5] applications.

Maneuvers in space are defined by the required velocity-increment of the spacecraft, or $\Delta v$. This velocity increment is generated by firing the on-board thruster, expelling propellant. The propellant mass required to generate a given $\Delta v$, relative to the spacecraft mass, is governed by Tsiolkovsky’s rocket equation:

$$\frac{m_p}{m_{sc}} = \exp \left( \frac{\Delta v}{v_{ex}} \right) - 1$$  \hspace{1cm} (1)

Here $v_{ex}$ is the exhaust velocity of the propellant expelled from the thruster, $m_p$ is the propellant mass and $m_{sc}$ the spacecraft dry mass (i.e. without propellant). A high $v_{ex}$ reduces the propellant mass required for a given $\Delta v$. A lower propellant mass translates to lower launch costs, as they scale with the payload mass. The advantage of electric propulsion then lies in its ability to produce very high exhaust velocities.

Conventional rocket propulsion uses the chemical energy released by a combustion reaction of the propellant to thermally expand the generated hot gas in a convergent-divergent, or 'De Laval’ nozzle and thereby accelerating it. The chemical energy available for conversion into kinetic energy is ultimately limited by the reaction enthalpy of the combustion reaction. There exists therefore a theoretical maximum of the exhaust velocity in chemical rockets, which lies around 5 km/s.

Electric propulsion (EP), more specifically electro-static and electro-magnetic thrusters, instead use the electric power on-board a spacecraft to accelerate the propellant, typically an ionized gas, to very high exhaust velocities by means of electro-magnetic forces [6], [7]. In this case there is no theoretical limit, readily producing exhaust velocities in excess of 15 km/s. The disadvantage is the reduced thrust-to-power ratio: $T/P = 2\eta/v_{ex}$, where $\eta$ is the overall efficiency of the engine. EP engines therefore excel once in orbit, where low thrust on the order of milli-Newton is sufficient for most maneuvers.

EP is an active field of research looking to improve mature technologies such as the HET and GIT as well as to develop new technologies such as electrode-less thrusters, pulsed plasma thrusters (PPT), electrosprays and field-enhanced electric propulsion (FEEP) [8], [9], [6]. Experimental work in EP is often involved with the characterization of thruster performance.

As may be clear from the previous paragraphs thrust is one of the most critical performance markers of EP devices, along with the specific Impulse ($I_{sp}$) and the thruster efficiency. Specific impulse is defined as the ratio of the thrust to the mass-flow-rate, normalized with $g_0 = 9.8065 m/s^2$ the standard acceleration...
due to gravity. It is a measure of the effective exhaust velocity. Direct thrust measurements by means of a thrust balance (TB) are the golden standard for measuring these quantities.

Direct thrust measurements of plasma thrusters have been commonplace in the EP community. While thrust measurements for chemical rocket motors are quite straightforward and based on load cells, thrust measurements of EP devices are more elaborate. EP devices also usually only operate in a high vacuum environment ($p \sim 10^{-5}$ mbar) confining experiments to a vacuum chamber. Due to their inherent low thrust-to-weight ratio they require both a rigid structure to support their weight as well as a sensitive, i.e. compliant structure that is able to transduce the low thrust values to a measurable signal. The solution of these opposite requirements generally comes in the form of a pendulum, either simple [10], [11], inverted [12], [13] or torsional [14], [15], [16] that converts thrust into displacement which is then measured. Each of these configurations has its advantages and disadvantages, which are summarized in Tab. I.

The simple pendulum is the most simple solution. It is naturally stable under external perturbations but also has the lowest sensitivity. Furthermore, its sensitivity is dependent on the pendulum length which is limited by the dimensions of the vacuum chamber that has to contain it. The inverted pendulum is the most common type of TB [17]. It is less stable than the simple pendulum, but has better sensitivity which is governed by the stiffness of the supporting flexures. The sensitivity is therefore much less dependent on the thruster mass. However, the stiffness of the flexures are prone to changes for instance, thermal drifts either due to changes in ambient temperature or due to thruster heat loads. Thermal management in a vacuum environment is challenging due to the lack of convection and the heat dissipated from the thruster can easily affect the thrust balance, including the flexures.

The inverted pendulum is also more compact than a simple pendulum. This has the advantage that the performance is less limited by the vacuum chamber dimensions. The disadvantage is that it also places the thruster, which is a source of heat and EMI, close to any sensitive components. This is of particular concern for a RF thrusters like the Helicon Plasma Thruster (HPT), where heat dissipation and EMI are significant.

The torsional pendulum has superior sensitivity. Since the plane of motion is perpendicular to the gravitational force the sensitivity is independent of the thruster mass. However the horizontal, often asymmetric, arrangement is not always easy to manage inside the limited space of a vacuum chamber. It would also place the thruster closer to the side walls of the chamber, resulting in plume-wall interaction that degrades thruster performance. These limitations are of particular concern for larger thrusters; torsional balances have therefore been most successfully applied for testing micro-propulsion devices, those providing continuous thrust as well as pulsed thrusters.

Polzin et al. [18] proposed a modification of the hanging pendulum that was dubbed the Variable...
### TABLE I
ADVANTAGES AND DISADVANTAGES OF THRUST BALANCE TYPES.

<table>
<thead>
<tr>
<th>Design</th>
<th>Advantage</th>
<th>Disadvantage</th>
</tr>
</thead>
<tbody>
<tr>
<td>Simple Pendulum</td>
<td>Stable under external</td>
<td>Lowest sensitivity, depends on</td>
</tr>
<tr>
<td></td>
<td>perturbations</td>
<td>pendulum length</td>
</tr>
<tr>
<td>Inverted Pendulum</td>
<td>High sensitivity</td>
<td>Sensitivity depends on flexures, prone</td>
</tr>
<tr>
<td></td>
<td>Compact</td>
<td>to changes</td>
</tr>
<tr>
<td>Torsional Pendulum</td>
<td>Superior sensitivity,</td>
<td>Asymmetric arrangement</td>
</tr>
<tr>
<td></td>
<td>independent of mass</td>
<td></td>
</tr>
</tbody>
</table>

Amplitude Hanging Pendulum with Extended Range (VAHPER). It includes a secondary, horizontal arm that is mechanically coupled to the main pendulum arm and results in a mechanical amplification of the displacement. The displacement measurement is performed on the secondary arm which allows for placing the sensor far away from the thruster without losing sensitivity. In their design, it was furthermore possible to change the location of the pivot of the secondary arm and thereby vary the amplification factor.

In this work we present a new thrust balance developed by the Space Propulsion and Plasmas Team (EP2) of the Carlos III university of Madrid (UC3M), based on the design of Polzin, to perform direct thrust measurements on an experimental helicon thruster platform.

Helicon plasma thrusters (HPT) \[19\], \[20\], \[21\] are electrode-less thrusters that use helicon waves to produce high density plasmas. These waves are excited at frequencies in the Megahertz range, typically 13.56 MHz in the presence of an applied magnetic field \[22\]. HPT rely furthermore on magnetic nozzles for plasma acceleration by converting the thermal electron energy into ion kinetic energy. Thrust is generated by the magnet repulsion between diamagnetic plasma currents and the magnetic circuit of the thruster \[8\], \[6\].

EP2 has been studying these devices for the past ten years, both theoretically \[23\], \[24\], \[25\], \[26\], \[27\] and experimentally \[28\], \[29\]. The current experimental platform, the HPTx, developed jointly by EP2 and SENER Aeroespacial is a flexible platform for investigating the effect of different design and operational parameters on the performance as well as the underlying physics. It can operate with either solenoids or permanent magnets, uses different propellants such as Argon and Xenon and can operate with up to 1 kW of RF power.

To date, all experimental characterization of the HTPx has been performed mainly with electrostatic...
probes. Current performance estimates are about 5-8 mN of thrust, 750-1200s of specific impulse and 7-10% of efficiency \cite{28}, \cite{29}. However, electrostatic probes only provide indirect measurements of thrust and specific impulse. The need for reliable thrust measurements has driven the development of a new TB, designed to primarily accommodate the HPTx prototype but also applicable to other thruster types.

The HPTx weighs about 5 kg thus has an estimated thrust-to-weight ratio of only 2 mN/kg, produces strong RF fields, requires electrical currents of up to 30 A, and presents thermal heat loads of up 1.5 kW. This imposes a set of stringent requirements on the design of the thrust balance. In this article we will present both a theoretical analysis of the TB design and its technical implementation as well as validation & calibration results and some preliminary thrust measurements.

The rest of the article is structured as follows. In section II a novel analysis of the dynamic response of this type of balance, using the Lagrange formalism, is introduced as well as a model for the damping system. Section III details the technical implementation of all the features of the thrust balance. Here we also introduce briefly the vacuum facility and the thruster. In section V we present results on the validation of the dynamical model, the damping system and the calibration method as well as actual thrust measurements obtained with the HPTx. The article ends with a discussion on the obtained results and the conclusions.

II. MODELLING

We start with defining our system. As can be seen in Fig. I the reference frame is centered at O with the $x$-axis rightwards, the $y$-axis upwards and the $z$-axis out of the plane. The mechanism consists of three articulated rigid elements numbered 1, 2, 3. The first element is an upside-down L-shaped component suspended at its corner from the fixed frame at $O$, by means of a flexure bearing with torsional spring constant $\kappa_1$. The second element, a straight beam, is anchored to the fixed frame at point $A$, also by a flexure bearing with constant $\kappa_2$. The joints in point $O$ and $A$ have one degree of freedom (DoF): rotation about $z$. The primary and secondary elements are interconnected with a third element, another straight beam, joined at $B$ and $C$ by flexure bearings with $\kappa_3$, $\kappa_4$, respectively. Points $B$ and $C$ have 3 DoF each, rotation around $z$ and translation in the $x-y$ plane. The thruster, with mass $M$, is suspended from the bottom of element 1, while a counterweight with mass $m$ is attached to the right end of element 2.

The positions of the centers of mass (CoM) of all elements $i$ are noted $r_i$, where we numbered the thruster and the counterweight 4 and 5, respectively. The point of element 2 where the displacement is measured is labeled $E$. Point $D$ is the point where the damping force acts. Whereas point $O$ and $A$ are fixed to the reference frame, points $B$, $C$, $D$ and $E$ can move; when \textit{not} in their initial position (i.e. when the vertical part of element 1 is parallel to the local gravity vector) they are denoted with a prime i.e.
The quantity of interest in conventional pendulum balances is the horizontal displacement of \( r_4 \) - here denoted \( x \) - and how it is related to the applied force \( T \). In reference [18] it was shown that in this particular arrangement the vertical displacement of \( E \), denoted \( y \), is a constant multiple of \( x \). However, as the relationship between \( x, y \) is dependent on the centre of thrust of the thruster, it makes more sense to speak of the relationship between the angles \( \theta_1, \theta_2 \), as this is independent of the thruster. It can be
shown that $\theta_2 = n\theta_1$, as we will see later. For the interested reader we have reproduced the equations from the static analysis of [18] using our nomenclature; they can be found in appendix [A]. When looking at eq. (30)-(37) in appendix [A], it becomes apparent that the angles $\theta_i$ are all a function of $\theta_1 \equiv \theta$ (here we have dropped the subscript for clarity). For the following analysis it is convenient to work with the differential angles $\delta_i(\theta) = \theta_i(\theta) - \theta_i(0)$ referred to the equilibrium position $\theta = 0$. The equilibrium positions for the angles $\theta_i$ are $\theta_1 = \theta_2 = 0$ and $\theta_3 = \theta_4 = \pi/2$.

To find the dynamic response of the system we can analyze it using Lagrangian mechanics. We consider the system formed by 5 elements. To simplify the calculations somewhat, we will neglect the motion of the connecting element $i = 3$, which is a reasonable approximation, considering its size and mass compared to the other elements. When considering small angles (as we will do), element 3 ideally experiences no rotational motion, further justifying this approximation. We first define the Lagrange function of the full system:

$$\mathcal{L} = E_k - U$$  \hspace{1cm} (2)

with $E_k$ the kinetic energy and $U$ the potential energy which are defined as:

$$E_k = \frac{1}{2} \sum_i m_i \dot{r}_i^2 + \omega_i^T \bar{I}_i \omega_i$$  \hspace{1cm} (3)

$$U = \sum_i \frac{1}{2} \kappa_i \delta_i^2 + m_i g \cdot r_i$$  \hspace{1cm} (4)

here $r_i$ are the position vectors of the centers of mass of the 5 elements, and $\dot{r}_i$ their time derivatives, $m_i$ are the corresponding masses. The $\omega_i$ are the angular velocities of the rigid elements (only defined for $i = 1, 2$) and $\bar{I}_i$ is the corresponding inertia matrix. Lastly, $\kappa_i$ are the torsional spring constants of the four pivots and $g = -g \hat{y}$ is the gravitational acceleration.

Now we consider the Euler-Lagrange equation, which reads:

$$\frac{d}{dt} \left( \frac{d\mathcal{L}}{dq_j} \right) - \frac{d\mathcal{L}}{dq_j} = Q_j$$  \hspace{1cm} (5)

where $q_j$ are the generalized variables and $Q_j$ are the generalized forces:

$$Q_j = \sum_i \mathbf{F}_i \cdot \frac{\partial \mathbf{r}_i}{\partial q_j}$$  \hspace{1cm} (6)

As we will see, all quantities in eq. (3) and (4) are a function of the angle $\theta$ and its time derivative $\dot{\theta}$, only. This angle is the single generalized coordinate that determines this system. Although the other quantities are non-linear functions of $\theta$, since $\theta \ll 1^\circ$, and we are interested in motion around the point $\theta = 0$, these functions can be linearized.
TABLE II
POSITION AND LINEAR VELOCITY FOR ELEMENTS $i$.

<table>
<thead>
<tr>
<th>$i$</th>
<th>$\mathbf{r}_i$</th>
<th>$\dot{\mathbf{r}}_i$</th>
</tr>
</thead>
<tbody>
<tr>
<td>1</td>
<td>$[\cos \theta \sin \theta] \begin{bmatrix} x_{1,0} \ y_{1,0} \end{bmatrix}$</td>
<td>$[-\sin \theta - \cos \theta] \begin{bmatrix} x_{1,0} \ y_{1,0} \end{bmatrix} \dot{\theta}$</td>
</tr>
<tr>
<td>2</td>
<td>$\mathbf{r}_A - L_2(\cos n \theta \hat{x} - \sin n \theta \hat{y})$</td>
<td>$L_2(\sin n \theta \hat{x} + \cos n \theta \hat{y}) n \dot{\theta}$</td>
</tr>
<tr>
<td>3</td>
<td>-</td>
<td>-</td>
</tr>
<tr>
<td>4</td>
<td>$-L_4(\sin \theta \hat{x} + \cos \theta \hat{y})$</td>
<td>$-L_4(\cos \theta \hat{x} - \sin \theta \hat{y}) \dot{\theta}$</td>
</tr>
<tr>
<td>5</td>
<td>$\mathbf{r}_A + L_5(\cos n \theta \hat{x} - \sin n \theta \hat{y})$</td>
<td>$-L_5(\sin n \theta \hat{x} + \cos n \theta \hat{y}) n \dot{\theta}$</td>
</tr>
</tbody>
</table>

The positions $\mathbf{r}_i$ are dependent on $\theta$ and $\theta_2$ only. The linearized angle $\theta_2$ and linearized differential angles are given in eq. (7)-(10) below. Using these expressions, eq. (5) can be expressed in terms of $\theta$ only.

$$\delta_1(\theta) = \theta + O(\theta^2)$$  

$$\delta_2(\theta) = n \theta + O(\theta^2) \equiv \theta_2 \quad \text{with} \quad n = 1 + n_3 + n_4$$  

$$\delta_3(\theta) = \theta_3'(0) \theta + O(\theta^2) \equiv n_3 \theta$$  

$$\delta_4(\theta) = \theta_4'(0) \theta + O(\theta^2) \equiv n_4 \theta$$  

Note that the prime denotes $d/d\theta$. The derivation of the various terms in the linearized equations can be found in appendix B.

Going back to eq. (5) we can substitute $q_j = \theta$. For now we consider a system without forcing or damping, so that $Q_j = 0$. This results in the following equation.

$$\frac{d}{dt} \left( \frac{dL}{d\dot{\theta}} \right) - \frac{dL}{d\theta} = 0$$  

Solving this equation will yield the equation of motion for the system. To solve this equation all $\mathbf{r}_i, \dot{\mathbf{r}}_i$ need to be known and are, for this purpose, tabulated in Tab. II. The angular velocities are $\omega_1 = -\dot{\theta}$ and $\omega_2 = -\dot{\theta}$ and zero for all other $i$. $L_i = |\mathbf{r}_i|$ are the lengths from the center of mass of element $i$ to $A$, or to $O$ for $i = 1, 4$. There is one exception: the center of mass of the L-shaped component ($i = 1$) has both an $x$ and a $y$ component in the equilibrium position, which are denoted $x_{1,0}$ and $y_{1,0}$. Calculation of $\mathbf{r}_1$ furthermore requires a rotation matrix as can be seen in Tab. II.

We now write out the terms of the Lagrange function. The different components are: the linear kinetic energy.
\[
\sum_{i=1}^{N} \frac{1}{2} m_i r_i^2 = \frac{1}{2} (m_1 L_1^2 + m_2 n^2 L_2^2 + m_4 L_4^2 + m_5 n^2 L_5^2) \dot{\theta}^2
\]  \quad (12)

the rotational kinetic energy,

\[
\sum_{i=1}^{N} \frac{1}{2} I_i \omega_i^2 = \frac{1}{2} (I_1 + n^2 I_2) \dot{\theta}^2
\]  \quad (13)

the gravitational potential energy,

\[
\sum_{i=1}^{N} m_i \mathbf{g} \cdot \mathbf{r}_i = -g [m_1 (y_{1,0} \cos \theta - x_{1,0} \sin \theta)
\]
\[
+ m_2 (y_A + L_2 \sin n\theta)
\]
\[
- m_4 L_4 \cos \theta
\]
\[
+ m_5 (y_A - L_5 \sin n\theta)]
\]  \quad (14)

and the elastic potential energy.

\[
\sum_{i=1}^{N} \frac{1}{2} \kappa_i \delta_i^2 = \frac{1}{2} (\kappa_1 + n^2 \kappa_2 + n_3^2 \kappa_3 + n_4^2 \kappa_4) \theta^2
\]  \quad (15)

From these equations it is evident that the kinetic energy is only dependent on \(\dot{\theta}\) and the potential energy is only dependent on \(\theta\), as expected. The Euler-Lagrange equation (5) for this system can now be written as:

\[
(m_1 L_1^2 + m_2 n^2 L_2^2 + m_4 L_4^2 + m_5 n^2 L_5^2 + I_1 + n^2 I_2) \ddot{\theta}
\]
\[
+ (\kappa_1 + n^2 \kappa_2 + n_3^2 \kappa_3 + n_4^2 \kappa_4 - g m_1 y_{1,0} + g m_4 L_4) \theta
\]
\[
= g (nm_2 L_2 - m_3 x_{1,0} - nm_5 L_5)
\]  \quad (16)

Note that the trigonometric functions are linearized, i.e. \(\sin x \approx \theta\) and \(\cos x \approx 1\) (for both \(x = \theta\) and \(x = n\theta\)) in the gravitational term. Also note that \(y_{1,0} < 0\), so the gravitational term of the first element is positive. It is interesting to see that the spring force of the flexures at \(A\) and \(C\) is multiplied by a factor \(n^2\) (which is equal to \(n_4^2\) and about \(10^3\) as shown in appendix B). The above expression represents a harmonic oscillator with an effective moment of inertia \(I\), an effective torsional spring constant \(\kappa\) and a constant offset \(\tau_0\):

\[
I \ddot{\theta} + \kappa \theta = \tau_0
\]  \quad (17)

The constant term is \(\tau_0 = g m_1 x_{1,0} + g m_5 L_5 - g m_2 L_2\) which are the gravitational torques. For the balance to be in the equilibrium position (\(\theta = 0\)) it is required that \(\tau_0\) is zero. This reflects the purpose of the counterweight; as its distance to the pivot \(A\), \(L_5\), is variable, it can be used to balance out the system by choosing \(L_5\) such that \(\tau_0 = 0\).
Now we will include the external forces. As shown in Fig. 1 there are two forces applied to the system, \( T \) the thrust operating at \( r_4 \) and \( F_D = -b \dot{r}_D \) the damping force, which acts on point \( D \) and is opposite and proportional to its velocity. To include these forces we now consider them as generalized forces \( Q_j \):

\[
Q_4 = \mathbf{F} \cdot \frac{\partial r_4}{\partial \theta} = TL_4 \tag{18}
\]

\[
Q_D = \mathbf{F} \cdot \frac{\partial r_D}{\partial \theta} = -bn^2 L_D^2 \dot{\theta} \tag{19}
\]

where \( r_D = r_A - L_D (\cos n \theta \hat{x} - \sin n \theta \hat{y}) \). The new equation of motion then becomes:

\[
I \ddot{\theta} + \beta \dot{\theta} + \kappa \theta = \tau(t) + \tau_0 \tag{20}
\]

with \( \beta = bn^2 L_D^2 \) being the torsional viscous damping factor, and \( \tau = TL_4 \) the torque due to the thrust.

It is now perfectly clear that in first approximation the balance behaves as a damped harmonic oscillator.

The equation of motion for the damped harmonic oscillator can be normalized as follows.

\[
\ddot{\theta} + 2\zeta \omega_0 \dot{\theta} + \omega_0^2 \theta = \frac{\tau}{I}, \quad \zeta = \frac{\beta}{2\sqrt{\kappa I}}, \quad \omega_0 = \sqrt{\frac{\kappa}{I}} \tag{21}
\]

Here we have introduced the damping ratio \( \zeta \) and the natural angular frequency \( \omega_0 \).

The other property of interest is the stiffness \( k \) of the system: the relationship between the measured displacement \( y \) and the applied thrust \( T \), such that \( T = ky \). In a steady state the equation of motion reduces to:

\[
\kappa \theta = TL_4 \tag{22}
\]

Combining this with eq. (8) and (30) (from appendix A) which we can simplify as \( y = L_{AE} \theta_2 \) we can find an expression for \( k \).

\[
k = \frac{\kappa}{nL_4L_{AE}} \tag{23}
\]

To have a response that closely follows the step input of the applied thrust and to prevent the system from oscillating eternally, a damping force is required. Generally damping ratios of \( 0.4 < \zeta < 0.8 \) give a good step response [17]. The damping is realized by means of Eddy current damping, which is contact-less and well suited for vacuum.

The principle is as follows. When a conductor moves through a magnetic field, Eddy currents are induced inside the conductor. Due to the magnetic field, the moving charges that make up these currents experience a Lorentz force which is proportional to and in opposite direction of the velocity. The induced current density and the resulting force are:

\[
\mathbf{J} = \sigma (\mathbf{E} + \mathbf{v} \times \mathbf{B}) \quad \mathbf{F}_d = \int_V \mathbf{J} \times \mathbf{B} \, dV \tag{24}
\]
For a thin plate of thickness \( \delta \) we can approximate the problem in 2D, assuming the current density and magnetic field constant over this dimension (\( z \)). For a velocity in the y-direction \( \mathbf{v} = v_y \hat{y} \) and a homogeneous axial magnetic field in the z-direction \( \mathbf{B} = B_z \hat{z} \), with pole projection \( S \). Ignoring the electric field for now, the Lorentz force is then proportional to:

\[
F_d \propto -\sigma v S \delta B_z^2
\]  

(25)

A sketch of the fields and current density is shown in figure 2. Due to the sharp gradient in the magnetic field at the edges of the pole projection area, charge accumulation occurs here. This charge results in an electric field perpendicular to the direction of motion of the conductor, in a way similar to the Hall-effect. Inside the pole projection area this electric field is opposite the component \( \mathbf{v} \times \mathbf{B} \) and here effectively limits the induced current density. Outside the pole projection area the electric field is the sole reason for the induced current density. Knowing both \( \mathbf{B} \) and \( \mathbf{E} \) the current density \( \mathbf{J} \) and correspondingly the force can be solved for. Furthermore, to ensure that the current density normal to the boundaries of the conductor is zero the method of image currents can be used. This problem was solved analytically by Bae [30] for square magnets. For circular magnets the integrals involved are rather complicated and it’s...
more convenient to solve it with commercial FEM software. We modeled the damping system of the
thrust balance using Comsol’s magnetic and electric field (mef) module. The system was modeled using
the dimensions and properties detailed in section III-C, the results are reported in section V.

III. TECHNICAL IMPLEMENTATION

A. Structure

The thrust balance structure as shown in Fig. 1 can also be recognized in Fig. 3 (although mirrored).
Point O and A are the pivots, item 2 in Fig. 3. These pivots are anchored to a large aluminium plate (top
white plate in Fig. 3). (As opposed to pivots B and C, not labeled in this figure, which are freely moving
as was explained in section II). This plate is placed on top of another aluminium plate (grey, bottom).
The relative attitude of the top plate with respect to the bottom-plate is controlled by the leveling system
described in the next section.

The bottom plate is suspended from the roof of the vacuum chamber (not shown for clarity). It also
includes the interface for the power lines, gas lines and cooling water lines (items, 14, 15, 16 respectively).

The secondary arm and the intermediate link are custom machined aluminium parts. The primary arm
is a combination of a machined aluminium part, an extruded 30 mm x 60 mm extruded aluminium profile
and a gun-drilled cold plate (item 9). The cold-plate doubles as the thermal and the mechanical interface
for the thruster. The pivots are C-flex type G10 for the main pivot (2 bearings at O) and D10 for the
other pivots (2 bearings at A, 1 bearing each at B, C).

The system further includes a calibration rig (items 7,8), liquid metal connectors (item 13) a viscous
damping system (item 5) and an optical displacement sensor (item 6) which will be elaborated on in the
rest of this section.

B. Leveling system

To be able to align the surface normal of the upper plate with the gravity vector the system uses two
motorized screw-jacks (item 12 in Fig. 3) in conjunction with a two-axis inclinometer. The screw-jacks
are fixed to the upper aluminium plate with the lead-screw projecting down, towards the bottom plate
and are connected to the bottom plate by means of spherical joints (item 19 in Fig. 3). Both joints allow
for 3 rotational degrees of freedom (DoF) and one of the joints allows for one additional (linear) DoF.
The screw-jacks allow for varying the distance between both plates, separately at each point of contact.
Both plates are connected at a third point by a rod of a fixed distance ending in a 4 DoF (3 rotational, 1
linear) joint. All joints are implemented using custom, lubricant-less bearings. This arrangement allows
for controlling the pitch and roll of the top plate with respect to the bottom plate. The pitch is changed
by co-rotating both screw-jacks, i.e. by generating a common change in the distance between the two 
plates at each contact point, while roll is changed by counter-rotating, i.e. by generating a differential 
change. Control of the screw-jacks is informed by the inclinometer that measures the angle with respect 
to the vertical.

C. Damping System

The implementation of the damping system consists of a conductive plate attached at the end of the 
secondary arm. Here the displacement (and hence the velocity) is the largest. Furthermore, including the 
damping force on the secondary arm takes advantage of the mechanical amplification. As can be seen 
from eq. 19, the damping scales with $n^2 \sim 10^3$. In this way, up to critical damping can be achieved for 
any thruster, with only a modest damping system.

Fig. 3. CAD drawing of the thrust balance design. 1) Primary arm. 2) Pivots $O$ and $A$. 3) Secondary arm. 4) Counterweight. 5) Eddy current damper. 6) Displacement sensor. 7) Voice coil. 8) Load cell. 9) Water cooled mounting plate. 10) Water cooled electronics plate. 11) Propellant line mounting point. 12) Motorized screw jack for auto-leveling. 13) Liquid metal connectors. 14) Power supply user interface (UI). 15) Propellant line UI. 16) Cooling water UI. 17) Power supply thruster interface. 18) Propellant supply thruster interface. 19) Pivot points for auto-leveling system.
The magnetic field is applied by two aligned permanent magnets on each side of the plate. The damping force scales with the conductivity of the metal. Of all affordable metals, copper has the highest conductivity of any metal, but it also has a high mass density. The added mass, with a considerable arm \( L_{AE} \approx 0.5 \) m, leads to an undesirable increase in the moment of inertia of the second arm. Pure aluminium has 60% the conductivity of copper but is only 30% as dense, making it the preferred option.

The reported conductivity of the alloy that was used, Al5745, is \( 2.04 \cdot 10^7 \) S/m. However, when we measured the conductivity it was found to be closer to \( 1.59 \cdot 10^7 \) S/m.

The conductive plate is made of aluminium and has the following dimensions: 30 mm x 60 mm x 2 mm. The two cylindrical magnets are 20 mm diameter, 5 mm thick, NdFeB grade N42 (maximum energy product 42 MGOe) magnets with a remanence of 1.3T and are spaced a distance \( s \) apart. To be able to vary the magnetic field strength, and thereby the damping, \( s \) is variable by means of small lead-screws.

D. Displacement Sensing

The quantity that must be measured is the displacement of point \( E \) (in Fig. 1). There are many different displacement sensors available: inductive, capacitive, interferometry, triangulation etc. For this particular balance we wanted a sensor that was impervious to EMI as the strong RF fields of a Helicon thruster are known to wreak havoc on (sensitive) electrical systems. The current design is outfitted with a confocal chromatic sensor CCS-Prima from STIL.

Confocal chromatic sensors consist of a white light source, a system of lenses (the ‘optical pen’) and a spectrometer. The white light is refracted by the optical pen; the focal distance is wavelength dependent. An object will reflect the wavelength with a focal distance corresponding to the position of the reflecting surface. By measuring the wavelength of the reflected light with a spectrometer, and given the relationship between the focal distance and wavelength, the distance of the object to the optical pen can be inferred.

The optical pen is mounted on the top plate, pointing upwards, about 25 mm distance from the horizontal arm. This distance can be controlled by a vacuum-rated manual linear translation stage. The optical pen is connected to the light source and spectrometer by an optical fiber, passing the vacuum/ambient barrier by means of an optical feed-through. Both the emitted and reflected light are coupled through the same fiber. The in-vacuum path only consist of passive elements (fiber, optical pen) while the light source, spectrometer and other electronics reside outside of vacuum chamber far away from the RF fields of the thruster. The range and resolution can be changed by changing the optical pen. For this particular application we are using the CL5-MG20 optical pen which has a measurement range...
of 12 mm and a resolution of 0.4 μm. It is unaffected by a tilt of the reflecting surface of up to 14° far exceeding the maximum deflection angle θ₂ < 3°.

E. Calibration System

Since the stiffness of the thrust balance is dependent upon the mass of the thruster, additional stiffness from gas and cooling lines as well as thermal drifts, it is important that the thrust balance can be calibrated repeatedly, preferably in-situ without breaking the vacuum. Calibration is done by applying a known force while measuring the displacement, for a range of loads spanning the desired measurement range.

We have chosen a calibration system based on a voice coil [32]. Voice coils consist of a small solenoid and a permanent magnet. The interaction of the magnetic fields of the solenoid and the magnet results in a repulsive force, linearly proportional to the current applied to the solenoid. The force per current ratio is constant only over a short part of the stroke length, and dependent on the relative position of the solenoid and the magnet. The actual force per current ratio could very well differ from the one specified by the manufacturer. Calculating the applied force from the applied current using the specified force per current ratio can therefore introduce an error. The voicecoil used is a Moticont LVCM-010-013-10 with a reported force per current of 0.29 N/A.

In the current design, the magnet of the voice coil is instead mounted on a calibrated load cell. This allows for directly measuring the applied force, circumventing the uncertainties in the force per current ratio. The load cell used is a custom version of the Novatech F329 with an extended range, up to 200 mN. Although the load cell is calibrated, its sensitivity can change slightly under loading, especially when loaded horizontally.

F. Gas-line & Power Feeds

The power and propellant supplies are generally separated from the thruster in an experimental setting; therefore both power supply and gas lines are needed between the fixed and mobile parts of the thrust balance. For the power lines, we opted for 4 supply lines which could power up to 2 electromagnets or for example the anode, keeper and the heater of a HET. For this purpose we also included a secondary gas line to, in the future, be able to operate thrusters that need a hollow cathode for neutralization, such as a HET or GIT.

The electromagnet on the HPTx is fed with up to 30 A, requiring at least gauge 10 wires (2.6 mm φ) according to MIL-STD-975. We settled for 3 mm φ enamelled copper cables that can handle temperatures up to 200°C. Due to their considerable diameter these wires would add excessive stiffness to the system. To mechanically decouple the power lines from the system we included liquid metal connectors (item 13,
Fig. 3). These consist of a metal receptacle filled with a liquid metal and metal rod partially submerged in the metal liquid. The receptacle is mounted on the fixed part of the thrust balance while the rods are mounted on the primary arm. As the viscous drag is negligible, the rod can move freely with respect to the receptacle while maintaining the electrical contact.

In the past, mercury was used as the liquid metal but this is undesirable because of its toxicity. Instead we used galinstan, an eutectic mixture of gallium, indium and tin. It is liquid at room temperature $T_{fus} = -19^\circ C$, has an electrical conductivity of $3.46 \times 10^6$ S/m and a vapour pressure of less than $10^{-8}$ mbar at $500^\circ C$ meaning, that it is suitable for testing in high vacuum. The problem with Galinstan is that it embrittles aluminium and other metals. Certain stainless steels (SS) and refractory metals have good resistance against embrittlement.

Although stainless steel (SS) is the most economic option, to accommodate the high currents we chose molybdenum for its higher conductivity $1.87 \times 10^7$ S/m, about 13 times that of SS. The receptacles are 15 mm diameter, 23 mm long cylinders with a 12 mm diameter, 12 mm deep cylindrical cavity.

We simulated the resistive dissipation in the liquid to see whether any notable heating would occur. The total dissipation in the liquid amounts to only 12 mW while the dissipation in the receptacle is little over 14 mW and that in the rod is 175 mW.

To minimize the impact of the gas-lines and to ensure that their impact is linear and limited to an increase in the overall stiffness we opted for thin walled PTFE (polytetrafluoroethylene) tubes of 6 mm outer diameter and 4 mm inner diameter. These tubes are mechanically fixed at two ends: at the top plate and at the cold plate and run parallel to the vertical part of the primary arm. In this way their contribution to the overall stiffness can be approximated with the Euler-Bernoulli beam bending theory.

The equivalent torsional stiffness of two 480 mm tubes with a Young’s modulus of 0.6 GPa is found to be 0.40 Nm/rad. As we will see later, this is less than 1% of the total torsional stiffness of the system.

The overall stiffness of the balance was also calibrated before and after installing the gas-lines and no significant increase in the stiffness was found.

G. Thermal Management

The HPTx prototype operates in the 0.3-1 kW range with an efficiency of around 10 %. The electromagnet has a resistance of about 0.8 Ω resulting in an Ohmic heating power of 720 W at 30 A. However, this increases with time due to the resistance increasing with temperature. The thermal loads to the balance are therefore of the order of 1-1.5 kW; the need for thermal management is obvious.

Moreover, with such high heat fluxes radiative cooling and/or a thermal block would not suffice. The
equilibrium temperature would far exceed the rated temperatures of some of the thruster components. Instead we have opted for water cooling.

To realise this, three separate water cooled cold-plates are installed: one integrated in the thruster mounting plate (item 9, Fig. 3), a second one on the top plate (item 10, Fig. 3), to cool the electronics and a smaller one passing by the liquid connectors. All three cold-plates are connected in series by means of 10 mm $\varnothing$ SS and PU (polyurethane) tubing. For a total cooling line length of about 10 m and a differential pressure of 4.5 bar we estimate a flow rate of 17.5 L/min and Reynolds number of $4.6 \times 10^4$ implying turbulent flow. The thermal resistance of the mounting cold plate is about $5.0 \, ^\circ\text{C/kW}$, that of the electronics plate $3.9 \, ^\circ\text{C/kW}$ and for the liquid connector cooling $24.8 \, ^\circ\text{C/W}$ providing sufficient cooling.

To minimize the effect of the cooling lines to and from the cold plate, the flow direction is kept perpendicular to the plane of motion of the balance. In this way the thrust balance will not be sensitive to any reaction forces of the water on the cooling lines. For that reason the cold-plate is also symmetric w.r.t. the centre line. To reduce the effect on the overall stiffness, the cooling lines run in semi-circular loops on each side of the centre line, from the top plate to the cold plate and back. The are made of 12 mm outer diameter, 10 mm inner diameter PU tubing. The overall stiffness of the balance was also calibrated before and after installing the cooling lines and only a small increase on the stiffness was found, on the order of a few percent.

The problem with water cooling is that the turbulent flow might cause vibrations which introduce noise into the measurement. Although water cooling is used in the shroud (an immobile part) of the thrust stand of [12], as far as we are aware there is no thrust balance design with water cooling of the mobile part(s). We found that the water cooling indeed introduces noise, increasing the RMS noise level from 0.5 $\mu$m to 2.9 $\mu$m. However, it also reduced the thermal drifts of the thrust balance and prevented both the thruster and balance from overheating.

### H. Data Acquisition and Control

The thrust balance is controlled by an on-board micro-controller. This reduces the amount of electrical feed-through connections (not including those for the thruster) to a minimum, 2 pins for power and 2 for data.

The electronics are exposed to the vacuum environment. Care is to taken to avoid the use of electrolytic capacitors which can lose their electrolyte over time due to evaporation. To prevent overheating the PCBs are mounted on the aforementioned cold plate. A metal cover is placed over the electronics to prevent electromagnetic interference (EMI).
The electronics are based on two Arduino micro-controllers and are divided over two PCBs, the main board containing the master Arduino Due and a secondary board with the slave Arduino Nano. The secondary board is mounted on top of the thruster mounting plate and further contains an accelerometer (ADXL355), 8 PT1000 resistance temperature detector (RTD) signal conditioners and a variable current source to power the voice coil of the calibration system. The main board also contains an accelerometer, 8 PT1000 signal conditioners as well as a dual axis inclinometer (ADIS16209), 3 stepper motor controllers and a signal conditioner for the load cell. Master and slave are connected by 6 wires using the SPI protocol. The master Arduino is connected to a PC by an RS485 to USB adaptor.

The stepper motors drive the screw-jacks mentioned in section III and together with the inclinometers allow for control of the pitch and roll of the top plate such that it can be maintained level with the local horizontal at all times. The accelerometers allow for measuring ambient vibrations which can be used to enhance the post-processing. The RTDs are used to track thermal drifts of various parts of the thrust balance. A block diagram of the full data & acquisition system can be seen in Fig. 4.

Fig. 4. Block diagram of data acquisition & control system.
IV. EXPERIMENTAL SETUP

To validate the thrust balance performance various measurements were performed, including some preliminary measurements using the HPTx. Ideally the thrust measurements would be compared to those of an already validated thrust balance. However, these are not yet available, mainly because most existing thrust balances are not compatible with the HPTx platform, which is one of the main drivers for the current work. What is available however, are combined probe measurements that can give an estimate of the thrust and specific impulse. These measurements estimate a thrust in the order of 5-8 mN.

A. Vacuum Facility

The vacuum facility that houses the thrust balance is a 3.5 m long, 1.5 m diameter stainless steel vacuum chamber at the EP2LAB at UC3M. The chamber has two magnetically levitated turbo-molecular pumps and three cryo-panels. The ultimate pressure is $5 \cdot 10^{-7}$ mbar; the background pressure with 10 sccm of Xenon is about $1.8 \cdot 10^{-5}$ mbar.

B. HPTx Prototype

The HPTx is an experimental platform developed by EP2 (jointly with SENER Aeroespacial). It is a laboratory breadboard model that allows for several operational parameters to be varied among which: RF power, mass flow rate, magnetic field. It consists of a solenoid with about 50 G/A that can handle currents of up to 30A; a 25 mm inner diameter, 120 mm long quartz tube; a ceramic injector at the back of this tube and a half-turn helical antenna fed by an RF power generator operating at a frequency of 13.56 MHz. Between the power generator and the thruster is a matching circuit to reduce power reflections due to impedance mismatch. A schematic of the prototype can be seen in Fig. 5 and the operational parameters are tabulated in Tab. III. More details on the HPTx and its predecessors can be found in [28], [29]. For the purpose of validating the thrust balance it is important to note that the thruster weighs 5.2 kg and is mounted to the balance using a mechanical interface that weighs 1.7 kg.

V. RESULTS

A. Validation

To validate the model we simulated the response for our particular balance and compared it to the measured response. All relevant properties and dimensions of the balance are tabulated in Tab. IV and V. The moments of inertia and the masses of all elements are obtained from a CAD model of the balance. Note that the mass of the primary arm includes the 1.7 kg cold-plate and that the mass of the thruster...
Fig. 5. Schematic of the HPTx setup.

TABLE III
HPTX OPERATIONAL PARAMETERS.

<table>
<thead>
<tr>
<th>parameter</th>
<th>nominal</th>
<th>range</th>
<th>unit</th>
</tr>
</thead>
<tbody>
<tr>
<td>RF power</td>
<td>450</td>
<td>300-600</td>
<td>W</td>
</tr>
<tr>
<td>B-field</td>
<td>1000</td>
<td>0-1500</td>
<td>G</td>
</tr>
<tr>
<td>mfr. Xe</td>
<td>10</td>
<td>5-30</td>
<td>sccm</td>
</tr>
<tr>
<td>mfr. Ar</td>
<td>20</td>
<td>10-50</td>
<td>sccm</td>
</tr>
</tbody>
</table>

here includes the 1.7 kg mechanical interface. The resulting center of mass is almost aligned with the vertical part of the arm; $x_{1,0} = 2.0$, $y_{1,0} = -375.0$ mm.

For the dimensions in Tab. V the linearization (see. appendix B) yields the following values: $\theta'_3(0) = -1$ which means that $n = \theta'_4(0) = 31.0$. The result $n = 31.0$ is also found using the static analysis in appendix.
TABLE IV

<table>
<thead>
<tr>
<th>i</th>
<th>( m_i ) [kg]</th>
<th>( L_i ) [mm]</th>
<th>( I_i ) [kg\cdot m^2]</th>
</tr>
</thead>
<tbody>
<tr>
<td>1</td>
<td>3.626</td>
<td>375.0</td>
<td>150 \times 10^{-3}</td>
</tr>
<tr>
<td>2</td>
<td>0.237</td>
<td>116.9</td>
<td>6.86 \times 10^{-3}</td>
</tr>
<tr>
<td>3</td>
<td>-</td>
<td>-</td>
<td>-</td>
</tr>
<tr>
<td>4</td>
<td>6.9</td>
<td>647.0</td>
<td>-</td>
</tr>
<tr>
<td>5</td>
<td>0.271</td>
<td>97.4</td>
<td>-</td>
</tr>
</tbody>
</table>

TABLE V

DIMENSIONS OF THE SCHEMATIC THRUST BALANCE.

<table>
<thead>
<tr>
<th>Point</th>
<th>( x ) [m]</th>
<th>( y ) [m]</th>
<th>( \kappa ) [N\cdot m/rad]</th>
</tr>
</thead>
<tbody>
<tr>
<td>O</td>
<td>0.000</td>
<td>0.000</td>
<td>0.19160</td>
</tr>
<tr>
<td>A</td>
<td>0.300</td>
<td>0.045</td>
<td>0.02330</td>
</tr>
<tr>
<td>B</td>
<td>0.310</td>
<td>0.000</td>
<td>0.01165</td>
</tr>
<tr>
<td>C</td>
<td>0.310</td>
<td>0.045</td>
<td>0.01165</td>
</tr>
<tr>
<td>D</td>
<td>-0.185</td>
<td>0.045</td>
<td>-</td>
</tr>
<tr>
<td>E</td>
<td>-0.120</td>
<td>0.045</td>
<td>-</td>
</tr>
<tr>
<td>M</td>
<td>0.000</td>
<td>0.674</td>
<td>-</td>
</tr>
</tbody>
</table>

Confirming the accuracy of the linearization.

Using equations (16) and (17) and the data in Tab. V and IV, we can calculate the effective moment of inertia \( I = 13.0 \text{ kg}\cdot\text{m}^2 \), and the effective torsional stiffness \( \kappa = 47.1 \text{ N}\cdot\text{m}/\text{rad} \) of the unloaded thrust balance. This results in a natural frequency of 0.30 Hz for the pendulum motion, equivalent to a period of about 3.3 seconds. When loaded with the HPTx the results are 15.9 kg\cdot m^2, 90.9 N\cdot m/rad, 0.38 Hz and 2.39 s.

To validate the dynamic response of the designed thrust balance, Fig. shows the calculated and measured response of the thrust balance when excited with a stepped load. The thrust balance was excited by the voice coil by stepping the applied current nearly instantaneously from zero to some constant value. The response was normalized and centred on \( t = 0 \), when the step is applied, and then fitted using the step response function for an under-damped harmonic oscillator.

\[
A(t) = 1 - e^{-\zeta \omega_0 t} \left[ \sin \left( \frac{\sqrt{1 - \zeta^2} \omega_0 t + \cos^{-1} \zeta}{\sqrt{1 - \zeta^2}} \right) \right]
\]  

(26)
The resulting values are $\omega_0 = 2.56$ rad/s corresponding to 0.41 Hz and $\zeta = 0.57$. The plot of the model is generated using the same value for $\zeta$. The relative difference in frequency between the measured and calculated response is about 8%.

The stiffness of the balance changes linearly with thruster mass. The static model, using eq. (38) and the dimensions and stiffness from Tab. V predicts a stiffness of $k = 0.753 \cdot M + 4.06$ mN/mm, ranging from about 4.8 N/m for a 1 kg thruster to about 42 mN/mm for a 50 kg thruster. For the HPTx weighing 5.2 kg, plus 1.7 kg of the cold plate and another 1.7 kg of the mechanical interface, a stiffness of 10.5 mN/mm is expected. Using eq. (23) our model predicts a stiffness of 10.4 mN/mm. Both values agree within less than 0.1%.

To validate the model of the damping system we simulated the damping coefficient $b$ for varying magnet spacing $s$ with Comsol’s magnetic and electric field (mef) module using the parameters described in section III-C. We also measured the damping ratio $\zeta$ by fitting eq. (26) to the measured step response.
for the same spacing \( s \). For each \( s \) we performed two square pulses, fitting eq. (26) on the positive edge and also on the negative edge (after multiplying by \(-1\)), resulting in 4 points per value of \( s \). The damping coefficient \( b \) can be converted to \( \zeta \) using:

\[
\zeta = \frac{(nL)^2}{2\sqrt{\kappa I}} \cdot b
\]  

which follows from eq. (19) and (21). The experimental data (\( \zeta \)) is scaled using eq. (27) with 2.97 s/kg (thruster) and 4.08 s/kg (no thruster). From the figure it is clear that the FEM model accurately predicts the damping ratio. As a control we also measured the magnetic field in the centre between the magnets for varying magnet spacing and compared it to the simulated value; this was also found to be in good agreement (\( R^2 = 0.995 \) and an RMSE (root mean squared error), normalized to the mean, of 2%). The damping system is capable of providing a damping ratio from 0.1 to more than 1 which is more than sufficient for practical application. We did find a slight difference between the results obtained from the positive and the negative edges, most clearly seen at \( s = 10 \) mm of the blue curve in Fig. 7. This is probably due to the fact that for the negative edge the plate does not start centered on the magnet. Regardless, this minor difference does not affect the previous conclusions.

The optimal damping ratio is that for which the settling time is minimal. The settling time is defined as the time it takes for a step response to fall within 2% of the final value. Using the parameters derived from our Lagrangian model we simulated the settling time for different values of the normalized damping coefficient \( \zeta \), resulting in the plot of Fig. 8. The optimal damping ratio is about \( \zeta = 0.78 \) with a settling time of 1.8s. We also inferred the settling time and damping ratio from the measured response of the system to a step input for various separation distances of the magnets which are the red circles in Fig. 8. As is apparent the measurement and simulation agree well. These results were obtained without a thruster mounted on the balance. Including the thruster mass in the model yields similar results, the optimum \( \zeta \) is still 0.78 and the settling time is slightly lower at 1.5 seconds. The discontinuities in the graph are due to the definition of the settling time and can be explained as follows: for a given value of \( \zeta \) there is a point where the step response \( A(t) \) will have crossed the lines \( A = 0.98 \) or \( A = 1.02 \) for the last time. This is the settling time. When \( \zeta \) is increased there will be a point at which this ultimate crossing will occur one half-period earlier, the settling time will then be reduced discontinuously by a half-period.

**B. Calibration**

Calibration of the system is necessary to obtain the actual stiffness of the balance which is needed for post-processing measurement data. Comparison of the calibrated stiffness with the stiffness predicted by the model also provides another venue for validation.
For the HPTx a maximum thrust level of around 8 mN was expected and therefore we intended to calibrate up to 20 mN. The force per current ratio of 0.29 N/A implies that the current applied to the voice coil should be varied from 0 to 70 mA. To verify this we first performed a linear fit of the applied current versus measured force of the voice coil and found instead a force per current ratio of 0.35 N/A a 21% difference. This significant difference could indicate that the calibration of the voice coil is affected by the horizontal orientation and the lateral loading due to the voice coil magnet. This could explain at least part of the difference in the predicted and measured thrust balance stiffness. Despite the different force per current value we maintained the maximum current at 70 mA for a calibration range of slightly less than 25 mN.

The calibration protocol was as follows: 0 to 70 mA is applied in steps of 10 mA. Each step lasts for about 20 seconds to allow the balance to settle and to be able to average sufficient data points in one
Fig. 8. Settling time $t_s$ as a function of damping ratio $\zeta$.

step. To account for drifts, each current step is followed by a zero step that also lasts 20 seconds. In this way each step has its own zero value. Furthermore, to check for hysteresis the procedure is repeated from 70 mA down to 0 mA. The displacement data is sampled at 10 Hz and the load cell data is sampled at 2 Hz. The resulting data is shown in Fig. 9. This data was taken with the HPTx mounted on the balance and with the cooling water running. The data is processed as follows. At each step as well as the subsequent ‘zero’-step, 150 displacement sensor data points and 30 load cell data points are taken. The displacement data is then averaged every 5 points to reduce it to 30 points. The zero values are subtracted from the preceding ‘high’ values. We then use a least squares fitting procedure to obtain a linear fit of the data. This results in the plot shown in Fig. 10, where we plotted the displacement versus the force, as well as the linear fit. In the upper window of the plot the standardized residuals are shown. These residuals should be normally distributed around zero. It can be seen that there is a pattern in the residuals. This is indicative of non-linearity. When doing a quadratic fit the residuals were normally distributed.
We quantified the non-linearity as the largest deviation of the quadratic fit with respect to the linear fit, relative to the full scale which resulted in a 0.4% non-linearity error. Since this uncertainty is so small we find the use of the linear fitting justified as it yields a single calibration constant that is easy to use. The inverse of the slope of the fit is $k_{cal} = 23.21 \pm 0.02$ mN/mm. However, this is the stiffness for forces applied at the position of the voice coil. This needs to be scaled by the ratio between the distance from $O$ to the voice coil (368 mm) and the distance from $O$ to the point of action of the thrust force $L_T$ (generally, but not necessarily: $L_T = L_A$) which is 674 mm; this ratio is 0.546. The stiffness of the thrust balance is therefore 12.7 mN/mm which is about 20% higher than that predicted by either model. In this particular calibration the relative uncertainty on the calibration constant is about 0.07%. In general we have found that the relative uncertainty is $\leq 1\%$. 

Fig. 9. Raw calibration data. Applied force $F$ and displacement $y$ as a function of time $t$. 
C. HPTx Thrust Measurements

In this section we report one of the first thrust measurements of the HPTx using the new thrust balance. In Fig. 11 the output of the displacement sensor during a firing of the HPTx is shown. The operating point was 15 sccm of Xenon, 450 W of RF power and a magnetic field of 1500 G. The blue line shows the raw data; as can be seen there is a thermal drift from the moment the magnetic field is turned on, which in first approximation is linear. It was found that subtracting the slope of the line connecting the point of the curve, right before turning on the magnetic field (40 s) and the point right after turning it off again (150 s) removed most of the drift as evidenced by the red line.

In Fig. 11 several events can be distinguished. Firstly at around 40 seconds the magnetic field is turned on, resulting in a negative thrust. This can be explained by the magnetic force between the solenoid and the magnet of the voice coil. A linear relationship was found between the magnetic field of the solenoid...
and the negative thrust generated, which seems to support this hypothesis. It is important to note that this negative thrust level is considered the ‘zero level’ of the measurement.

At about 64 seconds the propellant flow is turned on leading to a small increase in the thrust value, the so-called cold gas effect.

Little after 86 seconds the RF power is turned on starting the plasma discharge. This is the largest contribution to the thrust. A small overshoot can be seen after which the thrust stabilizes. The increased noise possibly comes from the cooling system. Currently the water cooling circuit for the thrust balance and that of the RF generator are both supplied from the facility water lines. The thrust balance cooling water runs continuously but the RF generator has a valve which opens/closes when the RF is on/off. Although they are connected at outlets several meters apart we suspect that opening and closing the valve causes pressure waves that induce vibrations in the thrust balance. The vibrations are worse when the
valve closes (RF OFF), probably due to the induced water hammer, at 132 s. Then subsequently at 145 s and 166 s propellant flow and magnetic field are turned off. Despite the oscillations when turning off and on the thruster the thrust balance signal is appreciably more stable than that of other thrust balances running a HPT [33], [34], [35].

To obtain the net displacement we first obtain the zero level by taking the mean of a 150 data points of the (detrended) data set, after turning on the magnetic field and before turning on the propellant flow where the curve is ‘flat’, between 45 and 60 seconds; we then obtain the mean of 330 data points of the steady-state part of the data between turning on and off the RF power, between 92 and 115 seconds. The mean values are denoted by the dash-dotted lines in Fig. 11. The difference between these values is the net displacement, 408 \( \mu \)m. Given the calibration constant of 12.7 mN/mm this is equal to 5.1 mN.

D. Uncertainty Estimate

In this section we report the analysis of the experimental uncertainty estimation. The final thrust value is the product of the following variables:

\[
T = y \cdot k_{\text{cal}} \cdot \frac{L_{\text{vc}}}{L_T}
\]

(28)

where \( L_{\text{vc}} = 368 \) mm is the distance from the voice coil to \( O \) and \( y \) the displacement measured by the sensor. All of these quantities come with an uncertainty. The uncertainty in \( L_T \) and \( L_{\text{vc}} \) are those in the measurement of both lengths and is \( \sigma_{L_T} = \sigma_{L_{\text{vc}}} = 1 \) mm. The uncertainty in \( k_{\text{cal}} \) is obtained concurrently with \( k_{\text{cal}} \) and for this particular measurement is \( \sigma_{k_{\text{cal}}} = 0.1 \) mm/mN. According to [17] the uncertainty in \( y \), \( \sigma_y \), can be estimated from the calibration data. However, we are of the opinion that in our case this underestimates the uncertainty. Therefore, we use the root sum of squares of the standard deviation of the zero level and thrust level. In this measurement of Fig. 11 this results in an uncertainty of 8 \( \mu \)m. The uncertainty in the thrust measurement is calculated from the root sum of squares of all the relative uncertainties.

\[
\frac{\sigma_T}{T} = \sqrt{\left( \frac{\sigma_y}{y} \right)^2 + \left( \frac{\sigma_{k_{\text{cal}}}}{k_{\text{cal}}} \right)^2 + \left( \frac{\sigma_{L_T}}{L_T} \right)^2 + \left( \frac{\sigma_{L_{\text{vc}}}}{L_{\text{vc}}} \right)^2}
\]

(29)

For the measurement of Fig. 11 the relative uncertainty in the thrust measurement is 2% which is dominated by the uncertainty in the displacement measurement. The thrust value is then \( T = 5.1 \pm 0.1 \) mN.

VI. DISCUSSION AND CONCLUSIONS

In this paper we have presented a mechanically amplified thrust balance designed specifically for helicon thrusters. This design has several advantages such as increased sensitivity; a more compact form
factor; a displacement sensor mounted on the main structure, far away from the source of disturbances;
among other advantages mentioned throughout this work.

A dynamic analysis using Lagrangian mechanics has been presented. This model reproduces the results
of the static analysis of Ref. [18]. On top of that it can be used to calculate the time-response of the
thrust balance and analyze the required damping force.

The model predicts an angular magnification of $31^\circ/\circ$ and a stiffness of 9.65 mN/mm in agreement
with the static model. The model shows that the system is equivalent to a damped harmonic oscillator
and predicts a natural frequency of 0.38 Hz when loaded with a 5.2 kg thruster (6.9 kg including the 1.7
kg mounting structure), within 8% of the measured value of 0.41 Hz.

The thrust balance includes a viscous damping system based on eddy currents, with a variable damping
ratio. The damping system was modeled using Comsol and the calculated damping ratio was in good
agreement with the measured value. It was shown that a damping ratio of 0.78 is optimal resulting in
the lowest settling time, 1.8 s.

The calibration system is based on a voice-coil mounted on a load-cell to avoid uncertainties in the
voice coil’s force constant. When verifying the force constant using this setup it was found that it is
21% higher than specified in the data sheet. This could indicate that the load cell calibration is affected
by the fact that it is loaded laterally. If the load cell indeed over-predicts the force it could explain the
difference between the model and the measurement. This needs be verified e.g. by cross-comparison with
a different calibration system. Coupling between the solenoid of the helicon thruster and the magnet of the
voice coil was observed, which poses a minor inconvenience during thrust measurements. The stiffness
of the thrust balance loaded with a 5 kg thruster and corrected for relative position of the calibration
system and the thruster is 12.7 mN/mm, almost 20% higher than expected from the models. However,
correcting the stiffness with the ratio between the force per current ratio of the voice coil as reported by
the manufacturer and the measured one, yields a stiffness of 10.4 mN/mm in exact agreement with the
prediction of our model. This lends credence to the hypothesis that the load cell sensitivity has increased
20%, probably due to loading and orientation, and thereby over-predicting the stiffness.

The design further includes liquid metal connectors to mitigate the contribution of the power cables
to the overall stiffness, on-board electronics to minimize electrical feed-through connections, leveling
system to keep the balance aligned with the horizontal, and a confocal chromatic displacement sensor
with 0.4 μm resolution and 12 mm range that is impervious to EMI. Another novelty is the active water
cooling of the thruster mounting plate. This acts as a thermal sink for the thruster shielding the thrust
balance from the heat load. We have shown that the water cooling increases the noise by a factor 6 from
0.5 μm to 2.9 μm (equivalent to 6 and 37 μN). Currently the water cooling is supplied from the facility’s
water net which does not have a regulated outlet pressure. This could be improved with a closed loop, dedicated chiller with more accurate pressure control.

The thrust balance performance has been validated by testing the HPTx platform. A displacement of 408 ± 8 μm was measured operating the thruster at 15 sccm of Xe, 450 W RF power and a magnetic field of 1500 G. This corresponds to a thrust level of 5.1 ± 0.1 mN. The relative uncertainty on the thrust measurement was 2% in this particular case and dominated by the noise induced by the RF generator valve. It is expected that this can be reduced to ≤ 1% by decoupling the water cooling system.

APPENDIX A

STATIC MODEL

\[ y = 2L_{AE} \sin \left( \frac{\theta_2}{2} \right) \] (30)
\[ \theta_2 = \theta_1 + \theta_3 + \theta_4 - \pi \] (31)
\[ \theta_1 = 2 \sin^{-1} \left( \frac{x}{2L_4} \right) \] (32)
\[ \theta_3 = \cos^{-1} \left( \frac{L_{OB}^2 + L_{AB'}^2 - L_{OA}^2}{2L_{OB}L_{AB'}} \right) \] (33)
\[ \theta_4 = \cos^{-1} \left( \frac{L_{AC}^2 + L_{BC}^2 - L_{AB'}^2}{2L_{AC}L_{BC}} \right) \] (34)
\[ L_{AB'}^2 = L_{OB}^2 + L_{OA}^2 - 2L_{OB}L_{OA} \cos (\gamma + \theta_1) \] (35)
\[ \gamma = \cos^{-1} \left( \frac{L_{OA}^2 + L_{OB}^2 - L_{BC}^2 - L_{AC}^2}{2L_{OA}L_{OB}} \right) \] (36)

where \( \gamma \equiv \angle AOB \).

The variables \( L_i \) denote the length from the centre of mass \( r_i \) to its corresponding point of rotation, while \( L_{jk} \) is the length between the points \( j, k \).

Note that in the previous equations, it is implicit that \( \theta_3, \theta_4 \) are initially right angles. It can be seen that \( y \) is a function of \( \theta_2 \) which is in turn a function of \( \theta_1 \), which is a function of \( x \).

The stiffness of the thrust balance can be obtained from a force balance at all four pivots (O, A, B, and C in Fig. [1]). The resulting relationship between the applied thrust \( T \) and the angle \( \theta_1 \) is given by:

\[ T = \frac{\tau_O + \tau_{B'}}{L_4} + Mg \sin(\theta_1) + \frac{(\tau_A + \tau_{C'}) L_{OB} \cos \left( \frac{\pi}{2} - \theta_3 \right)}{L_4L_{AC} \cos \left( \frac{\pi}{2} - \theta_4 \right)} \] (38)
where,
\[
\begin{align*}
\tau_O &= \kappa_1 \theta_1 \\
\tau_B &= \kappa_3 \left( \theta_3 - \frac{\pi}{2} \right) \\
\tau_A &= \kappa_2 \theta_2 \\
\tau_C &= \kappa_4 \left( \theta_4 - \frac{\pi}{2} \right)
\end{align*}
\]

\text{(39)}

\text{APPENDIX B}

\text{LINEARIZED EQUATIONS}

We can use a Taylor expansion to the first order to linearize the angles \( \theta_2, \theta_3 \) and \( \theta_4 \) around \( \theta = 0 \).

\[
\begin{align*}
\theta_3 &= \theta_3(0) + [\theta_3'(0)] \cdot \theta \\
\theta_4 &= \theta_4(0) + [\theta_4'(0)] \cdot \theta \\
\theta_2 &= \theta_2(0) + \theta_4(0) - \pi + \left[ 1 + \theta_3'(0) + \theta_4'(0) \right] \cdot \theta
\end{align*}
\]

\text{(40)-(42)}

Here the primes denote a derivative to \( \theta \). We simplify this notation by introducing the following nomenclature: \( n_3 = \theta_3', n_4 = \theta_4' \) and \( n = 1 + \theta_3'(0) + \theta_4'(0) \). We already have the equations for \( \theta_3, \theta_4 \) (eq. \text{(34),(35)}) we only need to find their derivatives. To this end we start by normalizing \( L_{OA}, L_{BC} \) and \( L_{AC} \) with \( L_{OB} \).

\[
\begin{align*}
p &= \frac{L_{OA}}{L_{OB}} \quad q &= \frac{L_{BC}}{L_{OB}} \quad s &= \frac{L_{AC}}{L_{OB}}
\end{align*}
\]

\text{(43)}

We also introduce the substitution:

\[
z(p, \theta, \gamma) = 1 + p^2 - 2p \cos(\theta + \gamma)
\]

\text{(44)}

We then rewrite eq. \text{(37), (34) and (35)} using the above expressions.

\[
\begin{align*}
\gamma &= \cos^{-1} \left( \frac{1 + p^2 - q^2 - s^2}{2p} \right) \\
\theta_3 &= \cos^{-1} \left( \frac{1 - p \cos(\theta + \gamma)}{\sqrt{z(p, \theta, \gamma)}} \right) \\
&\quad + \cos^{-1} \left( \frac{z(p, \theta, \gamma) + q^2 - s^2}{2q \sqrt{z(p, \theta, \gamma)}} \right) \\
\theta_4 &= \cos^{-1} \left( \frac{s^2 + q^2 - z(p, \theta, \gamma)}{2qs} \right)
\end{align*}
\]

\text{(45)-(48)}

We then take the derivative to \( \theta \).

\[
\begin{align*}
\frac{d\theta_3}{d\theta} &= \frac{1 - p \cos(\theta + \gamma)}{z(p, \theta, \gamma)} - 1 \\
&\quad - \frac{p \sin(\theta + \gamma) \left( 1 - \frac{q^2 - s^2}{z(p, \theta, \gamma)} \right)}{\sqrt{4q^2s^2 - \left[ z(p, \theta, \gamma) - q^2 - s^2 \right]^2}} \\
\frac{d\theta_4}{d\theta} &= \frac{2p \sin(\theta + \gamma)}{\sqrt{4q^2s^2 - \left[ q^2 + s^2 - z(p, \theta, \gamma) \right]^2}}
\end{align*}
\]

\text{(49)-(51)}

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When we evaluate $\theta_3, \theta_4$ and their derivatives at $\theta = 0$ all cosines and sines become a function of $\gamma$. When we substitute in eq. (37) we get rid of these trigonometric functions and are left with the following:

$$\theta_3(0) = \cos^{-1}\left(\frac{1 - p^2 + q^2 + s^2}{2\sqrt{q^2 + s^2}}\right)$$  \hspace{1cm} (52)

$$\theta_3'(0) = \cos^{-1}\left(\frac{q^2}{\sqrt{q^2 + s^2}}\right)$$  \hspace{1cm} (53)

$$\theta_4'(0) = \frac{1 - p^2 - q^2 - s^2}{2(q^2 + s^2)}$$  \hspace{1cm} (54)

$$\theta_4(0) = \frac{\pi}{2}$$  \hspace{1cm} (56)

For the dimensions given in Tab. [X] we find that $p = 0.9786$, $q = 0.1452$ and $s = 0.0323$ resulting in $\theta_3(0) = \pi/2$, $\theta_3'(0) = -1$ and $\theta_4'(0) = 31$. The fact that $\theta_3(0)$ and $\theta_4(0)$ are right angles is something that is designed for on purpose. In that case $\theta_3(0) + \theta_4(0) - \pi$ is zero and the linearized $\theta_2$ does not have an intercept. We can furthermore see that $\theta_3'(0)$ cancels out the 1 and that the angular magnification $n = n_4 = \theta_4'(0) = 31$. If we look at the differential angles for this case we find that:

$$\delta_1(\theta) = \theta$$  \hspace{1cm} (58)

$$\delta_2(\theta) = \theta_2 = n \cdot \theta$$  \hspace{1cm} (59)

$$\delta_3(\theta) = \theta_3 - \theta_3(0) = n_3 \cdot \theta$$  \hspace{1cm} (60)

$$\delta_3(\theta) = \theta_3 - \theta_3(0) = n_4 \cdot \theta$$  \hspace{1cm} (61)

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**Mick Wijnen** Biography text here.