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## Analysis of electroperforated materials using the quadrat counts method

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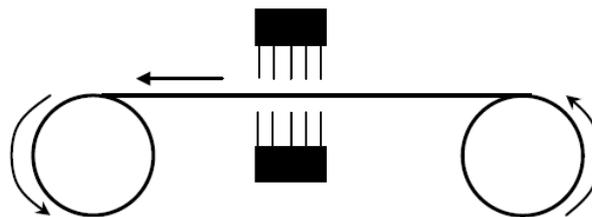
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**Abstract.** The electroperforation distribution in thin porous materials is investigated using the quadrat counts method (QCM), a classical statistical technique aimed to evaluate the deviation from complete spatial randomness (CSR). Perforations are created by means of electrical discharges generated by needle-like tungsten electrodes. The objective of perforating a thin porous material is to enhance its air permeability, a critical issue in many industrial applications involving paper, plastics, textiles, etc. Using image analysis techniques and specialized statistical software it is shown that the perforation locations follow, beyond a certain length scale, a homogeneous 2D Poisson distribution.

### 1. Introduction

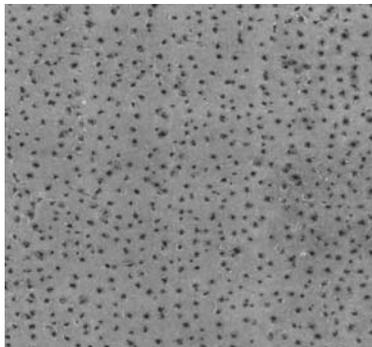
Electroperforation is a well-known method used to generate a hole pattern in thin porous materials like paper webs, plastics, textiles, etc, with the objective of modifying their air permeabilities [1]. It basically consists in the formation of high-energy electrical discharges passing through the material, which often moves at a high speed in between arrays of opposite needle-like electrodes (see Fig. 1). The method is fast and reliable so that it is well-suited for industrial applications. However, a major limitation of this technique is that there is a limit to the number of holes that can be created in a certain area due to the fact that the already generated holes offer an easier discharge path between the electrodes. Because of this self-limited hole generation process the porosity cannot be further increased beyond a certain level even under longer exposure times (lower velocities) or higher discharge frequencies. Figure 1 illustrates a typical perforation system with needle-like electrodes.



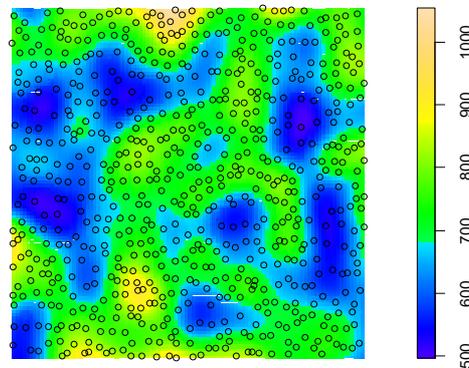
**Figure 1.** Electroperforation process of a web material using two arrays of electrodes.

Once the hole pattern has been created, it is necessary to characterize the hole distribution by means of exploratory statistical techniques. The hole pattern is regarded as a 2-D point process, which can be investigated using the Spatstat package for R language. R is a free software environment for statistical computing and can be downloaded from [2]. Spatstat can also be downloaded from the same website. Spatstat has been developed by Prof. A. Baddeley and Prof. R. Turner and is a specific package designed to be used for spatial data manipulation, exploratory data analysis, simulation, fitting and model diagnostics. In a few words, the aims of point process statistics are to describe the locations of randomly distributed objects in 1, 2, or 3-D space and, if possible, understand the mutual interactions that led to that distribution [3]. Numerous techniques have been developed to detect clustering (aggregation) or repulsion effects, as well as to investigate the distance scales involved with the purpose of inferring the physical underlying mechanism responsible for the pattern formation.

In this work, in order to characterize the hole distribution generated in a thin porous material, the quadrat counting method (QCM) is considered [3]. QCM is one of the most widely used tests in point pattern analysis and is included, together with a  $\chi^2$  (chi-square) goodness-of-fit test for homogeneous or inhomogeneous Poisson processes in the Spatstat package. In this method, the window containing the point pattern is divided into a grid of rectangular tiles or ‘quadrats’, and the number of points falling in each quadrat is counted. The quadrats are square by default but may have arbitrary shape. However, as it has been pointed out in literature, one major drawback of QCM is that the choice of the quadrat number is strongly linked to the spatial scale of the problem, which establishes a minimum quadrat size. This limitation has been investigated using the Morishita index (MI) [4]. Further analysis based on the Pearson residuals provided by Spatstat allows to evaluate the accuracy of having assumed a complete spatial randomness (CSR) model for the generation of holes.



**Figure 2.** Photograph of an electroperforated material. The dots are holes pierced through the thin porous material.

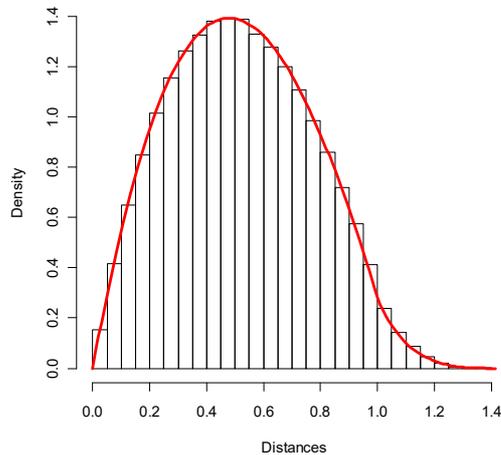


**Figure 3.** Density plot for a section of the electroperforated material similar to the one shown in Figure 2.  $N=695$  circles (holes) were counted.

## 2. Exploratory analysis of the perforation pattern

Figure 2 shows a typical example of an electroperforated material. For simplicity, the size of the square has been normalised to unity. A first characterization of the hole distribution is given by the so-called intensity plot. The color map in Figure 3 corresponds to the magnitude of the local intensity estimator (number of points weighted by an isotropic Gaussian smoothing kernel), which is the

average density of points, *i.e.* the expected number of points per unit area. Notice that for this particular realization the intensity is not homogeneous as it is expected for a Poisson process.



**Figure 4.** Histogram for the normalised distances between all holes. The size of the square has been normalized to 1x1. The histogram comprises  $(N^2-N)/2=(695^2-695)/2=241165$  values. The solid line corresponds to the theoretical probability density function given by expression (1).

Quantitative information about the hole distribution can be achieved by analyzing the interpoint distances. The histogram in Figure 4 shows that the distribution of the normalised distances  $d$  between all circles in Figure 2 follows the theoretical probability density (solid line) given by the expression [5]:

$$f(d) = \begin{cases} 4d \left[ \frac{\pi}{2} - 2d + \frac{d^2}{2} \right] & 0 \leq d \leq 1 \\ 4d \left[ \arcsin\left(\frac{1}{d}\right) - \arccos\left(\frac{1}{d}\right) - 1 - \frac{d^2}{2} + 2\sqrt{d^2 - 1} \right] & 1 < d \leq \sqrt{2} \end{cases} \quad (1)$$

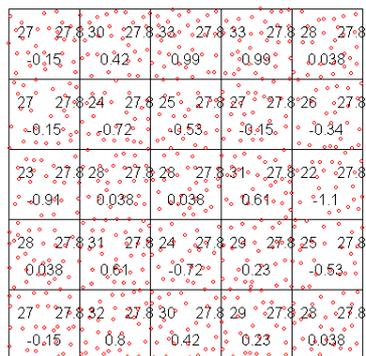
This function is obtained under the hypothesis that the x- and y-coordinates of each data point are uniformly distributed in the range [0,1]. It must be noted that the agreement is very good, what at first sight seems to indicate that the distribution is consistent with a CSR process. However, the main problem with this approach is that the histogram representation is not sensitive to the details of the distribution, in particular with respect to what happens at the smaller length scales (lower tail).

In order to get further insight into the hole pattern distribution it becomes necessary to use an alternative exploratory technique such as the already introduced QCM. In this method, the window containing the electroperforation pattern ( $N$  points) is divided into a grid of  $Q$  quadrats. The number of points falling in each quadrat ( $i$ ) is  $n[i]$ . One major drawback of the QCM is that the appropriate choice of the quadrat size is strongly linked to the spatial scale of the problem, which is often unknown. This apparent limitation can be overcome with the help of the Morishita index (MI):

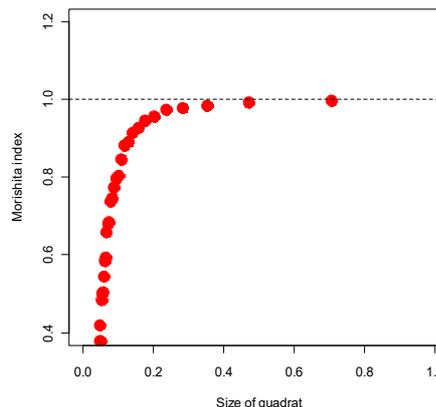
$$MI = Q \sum_{i=1}^Q \frac{n[i](n[i]-1)}{N(N-1)} \quad (2)$$

an index used to discern different scales of dependence in the point pattern data. The basic idea behind the QCM test is to find evidence against the null hypothesis of a CSR process. In what follows an

illustrative example of the aforementioned analysis is presented. Figure 5 shows the perforation distribution ( $N=695$ ) and a particular quadrat plot ( $Q=5 \times 5$ ) for the data shown in Figure 2. The choice of  $Q$  can severely affect the results of the analysis and therefore it must be taken into account to avoid misinterpretations. Figure 6 shows MI as a function of the size of the quadrats. For a completely random pattern  $MI \approx 1$ . In our example, CSR is clearly detected for  $Q \leq 25$  (linear dimension approximately  $\geq 0.2$ ). Values of  $MI < 1$  suggest repulsion or hard core potential effects [3].



**Figure 5.** Electroperforation distribution and an example of a quadrat plot ( $5 \times 5$ ).



**Figure 6.** Morishita index as a function of the quadrat size.

Assuming that the  $n[i]$  values are independent and identically-distributed Poisson random variables with the same expected value, the Pearson  $\chi^2$  test can be evaluated, which in turn can be used to reject or accept the CSR model. Figure 5 includes the observed number of points in each quadrat, the predicted number of points according to a fitted Poisson model (obtained by simulation with Spatstat) and the Pearson residuals ( $=[(\text{observed}) - (\text{expected})] / \sqrt{\text{expected}}$ ). A small Pearson residual indicates good agreement with the CSR model.

### 3. Conclusions

Electrical discharges are often used to perforate thin porous materials in industrial environments. In order to characterize the resulting hole distribution, statistical techniques are imperatively required. In this work, we have shown how the quadrat counts method in combination with the Morishita index can be applied to determine the minimum length scale from which the electroperforation pattern can be strictly regarded as a 2-D Poisson process.

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