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The influence of laminate stacking sequence on ballistic limit using a combined Experimental/FEM/Artificial Neural Networks (ANN) methodology

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Abstract

Composite laminates subjected to high velocity impacts are usually studied by means of experimental or numerical approaches. Nevertheless, these techniques are not appropriate to analyze the wide range of possibilities in the design of laminates (a great amount of time and economic resources are required); therefore, more efficient methods would be desirable.

This work presents the capability of an ANN approach to predict the change of the ballistic limit with the laminate stacking sequence, and hence to find the optimum laminate combination. In order to obtain a refined ANN tool, a combined methodology of experimental and finite element method has been used. The results of the experimentally validated FEM model, are used to provide the data to the ANN. Once trained, the ANN is able to predict accurately the ballistic limit of composite laminates studied. The ANN allows studying very efficiently the whole possibilities of laminate stacking sequence using the common orientations, in symmetric 12 plies laminates (4096 cases). In addition, a deeper comprehension of composite plates when subjected to high velocity impact has been achieved by means of the analysis of the results. Conclusions obtained can be used by composite design engineers to improve ballistic performance of composite plates.

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1. Introduction

The aerospace sector began to use composite materials by means of the introduction of fibreglass in the Boeing 707 during the 1950s [1]. Composite materials were at first used for tertiary structures, that are the ones which does not produce any reduction of aircraft flying capacity if they fail. Since the use of composite materials represents a major advantage in terms of weight reduction due to its higher specific stiffness and strength properties compared to aluminium, its use has been extended to secondary and primary aircraft structures. Nevertheless the replacing of metals in these structures is a major challenge for the aircraft industry due to the high performance and reliability needed. The first examples on secondary and primary structures were the use of composite materials for the vertical stabilizers in the Airbus A300 (1970s) and for the horizontal stabilizers in the Boeing 737 [2].

As it is said, a high performance is needed in secondary and primary structures since they could be subjected to a wide range of loads, including dynamic loads such as impacts. As an example, birds [2, 3], hail stones [4, 5, 6, 7] or runaway debris [8, 9] may impact against the structure at high velocity. These structures could also be subjected to low velocity impacts [10] it will not be the scope of the present work. Prior to the introduction of composite materials in critical aircraft structures, the analysis of how these materials behave when subjected to high velocity impact loads had to be performed to check safety flight requirements. Therefore first studies appear in the late 1960s and 1970s in which high velocity experimental impacts were done [11, 12]. Since those works, this scope has been widely studied using experimental tests; an interesting review can be seen in [13, 14]. The studies carried out have shown that composite materials are quite vulnerable to normal impacts, occurring damages that diminishes its load bearing capacity [15]. In addition the experimental
studies performed allowed to notice that the behaviour of composite materi-
als is influenced by many parameters, such as impact velocity [16], material
properties [15], laminate stacking sequence [17], laminate thickness [18], impact
trajectory [19] or temperature [20].

The understanding of the behaviour of composite materials when subjected
to high velocity impacts has improved with the use of numerical codes, in which
simulations have provided additional information that was impossible to obtain
experimentally. Moreover, numerical methods allow saving resources compared
to experimental techniques. The finite element method (FEM) has been used in
different works in which high velocity impact on composite materials are anal-
ysed. In order to obtain accurate results it is essential to use a proper material
model for the composite laminate. One approach is to use mesoescale models
at ply level in which different failure mechanisms are defined to reproduce the
material behaviour. For instance, López-Puente and his co-workers [9, 10] have
used a material model based on the Chang-Chang criteria [21] for modelling
high velocity impacts on composite laminates obtaining accurate results regard-
ing the residual velocity and damaged area. Similar approaches have been used
in other works but including the use of cohesive interfaces [22] or cohesive el-
ements [8] to model the delamination. Multiscale modelling has been used as
another approach to model composite laminate subjected to high velocity im-
pacts [23] obtaining good agreement when compared with other failure criteria.

The experimental and numerical techniques previously mentioned have been
proved to be useful to analyse composite material when subjected to high ve-
locity impact. However, the main drawback of these technique is that both
are very time consuming which results in a growth of the analysis cost. More-
over, the continuous improvements in the composite manufacturing engineering
have increased the range of possibilities in the designing of laminates (E.g. thin
plates vs thick plates, fibre and ply hybridization, new fibres and matrices, non-
conventional stacking sequence...). The use of the previously mentioned tech-
niques for analysing this wide range of possibilities will be inappropriate due to
the great amount of time and economic resources required. Therefore in order
to analyse these new types of composite laminates much more efficient methods would be desirable. Analytical modelling could be a proper technique to analyse these new types of composite laminates because its computational cost when the model is implemented is negligible and it has been used successfully to predict residual velocity and impact damage area on high velocity impact [24, 25]. Nevertheless, they require the assumption of certain hypothesis which limits its scope and its usage for the wide range of composite laminates. Another technique is the use of the Artificial Neural Networks (ANN) that are able to solve complex problems in a very low computational time. This technique has been used successfully on composite structures, both for quasistatic and dynamics loads. Naderpour et al. [26] has predicted the compressive strength of FRP-confined concrete with the use ANN. Kalhor et al. [27] have predicted the response of metal-FRP square tubes subjected to axial crush, studying the influence of FRP thickness and laminate stacking sequence. ANN model has been also used to predict low velocity impact against composite plates. For example, Malik et al. [28] predicted successfully the energy absorbed in these impacts using a ANN trained with the data provided by a FEM model. It has to be remarked that the previous papers correspond to quasistatic or low velocity impacts subjected to composite laminates. However, it has been found that prediction of composite materials subjected to high velocity impacts are scarce. For example, ANN was used by D. Fernández et al. [29] to predict the ballistic behaviour of composite plates analysing the influence of impact trajectory.

The works aforementioned have shown that ANN is an efficient method to predict the behaviour of composite laminates when subjected to quasistatic and low velocity impact loads but it has not been widely used to study composite laminates subjected to high velocity impacts. The possibilities of this method have not been totally explored: the use of a high efficient method can be useful in the designing process and ANN has not been yet applied to analyse the influence of laminate features in its behaviour when subjected to high velocity impacts.

The present work proposes a new methodology which is able to predict the
ballistic limit of composite laminate with different laminate stacking sequence
with a low computational effort in such a way that could be used as a design tool.
The methodology consists on combining the experimental, numerical (FEM) and
artificial neural network techniques. The experimental tests are used to validate
the FEM modelling. Once validated, the FEM model is used to provide data
for the ANN. The ANN is trained by means of the numerical results in order
to be able to predict with enough accuracy the ballistic limit of composite
laminates. The whole range of possibilities of laminate stacking sequence using
the standard orientation (0°, 45°, -45° and 90°) in symmetric 12 plies laminates
has been studied, which results in 4096 possible cases. The analysis of the
results has been used to find the optimum laminate stacking sequence and also
to improve the understanding of the behaviour of composite plates subjected to
high velocity impact. Conclusions can be used by composite design engineers
to improve ballistic performance of composite plates.

An outline of the remaining content of this paper is as follows. In the
next section (Section 2) a brief explanation of an ANN is included, centred in
the architecture that is the most used in the solid mechanics, the MultiLayer
Perceptron (MLP). In Section 3 it is detailed the methodology used in the
present work, describing the experimental test, the FEM model and the MLP
developed. The results are showed in Section 4, while a discussion focusing in
the designing purposes is included in Section 5. Finally, major conclusions are
listed in Section 6.

2. Artificial Neural Network (ANN)

In this work an artificial neural network (ANN) has been used to predict the
ballistic performance of laminates varying its laminate stacking sequence. An
ANN is a machine learning algorithm that is able to process the information
given to produce an answer. The validity of the answer depends on how the
information has been treated; this process is called training or learning. This
training process, as in the human brain, is done by learning by example in such
a way that the internal structure is modified during this process to assure the correct answer. The capability of learning is reached, also as in the human brain, by means of a highly interconnected internal structure. These characteristics made ANN suitable for solving highly non-linear problems such as pattern recognition, data classification or curve fitting problems than can be found in solid mechanics field.

Although there are many different types of artificial neural network, the multilayer perceptron architecture (MLP) is the most used in solid mechanics [29]. This architecture is the one selected in the present work. A MLP is an ANN composed by different layer of neurons interconnected in such a way that an individual neuron receives the signals of all neurons of the previous layer, process it and gives its signal to all neurons of the following layer. This process is called feed-forward algorithm and is the responsible of producing an output in the network. The behaviour of an individual neuron $i$ can be described as follows: $y_i = f_i \left( \sum \phi_{ij} x_j - b_i \right)$. The inputs ($x_j$) are multiplied by the weights ($\phi_{ij}$) and when the sum of the weighed inputs are higher than a threshold ($b_i$) the output of the neuron ($y_i$) is activated, by means of an activation function ($f_i$). The values of the weights and the threshold are the ones that are modified in the training process to minimize the output errors. For the activation function can be used different possibilities, such as the sigmoid function, hyperbolic tangent or linear function that have to be tested to select the proper one.

As it has been said, in a MLP neural network there are different layers of neurons which are classified in three different types: the input, hidden and output layers. The input and output layers are only composed by an unique layer which number of neurons is the same as the number of input and output variables used in the ANN. However, the number of layers and neurons for the hidden ones are not determinate by the problem and commonly these two parameters are chosen after an iterative process in which it is selected the value that produce the better results with an efficient computational cost.

Although an MLP is able to produce results once designed, it is not useful until the network is trained. In order to train the network it is needed a set
of known values of input and output \((x^*, y^*)_T\) to develop the learning process.

During this learning phase the internal structure of the network, in this case the values of the sets of weights and thresholds, are modified iteratively to minimize the error of the MLP. This process is called the back-forward error propagation algorithm. It can be distinguished the following steps on it:

1. The set of weights and thresholds are initialize by random values.
2. The ANN is run producing an output.
3. The error of the output \(e(s)\) is obtained with the following equation:

\[
e(s) = \frac{1}{2} \sum_{i=1}^{M} (y^*_i - y_i)^2
\]  

where \(y^*_i\) is the set of known output values and \(y_i\) are the values predicted by the ANN in the actual iteration.
4. The values of the weights and thresholds are modified according to the stochastic gradient descent expressed as:

\[
\phi_{ij}(s + 1) = \phi_{ij}(s) - \xi \frac{\partial e(s)}{\partial \phi_{ij}} \\
\]

\[
b_i(s + 1) = b_i(s) - \xi \frac{\partial e(s)}{\partial b_i}
\]

where \(\xi\) is the learning rate. The learning rate determines the influence of the error in the rate of changes occurred in the weights and thresholds in the learning process. This parameters affects the convergence and the stability of this process.
5. The previous steps from 2 to 4 are repeated iteratively. Each iteration is called an epoch. In this work the maximum number of epochs has been selected assuring that the global error of the ANN reaches a minimum.

3. **EXP- FEM -ANN combined methodology**

The methodology proposed in this paper is based on a combined approach of experimental, numerical (Finite Element Method) and machine learning (Ar-
Artificial Neural Network) techniques, which allows to obtain accurate and computational efficient results. The loop or path to apply this methodology is as follows:

- **Experimental tests.** In order to obtain experimental data for numerical validations, high velocity impact tests have been carried out in the University Carlos III of Madrid Impact Lab obtaining the ballistic performance of a CFRP tape laminate which stacking sequence is $(+45/-45/0/90/90/0)_s$.

- **Finite element method.** The data required to train the ANN is unapproachable from an experimental point of view, due to the high cost associated to manufacture and test laminates with different stacking sequences. For this reason, the data for the ANN learning should be provided by a less costly method, as the numerical; but prior to generate the cases, the model has to be proved to be representative. The steps followed are:
  
  - **Validation of the numerical model.** A numerical model of the ballistic impact is developed in the commercial finite element code LS-DYNA v.R7; the results are compared with the experimental tests performed to validate the proposed model.
  
  - **Virtual testing campaign.** Once the numerical model is validated, it is performed a virtual testing campaign obtaining the ballistic limit of different laminate stacking sequences, data needed to train the ANN.

- **Artificial Neural Network.** Through the development and training of a ANN, the influence of the stacking sequence in the ballistic limit is analysed. To this end, it is necessary:
  
  - **ANN learning process.** An ANN is developed in Python and it is trained by the data obtained from the virtual testing campaign. A parametrical study of the characteristics (number of neurons and hidden layers, epochs etc…) of the ANN has been performed to obtain the optimum network.
- **ANN prediction.** Once trained the ANN is able to predict the ballistic limit for the whole spectrum of lay-ups sequences using the common (0°, 45°, -45° and 90°) in symmetric 12 plies laminates (2048 different laminate stacking sequences).

Each technique used in the present paper (Experimental, FEM and ANN) is described in detail below. A scheme of the methodology is described in Fig. 1 in which it can be distinguished five main steps.

![Scheme of the methodology](image)

**Figure 1: Scheme of the methodology**

The analysis of the results leads to finding the optimum laminate; additionally, it has been achieved a deeper understanding of the process that would guide composite designer to improve laminate ballistic performance.

### 3.1. Experimental procedure

The experimental tests used to validate the numerical model consist in composite laminates subjected to high velocity steel sphere impacts. For the projectile, it has been selected a 7.5 mm diameter tempered steel sphere which is accelerated using a pneumatic canon. The impact velocity was varied in a range from 60 to 500 m/s changing the helium pressure which impels the projectile.

The laminates were manufactured from prepregs made by Hexcel Composites, using AS4 carbon fiber and 8552 epoxy matrix; 12 plies were stacked using the following laminate stacking sequence: 

\[
(+45/-45/0/90/90/0)_s
\]

which leads to a
2.4 mm thickness. The specimen dimension was $110 \times 110 \, mm \times mm$; this value was selected assuring that the damage does not reach the plate contour. The laminates were simply supported on its four back face edges. To have enough information to compare with the numerical model, the experimental residual velocity and the damaged area of the laminates have been obtained. The first one has been measured by means of a high speed video camera (Photron Ultima APX-RS at 36 kfps) and the damaged area has been quantified using the ultrasonic C-Scan technique with a $5 \, MHz$ transducer. The results of the experimental test are presented in detail in [? ]

### 3.2. Finite element model

The numerical model used is implemented in the commercial finite element code LS-DYNA v R7 in which it is simulated the steel impactor and the composite laminate. The projectile used in the experimental test was made of tempered steel and since no plastic deformation nor damage has been observed after the impacts a linear elastic material ($\rho = 7850 \, kg/m^3$, $E = 210 \, GPa$ and $\nu = 0.3$) has been chosen for the simulations.

CFRP specimens are composed by 12 plies; each one represented by an element through the thickness with the proper orientation. In order to model the behaviour of the unidirectional carbon/epoxy laminate a combination of two approaches has been used: one to take into account the intra-laminar damages and the other to consider the inter-laminar damages.

The intralaminar damage model considers different type of damages based in the Chang-Chang model [21]; distinguishing between fiber and matrix failure for tension and compression. The plies behaves as an orthotropic linear elastic material until failure. Once a certain failure mode occurs, a damaged model degrades the stiffness of the material involved in this failure mechanism in such a way that they are reduced keeping constant the associated stress component [30]. Finally, the damaged elements are removed using a maximum strain criteria in order to avoid numerical instabilities due to excessive distortion. The interlaminar damage is taken into account through a cohesive interaction based on
a traction-separation law, in which it is necessary to define a damage onset and
a damage evolution law. The cohesive interaction is placed in between each ply
of composite material. The damage onset is controlled by a quadratic traction
criteria while the damage evolution is controlled by a linear law type based on
energies where the mixed mode is controlled by a power law. Both interlaminar
and intralaminar properties of the laminate can be found in an article of the
same author of the current work [22] where a similar computational approach
has been used to model the same composite material (AS4 fibre and 8552 epoxy
resin).

Regarding the meshing, the element in-plane size was $0.73 \times 0.73\text{mm} \times \text{mm}$
which lead to 22500 solid elements for each ply of the composite laminate,
resulting in a total of 270000 elements. The boundary conditions are the same
as it can be found in the experimental tests: simply supported on its four back
face edges.

3.3. MultiLayer Perceptron (MLP) development

A MLP has been coded in the software Python to predict the ballistic limit
of CFRP laminates. Since the study has been done to analyse the influence of
laminate stacking sequence consequently, the input variables of the present MLP
were the orientations of each ply of the laminate. Due to the symmetry of the 12
ply laminate, only 6 variable were independent. The output variable was unique
and corresponded to the ballistic limit of the laminate. In the present study
the allowed ply orientations were the four common ones used in the aeronautic
industry ($0^\circ, 45^\circ, -45^\circ$ and $90^\circ$). Therefore the ballistic limit of $4^6 = 4096$ different
laminates has to be predicted. This value can be reduced to half because of the
symmetry of the problem (i.e $(0)_{6S}$ is symmetric with $(90)_{6S}$), so 2048 laminates
have been studied. As it has been said the results of the MLP are only valid when
the learning process is performed. In this work, it has been selected different
laminate stacking sequences whose ballistic limit is obtained by means of the
previously FEM model developed. The pair of data, laminate stacking sequence
and ballistic limit, were provided in the training phase to the MLP. In order
to assure a proper learning process it is required the use of enough quantity of
data, in this case, 40 different cases have been selected in the learning process.
These cases were divided in two groups: 37 cases have been used to train the
network while the other 3 have been chosen to carry the testing validation. The
testing validation consists on comparing the results of the 3 cases (whose results
were not provided to the MLP for the learning process) with the prediction of
the MLP, checking the accuracy of the MLP. In the Section 4 it will be shown
that the training phase developed is appropriate to achieve accurate results.

The MLP architecture is characterized by the number of neurons arranged in
the input, hidden and the output layer. The input layer has the same number
of neurons as input variables, in this case 6 (the ply orientations); while the
output layer has, in this case, only one neuron which correspond to the ballistic
limit. As it is said previously, the number of hidden layers and its number
of neurons are not determined by the problem. In this case only one hidden
layer has been selected, as it is recommended in the paper of D. Fernandez-Fdz
et al.[29]. The number of neurons in the hidden layer has been chosen after
an optimization process, following the same procedure as in [28], which results
can be seen in Fig. 2. As the number of neurons is increased a significant
reduction in the error can be seen, exhibiting that the error oscillates around
a 6%. As it is done in other works [29, 28], it has been selected the value of
neurons which produced the minimum error; in this case the MLP with 9 hidden
neurons produce an average error in the learning process of only a 3.7 %. As
it was mentioned previously, one of the major advantages of the ANN is its
highly computational efficiency. To check this, the computational time in the
prediction of the ballistic limit of the laminates for each architecture is plotted
in Fig. 2. A linear tendency can be seen and it has to be remarked that the
chosen case, only has a computational time of 9 s for predicting the ballistic
limit of 2048 different laminates. Hence, it can be concluded that the MLP
fulfils the requirements of accuracy and efficiency.

Concerning the activation function, that it was previously explained in Sec-
tion 2, it has been used a linear function for the input layer and a tangent
hyperbolic function in both the hidden and output layer. In order to use this function the input and output data has to be normalized between -1 and 1. A value of 1 has been selected in the learning rate ($\xi$) of the hidden layer and 0.1 for the output layer. It has to be mentioned that these values are in accordance with the values used in other works [29]. Finally, a value of 1000 epochs (number of iteration) has been selected assuring an appropriate learning process avoiding the overlearning of the MLP. Fig. 3 shows an scheme of the MLP developed.

Figure 2: Average error and computational time versus the number of hidden neurons

Figure 3: Scheme of the MLP developed
4. Results

In this section it is included the experimental, numerical and ANN results obtained in the prediction of the ballistic limit of composite laminates with different laminate stacking sequence. The results are divided in three sections: the validation of the numerical model comparing the numerical and the experimental results, the ANN learning process by means of the data obtained from the virtual testing campaign and finally the ballistic limit prediction.

4.1. Experimental and numerical results: Validation

In order to validate the numerical model, three variables have been selected to compare the numerical results with the experimental ones: the residual velocity, the ballistic limit and the delaminated area after the impact. The residual velocity is the key parameter to assure the correct simulation of the process since it represents the energy dissipated by the laminate through its failure mechanisms.

The ballistic limit \( v_{\text{bl}} \) is obtained from the residual velocity using the Lambert-Jonas equation [31] for both experimental and numerical results. This equation relates residual velocity with impact velocity \( v_{\text{res}} = a(v_{\text{imp}}^{1/p} - v_{\text{bl}}^{1/p})^p \) being \( a, v_{\text{bl}} \) and \( p \) three fitting parameters. When a composite laminate is subjected to an impact delamination is one of the failure mechanism that appears. Although in terms of energy dissipation it could be neglected for high velocity impacts [24, 25], its occurrence could modify remarkably the kinematics of the impact and therefore it may influence the apparition of other failure mechanisms, such as fibre failure.

In Fig 4 (a) it can be observed the residual velocity versus the impact velocity for both numerical and experimental results. In the experimental tests, the residual velocity is obtained by means of a high speed video camera. It can be seen a very good agreement between both results. Below the ballistic limit the residual velocity is equal to zero, while once the ballistic limit is reached it is seen a steep slope of the residual velocity. Finally, as the impact velocity increases residual velocity tends to have a constant slope. In Fig. 4 (a), it is
included a detailed view of the region in the vicinity of the ballistic limit where
it can be seen an accurate prediction of the residual velocity which implies a
proper ballistic limit determination. To check this, it is obtained the Lambert-
Jonas equation for experimental and numerical results, included in Fig 4 (a),
in which the ballistic limit is obtained. The difference of the ballistic limit of
experimental and numerical results is only a 2.1 %.

Fig 4 (b) shows how the impact velocity affects the delaminated area gen-
erated in the laminate. As the impact velocity increases, the delamination
becomes higher, reaching a maximum value when the impact velocity is close
to the ballistic limit. Afterwards, the trend is the opposite, as the impact ve-
locity increases, the delaminated area decreases tending to an asymptote which
is the value of the frontal area of the projectile [20]. From the comparison it
is possible to observe a good correlation between experimental and numerical
delaminated area through all the range of velocities considered. Another option
to validate the numerical model is paying attention not only on the value of the
delaminated area, but also to its shape. Fig 5 shows the experimental C-scan
images obtained by means of an ultrasonic inspection (5MHz pulser), and the
corresponding damaged area obtained numerically for two different impact ve-
locities. It has to be noticed that in the experimental images it can be seen two
different failure mechanism: the delaminated area and the debonding of the last
ply; while in the numerical images only delamination is obtained. It is possi-
ble to state that the shape of delamination is also well captured. At velocities
close to the ballistic limit the shape of the delaminated area presents the typical
peanut shell patterns. As the impact velocity increases its delaminated shape
is reduced and becomes rounder. According to the results obtained, it can be
concluded that the numerical model reproduces the ballistic performance of the
laminate and therefore it will be used in the learning process of the ANN.
4.2. ANN training process

Once the numerical model is validated, a virtual testing campaign has been performed using the FEM model to provide the data required for the learning process of the MLP. As it was already said, 40 different laminate stacking sequences have been considered. For each laminate, at least a set of 6 simulations with different impact velocity have been performed in order to assure a correct determination of the ballistic limit using Lambert-Jonas equation. As an example, Fig. 6 shows the corresponding residual velocity versus impact velocity for three of the forty laminates considered: (0/0/0/90/90/90)$_S$, (45/45/45/90/45/45)$_S$ and (0/0/45/−45/0/0)$_S$. In addition the Lambert-Jonas equation obtained is included in each figure. The parameter of Lambert-Jonas equation has been obtained by curve fitting of $V_r - V_i$ curve.
In the learning process, 37 out of the 40 cases have been provided to the ANN and the other 3 have been used to do the testing validation of the learning process. The testing validation consist on comparing the results of the 3 cases (whose results are not provided to the MLP for the learning process) with the prediction of the MLP, checking the accuracy of the MLP.

Fig 7 shows the ballistic limit obtained with the FEM model versus the same results predicted by the ANN, distinguishing the 37 learned cases and the 3 testing validated cases. As it can be seen all the cases are inside the marked region that represent a maximum error of a 10% of error, being the maximum error in the ballistic limit ANN prediction a 9 % while the average error is only
a 3.7 %. If only the three cases of the testing validation are taking into account
the average error is about a 7 %. Therefore, it can be concluded that the ANN
provides a good prediction of the ballistic limit and it is suited to determine the
ballistic limit for different laminate stacking sequence and it could be used to
find the optimum configuration.

Figure 7: Ballistic limit prediction of FEM model and ANN obtained in the learning process

4.3. ANN ballistic limit prediction

Once the ANN have learned, the ballistic limit of all the possible configurations of laminate stacking sequence with orientations of 0, ±45 and 90 have been obtained. As it has been already said, there are $4^6 = 4096$ different laminate stacking sequence for symmetrical laminates of 12 plies, but due to symmetry only half of the laminates has to be studied. In Fig. 8 (a) it is included the ballistic limit for all the configurations studied obtaining values from $V_{bl} = 92 \text{ m/s}$ to 130 m/s. It can be seen that varying laminate stacking sequence, which has no consequence in the weight, the ballistic limit can be increased a 41 %, which in terms of energy means almost double the ballistic performance; this can be
seen as a promising result for designing purposes. In addition, the figure shows a particular pattern which can be explained due to the way the laminates are sorted. First half of the laminates corresponds to the ones whose outer layer is 0° while the rest are the ones that is 45°. It can be seen that the ballistic limit is clearly influenced by the orientation of the first ply; while a certain amount of laminates of the first half are above 120 m/s only a few of the second half reach this value. In each half, it can also be seen four different trends because the laminates are also ordered with the orientation of the second outer layer. In the figure it is indicated the different groups in order to clarify it. Fig. 8 (b) shows a detailed view of the group of laminates whose two outer layers are (0/90) in which the best results can be seen. An increasing trend can be seen as the third outer layer is changing from 0° to 90°. Therefore, the better results in terms of the maximum value and minimum variability correspond to the laminate family (0/90/90/a/b/c)_S where a, b, and c can be any of the different ply orientation studied (0°, ±45° and 90°). Moreover, the best laminate belongs to this family and is (0/90/90/0/90/0)_S which has a ballistic limit of 130.2 m/s. All the previous findings lead to think that the orientation of the outer layers plays an important role for the laminate ballistic limit. Therefore, not only the orientation of the plies but also their position with respect to the mid-plane is a factor that influences the laminate ballistic limit. In order to understand these results a deeper analysis for all the laminates studied has been performed.

5. Discussion

As it has been seen previously, the laminate stacking sequence has a clear influence in the ballistic limit, with differences that can reach more than a 40%. The differences are influenced not only by the orientation of the plies but also by their position with respect to the mid-plane. As it is known, the distance with respect to the mid-plane plays an important role in the determination of the laminate bending stiffness matrix $D$, and therefore this matrix may be a major responsible for the ballistic limit. Matrix $D$ is obtained by $D = \frac{1}{3} \sum_{i=1}^{n} \bar{Q}(\gamma_i^3 -$
\[ z^3_{i-1} \] where \( n \) is the number of plies of the laminate, \( \overline{Q} \) is the ply stiffness matrix, and \( z_i \) and \( z_{i-1} \) is the top location and the bottom location of the ply with respect the mid-plane. Consequently the ply stiffness matrix is multiplied by a magnitude that increases with the cube of the distance to the mid-plane.

In order to capture the influence of the bending stiffness matrix with the ballistic limit, Fig 9 shows the ballistic limit versus the equivalent bending stiffness \( (D^*) \). The equivalent bending stiffness is defined as \([32, 33]\) \[ D^* = \sqrt{\frac{A+1}{2} D_{11} D_{22}} \] where \( A = \frac{D_{11} + 2D_{66}}{D_{11} + D_{22}} \) and \( D_{ij} \) corresponds to the position of the \( i \) row and \( j \) column in the matrix \( D \). This parameter is related with the response of composite laminate when subjected to low velocity impacts \([34, 35]\). As it can be seen, there is no clear trend in this graph, although it can be noticed that there are a wide range of variability in the ballistic limit in the region of maximum equivalent bending stiffness \( D^* > 65 \text{ Nm} \), including the maximum and minimum values of the ballistic limit. This effect can be explained because the definition of the equivalent bending stiffness takes into account different terms, such as the component of the bending stiffness with respect to the in-plane axis \( (D_{11} \text{ and } D_{22}) \) but also the bending stiffness values related to the torsional moments \( (D_{66}) \) and the coupling effects \( (D_{12}) \). It is reasonable to think that due to the physics of the problem, in which the impact (at least in the velocity near to ballistic limit) mainly induces bending deformation in the plate, the first terms are playing a major role in this problem.

Therefore, the relation of the ballistic limit versus these parameters \( (D_{11} \text{ and } D_{22}) \) has been analysed; first regarding the magnitude of an average of them and finally comparing the differences between both. In Fig. 10 (a) it is shown the relation between the ballistic limit of the laminates with respect to the geometric mean of the parameters \( D_{11} \) and \( D_{22} \). Although a variability can be observed, it can be seen a trend that relates higher values of the ballistic limit with an increase in the geometrical mean of \( D_{11} \) and \( D_{22} \). Thus, it confirms that the physics of the problem is influenced by the in-plane components of the laminate bending stiffness matrix. This effect produces a global bending in the plate obtaining the higher deformation in the outer plies in the 11 and
22 direction. The best way to increase the value of $D_{11}$ and $D_{22}$ is to use plies at 0° and 90° since the fibres are oriented in the direction $x$ and $y$ of the plate (that correspond to the axes 1 and 2). It has to be mentioned that the 0° and 90° plies have to be placed preferably in the outer layer since outer plies have more influence in the laminate bending stiffness matrix than inner ones. Hence, these laminates have the fibres in the same direction and location as the maximum strain are observed, becoming more efficient to carry the stress-strain state induced by the impact. Regarding the lower values of $D_{11}$ and $D_{22}$, it has to be said that they are related to laminates whose outer layer are composed mainly by ± 45° plies and then they are not so efficient for the present impact load conditions.

In Fig. 10 (b) it is shown the relation between the ballistic limit of the laminates with respect to the absolute value of the logarithm of the ratio of the parameters $D_{11}$ and $D_{22}$. A value $|\log \frac{D_{11}}{D_{22}}| = 0$ means that both values are equal while higher values means increasing differences. It can be noticed that the higher values of the ballistic limit are in the region where $D_{11}$ and $D_{22}$ are similar between them while in the region of maximum differences of $D_{11}$ and $D_{22}$, there is no laminate that reach a ballistic limit of 120 m/s. This can be explained because a difference in the stiffness would produce uneven plate deformation and may trigger earlier the failure mechanism in one direction than in other. Nevertheless in the region of $|\log \frac{D_{11}}{D_{22}}| = 0$ it can be seen also the minimum values of the ballistic limit, that correspond to laminates that have mainly plies at ± 45° that produce no difference regarding direction 11 and 22 in matrix D. As it has been explained previously for Fig. 10 (a) the use of ± 45° does not increase enough the value of the parameters $D_{11}$ and $D_{22}$ leading to lower ballistic limit.

To understand finally the influence of $D_{11}$ and $D_{22}$ parameters with the ballistic limit it has been included the Fig. 11 in which it has been analysed the ratio of the parameters $D_{11}$ and $D_{22}$ with respect to the geometric mean of these parameters, while the ballistic limit is depicted with a contour map in each marker. It can be seen, as the other figures show, that the laminates with a
higher ballistic limit are in the region with a higher value of the geometric mean of the parameters $D_{11}$ and $D_{22}$, but with similar values ($|\log \frac{D_{11}}{D_{22}}| = 0$). This can be explained because the most efficient way to bear the load is to have the higher bending stiffness as possible for both main directions but with similar values avoiding uneven deformation and failures.

In order to confirm this explanation it has been included in Table 1 a comparison of several laminate stacking sequences, showing the $D_{11}$ and $D_{22}$ parameters and its ballistic limit. The first two laminates included in the table, laminate A $(0/90/90/0/90/0)_S$ and B $(0/90/90/0/0)_S$, belong to the optimum laminate family (Fig. 8 (b)); as it was said previously these laminates are $(0/90/90/a/b/c)_S$ where $a$, $b$, and $c$ can be any of the different ply orientation studied ($0^\circ$, $\pm 45^\circ$ and $90^\circ$). Laminate A presents the maximum ballistic limit predicted by the ANN technique and its performance under impact can be explained because its higher geometrical mean of $D_{11}$ and $D_{22}$ ($\sqrt{D_{11}D_{22}} = 85.7 \text{ Nm}$) and the lower differences between these values ($|\log \frac{D_{11}}{D_{22}}| = 0.05$) compared with the other laminates. Laminate B shows also a high value of $\sqrt{D_{11}D_{22}}$, nevertheless the increment in the difference between $D_{11}$ and $D_{22}$ ($|\log \frac{D_{11}}{D_{22}}| = 0.16$) leads to the reduction in the ballistic limit. The third laminate included, laminate C $(0/90/0/90/0/0)_S$, presents higher value of $|\log \frac{D_{11}}{D_{22}}| = 0.46$ which means a lower ballistic limit, although the geometrical mean is similar to laminates A and B $\sqrt{D_{11}D_{22}} = 83.5 \text{ Nm}$. Finally, it is included laminate D, $(45/-45/45/45/-45)_S$ whose reduction of the $\sqrt{D_{11}D_{22}}$ is produced by the presence of the $\pm 45^\circ$ plies in the laminate stacking sequence, leading to an important reduction in the ballistic limit even without differences between $D_{11}$ and $D_{22}$ ($|\log \frac{D_{11}}{D_{22}}| = 0$). As it can be seen, the ANN technique allows to find the optimum laminate stacking sequence and to achieve a deeper comprehension of composite plates when subjected to high velocity impact with a low-time consumption. Conclusions obtained can be used by composite design engineers to improve ballistic performance of composite plates.
Table 1: Comparison of laminate stacking sequence, ballistic limit and $D_{11}$ and $D_{22}$ parameters.

| Laminate | Stacking sequence        | $D_{11}$ [Nm] | $D_{22}$ [Nm] | $\sqrt{D_{11}D_{22}}$ [Nm] | $|\log \frac{D_{11}}{D_{22}}|$ | $v_{bl}$ [m/s] |
|----------|--------------------------|---------------|---------------|-----------------------------|-------------------------------|---------------|
| A        | (0/90/90/0/90/0)$_S$     | 87.8          | 83.7          | 85.7                        | 0.05                          | 130.2         |
| B        | (0/90/90/0/0)$_S$        | 92.7          | 78.8          | 85.5                        | 0.16                          | 114.9         |
| C        | (0/90/0/90/0)$_S$        | 105.3         | 66.2          | 83.5                        | 0.46                          | 109.9         |
| D        | (45/-45/45/45/-45)$_S$   | 50.3          | 50.3          | 50.3                        | 0.0                           | 99.4          |

6. Conclusions

In this work, a combined methodology of experimental, Finite Element Method and Artificial Neural Network has been used to predict the ballistic limit of composite laminates varying its laminate stacking sequence. Several conclusions must be drawn after the analysis and discussion of the results.

- The proposed combined methodology allows to develop an ANN that predict ballistic limit of composite plates with different laminate stacking sequence successfully.
- The numerical model, validated by experimental tests, is able to reproduce the influence of the laminate stacking sequence in the ballistic limit.
- The ANN has been trained with a virtual testing campaign performed with the FEM model. It is obtained a 3.7% error in the learning process, while a 7% for the testing validation cases. The optimum results has been obtained with only one hidden layer composed by 9 neurons.
- Once the ANN is trained, the whole range of possibilities of laminate stacking sequence using the common orientation in symmetric 12 plies laminates have been studied. It has been observed that the ballistic limit can be increased up to a 41% percent without any weight increase. The optimum laminates are the ones with the stacking sequence $(0/90/90/a/b/c)_S$.
- The ANN helps to understand the problem of ballistic limit optimization. The key point to obtain the optimum laminates (for the conditions studied
in this work) is to use 0° and 90° for the outer plies in such a way that the values of $D_{11}$ and $D_{22}$ are high and similar between them.

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References


Figure 8: (a) ANN ballistic limit prediction for the whole range of possibilities of laminate stacking sequence using the common (0°, 45°, -45° and 90°) in symmetric 12 plies laminates. (b) Detail of the ANN ballistic limit prediction for the laminates which outer plies are 0/90°.
Figure 9: ANN ballistic limit prediction versus the equivalent bending stiffness ($D^*$).
Figure 10: ANN ballistic limit prediction versus (a) geometric mean of the parameters $D_{11}$ and $D_{22}$ (b) the absolute value of the logarithm of ratio of the parameters $D_{11}$ and $D_{22}$. 
Figure 11: ANN ballistic limit prediction versus geometric mean of the parameters $D_{11}$ and $D_{22}$, the absolute value of the logarithm of ratio of the parameters $D_{11}$ and $D_{22}$. 