Theoretical and finite element approaches to model the phenomenon of high-velocity impact on GFRP plates

by

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If you want you’ll find the way, if you don’t you’ll find the excuse
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- Alonso L., García-Castillo S.K., Navarro C. Study of the influence of the peels on the behaviour of sandwich structures subjected to high-velocity impact. *MatComp2019*. This publication is partially included in Chapter 4.


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Abstract

Composite materials are widely used nowadays to build structures and protections. Their good mechanical properties along with their light weight has lead to an increase in the use of these materials over the last decades. Specifically, Glass Fibre Reinforced Polymers (GFRP) plates are really resistant to corrosion and permeable to electromagnetic waves. Because of that, GFRP are suitable for naval and telecommunication applications.

Although these structures may not be designed as protections they may be subjected to different kind of impacts during their service life. Since some elements can be critical, safety must be ensured and thus engineers need a deeper knowledge of the impact performance. That is why the development of analytical, theoretical and finite element models is a hot topic in order to predict critical design factors such as the ballistic limit.

This doctoral thesis focuses on the modelling of the high-velocity impact phenomenon through different approaches in order to reproduce better the effects that take place during a high-velocity impact.

First of all, a wide set of experiments have been carried out to obtain the data necessary for the models. Characterization tests have been carried out to know the mechanical properties needed for the models. Furthermore, ballistic impact tests have been performed on 9 mm thick specimens. In addition, non-destructive inspection tests have been carried out to evaluate the damaged area after impact.

After that, two non-dimensional energy-based theoretical models are presented in order to predict the ballistic behaviour of woven GFRP plates. Some important non-dimensional groups that are useful to analyze the behaviour and validity of the models appear naturally using this formulation. Hypotheses based on physical phenomena are also taken into account in the formulation.

Then, a finite element model is developed by means of a new constitutive model and a continuum damage mechanics approach to predict the performance of GFRP plates under high-velocity impact. In addition, the failure modes of this model are formulated in such a way that allows to associate the energy dissipated by each of them to the energy absorption mechanisms defined in the theoretical models.

Finally, some of the new hypotheses assumed in the theoretical models are validated with the finite element one. Furthermore, the energy absorption mechanisms are compared between both approaches to draw enlightening conclusions.
Los materiales compuestos se utilizan ampliamente en la actualidad para construir estructuras y protecciones. Sus buenas propiedades mecánicas junto con su bajo peso específico han favorecido un incremento en el uso de estos materiales durante las últimas décadas. En concreto, los laminados reforzados con fibra de vidrio (GFRP en inglés) son muy resistentes a la corrosión y permeables a las ondas electromagnéticas. Por esta razón, los GFRP son indicados para aplicaciones navales y de telecomunicaciones.

Aunque estas estructuras pueden no estar diseñadas como protecciones, son susceptibles de estar sometidas a diferentes tipos de impacto durante su vida útil. Dado que algunos elementos pueden ser críticos, se debe asegurar la seguridad y por lo tanto, los ingenieros necesitan un conocimiento más profundo del comportamiento frente a impacto. Es por ello que el desarrollo de modelos teóricos y de elementos finitos es una línea de investigación de interés que permite predecir factores de diseño críticos como el límite balístico.

Esta tesis doctoral está enfocada en la modelización del fenómeno de impacto a alta velocidad a través de diferentes métodos para poder reproducir de la manera más fidedigna los efectos que tienen lugar durante un impacto a alta velocidad.

Primero, se ha llevado a cabo un gran número de experimentos con el objetivo de obtener los datos necesarios para los modelos desarrollados. Se han realizado experimentos de caracterización mecánica para determinar las propiedades requeridas por los modelos. Además, se han llevado a cabo ensayos de impacto balístico en especímenes de 9 mm de espesor. Igualmente, se han realizado ensayos de inspección no destructiva (IND) para evaluar el área dañada del laminado después de impacto.

Posteriormente, se presentan dos modelos teóricos no dimensionales basados en criterios energéticos para predecir el comportamiento balístico en laminados GFRP. Algunos grupos no dimensionales que son útiles para analizar el comportamiento y la validez de los modelos aparecen de forma natural al utilizar esta formulación. En la formulación se tienen en cuenta hipótesis basadas en fenómenos físicos.

A continuación, se desarrolla un modelo de elementos finitos mediante un modelo constitutivo a través de un modelo de daño continuo para predecir el comportamiento de los laminados GFRP frente a impactos de alta velocidad. Además, los modos de fallo
están formulados de tal forma que la energía disipada por cada uno de ellos se puede asociar a los mecanismos de absorción de energía definidos en los modelos teóricos.

Por último, algunas de las nuevas hipótesis asumidas en los modelos teóricos se han validado con el modelo de elementos finitos. Los mecanismos de absorción de energía se comparan entre los modelos teóricos y el modelo de elementos finitos para extraer conclusiones esclarecedoras.
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1.1 Motivation

Many design criteria have been found since ancient times for the calculation of structural elements. These criteria must meet the operating requirements established. Between them, the mechanical behaviour of the materials used in the construction of these structures is a fundamental aspect.

The structures and components built nowadays are used in environments where they may withstand impacts from different objects apart from their daily use. Therefore, it is important to ensure the safety of this structures under extraordinary circumstances. Moreover, the failure of critical components may lead to irreparable disasters with economic and specially human losses.

Nowadays, laminated composites are widely used in the manufacturing of different structural components in the aerospace, naval and civil industries among others. These materials have the best cohesion properties of the matrix (epoxy, vinylester, polyester) as well as the resistance provided by the fibres (Kevlar, carbon, glass).

For instance, the aerospace sector has grown exponentially in the last decades and aircrafts have to be resistant enough to bear impacts from space objects. The structural design of the International Space Station (ISS), which is an aircraft of large dimensions and intended to be used for a long time, was carried out mainly with composite materials. The high probability of small debris as well as the inability to detect those objects due to their small size led to a design with different layers in order to protect the aircraft. NASA developed a shield made of the following layers from the outside in: aluminum, ceramic with Kevlar fibre and aluminum again.

Specifically, E-glass composite materials are widely used in sectors such as maritime, aerospace or transport. Other applications of glass fibre are the construction of pressurized pipes, boat hulls and storage containers.
Motivation

For instance, the recent FibreShip (2019) project is focused on the construction of large ships using GFRP’s. Many companies and universities have been involved in this project. The use of this material instead of the traditional steels presents some advantages such as 30 % weight reduction and consequently 15 % fuel consume reduction, 12 % load capacity increase, 5 % operating cost decrease, better stability and sustainability of the ship and service life increase as a consequence of being immune to corrosion.

In order to study the response of these structures under high-velocity impact a lot of works can be found in the literature. Fujii et al. (2002), Buitrago-Pérez et al. (2010), Briesciani et al. (2016) studied the ballistic behaviour of composites made of Carbon Fibre Reinforced Polymers (CFRP), glass/polyester structures and Kevlar plain-woven composites respectively by means of experimental tests. Many authors such as Gellert et al. (2000), Wen (2000), Deka et al. (2008) have also studied experimentally such parameters as thickness on the influence of the ballistic behaviour in glass-fibre and Fibre Reinforced Polymers (FRP) composites.

Others authors have studied the ballistic performance of composite materials by means of analytical and theoretical models which have been proved to provide good predictions through a relatively simple mathematical modelling of the problem. Analytical models for polymer composites have been proposed by Moyre et al. (2000), Wen (2001), Naik and Shrirao (2004), Naik and Doshi (2005) developed analytical models for thin and thick woven composites. López-Puente et al. (2007) proposed a model for woven CFRP plates. These models are useful because once they are formulated and tested they allow to predict such aspects as the ballistic limit with accuracy, speed and few resources.

Finite element approaches are another tool to tackle the phenomenon of high-velocity impact. These studies have been proved to be successful to predict the performance of composite materials. Xiao et al. (2007) developed a numerical model to predict progressive damage in plain-weave glass composites. Maimí et al. (2007a) developed a constitutive model to predict intra-laminar failure in unidirectional laminates. Ansari et al. (2017) proposed a continuum damage formulation to differentiate out of plane from in-plane damage for woven GFRP plates. These models can complement or replace analytical and theoretical models to study the phenomenon of high-velocity impact.

Within this context, it is clear that composite materials are used in many applications and thus, a deep understanding of the behaviour of these structures is necessary. Furthermore, when human lives are involved, all the available resources must be used to do research on the subject. For this reason, a full study in woven E-glass fibre/polyester plates against high-velocity impact is carried out in this thesis.

In addition, the three approaches mentioned above have been followed: experimental, theoretical and numerical. In this way we make sure that rigorous conclusions can be drawn in order to improve the design and thus the performance in service of these structures.
1.2 Objectives

The main objective of this work is to develop theoretical and finite element models to predict the high-velocity impact behaviour on woven GFRP plates. In order to accomplish this goal, several partial objectives have been reached during this work. They are defined in the following items:

- Development of two theoretical models based on energy criteria for thin and thick woven GFRP. These models are intended to be used to obtain results such as the ballistic limit or the energy absorbed by each energy absorption mechanism. In the literature different models have been found depending on the thickness because the energy absorption mechanisms are different. Therefore, the need for formulating two theoretical models to predict the behaviour of these laminates in the range of thicknesses is latent.

- Development of a finite element model by means of a subroutine to define the constitutive material response. The failure criteria must be adapted to the theoretical model so that they can be compared in different aspects. The main requirements of this model are to capture the ballistic response of the laminates studied and to complement the results given by the theoretical models.

- Carrying out impact tests on woven GFRP plates to obtain the ballistic response of this material in a wide range of thicknesses. Also, carrying out characterization tests in order to obtain the properties needed for all the models developed as well as non-destructive tests to evaluate relevant results such as the damage area of a laminate after impact.

- Validation of the theoretical and finite element models by means of the experimental ballistic results carried out in this work and from the literature (Buitrago-Pérez et al. (2010)).

- Validation of some of the hypotheses assumed in the theoretical models by means of the finite element model. In this way, the finite element model can be used for something more than predicting the ballistic response.

- Finding the threshold from which a laminate can be considered either thin or thick.

- Obtaining results from the theoretical models in order to validate them from a physical point of view. It can be accomplished by means of parametric studies.

- Comparison between the energies obtained by the theoretical models with the ones obtained by the finite element model. In this way the new failure criteria implemented can be tested.
1.3 Content description

With the aim to give a structure to the document, the chapters that make up this thesis are briefly explained in this section.

In Chapter 1 the motivation to carry out this work as well as the expected objectives are stated.

In Chapter 2 the procedures followed to perform the ballistic, characterization and non-destructive tests as well as the results obtained are explained.

In Chapter 3 two new theoretical models based on energy criteria for woven GFRP plates are formulated in a non-dimensional way. For both cases, first the energy absorption mechanisms considered are explained and then the governing equation is obtained.

In Chapter 4 a finite element model to predict the ballistic response of GFRP plates is presented. A new 3D continuum mechanics model is developed with failure criteria associated to the energy absorption mechanisms described in the theoretical models.

In Chapter 5 the validation of the theoretical and finite element models is carried out by means of the comparison with the ballistic response obtained in the experiments.

In Chapter 6 the main results obtained from the theoretical and finite element models are presented. There are also comparisons between them, which are used to prove that they are reliable.

In Chapter 7 the most important conclusions inferred from this work as well as the future work to continue this research are stated. Furthermore, the publications and conference contributions derived from this thesis are cited.

Chapter 8 is the same Chapter 7 written in Spanish.

In Appendix A all the details of the mathematical formulation of the two theoretical models are explained. In Appendix B the subroutine developed for the finite element model is explained in more detail with aspects such as the evolution of the softening functions through simple examples. In Appendix C, the blueprints of the tool designed for friction and shear tests are presented.
The aim of this chapter is to provide a detailed explanation of the tests performed within the framework of this thesis. First, we gathered the known properties of the composite plates obtained in previous works García-Castillo et al. (2012) and Buitrago-Pérez (2012). Then, we carried out characterization tests in order to obtain the properties needed for the models developed. With the objective of performing some of these tests, different devices had to be designed. Furthermore, we performed high-velocity impact tests on laminates 9 mm thick, which combined to the experimental results found in the literature (Buitrago-Pérez et al. (2010)), establishes a wide range of thicknesses tested. Finally, we used the C-Scan technique to determine, among other data, the damaged area in the wide range of laminates available.

2.1 Introduction

Many previous works have examined the impact behaviour of composite materials by means of experimental tests such as Focht and Vinson (2002), Fujii et al. (2002), Hazell et al. (2008), Buitrago-Pérez et al. (2010), Yashiro et al. (2013), García-Castillo et al. (2014) and Briesciani et al. (2016). These tests obtain the response of composite laminates under impulsive loads. However, they are costly and time consuming and they are only valid for the configuration (plate and projectile) tested.

Furthermore, other tests such as characterization or non-destructive tests can be carried out in order to obtain relevant information such as the damage inside a plate after impact. Depending on the damaged area, the performance in service can be compromised.

Laminate thickness is one of the main parameters affecting perforation, which bears an intimate relationship with areal density (composite mass per unit area). Numerous authors have studied the influence of these parameters on both the projectile perforation velocity and the perforation-threshold energy for laminates with different fibres and resin matrices. Some of these authors are Gellert et al. (2000), Wen (2001), Deka et al. (2008),
Grucijic et al. (2009), Gogineni et al. (2012), García-Castillo et al. (2012), Rahimi et al. (2015). Examination of these relationships has produced contradictory results, and both linear and non-linear relationships between minimum perforation velocity or minimum perforation energy and thickness have been observed in composite plates as can be seen in Cantwell and Morton (1990), Gellert et al. (2000), Wen (2001), Naik et al. (2006), He et al. (2007), Deka et al. (2008), García-Castillo et al. (2012), Rahimi et al. (2015).

In this regard, the behaviour of perforation velocity in relation to thickness has been seen to vary in glass-fibre laminates. Gellert et al. (2000) tested glass-fibre laminates of various thicknesses (4.5–20.0 mm) struck with three nose shapes rounds. A bilinear relationship was found between perforation-threshold energy and plate thickness. By proposing a simple model to explain this relationship, Gellert et al. (2000) stated that the projectile geometry does not affect the energy absorption mechanisms in thin GFRP plates.

By means of a wide set of tests carried out in thick laminates subjected to impact, Wen (2001) related the thickness specimen with the ballistic limit. The projectiles used in this study varied in mass and shape. There were found linear and non-linear relationships between these parameters when changing the mass and the diameter of the flat-ended low-mass projectiles used in this study for E-glass/polyester laminates. The trend followed was the same for hemispherical-ended projectiles. Deka et al. (2008) studied the loss of mechanical properties on E-glass/polypropylene plates struck in high-velocity regime through a finite element model. Cylindrical flat-nosed projectiles were used in this study. Deka et al. (2008) used a progressive-failure model based on Hashin’s failure criteria in order to estimate the energy absorbed by the laminate. A non-linear relationship was observed between perforation velocity and thickness. In addition, He et al. (2007), found that ballistic limit and thickness were related by a non-linear relationship based on the results provided by an analytical model. This study was carried out for thin glass-fibre laminates (less than 7 mm). The relationship found by Naik et al. (2006) was linear though for woven E-glass/epoxy plates. The impactors used in this study were flat-ended and the thicknesses studied ranged from 1.0 mm to 2.6 mm. Naik et al. (2006) applied an analytical model based on energy considerations in order to predict the variation of perforation velocity as a function of thickness. Furthermore, the analytical model developed by García-Castillo et al. (2012) predicted a linear variation for thin woven E-glass/polyester laminates for thicknesses ranging from 3 mm to 6 mm when they were impacted by spherical projectiles.

Laminate damaged area is another important parameter taken into consideration in studies of high-velocity impact as it is directly related to the residual strength of the component after impact. It has been observed that the damage generated is quite widespread in ballistic impacts on glass-fibre composites (De Luca et al. (1998)). It makes the damaged area a critical parameter to be taken into consideration in structural design. The ultrasonic C-Scan is a non-destructive technique that has the ability to
detect and quantify damaged extension in all thicknesses of composite laminates as it is shown in the studies carried out by Ball and Almond (1998) and Zheng et al. (2015). Gellert et al. (2000) demonstrated that the damaged extension depends on the laminate thickness. Delamination is cone shaped and opens towards the exit side of the plate if it is thin enough. This cone increases in diameter and height when increasing target thickness. If the thickness is increased up to a certain point, a delamination cone opening towards the impact side also appears.

This section aims at characterising woven E-glass fibre/polyester laminates. To this end, experimental analysis have been carried out through characterization, non-destructive and impact tests. Different aspects such as the ballistic limit, the influence of areal density on minimum perforation velocity, the contact time, and the extension of the damaged area (front and back sides of plates) are studied.

2.2 Characterization tests

Characterization tests are meant to provide the mechanical properties of a certain material to be able to predict its behaviour under any mechanical load. For this reason, some material properties such as the failure shear stress, the compressive failure strain or the friction force between the projectile and the laminate have to be determined. In addition, the theoretical and finite element models developed in this thesis need these parameters as inputs.

2.2.1 Laminates

The woven laminates studied in this thesis were manufactured from E-glass fibres embedded in a polyester matrix. In each lamina the fibres are interlaced perpendicularly (in the same way in all the laminae). The fibres are also aligned with the laminate border forming a woven alignment. The number of fibres is the same in both in-plane perpendicular directions.

The woven E-glass plies used (RT 800) were manufactured by SAINT-GOBAIN VETROTEX. Some of the advantages of this enforcement are its widespread compatibility with different kinds of resins as well as its dimensional stability and mechanical properties.

The matrix used was composed of a polyester resin (AROPOL FS6902). It was manufactured by ASHLAND. The more meaningful characteristics of this kind of matrix are its flexibility, which is beneficial to withstand demanding load conditions, and its remarkable wettability, allowing an optimal bonding of the E-glass fibre plies. Flexibility and wettability properties can be found in García-Castillo (2007).

Laminate processing was carried out by SISTEMAS Y PROCESOS AVANZADOS that produced the laminates in sets of square geometry pieces of 500 mm x 500 mm for different thicknesses.
Experimental procedure

For the characterization of the laminate properties such as density, tests were carried out by García-Castillo et al. (2012) in the Science and Material Engineering Laboratory at University Carlos III of Madrid according to MPIF-Standard-42 (1980). Moreover, other tests were performed in order to measure mechanical properties of interest such as the failure stress in the direction of the fibre or the failure strain García-Castillo et al. (2012).

It is also essential to take into account the fact that the laminate mechanical properties depend on the strain rate. The influence of strain rate on the mechanical properties has been studied for several authors such as Barré et al. (1996) for laminates made of an epoxy matrix reinforced with graphite, Ochola et al. (2004) for laminates made of a polymeric matrix reinforced with glass-fibre and Moyre et al. (2000) for laminates made of a phenolic matrix reinforced with nylon. Those studies conclude that there are remarkable differences in the laminate properties depending on the strain rate.

The tests undertaken by García-Castillo et al. (2012) were carried out in quasi-static conditions. As mentioned above, the mechanical properties of the laminate are different at high strain rates. The properties for these laminates for quasi-static conditions are summoned up in Table 2.1. The dynamic properties are obtained applying some coefficients to the static properties as indicated in García-Castillo (2007) as a first approach.

<table>
<thead>
<tr>
<th>Property</th>
<th>Nomenclature</th>
<th>Value</th>
</tr>
</thead>
<tbody>
<tr>
<td>Laminate density [kg m(^{-3})]</td>
<td>(\rho_l)</td>
<td>1980</td>
</tr>
<tr>
<td>Young’s modulus in directions 11 and 22 [GPa]</td>
<td>(E_{11} = E_{22} = E)</td>
<td>10.1</td>
</tr>
<tr>
<td>Shear modulus in direction 12 [GPa]</td>
<td>(G_{12})</td>
<td>4.4</td>
</tr>
<tr>
<td>In-plane failure strain</td>
<td>(\varepsilon_r)</td>
<td>0.03627</td>
</tr>
<tr>
<td>Poisson’s coefficient</td>
<td>(\nu)</td>
<td>0.16</td>
</tr>
<tr>
<td>Failure stress in directions 11 and 22 [MPa]</td>
<td>(X_{11r} = X_{22r} = \sigma_r)</td>
<td>367</td>
</tr>
<tr>
<td>Energy absorbed by matrix cracking per unit volume [J m(^{-3})]</td>
<td>(E_{MT})</td>
<td>(10^6)</td>
</tr>
<tr>
<td>Critical dynamic-strain energy-release rate in mode II [J m(^{-2})]</td>
<td>(G_{IICD})</td>
<td>3000</td>
</tr>
<tr>
<td>Yarn width [mm]</td>
<td>(B)</td>
<td>0.5</td>
</tr>
<tr>
<td>Stress wave transmission factor</td>
<td>(b)</td>
<td>0.9</td>
</tr>
<tr>
<td>Shape factor of delamination</td>
<td>(\alpha_{DL})</td>
<td>1</td>
</tr>
<tr>
<td>Shape factor of matrix cracking</td>
<td>(\alpha_{MC})</td>
<td>1</td>
</tr>
</tbody>
</table>

*Table 2.1 Summary of the static properties of woven E-glass fibre/polyester laminates*

2.2.2 Shear and friction tests

As mentioned before, some properties along the thickness direction needed to be obtained so that all the inputs for the models could be known. Therefore, we carried out friction and shear tests in order to obtain these properties.
2.2 Characterization tests

To do so, we used a universal test 8802 INSTRON machine. This machine is able to apply a maximum force of 100 kN and it has a maximum stroke of the piston of 75 mm.

In order to perform these tests in optimal conditions as it is said in ASTM-Standard-D732-02 (2002) we designed an specific tool based on Pandya et al. (2015). The assembly of this tool (Figure C.7) as well as the blueprints of the parts can be seen in Appendix C. Although some clear differences can be appreciated. It was designed to meet the requirements for the tests performed. The designed tool has a cylindrical shape instead of the typical square shape so that it can be adapted to the 8802 INSTRON machine. Such aspects as the spherical shape of the projectile were taken into account by means of the punch tools. In addition, four highlights were added to the lower cover to place the specimens.

In Figure C.1, the shear punch can be seen. It was manufactured in such a way that a cylinder of 6 mm diameter emerges from its end with the aim of introducing the punch inside the laminate so that it can guide the movement.

In Figure C.2, the friction punch is shown. This punch is quite similar to the shear one. However, it has a concavity at the end with the goal of guiding the projectile in its way through the laminate.

In Figure C.3, the coupling to the test machine is shown. This element was welded with the lower cover and both were connected with the lower part of the test machine. Furthermore, the lower cover has four highlights in order to center the laminate and four M4 threaded holes to place the fixing screws.

In Figure C.4, it can be appreciated that the upper cover was connected to the lower one by means of the screws that go through the four through holes. Their purpose was to ensure that the laminate did not move during the test.

Shear tests

The main goal of the shear tests is to determine properties such as the failure shear stress and the shear moduli in directions 13 and 23. These properties are used in the theoretical and finite element models. Some studies show that this failure shear stress depends on the thickness for glass and carbon-fibre laminates Pandya et al. (2015). Consequently, tests were made for thicknesses of 9 mm and 12 mm.

In order to perform these tests in optimal conditions, we followed ASTM-Standard-D732-02 (2002). This method is intended to provide a general procedure to determine the failure shear stress of test specimens in a thickness range from 1.27 mm to 12.7 mm. This method explains that a square specimen with a hole in the center should be clamped in a testing machine which can guarantee a constant movement when applying the force. By means of a shear punch that fits into the hole of the specimen and that is connected to the testing machine, the force is applied. Both force and displacement are recorded. An important aspect during the test is that the specimen must be clamped
Experimental procedure

at any moment except for the part which moves with the shear punch to avoid bending effects.

The geometry of the laminates tested is square (50 mm x 50 mm) and with a through hole in the center. The diameter of the hole is 6 mm. The only purpose of this hole is to guide the shear punch in its movement ensuring a uniform push in the normal direction to the surface of the laminate. The thicknesses tested were 9 mm and 12 mm. The hole was made according to ASTM-Standard-D732-02 (2002). The characteristics of the tests performed can be seen in Table 2.2.

<table>
<thead>
<tr>
<th>Nº Experiments</th>
<th>Thickness [mm]</th>
<th>Hole diameter [mm]</th>
<th>Depth</th>
</tr>
</thead>
<tbody>
<tr>
<td>2</td>
<td>9</td>
<td>6</td>
<td>Through</td>
</tr>
<tr>
<td>2</td>
<td>12</td>
<td>6</td>
<td>Through</td>
</tr>
</tbody>
</table>

*Table 2.2 Friction test data*

An scheme of the relative disposition between the specimen and the machine designed during the test can be seen in Figure 2.1.

![](image)

*Fig. 2.1 Relative disposition between the laminate and the test machine at the beginning of the shear test according to ASTM-Standard-D732-02 (2002)*

The following procedure was followed to perform the tests with the tool designed.

The first step to carry out the tests was to place the corresponding specimen on the lower cover (Figure C.5) with the help of the highlights to center it. The coupling element (Figure C.3) was placed on the universal machine test.

Then, the upper cover (Figure C.4) was placed on and was connected to the lower part with the help of M4 screws (Figure C.6). The main objective was that the specimen was embedded during the test to avoid bending effects. That was achieved by fixing the specimen between the upper and the lower covers through the tightening of the screws. At the beginning of the test, the only free surface is a circle of 12.8 mm diameter.
2.2 Characterization tests

The test started once the 6 mm diameter cylinder was completely introduced in the laminate. At that moment, the tool surface in contact with the upper surface of the specimen was a circular crown. The outer and the inner diameters were 12.5 mm and 6 mm respectively. Once the test started the lower stem moved up with a velocity of 1 mm min$^{-1}$, which is the established by ASTM-Standard-D732-02 (2002). The velocity was so low to guarantee the test was carried out in quasi-static conditions to avoid inertial effects.

From each test, the data system acquisition of the machine allows to record the force and the displacement as a function of time with a frequency of 10 Hz.

The failure shear stress is calculated with equation (2.1)

$$S_{sp} = \frac{F_{max}}{\pi De}$$ (2.1)

where $S_{sp}$ is the failure shear stress in directions 13 and 23, $F_{max}$ is the maximum force that the universal test machine records during the test, $D$ is the diameter of the punch and $e$ is the laminate thickness.

Furthermore, an approximate shear modulus is calculated with equations (2.2) and (2.3).

$$\gamma = \frac{x}{r}$$ (2.2)

$$F_s = \gamma GA_T$$ (2.3)

where $\gamma$ is an estimated angular strain, $A_T$ is the transverse area affected and $F$ is the force recorded by the machine. These magnitudes are shown in Figure 2.2. Two tests were carried out and a representative curve is shown in Figure 2.3. Furthermore, this test configuration is similar to what happens during an impact so it seems a reasonable approach to calculate the shear moduli in this way.

![Fig. 2.2 Detail of the shear test configuration with the variables involved](image-url)
From Figure 2.3, $G_{13}$ can be obtained. Since an linear-elastic behaviour until failure will be assumed in the models, a representative curve of the tests has been used to calculate $G_{13}$ as the slope of the linear approach. This can obviously be improved in future models to consider the non-linear behaviour of the material in these shear directions.

An additional hypothesis assumed is that shear fibre failure stresses $S_{13}$ are different from the shear failure matrix stresses $S_{13s}$. Furthermore $S_{13s}$ is lower than $S_{13}$ because it is considered that the failure of matrix takes place before the failure due to the fibres in shear. This approach was used by Xiao et al. (2007). It may be backed up experimentally as it can be slightly appreciated in Figure 2.3, for directions 13 and 23. Direction 12 is assumed to have the same behaviour with the same shear modulus and shear failure stresses than the other two shear directions. From a certain stress corresponding to 0.12 GPa approximately the curve becomes a bit unstable and little peaks appear before the complete failure. This point is associated with matrix shear failure and the maximum value with fibre shear failure stress.

In the research carried out by Pandya et al. (2015) it is concluded that the failure shear stress depends on the thickness of the laminate. In this study, the maximum failure shear stress obtained is reached for a certain thickness. For higher and lower thicknesses, this stress decreases.

In the tests performed in this work a final failure shear stress is obtained. This is due to the fact that only two tests were made for each thickness and the value of this stress is similar in all the tests.

The new static properties determined in shear tests are collected in Table 2.3
2.2 Characterization tests

<table>
<thead>
<tr>
<th>Property</th>
<th>Nomenclature</th>
<th>Value</th>
</tr>
</thead>
<tbody>
<tr>
<td>Shear modulus in directions 23 and 13 [GPa]</td>
<td>$G_{23} = G_{13}$</td>
<td>0.48</td>
</tr>
<tr>
<td>Matrix shear failure stress in directions 12, 23 and 13 [MPa]</td>
<td>$S_{12} = S_{23} = S_{13}$</td>
<td>120</td>
</tr>
<tr>
<td>Fibre shear failure stress in directions 12, 23 and 13 [MPa]</td>
<td>$S_{12} = S_{23} = S_{13}$</td>
<td>136</td>
</tr>
</tbody>
</table>

Table 2.3 Static properties of woven E-glass fibre/polyester laminates determined by shear tests according to ASTM-Standard-D732-02 (2002)

Friction tests

The main goal of the friction tests is to establish the relation force-displacement during the movement of the projectile. The curve obtained from this relationship is used in the theoretical model for the energy absorbed by friction. It allows to obtain the energy absorbed in one of the stages of the theoretical model.

In order to perform these tests in optimal conditions, we followed ASTM-Standard-D732-02 (2002). This Standard can also be applied for this kind of tests and other authors have used it (Pandya et al. (2015)).

The geometry of the laminates tested is square (50 mm x 50 mm) with a through hole in the center. The diameter of the hole is slightly smaller than the projectile’s. The thicknesses tested were 9 mm and 12 mm.

The data related to the tests performed are collected in Table 2.4.

<table>
<thead>
<tr>
<th>Nº Experiments</th>
<th>Thickness [mm]</th>
<th>Projectile diameter [mm]</th>
<th>Hole diameter [mm]</th>
<th>Depth</th>
</tr>
</thead>
<tbody>
<tr>
<td>6</td>
<td>9</td>
<td>7.5</td>
<td>7.4</td>
<td>Through</td>
</tr>
<tr>
<td>7</td>
<td>12</td>
<td>7.5</td>
<td>7.4</td>
<td>Through</td>
</tr>
</tbody>
</table>

Table 2.4 Friction test data

A scheme of the relative disposition between the specimen and the machine designed during the test can be seen in Figure 2.4. The same general procedure than the shear tests’ was followed to carry out all the experiments. There was only a different part related to the projectile used. The projectile was placed on the hole in the upper surface. The shear punch was placed in the upper jaw of the universal testing machine. The concavity of the punch was located on the projectile making contact with it so that the punch guided the movement. Once the test started the lower stem moved up with a velocity of 1 mm min$^{-1}$, which is the established by ASTM-Standard-D732-02 (2002) as in the shear tests. From each test, the system acquisition data of the machine allows to record the friction force and the displacement as a function of time with a frequency of 10 Hz. From this data, the force-displacement curve can be represented.
The curves for each thickness for different tests are quite similar, practically equal. Since the objective was to obtain a single friction force-displacement curve for each thickness, an average with all the tests was calculated. The average of the curves can be seen in Figure 2.5a for a 9 mm thickness test and Figure 2.5b for a 12 mm thickness test.

In view of the results, some conclusions can be inferred. For both thicknesses, the force increases initially until the whole projectile is inside the laminate. At this point, the contact surface between the projectile and the laminate reaches a maximum. When the projectile begins to come out of the specimen by the rear face, the friction force starts to decrease. The shape of the curves obtained is triangular. The peak that the
2.2 Characterization tests

curves present in the 12 mm thick specimens is higher than the one for 9 mm thick. It makes sense because the larger the thickness the more time the projectile remains inside the laminate, which is the time where the friction force is maximum.

Pandya et al. (2015) carried out similar experiments with E-glass fibre/epoxy laminates. In those tests, the maximum friction force oscillates between 3 kN and 4 kN for thicknesses between 10 mm and 20 mm. The order of magnitude of this force is similar than in Figure 2.5a. Nevertheless, the shape of the curve is different due to the geometry of the projectile. In the tests carried out by Pandya et al. (2015) they used a cylindrical flat-ended projectile. Therefore, the contact area between the projectile and the laminate is always growing monotonously and so does the force-displacement curve.

Then, an adjustment by means of least squares provides equations with their valid intervals of application. All of them have a fit coefficient greater than 0.98.

\[
F = \begin{cases} 
0.4569x + 0.1014, & \text{if } 0 < x \leq 1.17, \\
3.1815x - 2.7433, & \text{if } 1.17 < x \leq 3.72, \\
0.1002x^2 - 2.624x + 16.952, & \text{if } 3.72 < x < 9.
\end{cases}
\]  (2.4)

\[
F = \begin{cases} 
-0.094x^2 + 0.5304x + 0.0671, & \text{if } 0 < x \leq 2.68, \\
4.1184x - 10.264, & \text{if } 2.68 < x \leq 5.60, \\
0.112x^2 - 3.8096x + 30.683, & \text{if } 5.60 < x < 12.
\end{cases}
\]  (2.5)

where \(x\) is the spatial coordinate in mm with the origin in the front face of the laminate and \(F\) is the friction force in kN. Equations (2.4) and (2.5) are valid for 9 mm and 12 mm thick specimens respectively.

2.2.3 Compression tests

Out of plane tests

The objective of the compression tests is to determine the Young’s modulus and the failure stress and strain in compression through the thickness direction.

In order to design the procedure for this test, ASTM-Standard-D695-96 (1995) was followed. This method is intended to provide a general procedure to determine the compressive failure stress of test specimens. This method explains that a specimen with at least two parallel faces should be subjected to compressive loads by means of a testing machine which can guarantee a constant movement when applying the force. ASTM-Standard-D695-96 (1995) proposes a velocity of 1.3 mm min\(^{-1}\) to ensure quasi-static conditions during the test. The machine used should be able to record the force applied and the displacement as well as guarantee that the faces remain parallel during the whole test. Some considerations has to be taken into account such as the uniauxiality of the test,
the avoidance of significant stress concentrations, the prevention of bending and Euler buckling and the avoidance of the barrel effect (Ferguson et al. (1998)).

To be completely reliable, such tests for FRP materials has to be carried out with specimens of at least 40 mm of thickness. However, Ferguson et al. (1998) stated that some properties such as the Young’s modulus can be obtained with shorter blocks although stress concentrations can arise at specimen ends.

The best way we found to manufacture the specimens needed with the available thread machine was to make them with a cube geometry. The cube dimensions were 12 mm x 12 mm x 12 mm. A set of five tests was carried out. Although it is advisable to work with thicker specimens and with a more refined geometry, this geometry can lead to good results. Tarfaoui et al. (2008) also used this geometry (cubes of 13 mm x 13 mm x 13 mm) to measure similar properties at different strain rates in GFRP’s with the Hopkinson bar.

To carry out the experiments, we used a universal test 8802 INSTRON machine (Figure 2.6). This machine is able to apply a maximum force of 250 kN and it has a maximum stroke of the piston of 75 mm.

![8802 INSTRON connected to the out of plane compression tool](image)

However, the tests could not be performed only with this machine for two reasons. First, it could not be ensured that the supports of the machine remained undeformed when they withstand high forces. Second, if there had been any small manufacturing defect in the specimens, this machine would not have been able to correct them. For example, if the thickness was not perfectly uniform, problems such as stress concentration and appearance of high shear stresses could arise. It could lead to wrong and non-reliable results.

In order to perform these tests in optimal conditions we used an specific device that fit into the 8802 INSTRON machine as can be seen in Figure 2.7. This machine was
2.2 Characterization tests

designed for compression tests and has the shape of a chapel. In this case, the central piston (the disk above) has a guidance system with an internal spring and a maximum stroke of 25 mm. The device was placed between the compression plates of the 8802 INSTRON machine and the displacement alarm was set so that the maximum stroke was not reached. The compression disks are cubes of 50 mm x 50 mm made of hardened steel with 54 HRc hardness. This hardness was enough to ensure that there was not deformation in the disks during the tests. Furthermore, the upper disk has a small ball joint so that the possible manufacturing defects can be solved and the load is applied always parallel to the surface. These characteristics made possible the performance of the tests.

![Specific device for the out of plane compression tests with a specimen tested](image)

*Fig. 2.7 Specific device for the out of plane compression tests with a specimen tested*

The procedure to carry out the tests was really simple. First, the device was placed on the 8802 INSTRON machine, and the 8802 INSTRON machine was connected to the computer. The machine provides the force and the time. The displacement was measured with the extensometer (LVDT) shown in Figure 2.6. Also, a camera was placed parallel to the machine to record the whole test. After that, the specimens were cleaned and smeared with grease in the surfaces in contact with the steel disks in order to avoid the barrel effect. Then, the disks were adjusted until they touched the specimen and the test started with a velocity of $1 \text{ mm min}^{-1}$ to ensure that the test was performed in quasi-static conditions.

As can be seen in Figure 2.7, the breakage shape is conical as it is supposed to be in laminates similar to E-glass fibre/polyester’s. For example, Ferguson et al. (1998) predicted a similar breakage shape for 0/90 woven glass/epoxy laminates.

Since the test is assumed to be uniaxial, the stress-strain curve along the thickness direction can be represented from the data recorded. Stress and strain are calculated with equations (2.6) and (2.7).

\[
\sigma_3 = \frac{F_c}{A_c} \quad (2.6)
\]
Experimental procedure

\[ \epsilon_3 = \frac{x}{L_0} \]  

(2.7)

where \( A_c \) is the area in contact with the steel disk (12 mm x 12 mm), \( F \) is the force recorded by the machine, \( L_0 \) is the initial thickness of the laminate (12 mm) and \( x \) is the displacement of the machine in a reference system with origin in the starting test position.

In Figure 2.8 a representative stress-strain curve of one of the tests performed is represented. The relationship between these variables is linear until failure as it is supposed to be for this kind of laminates.

![Out of plane compressive stress vs strain curve along the thickness direction](image)

Fig. 2.8 Out of plane compressive stress vs strain curve along the thickness direction

The slope of this curve represents the compressive Young’s modulus in the thickness direction. And the stress before failure can be interpreted as the compressive failure stress in the thickness direction. These values are summoned in Table 2.5.

<table>
<thead>
<tr>
<th>Property</th>
<th>Nomenclature</th>
<th>Value</th>
</tr>
</thead>
<tbody>
<tr>
<td>Compressive Young’s modulus in direction 33</td>
<td>( E_{33} )</td>
<td>4.5</td>
</tr>
<tr>
<td>Compressive failure stress in direction 33</td>
<td>( X_{33} )</td>
<td>464</td>
</tr>
<tr>
<td>Compressive failure strain in direction 33</td>
<td>( \epsilon_{3c} )</td>
<td>0.1085</td>
</tr>
</tbody>
</table>

Table 2.5 Static properties of woven E-glass fibre/polyester laminates determined by out of plane compression tests according to ASTM-Standard-D695-96 (1995)

In-plane compression tests

In-plane compression tests were carried out with the aim of characterizing the compressive failure stress in directions 11 and 22.
2.2 Characterization tests

There are different standard tests to measure the compressive properties of composite materials, ASTM-Standard-D3410/D-3410M-95 (2010) and ASTM-Standard-D695-96 (1995). The geometry of the specimens tested was that indicated by ASTM-Standard-D3410/D-3410M-95 (2010), which is the one recommended for GFRP.

ASTM-Standard-D3410/D-3410M-95 (2010) establishes the minimum thickness needed for a single specimen as a function of the Young’s modulus and the compressive failure stress expected. We chose a thickness of 3 mm for the specimens tested, which seems a reasonable thickness taking into account the static Young’s modulus (10.1 GPa) and the compressive failure stress expected (between 200 MPa and 600 MPa). This estimation was made looking for a similar material (glass-fibre/epoxy composites) used by Naik and Doshi (2008). Once these values have been estimated, ASTM-Standard-D3410/D-3410M-95 (2010) provides a minimum value for the thickness specimen that should be used. In this case, this value was between 1 mm and 3 mm.

The free gap a specimen should have during the test is also specified in ASTM-Standard-D3410/D-3410M-95 (2010). It is also related to the chosen thickness. The free length must be short enough to avoid the possible problems of buckling caused by the high compressive load. However, this length must be long enough to assume a uniaxial stress profile and to avoid the Poisson effect in the vicinity of the clamped region. We chose a free gap of 10 mm for the tests.

The specimens tested were rectangles of 25 mm x 140 mm. The thickness of the specimens was 3 mm.

In order to carry out the experiments, we used a universal test 8802 INSTRON machine. This machine is able to apply a maximum force of 100 kN and it has a maximum stroke of the piston of 75 mm. The specimen is fixed with the aid of the jaws of the 8802 INSTRON machine. There must be a free length to apply the compressive force.

We used rosettes to measure the strain in the loading direction as well as in the perpendicular direction to check if the buckling effect took place. The rosettes were connected also to the computer. The acquired data were the strains of both faces at any time.

In Figure 2.9, the longitudinal strains on both faces of the specimen are shown. The longitudinal strains in test 1 (Figure 2.9a) are quite different. The free length chosen was 14 mm for this test. Since one face was being deformed more than the other, it can be assumed the buckling phenomenon was taking place. Actually, this effect was really important at the view of the differences between strains.

To mitigate this effect, we reduced the free length of the specimen to 10 mm. Figure 2.9b shows that the buckling effect did not come about because the strains of the front and rear faces almost overlap. Nevertheless, there were small differences and they could not be corrected due to physical limitations of the 8802 INSTRON machine.
Experimental procedure

The data of the second test is considered valid because it was the test that reproduces better the ideal conditions suggested by ASTM-Standard-D3410/D-3410M-95 (2010) for a compression test on composite materials. A proof of that is the good agreement between the strains on both faces.

Figure 2.10b shows a noticeable difference regarding to the direction of fibre failure along the thickness for both tests. In the first test the failure direction forms an angle of 65° with direction 3. Obviously, the reason why the direction is not 0° is the excessive free length that provoked the buckling phenomenon. That explains the difference between the longitudinal strains in Figure 2.9a.

Fig. 2.9 Longitudinal strain vs time for the in-plane compression tests with a gap length of (a) 14 mm (b) 10 mm

Fig. 2.10 (a) Disposition of the in-plane compression tests according to ASTM-Standard-D3410/D-3410M-95 (2010) (b) Failure direction of the two specimens tested in test 1 (right) and test 2 (left)
In Figure 2.10b the direction of fibre failure is almost $0^\circ$ aligned with direction 3 since buckling was not appreciated. This explains that the strains in both faces are practically the same (Figure 2.9b).

Assuming the cross section of the specimens remained constant for the entire test and using the force data recorded, the compressive failure stress may be calculated as the maximum force divided by the cross section.

$$\sigma_{11c} = \frac{F}{A_{cs}}$$  \hspace{1cm} (2.8)

where $\sigma_{11c}$ is the compressive stress in direction 11 or 22, $F$ is the force recorded by the test machine and $A_{cs}$ is the cross section of the specimens. Substituting the maximum value of the force recorded in (2.8), the compressive failure stress in directions 11 $X_{11c}$ and 22 $X_{22c}$ is obtained.

The new static properties determined are presented in Table 2.6

<table>
<thead>
<tr>
<th>Property</th>
<th>Nomenclature</th>
<th>Value</th>
</tr>
</thead>
<tbody>
<tr>
<td>Compressive failure stress in directions 11 and 22 [MPa]</td>
<td>$X_{11c} = X_{22c}$</td>
<td>309</td>
</tr>
</tbody>
</table>

Table 2.6 Static properties of woven E-glass fibre/polyester laminates determined by in-plane compression tests according to ASTM-Standard-D3410/D-3410M-95 (2010)

### 2.3 Impact tests

#### 2.3.1 Experimental procedure

High-velocity impacts were carried out on E-glass fibre/polyester laminates. The area and the thickness of the laminates studied were 150 mm x 150 mm and 9 mm respectively.

Other results were collected from the literature for the same material and the same area but for thicknesses of 3 mm García-Castillo (2007), 6 mm and 12 mm Buitrago-Pérez et al. (2010).

Results of 3 mm, 6 mm and 12 mm show that these laminates have a really sensitive behaviour when changing the thickness against high-velocity impacts García-Castillo et al. (2012). For that reason, it was necessary to have experimental results for 9 mm thick laminates so that a complete range of results between 3 mm and 12 mm could be studied.

The size of the specimens is large enough so that the damaged area does not reach the border of the laminate as can be seen in the specimens tested by Buitrago-Pérez et al. (2010). In Figure 2.11, it can be appreciated the comparison between the front and the rear faces of a 9 mm thick laminate tested in this work at velocities near the
Experimental procedure

ballistic limit. The damaged area does not reach the borders of the specimens so the results are reliable.

Regarding to the projectile used, the mechanical properties and the geometry are the same of the one used by Buitrago-Pérez et al. (2010) and García-Castillo (2007). The projectile is a steel sphere with a diameter of 7.5 mm and a density of 7800 kg m$^{-3}$. Since the projectile is made of steel, plastic deformations were not appreciated after the impact tests.

![Fig. 2.11 Damaged area of the front (left) and rear (right) faces in a 9 mm thick specimen for a velocity around the ballistic limit](image)

In order to carry out the impact tests in 9 mm thick laminates, a gas cannon manufactured by SABRE BALLISTIC (A1G+ model) was used to impulse the projectile at the required velocity. This cannon consist on two clearly differentiated zones:

- Load and accommodation area of the projectile: It is the pressure chamber that propels the projectile against the target. The higher the pressure inside the chamber, the higher the velocity of the projectile. There is an approximated known relationship between the pressure inside the chamber and the output velocity of the projectile. This relationship was studied by Buitrago-Pérez et al. (2010). In Table 2.7 it is shown the correspondence between the velocities reached in the tests performed in this work and the pressures needed to obtain these velocities. This area is isolated from the environmental conditions which could influence the projectile during the impact.

- Frame of essays: Enclosure where the laminate is clamped. It is composed by different grips that allow to test laminates of different thickness. It provides a wide versatility in terms of configurations that can be tested.

Recording the tests with a high-velocity camera was the way used to pick up essential information such as the impact and residual velocities. The camera used was a Photron Fastcam ATX, which is able to take 250000 frames per second (fps). In these test the acquisition data velocity was 90000 fps. The camera was placed parallel to the frame of
essays. In Figure 2.12, an instant of time during an impact is represented. This frame corresponds to an instant before the impact. An ARRI ARRISON 12 PLUS lamp was used for lighting. This lamp has a power of 1200 W.

The general procedure to carry out the experiments is described below. First, the projectile wrapped in a sabot was introduced in the load and accommodation area. The purpose of the sabot was to lead the projectile to the right path. Then, the pressure of the gas used was regulated and the projectile was propelled against the laminate. In this case, Helium was the gas used because it is inert and non-flammable. The projectile went through the tunnel and reached the laminate, which was embedded by its borders. The camera recorded the whole event.

<table>
<thead>
<tr>
<th>Nº Experiment</th>
<th>Pressure [bar]</th>
<th>Impact velocity [m s(^{-1})]</th>
</tr>
</thead>
<tbody>
<tr>
<td>1</td>
<td>38</td>
<td>362</td>
</tr>
<tr>
<td>2</td>
<td>40</td>
<td>383</td>
</tr>
<tr>
<td>3</td>
<td>42</td>
<td>397</td>
</tr>
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<td>4</td>
<td>44</td>
<td>402</td>
</tr>
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<td>5</td>
<td>48</td>
<td>430</td>
</tr>
<tr>
<td>6</td>
<td>50</td>
<td>431</td>
</tr>
<tr>
<td>7</td>
<td>52</td>
<td>441</td>
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<tr>
<td>8</td>
<td>56</td>
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<td>617</td>
</tr>
<tr>
<td>18</td>
<td>140</td>
<td>641</td>
</tr>
</tbody>
</table>

Table 2.7 Correspondence between Helium pressure and the impact velocity of the projectile

### 2.3.2 Data processing

A set of 18 impact tests were performed so that a reliable value of the ballistic limit could be obtained. The procedure to determine the impact and residual velocities is detailed below. The software Photron Fastcam Viewer allows to freeze the image at any frame as in Figure 2.12.
Experimental procedure

Fig. 2.12 Initial instant to start the measurement for an impact on a 9 mm thick specimen

The most backward pixel belonging to the projectile was selected when it appeared in the video. The software provided the coordinates of this point and also the time. The new coordinates and time for this point were measured just before the projectile stroke the laminate. The same procedure was used to calculate the residual velocity. Assuming that the velocity did not vary during this time, it can be easily calculated because an equivalence between the images and the real measurements exists and it is known. This equivalence was calculated processing the image in Figure 2.13.

Fig. 2.13 Ruler that provides the equivalence between the length in the image and the real length

Equation (2.9) can be used to calculate the impact and the residual velocities under conditions of constant velocity.

\[ v = \frac{x_2 - x_1}{t_2 - t_1} n \]  

(2.9)

where \( v \) is the velocity, \( x_1 \) is the position of the projectile in the first instant, \( t_1 \) is the time in the first instant, \( x_2 \) is the position of the projectile in the last instant, \( t_2 \) is the time in the last instant and \( n \) is the conversion factor to turn pixels into meters.

2.3.3 Results

The experimental data obtained in the impact tests are shown in Figure 2.14. Applying the Lambert and Jonas equation (2.10), which was validated by Kasano (1999), the ballistic limit was obtained for 9 mm thick laminates. We used this equation because we
2.3 Impact tests

did not have enough specimens to calculate the ballistic limit by means of probabilistic methods. In addition, impact velocity versus residual velocity curve was represented.

$$V_r = \begin{cases} 
0, & \text{if } 0 < V_0 < V_{bl}, \\
A(V_0^p - V_{bl}^p)^{\frac{1}{p}}, & \text{if } V_0 > V_{bl}. 
\end{cases}$$  \hspace{1cm} (2.10)

where \( V_0 \) is the impact velocity of the projectile, \( V_{bl} \) is the ballistic limit, \( V_r \) is the residual velocity of the projectile, \( A \) is an empirical parameter, which varies with thickness as it was demonstrated by Buitrago-Pérez et al. (2010) and \( p \) is a constant.

![Graph](image)

**Fig. 2.14** Experimental adjusted impact velocity vs residual velocity curve for 9 mm thick laminates

According to the impact tests performed, it can be stated the ballistic limit for E-glass fibre/polyester laminates is approximately \( 428 \pm 2 \text{ m s}^{-1} \).

<table>
<thead>
<tr>
<th>Thickness [mm]</th>
<th>Ballistic limit [m s(^{-1})]</th>
<th>( p )</th>
<th>( A )</th>
</tr>
</thead>
<tbody>
<tr>
<td>3</td>
<td>212</td>
<td>2</td>
<td>1</td>
</tr>
<tr>
<td>6</td>
<td>332</td>
<td>2</td>
<td>0.9</td>
</tr>
<tr>
<td>9</td>
<td>428</td>
<td>2</td>
<td>0.8</td>
</tr>
<tr>
<td>12</td>
<td>550</td>
<td>2</td>
<td>0.8</td>
</tr>
</tbody>
</table>

**Table 2.8** Ballistic limit and empirical parameters for different thicknesses of woven E-glass fibre/polyester composite plates

In Table 2.8 the data corresponding to the ballistic limit and the parameters \( A \) and \( p \) are presented. These data are collected from Buitrago-Pérez et al. (2010) for 6 mm and
Experimental procedure

12 mm, García-Castillo (2007) for 3 mm. The data for 9 mm have been obtained in this work.

In Figure 2.15, the experimental data which Table 2.8 was obtained with is represented. Likewise, the adjustment curves for the experimental data of each thickness can also be seen.

![Fig. 2.15 Experimental adjusted impact velocity vs residual velocity curves for 3 mm García-Castillo (2007), 6 mm, 12 mm Buitrago-Pérez (2012) and 9 mm thick laminates](image)

With the aim of comparing the ballistic limit corresponding to each thickness, Figure 2.16 is shown. This curve was obtained by Buitrago-Pérez et al. (2010) with the ballistic limit of three different thicknesses.

![Fig. 2.16 Ballistic limit as a function of thickness (Buitrago-Pérez et al. (2010))](image)
2.3 Impact tests

Figure 2.16 shows a linear relationship between the thickness of the specimen and its ballistic limit. Consequently, it would be reasonable if the ballistic limit for 9 mm thick specimens was on this curve.

Once the minimum perforation velocity has been determined for the four thicknesses of the laminates studied, these can be plotted against the thickness and the minimum perforation energy versus thickness can be represented, as can be seen in Figure 2.17. The curve obtained taking into account the four thicknesses with their ballistic limits is almost equal as before adding the data for 9 mm thickness. Therefore, the new ballistic limit is on the curve obtained by Buitrago-Pérez et al. (2010).

![Fig. 2.17 Minimum velocity and energy of perforation vs thickness for woven E-glass/polyester plates](image)

The results of the experiments show a linear dependence between the minimum velocity of perforation and the thickness (from 3 mm to 12 mm). Similar behaviour of perforation velocity versus thickness have been described by other researchers for Kevlar fibre laminates (Rosenberg and Dekel (2012)) and for thin laminates of E-glass/polyester (García-Castillo et al. (2012)). However, this result contradicts those of other authors who propose a non-linear relationship between perforation velocity and thickness Gellert et al. (2000).

In addition, the minimum perforation energy can also be calculated and it is proportional to the square of the velocity, meaning that perforation energy has a quadratic dependence on thickness (Figure 2.17). Within the thickness range from 3 mm to 12 mm the experimental results would appear to contradict other studies such as those undertaken by Gellert et al. (2000), which proposed a bilinear relationship between perforation energy and thickness.
2.4 Non-destructive inspection tests

2.4.1 Experimental procedure

The main objective of these tests is to measure the damaged area of the laminates tested in this and previous works. Relating different variables with the damage area we expect to find useful relationships that help to understand better the behaviour of the laminates under high-velocity impact loads.

The laminates analyzed were the same tested for the impact tests. The area of the laminates was 150 mm x 150 mm and it was shown to be large enough to ensure that waves did not reach the borders in any case. Therefore, the phenomenon is ruled by the local behaviour of the material regardless the boundary conditions and the damaged area is not influenced by them. Four different thicknesses were considered for this study: 3 mm, 6 mm, 9 mm, 12 mm. A total number of 64 samples was inspected using the Non-Destructive Inspection technique (NDI).

The impact tests carried out during the study were performed on laminates with thickness of 9 mm. These impact tests correspond to 17 samples. For the other thicknesses to be considered the experimental data (same material and geometry) were taken from a previous studies, García-Castillo (2007) and Buitrago-Pérez et al. (2010).

Once the impact tests had been performed, NDI was carried out on all impacted specimens for the four thicknesses. Although the composite laminate is translucent, the extension of damaged area was not determined using optical techniques as differences between the damaged extension at the front and rear face of the 3 mm thickness plates were not observed using this technique for glass-fibre composites (Da Silva et al. (2004)). Rather, the NDI technique used to study the damage caused by the impact was the C-Scan technique with water as coupling medium and non-contact ultrasounds. The experimental equipment is manufactured by TECNITEST and comprises the following components:

- A computer to record the information provided by the transducer.

- A Probe USPC7100 LA to emit the ultrasonic pulse and receive the signal. For the purposes of this study a transducer of 5 MHz frequency and 10 mm diameter was chosen.

- An automatic system to move the transducer for the inspections (Figure 2.18).
2.4 Non-destructive inspection tests

2.4.2 Results

The absorbed energy is an interesting measurement to evaluate the capacity and resistance of a laminate under impulsive loads. The representation of the absorbed energy versus the impact energy for all the test available as in Figure 2.19 can lead to unexpected conclusions.

![Experimental set up for C-Scan inspection of impacted monolithic plates](image1)

*Fig. 2.18 Experimental set up for C-Scan inspection of impacted monolithic plates*

![Absorbed energy/thickness vs impact energy for different thicknesses for woven E-glass/polyester plates](image2)

*Fig. 2.19 Absorbed energy/thickness vs impact energy for different thicknesses for woven E-glass/polyester plates*

Figure 2.19 shows the ratio of the absorbed energy/thickness versus the impact energy for all experimental tests available. The absorbed energy is divided by the thickness of the specimen in order to avoid inertial effects that may appear due to the mass of the laminate. In addition, the minimum perforation energy, $E_{BL}$, which is represented by a vertical line for each thickness, can be observed. The graph shows that, for higher impact energies than 100–150 J, a roughly linear relationship exists between the absorbed energy per unit of mass and the impact energy. Furthermore, this result is independent of thickness. These impact energies coincide with the minimum perforation energies for
Experimental procedure

laminates with thicknesses of 6 mm and 9 mm, respectively, as can been see in Figure 2.17.

In this work the contact time between the projectile and the target is also considered and defined. This parameter, defined as the time needed to perforate the laminate, may be calculated using high-speed video in tests which the perforation of the laminate takes place in (Figure 2.20). Figure 2.20a shows the contact time versus the impact energy and Figure 2.20b the first magnitude versus the impact energy divided by the thickness of the specimen. In Figures 2.20a and 2.20b, the minimum energy of perforation is represented by means of a line for each thickness. For all laminates studied, the maximum contact time is reached when perforation energy is close to the minimum and diminishes with the inverse of impact energy (Figure 2.20).

Moreover, in order to evaluate the influence of specimen thickness, the variation of the contact time was analyzed as a function of the impact energy divided by thickness. The trend of the curves in Figure 2.20a is the same as in Figure 2.20b. In spite of the fact that the impact energy has been divided by thickness (Figure 2.20b) the contact time curve is not the same for all thicknesses. It means that for the same impact energy per unit of mass, the greater the thickness, the greater the contact time. So contact time increases with thickness, subtracting the mass effect.

Another aspect of interest in this study is the damaged area in the specimens tested. All specimens were examined using C-Scan technique, including those with different thickness. Both the front and rear faces of the specimens were examined.

Figure 2.21 shows the damaged areas on both sides of the specimens with a thickness of 9 mm for three different cases, namely below, around and above the ballistic limit. It may
be observed that the damaged area presents a circular shape in all cases. Measurements of four different diameters (in different directions) were taken for both faces in each specimen and the maximum deviation of the diameters on the front and rear faces was seen to be 12\% and 14\%, respectively. Therefore, it would appear that it is reasonable to consider the damaged area as being approximately circular. Bearing this in mind, the results shown below are presented as a function of the diameter of the damaged area, rather than of the damaged area itself. In Figure 2.21, two stains can be seen in the lateral areas of the specimen. However, these stains correspond to the effect of the supports used to hold the specimen during the inspection and such are not considered damaged areas.

Fig. 2.21 Damaged area of the front (left) and rear (right) faces in 9 mm thick specimens for an impact velocity (a) below the ballistic limit (b) around the ballistic limit (c) above the ballistic limit

In Figure 2.21, the maximum damaged area is reached around the ballistic limit. This result appears to be logical, as the maximum contact time also occurs around the ballistic limit (Figure 2.20). In general, as can be seen in Figure 2.21, the maximum
Experimental procedure

diameter of the damaged area occurs around the perforation energy for both faces of the specimen for the various thicknesses considered.

For low thicknesses (3 mm and 6 mm) it can be observed (Figure 2.22) that the diameter of the damaged area grows quickly until the ballistic energy limit is reached. For impact energies greater than those of the energy corresponding to the ballistic limit, the diameter of the damaged area decreases slowly. However, for higher thicknesses (9 mm and 12 mm), while the diameter of the damaged area increases up to the point where the ballistic energy limit is reached, a significant decrease below this value of energy cannot be appreciated. This may be because there are very few experiments related to impact energies above the ballistic limit for specimens with these thicknesses.

![Fig. 2.22 Diameter of the damaged area of the (a) front (b) rear face vs impact energy for a variety of thicknesses](image)

Moreover, the diameter of the damaged area in the specimens precisely at the ballistic limit is an aspect of interest here. This is because the maximum extension of damaged area is found at minimum perforation velocity when composite laminates are subjected to high-velocity impact. Figure 2.23 represents the diameter in both front and rear faces versus thickness of the specimen for the minimum perforation energy. A linear relationship was observed between the damaged area diameters on the front and rear faces for the range of thicknesses considered in this work.
The diameter of the damaged area increases with thickness for velocities close to the ballistic limit. Furthermore, Figure 2.23 shows that both lines have practically the same slope, with an approximate difference of 3.8%. It implies that a certain relationship exists between the damaged area on the front and rear faces for velocities close to the ballistic limit. In this case, the diameter of the damaged area on the rear side of the specimen is equal to the diameter of the damaged area on the front side with an offset of 0.067 m.
The objective of this chapter is to provide two theoretical models for high-velocity impacts on composite materials, on woven E-glass/polyester laminates specifically. The main goal is that the models be based on hypotheses physically reasonable and capture the effects taking place during impact. First, a literature review on the subject of theoretical and analytical models is carried out and the need for formulating two different models is explained. Then, two new non-dimensional theoretical models are developed, for thin and thick plates. The theoretical model for thin and thick laminates are influenced by the models developed by Alonso et al. (2018a) García-Castillo et al. (2013) and Naik and Doshi (2005) respectively.

3.1 Introduction

There are different approaches to study the impact behaviour of composite materials subjected to impulsive loads. Experimental and finite element approaches are widely used and provide good results. However, time and computational cost are important disadvantages. To avoid these problems, analytical and theoretical models can be useful. They allow to understand the physics of the problem and provide in some cases a sufficiently accurate solution with a lower computational cost than numerical methods.

As a consequence of the above, there is continual interest in developing analytical and theoretical models to predict the ballistic limit of laminate plates among other things. There are mainly two approaches to tackle the theoretical impact modelling in terms of the formulation: models based on energy balances and based on momentum transfer. The work of this thesis is focused on the first ones.

Many authors have developed both simple and sophisticated models based on momentum transfer. For example, in the model of Mamivand and Liaghat (2010) one-dimensional elastic wave theory is applied and a transference of linear momentum between the projectile and the successive layers is assumed. Briesciani et al. (2015) developed a model for
3.1 Introduction

Ballistic impacts against plain woven fabrics with a polymeric matrix taking into account flexion effects inside the energy absorption mechanisms.

Analytical and theoretical models are also widely used nowadays to study the phenomenon of high-velocity impact on composite materials, metals and ceramics. For example, in Rosenberg and Dekel (2012) a previous model for metals developed by Rosenberg is formulated. In this model the energy lost by the projectile is considered to be absorbed by plastic work which depends on a resistive stress. In addition, this model allows to determine analytically the ballistic limit. Metal models were used to design armours in the military environment and for defense applications. Florence and Arhens (1967) developed a model to predict the ballistic limit on ceramic materials by means of an energy balance between the energy absorbed by the laminate and the kinetic energy of the projectile. Along last decades analytical and theoretical models have been developed and improved.

Regarding composite laminate modelling, the evolution has been remarkable in last decades. Zhu et al. (1992) developed an analytical model valid for Kevlar 29/polyester targets. This model is divided into three stages, which are indentation, perforation and exit. Different energy absorption mechanisms such as fibre failure or friction are taken into account and the laminated plate theory is used. Navarro (1998) stated that models of brittle composite materials should be different from ductile composite materials because the energy absorption mechanisms are different and thus their behaviour. Furthermore, he developed an analytical model applying the one-dimensional elastic wave theory. The analytical model for polymer composites proposed by Moyre et al. (2000) considers the energy absorbed by the laminate divided into kinetic energy transferred to the laminate, elastic deformation of fibres and tensile failure of fibres and it allows to obtain an analytical expression for the ballistic limit velocity. Wen (2000) and Wen (2001) developed an analytical model for the penetration and perforation of thick FRP laminates struck by projectiles with different ending shapes such as flat, conical or truncated. This model assumes the deformations are localized in the impact zone and it is formulated from the resistive pressure and the dynamic resistive pressure in a small region near the impact zone and its surroundings. Naik and Shrirao (2004) and Naik et al. (2006) developed a model for woven fabric composites. This model is based on an energy balance considering delamination, matrix cracking, shear plugging, elastic deformation of fibres, tensile failure of fibres and the kinetic energy transferred from the projectile to the laminate. This model is formulated incrementally, and thus time is divided in small increments and the kinematic variables are calculated in each step. Naik and Doshi (2005) formulated another model for thick woven composites taking into account the wave propagation through the thickness direction and additional energy absorption mechanisms which take place such as compression. López-Puente et al. (2007) developed a theoretical model for thin woven CFRP plates. By means of an energy balance and taking into account the geometry of the projectile, a differential equation is obtained that
Theoretical model

can be solved using perturbation methods. This model fits the experimental data for high velocity impacts. García-Castillo et al. (2013) developed a theoretical model based on an energy balance which leads to a differential equation solved by numerical methods.

In the following sections two new non-dimensional theoretical models are formulated, for thin and thick laminates. In the literature review, different models have been found. It is remarkable the lack of agreement when considering a laminate as thin or thick. That is one of the main reasons why in this chapter two different models are presented depending on the thickness. In the results, the threshold to consider a laminate either thin or thick is discussed.

3.2 Model description

Two theoretical models for woven GFRP laminates have been developed. Both models consider the kinetic energy of the projectile impact is consumed during the perforation process by several energy absorption mechanisms. Depending on the thickness, some of these mechanisms may be disregarded or taken into account because the behaviour of the laminate changes. To give two clear examples of this change, we may include the moving mass of the laminate for thin laminates and the phenomenon of local compression for thick ones. When a laminate is very thin, there is experimental evidence that it behaves as a thin membrane before it breaks. Therefore, the energy of this moving mass has to be taken into account. On the contrary, when the laminate is thick enough, this effect cannot be appreciated, instead, there is a local compression in the region just below the projectile and its surroundings. But some questions appear after this dissertation. The threshold where we can assume the change of behaviour is the most clear, but knowing what this threshold is influenced by is also interesting. For example, it could be influenced by the size of the impactor.

To identify the parameters on which the problem depends in a clearer way, a non-dimensional formulation is used, applying the Vaschy-Buckingham $\Pi$ Theorem.

In summary, the principal objective of the models presented is to provide a tool to predict aspects such as the ballistic limit or the residual velocity giving an insight look at the physics but also giving response to unanswered questions.

3.2.1 Theoretical model for thin woven laminates

The theoretical model for thin woven laminates is influenced by a previous model developed by Alonso et al. (2018a). Nevertheless, the new model proposed has been formulated including additional considerations.

The following hypotheses have been assumed in the model formulation:
3.2 Model description

- The projectile is rigid and thus it remains undeformable during the entire impact process. There is experimental evidence that the projectile is not deformed after any of the impacts.
- The laminate is considered linear-elastic and $x$-axially symmetric, being $x$ the thickness direction.
- All wave velocities considered remain constant over the whole impact process.
- The spherical geometry of the projectile is taken into account for the different energy absorption mechanisms.
- The back side of the laminate is subjected to acceleration by the projectile and it moves with a different velocity than the projectile’s. Therefore, there is a relative displacement between the projectile and the laminate as they move.
- The stop condition of the model is that the relative displacement between the projectile and the laminate equals the initial thickness.
- The energies absorbed by tensile failure and elastic deformation of fibres are treated independently.
- The energies absorbed by friction and shear plugging are considered negligible.
- The energy absorbed by heat transfer between the projectile and the laminate is disregarded.

The problem itself depends on three elemental magnitudes which are mass $[M]$, length $[L]$ and time $[T]$. They may be written in their non-dimensional form as follows:

\[
[M] = \rho_p \phi_p^3 \tag{3.1}
\]

\[
[L] = \phi_p \tag{3.2}
\]

\[
[T] = \frac{\phi_p}{V_i} \tag{3.3}
\]

where $\rho_p$ is the projectile density, $\phi_p$ is the projectile diameter and $V_i$ is the impact velocity. The non-dimensional time $\tau$ is defined as the integration variable.

The problem depends on 16 fundamental parameters which are presented in Table 3.1 as long as their corresponding $\Pi$ groups. It bears noting that some of the parameters are already non-dimensional, so the parameter is directly the $\Pi$ group in those cases.
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<table>
<thead>
<tr>
<th>Parameter</th>
<th>Nomenclature</th>
<th>II group</th>
</tr>
</thead>
<tbody>
<tr>
<td>Projectile diameter</td>
<td>$\phi_p$</td>
<td></td>
</tr>
<tr>
<td>Projectile density</td>
<td>$\rho_p$</td>
<td></td>
</tr>
<tr>
<td>Impact velocity</td>
<td>$V_i$</td>
<td></td>
</tr>
<tr>
<td>Laminate thickness</td>
<td>$e$</td>
<td>$\Pi_e = \frac{e}{\phi_p}$</td>
</tr>
<tr>
<td>In-plane Young’s modulus</td>
<td>$E$</td>
<td>$\Pi_E = \frac{E}{\rho_p V_i^2}$</td>
</tr>
<tr>
<td>In-plane failure strain</td>
<td>$\varepsilon_r$</td>
<td>$\varepsilon_r$</td>
</tr>
<tr>
<td>In-plane failure stress</td>
<td>$\sigma_r$</td>
<td>$\Pi_{\sigma_r} = \frac{\sigma_r}{\rho_p V_i^2}$</td>
</tr>
<tr>
<td>Laminate density</td>
<td>$\rho_l$</td>
<td>$\Pi_{\rho_l} = \frac{\rho_l}{\rho_p}$</td>
</tr>
<tr>
<td>Energy absorbed by matrix cracking per unit volume</td>
<td>$E_{MT}$</td>
<td>$\Pi_{E_{MT}} = \frac{E_{MT}}{\rho_p \phi_p V_i^2}$</td>
</tr>
<tr>
<td>Critical dynamic-strain energy-release rate in mode II</td>
<td>$G_{ICD}$</td>
<td>$\Pi_{G_{ICD}} = \frac{G_{ICD}}{\rho_p \phi_p V_i^2}$</td>
</tr>
<tr>
<td>Shape factor of delamination</td>
<td>$\alpha_{DL}$</td>
<td>$\alpha_{DL}$</td>
</tr>
<tr>
<td>Shape factor of matrix cracking</td>
<td>$\alpha_{MC}$</td>
<td>$\alpha_{MC}$</td>
</tr>
<tr>
<td>Yarn width</td>
<td>$B$</td>
<td>$\Pi_B = \frac{B}{\phi_p}$</td>
</tr>
<tr>
<td>Stress wave transmission factor</td>
<td>$b$</td>
<td>$b$</td>
</tr>
<tr>
<td>Poisson’s ratio</td>
<td>$\nu$</td>
<td>$\Pi_{\nu} = \left[12(1-\nu^2)\right]^{1/6}$</td>
</tr>
<tr>
<td>Constant</td>
<td>$c$</td>
<td>$\Pi_c = c\phi_p^{1/2} \rho_p^{1/6} V_i^{1/3}$</td>
</tr>
</tbody>
</table>

*Table 3.1 Summary of the parameters and II groups of the problem*

According to the Vaschy-Buckingham II Theorem, the number of parameters on which the problem depends has been reduced from 16 to 13. Some of the most interesting II groups will be analyzed in the results section.

From now on, the non-dimensional character of the variables will be represented with a bar over the variable. For example, $\bar{v}(\tau)$ represents the non-dimensional velocity of the projectile.

The kinematic variables of the problem are: the position $\bar{x}(\tau)$, the velocity $\bar{v}(\tau)$ and the acceleration $\bar{a}(\tau)$ of the projectile.

In the following subsections the different energy absorption mechanisms are explained and formulated. Figure 3.1 is a scheme of a generic instant during the impact process where the variables and parameters described in the following sections can be easily understood.
3.2 Model description

![Scheme of the impact phenomenon at a generic instant of time](image)

**Energy absorbed by laminate acceleration**

The first concept that must be understood is the wave propagation when a projectile strikes a laminate. Once the projectile comes into contact with the laminate in the first place, there are certain waves that propagate in different directions. Since the laminate is considered as thin, the waves along the thickness direction are disregarded.

Applying one-dimensional wave theory, two different waves spread out along the fibre direction when a fibre is impacted transversely Navarro (1998). The longitudinal wave, $V_l$, is the fastest because it propagates at the sound speed of the fibre material as can be seen in equation (3.4). However, the wave responsible for the laminate acceleration is the transverse wave, $V_t$. This wave is slower and provides the thin laminate its tent shape. The fundamental parameters that determine the velocity of the transverse wave are the tensile force and the mass per unit length of the unstrained fibre. The original expressions for these waves were proposed initially by Smith et al. (1958).

$$C_{V_t} = \sqrt{\frac{\Pi E}{\Pi_{pl}}}$$ (3.4)
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\[ C_{V_l} = \sqrt{(1 + \varepsilon_r) \frac{\Pi_{\sigma_r}}{\Pi_{\rho_l}}} - \sqrt{\frac{\Pi_E}{\Pi_{\rho_l}}} \varepsilon_r \]  

(3.5)

It has to be noticed that \( C_{V_l} \) and \( C_{V_t} \) are not directly \( \Pi \) groups but a combination of them. They may be written with their dependencies as in equations (3.6) and (3.7).

\[ C_{V_l} = C_{V_l}(\Pi_E, \Pi_{\rho_l}) \]  

(3.6)

\[ C_{V_t} = C_{V_t}(\varepsilon_r, \Pi_{\sigma_r}, \Pi_E, \Pi_{\rho_l}) \]  

(3.7)

Therefore, the radius of the moving zone of the laminate, \( \bar{R}_t(\tau) \), is given by equation (3.8) since the transverse wave is the responsible for this movement. The radius of the longitudinal wave is given by (3.9).

\[ \bar{R}_t(\tau) = C_{V_t} \tau \]  

(3.8)

\[ \bar{R}_l(\tau) = C_{V_l} \tau \]  

(3.9)

If the matrix is considered to move in solidarity with the fibres, the mass of the zone of the laminate accelerated, \( \bar{m}_L(\tau) \), can be easily determined:

\[ \bar{m}_L(\tau) = \pi \Pi_\varepsilon \Pi_{\rho_l} \bar{R}_t(\tau)^2 \]  

(3.10)

The points of the material are assumed to move in the radial direction with a constant velocity from the point the longitudinal wave has reached to the transverse radius and from this point with a linear decreasing velocity until reaching the value of zero at the impact point. These points are assumed to move transversely with the velocity of the projectile at the impact point and with a decreasing velocity from this point to the point the transverse wave has reached. This explanation is illustrated in Figure 3.2.

![Fig. 3.2 Scheme of the movement of the laminate points in the radial and transverse directions during an impact](image)

The last consideration before the kinetic energy of the accelerated laminate can be calculated is that there is a relative displacement between the projectile and the laminate. Consequently, the laminate does not move exactly at the same velocity as the projectile, only the point in contact. Instead, it moves with a certain percentage of the projectile velocity. This percentage should depend on different aspects such as the laminate stiffness or the projectile velocity.
Here it is proposed a phenomenological function $k(t, v(t)) \in (0, 1]$ which represents the percentage of the projectile velocity at which the laminate moves. In other words, in the contact point between the two bodies the velocity is the one of the projectile and a gradient is assumed up to the point reached by the transverse wave, where the transverse velocity is zero. The way to quantify all the kinetic energy of the laminate is through the function $k(t, v(t))$ as can be seen in Figure 3.3. The velocity of the laminate must be understood as the velocity applied in the resultant of the cone, the center of mass.

$$k(t, v(t)) = cD^{1/6} \left( \frac{v(t)}{V_i} \right)^2 \left( \frac{V_it}{e} \right)^{1/2}, k \in (0, 1]$$  \hspace{1cm} (3.11)

where $D$ is the flexural rigidity of a plate defined in equation (3.12) and $c$ is a constant value of 0.25 [N m]$^{-1/6}$.

$$D = \frac{Ee^3}{12(1-\nu^2)}$$  \hspace{1cm} (3.12)

Trying to give a physical sense to the function $k(t, v(t))$, it can be subdivided in three parts. Looking at the flexural rigidity, it makes sense that the higher $D$, the higher $k(t, v(t))$. Therefore, the shape of tent will be more difficult to appreciate because the relative displacement will be very high. This behaviour could be seen in carbon fibre reinforced laminates where the stiffness is higher and a plug is formed instead of a cone. The relationship between $v(t)$ and $V_i$ seems to be reasonable because at the beginning of the impact when there is no damage this ratio is higher which means that $k(t, v(t))$ increases and the laminate moves almost like the projectile. But, when the fibres and the matrix start to break the relative displacement increases and also the difference between velocities, therefore this ratio is lower. Last, the ratio between the radius of the transverse wave and the thickness has a clear meaning. The lower this ratio, the lower the function $k(t, v(t))$ and it happens when the thickness increases. So $k(t, v(t))$ is lower and it is more difficult to appreciate the projectile and the laminate moving together when the thickness increases.
Rewriting $k(t, v(t))$ as a function of the $\Pi$ groups of the problems leads to equation (3.13).

$$\bar{k}(\tau, \bar{v}(\tau)) = \frac{\Pi_c}{\Pi_L} \Pi E^{1/6} C V_i^{1/2} \bar{v}(\tau)^2 \tau^{1/2}, \bar{k} \in (0, 1]$$  \hspace{1cm} (3.13)

Finally, the energy absorbed by laminate acceleration is given by equation (3.14).

$$\bar{E}_L(\tau) = \frac{1}{2} \bar{m}_L(\tau) \bar{v}_l(\tau)^2$$  \hspace{1cm} (3.14)

where $\bar{v}_l(\tau)$ is the laminate velocity defined in equation (3.15).

$$\bar{v}_l(\tau) = \bar{k}(\tau, \bar{v}(\tau)) \bar{v}(\tau)$$  \hspace{1cm} (3.15)

Substituting (3.8) in (3.10) and then (3.10) and (3.15) in (3.14) the most convenient form for the energy absorbed by laminate acceleration is obtained:

$$\bar{E}_L(\tau) = \frac{1}{2} \pi \Pi_L \Pi_{\mu_l} C V_i^2 \tau^2 \bar{k}(\tau, \bar{v}(\tau))^2 \bar{v}(\tau)^2$$  \hspace{1cm} (3.16)

**Energy absorbed by elastic deformation of fibres**

In the previous section it was seen that the fibres which are affected by the transverse wave exhibit a certain deformation because they increase their original length as a consequence of this wave provoked by the impactor.

The first consideration to take into account is the geometry of the projectile. Since the projectile is a steel sphere, and there is a relative displacement between the projectile and the laminate, the projected diameter of the projectile $\bar{\phi}$ can be written as a function of the relative displacement $\bar{\delta}$.

$$\bar{\phi}(\bar{\delta}) = \begin{cases} 
2\sqrt{\bar{\delta} - \bar{\delta}^2}, & \text{if } \bar{\delta} < 0.5 \\
1, & \text{if } \bar{\delta} \geq 0.5 
\end{cases}$$  \hspace{1cm} (3.17)

The relative displacement $\bar{\delta}$ is defined as the subtraction of the distance travelled by the projectile $\bar{x}(\tau)$ and the accelerated part of the laminate $\bar{x}_{lam}(\tau)$. The distance travelled by the projectile is defined in equation (3.18).

$$\bar{x}(\tau) = \int_0^\tau \bar{v}(\tau) d\tau$$  \hspace{1cm} (3.18)

In order to define the distance travelled by the laminate, the function $\bar{k}(\tau, \bar{v}(\tau))$ defined in last section is fundamental.

$$\bar{x}_{lam}(\tau) = \int_0^\tau \bar{k}(\tau, \bar{v}(\tau)) \bar{v}(\tau) d\tau$$  \hspace{1cm} (3.19)
The relative displacement can be written as in equation (3.20).

\[ \bar{\delta}(\tau, \bar{x}(\tau)) = \bar{x}(\tau) - \int_0^\tau \bar{k}(\tau, \bar{\nu}(\tau)) \bar{\nu}(\tau) d\tau \] (3.20)

In order to formulate this energy absorption mechanism, the approach proposed by Moyre et al. (2000) is considered. This approach proposes the energy per unit volume absorbed by elastic deformation of fibres is the area below the stress-strain curve in directions 11 or 22. The total energy absorbed by elastic deformation is given by (3.21).

\[ \bar{E}_{ED} = \int_{\bar{V}_0}^{\bar{V}} \left( \int_0^\varepsilon \bar{\sigma}(\varepsilon) d\varepsilon \right) d\bar{V} \] (3.21)

where \( \bar{V}_0 \) is an initial volume and \( \bar{V} \) is the volume included up to the distance travelled by the transverse wave. There is a subtlety in the definition of \( \bar{E}_{ED} \) that has been made. The lower limit of integration is not zero. This is because the elastic deformation of fibres is treated independently of tensile failure of fibres. Therefore, the region taken into account by this mechanism includes from the projectile diameter indented in the laminate to the distance travelled by the transverse wave. Another aspect to take into consideration is that it may occur that the indented projectile diameter grows faster than the transverse wave. In that case, the energy absorbed by elastic deformation of fibres is zero until the wave exceeds the indented projectile diameter, which varies also with time.

Since the material behaviour is considered linear-elastic until failure, this energy can be rewritten ((3.23)). To consider radial coordinates seems the more reasonable approach.

\[ d\bar{V} = 2\pi \bar{r} d\bar{r} \] (3.22)

where \( \bar{r} \) is the in-plane radial coordinate. Since waves along the thickness direction are not considered, the whole thickness intervenes in the volume differential from the beginning. In equation (3.23), the dependency of this energy absorption mechanism with time can be clearly seen.

\[ \bar{E}_{ED}(\tau) = \pi \Pi_E \int_{\bar{R}_d(\tau)}^{\bar{R}_i(\tau)} \varepsilon^2 \bar{r} d\bar{r} \] (3.23)

To quantify the deformation inside the integral a profile of deformations has to be considered. In this case, a linear profile of deformations is assumed. The maximum strain, which is the failure strain in directions 11 and 22, is located at the projectile periphery in contact with the laminate. The minimum strain is zero at the point that
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the transverse wave has reached. Then, the deformation is formally defined by (3.24).

\[
\varepsilon = \begin{cases}
\varepsilon_r, & \text{if } \bar{r} \leq \frac{\phi(\delta)}{2} \\
\frac{2(\bar{R}_t(\tau) - \bar{r})}{2\bar{R}_t(\tau) - \phi(\delta)}, & \text{if } \frac{\phi(\delta)}{2} < \bar{r} < \bar{R}_t(\tau) \\
0, & \text{if } \bar{r} \geq \bar{R}_t(\tau)
\end{cases}
\] (3.24)

Therefore, the energy absorbed by elastic deformation of fibres adopts the following form:

\[
\bar{E}_{ED}(\tau) = \pi \Pi_e \Pi E \varepsilon_r^2 \int_{\frac{\phi(\delta)}{2}}^{\bar{R}_t(\tau)} \left[ \frac{2(\bar{R}_t(\tau) - \bar{r})}{2\bar{R}_t(\tau) - \phi(\delta)} \right] \bar{r} d\bar{r}
\] (3.25)

Integrating and rearranging equation (3.25) the final expression for this mechanism is given by equation (3.26).

\[
\bar{E}_{ED}(\tau) = \frac{\pi \Pi_e \Pi E \varepsilon_r^2}{48} \frac{16\bar{R}_t(\tau)^4 - 24\bar{R}_t(\tau)^2\phi(\delta)^2 + 16\bar{R}_t(\tau)\phi(\delta)^3 - 3\phi(\delta)^4}{(2\bar{R}_t(\tau) - \phi(\delta))^2}
\] (3.26)

Energy absorbed by tensile failure of fibres

In this model, it is assumed that the fibres which fail by tensile failure are the ones just below the projectile. To make this assumption an additional hypotheses must be added. This is, that the diameter of the projectile is large enough to impact entirely against two fibres of the laminate (0° and 90°). Only these two fibres are considered because they are the ones which offer the highest resistance to the penetration of the projectile. The energy per unit volume is again the area below the stress-strain curve in directions 11 or 22. Considering that the fibres break as the projectile goes through them, a differential energy can be expressed as a function of a differential relative displacement.

\[
d\bar{E}_{TF} = \Pi_B d\bar{\delta} \int_0^{2\bar{R}_t(\tau)} \left( \int_0^{\varepsilon_a} \sigma(\varepsilon) d\varepsilon \right) d\bar{r}
\] (3.27)

where \(\varepsilon_a\) is the attenuated strain. In this case the differential volume considered is the distance the longitudinal wave has travelled, the width of the fibre and the relative displacement between the projectile and the laminate. The factor 2 in the upper integration limit in (3.27) is due to symmetry reasons.

\[
d\bar{V} = \Pi_B d\bar{\delta} d\bar{r}
\] (3.28)

The model considers a phenomenon reported by Naik et al. (2006) which corresponds to the attenuation wave factor. This phenomenon considers the maximum stress is reached at the impact point and then it decreases until reaching the value of zero at the longitudinal wave radius. The main reason for this phenomenon is the reflection and transmission of waves when they encounter material boundaries. Naik et al. (2006)
3.2 Model description

provides a further explanation about this matter and explains this factor is based on studies for each material because it depends on the mechanical, physical and geometrical properties of the fibres and the matrix. This factor can be considered either in stresses or strains. To take into account this effect it is enough to include the stress attenuation factor in the upper integration limit. The strain is attenuated automatically. The function which describes this effect is included in equation (3.29).

\[ \varepsilon_a = \varepsilon_r b \bar{n}_{\bar{m}} \]

(3.29)

If (3.27) is divided by \(d\tau\), a linear-elastic behaviour is assumed and the inner integral is calculated, equation (3.30) is obtained.

\[ \frac{d\tilde{E}_{TF}}{d\tau} = \frac{\Pi B}{2} \Pi E \varepsilon_r^2 \frac{d\bar{d}}{d\tau} \int_0^{2\bar{R}_l(\tau)} b \bar{n}_{\bar{m}} \, d\bar{r} \tag{3.30} \]

Solving the last integral in equation (3.30) and substituting the derivative of the relative displacement with respect to non-dimensional time, a final expression for the derivative of the energy absorbed by tensile failure of fibres is obtained.

\[ \frac{d\tilde{E}_{TF}}{d\tau} = \frac{\Pi B}{2} \Pi E \varepsilon_r^2 \left(1 - \bar{k}(\tau, \bar{v}(\tau)))\bar{v}(\tau)\left[\frac{4C_{V_1}}{b \bar{n}_{\bar{m}}} - 1\right]\right) \]

(3.31)

Integration of both sides of equation (3.30) provides a final expression for the energy absorbed by tensile failure of fibres.

\[ \tilde{E}_{TF} = \frac{\Pi B}{2} \Pi E \varepsilon_r^2 \left[\frac{4C_{V_1}}{b \bar{n}_{\bar{m}}} - 1\right] \int_0^{\tau} \left(1 - \bar{k}(\tau, \bar{v}(\tau)))\bar{v}(\tau)\right) d\tau \tag{3.32} \]

Energy absorbed by delamination

During the impact process, the movement of the projectile provokes a certain bending in the laminate contributing to the detachment of the adjacent plies. When the stresses are higher than the adhesive resistance, the plies are separated and this failure is translated to damaged area. It has been experimentally observed that the damaged area can be assumed circular for this material (Alonso et al. (2018b)). The radius of the damaged area matches the distance travelled by the transverse wave, so the damaged area is maximum at the ballistic limit because the contact time is the highest. Therefore, the energy absorbed by delamination is defined by means of a specific energy per unit area, \(G_{IICD}\), which is the energy a crack needs to propagate in a specific direction. The specific energy used is the one in mode II because this mode corresponds to a crack propagation due to in-plane shear, which provoke delamination. A certain area and a shape factor form, \(a_{DL}\), are also used. In this case, the shape factor form is 1 because
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the damaged area has a circular shape.

\[ \tilde{E}_{DL}(\tau) = \pi \Pi_{G_{ICD}} C_{V_i} \tau^2 \alpha_{DL} \]  (3.33)

Energy absorbed by matrix cracking

Matrix cracking and delamination are intimately related. At the same time the detachment of the plies takes place, the matrix is also damaged. Some authors as Naik et al. (2006) assumes the matrix breakage is produced as a consequence of the transverse wave, which is the responsible for the radial strain. Assuming this approach is reasonable, the energy absorbed by matrix cracking is defined by the specific energy dissipated by matrix cracking per unit volume, \( E_{MT} \), a volume and a shape factor form, \( \alpha_{MC} \), which in this case is 1 for the reason explained above.

\[ \tilde{E}_{MC}(\tau) = \pi \Pi_{E_{MT}} C_{V_i} \tau^2 \alpha_{MC} \]  (3.34)

Energy balance and governing equation of the model

The model is formulated by means of an energy balance. This balance may be written in its dimensional form as in equation (3.35). It is an instantaneous balance that is valid for any instant of time. It means the initial kinetic energy of the projectile \( E_0 \) is equal to the kinetic energy of the projectile at any instant of time \( E_{p}(t) \) plus the energy absorbed by all the mechanisms described before at that time \( E_{AB}(t) \).

\[ E_0 = E_{p}(t) + E_{AB}(t) \]  (3.35)

In the non-dimensional formulation, the balance adopts the following form:

\[ 1 = \bar{v}(\tau)^2 + \frac{12}{\pi} \bar{E}_{AB}(\tau) \]  (3.36)

where \( \bar{E}_{AB}(\tau) \) is defined as:

\[ \bar{E}_{AB}(\tau) = \bar{E}_{L}(\tau) + \bar{E}_{ED}(\tau) + \bar{E}_{TF}(\tau) + \bar{E}_{DL}(\tau) + \bar{E}_{MC}(\tau) \]  (3.37)

Derivation of this balance with respect to non-dimensional time provides the expression of equation (3.38).

\[ 0 = 2 \bar{v}(\tau) \bar{a}(\tau) + \frac{d}{d\tau} [\bar{E}_{L}(\tau) \bar{E}_{ED}(\tau) + \bar{E}_{TF}(\tau) + \bar{E}_{DL}(\tau) + \bar{E}_{MC}(\tau)] \frac{12}{\pi} \]  (3.38)

Two functions are defined in order to facilitate the handling of the equations (3.39) and (3.40).

\[ \bar{h}(\tau, \bar{v}(\tau)) = \frac{d\bar{E}_{TF}(\tau)}{d\tau} \frac{1}{\bar{v}(\tau)} \]  (3.39)
3.2 Model description

\[ \bar{g}(\tau, \bar{x}(\tau), \bar{v}(\tau)) = -\frac{d}{d\tau} [\bar{E}_{ED}(\tau) + \bar{E}_{DL}(\tau) + \bar{E}_{MC}(\tau)] \quad (3.40) \]

Introduction of these functions gives the form of the balance written in equation (3.41).

\[ 0 = 2\bar{v}(\tau)\bar{a}(\tau) + \bar{h}(\tau, \bar{v}(\tau))\bar{v}(\tau) - \bar{g}(\tau, \bar{x}(\tau), \bar{v}(\tau)) + \frac{1}{2}\pi \Pi_e \Pi_{\rho_l} C_{V_t} \bar{v}(\tau)^2. \quad (3.41) \]

\[ [(2\tau k(\tau, \bar{v}(\tau))^2 + 2\tau^2 \bar{k}(\tau, \bar{v}(\tau))\frac{d\bar{k}(\tau, \bar{v}(\tau))}{d\tau})\bar{v}(\tau)^2 + 2\tau^2 \bar{v}(\tau)\bar{a}(\tau)]\frac{12}{\pi} \]

Reorganizing the equation (3.41), the non-linear second-order differential equation which governs the problem is obtained, once the initial conditions have been stated.

\[ \bar{a}(\tau) = \frac{\bar{g}(\tau, \bar{x}(\tau), \bar{v}(\tau)) - \bar{h}(\tau, \bar{v}(\tau))\bar{v}(\tau)}{\bar{v}(\tau) + \pi \Pi_e \Pi_{\rho_l} C_{V_t} \bar{v}(\tau)^2 [2\tau^{5/2} \bar{k}(\tau, \bar{v}(\tau)) \Pi_{\Pi_e} \Pi_{\Pi_{E}}^{1/2} C_{V_t}^{1/2} \bar{v}(\tau)^3 + \tau^2 \bar{k}(\tau, \bar{v}(\tau))^2 \bar{v}(\tau)]} + \]

\[ \frac{\pi \Pi_e \Pi_{\rho_l} C_{V_t} \bar{v}(\tau)^2 [\tau \bar{k}(\tau, \bar{v}(\tau))^2 \bar{v}(\tau)^2 + \tau^3 \bar{k}(\tau, \bar{v}(\tau)) \Pi_{\Pi_e} \Pi_{\Pi_{E}}^{1/2} C_{V_t}^{1/2} \bar{v}(\tau)^4]}{\bar{v}(\tau) + \pi \Pi_e \Pi_{\rho_l} C_{V_t} \bar{v}(\tau)^2 [2\tau^{5/2} \bar{k}(\tau, \bar{v}(\tau)) \Pi_{\Pi_e} \Pi_{\Pi_{E}}^{1/2} C_{V_t}^{1/2} \bar{v}(\tau)^3 + \tau^2 \bar{k}(\tau, \bar{v}(\tau))^2 \bar{v}(\tau)]} \]

\[ \bar{x}(0) = 0 \]

\[ \bar{v}(0) = 1 \quad (3.42) \]

The values of the parameters in equation (3.42) obtained in other works can be found in Table 2.1 and the ones obtained in this work throughout Chapter 2.

3.2.2 Theoretical model for thick woven laminates

The theoretical model for thick woven laminates is influenced by previous models developed by Naik and Doshi (2005) and Alonso et al. (2018a). However, new considerations have been made to reformulate the whole problem.

As in the thin laminates model, the problem depends on three elemental magnitudes which are mass \([M]\), length \([L]\) and time \([T]\). Nevertheless, this model has independent stages. The outputs of the previous stage are the inputs for the next. Therefore, each stage has been formulated in their non-dimensional form with its characteristic parameters.

The first stage starts when the projectile impacts on the plate. From this moment, several waves propagate in different directions. Unlike to the thin laminates model, compressive waves along the thickness direction are considered. During this stage different energy absorption mechanisms are considered: compression, tensile failure of fibres, shear plugging, transference of kinetic energy, matrix cracking and delamination. This stage ends when the complete failure of the laminate takes place. At that moment, a plug with a certain kinetic energy is formed. In the second stage, which is assumed to be instantaneous, the projectile transfers linear momentum to the plug in order to move both bodies at the same velocity. A perfectly inelastic shock between the projectile and the plug is assumed in order to obtain this velocity. With both bodies moving at
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developed. In this last stage only the friction between the projectile and the laminate, which was obtained experimentally, is taken into account.

The following hypotheses have been assumed in the model formulation:

- The projectile is rigid and thus it remains undeformable during the entire impact process. There is experimental evidence that the projectile does not present deformation after any of the impacts.

- The laminate is considered linear-elastic and \( x \)-axially symmetric, being \( x \) the thickness direction.

- All wave velocities remain constant during the impact process. Furthermore, the compressive wave along the thickness direction is considered.

- The impact event is divided into the three stages described below.

- The spherical geometry of the projectile is taken into account in the formulation.

- In the first stage, the energy absorbed by compression is divided in two regions (Figure 3.4), the fibres that are directly under the projectile and the rest of the fibres indirectly affected by the movement of the projectile.

- The first stage ends when the complete failure of the laminate takes place.

- The failure criterion which determines the transition between the first two stages is assumed to be compression.

- The second stage is assumed to be instantaneous and ends when the transfer of momentum between the projectile and the laminate is finished.

- The third stage only takes into account the friction between the projectile and the laminate and it ends when the projectile exits the laminate.

- The energy absorbed by heat transfer between the projectile and the laminate is assumed to be negligible.

In the following sections, all the stages are fully described.

Stage 1

In the first stage, mass, length and time may be written in their non-dimensional form as follows:

\[
[M] = \rho_p \phi_p^3
\]  
\[ (3.43) \]

\[
[L] = \phi_p 
\]  
\[ (3.44) \]

\[
[T] = \frac{\phi_p}{V_i} 
\]  
\[ (3.45) \]
where $\rho_p$ is the projectile density, $\phi_p$ is the projectile diameter and $V_i$ is the impact velocity. The non-dimensional time $\tau$ is defined as the integration variable.

The problem depends on 17 fundamental parameters which are presented in Table 3.2 along with their corresponding $\Pi$ groups. It bears noting that some of the parameters are already non-dimensional, so the parameter is directly the $\Pi$ group in those cases.

<table>
<thead>
<tr>
<th>Parameter</th>
<th>Nomenclature</th>
<th>$\Pi$ group</th>
</tr>
</thead>
<tbody>
<tr>
<td>Projectile diameter</td>
<td>$\phi_p$</td>
<td></td>
</tr>
<tr>
<td>Projectile density</td>
<td>$\rho_p$</td>
<td>$\Pi_\rho$</td>
</tr>
<tr>
<td>Impact velocity</td>
<td>$V_i$</td>
<td></td>
</tr>
<tr>
<td>Laminate thickness</td>
<td>$e$</td>
<td>$\Pi_e = \frac{e}{\phi_p}$</td>
</tr>
<tr>
<td>In-plane Young’s modulus</td>
<td>$E$</td>
<td>$\Pi_E = \frac{E}{\rho_p V_i^2}$</td>
</tr>
<tr>
<td>Compressive out of plane Young’s modulus</td>
<td>$E_c$</td>
<td>$\Pi_{E_c} = \frac{E_c}{\rho_p V_i^2}$</td>
</tr>
<tr>
<td>In-plane failure strain</td>
<td>$\varepsilon_r$</td>
<td>$\varepsilon_r$</td>
</tr>
<tr>
<td>Compressive in-plane failure strain</td>
<td>$\varepsilon_{rc}$</td>
<td>$\varepsilon_{rc}$</td>
</tr>
<tr>
<td>Out of plane failure shear stress</td>
<td>$S_{SP}$</td>
<td>$\Pi_{S_{SP}} = \frac{S_{SP}}{\rho_p V_i^2}$</td>
</tr>
<tr>
<td>Laminate density</td>
<td>$\rho_l$</td>
<td>$\Pi_{\rho_l} = \frac{\rho_l}{\rho_p}$</td>
</tr>
<tr>
<td>Energy absorbed by matrix cracking per unit volume</td>
<td>$E_{MT}$</td>
<td>$\Pi E_{MT} = \frac{E_{MT}}{\rho_p V_i^2}$</td>
</tr>
<tr>
<td>Critical dynamic-strain energy-release rate in mode II</td>
<td>$G_{IICD}$</td>
<td>$\Pi G_{IICD} = \frac{G_{IICD}}{\rho_p \phi_p V_i^2}$</td>
</tr>
<tr>
<td>Shape factor of delamination</td>
<td>$\alpha_{DL}$</td>
<td>$\alpha_{DL}$</td>
</tr>
<tr>
<td>Shape factor of matrix cracking</td>
<td>$\alpha_{MC}$</td>
<td>$\alpha_{MC}$</td>
</tr>
<tr>
<td>Yarn width</td>
<td>$B$</td>
<td>$\Pi_B = \frac{B}{\rho_p}$</td>
</tr>
<tr>
<td>Stress wave transmission factor</td>
<td>$b$</td>
<td>$b$</td>
</tr>
</tbody>
</table>

Table 3.2 Summary of the parameters and $\Pi$ groups of the first stage

As in the thin laminates model, the number of parameters governing the problem has been reduced by 3, from 17 to 14. The nomenclature to indicate the non-dimensional character of the variables is the same than for the thin model, a bar over the variable.

In the following subsections, the energy absorption mechanisms considered in this stage are explained and formulated. Figure 3.4 shows the variables and parameters involved during this stage in a generic instant of time.
Energy absorbed by compression in Region 1

When a projectile impacts on a laminate and the thickness is large enough, the behaviour of the plate as a membrane can no longer be appreciated. Instead, the phenomenon of local compression is more evident. In order to calculate all the contributions of compression, it is divided into two regions. A first region that is the one just below the projectile where the fibres are compressed from the projectile to the distance travelled by the through thickness compressive wave. This is the direct compression, which is caused by the movement of the projectile. And a second region whose domain goes from the perimeter of the projectile to the distance travelled by the transverse wave. This is the indirect compression because it is not caused directly by the projectile but to the effect of it and the transverse wave on the surroundings. As it is an indirect compression, all the thickness is taken into account from the beginning. These two regions can be seen in Figure 3.4. In the first region, the induced stresses provoke the material failure at some point, when the compressive failure strain is reached.

Wave propagation is a crucial aspect in the formulation of thick laminates model. Since this model is intended to be used for thick laminates, the through thickness compressive wave, which rules the phenomenon of compression, is considered unlike in
the thin laminates model. The longitudinal and the transverse wave are defined in (3.4) and (3.5). The expression of the through thickness compressive wave is given by:

\[ C_{V_{xl}} = \sqrt{\frac{\Pi_{E_c}}{\Pi_{\rho_l}}} \]  

(3.46)

\( C_{V_{xl}} \) may be written as a function of different \( \Pi \) groups because it is not directly a \( \Pi \) group but a combination of them.

\[ C_{V_{xl}} = C_{V_{xl}}(\Pi_{E_c}, \Pi_{\rho_l}) \]  

(3.47)

Consequently, the distance travelled by the through thickness compressive wave is given by (3.5).

\[ \bar{R}_{xl}(\tau) = C_{V_{xl}} \tau \]  

(3.48)

The energy absorbed by compression in region 1, \( \bar{E}_{C1} \), may be calculated as the energy below the compressive stress-strain curve in the thickness direction integrated in the volume of the region.

\[ \bar{E}_{C1} = \int_{0}^{\bar{V}} \left( \int_{0}^{\bar{\varepsilon}_{rc}} \bar{\sigma}_c(\varepsilon_c) \, d\varepsilon_c \right) \, d\bar{V} \]  

(3.49)

At this point, the definition of the projected diameter in contact with the laminate has to be formulated since it is variable. In this problem, since the plate does not behave as a membrane due to its thickness, the relative displacement between the projectile and the laminate is considered to be directly the distance the projectile has travelled. Thus, the projected diameter of the projectile can be written as a function of its position.

\[ \bar{\phi}(\bar{x}) = \begin{cases} 
2\sqrt{\bar{x} - \bar{x}^2}, & \text{if } \bar{x} < 0.5 \\
1, & \text{if } \bar{x} \geq 0.5 
\end{cases} \]  

(3.50)

Assuming that during a differential increment of time, the projected area of the projectile remains constant, the volume differential can be written as:

\[ d\bar{V} = \frac{\pi}{4} C_{V_{xl}} \bar{\phi}(\bar{x})^2 \, d\tau \]  

(3.51)

Introducing (3.51) in (3.49) and integrating leads to the expression for energy absorbed by compression in region 1.

\[ \bar{E}_{C1} = \frac{\pi}{8} \Pi_{E_c} \bar{\varepsilon}_{rc} \bar{\phi}(\bar{x})^2 C_{V_{xl}} \tau \]  

(3.52)
Theoretical model

Energy absorbed by compression in Region 2

Along with the compression just below the projectile, there is another region also subjected to compression. In its way through the laminate, the projectile provokes that the upper fibres press the lower fibres generating indirect compression in this zone as can be seen in Figure 3.4. The energy absorbed by this mechanism is calculated as the area below the compressive stress-strain curve in direction 33 integrated in the volume of the region.

\[
\bar{E}_{C2} = \int_{V_0}^{\bar{V}} \left( \int_0^{\bar{\varepsilon}_{ec}} \bar{\sigma}_{c}(\varepsilon_c) \, d\varepsilon_c \right) \, d\bar{V} \tag{3.53}
\]

where \(\bar{V}_0\) is an initial volume and \(\bar{V}\) is the volume included up to the distance travelled by the transverse wave in the in-plane direction. Since the compression in this zone is not caused by the compressive wave because it is an indirect compression provoked by the projectile movement and the transverse wave, the expression for the volume differential is the following:

\[
d\bar{V} = 2\Pi_c \bar{r} d\bar{r} \tag{3.54}
\]

Substituting (3.54) in (3.53) leads to equation (3.55).

\[
\bar{E}_{C2}(\tau) = \pi \Pi_c \Pi_{E_c} \int_{\bar{\phi}(\bar{x})}^{\bar{R}_t(\tau)} \varepsilon_c^2 \bar{r} \, d\bar{r} \tag{3.55}
\]

It is assumed that the maximum compression is reached in the periphery of the projectile and it is equal to the compressive failure strain through the thickness direction and that the minimum compression is zero at the radius of the transverse wave. Between these two points, a linear profile of deformations is assumed. Therefore, the deformation is formally defined by (3.56).

\[
\varepsilon_c = \begin{cases} 
\bar{\varepsilon}_{rc}, & \text{if } \bar{r} \leq \frac{\bar{\phi}(\bar{x})}{2} \\
\frac{2(\bar{R}_t(\tau) - \bar{r})}{2\bar{R}_t(\tau) - \bar{\phi}(\bar{x})}, & \text{if } \frac{\bar{\phi}(\bar{x})}{2} < \bar{r} < \bar{R}_t(\tau) \\
0, & \text{if } \bar{r} \geq \bar{R}_t(\tau) 
\end{cases} \tag{3.56}
\]

Consequently, the energy absorbed by compression in region 2 adopts the following form:

\[
\bar{E}_{C2}(\tau) = \pi \Pi_c \Pi_{E_c} \varepsilon_{rc}^2 \int_{\bar{\phi}(\bar{x})}^{\bar{R}_t(\tau)} \left[ \frac{2(\bar{R}_t(\tau) - \bar{r})}{2\bar{R}_t(\tau) - \bar{\phi}(\bar{x})} \right]^2 \bar{r} \, d\bar{r} \tag{3.57}
\]

Integrating and rearranging equation (3.57) the final expression for this mechanism is given by equation (3.58).

\[
\bar{E}_{C2}(\tau) = \frac{\pi \Pi_c \Pi_{E_c} \varepsilon_{rc}^2}{48} \frac{16\bar{R}_t(\tau)^4 - 240\bar{R}_t(\tau)^2 \bar{\phi}(\bar{x})^2 + 16\bar{R}_t(\tau)\bar{\phi}(\bar{x})^3 - 3\bar{\phi}(\bar{x})^4}{(2\bar{R}_t(\tau) - \bar{\phi}(\bar{x}))^2} \tag{3.58}
\]
3.2 Model description

Energy absorbed by tensile failure of fibres

As the projectile impacts on the laminate, compression of the target takes place just below the projectile (region 1) and in its surroundings (region 2). As a result of this compression the fibres which are directly in contact with the projectile are strained in the radial direction as can be seen in Figure 3.4. Therefore, these fibres may fail as a consequence of the in-plane stress.

The hypotheses considered to formulate this energy mechanism are the same than for the thin laminates model. The only difference is that there is not a movement of the plate as a membrane and thus the relative displacement between the projectile and the laminate is considered to be directly the distance travelled by the projectile. A differential energy as a function of a differential relative displacement is expressed below.

\[ d\bar{E}_{TF} = \Pi_B d\bar{x} \int_0^{2\bar{R}_l(\tau)} \left( \int_0^{\varepsilon_a} \bar{\sigma}(\varepsilon)\, d\varepsilon \right) \, d\bar{r} \]  

(3.59)

where \( \varepsilon_a \) is the attenuated strain defined in (3.29). As in the model for thin laminates, the attenuation wave factor is considered (Naik et al. (2006)). The differential volume considered is defined by the distance travelled by the longitudinal wave, the width of the fibre and the distance travelled by the projectile through the thickness direction.

\[ d\bar{V} = \Pi_B d\bar{x} d\bar{r} \]  

(3.60)

If (3.59) is divided by \( d\tau \), a linear-elastic behaviour is assumed and the inner integral is calculated, equation (3.61) is obtained.

\[ \frac{d\bar{E}_{TF}}{d\tau} = \frac{\Pi_B}{2} \Pi_E \varepsilon_T^2 \frac{d\bar{x}}{d\tau} \int_0^{2\bar{R}_l(\tau)} b^{\frac{4CV_{v,T}}{b}} \, d\bar{r} \]  

(3.61)

Solving the last integral in equation (3.61), a final expression for the derivative of the energy absorbed by tensile failure of fibres is obtained.

\[ \frac{d\bar{E}_{TF}}{d\tau} = \frac{\Pi_B}{2} \Pi_E \varepsilon_T^2 \bar{v}(\tau) \left[ b^{\frac{4CV_{v,T}}{b}} - 1 \right] \int_0^\tau \bar{v}(\tau)\, d\tau \]  

(3.62)

Integration of both sides of equation (3.62) provides a final expression for the energy absorbed by tensile failure of fibres.

\[ \bar{E}_{TF} = \frac{\Pi_B}{2} \Pi_E \varepsilon_T^2 \left[ b^{\frac{4CV_{v,T}}{b}} - 1 \right] \int_0^\tau \bar{v}(\tau)\, d\tau \]  

(3.63)

Energy absorbed by transference of kinetic energy

In the model for thin laminates, the plate is supposed to behave as a membrane in all its thickness absorbing kinetic energy of the projectile. On the contrary, for thick laminates,
Theoretical model

this behaviour is not observed. However, the projectile transfers some kinetic energy to the laminate. The hypothesis made to quantify this energy is to impose a contact condition. And that is that the part of the laminate in contact with the projectile moves with the same velocity of the projectile. The point reached by the compressive wave through the thickness direction has a velocity equal to zero. Between these two points, a profile of velocities has to be assumed. In this case, a linear profile has been chosen. Only the material just below the projected projectile diameter, which changes during the impact (3.50), is taken into account for this transference.

Each differential of mass moves with a certain velocity which changes with time. In order to count and add the kinetic energy of all the differentials at any time equation (3.64) is defined.

\[
\hat{E}_{CL}(\tau) = \frac{1}{2} \int_0^{\tilde{m}(\tau)} \bar{v}(\bar{s}, \tau)^2 d\tilde{m}
\]

(3.64)

where \( \bar{v}(\bar{s}, \tau) \) is the velocity as a function of a mute spatial variable \( \bar{s} \) and time, \( d\tilde{m} \) is a differential of mass and \( \tilde{m}(\tau) \) is the mass affected by this phenomenon.

The differential of mass can be expressed as a function of the spatial variable.

\[
d\tilde{m} = \frac{\pi}{4} \rho l \bar{\phi}(\bar{x})^2 d\bar{s}
\]

(3.65)

The velocity in equation (3.64) can be expressed as a function of the velocity of the projectile and the spatial variable assuming a linear profile of velocities as it was explained before.

\[
\bar{v}(\bar{s}, \tau) = \bar{v}(\tau) \left[ 1 - \frac{\bar{s}}{R_{xl}(\tau)} \right]
\]

(3.66)

Replacing (3.65) and (3.66) in (3.64), the kinetic energy absorbed by the laminate adopts the following expression:

\[
\hat{E}_{CL}(\tau) = \frac{\pi}{8} \Pi_{\rho l} \bar{\phi}(\bar{x})^2 \int_0^{\bar{R}_{xl}(\tau)} \left[ 1 - \frac{\bar{s}}{R_{xl}(\tau)} \right]^2 d\bar{s}
\]

(3.67)

Integrating, the final expression for the energy absorbed by transference of kinetic energy is obtained.

\[
\hat{E}_{CL}(\tau) = \frac{\pi}{24} \Pi_{\rho l} \bar{v}(\tau)^2 \bar{\phi}(\bar{x})^2 C_{V_{xl}} \tau
\]

(3.68)

Energy absorbed by shear plugging

Under high-velocity impact, thick laminates withstand high shear stresses whether they fail by this mechanism or not. The forces responsible for these stresses are generated around the projectile diameter in contact with the laminate. If these shear stresses are high enough and they surpass the maximum shear strength of the material in directions 13 and 23, the laminate can fail by shear plugging.
3.2 Model description

A differential energy absorbed by this mechanism may be obtained by multiplying the annular area by the maximum shear stress of the material in directions 13 and 23 and by the displacement of the projectile.

\[ d\bar{E}_{SP} = \pi \bar{\phi}(\bar{x})\Pi_{\ell} \Pi_{SP} d\bar{x} \]  
(3.69)

If (3.69) is divided by \( d\tau \), equation (3.70) is obtained.

\[ \frac{d\bar{E}_{SP}}{d\tau} = \pi \bar{\phi}(\bar{x})\Pi_{\ell} \Pi_{SP} \frac{d\bar{x}}{d\tau} \]  
(3.70)

Rearranging (3.70), the energy absorbed by shear plugging is expressed as:

\[ \bar{E}_{SP} = \int_{0}^{\tau} \pi \bar{\phi}(\bar{x})\Pi_{\ell} \Pi_{SP} \bar{v}(\tau) d\tau \]  
(3.71)

Energy absorbed by delamination and matrix cracking

The energy absorbed by delamination and matrix cracking are exactly the same than the ones described in the thin laminates model. The expressions are given by (3.33) and (3.34) respectively. The only qualitative difference is that it has been observed experimentally that there is more difference in the damaged area between the front and rear faces for thicker laminates. However, the equations are still the same.

Energy balance and governing equation of the first stage

This stage is formulated by means of an energy balance. The equation which defines this balance is the same as for the thin laminates model, (3.35) for the dimensional case and (3.36) for the non-dimensional case. But in this case, the sum of all the absorption energy mechanisms is obviously different and it is defined in equation (3.72).

\[ \bar{E}_{AB}(\tau) = \bar{E}_{C1}(\tau) + \bar{E}_{C2}(\tau) + \bar{E}_{TF}(\tau) + \bar{E}_{CL}(\tau) + \bar{E}_{SP}(\tau) + \bar{E}_{DL}(\tau) + \bar{E}_{MC}(\tau) \]  
(3.72)

Derivation (3.72) with respect to non-dimensional time provides the expression of equation (3.73).

\[ 0 = 2\bar{v}(\tau)\bar{a}(\tau) + \frac{d}{d\tau} \left[ \bar{E}_{C1}(\tau) + \bar{E}_{C2}(\tau) + \bar{E}_{TF}(\tau) + \bar{E}_{CL}(\tau) + \bar{E}_{SP}(\tau) + \bar{E}_{DL}(\tau) + \bar{E}_{MC}(\tau) \right] \frac{12}{\pi} \]  
(3.73)

In this case three functions need to be defined to make algebra easier.

\[ \bar{h}(\tau, \bar{v}(\tau)) = \frac{d}{d\tau} \left[ \bar{E}_{TF}(\tau) + \bar{E}_{SP}(\tau) \right] \frac{1}{\bar{v}(\tau)} \]  
(3.74)

\[ \bar{g}(\tau, \bar{x}(\tau), \bar{v}(\tau)) = -\frac{d}{d\tau} \left[ \bar{E}_{C2}(\tau) + \bar{E}_{DL}(\tau) + \bar{E}_{MC}(\tau) \right] \]  
(3.75)
Theoretical model

\[ \bar{f}(\tau, \bar{x}(\tau)) = \frac{dE_{C1}(\tau)}{d\tau} \]  (3.76)

Introduction of these functions gives the form of the balance written in equation (3.77).

\[ 0 = \frac{\pi}{6} \bar{v}(\tau) \bar{a}(\tau) + \bar{h}(\tau, \bar{v}(\tau)) \bar{v}(\tau) - \bar{g}(\tau, \bar{x}(\tau), \bar{v}(\tau)) + \bar{f}(\tau, \bar{x}(\tau)) + \frac{\pi}{24} \Pi \rho l C V x l \left[ (2\bar{v}(\tau) \bar{a}(\tau) \bar{\phi}(\bar{x}))^2 + 2\bar{v}(\tau) \bar{\phi}(\bar{x}) \frac{d\bar{\phi}}{d\tau} \bar{v}(\tau) \right] \]

Reorganizing equation (3.77), the non-linear second-order differential equation which governs the first stage is obtained, with its corresponding initial conditions.

\[ \bar{a}(\tau) = \frac{\bar{g}(\tau, \bar{x}(\tau), \bar{v}(\tau)) - \bar{h}(\tau, \bar{v}(\tau)) - \bar{f}(\tau, \bar{x}(\tau)) - \frac{\pi}{24} \Pi \rho l C V x l \left[ 2\bar{v}(\tau) \bar{\phi}(\bar{x}) \frac{d\bar{\phi}}{d\tau} \bar{v}(\tau) + \bar{v}(\tau) \bar{\phi}(\bar{x})^2 \right]}{\frac{\pi}{6} \bar{v}(\tau) + \frac{\pi}{12} \Pi \rho l C V x l \bar{v}(\tau) \bar{\phi}(\bar{x})^2} \]

\[ \bar{x}(0) = 0 \]
\[ \bar{v}(0) = 1 \]

(3.78)

It has to be noticed that this stage finishes when the stop condition given by equation (3.79) is met. At this moment, the velocity of the projectile is the initial velocity for the second stage.

\[ \bar{x}(\tau) = \Pi_e \varepsilon_{rc} \]  (3.79)

The values of the parameters in equation (3.78) obtained in other works can be found in Table 2.1 and the ones obtained in this work throughout Chapter 2.

Stage 2

At the end of the first stage, just before failure, the projectile moves with a certain linear momentum. Also, some kinetic energy has been transferred to a small part of the laminate. This part is just below the indented projected projectile diameter up to the radius of the compressive wave. Therefore, this part moves with a certain linear momentum too. Just after failure, the second stage, which is instantaneous, takes place. This stage is illustrated in Figure 3.5.

Fig. 3.5 Scheme of the impact phenomenon at a precise instant of time of the second stage
In the second stage, a perfectly inelastic shock is assumed. The first body which is involved in the shock is the projectile and the second one is the plug formed. This last is the cylinder with a diameter equal to the projectile’s and a height equal to the thickness. After the shock, the two bodies move at the same velocity. At the moment of transition between the first and the second stage there are not exterior forces acting in the two bodies. Therefore, the momentum conservation principle can be applied.

\[ \vec{p}_2 = \vec{p}_3 \]  

(3.80)

where \( \vec{p}_2 \) and \( \vec{p}_3 \) are the linear momentums at the beginning of the second and third stages respectively. Since the transference of linear momentum takes place in one direction, equation (3.80) can be expressed as a scalar equation.

\[ p_{11} + m_p V_{i2} = (m_p + m_l)V_{i3} \]  

(3.81)

where \( p_{11} \) is the accumulated linear momentum of the laminate at the end of the first stage, \( V_{i2} \) is the initial velocity of the second stage which is the final velocity of the first one, \( m_p \) is the mass of the projectile, \( m_l \) is the mass of the plug formed and \( V_{i3} \) is the final velocity of the second stage and the initial velocity of the third stage.

Although the only unknown parameter is \( V_{i3} \), to maintain the non-dimensional formulation, the Vaschy-Buckingham II Theorem is given an "artificial use". Mass and length may be written in their non-dimensional form like in equations (3.1) and (3.2). Time may be written as follows:

\[ [T] = \frac{\phi_p}{V_{i2}} \]  

(3.82)

The problem depends on 8 fundamental parameters and thus 5 "artificial" II groups can be formed. They are presented in Table 3.3.

<table>
<thead>
<tr>
<th>Parameter</th>
<th>Nomenclature</th>
<th>II group</th>
</tr>
</thead>
<tbody>
<tr>
<td>Projectile diameter</td>
<td>( \phi_p )</td>
<td></td>
</tr>
<tr>
<td>Projectile density</td>
<td>( \rho_p )</td>
<td></td>
</tr>
<tr>
<td>Initial velocity</td>
<td>( V_{i2} )</td>
<td></td>
</tr>
<tr>
<td>Laminate thickness</td>
<td>( e )</td>
<td>( \Pi_e = \frac{e}{\phi_p V_{i2}} )</td>
</tr>
<tr>
<td>Compressive out of plane Young’s modulus</td>
<td>( E_c )</td>
<td>( \Pi_{E_c} = \frac{E_c}{\phi_p V_{i2}} )</td>
</tr>
<tr>
<td>Projected diameter at the end of stage 1</td>
<td>( \phi_1 )</td>
<td>( \Pi_{\phi_1} = \frac{\phi_1}{\phi_p} )</td>
</tr>
<tr>
<td>Time at the end of stage 1</td>
<td>( t_1 )</td>
<td>( \Pi_{t_1} = \frac{t_1 \phi_p}{V_{i2}} )</td>
</tr>
<tr>
<td>Laminate density</td>
<td>( \rho_l )</td>
<td>( \Pi_{\rho_l} = \frac{\rho_l}{\rho_p} )</td>
</tr>
</tbody>
</table>

Table 3.3 Summary of the parameters and the II groups of the problem
Theoretical model

\( p_{1l} \) can be calculated by adding all the linear momentums of each differential of mass.

\[
p_{1l} = \int_0^{m_1} v(s) \, dm
\]  
(3.83)

where \( m_1 \) is the mass that has absorbed energy by transference of kinetic energy in the first stage, \( v(s) \) is the velocity which depends on a spatial variable and \( dm \) is a differential of mass. All these parameters are evaluated at the end of the first stage. Therefore the differential of mass adopts the following form:

\[
dm = \frac{\pi}{4} \rho_1 \phi_1^2 ds
\]  
(3.84)

The hypotheses of a linear profile of velocities was assumed for the energy absorbed by transference of kinetic energy. So \( v(s) \) may be written as in equation (3.85).

\[
v(s) = V_{i2} \left[ 1 - \frac{s}{R_{xl}(t_1)} \right]
\]  
(3.85)

Replacing (3.84) and (3.85) in (3.83), a new expression for the linear momentum of the laminate at the beginning of the second stage is obtained.

\[
p_{1l} = \frac{\pi}{4} \rho_1 V_{i2} \phi_1^2 \int_0^{R_{xl}(t_1)} \left[ 1 - \frac{s}{R_{xl}(t_1)} \right] ds
\]  
(3.86)

Integrating (3.86), a final expression is obtained.

\[
p_{1l} = \frac{\pi}{8} \rho_1 V_{x2} V_{i2} \phi_1^2 t_1
\]  
(3.87)

The expressions for the mass of the projectile as well as the mass of the laminate affected by the transference of kinetic energy during the first stage are defined below.

\[
m_p = \frac{\pi}{6} \phi_p^3 \rho_l
\]  
(3.88)

\[
m_l = \frac{\pi}{4} \phi_p^2 e \rho_l
\]  
(3.89)

Substituting (3.86), (3.88) and (3.89) in (3.81), dividing the two sides of (3.81) by \( \phi_p^3 \rho_l V_{i2} \), rearranging and treating \( V_{i3} \) as the variable to obtain, equation (3.90) is reached.

\[
\bar{V}_{i3} = \frac{\Pi_{p_1} \Pi_{t_1} \phi_1^2 C_{V_{x3}} + 4}{6 \Pi_{p_1} \Pi_{p_1} + 4}
\]  
(3.90)

Stage 3

In this last stage, the projectile moves in solidarity with the plug with an initial velocity which is assumed to be the final velocity obtained in the second stage. The only energy
absorption mechanism that takes place in this stage is the friction between the projectile and the laminate. The friction force-displacement curve for each thickness was measured experimentally as it was explained in Chapter 2.

The problem depends on three elemental magnitudes: mass and length may be written in their non-dimensional form like in equations (3.1) and (3.2) and time as follows:

$$[T] = \frac{\phi_p}{V_{i3}}$$

where $V_{i3}$ is the initial velocity of this stage.

The problem depends on 16 fundamental parameters which are presented in Table 3.4 with their corresponding $\Pi$ groups. According to the Vaschy-Buckingham $\Pi$ Theorem, the number of parameters on which the problem depends has been reduced from 16 to 13.

<table>
<thead>
<tr>
<th>Parameter</th>
<th>Nomenclature</th>
<th>$\Pi$ group</th>
</tr>
</thead>
<tbody>
<tr>
<td>Projectile diameter</td>
<td>$\phi_p$</td>
<td></td>
</tr>
<tr>
<td>Projectile density</td>
<td>$\rho_p$</td>
<td></td>
</tr>
<tr>
<td>Initial velocity</td>
<td>$V_{i3}$</td>
<td></td>
</tr>
<tr>
<td>Laminate thickness</td>
<td>$e$</td>
<td>$\Pi_e = \frac{e}{\phi_p}$</td>
</tr>
<tr>
<td>Laminate density</td>
<td>$\rho_l$</td>
<td>$\Pi_{\rho_l} = \frac{\rho_l}{\rho_p}$</td>
</tr>
<tr>
<td>Friction parameter 1</td>
<td>$C_1$</td>
<td>$\Pi_{C_1} = \frac{C_1}{\rho_p V_{i3}^2}$</td>
</tr>
<tr>
<td>Friction parameter 2</td>
<td>$C_2$</td>
<td>$\Pi_{C_2} = \frac{C_2}{\rho_p \phi_p V_{i3}^2}$</td>
</tr>
<tr>
<td>Friction parameter 3</td>
<td>$C_3$</td>
<td>$\Pi_{C_3} = \frac{C_3}{\rho_p \phi_p V_{i3}^2}$</td>
</tr>
<tr>
<td>Friction parameter 4</td>
<td>$C_4$</td>
<td>$\Pi_{C_4} = \frac{C_4}{\rho_p V_{i3}^2}$</td>
</tr>
<tr>
<td>Friction parameter 5</td>
<td>$C_5$</td>
<td>$\Pi_{C_5} = \frac{C_5}{\rho_p \phi_p V_{i3}^2}$</td>
</tr>
<tr>
<td>Friction parameter 6</td>
<td>$C_6$</td>
<td>$\Pi_{C_6} = \frac{C_6}{\rho_p \phi_p V_{i3}^2}$</td>
</tr>
<tr>
<td>Friction parameter 7</td>
<td>$C_7$</td>
<td>$\Pi_{C_7} = \frac{C_7}{\rho_p V_{i3}^2}$</td>
</tr>
<tr>
<td>Friction parameter 8</td>
<td>$C_8$</td>
<td>$\Pi_{C_8} = \frac{C_8}{\rho_p \phi_p V_{i3}^2}$</td>
</tr>
<tr>
<td>Friction parameter 9</td>
<td>$C_9$</td>
<td>$\Pi_{C_9} = \frac{C_9}{\rho_p \phi_p V_{i3}^2}$</td>
</tr>
<tr>
<td>Length parameter 1</td>
<td>$L_1$</td>
<td>$\Pi_{L_1} = \frac{L_1}{\phi_p}$</td>
</tr>
<tr>
<td>Length parameter 2</td>
<td>$L_2$</td>
<td>$\Pi_{L_2} = \frac{L_2}{\phi_p}$</td>
</tr>
</tbody>
</table>

Table 3.4 Summary of the parameters and $\Pi$ groups of the problem in the third stage

$\Pi_{C_i}$ for $i = 1, 9$ represents the coefficients of the experimental friction force-displacement curve in their non-dimensional form and they can be found in Chapter 2. $\Pi_{L_i}$ for $i = 1, 2$ represents the non-dimensional displacements which bracket the intervals of the experimental friction force-displacement curve because it is defined by pieces.
Theoretical model

A generic time during this stage with the variables involved is illustrated in Figure 3.6.

\[ \dot{x}(\tau) = \begin{bmatrix} \ddot{x}(\tau) \\ \ddot{y}(\tau) \end{bmatrix} \]

**Fig. 3.6 Scheme of the impact phenomenon at a generic instant of time in the third stage**

The amount of energy the set projectile plus plug loses through their path across the laminate may be expressed at any time by equation (3.92).

\[ \Delta E_f(t) = \frac{1}{2}(m_p + m_l)(V_{i3} - v(t)^2) \quad (3.92) \]

In its non-dimensional form, with \( m_p \) and \( m_l \) defined in equations (3.88) and (3.89) respectively, equation (3.92) turns into equation (3.93).

\[ \Delta \bar{E}_f(\tau) = \frac{\pi}{4} \left( \frac{1}{3} + \frac{1}{2} \bar{\Pi}_e \bar{\Pi}_p \right) (1 - \bar{\upsilon}(\tau)^2) \quad (3.93) \]

This lost energy is the energy below the experimental friction force-displacement curve since only friction is acting. The hole for the friction tests was slightly smaller than the projectile's. For this reason, the ranges of the curve are 9 mm and 12 mm. As can be seen in Figure 3.6, a half of the projectile is indented approximately in the starting position, and the final position takes place when the other half of the projectile has crossed all the plate. An additional hypotheses needs to be assumed to be able to use this curve and this is that although the failure during stage 1 does not occur exactly with an indentation equal to the radius, it is approximately equal to it. In this way, from the beginning of stage 3 we are at the position where the friction curve starts. With a simple calculation using equations (3.79) and (3.50), the indented diameter at the moment of failure can be determined for 9 mm and 12 mm, which are \( \phi_{f9} = 6.58 \) mm and \( \phi_{f12} = 7.14 \) mm respectively, not so far from the 7.5 mm given by the projectile diameter.

Making the last assumption, the loss of energy can also be calculated from the beginning of the third stage by means of the friction force-displacement curve (3.94). An additional consideration is that the reference system in this last stage is moved so that the initial position of the third stage be zero.

\[ \Delta \bar{E}_f(\tau) = \int_0^{\bar{s}(\tau)} \bar{f}(\bar{s}) \, d\bar{s} \quad (3.94) \]
3.2 Model description

where $\bar{f}(\bar{s})$ is the friction force-displacement curve as a function of a mute spatial variable $\bar{s}$.

$$
\bar{f}(\bar{s}) = \begin{cases} 
\Pi C_1 \bar{s}^2 + \Pi C_2 \bar{s} + \Pi C_3, & \text{if } 0 < \bar{s} \leq \Pi L_1 \\
\Pi C_4 \bar{s}^2 + \Pi C_5 \bar{s} + \Pi C_6, & \text{if } \Pi L_1 < \bar{s} \leq \Pi L_2 \\
\Pi C_7 \bar{s}^2 + \Pi C_8 \bar{s} + \Pi C_9, & \text{if } \Pi L_2 < \bar{s} < \Pi e
\end{cases}
$$

(3.95)

Combining equations (3.93) and (3.94) and deriving with respect to non-dimensional time, equation (3.96) is obtained.

$$
\pi \frac{2}{\left(1 + \frac{1}{3} + \frac{1}{2} \Pi e \Pi l\right)} \bar{a}(\tau) = -\bar{f}(\bar{x}(\tau))
$$

(3.96)

Stating the initial conditions, a second-order differential equation is obtained.

$$
\bar{a}(\tau) = \frac{-\bar{f}(\bar{x}(\tau))}{\frac{\pi}{2} \left(\frac{1}{3} + \frac{1}{2} \Pi e \Pi l\right)} \\
\bar{x}(0) = 0 \\
\bar{v}(0) = 1
$$

(3.97)

This last stage finishes when the condition given by equation (3.97) is fulfilled, which is the moment when the projectile is not any more in contact with the plate as can be seen in Figure 3.6.

$$
\bar{x}(\tau) = \Pi e
$$

(3.98)
The objective of this chapter is to develop a finite element model for high-velocity impact on woven GFRP laminates. This model is intended to be used not only to predict such aspects as the ballistic limit or the residual velocities but to check some of the new hypotheses assumed in the theoretical models. First, different models in the literature were compared to understand why different failure criteria and constitutive models are used for different materials. In view of these models, a new continuum damage model, which has an equivalence with the theoretical models, is defined. The finite element model is used for all the thicknesses available.

4.1 Introduction

Numerical models developed by finite element approaches have been proved to be successful to model the impact phenomenon and other effects on composite structures such as Silva et al. (2005), Gower et al. (2008), García-Castillo et al. (2013), Chu et al. (2016) Briesciani et al. (2016) and Ansari et al. (2017).

Iannucci et al. (2001) proposed a progressive damage model based on an energy dissipation approach for woven glass-fibre composites. This model takes into account strain rate effects and predicts fibre fracture and matrix cracking in the in-plane directions. It was validated with beam impact experiments. Iannucci and Willows (2005) also developed a similar approach for woven carbon composites under high strain loadings considering different damage variables for tension and compression.

Xiao et al. (2007) developed a numerical model to predict progressive damage and delamination under quasi-static punch-shear loading in plain weave-glass composites. An anisotropic damage mechanics approach was considered taking into account different failure modes. Miamí et al. (2007a) and Miamí et al. (2007b) developed a constitutive model to predict the quasi-static onset and growth of intra-laminar failure mechanisms in unidirectional laminates. The model is based in four possible fracture planes using a
4.2 Constitutive modelling

simplification of LaRC04 failure criteria, which is fully explained in Davila et al. (2005). Briesciani et al. (2016) analyzed the impact behaviour of tungsten blunt projectiles on Kevlar 29 plain-woven fabrics using two different approaches, a macro-homogeneous model and a meso-heterogeneous one. Ansari et al. (2017) proposed a continuum damage formulation to take into account both in-plane and out of plane failure modes for woven GFRP plates, modifying Hashin’s failure criteria. This model was used to predict the ballistic performance of GFRP’s against conical-nosed steel projectiles. Apart from continuum damage approaches, there are also discrete damage models to estimate matrix damage evolution and fibre failure in laminated composite plates such as the ones proposed by Moure et al. (2015) and Barbero and Cortes (2010). The only inconvenient of numerical simulations is that they are computational and time consuming.

After going through these models what is clear is the division between in-plane and out of plane failure criteria. The last have a more pronounced importance when the thickness is large enough to appreciate out of plane effects such as out of plane compression and out of plane shear stresses. Furthermore, the composite modelling can be accomplished by means of macro, meso or micro-mechanical approaches depending on the way the constituents are defined as well as the spatial discretization.

Within this context, in the following sections a formulation is developed to simulate the phenomenon of damage in woven glass-fibre composites under out of plane dynamic loads. First, the constitutive response of the material is defined by means of a macro-mechanical continuum damage model. This damaged model is adapted to take into account the different failure mechanisms which are described in the theoretical model. To do so, each failure mechanism is associated with an energy absorption mechanism in the theoretical model trying to give a physical meaning and to capture all the possible failure mechanisms. The model is intended to be used within the framework of explicit finite element method and it is coded as Abaqus/Explicit VUMAT user-written subroutine. Furthermore, in order to capture the inter-ply damage, a cohesive damage model is implemented to account for delamination. In this case, an Abaqus module has been used to implement the behaviour of cohesive elements.

Such aspects as geometry, type of elements, boundary conditions, mesh and element size are explained after the constitutive model. Along with these, a rate dependent friction model is implemented instead of the classic Coulomb model.

4.2 Constitutive modelling

In order to define the laminate behaviour in both the elastic and the damaged zone, a continuum damage mechanics approach is defined. This model considers the failure for the different energy absorption mechanisms as follows.

There are different approaches to define the functions of damage evolution. When a failure criterion is met in a certain direction and damage starts, the decay in the
Finite element model

stress-strain curve may have different shapes. In this case, evolution of damage is chosen to provoke a linear decay in the stress-strain curve once the damage is reached. These softening functions are defined in terms of the strains and the fracture toughnesses of the material at each particular direction.

Once the damage variables are defined, the strain tensor \( \varepsilon \) can be obtained from the differentiation of the complementary free energy density \( G_{\text{free}} \) with respect to the stress tensor \( \sigma \). Deriving a second time this energy with respect to the stress tensor, the flexibility matrix \( [S] \) is obtained, which is the inverse of the stiffness matrix \( [D]^{-1} \).

\[
\varepsilon = S : \sigma \tag{4.1}
\]

which may be written in Voigt’s notation as:

\[
\begin{bmatrix}
\varepsilon_{11} \\
\varepsilon_{22} \\
\varepsilon_{33} \\
\gamma_{12} \\
\gamma_{23} \\
\gamma_{13}
\end{bmatrix} =
\begin{bmatrix}
\frac{1}{E_{11}(1-d_1)} & -\frac{\nu_{12}}{E_{11}} & -\frac{\nu_{13}}{E_{11}} & 0 & 0 & 0 \\
-\frac{\nu_{12}}{E_{22}} & \frac{1}{E_{22}(1-d_2)} & -\frac{\nu_{23}}{E_{22}} & 0 & 0 & 0 \\
-\frac{\nu_{13}}{E_{33}} & -\frac{\nu_{23}}{E_{33}} & \frac{1}{E_{33}(1-d_3)} & 0 & 0 & 0 \\
0 & 0 & 0 & \frac{1}{G_{12}(1-d_4)} & 0 & 0 \\
0 & 0 & 0 & 0 & \frac{1}{G_{23}(1-d_5)} & 0 \\
0 & 0 & 0 & 0 & 0 & \frac{1}{G_{13}(1-d_6)}
\end{bmatrix}
\begin{bmatrix}
\sigma_{11} \\
\sigma_{22} \\
\sigma_{33} \\
\sigma_{12} \\
\sigma_{23} \\
\sigma_{13}
\end{bmatrix}
\tag{4.2}
\]

where \( d_i \) for \( i = 1, 2, 3, 4, 5, 6 \) are damage parameters associated to different failure mechanisms. The deletion of elements takes place when any of the damage parameters reaches the value of 1 because it means the stiffness of the element is zero in a certain direction. These variables are introduced and developed along next subsections. Moreover, \( E_{ij}, \nu_{ij} \) and \( G_{ij} \) are the Young’s moduli, Poisson’s ratios and shear moduli associated to \( i = 1, 2, 3 \) and \( j = 1, 2, 3 \) directions. In addition, any shear strain meets the relationship \( \gamma_{ij} = 2\varepsilon_{ij} \).

4.2.1 In-plane damage model

Different in-plane failure mechanisms are taken into account in this model. On one hand, we have the fibre failures in directions 11 and 22, which can take place in tension or compression. These two failure mechanisms are associated to elastic deformation of fibres and tensile failure of fibres in the theoretical model. On the other hand, we have the matrix cracking failure in direction 12, which is obviously associated to matrix cracking in the theoretical model.

For each damage mode, the material response is linear-elastic until the onset of damage. Then it softens according to a linear law. This scheme can be seen in Figure 4.1, where \( l_{\text{char}} \) is the characteristic length of the element calculated as the cubic root of the volume.
4.2 Constitutive modelling

Fig. 4.1 Scheme of the softening behaviour implemented for a simple case of pure traction in direction 11

The fibre failure criterion for woven laminates derives from the model proposed by Chang and Chang (1987), which has been widely used in the literature by many authors such as Hou et al. (2000), Iannucci and Willows (2005), López-Puente et al. (2008).

\[
\left( \frac{\sigma_{11}}{S_{11i}} \right)^2 + \left( \frac{\sigma_{12}}{S_{12}} \right)^2 + \left( \frac{\sigma_{13}}{S_{13}} \right)^2 = 1 \quad (4.3)
\]

\[
\left( \frac{\sigma_{22}}{S_{22i}} \right)^2 + \left( \frac{\sigma_{12}}{S_{12}} \right)^2 + \left( \frac{\sigma_{13}}{S_{13}} \right)^2 = 1 \quad (4.4)
\]

where the sub-index \(i = t, c\) accounts for tension and compression. The functions of evolution of damage for fibre failure are defined in (4.5) and (4.6).

\[
d_1 = d_1^+ \frac{\langle \sigma_{11} \rangle}{|\sigma_{11}|} + d_1^- \frac{-\langle \sigma_{11} \rangle}{|\sigma_{11}|} \quad (4.5)
\]

\[
d_2 = d_2^+ \frac{\langle \sigma_{22} \rangle}{|\sigma_{22}|} + d_2^- \frac{-\langle \sigma_{22} \rangle}{|\sigma_{22}|} \quad (4.6)
\]

where \(\langle x \rangle\) is the Macaulay operator defined as \(\langle x \rangle = (x + |x|)/2\), \(d_1^+\) and \(d_2^+\) are the damage variables associated with longitudinal fibre failure in tension and \(d_1^-\) and \(d_2^-\) in compression. The Macaulay operator allows to treat the damage caused by tension separately from the damaged caused by compression as in the model proposed by Lopes et al. (2009). By formulating the damage variables in this way, the effect of cracks closure, which may take place during compression is contemplated.

\[
d_{i+} = \frac{\varepsilon_{if} (\varepsilon_{ii} - \varepsilon_{iimax})}{\varepsilon_{ii} (\varepsilon_{if} - \varepsilon_{iimax})} \quad (4.7)
\]
Finite element model

\[ d_{i-} = \frac{\varepsilon_{i fc} (\varepsilon_{iic} - \varepsilon_{iimaxc})}{\varepsilon_{iic} (\varepsilon_{i fc} - \varepsilon_{iimaxc})} \quad (4.8) \]

where the sub-index \( i = 1, 2 \) accounts for directions 11 and 22, \( \varepsilon_{i f}, \varepsilon_{ii}, \varepsilon_{iimax} \) are clearly defined in Figure 4.1 and the sub-index \( c \) means compression. Obviously, the behaviour is not so simple as in Figure 4.1 because in equation (4.3) three different stresses are contributing, but the methodology is the same. A further explanation of the softening behaviour is given in Appendix B.

The procedure to update the stresses is always the following. For instance, for the case of fibre failure in tension in direction 11, once the failure criterion is met, \( \varepsilon_{11max} \) and \( X_{11} \) are obtained. Then, \( \varepsilon_{1f} \) is obtained by means of the fracture toughness \( \Gamma_{11} \) as follows because \( \varepsilon_{1f} \) must be such that the energy dissipated per unit volume remains constant in spite of the characteristic length of the element.

\[ \varepsilon_{1f} = \frac{2\Gamma_{11}}{X_{11}l_{char}} \quad (4.9) \]

In shear directions, the procedure is the same but using the angular strain \( \gamma_{ij} \). For example, the failure angular strain in direction 12 is:

\[ \gamma_{12f} = \frac{2\Gamma_{12}}{S_{12}l_{char}} \quad (4.10) \]

The fracture toughnesses \( \Gamma_{ij} \) associated to \( i = 1, 2, 3 \) and \( j = 1, 2, 3 \) directions have been estimated from a woven E-glass/epoxy composite by Martínez-Hergueta et al. (2019) and are collected in Table 4.1.

<table>
<thead>
<tr>
<th>Units</th>
<th>( \Gamma_{11} )</th>
<th>( \Gamma_{22} )</th>
<th>( \Gamma_{33} )</th>
<th>( \Gamma_{12} )</th>
<th>( \Gamma_{23} )</th>
<th>( \Gamma_{13} )</th>
</tr>
</thead>
<tbody>
<tr>
<td>[N mm(^{-1})]</td>
<td>30</td>
<td>30</td>
<td>100</td>
<td>5</td>
<td>30</td>
<td>30</td>
</tr>
</tbody>
</table>

*Table 4.1 Fracture toughnesses for woven E-glass fibre/polyester laminates estimated from Martínez-Hergueta et al. (2019)*

The expression for the onset of matrix cracking failure criterion or in-plane shear failure mode derives from the principle of progressive failure of Hashin (1980) and damage mechanics of Matzenmiller et al. (1995) as it is used in Xiao et al. (2007). However, it is slightly different because this one is not formulated in strains but in stresses, so there are more contributions taken into account.

\[ \left( \frac{\sigma_{12}}{S_{12s}} \right)^2 = 1 \quad (4.11) \]

where \( S_{12s} \) is different from \( S_{12} \) used in equation (4.3). This approach was used by Xiao et al. (2007), it considers the fibre shear failure stress \( S_{12} \) is different from the matrix shear failure stress \( S_{12s} \). It is further explained in Chapter 2.
4.2 Constitutive modelling

The function of damage evolution is defined in (4.12).

\[
d_4 = \frac{\gamma_{12}(\gamma_{12} - \gamma_{12\text{max}})}{\gamma_{12}(\gamma_{12f} - \gamma_{12\text{max}})}
\]  

(4.12)

It bears noting that there is no difference between tension and compression. This is because this model assumes that the shear damage variables are not influenced by the closure effect as in the model proposed by Lopes et al. (2009).

4.2.2 Out of plane damage model

The out of plane damage model considers two different failure mechanisms that give rise to three damage variables. These are matrix and fibre failure through the thickness direction and crush failure mode. These modes are associated with shear plugging and compression through the thickness direction respectively in the theoretical model for thick laminates proposed in Alonso et al. (2018a). Since this continuum damage model is intended to be used for a wide range of thicknesses, these failure mechanisms are considered from the beginning. Compression and shear plugging take more importance as the ratio \( \Pi_e \) increases as it was demonstrated by Alonso et al. (2018a). The range of thicknesses analyzed experimentally goes from 3 mm to 12 mm. It is expected that for specimens of 3 mm thick (\( \Pi_e = 0.4 \)) the out of plane damage mechanisms are not remarkable. Under this assumption they were neglected in the theoretical model. Nevertheless, these mechanisms are supposed to be the most important for specimens of 12 mm thick (\( \Pi_e = 1.6 \)). In between these two thicknesses, a transition between in-plane and out of plane energy absorption mechanisms should be noticed.

The material response is the same as in the in-plane damage model as Figure 4.1 shows.

The crush failure criterion due to high through thickness compressive pressure is modelled in equation (4.13).

\[
\left( \frac{\sigma_{33}}{X_{33}} \right)^2 = 1
\]  

(4.13)

The damage evolution function is defined in (4.14).

\[
d_3 = \frac{\varepsilon_{33}(\varepsilon_{33} - \varepsilon_{33\text{max}})}{\varepsilon_{33}(\varepsilon_{33f} - \varepsilon_{33\text{max}})}
\]  

(4.14)

In order to define matrix and fibre failure through the thickness direction, a single criterion which activates two damage variables at the same time is defined.

\[
\left( \frac{\sigma_{13}}{S_{13s}} \right)^2 + \left( \frac{\sigma_{23}}{S_{23s}} \right)^2 = 1
\]  

(4.15)
Then two damage evolution functions are defined.

\[ d_5 = \frac{\gamma_{23} f (\gamma_{23} - \gamma_{23\text{max}})}{\gamma_{23} (\gamma_{23} f - \gamma_{23\text{max}})} \]  
(4.16)

\[ d_6 = \frac{\gamma_{13} f (\gamma_{13} - \gamma_{13\text{max}})}{\gamma_{13} (\gamma_{13} f - \gamma_{13\text{max}})} \]  
(4.17)

Usually (4.13) and (4.15) are part of the same failure criterion as in the model proposed by Chang and Chang (1987). However, in this case they are separated to be able to differentiate between the failure caused by compression and by shear stresses through the thickness direction. Furthermore, due to the symmetry of the problem the onset of damage has been chosen the same for the activation of (4.16) and (4.17) although the evolution of damage can be different afterwards.

### 4.2.3 Inter-laminar damage model

The inter-laminar damage model implemented has been already used for similar woven laminates by Martínez-Hergueta et al. (2019). Delamination between layers is simulated using a classical cohesive zone method. The behaviour of the cohesive element used is controlled by a traction-separation law Turon et al. (2007). Damage is assumed to initiate when a quadratic interaction function involving the nominal stress ratios reaches a value of one (Abaqus6.14 (2014)).

\[ \left( \frac{t_n}{N} \right)^2 + \left( \frac{t_s}{S} \right)^2 + \left( \frac{t_t}{S} \right)^2 = 1 \]  
(4.18)

where \( t_n, t_s \) and \( t_t \) are the normal and shear stresses in the first and second shear directions respectively and \( N, S \) are the normal and shear strengths cohesive elements withstand before damage. Since delamination is provoked by the separation of adjacent plies, it makes no sense to start the damage under a pure compressive load. That is why the Macaulay operator is included in (4.18).

Once the damage criterion is met, damage evolution is controlled by the inter-ply fracture toughness \( \Gamma^C \) by means of the Benzeggagh-Kenane (BK) fracture criterion, which is quite useful if the critical fracture energies during deformation purely along the first and the second shear directions are the same. This assumption has been made by other authors for similar materials like in the study carried out by Martínez-Hergueta et al. (2019). BK law takes into account the fracture energy dissipation on the mode mixity.

\[ \Gamma^C = \Gamma_n^C + (\Gamma_s^C - \Gamma_n^C) \left( \frac{\Gamma_s + \Gamma_t}{\Gamma_n + \Gamma_s + \Gamma_t} \right)^{\eta} \]  
(4.19)

where \( \Gamma_n^C \) and \( \Gamma_s^C \) are the critical energy release rates for delamination in modes I and II, which correspond to pure tension and shear mode respectively. \( \Gamma_n, \Gamma_s \) and \( \Gamma_t \)
account for the work dissipated due to the displacements in normal and shear directions provoked by normal and shear stresses. $\eta$ is a characteristic parameter of the BK law which gives a measure of the increase in toughness with the amount of mode mixity (Abaqus6.14 (2014)).

The properties related to the strengths used for the cohesive elements have been estimated from the properties of the resin that serves as adhesive between plies and they are published in García-Castillo (2007). In addition, fracture toughnesses have been estimated from the values found in the literature for similar materials. The energies given by Muñoz et al. (2015) and Martínez-Hergueta et al. (2019) have been taken into account to have an idea of the order of magnitude. Keeping that in mind, a low enough value for these energies has been assumed to avoid numerical problems since these properties do not have an important influence on the ballistic response (Muñoz et al. (2015)). Inter-ply properties are collected in Table 4.2.

<table>
<thead>
<tr>
<th>Property</th>
<th>Nomenclature</th>
<th>Value</th>
</tr>
</thead>
<tbody>
<tr>
<td>Normal Interface Strength [MPa]</td>
<td>N</td>
<td>55</td>
</tr>
<tr>
<td>Shear Interface Strength [MPa]</td>
<td>S</td>
<td>55</td>
</tr>
<tr>
<td>Mode I fracture toughness [N mm(^{-1})]</td>
<td>$\Gamma^C_{n}$</td>
<td>0.01</td>
</tr>
<tr>
<td>Mode II fracture toughness [N mm(^{-1})]</td>
<td>$\Gamma^C_{s}$</td>
<td>0.01</td>
</tr>
<tr>
<td>Benzeggagh-Kenane parameter</td>
<td>$\eta$</td>
<td>1.75</td>
</tr>
</tbody>
</table>

Table 4.2 Cohesive element properties estimated from García-Castillo (2007), Muñoz et al. (2015) and Martínez-Hergueta et al. (2019)

4.3 Definition of the finite element model

The finite element simulations were carried out with four models using Lagrangian 3D elements. The dimensions of the plates simulated were the same as the ones tested experimentally (150 mm x 150 mm) with values of thicknesses of 3 mm, 6 mm, 9 mm and 12 mm. These tests were carried out by Buitrago-Pérez et al. (2010) and Alonso et al. (2018a). The type of element used was trilinear elements with reduced integration (C3D8R in Abaqus notation) to simulate the plies. Cohesive elements were also used between plies, 1 element in the thickness direction for each union, with a thickness of 0.001 mm (COH3DS8 in Abaqus notation). The target sweep mesh developed is shown in Figure 4.2, where 10, 20, 45 and 60 elements were used along the thickness direction for plates of 3 mm 6 mm, 9 mm and 12 mm thick respectively. This longitudinal density of elements meets the criterion established by Abaqus6.14 (2014) when modelling any structure carrying bending loads. The mesh is divided in two regions, a fine mesh where a lot of elements are spread out in a square of 40 mm x 40 mm, where the projectile impacts and the rest. To reduce the computational time, a constant gradient can be
appreciated increasing the size of elements in Figure 4.2. The number of elements used in the two regions for the different specimens are collected in Table 4.3.

<table>
<thead>
<tr>
<th>Thickness [mm]</th>
<th>Refined region</th>
<th>Total</th>
</tr>
</thead>
<tbody>
<tr>
<td></td>
<td>C3D8R</td>
<td>COH3DR</td>
</tr>
<tr>
<td>3</td>
<td>25600</td>
<td>25600</td>
</tr>
<tr>
<td>6</td>
<td>128000</td>
<td>57600</td>
</tr>
<tr>
<td>9</td>
<td>796005</td>
<td>247646</td>
</tr>
<tr>
<td>12</td>
<td>1061340</td>
<td>336091</td>
</tr>
</tbody>
</table>

*Table 4.3 Type and number of elements used in the definition of the finite element model for the four thicknesses*

The mesh is radially symmetrical to avoid appearance of spurious generation of cracks (García-González et al. (2015)). A convergence study varying the areal mesh density was carried out. More elements were needed to reach the convergence for the thickest laminates (9 mm and 12 mm).

*Fig. 4.2 Mesh configuration used in the simulations*

The boundary condition applied to the model is to clamp the nodes on the laminate edges. Since there were not permanent deformations on the projectiles after any of the impacts, the impactor was simulated with an analytical surface (rigid body), which is undeformable.
4.3 Definition of the finite element model

Regarding to the contact interaction between impactor and target, an exponential decay friction model was implemented instead the classical Coulomb model. In the Coulomb model the static friction coefficient corresponds to the value given at zero slip rate, and the kinetic friction coefficient corresponds to the value given at the highest slip rate (Abaqus6.14 (2014)). This model assumes and exponential decay between these two values. It is indicated to use this model when the range of velocities is very high and to use a constant friction coefficient for the whole range is not quite realistic. The friction coefficient at any slip rate is given by equation (4.20).

\[
\mu = \mu_k + (\mu_s - \mu_k)e^{-d_c \dot{\gamma}_{eq}} \quad (4.20)
\]

where \(\mu_k\) is the kinetic friction coefficient, \(\mu_s\) is the static friction coefficient, \(d_c\) is a user-defined decay coefficient, and \(\dot{\gamma}_{eq}\) is the slip rate. Giving a static value and two points of the curve the model is defined. This ensures that the horizontal asymptote for the highest slip rate gives a friction coefficient really low \(\mu_k \approx 0.01\) and the friction coefficient for the lowest impact velocity of the experiments is \(\mu = 0.3\).
The aim of this chapter is to validate the theoretical models as well as the finite element model. Before obtaining results from the models for the Chapter 6, they have to be validated so that the results obtained can be reliable. To do so, both models are compared between them and also with the experimental results. The ballistic limit and the residual velocities are the two checkings that are used to validate the models.

5.1 Validation

Validation is an essential step when theoretical and finite element models are formulated and developed. In this case, it has to be proved they predict the behaviour of the laminates studied under high-velocity impact in a reasonable accurate way. In order to accomplish this, it has to be reminded that the fundamental parameter in perforation tests is the ballistic limit since it is the velocity which guarantees the integrity of the structure. Therefore, the data available provided by the experimental tests, in terms of impact and residual velocities, is compared to the ones provided by the models.

Figure 5.1a shows the comparison between the experimental data and the results provided by the theoretical model for thin laminates for the four thicknesses tested. As can be seen in Figure 5.1a, the theoretical model for thin laminates fits really well the experimental data for thicknesses of 3 mm and 6 mm. It captures both the ballistic limit and the residual velocities. However, it does not provide acceptable results for higher thicknesses. These differences can be attributed to a change in the behaviour due to a change in the energy absorption mechanisms from a certain thickness. As a brief reminder, the energy absorption mechanisms considered for the theoretical model for thin laminates are: elastic deformation of fibres, tensile failure of fibres, acceleration of the laminate, matrix cracking and delamination, and for thick laminates: compression, transference of kinetic energy, tensile failure of fibres, shear plugging, matrix cracking,
5.1 Validation

delamination, transference of linear momentum and friction. Further analysis of this effect will be done in the results section.

Figure 5.1b is analogous to Figure 5.1a but the theoretical results are provided by the theoretical model for thick laminates. Unlike in Figure 5.1a, a good agreement between the experimental data and the theoretical results for thicknesses of 9 mm and 12 mm can be seen in Figure 5.1b. Nevertheless, this model does not provide good results for lower thicknesses.

Figure 5.2 shows the comparison between the experimental data and the results provided by the finite element model. The agreement is good enough in terms of the ballistic limit. The results for the residual velocities are good up to a certain point which correspond approximately to a velocity of 550 m s$^{-1}$. From this point, the results for residual velocities diverge, being the finite element results lower than the experimental ones. It means the finite element model predicts a greater absorption of energy than the experiments. Since this behaviour always takes place from a very high velocity, it could be explained as an effect of the temperatures reached during the perforation process. Many authors such as Hufenbach et al. (2011), Bai et al. (2008) and Ou et al. (2016) have explained the influence of temperature on the mechanical properties of composite materials. When such high velocities are reached, high temperatures arise within the laminate and they can provoke the relaxation phenomenon. This effect is provoked mainly by the matrix in which the fibres are embedded. In some cases such as polymer matrices, the sensitivity of mechanical properties with temperature is really high. Through the so-called relaxation phenomenon, these properties normally decrease with temperature, reducing the stiffness of the material and thus its capacity to absorb
Validation

energy. Furthermore, matrix heat conductivity is low, so for short time (high-velocity impact) energy dissipation is nearly an adiabatic process with high temperature increase.

![Fig. 5.2 Impact velocity vs residual velocity experimental results along with the predictions of the finite element model](image)

Regarding to the ballistic limit, the predictions made by all the models are summoned in Table 5.1. In view of these results, it is reasonable that the thin laminates model should be used to predict the results for thicknesses of 3 mm and 6 mm. On the other hand, the thick laminates model should be used to predict the results for thicknesses of 9 mm and 12 mm. Figure 5.3 shows the experimental results as well as the final predictions of the theoretical and finite element models.

<table>
<thead>
<tr>
<th>Thickness [mm]</th>
<th>$V_{BL_{exp}}$ [m s$^{-1}$]</th>
<th>$V_{BL_{thin}}$ [m s$^{-1}$]</th>
<th>$V_{BL_{thick}}$ [m s$^{-1}$]</th>
<th>$V_{BL_{FEM}}$ [m s$^{-1}$]</th>
</tr>
</thead>
<tbody>
<tr>
<td>3</td>
<td>212</td>
<td>218.5</td>
<td>175.5</td>
<td>203</td>
</tr>
<tr>
<td>6</td>
<td>332</td>
<td>362.5</td>
<td>294.5</td>
<td>309</td>
</tr>
<tr>
<td>9</td>
<td>428</td>
<td>489.5</td>
<td>421.5</td>
<td>430</td>
</tr>
<tr>
<td>12</td>
<td>550</td>
<td>606.5</td>
<td>543.5</td>
<td>553</td>
</tr>
</tbody>
</table>

Table 5.1 Predictions of the ballistic limits by the models developed along with the experimental ones
5.1 Validation

Fig. 5.3 Impact velocity vs residual velocity experimental results along with the final predictions of the theoretical and finite element models.
The aim of this chapter is to obtain results of interest by means of the theoretical and finite element models developed. First, different Π groups are studied to understand how the theoretical models work and to extract important conclusions such as the threshold from which a laminate can be considered either thin or thick. After that, the finite element model has been used to validate some of the new and most important hypothesis assumed in the theoretical models. In addition, the comparison between the energy absorption mechanisms of the theoretical models and those of the finite element model leads to an interesting discussion.

6.1 Parametric study

In this section different Π groups are analyzed to prove the physical consistency of the theoretical models when varying these groups. In this parametric study the thickness ratio Π∗e will be analyzed for all the thicknesses available. It is necessary to do the study for all the thicknesses since the main objective is to determine when a laminate can be considered as thin or thick. Furthermore, two of the most important Π groups are analyzed for thin and thick laminates. Π∗E and εr for the model for thin laminates and Π∗Ec and Π∗Sp for the one for thick laminates.

6.1.1 Analysis of the thickness Π group

The first Π group analyzed is the thickness ratio Π∗e. This group is crucial to understand the threshold thickness from which the behaviour of a laminate changes. In order to find this threshold, the following procedure is followed. First, the ballistic limit is taken as the point to compare the two theoretical models because perforation is the most critical condition in these structures. Second, the values of the ballistic limit obtained experimentally are assumed to be the real ones. Finally, both models are used to calculate the ballistic limit of the four available thicknesses and the error with respect to the
experimental value is obtained. These results are presented in Table 6.1. The lines which connect the four points representing the errors intersect at some point as can be seen in Figure 6.1.

![Graph showing theoretical model errors](image)

**Fig. 6.1 Representation of the theoretical models errors with respect to the experimental data**

<table>
<thead>
<tr>
<th>Thickness [mm]</th>
<th>Thin theoretical model error [%]</th>
<th>Thick theoretical model error [%]</th>
</tr>
</thead>
<tbody>
<tr>
<td>3</td>
<td>3.1</td>
<td>17.2</td>
</tr>
<tr>
<td>6</td>
<td>9.2</td>
<td>11.3</td>
</tr>
<tr>
<td>9</td>
<td>14.4</td>
<td>1.5</td>
</tr>
<tr>
<td>12</td>
<td>10.3</td>
<td>1.2</td>
</tr>
</tbody>
</table>

*Table 6.1 Theoretical models errors at the ballistic limit*

The point of intersection between the two curves corresponds to a geometry ratio of $\Pi_e = 0.85$, which is associated to a thickness equal to 6.4 mm. This value is slightly lower than 1. It means that from this point down the thin theoretical model predicts better results and from this point up the thick theoretical model makes better predictions. This result is similar to the one reported by Rosenberg and Dekel (2012) for metal plates subjected to high-velocity impact. These authors stated that the relevant parameter in impact modelling is the geometry ratio, at least for metals. Furthermore, the classification between thin and thick laminates analytical models in the literature (Naik and Shrirao (2004), Naik and Doshi (2005)) gives an idea of the importance of this ratio. In view of
Results

Figure 6.1, it may be concluded that woven E-glass/polyester laminates can be classified as:

- Thin plates: $\Pi_e << 1$
- Thick plates: $\Pi_e >> 1$

In addition, different simulations varying thickness and diameter have been carried out. Table 6.2, Table 6.3 and Table 6.4 show the ballistic limit predicted by both theoretical models for geometry ratios of 1, 0.85 and 1.15 respectively.

<table>
<thead>
<tr>
<th>Projectile diameter [mm]</th>
<th>$V_{BL\text{thin}}$ [m s$^{-1}$]</th>
<th>$V_{BL\text{thick}}$ [m s$^{-1}$]</th>
</tr>
</thead>
<tbody>
<tr>
<td>5</td>
<td>432.5</td>
<td>381.5</td>
</tr>
<tr>
<td>6.5</td>
<td>426.5</td>
<td>355.5</td>
</tr>
<tr>
<td>7.5</td>
<td>427.5</td>
<td>349.5</td>
</tr>
<tr>
<td>8.5</td>
<td>430.5</td>
<td>345.5</td>
</tr>
<tr>
<td>10</td>
<td>437.5</td>
<td>343.5</td>
</tr>
</tbody>
</table>

*Table 6.2 Predictions of the ballistic limit with both theoretical models when varying the projectile diameter and the thickness, maintaining $\Pi_e = 1$*

<table>
<thead>
<tr>
<th>Projectile diameter [mm]</th>
<th>$V_{BL\text{thin}}$ [m s$^{-1}$]</th>
<th>$V_{BL\text{thick}}$ [m s$^{-1}$]</th>
</tr>
</thead>
<tbody>
<tr>
<td>5</td>
<td>379.5</td>
<td>319.5</td>
</tr>
<tr>
<td>6.5</td>
<td>379.5</td>
<td>307.5</td>
</tr>
<tr>
<td>7.5</td>
<td>382.5</td>
<td>304.5</td>
</tr>
<tr>
<td>8.5</td>
<td>387.5</td>
<td>303.5</td>
</tr>
<tr>
<td>10</td>
<td>395.5</td>
<td>303.5</td>
</tr>
</tbody>
</table>

*Table 6.3 Predictions of the ballistic limit with both theoretical models when varying the projectile diameter and the thickness, maintaining $\Pi_e = 0.85$*

<table>
<thead>
<tr>
<th>Projectile diameter [mm]</th>
<th>$V_{BL\text{thin}}$ [m s$^{-1}$]</th>
<th>$V_{BL\text{thick}}$ [m s$^{-1}$]</th>
</tr>
</thead>
<tbody>
<tr>
<td>5</td>
<td>485.5</td>
<td>455.5</td>
</tr>
<tr>
<td>6.5</td>
<td>474.5</td>
<td>409.5</td>
</tr>
<tr>
<td>7.5</td>
<td>472.5</td>
<td>396.5</td>
</tr>
<tr>
<td>8.5</td>
<td>474.5</td>
<td>388.5</td>
</tr>
<tr>
<td>10</td>
<td>478.5</td>
<td>384.5</td>
</tr>
</tbody>
</table>

*Table 6.4 Predictions of the ballistic limit with both theoretical models when varying the projectile diameter and the thickness, maintaining $\Pi_e = 1.15$*
The results of Table 6.2, Table 6.3 and Table 6.4 are summarized in Figure 6.2.

![Figure 6.2 Graphic representation of Tables 6.2, 6.3 and 6.4](image)

A conclusion that can be inferred from Figure 6.2 is that the theoretical model for thick laminates always predicts a lower ballistic limit than the one for thin laminates. This is due to the fact that in the thin laminates model energy absorption mechanisms such as compression are disregarded and then the total energy absorbed is lower giving a higher ballistic limit. Furthermore, the higher $\Pi_e$, the higher the predicted ballistic limit for both models. This fact is obvious because the effect of increasing $\Pi_e$ is the same as increasing the thickness maintaining the projectile diameter.

Although the geometry ratio is the same within each of these tables, the projectile diameter and the thickness change. Since one of the parameters used to make the problem non-dimensional is this diameter, different $\Pi$ groups are affected when varying it. Therefore, there is no reason to think the ballistic limit will be the same if $\Pi_e$ is maintained, because other $\Pi$ groups change. However, it can be observed that the ballistic limit is approximately the same within the same $\Pi_e$ group and the same theoretical model. And this is fulfilled for different values of $\Pi_e$ as Figure 6.2 shows. This observation suggests the ballistic limit depends exclusively on the relation of masses between the impactor and the target, which is analogous to $\Pi_e$. 
6.1.2 Analysis of the most important Π groups for thin laminates

Analysis of the in-plane Young’s modulus Π group

The Π group involving the in-plane Young’s modulus, $\Pi_E$, is an important parameter which affect the performance of a laminate under high-velocity impact. This parameter is dominant in the theoretical model for thin laminates because it directly affects the energy absorption by elastic deformation and tensile failure of fibres (3.26), (3.32), which are the most important mechanism when a laminate is thin (Alonso et al. (2018a)). It is clear that the in-plane Young’s modulus can be a measure of the stiffness of a plate. For instance, woven carbon-fibre composites have normally higher stiffnesses than E-glass composites. For this reason, by means of the theoretical model, a parametric study of $\Pi_E$ has been done taking into account a wide range of stiffnesses. The results are presented in Figure 6.3.

![Figure 6.3 Impact velocity vs residual velocity for different Young’s moduli for (a) 3 mm (b) 6 mm thick laminates](image)

In Figure 6.3 it can be seen that the trend of the curves is the same for the two thicknesses. First, the higher $\Pi_E$, the higher the ballistic limit for both cases. This result can be justified mathematically. If $\Pi_E$ increases, the energy absorbed by elastic deformation and tensile failure of fibres also increases because they are proportional. Furthermore, $\Pi_E$ is also involved in the velocity of the waves considered in the theoretical model for thin laminates. Specifically, the velocity of transverse waves (3.5) increases if $\Pi_E$ increases and it accelerates a larger mass of the laminate. In addition, for very high velocities all the curves collapse into one. This result seems obvious because in the limit all the curves must be the same. But also, elastic deformation of fibres starts acting when the transverse wave is higher than the radius of the indented projectile.
6.1 Parametric study

For very high velocities, this radius grows faster than the waves for any $\Pi_E$ and the contact time is lower. Therefore, elastic deformation of fibres reduces its importance. In Table 6.5, the absorbed energies are compared for two velocities to appreciate this effect. These energies are calculated for the initial Young’s modulus $E = 15.2$ GPa, but the effect is the same for other values. For very high velocities, Table 6.5 shows that elastic deformation of fibres diminishes its importance and the phenomenon is controlled by the acceleration of the laminate.

<table>
<thead>
<tr>
<th>Thickness [mm]</th>
<th>Impact velocity [m s$^{-1}$]</th>
<th>$E_L(t)$ [J]</th>
<th>$E_{ED}(t)$ [J]</th>
<th>$E_{TF}(t)$ [J]</th>
<th>$E_{DL}(t)$ [J]</th>
<th>$E_{MC}(t)$ [J]</th>
</tr>
</thead>
<tbody>
<tr>
<td>3</td>
<td>250</td>
<td>4.51</td>
<td>16.26</td>
<td>10.89</td>
<td>1.86</td>
<td>1.86</td>
</tr>
<tr>
<td>3</td>
<td>500</td>
<td>19.81</td>
<td>2.24</td>
<td>7.98</td>
<td>0.32</td>
<td>0.32</td>
</tr>
<tr>
<td>6</td>
<td>400</td>
<td>16.86</td>
<td>54.19</td>
<td>23.12</td>
<td>3.27</td>
<td>6.53</td>
</tr>
<tr>
<td>6</td>
<td>600</td>
<td>100.48</td>
<td>21.22</td>
<td>20.51</td>
<td>1.21</td>
<td>2.41</td>
</tr>
</tbody>
</table>

Table 6.5 Total energies absorbed by each energy absorption mechanism for different thicknesses and velocities for a Young’s modulus of $E = 15.2$ GPa

Analysis of the in-plane failure strain $\Pi$ group

The in-plane failure strain, $\varepsilon_r$, is another crucial aspect on a plate performance under impact. The in-plane deformation that the laminate can stand before breaking is also involved in the energy absorbed by elastic deformation and tensile failure of fibres as well as in the velocity of waves. For this reason, a parametric study of $\varepsilon_r$ has been carried out. Figure 6.4 shows the results obtained for a wide range of in-plane failure strains.

![Impact velocity vs residual velocity for different in-plane failure strains for (a) 3 mm (b) 6 mm thick laminates](image)

Fig. 6.4 Impact velocity vs residual velocity for different in-plane failure strains for (a) 3 mm (b) 6 mm thick laminates

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Results

The curves obtained are similar as for the parametric study of $\Pi_E$. Figure 6.4 shows that the higher the in-plane failure strain, the higher the ballistic limit for the two thicknesses analyzed. Figure 6.4b also shows that the curves intersect at a certain point around 515 m s$^{-1}$. To explain this, it is important to remind that the velocity of the transverse wave decreases if $\varepsilon_r$ increases, thus affecting to all the energy absorption mechanisms. If $\varepsilon_r$ increases, elastic deformation of fibres increases and the energy absorbed by the acceleration of the laminate decreases. These two mechanisms, which are the most important for high velocities, compensate each other. For high velocities, mass effects become dominant.

<table>
<thead>
<tr>
<th>In-plane failure strain</th>
<th>Impact velocity [m s$^{-1}$]</th>
<th>$E_L(t)$ [J]</th>
<th>$E_{ED}(t)$ [J]</th>
<th>$E_{TF}(t)$ [J]</th>
<th>$E_{DL}(t)$ [J]</th>
<th>$E_{MC}(t)$ [J]</th>
</tr>
</thead>
<tbody>
<tr>
<td>0.0362675</td>
<td>500</td>
<td>103.60</td>
<td>11.97</td>
<td>5.50</td>
<td>2.86</td>
<td>5.72</td>
</tr>
<tr>
<td>0.14507</td>
<td>500</td>
<td>7.51</td>
<td>47.45</td>
<td>85.01</td>
<td>0.67</td>
<td>1.34</td>
</tr>
<tr>
<td>0.0362675</td>
<td>700</td>
<td>168.70</td>
<td>5.31</td>
<td>4.88</td>
<td>1.21</td>
<td>2.42</td>
</tr>
<tr>
<td>0.14507</td>
<td>700</td>
<td>36.01</td>
<td>18.47</td>
<td>74.40</td>
<td>0.29</td>
<td>0.58</td>
</tr>
</tbody>
</table>

Table 6.6 Total energies absorbed by each energy absorption mechanism for 6 mm thick apecimens with different in-plane failure strains and impact velocities

6.1.3 Analysis of the most important $\Pi$ groups for thick laminates

Analysis of the out of plane compressive Young’s modulus $\Pi$ group

Thick enough laminates have shown the most important energy absorption mechanisms are compression and shear plugging (Alonso et al. (2018a)). Therefore, it seems reasonable to carry out a parametric study of the parameters which control those mechanisms. The $\Pi$ group involving the compressive Young’s modulus through the thickness direction, $\Pi_{E_c}$, is a dominant parameter in the theoretical model for thick laminates since it directly affects to the energy absorbed by compression (3.52), (3.58) and to the compressive wave in the through thickness direction (3.46). The results given by the theoretical model when varying the through thickness Young’s modulus are presented in Figure 6.5.

From the curves obtained in Figure 6.5 it can be inferred that the higher the compressive through thickness Young’s modulus, the higher the ballistic limit. At a certain velocity, all the curves tend into a common curve as expected. Obviously this convergence is obtained for 9 mm thick or thicker specimens as can be seen if Figures 6.5a and 6.5b are compared. In order to see the influence of $\Pi_{E_c}$ in the energy absorption mechanisms, those are compared for a 9 mm thick specimen for a velocity of 850 m s$^{-1}$. These results are collected in Table 6.7.
6.1 Parametric study

Fig. 6.5 Impact velocity vs residual velocity for different compressive through thickness Young’s moduli for laminates of (a) 9 mm (b) 12 mm

<table>
<thead>
<tr>
<th>$E_c$ [GPa]</th>
<th>$E_{C1}(t)$ [J]</th>
<th>$E_{C2}(t)$ [J]</th>
<th>$E_{TF}(t)$ [J]</th>
<th>$E_{CL}(t)$ [J]</th>
<th>$E_{SP}(t)$ [J]</th>
<th>$E_{DL}(t)$ [J]</th>
<th>$E_{MC}(t)$ [J]</th>
</tr>
</thead>
<tbody>
<tr>
<td>1.35</td>
<td>2.10</td>
<td>0</td>
<td>2.12</td>
<td>14.00</td>
<td>51.35</td>
<td>0.02</td>
<td>0.05</td>
</tr>
<tr>
<td>3.375</td>
<td>8.34</td>
<td>0</td>
<td>2.12</td>
<td>21.68</td>
<td>51.35</td>
<td>0.02</td>
<td>0.05</td>
</tr>
<tr>
<td>6.75</td>
<td>23.78</td>
<td>0</td>
<td>2.12</td>
<td>29.56</td>
<td>51.35</td>
<td>0.02</td>
<td>0.05</td>
</tr>
<tr>
<td>13.5</td>
<td>68.56</td>
<td>0</td>
<td>2.14</td>
<td>38.20</td>
<td>51.35</td>
<td>0.02</td>
<td>0.05</td>
</tr>
<tr>
<td>33.75</td>
<td>299.87</td>
<td>0</td>
<td>2.22</td>
<td>34.03</td>
<td>51.35</td>
<td>0.02</td>
<td>0.07</td>
</tr>
</tbody>
</table>

Table 6.7 Total energies absorbed by each energy absorption mechanism for 9 mm thick specimens with different compressive through thickness Young’s moduli for an impact velocity of 850 m s$^{-1}$

In Table 6.7 it can be seen that the energies affected by a change in $E_c$ are compression in region 1, which is directly proportional to $E_c$ (3.52) and the transference of kinetic energy to the laminate (3.68). This last increases as a consequence of the through thickness wave velocity (3.46). This wave velocity increases with $E_c$ and thus the energy transferred to the laminate is higher. Since these two energy absorption mechanisms increase while the others remain constant, it makes sense that the ballistic limit increases with $E_c$.

Another remarkable observation is that the energy absorbed by compression in region 2 is zero for all the cases. This happens because the impact velocity is so high that the second stage is reached before the transverse wave reaches the indented projectile diameter.
Results

Analysis of the out of plane failure shear stress \( \Pi \) group

Out of plane effects gain importance in the impact phenomenon as the thickness grows. Shear plugging is controlled by out of plane shear stresses, which reach quite high values for high thicknesses. In fact, this energy absorption mechanisms is one of the most important when a laminate is thick (Alonso et al. (2018a)). Thus, to know how the ballistic behaviour of a laminate changes with \( \Pi_{S_{SP}} \) is a question of interest. This parameter directly affects the energy absorbed by shear plugging because they are proportional (3.71). The results provided by the theoretical model for thick laminates varying the out of plane failure shear stress are shown in Figure 6.6.

![Fig. 6.6 Impact velocity vs residual velocity for different out of plane failure shear stresses for (a) 9 mm (b) 12 mm thick laminates](image)

Figures 6.6a and 6.6b reveal that the higher the out of plane failure shear stress, the higher the ballistic limit. The same tendency as in the other parametric studies is observed, all the curves collapse into one for very high velocities.

<table>
<thead>
<tr>
<th>( S_{SP} ) [MPa]</th>
<th>( E_{C1}(t) ) [J]</th>
<th>( E_{C2}(t) ) [J]</th>
<th>( E_{TF}(t) ) [J]</th>
<th>( E_{CL}(t) ) [J]</th>
<th>( E_{SP}(t) ) [J]</th>
<th>( E_{DL}(t) ) [J]</th>
<th>( E_{MC}(t) ) [J]</th>
</tr>
</thead>
<tbody>
<tr>
<td>102.15</td>
<td>51.44</td>
<td>0</td>
<td>4.46</td>
<td>28.18</td>
<td>52.49</td>
<td>0.06</td>
<td>0.24</td>
</tr>
<tr>
<td>136.2</td>
<td>52.16</td>
<td>0</td>
<td>4.48</td>
<td>26.50</td>
<td>69.99</td>
<td>0.06</td>
<td>0.24</td>
</tr>
<tr>
<td>204.3</td>
<td>53.76</td>
<td>0</td>
<td>4.53</td>
<td>23.02</td>
<td>104.98</td>
<td>0.06</td>
<td>0.26</td>
</tr>
<tr>
<td>306.45</td>
<td>56.75</td>
<td>0</td>
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<td>17.46</td>
<td>157.47</td>
<td>0.07</td>
<td>0.29</td>
</tr>
<tr>
<td>408.6</td>
<td>60.97</td>
<td>0</td>
<td>4.68</td>
<td>11.32</td>
<td>209.97</td>
<td>0.08</td>
<td>0.33</td>
</tr>
</tbody>
</table>

Table 6.8 Total energies absorbed by each energy absorption mechanism for 12 mm thick specimens with different out of plane failure shear stresses for an impact velocity of 650 m s\(^{-1}\)
6.2 Validation of the hypotheses assumed in the theoretical models

By means of the variation of $S_{SP}$ for an impact velocity of 650 m s$^{-1}$, the energy absorption mechanisms are analyzed. These results are shown in Table 6.8, which shows that $S_{SP}$ affects all the energy absorption mechanisms. However, the most affected mechanism is shear plugging. In view of equation (3.71) this result is logical, so an increase in $S_{SP}$ provokes an increase in the energy absorbed by this mechanism and consequently the ballistic limit.

It may be noticed that the energy absorbed by compression in region 2 is zero for all the cases. This is because the impact velocity is very high for both cases and the indented projectile diameter grows faster than the transverse wave as in the case of $\Pi_{E_c}$.

6.2 Validation of the hypotheses assumed in the theoretical models

6.2.1 Hypotheses of the theoretical model for thin laminates

Analysis of the behaviour of the phenomenological function $\bar{k}$

One of the most important assumptions in the theoretical model for thin laminates is the function $\bar{k}$, which represents the velocity at which the center of mass of the accelerated laminate is moving (3.13).

To give an insight look and a physical interpretation for this function different results varying $\Pi$ groups have been obtained. In Figure 6.7, $\bar{k}$ is represented when varying $\Pi_E$ for a 3 mm thick plate for velocities of 250 m s$^{-1}$ and 450 m s$^{-1}$. In Figure 6.8, $\bar{k}$ is represented when varying $\Pi_E$ for a 6 mm thick plate for velocities of 350 m s$^{-1}$ and 500 m s$^{-1}$. These velocities have been chosen to analyze the ballistic limit and a higher velocity. $\bar{k}$ is represented versus the non-dimensional time, which has been non-dimensionalized with the maximum time for each case to be able to compare better the shape of $\bar{k}$.

Figures 6.7a and 6.8a show that $\bar{k}$ has the same parabolic shape for velocities near the ballistic limit. Furthermore, $\bar{k}$ decreases quicker when the Young’s modulus is higher. It means that for higher Young’s moduli, the projectile penetrates quicker into the laminate because the behaviour as a membrane is less evident, the laminate moves less. It makes sense because the stiffness is higher. In addition, there are cases where $\bar{k}$ reaches the value of zero for high Young’s moduli. It means the projectile does not penetrate the laminate, which makes sense if the velocity is fixed and the Young’s modulus increases.
Results

Fig. 6.7 $\bar{k}$ vs non-dimensional time for different $\Pi_E$ for 3 mm thick laminates for impact velocities of (a) 250 m s$^{-1}$ (b) 450 m s$^{-1}$

Figures 6.7b and 6.8b show that $\bar{k}$ still present a parabolic shape for velocities above the ballistic limit. The behaviour is the same than for velocities around the ballistic limit but since the projectile comes out the laminate sooner, $\bar{k}$ does not have time to decrease. Therefore, the higher the stiffness, the lower the $\bar{k}$. But this behaviour can be intuited with the slopes of $\bar{k}$ at time 1 that show which ones would decrease quicker.

Fig. 6.8 $\bar{k}$ vs non-dimensional time for different $\Pi_E$ for 6 mm thick laminates for impact velocities of (a) 350 m s$^{-1}$ (b) 500 m s$^{-1}$
6.2 Validation of the hypotheses assumed in the theoretical models

In Figure 6.9, $\bar{k}$ is represented when varying $\varepsilon_r$ for a 3 mm thick plate for velocities of 250 m s$^{-1}$ and 450 m s$^{-1}$. In Figure 6.10, $\bar{k}$ is represented when varying $\varepsilon_r$ for a 6 mm thick plate for velocities of 350 m s$^{-1}$ and 500 m s$^{-1}$.

As can be seen in Figure 6.4, the plate absorbs more energy if the failure strain is higher, for the velocities analyzed. If the failure strain is higher, the fibres can absorb more energy before they break and since this mechanism is the dominant for these
Results

velocities, \( \bar{k} \) reaches the value of zero and the projectile stops sooner. Furthermore, the volume of laminate accelerated is lower because the transverse wave is slower. For this reason, the values of \( \bar{k} \) are lower and although the energy absorbed is higher, the relative displacement is higher. In fact, there are some cases in 6.9a and 6.10a where \( \bar{k} \) reaches zero and the penetration does not take place.

For all the cases analyzed a general explanation of the parabolic shape can be provided. At the beginning of the impact the projectile transfers energy to the laminate and the laminate starts moving trying to 'follow' the projectile, so \( \bar{k} \) increases. But at some point of deformation, the fibres and the matrix start to fail and the laminate can no longer 'follow' the projectile, so \( \bar{k} \) decreases and the projectile penetrates more easily.

Analysis of the behaviour of the variable representing the relative displacement, \( \bar{\delta} \)

It may seem that the discussion of \( \bar{k} \) is not enough since it has been done only with the theoretical model although the results seem to have a physical meaning. For this reason a theoretical and a finite element validation of \( \bar{\delta} \), which is directly related to \( \bar{k} \) (3.20) is discussed.

The theoretical model provides (3.20) directly but we had to set up a general procedure to measure it in the finite element model. Figure 6.11 helps to understand the procedure followed. First, the instant of time where the velocity of the projectile does not change any more is checked because this is the point from which there is not more interaction between the two bodies. At that moment, four nodes (one corresponding at each branch as can be seen in Figure 6.11) which coincide approximately with the most advanced coordinate of the projectile \( x(t) \) are chosen. An average of the displacements of these four points in the through thickness direction is done and the resultant displacement is assumed to be the displacement of the laminate \( x_{\text{average}}(t) \). The displacement of the projectile as a rigid body is also known.

![Fig. 6.11 Instant of time in one of the finite element simulations from which the projectile does not interact any more with the laminate](image)

Therefore, the finite element relative displacement is calculated by (6.1).

\[
\delta_{\text{num}}(t) = x(t) - x_{\text{average}}(t) - x_{\text{offset}}
\]  

(6.1)
where \( x_{offset} \) is the distance between the projectile and the laminate at the beginning of the simulation because it is convenient that they are not in contact from the first moment.

Different measures have been taken for 3 mm and 6 mm thick laminates. To homogenize the results, the variables presented are in their non-dimensional form, using the maximum time of each simulation for time and the maximum theoretical and finite element relative displacements respectively for \( \delta \) and \( \delta_{num} \).

---

**Fig. 6.12** Relative displacement vs non-dimensional time for 3 mm thick laminates for impact velocities of (a) 263 m s\(^{-1}\) (b) 320 m s\(^{-1}\)

**Fig. 6.13** Relative displacement vs non-dimensional time for 6 mm thick laminates for impact velocities of (a) 393 m s\(^{-1}\) (b) 487 m s\(^{-1}\)
Results

Results are presented in Figures 6.12 and 6.13 for different velocities and thicknesses. Figure 6.12 shows a really good agreement between the relative displacement obtained theoretically and numerically for different velocities. However, the agreement is not so good for laminates of 6 mm (Figure 6.13) because the finite element model provides higher values for the relative displacement although the tendency is still the same.

Nevertheless, such good agreements reinforce the hypotheses assumed in the theoretical model, for $\bar{k}$ and $\bar{\delta}$. It also makes sense to find the worse agreement for thicker laminates because the laminate begins to behave as a thick one at some point (Alonso et al. (2018a)) and the hypothesis of the relative displacement is not valid for these laminates. At this moment, the behaviour as a membrane is more difficult to be noticed.

6.2.2 Hypotheses of the theoretical model for thick laminates

Analysis of the profile of velocities across the thickness direction

One of the hypotheses assumed in the theoretical model for thick laminates is the existence of a profile of velocities across the thickness direction. This profile was supposed to be linear between the contact point and the distance travelled by the compressive through thickness wave. In order to check this assumption, the following procedure was followed in the finite element model. First, a path of nodes is created across the thickness direction. This path goes from the upper layer to the bottom layer. The profile of velocities changes in each increment of time and also some of the elements belonging to the path are deleted at some point. Those aspect must be taken into account to obtain reliable results. In this way the velocity versus position can be known for each increment and a representative curve is obtained for each time increment. In Figures 6.14 and 6.15 the first point represented for each time increment is a point a bit ahead of the most advanced part of the projectile to avoid problems that can introduce false data such as deletion of elements. The last point, is that reached by the compressive wave through the thickness direction at each increment. The velocity of each element in the thickness direction, $V_z$, is obtained for each increment of time. The velocity is non-dimensionalized with the maximum velocity at each time increment, which corresponds to the velocity in the closest node to the projectile. The spatial coordinates, $z$, are represented in their undeformed way and they are non-dimensionalized with the thickness.

Four cases have been studied corresponding to 9 mm and 12 mm thick specimens at two different impact velocities, approximately the ballistic limit and slightly above it. In Figure 6.14 the velocity across the path versus the thickness coordinate is represented for 9 mm thick specimens for velocities of 465 m s$^{-1}$ and 501 m s$^{-1}$ in Figures 6.14a and 6.14b respectively. The same is represented in Figure 6.15 but for 12 mm thick specimens for velocities of 553 m s$^{-1}$ and 625 m s$^{-1}$ in Figures 6.15a and 6.15b respectively.
6.2 Validation of the hypotheses assumed in the theoretical models

Fig. 6.14 Non-dimensional velocity of a path of nodes across the thickness vs non-dimensional position across the thickness for different instants of time for a 9 mm thick specimen for impact velocities of (a) $465 \text{ m s}^{-1}$ (b) $501 \text{ m s}^{-1}$

The profile of velocities is represented until the transition from the first stage to the second stage is met. Therefore, in Figures 6.14 and 6.15 the final time corresponds with the time at which equation 3.79 is satisfied (Figure 3.4) because it only makes sense to compare the shape of this profile in the first stage.

Fig. 6.15 Non-dimensional velocity of a path of nodes across the thickness vs non-dimensional position across the thickness for different instants of time for a 12 mm thick specimen for impact velocities of (a) $553 \text{ m s}^{-1}$ (b) $625 \text{ m s}^{-1}$
The shape of the profiles of velocities across the thickness is really similar for all the cases studied. Therefore, from now on, the conclusions obtained are valid for all the cases. From Figures 6.14 and 6.15 it can be inferred that the velocity always decreases for points farther away from the impact point. Obviously, the curve moves to the right when time increases because the projectile advances and so the compressive wave but the shape remains similar. Regarding the shape of the profile of velocities it cannot be said that it is exactly a line. However, a line with approximately the same slope for all the cases seems to fit reasonably well. In view of these results, the hypotheses of a linear gradient of velocities from the point in contact between the two bodies and the distance travelled by the through thickness compressive wave can be assumed to be valid.

### 6.3 Analysis of failure mechanisms and energy absorption

One of the most important assumptions made in the finite element method approach was to define the failure criteria with certain equivalences to the theoretical model. That was made to associate a group of failure criteria in the finite element model to a failure mode or energy absorption mechanism in the theoretical model. In this section the amount of energy absorbed for each mechanism is analyzed. First, the procedure to obtain the energies in both models is explained. Then, the equivalences for the energy absorption mechanisms between the finite element and the theoretical models are explained for each case. Finally, the results are discussed.

The measurements of the energies absorbed by each damage mechanism in the theoretical model are obtained immediately since there are analytical expressions to calculate them. However, a general procedure has to be used in the finite element model. Due to the memory of the computer used, it was not practical to make an addition of the energies of all the elements. Therefore, a set of elements was created, selecting 15 elements per layer of elements making a total of 150 elements for the 3 mm thick model, 300 elements for the 6 mm thick model, 450 elements for the 9 mm thick model and 600 elements for the 12 mm thick model. The elements were chosen randomly in the surroundings of the impact zone, decreasing the density of elements as the distance to the impact increases. This set of elements is assumed to be a representative sample to measure the energies absorbed during an impact. The energy absorbed by each element is fully explained in Appendix B.

In the theoretical model for thin laminates, out of plane damage mechanisms are not taken into account. Therefore, the more reasonable equivalences found are the following. Fibre failure (4.3) and (4.4) are the failure mechanisms associated to elastic deformation and tensile failure of fibres in the theoretical model. Matrix failure (4.13) is associated with matrix cracking. The laminate acceleration in the theoretical model is associated to the kinetic energy of the elements set. And delamination can be seen with the cohesive
elements. Since out of plane energy absorption mechanisms are not contemplated in the theoretical model for thin laminates, the results obviously will not be the same.

For the theoretical model for thick laminates, the following equivalences have been set. Fibre failure (4.3) and (4.4) are the failure mechanisms associated to tensile failure of fibres in the theoretical model. Matrix failure (4.13) is associated with matrix cracking as in the theoretical model for thin laminates. Crush failure and failure caused by shear stresses (4.11) and (4.15) are associated to compression and shear plugging respectively. Delamination is associated to the cohesive elements. In this case the kinetic energy of the set of elements is associated to the kinetic energy transferred to the laminate during the first stage plus the instant transference of kinetic energy in the second stage. Finally, the energy absorbed by friction can be compared because there is a specific mechanism for friction in both models.

This comparison is carried out to see the general trends of the energies absorbed in both models and infer conclusions of the behaviour of the models.

The study of the energies has been carried out at velocities near the ballistic limit for both thicknesses. The velocity which determines if the perforation takes place or not is the most important to ensure safety and structural integrity in any application. That is why the two velocities analyzed are 197 m s$^{-1}$ and 240 m s$^{-1}$ for 3 mm, 318 m s$^{-1}$ and 337 m s$^{-1}$ for 6 mm, 402 m s$^{-1}$ and 465 m s$^{-1}$ for 9 mm, 517 m s$^{-1}$ and 607 m s$^{-1}$ for 12 mm. The comparison of the energies absorbed for 3 mm and 6 mm thick specimens can be seen in Figures 6.16 and 6.17.

![Fig. 6.16 Comparison between the energy absorption mechanisms provided by the theoretical and finite element models for a 3 mm thick specimen subjected to impact velocities of (a) 197 m s$^{-1}$ (b) 240 m s$^{-1}$](image-url)
Results

Fig. 6.17 Comparison between the energy absorption mechanisms provided by the theoretical and finite element models for a 6 mm thick specimen subjected to impact velocities of (a) 318 m s\(^{-1}\) (b) 337 m s\(^{-1}\).

For a velocity just below the ballistic limit for 3 mm, Figure 6.16a shows that the major energy absorption mechanism for both models is the failure of fibres (Fib.) which is associated to elastic deformation and tensile failure of fibres in the theoretical model. This mechanism is almost constant in Figures 6.16 and 6.17. Also matrix cracking (Mat.) is really similar between both models and compression (Comp.) and shear (Shear) contribution are low in the finite element model compared to the failure of fibres. For a velocity just above the ballistic limit, Figure 6.16b shows that both models predict the same mechanism as the most important one, the failure of fibres although the compression is a bit higher in this case. The acceleration of the laminate (Acc.) is higher in the finite element model. This is due to the fact that it is calculated as the kinetic energy of the elements in the finite element model, so if we run the simulation for more time, the elements will go back to their original positions decreasing a bit this kinetic energy of the elements and becoming more similar to the theoretical model. In a general view, it can be inferred that both models predict similar behaviours in terms of the relative importance of the energy absorption mechanisms.

For 6 mm the results does not fit so well for both velocities as can be seen in Figure 6.17. It is true that failure of fibres is still one of the most important energy absorption mechanisms but it shares this role with compression. However, this results make sense with the hypotheses assumed in the theoretical model and with the results presented by Alonso et al. (2018a). As long as the thickness increases, the phenomena of compression and shear stresses gain importance. In fact, in this work it has been demonstrated that for this kind of laminates the threshold to start considering a laminate as thick instead of thin is around \(\Pi_c = 0.85\), which correspond to a thickness of 6.4 mm for the projectile diameter used. Therefore, since 6 mm specimens are really close to this thickness, it
is logical that these out of plane mechanisms should be more important because the thin laminates approximation is about to be no longer valid. Related to the laminate acceleration, it is zero in the theoretical model because for these velocities the perforation does not take place.

The comparison of the absorbed energies for 9 mm and 12 mm thick specimens can be seen in Figures 6.18 and 6.19.

**Fig. 6.18** Comparison between the energy absorption mechanisms provided by the theoretical and finite element models for a 9 mm thick specimen subjected to impact velocities of (a) $402 \text{ m s}^{-1}$ (b) $465 \text{ m s}^{-1}$

**Fig. 6.19** Comparison between the energy absorption mechanisms provided by the theoretical and finite element models for a 12 mm thick specimen subjected to impact velocities of (a) $517 \text{ m s}^{-1}$ (b) $607 \text{ m s}^{-1}$
For a velocity just below the ballistic limit for 9 mm, Figure 6.18a shows that the major energy absorption mechanism for both models is compression. This mechanism is followed by shear plugging in the theoretical model. The rest of the energy absorption mechanisms are minor. For a velocity slightly above the ballistic limit, Figure 6.18b shows that compression is still the most important energy absorption mechanism in the finite element model. However, this role is shared between compression and shear plugging in the theoretical model. Matrix cracking and delamination are very low for both models in Figures 6.18a and 6.18b. Friction gains important in both models as long as the impact velocity increases and the values for it are similar.

For 12 mm shear plugging becomes in the most important energy absorption mechanism as can be seen in Figure 6.19. The finite element model keeps predicting compression as the most important energy absorption mechanism. But what is more important here is that the out of plane damage mechanisms (compression and shear plugging) are the dominant mechanisms for thick laminates (Figures 6.18 and 6.19) for both models. Furthermore, it can be seen that the prediction of the energy absorbed by kinetic energy in the theoretical model grows a lot when the impact velocity is above the ballistic limit. This is due to the fact that the transfer of linear momentum between the projectile and the plate increases a lot if there is perforation.

If thin laminates (Figures 6.16 and 6.17) are compared to thick laminates (Figures 6.18 and 6.19) an evolution from in-plane failure mechanisms (basically energy absorbed by the fibres) to out of plane failure mechanisms (compression and shear plugging) is observed when increasing the thickness. Both the theoretical and the finite element models predict the same trend in terms of the importance of the energy absorption mechanisms, which is the most important result in this study of the energies absorbed.

### 6.4 Comparison with the Hertz model

A brief summary of the classic Hertz model is presented in order to understand the results shown in this subsection.

Over the last decades, different models to describe the contact forces between two bodies have been proposed. Hertz (1881) was one of the first to describe the contact phenomenon. Other more complex models have been proposed to take into account different effects such as plastic dissipation. Some examples are the models proposed by Hunt and Crossley (1975), Bo and Persson (2000).

Hertz contact law can be applied to the impact problem on plates if the relative displacement between the striker and the plate $\delta$ is much smaller than the dimensions of the bodies in contact and the contact area itself (Mahfuz et al. (1998)). The contact force $F$ can be written as:

$$F = K_c \delta^2$$  \hspace{1cm} (6.2)
where $K_c$ is a stiffness which depends on the geometry and the material properties of both the impactor and the laminate. This $K_c$ is given by equation (3.40).

\[
K_c = \frac{4 \left( \frac{\varphi_p}{2} \right)^{\frac{3}{2}}}{3\pi(k_1 + k_2)}
\]

(6.3)

where $k_1$ and $k_2$ are the stiffness measures of the projectile and the plate respectively.

\[
k_1 = \frac{1 - \nu_p^2}{\pi E_p}
\]

(6.4)

\[
k_2 = \frac{1}{\pi E_{33}}
\]

(6.5)

where $\nu_p$ and $E_p$ are the Poisson’s ratio and the Young’s modulus of the impactor (steel in this case), and $E_{33}$ is the Young’s modulus of the plate through the thickness direction.

The prediction of the contact force by the Hertz law is compared and discussed below with the predictions obtained by the theoretical model proposed.

The force given by the theoretical model for thin laminates has been compared to the force given by the classic Hertz model in order to draw conclusions and understand better the model proposed. To calculate the force in the theoretical model, the acceleration obtained in (3.42) is multiplied by the mass of the projectile. To obtain the force in the Hertz model, the relative displacement must be provided by the theoretical model. Figures 6.20 and 6.21 show the force versus the non-dimensional time (non-dimensionalized with the contact time) for different velocities for 3 mm and 6 mm thick plates. Figures 6.20a and 6.21a show the force for a velocity below the ballistic limit. A good agreement can be noticed between the prediction of both forces until the 30 % and 43 % of the total impact time for 3 mm and 6 mm respectively. After these times the forces diverge because the acceleration decreases in the theoretical model, which is logical taking into account that the projectile stops while the force given by the Hertz model continues its tendency. It is surprising how alike are the forces for a great amount of the total time. In Figures 6.20a and 6.21a the last point is associated to an indented diameter of $\varphi_i = 4.58$ mm and $\varphi_i = 5.35$ mm respectively. Finally, it is worth mentioning the theoretical model force presents a discontinuity because at that moment, elastic deformation of fibres starts acting.

For higher velocities, approaching the ballistic limit, the forces are alike for a much smaller range, almost 5 % and 2 % in Figures 6.20b and 6.21b. In Figures 6.20b and 6.21b the final point is associated to an indented diameter of $\varphi_i = 2.71$ mm and $\varphi_i = 2.10$ mm respectively. But it makes sense that at the beginning of the impact when everything can be considered as elastic, the forces generated are similar. Hertz theory is not adequate for high velocities and dissipation of energy. In fact, it has been surprising that below the ballistic limit the range of valid time is so wide.
Results

The good agreement between the forces given by the theoretical model and the Hertz model for a certain range of time reinforces the validity of the other results given by the theoretical model.

Fig. 6.20 Force vs non-dimensional time for 3 mm thick laminates for impact velocities of (a) 170 m s$^{-1}$ (b) 220 m s$^{-1}$

Fig. 6.21 Force vs non-dimensional time for 6 mm thick laminates for impact velocities of (a) 200 m s$^{-1}$ (b) 300 m s$^{-1}$
7

Conclusions and future work

7.1 Conclusions

In this work, the high-velocity impact phenomenon on woven E-glass fibre/polyester plates was studied by means of theoretical and finite element models as well as experimental tests.

Shear, friction, in-plane compression and out of plane compression tests were carried out in order to obtain the properties needed for the models developed. Furthermore, impact tests were carried out in 9 mm thick specimens in order to know the ballistic response in a wide range of thicknesses (from 3 mm to 12 mm). Non-destructive tests were performed to measure aspects of interest such as the damaged area and delamination after impact.

Two non-dimensional energy-based theoretical modes were developed considering different energy absorption mechanisms. The governing equation of the two problems is a non-linear second-order differential equation that can be solved by numerical integration. These models are formulated with new phenomenological physical based hypotheses in order to provide a more reliable approach to what is really happening during an impact.

On the other hand, a finite element model was implemented in Abaqus/Explicit by means of a VUMAT user-written subroutine. The model was implemented using Langrangian 3D elements, trilinear elements with reduced integration as well as cohesive elements to take into account delamination. A continuum damage model was defined to divide the in-plane damage from the out of plane damage. The failure criteria of the finite element model are a mixture of different criteria previously used. We tried to associate somehow the failure mechanisms in the finite element model to the different energy absorption mechanisms in the theoretical models. The evolution of the damage variables was defined by means of fracture throughnesses.
Conclusions and future work

The results of the ballistic limits as well as the residual velocities show a good agreement between the theoretical models, the finite element model and the experimental results.

Once the models were validated by means of these experimental results, five representative Π groups were studied to see the physical consistency of the theoretical models. The first one is the thickness ratio. By comparing the error in the ballistic limit of the two models with respect to the experimental results, the threshold from which a laminate changes its behaviour is found, and it is really close to 6 mm. Two representative Π groups were studied for thin laminates, which are $Π_E$ and $ε_r$. It was observed that the higher these parameters, the higher the ballistic limit. It seems to be in agreement with reality because the higher these parameters the more energy the laminate can absorb. Since the importance of the energy absorption mechanisms changes for thick laminates, the Π groups analyzed were $Π_{E_c}$ and $Π_{S_{SP}}$. It was observed the higher these parameters, the higher the ballistic limit because the energy absorbed by compression and shear plugging increases.

Furthermore, some of the hypotheses assumed in the theoretical models were checked with the finite element model. First, a study of the phenomenological function $\bar{k}$ was carried out by means of the theoretical model. The results of $\bar{k}$ when varying $Π_E$ show that $\bar{k}$ is lower for higher values of $Π_E$. Since the in-plane stiffness is increased it is logical that $\bar{k}$ decreases because the relative displacement is higher from the beginning. Nevertheless, a trade off between mass effects and elastic deformation of fibres is observed when varying $ε_r$, resulting in lower $\bar{k}$ for higher $ε_r$. Then, the relative displacement and thus the phenomenological function $\bar{k}$, were compared between both models obtaining a very good agreement for 3 mm and a bit worse for 6 mm. This backs up the idea that these hypotheses can only be used for thin laminates and it supports the physical meaning of the hypotheses assumed. Along with this, the profile of velocities through the thickness direction assumed linear in the theoretical model for thick laminates was checked. The results show that the linear approximation fits well with the profile obtained with the finite element model for the two thicknesses analyzed for all times.

After that, the energy absorption mechanisms provided by the theoretical and finite element models were compared leading to an interesting discussion. In general terms, energy absorbed by fibres is the most important mechanism for thin laminates (3mm) in both models. However, when the thickness increases up to 6 mm compression and shear plugging start to being crucial mechanisms in the finite element model. This behaviour makes sense because 6 mm is really close to the threshold to consider a laminate as thick ($Π_e = 0.85$). The other energy absorption mechanisms are minor for these thicknesses. From 6 mm thickness on, the out of plane energy absorption mechanisms (compression and shear plugging) are the ones that control in the theoretical and the finite element models. Obviously the out of plane effects increase their importance with thickness.
Friction is also a bit more important for thick than for thinner specimens and the theoretical and finite element models follow the same trend.

In addition, a comparison between the force given by the theoretical model for thin laminates and the one given by the classic Hertz model (with the input of the relative displacement given by the theoretical model) was carried out. The forces are alike for an approximately 30-40 % of the total time below the ballistic limit, when there is not penetration. However, for higher velocities this interval was reduced to 2-5 % of the total time because the Hertz model makes worse predictions as long as the indented diameter increases.

### 7.2 Future work

From the results and conclusions obtained in this research, the following future work on the subject are proposed:

- Characterization of the woven E-glass fibre/polyester laminates available at high strain rates. This would provide a more reliable constitutive response of the laminates under high-velocity impact and it could be implemented in the finite element model.

- Taking into account the possible non-linearities in shear directions in the constitutive response for the finite element model.

- Carrying out characterization tests at different testing temperatures so that more reliable mechanical properties could be collected and the relaxation phenomenon could be implemented for very high impact velocities.

- Modification of the theoretical models to take into account different geometries of the projectile such as flat-ended, conical or hemispherical. For some cases it would be necessary to modify some of the hypotheses assumed such as the moment when elastic deformation of fibres starts acting or that the projectile impacts on a whole fibre.

- Carrying out a study for the same material but impacting with an inclination angle. The finite element model should work independently of the angle. However, the theoretical models should be subjected to changes. Firstly, the distance covered by the projectile would be higher. For the thin laminate model, the relative displacement as long as the function $\bar{k}$ should be reformulated since the indentation would not be symmetrical any more.

- Utilization of the subroutine developed to simulate other structures such as sandwich laminates. The subroutine could be used for the constitutive response of the skins.
8

Conclusiones y trabajos futuros

8.1 Conclusiones

En este trabajo se ha estudiado el fenómeno de impacto a alta velocidad en laminados de tejido de fibra de vidrio tipo E en matriz poliéster mediante modelos teóricos y de elementos finitos así como con la ayuda de ensayos experimentales.

Se han realizado ensayos de cortadura, de fricción, de compresión en el plano y fuera del plano para obtener las propiedades mecánicas necesarias para los modelos desarrollados. Además, se llevaron a cabo experimentos de impacto a alta velocidad en probetas de 9 mm de espesor para disponer de la respuesta ante impacto balístico dentro de un amplio rango de espesores (de 3 mm hasta 12 mm). También se realizaron ensayos de inspección no destructiva para medir aspectos de interés como el área dañada y la delaminación después de impacto.

Se desarrollaron dos modelos teóricos no dimensionales basados en criterios energéticos considerando diferentes mecanismos de absorción de energía. Las ecuaciones que controlan los dos problemas son dos ecuaciones diferenciales no lineales de segundo orden que se pueden resolver por integración numérica. Estos modelos están formulados mediante nuevas hipótesis fenomenológicas basadas en fenómenos físicos con el objetivo de reproducir de manera más fiable lo que ocurre realmente durante un impacto.

Por otro lado, se implementó un modelo de elementos finitos en Abaqus/Explicit a través de una subrutina de usuario VUMAT. El modelo fue implementado usando elementos lagrangianos 3D, trilineales con integración reducida así como elementos cohesivos para tener en cuenta la delaminación. Se definió un modelo de daño continuo para separar el daño en el plano del daño fuera del plano. Los criterios de fallo utilizados en el modelo de elementos finitos son una mezcla de criterios previamente utilizados. Se intentaron asociar los criterios de fallo del modelo de elementos finitos a los diferentes mecanismos de absorción de energía en los modelos teóricos. La evolución de las variables de daño se definió a través de las energías de fractura.
8.1 Conclusiones

Los resultados de los límites balísticos junto con los de las velocidades residuales muestran un buen ajuste entre los modelos teóricos, el modelo de elementos finitos y los resultados experimentales.

Una vez los modelos fueron validados con los resultados experimentales, se estudiaron cinco grupos \( \Pi \) representativos para comprobar la consistencia física de los modelos teóricos. El primero es el grupo \( \Pi \) relativo al espesor. Comparando el error en el límite balístico de los dos modelos con respecto a los resultados experimentales, se determina el umbral a partir del cual un laminado cambia su comportamiento, muy cerca de 6 mm. Se estudiaron dos grupos \( \Pi \) representativos para laminados delgados, \( \Pi_E \) y \( \varepsilon_r \). Se observó que al incrementar el valor de estos parámetros, el límite balístico también aumentaba. Este resultado parece ser coherente porque cuanto más altos son estos parámetros, la energía que un laminado puede absorber aumenta. Al cambiar la importancia de los mecanismos de absorción de energía para laminados gruesos, los grupos \( \Pi \) analizados en este caso fueron \( \Pi_{Ec} \) y \( \Pi_{Ssp} \). Se observó que al aumentar estos parámetros aumentaba también el límite balístico debido a una mayor absorción de energía por compresión y debido al tapón de cortadura.

Además, algunas de las hipótesis asumidas en los modelos teóricos se han comprobado con el modelo de elementos finitos. Primero se ha llevado a cabo un estudio de la función fenomenológica \( \tilde{k} \) utilizando el modelo teórico. Los resultados de \( \tilde{k} \) al variar \( \Pi_E \) muestran un decrecimiento de \( \tilde{k} \) al aumentar \( \Pi_E \). Al incrementarse la rigidez en el plano, es lógico que \( \tilde{k} \) decrezca porque el desplazamiento relativo es mayor desde el principio. Sin embargo, se observa una compensación entre los efectos máxicos y la deformación elástica de fibras cuando se varía \( \varepsilon_r \), resultando en \( \tilde{k} \) menores cuando \( \varepsilon_r \) aumenta. A continuación se compararon el desplazamiento relativo y por lo tanto, la función fenomenológica \( \tilde{k} \) entre el modelo teórico y el de elementos finitos obteniendo un muy buen ajuste para 3 mm y algo peor para 6 mm. Esto refuerza la idea de que estas hipótesis solo se pueden asumir para laminados delgados y proporciona otra evidencia que apoya la consistencia física de las hipótesis asumidas. Junto con esto, se comprueba el perfil de velocidades a través de la dirección del espesor asumida lineal en el modelo teórico para laminados gruesos. Los resultados muestran que la aproximación lineal se ajusta al perfil obtenido con el modelo de elementos finitos para los dos espesores estudiados en todos los instantes de tiempo.

Posteriormente, los mecanismos de absorción de energía de los modelos teóricos y del modelo de elementos finitos se comparan, llegando a una discusión interesante. En términos generales, la energía absorbida por las fibras es el mecanismo de absorción de energía más importante para laminados delgados (3 mm) en ambos modelos. Sin embargo, cuando el espesor aumenta a 6 mm la compresión y el tapón de cortadura empiezan a ser mecanismos determinantes en el modelo de elementos finitos. Este comportamiento tiene sentido debido a que 6 mm es un espesor muy cercano al umbral donde un laminado se puede considerar grueso (\( \Pi_e = 0.85 \)). Los otros mecanismos de absorción de energía
Conclusiones y trabajos futuros

son de una importancia menor para estos espesores. A partir de 6 mm de espesor en adelante, los mecanismos de absorción de energía fuera del plano (compresión y tapón de cortadura) son los mecanismos dominantes en el modelo teórico y en el de elementos finitos. Obviamente, los efectos fuera del plano adquieren más importancia conforme el espesor crece. También la fricción es algo más importante en comparación con laminados más delgados y la predicción de los modelos teórico y de elementos finitos sigue la misma tendencia.

Por último, se llevó a cabo una comparación entre la fuerza predicha por el modelo teórico para laminados delgados y por el modelo clásico de Hertz (con la variable de entrada del desplazamiento relativo proporcionada por el modelo teórico). Las fuerzas son parecidas para aproximadamente el 30-40 % del tiempo total de impacto para una velocidad por debajo del límite balístico, cuando no existe penetración. Sin embargo, para velocidades más altas este intervalo se reduce al 2-5 % del tiempo total de impacto debido a que el modelo de Hertz proporciona peores predicciones conforme el diámetro indentado aumenta.

8.2 Trabajos futuros

A partir de los resultados y conclusiones obtenidos en esta investigación, se proponen los siguientes trabajos futuros en el campo:

• Caracterización de los laminados de tejido de fibra de vidrio tipo E en matriz poliéster a altas velocidades de deformación. Esto proporcionaría una respuesta constitutiva de los laminados más fiable para impactos de alta velocidad y se podría implementar en el modelo de elementos finitos.

• Tener en cuenta en la respuesta constitutiva las posibles no linealidades en las direcciones de cortadura para el modelo de elementos finitos.

• Realizar ensayos de caracterización a diferentes temperaturas para obtener propiedades mecánicas más fiables a altas temperaturas y de este modo poder tener en cuenta el fenómeno de relajación para velocidades de impacto muy altas.

• Modificación de los modelos teóricos para tener en cuenta diferentes geometrías del proyectil como cilíndrico, cónico o hemiesférico. Para algunos casos sería necesario modificar algunas de las hipótesis asumidas como el momento en el que el mecanismo de absorción de energía por deformación elástico de fibras empieza a actuar o que el proyectil impacta sobre una fibra completa.

• Llevar a cabo un estudio para los mismos laminados pero impactando con un ángulo de inclinación. El modelo de elementos finitos debería funcionar independientemente del ángulo propuesto. Sin embargo, los modelos teóricos estarían sujetos a cambios. En primer lugar, la distancia recorrida por el proyectil sería mayor. Para el modelo
8.2 Trabajos futuros

de laminados delgados, se debería reformular el desplazamiento relativo junto con
la función $\tilde{k}$ ya que la indentación no sería simétrica en este caso.

- Utilización de la subrutina desarrollada para simular otro tipo de estructuras como
las sándwich. La subrutina podría utilizarse para la respuesta constitutiva de las
pieles.


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Mathematical development of the theoretical models

With the goal to make this document self-contained, the mathematical development employed throughout the formulation of the theoretical models is detailed in this appendix. Attention is focused on the derivatives and the functions that appear naturally when deriving the energy balance to obtain the governing equation of the problem.

A.1 Theoretical model for thin woven laminates

Derivation of equation (3.37) with respect to non-dimensional time provides equation (3.38). In this last equation, the derivatives of all energy absorption mechanisms appear. Derivation of equation (3.16) gives the derivative of the energy absorbed by laminate acceleration.

\[
\frac{dE_L(\tau)}{d\tau} = \frac{1}{2}\pi \Pi_r \Pi_{\rho l} C_{V_s}^2 \left[(2\tau \bar{k}(\tau, \bar{v}(\tau))^2 + 2\tau^2 \bar{k}(\tau, \bar{v}(\tau)) \frac{d\bar{k}(\tau, \bar{v}(\tau))}{d\tau} \bar{v}(\tau)^2 + 2\tau^2 \bar{k}(\tau, \bar{v}(\tau))^2 \bar{v}(\tau)\bar{a}(\tau)\right]
\]

(A.1)

The derivative of the phenomenological function \( \bar{k}(\tau, \bar{v}(\tau)) \) adopts the following expression:

\[
\frac{d\bar{k}(\tau, \bar{v}(\tau))}{d\tau} = \frac{\Pi_c}{\Pi_{\nu}} \Pi_E^{1/6} \left[2\bar{v}(\tau)\bar{a}(\tau)\tau^{1/2} + \frac{1}{2} \bar{v}(\tau)^2 \tau^{-1/2}\right]
\]

(A.2)

Derivation of equation (3.26) gives the derivative of the energy absorbed by elastic deformation of fibres, which is formulated as follows:

\[
\frac{\bar{E}_{ED}(\tau)}{d\tau} = \bar{C}_{ED} \frac{\bar{N}_{ED1}\bar{N}_{ED2} - \bar{N}_{ED3}\bar{N}_{ED4}}{\bar{D}_{ED}}
\]

(A.3)
where $\bar{C}_{ED}$, $\bar{N}_{ED1}$, $\bar{N}_{ED2}$, $\bar{N}_{ED3}$, $\bar{N}_{ED4}$ and $\bar{D}_{ED}$ are functions defined in (A.4), (A.5), (A.6), (A.7), (A.8) and (A.9) respectively.

$$\bar{C}_{ED}(\tau) = \frac{\pi \Pi_e \Pi_E \varepsilon_r^2}{48}$$  \hspace{1cm} (A.4)

$$\bar{N}_{ED1}(\tau) = 64C_V \tau^3 - 24 \left[ 2C_V^2 \tau \bar{\phi}(\delta)^2 + 2C_V \tau^2 \bar{\phi}(\delta) \frac{d\bar{\phi}(\delta)}{d\tau} \right] + 16 \left[ C_V \bar{\phi}(\delta)^3 + 3C_V \tau \bar{\phi}(\delta)^2 \frac{d\bar{\phi}(\delta)}{d\tau} \right] - 12 \bar{\phi}(\delta) \frac{d\bar{\phi}(\delta)}{d\tau}$$  \hspace{1cm} (A.5)

$$\bar{N}_{ED2}(\tau) = \left[ 2C_V \tau - \bar{\phi}(\delta) \right]^2$$  \hspace{1cm} (A.6)

$$\bar{N}_{ED3}(\tau) = 16C_V \tau^4 - 24C_V \tau^2 \bar{\phi}(\delta)^2 + 16C_V \tau \bar{\phi}(\delta)^3 - 3\bar{\phi}(\delta)^4$$  \hspace{1cm} (A.7)

$$\bar{N}_{ED4}(\tau) = 2[2C_V \tau - \bar{\phi}(\delta)] \left[ 2C_V - \frac{d\bar{\phi}(\delta)}{d\tau} \right]$$  \hspace{1cm} (A.8)

$$\bar{D}_{ED}(\tau) = \left[ 2C_V \tau - \bar{\phi}(\delta) \right]^4$$  \hspace{1cm} (A.9)

The indented diameter is defined in terms of the relative displacement (3.17), and this relative displacement depends on time (3.20). Figure 3.1 shows all these variables for a generic time during an impact. By the chain rule, when the derivative of the indented diameter with respect to non-dimensional time appears, the derivatives in equations (A.10) and (A.11) has to be defined.

$$\frac{d\bar{\phi}(\delta)}{d\delta} = \begin{cases} 
1 - 2\delta \frac{1}{\sqrt{\delta - \delta^2}}, & \text{if } \delta < 0.5 \\
0, & \text{if } \delta \geq 0.5
\end{cases}$$  \hspace{1cm} (A.10)

Applying the Fundamental Theorem of Calculus to equation (3.20):

$$\frac{d\bar{\delta}}{d\tau} = \bar{v}(\tau)(1 - \bar{k}(\tau, \bar{v}(\tau)))$$  \hspace{1cm} (A.11)

The derivative of the energy absorbed by tensile failure of fibres with respect to non-dimensional time was already defined in (3.31). The derivatives of the energy absorbed by delamination and matrix cracking are defined in equations (A.12) and (A.13) respectively.

$$\bar{E}_{DL}(\tau) = 2\pi \Pi_{G_{11}} C_V \tau \alpha_{DL}$$  \hspace{1cm} (A.12)

$$\bar{E}_{MC}(\tau) = 2\pi \Pi_e \Pi_{E_{MT}} C_V \tau \alpha_{MC}$$  \hspace{1cm} (A.13)

By means of the mathematical development presented in this section, the functions $\bar{h}(\tau, \bar{v}(\tau))$ and $\bar{g}(\tau, \bar{x}(\tau), \bar{v}(\tau))$ defined in (3.39) and (3.40) respectively can be written and fully understood. Furthermore, the governing equation of the model (3.42) is obtained through these equations.
A.2 Theoretical model for thick woven laminates

Derivation of equation (3.72) with respect to non-dimensional time provides equation (3.78). In this last equation, the derivatives of all energy absorption mechanisms for the first stage appear. Derivation of equation (3.52) with respect to non-dimensional time provides the derivative of the energy absorbed by compression in region 1.

\[ \frac{d\bar{E}_C1(\tau)}{d\tau} = \frac{\pi}{8} \Pi_{E_c} \bar{\varepsilon}_{re}^2 C_{Vs} \left[ \bar{\phi}(\bar{x})^2 + 2\tau \bar{\phi}(\bar{x}) \frac{d\bar{\Phi}(\bar{x})}{d\bar{x}} \bar{v}(\tau) \right] \]  

(A.14)

In this model, the indented diameter is defined in terms of the position of the projectile (3.50) for the reasons explained in the thick model section. All the variables involved in the penetration process can be seen in Figure 3.4. By the chain rule, the derivative of the indented diameter with respect to non-dimensional time can be decomposed in equations (A.15) and (A.16).

\[ \frac{d\bar{\phi}(\bar{x})}{d\bar{x}} = \begin{cases}  \frac{1-2\bar{x}}{\sqrt{\bar{x}-\bar{x}^2}}, & \text{if } \bar{x} < 0.5 \\ 0, & \text{if } \bar{x} \geq 0.5 \end{cases} \]  

(A.15)

And the derivative of the position with respect to non-dimensional time is directly the velocity.

\[ \frac{d\bar{x}}{d\tau} = \bar{v}(\tau) \]  

(A.16)

Derivation of equation (3.58) gives the derivative of the energy absorbed by compression in region 2, which is defined with the aid of the following auxiliar functions:

\[ \frac{\bar{E}_C2(\tau)}{d\tau} = \bar{C}_{C2} \frac{\bar{N}_{C2} \bar{N}_{C2} - \bar{N}_{C2} \bar{N}_{C2}}{\bar{D}_{C2}} \]  

(A.17)

where \( \bar{C}_{C2} \), \( \bar{N}_{C2} \), \( \bar{N}_{C2} \), \( \bar{N}_{C2} \) and \( \bar{D}_{C2} \) are functions defined in (A.18), (A.19), (A.20), (A.21), (A.22) and (A.23) respectively.

\[ \bar{C}_{C2}(\tau) = \frac{\pi \Pi_{E_c} \bar{\varepsilon}_{re}^2}{48} \]  

(A.18)

\[ \bar{N}_{C21}(\tau) = 64C_{Vs}^4 \tau^3 - 24 \left[ 2C_{Vs}^2 \tau \bar{\phi}(\bar{x})^2 + 2C_{Vs}^2 \tau^2 \bar{\phi}(\bar{x}) \frac{d\bar{\Phi}(\bar{x})}{d\bar{x}} \bar{v}(\tau) \right] + 
16 \left[ C_{Vs} \bar{\phi}(\bar{x})^3 + 3C_{Vs} \tau \bar{\phi}(\bar{x}) \frac{d\bar{\Phi}(\bar{x})}{d\bar{x}} \bar{v}(\tau) \right] - 12\bar{\phi}(\bar{x})^3 \frac{d\bar{\phi}(\bar{x})}{d\bar{x}} \bar{v}(\tau) \]  

(A.19)

\[ \bar{N}_{C22}(\tau) = \left[ 2C_{Vs} \tau - \bar{\phi}(\bar{x}) \right]^2 \]  

(A.20)

\[ \bar{N}_{C23}(\tau) = 16C_{Vs}^4 \tau^4 - 24C_{Vs}^2 \tau^2 \bar{\phi}(\bar{x})^2 + 16C_{Vs} \tau \bar{\phi}(\bar{x})^3 - 3\bar{\phi}(\bar{x})^4 \]  

(A.21)
Mathematical development of the theoretical models

\[ \tilde{N}_{C24}(\tau) = 2[2C_{Vt}\tau - \tilde{\phi}(\tilde{x})]\left[2C_{Vt} - \frac{d\tilde{\phi}(\tilde{x})}{d\tilde{x}}\tilde{v}(\tau)\right] \quad (A.22) \]

\[ \tilde{D}_{C2}(\tau) = [2C_{Vt}\tau - \tilde{\phi}(\tilde{x})]^4 \quad (A.23) \]

Finally, the derivative of the energy absorbed by transference of kinetic energy from the projectile to the laminate (3.68) is expressed in equation (A.24).

\[ \frac{d\tilde{E}_{CL}(\tau)}{d\tau} = \frac{\pi}{24} \Pi_{ri}C_{Vs_i}\left[(2\tilde{v}(\tau)\bar{a}(\tau)\tilde{\phi}(\tilde{x})^2 + 2\tilde{v}(\tau)^2\tilde{\phi}(\tilde{x})\frac{d\tilde{\phi}(\tilde{x})}{d\tilde{x}}\tilde{v}(\tau))\tau + \tilde{v}(\tau)^2\tilde{\phi}(\tilde{x})^2\right] \quad (A.24) \]

Once the derivatives of all the energy absorption mechanisms are defined, the governing equation of the first stage can be reached (3.78). In order to facilitate its formulation, functions \( \tilde{h}(\tau, \tilde{v}(\tau)), \tilde{g}(\tau, \tilde{x}(\tau), \tilde{v}(\tau)) \) and \( \tilde{f}(\tau, \tilde{x}(\tau)) \) which are combinations of the derivatives here developed are defined in (3.74), (3.75) and (3.76) respectively.
In this appendix, a more detailed explanation of some aspects of the finite element model is presented. Specifically, the way the failure criteria work is more developed as well as the activation and growth of damage variables through simple load examples. Moreover, the subroutine VUMAT is explained step by step.

Before explaining how the subroutine works, two simple examples will be analyzed for a better understanding of the damage variables. If we consider a single cube as is shown in Figure B.1, and we impose a constant velocity which varies according to Figure B.2a in one face while pinning the other, it is clear that tension and compression stresses will appear.

Since stresses in direction 11 are dominant, the failure criterion which controls the onset of damage is (4.3). In addition, equation (4.3) can be simplified to equation (B.1)
Description of the behaviour of the finite element model implemented

in this case because stresses in directions 12 and 13 do not arise.

\[
\left( \frac{\sigma_{11}}{X_{11i}} \right)^2 = 1
\]  

(B.1)

where the sub-index \( i = t, c \) accounts for tension and compression. Imposition of a velocity in the nodes of the upper face as the one shown in Figure B.2a provides the constitutive response shown in Figure B.2b.

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In view of Figure B.2, the phenomenon of closure of cracks can be explained. From the beginning to the point 2 a positive velocity is applied. The point 1 is reached when equation (B.1) is fulfilled and the damage starts. From this point to the point 2 the damage grows. From the point 2 to the point 5 a negative velocity is applied. First, the curve comes back through the point 3 with a different slope because the material is damaged in direction 11 and the stiffness decreases. But since a state of compression is taking place after the point 3, the slope is again the initial one as no damage had occurred. At the point 4, (B.1) is fulfilled again and the damage by compression starts and grows until the point 5. From the point 5 to the point 6 a positive velocity is applied again until there is no stress applied.

The variables of damage evolution in direction 11 for this example can be seen in Figure B.3.
The final damage variable to update the stresses in direction 11 is $d_{1f}$. This variable is equal to the damage variable in tension, $d_{1t}$, when the stress state is positive in direction 11. On the other hand, it is equal to the damage variable in compression, $d_{1c}$, when the stress state is negative in that direction. This behaviour can be seen in Figure B.3 where $d_{1f}$ is the orange curve and $d_{1t}$, $d_{1c}$ are the blue and the red curves respectively.

Another example where shear stresses are generated in direction 12 is presented in Figure B.4.

A constant velocity in direction 22 is applied on the nodes of the cube as it is shown in Figure B.4. The velocity in the nodes of the upper face is the same in modulus and direction but opposite to those in the lower face. Furthermore, the nodes are pinned except for direction 22. With this kind of configuration the highest stresses are mostly
Description of the behaviour of the finite element model implemented

generated in direction 12. Thus, the failure criterion which rules the onset of damage is (4.11). The evolution of the imposed velocity as well as the constitutive response can be seen in Figure B.5.

Since there is no difference between tension and compression in shear directions, it is considered that the closure effect does not take place in these cases. From the beginning to the point 2 a positive velocity is applied. The point 1 is reached when the criterion given by equation (4.11) is met and the damage grows from the point 1 to the point 2. From the point 2 to the point 5 a negative velocity is applied. First, the curve comes back through the point 3 with a different slope because the material is damaged in direction 12 and the stiffness decreases. But in this case, unlike in the first example, the slope continues to be the same from the point 3 on. At the point 4, (4.11) is met again and the damage continues growing until the point 5. Finally, a positive velocity is applied until there is no stress applied (point 6).

The variable of damage evolution in direction 12 for this example can be seen in Figure B.6.
Fig. B.6 Variables of damage evolution in direction 12 for the load example given by B.5b

In order to see and example where a failure criterion is met with the contribution of more than one significant stress, a stress state implying $\sigma_{13}$ and $\sigma_{23}$ is considered. If these stresses are predominant, the failure criterion defined by (4.15) controls the onset of damage.

Figure B.7 shows the path this stress state provokes.

Fig. B.7 Constitutive response generated with the application of the stress state mentioned above

In Figure B.7 the failure shear stresses $S_{13s}$ and $S_{23s}$ together with the strain at that point and the fracture toughnesses in each direction define the initial triangles remarked with broken lines. Since two stresses are contributing, (4.15) is met at $S_{13sn}$ and $S_{23sn}$. The fracture toughnesses are maintained, therefore the triangles change being the maximum strain increased up to $\gamma_{13f}$ and $\gamma_{23f}$. When this point is reached, the
Description of the behaviour of the finite element model implemented

damage variables $d_5$ and $d_6$ are activated and the can evolve different in this case. These damage variables control the update of the $\sigma_{13}$ and $\sigma_{23}$ stresses. If any of the two damage variables reaches the value of 1, the element is deleted. It bears noting that depending on the stress state, the triangles can be different because they depend on the contribution of each stress when reaching (4.15).

If the criterion involves three different contributions of stresses as in equation (4.3), the procedure to obtain the new triangles is the same as the one explained before but with three components. But in this case, the criterion only activates one damage variable, $d_1$, which uploads $\sigma_{11}$, although more stresses are involved in the onset criterion. Therefore, only the triangle in 11 direction is recalculated and from this point $d_1$ evolves.

Given these examples, the subroutine can be explained now step by step:

- Abaqus provides the tensor of increase in strains. With these data as well as the material properties, all the failure criteria are checked.

- Secondly, if any criterion is met, the strain at this direction is recorded as well as the characteristic length of the element. The maximum elastic strain is the strain in the last increment and the triangle is defined with that strain, the fracture toughness and the characteristic length. The damage variable associated to this direction will be activated.

- After that, the triangles are defined in directions in which the failure criterion is met. Therefore, the damage variables can be calculated in these directions. The current strain is used along with the maximum elastic strain and the final strain before deletion ($\gamma_{23f}$ and $\gamma_{13f}$ in Figure B.2b), to calculate the corresponding damage variable. If the direction is 11, 22 or 33, depending on the sign of the current strain, the damage variable in tension or compression is calculated and it will be used to update the corresponding stress.

- Next, all the stresses are updated with the damage variables. In addition, if a failure criterion was met in any direction, the updated stress is recorded to be used as the maximum stress in the failure criterion of the next increment.

- Finally, the absorbed energies are calculated as it is explained below and the stresses are sent to Abaqus so that it can provide the next tensor of increase in strains.

The energy of each element is calculated with the following procedure in the finite element model. When the element is deleted because a damage variable has reached the value of 1, the energy related to this failure criterion is the energy below the stress-strain curve and it is plastic energy because it is not recoverable. However, at that point, the stress-strain curve in other directions may not be zero, so the elastic and plastic energies in those directions have to be taken into account at the moment of deletion. For instance, if $d_{3f} = 1$ in one element and at that point in direction 11 we are in the point which
Figure B.8 shows, the energy in direction 11 is calculated as the sum of the elastic energy plus the plastic energy. Obviously, all these energies has to be multiplied by the volume of the element.

\[ d_{1} = 0 \]
\[ d_{1} = 1 \]
\[ E_{11}(1 - d_{1}) \]
\[ E_{11} \]
\[ \sigma_{11} \]
\[ \epsilon_{11} \]
\[ \epsilon_{1f} \]

**Fig. B.8 Scheme of how the energies are computed in the finite element model once the element is deleted**
In this appendix, the planes of the different components of the designed tool for the punch and shear tests are presented. Furthermore, a final view of the whole tool is also attached.
Fig. C.1 Shear punch
Fig. C.2 Friction punch
Fig. C.3 Coupling element
Planes of the designed tool for the punch and shear tests

Fig. C.4 Upper cover
Fig. C.5 Lower cover
Planes of the designed tool for the punch and shear tests

Fig. C.6 Screw
Fig. C.7 Assembly of the tool