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INCENTIVE COMPATIBILITY AND PRICING UNDER MORAL HAZARD *

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Abstract

We study a simple insurance economy with moral hazard, in which random contracts overcome the non-convexities generated by the incentive-compatibility constraints. The novelty is that we use *linear programming and duality theory* to study the relation between incentive compatibility and pricing. Using linear programming has the advantage that we can impose the incentive-compatibility constraints on the agents that are uninformed (the insurance firms). In contrast, most of the general equilibrium literature imposes them on the informed agents (the consumers). We derive the two welfare theorems, establish the existence of a competitive equilibrium, and characterize the equilibrium prices and allocations. Our competitive equilibrium has two key properties: (i) the equilibrium prices reflect all the relevant information, including the welfare costs arising from the incentive-compatibility constraints; (ii) the equilibrium allocations are the same as when the incentive-compatibility constraints are imposed on the consumers.

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1 Introduction

In their pathbreaking contribution, Prescott and Townsend [20, 21] show how to extend the Arrow–Debreu model to a large class of economies with asymmetric information. In these economies, asymmetric information is realized ex post, that is after the agents have traded. This class includes economies with moral hazard, where agents choose their effort after they have traded.[1] In particular, Prescott and Townsend define allocations in the space of lotteries over bundles of state–contingent commodities. They then derive the welfare theorems and show that a competitive equilibrium exists. The key modeling choice of Prescott and Townsend is to impose the incentive–compatibility constraints arising from asymmetric information on the consumers, and not on the firms. This modeling choice highly simplifies the analysis and allows to make initial progress because consumers are the informed agents and firms are the uninformed agents. A typical example is an insurance economy with moral hazard where consumers are subject to idiosyncratic risk. As in the full–information benchmark, firms supply actuarially fair insurance plans and any actuarially fair insurance plan is budget feasible. Consumers choose from the actuarially fair insurance plans under the incentive–compatibility constraint. As a result the second best is attained.[2]

The motivation for our paper is a potential conceptual problem with imposing the incentive–compatibility constraints on the consumers: it is unclear how these incentive–compatibility constraints are enforced in the decentralized economy. In the standard general equilibrium model, all the relevant information is conveyed through prices. In particular, prices reflect the costs arising from the resource constraints. With asymmetric information, resource constraints are accompanied by incentive–compatibility constraints. But imposing the incentive–compatibility constraints on the consumers implies that they do not affect the equilibrium prices, as the consumers are the informed agents. Our paper therefore takes a different approach and imposes the incentive–compatibility constraints on the firms. We will show that the properties of the equilibrium allocation remain unchanged. Crucially, however, the incentive–compatibility constraints will affect the equilibrium prices and so the equilibrium prices will reflect *all* the relevant information.

We make our point in a simple insurance economy with moral hazard. There is a

[1]However, it does not include economies with adverse selection, where agents learn their types before they trade (ex ante asymmetric information).

[2]The recent work of Kehoe, Levine and Prescott [16] builds on the Prescott–Townsend approach to study exchange economies with ex post private information about endowments. Again, the key modeling choice is to impose the incentive–compatibility constraints on the consumers, rather than on the firms.

continuum of ex ante identical consumers and a finite number of idiosyncratic endowment states. Each consumer can exert high or low effort at a direct utility cost. High effort reduces the probability of ending up in a poor state. The commodities are insurance contracts, which are signed between a consumer and a firm. Insurance contracts specify a vector of state–contingent net trades and an effort level for the consumer. We assume that net trades are perfectly verifiable and fully enforceable. It therefore suffices to consider exclusive contractual relations in which consumers can buy insurance from at most one firm.[3] Firms have access to a constant–returns–to–scale insurance technology and they face both technological and incentive–compatibility constraints. The incentive–compatibility constraints require that the contracts give the consumers the incentives to conform to the effort specifications. We allow for random contracts (or lotteries) to overcome the non–convexities generated by the incentive–compatibility constraints.

The crucial insight is that with random contracts, incentive–efficient allocations are the optimal solutions to a linear programming problem.[4] To study the decentralization of incentive–efficient allocations as competitive equilibria, we then proceed as follows. First, we use the primal problem, its dual, and their corresponding complementary slackness conditions to obtain a characterization of the incentive–efficient allocations. Second, we show that the competitive equilibrium allocation solves the primal problem. Thus, the first welfare theorem holds. We also establish an equivalence between the competitive equilibrium prices and the solutions to the dual problem. This equivalence allows us to derive the prices that decentralize incentive–efficient allocations. Thus, the second welfare theorem holds. Third, the existence of optimal solutions to the primal and dual problems directly implies the existence of a competitive equilibrium. In a companion paper (Jerez [14]), we establish the existence of optimal solutions to the primal and dual problems.

Our application of linear programming draws heavily on the work of Makowski and Ostroy's [18], who develop the linear programming methodology for large economies with full information. Specifically, they use a measure–theoretic description of the economy to show that efficient allocations solve a linear programming problem. Then they establish an equivalence between the competitive equilibrium allocations and prices, on the one hand, and the solutions to the primal and dual problems, on the other hand. Gretsky, Ostroy and Zame [10] present a similar analysis for large

[3]Bisin and Gottardi [3] and Bisin and Guaitoli [5] depart from this exclusive benchmark and study moral hazard economies with non–verifiable trades. Dubey, Geanakoplos and Shubik [6] study environments where asymmetric information arises from the possibility of default.

[4]Incentive–efficient allocations are the Pareto–optimal allocations in the set of technologically feasible and incentive–compatible allocations.

assignment economies.

The key advantage of our methodology is that the equilibrium prices do reflect all the relevant information because we impose the incentive–compatibility constraint on the uninformed agents (the firms), and not on the informed agents (the consumers). The equilibrium prices then internalize the welfare costs arising from both the technological constraints and the incentive–compatibility constraints. For example, actuarially fair contracts which specify a high effort generate identical technological costs but different incentive costs. Providing more insurance implies higher incentive costs because it raises the consumers' incentive to shirk. This raises the equilibrium price of an actuarially fair contract. Consumers then don't purchase the full–insurance contract because it is not budget feasible, and firms don't offer it because it is not incentive compatible (with full insurance consumers always shirk). As a result, the competitive equilibrium allocation provides only partial insurance. Note that the amount of insurance is then unaffected by our assumption to impose the incentive–compatibility constraints on the firms.

There is also a formal difference between our paper and that of Prescott and Townsend [20]. With their approach, the competitive equilibrium prices are the same as in the full–information benchmark, so they are linear on the agents' net trade sets. With our approach, the competitive equilibrium prices are not the same as in the full–information benchmark. Instead, they are non-linear on the agents' net trade sets. The reason is that they must internalize the welfare costs arising from the incentive–compatibility constraints, and the welfare cost of incentives are non–linear and may even be non–convex. Note that this feature of our model is perfectly consistent with standard general equilibrium analysis, because prices remain linear in the commodities, the insurance contracts.

This paper complements the work of a companion paper (Jerez [14]), in which we used a similar methodology to study incentive–efficient allocations in economies with adverse selection and moral hazard. We showed that, with adverse selection, the welfare costs arising from the incentive–compatibility constraints are *external*. For instance, providing more insurance to a low–risk consumer generates external incentive costs because it raises the incentives of the high–risk consumers to lie about their type.[5] With moral hazard, the welfare costs arising from the incentive–compatibility constraints are *not external*. For instance, providing more insurance conditional on a high–effort specification generates incentive costs because the consumer that receives the high–effort specification has a higher incentive to shirk. Crucially, how-

[5]The presence of external welfare effects in economies with adverse selection is also discussed by Bisin and Gottardi [4], Greenwald and Stiglitz [9] and Arnott, Greenwald and Stiglitz [1].

ever, the incentives of the other consumers are unaffected. In Jerez [14] we showed that the welfare effects arising from the incentive–compatibility constraints may be non–convex, and so the incentive–efficient allocations may be random. We then used linear programming and duality theory to obtain a complete characterization of the incentive–efficient allocations with adverse selection and with moral hazard.

The paper is organized as follows. In Section 2 we describe the economy. In Section 3 we present the general equilibrium model. In Section 4 we review the dual characterization of the incentive–efficient allocations in Jerez [14]. In Section 5 we define a competitive equilibrium and characterize the competitive equilibrium prices and allocations. In Section 6 derive the two welfare theorems and establish the existence of a competitive equilibrium. In Section 7 we compare our approach with the approach of Prescott and Townsend [20]. Section 8 concludes. The proofs are deferred to the Appendix.

2 The Economy

There is a continuum of identical consumers with measure one and a single consumption good. Consumers are subject to idiosyncratic endowment shocks. Shocks are independent across consumers and render no aggregate uncertainty.[6] Each consumer faces S idiosyncratic states, $s = 1, \ldots, S$. Her endowment of the consumption good in state s is denoted by ω_s, and satisfies $\omega_s \leq \omega_{s'}$ if $s < s'$ (endowments are lower in lower states). Each consumer is moreover endowed with one unit of time that she allocates between leisure activities and effort in preventing the realization of a low state. The effort of the consumer can be either high or low, with the set of effort levels denoted by $E = \{e_L, e_H\}$, where $0 \leq e_L < e_H$. We denote the probability of state s with effort e_i by θ_{is}. We assume that the likelihood ratio $\left\{\frac{\theta_{Hs}}{\theta_{Ls}}\right\}$ increases with the state s. So high effort reduces the probability of ending up in a low state. Consumers have von Neumann–Morgerstern preferences as defined by the utility function $u : E \times \Re_+ \rightarrow \Re$. The utility of consumption c under effort e_i is given by $U_i(c) = u(e_i, c)$, where U_i is

[6] We assume that the law of large numbers holds. In the standard continuum model, where the set of consumers is the unit interval with Lebesgue measure, it not possible to obtain non–trivial processes of i.i.d. random endowments that yield the "exact" law of large numbers (see Judge [15]). Thus, the standard continuum model is not a good approximation of the limit of a sequence of large finite economies with independent shocks across consumers. Sun [23] shows that a limit model can be constructed using hyperfinite models from non–standard analysis. The non–standard model is asymptotically implementable in a setting with a large but finite number of agents so, in Sun's words, it is "elementarily equivalent" to the standard continuum model. For alternative approaches to this problem see Feldman and Gilles [7] and Hammond and Lisboa [11].

assumed twice continuously differentiable, strictly increasing, and strictly concave with $\lim_{c\to 0} U_i'(c) = \infty$ and $\lim_{c\to\infty} U_i'(c) = 0$. Effort is costly, so $U_L(c) - U_H(c) > d$ for all $c \in \Re_+$ and some positive constant d.

There is a finite number of firms which provide insurance services and are large relative to the non–atomic consumers. Each firm insures a positive mass of agents, thus facing no aggregate risk. We assume that insurance claims are perfectly verifiable and fully enforceable. It therefore suffices to consider exclusive contractual relations in which consumers can buy insurance from at most one firm.

The timing of the model is as follows. At some initial date, the insurance market opens and consumers buy insurance from the firms. After the trading period, consumers choose their effort level. Then, endowment shocks are realized. Finally, insurance contracts are enforced, and consumption takes place. There is no ex post trade. The structure of uncertainty is common knowledge and the realization of the endowment shocks is observable. However, effort is private information.

3 The General Equilibrium Model

In this section, we describe the commodity space, the consumption and production sets, and the consumers' utility over consumption bundles. We then define allocations and prices. We begin with some preliminary notation.

Notation

Let Z be the consumer's net trade set, and denote its elements by $z = (z_1, \ldots, z_S)$:

$$Z = \left\{ z \in R^S : z_s \geq -\omega_s,\ s = 1, \ldots, S \right\}.$$

Let $C(Z)$ denote the space of continuous real–valued functions on Z, endowed with the topology of uniform convergence on compact sets. The topological dual of $C(Z)$ is the space of signed Borel measures on Z which are finite on compact sets and have compact support.[7] We denote the dual space by $M_c(Z)$, and let it be endowed with the weak–star topology. Then, $C(Z)$ is also the dual of $M_c(Z)$. The dual pair of spaces $(C(Z), M_c(Z))$ is endowed with the standard bilinear form:

$$\langle f, \mu \rangle \equiv \int_Z f(z) d\mu(z), \quad f \in C(Z),\ \mu \in M_c(Z).$$

Here, the bracket notation highlights the infinite dimensional nature of the spaces in the pairing. We denote the total variation of a measure $\mu \in M_c(Z)$ by $||\mu||$.

[7]See Hewitt [12].

For any $z \in Z$, the expected net trade of a consumer with effort e_i is

$$r_i(z) \equiv \sum_{s=1}^{S} \theta_{is} z_s, \tag{3.1}$$

and her expected utility is

$$EU_i(z) \equiv \sum_{s=1}^{S} \theta_{is} U_i(\omega_s + z_s). \tag{3.2}$$

Hence, $EU_i, r_i \in C(Z)$ for each $i = L, H$.

Commodities

The commodities are insurance contracts, which are signed between a consumer and a firm. An insurance contracts specifies a vector of state–contingent net trades and an effort level for the consumer. Both specifications are allowed to be random and are given as follows.[8] First, the consumer is assigned a lottery which specifies an effort level. After the consumer chooses her effort and conditional on the effort specification received, a second lottery specifies a vector of state–contingent net trades.

We take as the commodity space the product space

$$L = M_c(Z) \times M_c(Z),$$

endowed with the product topology. We describe an insurance contract by a bundle $x = (x_L, x_H) \in L^+$ such that

$$||x_L|| + ||x_H|| = \int_Z dx_L(z) + \int_Z dx_H(z) = 1. \tag{3.3}$$

Here, $||x_i||$ represents the probability that the contract specifies effort e_i, and the equality in (3.3) is an adding–up condition. In addition, the probability measure $\frac{1}{||x_i||} x_i$ represents the random net trade assigned conditional on specification e_i. Note that the uncertainty involved in a contract resolves in two steps. In the first step, consumers may be uncertain about the effort that the contract will specify. This occurs when both $||x_L||$ and $||x_H||$ are positive. In the second step, consumers find out their effort specifications but, in deciding whether to conform or not to such specifications, they may still be uncertain about the net trade that the contract will

[8] It is well known since the seminal work of Prescott and Townsend [20] that lotteries may play a role in the presence of incentive–compatibility constraints. In Jerez [14] we derived conditions under which random effort specifications and random net trades are optimal in this model. See also Bennardo and Chiappori [2].

specify (and thus about their state–contingent consumption plan). This occurs when $\frac{1}{||x_i||}x_i$ is a non–degenerate probability measure.

Remark: We could also take as the commodity space the space of compactly supported measures over pairs of effort and net trade, $M_c(E \times Z)$. That is, we could define a contract as a probability measure on $E \times Z$. Since the set of effort levels E has two elements, the two definitions of the commodity space are equivalent. Our choice of the commodity space has the advantage that it directly implies that incentive–efficient allocations are the optimal solutions to a linear programming problem (see Section 4). Our choice of the commodity space is also equivalent to the one of Prescott and Townsend [20], who define the commodity space to be the space of measures over triples of effort, consumption and endowment. The difference with respect to Prescott and Townsend [20] is that they assume that the full–information consumption set, and thus net trade set Z, is a finite set. With this assumption, the commodity space is finite dimensional since it is isomorphic to the Euclidean space. We consider the general case in which the net trade set need not be a finite set.[9]

Consumption Sets

The consumption set X is the set of insurance contracts:

$$X = \left\{ (x_L, x_H) \in L^+ : \quad ||x_L|| + ||x_H|| = 1 \right\}. \tag{3.4}$$

The exclusivity assumption implies that consumers can sign at most one contract. Note that the consumer can always choose to be uninsured with $z = 0$ and exert any effort level e_i. In this case, $x_i = \delta_0$ and $x_j = 0$ for $j \neq i$ (with δ_0 denoting the Dirac measure at $z = 0$).

Preferences

We now define the consumers' expected utility over insurance contracts. Remember that the expected utility of a consumer with effort e_i and net trade z is $EU_i(z)$, as defined in equation (3.2). Therefore, the expected utility from contract x is[10]

$$\langle EU, x \rangle = \langle EU_L, x_L \rangle + \langle EU_H, x_H \rangle = \int_Z EU_L(z) dx_L(z) + \int_Z EU_H(z) dx_H(z). \tag{3.5}$$

Since contracts are random, the consumer's expected utility is linear in the contracts.

[9]See also Kehoe, Levine and Prescott [16].

[10]Here, $EU = (EU_L, EU_H) \in C(Z) \times C(Z)$.

Production Sets

Each firm supplies a single insurance contract.[11] We describe a production plan by a bundle $y = (y_L, y_H) \in L^+$. Here, (i) the probability measure $\frac{1}{||y||}y$ describes the contract supplied by the firm, and (ii) $||y||$ is the total mass of contracts supplied. The law of large numbers implies that, when the firm supplies a contract to a positive mass of customers, it faces no uncertainty. We assume that the firm assigns random contracts across customers in order to preserve this lack of uncertainty. Then $||y_i||$ represents the ex post mass of customers who are specified effort e_i, and $\frac{1}{||y_i||}y_i$ represents the net trade distribution of these customers once the outcomes of their individual random contracts are realized.

Remember that the expected net trade of a customer with effort e_i and net trade z is $r_i(z)$, as defined in equation (3.1). The net transfer of resources that the firm makes to its customers under production plan y is then

$$\langle r_L, y_L\rangle + \langle r_H, y_H\rangle = \int_Z r_L(z)dy_L(z) + \int_Z r_H(z)dy_H(z). \tag{3.6}$$

A production plan y is technologically feasible if the net transfer of resources that the firm makes to its customers is non–positive:

$$\langle r_L, y_L\rangle + \langle r_H, y_H\rangle \leq 0. \tag{3.7}$$

Since the firm cannot observe the effort choice of its customers, the firm also faces incentive–compatibility constraints. Under production plan y, the utility of a customer who is specified effort e_i and chooses effort e_j is

$$\langle EU_j, \frac{y_i}{||y_i||}\rangle = \frac{1}{||y_i||}\int_Z EU_j(z)dy_i(z). \tag{3.8}$$

A production plan y is incentive compatible if it is not in the interest of the customers to deviate from their effort specifications:

$$\langle EU_i, y_i\rangle \;\geq\; \langle EU_j, y_i\rangle, \quad j \neq i, \quad i,j = L, H. \tag{3.9}$$

The production set Y is the set of production plans satisfying the technological constraint and the incentive–compatibility constraints:

$$\begin{aligned} Y \;=\; \Big\{ & (y_L, y_H) \in L^+ : \langle r_L, y_L\rangle + \langle r_H, y_H\rangle \leq 0, \\ & \langle EU_i - EU_j, y_i\rangle \geq 0,\; j \neq i,\; i,j = L, H\Big\}. \end{aligned} \tag{3.10}$$

[11] Since consumers are ex ante identical, we shall restrict our attention to symmetric allocations.

Since all the constraints are linear, the production set Y displays constant returns to scale and is convex (i.e., Y is a convex cone). Note that $0 \in Y$, so the firm can choose to be inactive.

Allocations, Feasible Allocations and Incentive Efficient Allocations

Since the production set displays constant returns to scale, we assume that there is a single firm in the economy. A symmetric allocation is a consumption bundle for the consumers and a production plan for the firm; i.e., a pair $(x, y) \in L^2$. An allocation (x, y) is *feasible* if it satisfies:

(a) $x \in X$ and $y \in Y$, and

(b) $y = x$.

Condition (a) requires that the allocation be individually feasible. Condition (b) requires that the allocation be feasible in the aggregate. That is, it requires that the insurance contract demanded by consumers coincides with the contract supplied by the firm, and that the mass of contracts supplied by the firm is equal to the total mass of consumers in the economy.

An feasible allocation (x, y) is *incentive efficient* if there is no other feasible allocation (x', y') that implies higher expected utility for the consumers, so $\langle EU, x'\rangle > \langle EU, x\rangle$.

Prices

The price space P is set of continuous linear functionals on the commodity space (the *dual* space):

$$P \equiv L^* = C(Z) \times C(Z),$$

and is endowed with the product topology. A price system is then a pair of continuous functions on Z, and is denoted by $p = (p_L, p_H)$. For a given $p \in P$, the value of a commodity bundle $x \in L^+$ is given by the inner product:

$$\langle p, x\rangle = \langle p_L, x_L\rangle + \langle p_H, x_H\rangle = \int_Z p_L(z)dx_L(z) + \int_Z p_H(z)dx_H(z). \tag{3.11}$$

For instance, the price of a deterministic insurance contract which specifies effort e_i and net trade z is $p_i(z)$.[12] That is, prices depend both on the effort and the net trade specified by the contract. On the other hand, a random contract specifies different

[12]Denote the Dirac measure at z by δ_z. The contract is given by $x_i = \delta_z$ and $x_j = 0$. Its price is: $\langle p, x\rangle = \langle p_i, x_i\rangle = \langle p_i, \delta_z\rangle = p_i(z)$.

pairs of effort and net trade with positive probability. Equation (3.11) says that the price of a random contract is calculated by adding the values of each individual component using the corresponding probability weights (i.e. integrating $p_i(z)$ over z with respect to the measure x_i for each e_i, and summing over e_i).[13]

Moral Hazard versus Full Information

There are some key differences between the moral hazard model and the full–information model of Arrow and Debreu. In the Arrow–Debreu model, the commodities are state–contingent consumption goods. The environment we study features a single consumption good and a finite number S of idiosyncratic states. Hence, the commodity space and the price space under full information are given by the Euclidean space. With moral hazard, the commodities are insurance contracts. Insurance contracts are rather different from state–contingent consumption goods. In the spirit of the Arrow–Debreu model, an insurance contract specifies a state–contingent consumption plan for the consumer (conditional on certain effort specifications). Crucially, however, an insurance contract is a "package" which is indivisible. That is, agents cannot separately buy (sell) the components of the contract. For instance, consumers cannot separately buy units of the consumption good in state 1. Neither can agents buy (sell) a fraction of the contract. For instance, consumers cannot buy half of a contract. They can buy a contract or they can buy none. Each insurance contract is then effectively a different indivisible commodity. That is, the moral hazard model features a continuum of indivisible commodities. As a result, the commodity space and the price space under moral hazard are infinite dimensional vector spaces.

Since two different insurance contracts are two different commodities the prices of these contracts need not be related. For instance, take two deterministic contracts, x_1 and x_2, which prescribe the same effort level e_i and assign net trades z and tz (for some $t > 0$ with $t \neq 1$). Their respective prices are $p_i(z)$ and $p_i(tz)$. These prices need not be related. In particular, the price of x_2 need not be t times the price of x_1, so $p_i(tz) = tp_i(z)$. In other words, *the function p_i need not be linear.* Clearly, the set of price systems which are linear on Z is only a subset of the price space P. Note that *this feature of our model is perfectly consistent with standard general equilibrium analysis, because prices are linear in the commodities, the insurance contracts.*[14] The crucial departure from the Arrow–Debreu model is that, with exclusive contracts, the commodity space under moral hazard (and thus the price space) is a space of higher

[13] See also Prescott and Townsend [20].

[14] Prices are (i) additive: $\langle p, x_1 + x_2 \rangle = \langle p, x_1 \rangle + \langle p, x_2 \rangle$ for all $x_1, x_2 \in L^+$; and (ii) homogeneous: $\langle p, tx \rangle = t\langle p, x \rangle$ for all $x \in L^+$ and all $t \in \Re_+$.

dimension than under full information.[15]

4 A Dual Characterization of Incentive Efficiency

In this section, we show that incentive–efficient allocations are the optimal solutions to a linear programming problem.

The problem of the planner is to choose a feasible allocation in order to maximize the expected utility of the consumers. The aggregate feasibility constraint $y = x$ can be substituted into the firm's individual feasibility constraint $y \in Y$. The inner product notation $\langle \cdot , \cdot \rangle$ can be extended to the adding–up constraint in the definition of the consumption set X. To this purpose we denote the characteristic function on Z by $\mathcal{I} : Z \to \{0, 1\}$ and write $||x_i|| = \langle \mathcal{I}, x_i \rangle$ for $i = L, H$. The problem of the planner is to choose $(x_L, x_H) \in M_c(Z) \times M_c(Z)$ to solve

$$(D) \qquad \sup \quad \langle EU_L, x_L \rangle + \langle EU_H, x_H \rangle$$

s.t.

$$\begin{aligned} \langle \mathcal{I}, x_L \rangle + \langle \mathcal{I}, x_H \rangle &= 1, & (4.12) \\ -\langle EU_L, x_L \rangle + \langle EU_H, x_L \rangle &\leq 0, & (4.13) \\ \langle EU_L, x_H \rangle - \langle EU_H, x_H \rangle &\leq 0, & (4.14) \\ \langle r_L, x_L \rangle + \langle r_H, x_H \rangle &\leq 0, & (4.15) \\ x_L, x_H &\geq 0. & (4.16) \end{aligned}$$

That is, the problem of the planner is to choose an insurance contract that is technologically feasible and incentive compatible and maximizes the expected utility of the consumers. Problem (D) is a linear programming problem. Standard results in linear programming theory show that problem (D) is *dual* to another linear programming problem, known as the *primal* problem or problem (P). Whereas problem (D) is a maximization problem which is posed in an infinite dimensional space and has a finite number of constraints, problem (P) is a minimization problem which has a finite number of variables and an infinite number of constraints. In optimization theory, these kind of problems are known as Linear Semi–Infinite Programming (LSIP)

[15] In Jerez [14] we argue that this is a key feature of general equilibrium models with asymmetric information and exclusive contracts. Models with asymmetric information and non–exclusive do not have this feature (Bisin and Gottardi [3]). With non–exclusive contracts, the commodity space is the same both under full and under asymmetric information.

problems.[16] The primal and dual problems are related because the primal variables are also the shadow prices of the dual constraints, and vice versa.

Problem (P), which is derived in detail in the Appendix, consists of finding a quadruple $(\alpha, \beta_L, \beta_H, q) \in \mathbf{R}^4$ to solve

$$(P) \qquad \inf \quad \alpha$$

s.t.

$$\alpha \geq EU_L(z) - qr_L(z) - \beta_L[EU_H(z) - EU_L(z)] \quad \forall z \in Z, \tag{4.17}$$

$$\alpha \geq EU_H(z) - qr_H(z) - \beta_H[EU_L(z) - EU_H(z)] \quad \forall z \in Z, \tag{4.18}$$

$$\beta_L, \beta_H, q \geq 0, \tag{4.19}$$

where α is the shadow price of the adding–up constraint (4.12), β_L and β_H are the shadow prices of the incentive–compatibility constraints (4.13) and (4.14), and q is the shadow price of the technological constraint (4.15).

In Jerez [14] we have shown that problems (P) and (D) have optimal solutions, and that their optimal values coincide.[17] We have also shown that the space of dual variables can be restricted without loss of generality to measures with finite support. Let M_F denote the set of finitely supported measures on Z. Consider the dual problem when (x_L, x_H) is restricted to lie in the space $M_F \times M_F$, and denote the restricted problem by (D_F). The optimal values of problems (D) and (D_F) coincide.[18]

First–best Allocations vs. Incentive–Efficient Allocations

We now proceed to characterize the incentive–efficient allocations. Consider for a moment the case of full information. With full information, there are no incentive–compatibility constraints in the planner's problem. Since uncertainty is purely idiosyncratic, it is optimal that all consumers be fully insured and consume their expected endowment. That is, the first–best contract implies full insurance and is actuarially fair. If the disutility of the high effort is not too large relative to the low effort (or if the expected endowment under high effort is sufficiently larger than under low effort),

[16]An LSIP problem is an optimization problem with linear objective and linear constraints in which either number of variables or the number of constraints is finite.

[17]Unlike an ordinary linear programming problem, an LSIP problem need not have optimal solutions when its feasible set is non–empty. Neither need the primal and dual LSIP problems have the same optimal value, as a "positive duality gap" may occur. See Goberna and López [8].

[18]Carathèodory's Theorem implies that it is enough to consider pairs (x_L, x_H) such that the union of the supports of x_L and x_H has at most $n+1$ elements, with n denoting the number of binding constraints in problem (D). As we argue latter, only constraints (4.12) and (4.15) bind in problem (D), so $n = 2$.

it is optimal that all consumers provide the high effort. The problem arises because, when effort is private information, a consumer who is fully insured will shirk when high effort is specified. Allocations which specify high effort with positive probability can then only provide partial insurance.

We characterize the incentive–efficient allocations by appealing to the complementary slackness theorem of linear programming (see Krabs [17, Theorem I.3.3]).

Theorem 4.1 *(Complementary Slackness)*
Feasible solutions (x_L, x_H) *and* $(\alpha, \beta_L, \beta_H, q)$ *for problems* (D) *and* (P)*, respectively, are optimal if and only if they satisfy the complementary slackness conditions:*

$$q(\langle r_L, x_L\rangle + \langle r_H, x_H\rangle) = 0, \tag{4.20}$$

$$\beta_L(\langle EU_H, x_L\rangle - \langle EU_L, x_L\rangle) = 0, \tag{4.21}$$

$$\beta_H(\langle EU_L, x_H\rangle - \langle EU_H, x_H\rangle) = 0, \tag{4.22}$$

$$\alpha = EU_L(z) - qr_L(z) - \beta_L[EU_H(z) - EU_L(z)] \quad \textit{if} \;\; x_L(z) > 0, \tag{4.23}$$

$$\alpha = EU_H(z) - qr_H(z) - \beta_H[EU_L(z) - EU_H(z)] \quad \textit{if} \;\; x_H(z) > 0. \tag{4.24}$$

Condition (4.20) states that the optimal shadow price q is a complementary multiplier for the technological constraint (4.15). Since the monotonicity of preferences implies that q is positive, (4.20) implies that the aggregate net trade is zero and thus that the incentive–efficient contract is actuarially fair. Conditions (4.21) and (4.22) state that the optimal shadow prices β_L and β_H are complementary multipliers for the incentive–compatibility constraints (4.13) and (4.14), respectively. Since the first–best contract is not incentive compatible, the incentive–compatibility constraint (4.14) binds with $\beta_H > 0$. Hence, condition (4.22) implies that when e_H is specified the agent is indifferent between exerting effort and shirking. Since implementing a low–effort specification is trivial, the incentive–compatibility constraint (4.13) does not bind and $\beta_L = 0$. Finally, conditions (4.23)–(4.24) state that the optimal measures x_L and x_H are complementary multiplier vectors for the primal constraint systems (4.17) and (4.18), respectively. To interpret these conditions we need to take a closer look at the primal constraint systems.

For a given net trade $z \in Z$, the expression on the righthand side of (4.17),

$$EU_L(z) - qr_L(z), \tag{4.25}$$

is the difference between consumers' expected utility and the value of their aggregate net trade when their effort is low. Therefore, this expression the represents the *net* *contribution to social welfare when consumers are specified effort* e_L *and net trade* z. Similarly, the expression on the righthand side of (4.18),

$$EU_H(z_H) - qr_H(z_H) - \beta_H[EU_L(z_H) - EU_H(z_H)], \tag{4.26}$$

represents the *net contribution to social welfare when consumers are specified effort* e_H *and net trade* z. In addition to the consumers' expected utility and the value of their aggregate net trade, an additional welfare effect arises when the high effort is specified. This welfare effect is associated with the incentives of the consumers to conform to the high–effort specification. If the net trade z is such that consumers prefer to shirk, the welfare effect is *negative* and is proportional to the utility gain from shirking. If the net trade z is such that consumers prefer not to shirk, the welfare effect is *positive* and is proportional to the utility loss from shirking. If consumers are indifferent between exerting effort and shirking, there is no welfare effect associated with the consumer's incentives.

The primal constraint systems (4.17) and (4.18) imply that the net contribution to social welfare for any effort e_i and any net trade z is bounded above by α. On the other hand, the complementary slackness conditions (4.23)–(4.24) state that contract (x_L, x_H) puts all the probability weight on pairs of effort and net trade for which the net contribution to social welfare is equal to α. It thus follows that (x_L, x_H) *puts all the probability weight on pairs of effort and net trade that maximize the net contribution to social welfare*. The optimal shadow price α then measures the maximal net contribution to social welfare.[19]

The complementary slackness conditions (4.20)–(4.24) allow to derive the properties of the incentive–efficient allocations. It is easy to verify that the net contribution to social welfare with low effort (4.25) is a strictly concave function of z and it is maximized when z provides full insurance. Hence, if $||x_L|| > 0$ then x_L is degenerate and provides full insurance (random net trade assignments are never optimal conditional on a low–effort specification). The net contribution to social welfare with high effort (4.26) is *not* a strictly concave function, and may have more than one maximum. Hence, x_H may be a non–degenerate measure (random net trade assignments may be optimal conditional on a high–effort specification). The planner can use random net trade assignments to exploit differences in preferences for risk with high and low effort. If risk aversion decreases fast enough with the level of effort,

[19] Our notion of the maximal net contribution to social welfare for the moral–hazard economy is the parallel of Makowski and Ostroy's [18] notion of the conjugate or indirect utility for economies with full information. These authors have shown how the fact that the constraints of the primal program (the "pricing problem" in their terminology) can be incorporated into the objective function is characteristic of the LP version of General Equilibrium.

random net trade assignments make the deviation to low effort more costly (i.e., they reduce the negative welfare effect of the assignment). In Jerez [14] we have shown that, if utility is separable in consumption and effort, or if the coefficient of absolute risk aversion does not increase with effort, the net contribution to social welfare with high effort (4.26) is a strictly concave function. In this case, x_H is degenerate and provides partial insurance.

Random effort can also be optimal. This is the case when the maximal value of the net contribution to social welfare with high and low effort is the same. In Jerez [14] we have shown that random effort is optimal if the consumers' expected endowment is large enough, or if the disutility of effort increases fast enough with consumption. In these instances, consumers are willing to give up some consumption to reduce their effort. The tradeoff between consumption and effort is resolved by allowing the consumers to provide low effort with some positive probability at the cost of reducing their expected consumption.[20]

5 Competitive equilibrium

In this section, we define a competitive equilibrium. We then use linear programming techniques to characterize the competitive equilibrium prices and allocations.

Competitive equilibrium

A competitive equilibrium is defined in the standard way.

Definition 5.1 *A competitive equilibrium is an allocation* $(x^*, y^*) \in L^2$ *and a price system* $p^* \in L^*$ *such that:*

(i) Consumers maximize their expected utility subject to their budget constraint:

$$\begin{array}{rcl} \langle EU, x^* \rangle & = & \sup_{x \in X} \langle EU, x \rangle \\ \text{s.t.} & & \langle p^*, x \rangle \leq 0. \end{array}$$

(ii) The firm maximizes profits in the production set:

$$\langle p^*, y^* \rangle = \sup_{y \in Y} \langle p^*, y \rangle.$$

(iii) Markets clear: $x^* = y^*$.

[20]See also Bennardo and Chiappori [2].

In order to characterize the competitive equilibrium prices and allocations, we analyze the optimal decisions of the firm and the consumers. We then relate these optimal decisions through the market clearing condition.

Optimal production plans

The firm chooses $y = (y_L, y_H) \in L$ to solve the following linear programming problem:

(D_f) $\quad \sup \quad \langle p_L, y_L\rangle + \langle p_H, y_H\rangle$

s.t.

$$-\langle EU_L, y_L\rangle + \langle EU_H, y_L\rangle \le 0, \tag{5.27}$$
$$\langle EU_L, y_H\rangle - \langle EU_H, y_H\rangle \le 0, \tag{5.28}$$
$$\langle r_L, y_L\rangle + \langle r_H, y_H\rangle \le 0, \tag{5.29}$$
$$y_L, y_H \ge 0. \tag{5.30}$$

Problem (D_f) is the dual of another linear programming problem. The primal problem (P_f) consists of finding a triple $(\beta_L^f, \beta_H^f, q^f) \in \mathbf{R}^3$ to solve

(P_f) $\quad \inf \quad 0$

s.t.

$$0 \ge p_L(z) - \beta_L^f(EU_H(z) - EU_L(z)) - q^f r_L(z) \quad \forall z \in Z, \tag{5.31}$$
$$0 \ge p_H(z) - \beta_H^f(EU_L(z) - EU_H(z)) - q^f r_H(z) \quad \forall z \in Z, \tag{5.32}$$
$$\beta_H^f, \beta_L^f, q^f \ge 0, \tag{5.33}$$

where (β_L^f, β_H^f) and q^f denote the shadow prices of the incentive–compatibility constraints (5.27)–(5.28) and the technological constraint (5.29), respectively.

The fact that Y is a cone and $0 \in Y$ directly implies that an optimal production plan yields zero profits.

Lemma 5.1 *Let y be an optimal solution for problem (D_f) then*

$$\langle p, y\rangle = \langle p_L, y_L\rangle + \langle p_H, y_H\rangle = 0.$$

Therefore, if an optimal solution (D_f) for problem exist, the optimal value of problem (D_f) is zero. Any feasible solution for problem (P_f) is optimal by definition since the value of problem (P_f) is always zero. Therefore, if a feasible solution for problem (P_f) exists, the optimal value of problem (P_f) is also zero.[21]

According to the complementary slackness theorem, feasible solutions (y_L, y_H) and $(\beta_L^f, \beta_H^f, q^f)$ for problems (D_f) and (P_f), respectively, are optimal if and only if they satisfy the complementary slackness conditions:

$$q^f(\langle r_L, y_L\rangle + \langle r_H, y_H\rangle) = 0, \tag{5.34}$$

$$\beta_L^f(\langle EU_H, y_L\rangle - \langle EU_L, y_L\rangle) = 0, \tag{5.35}$$

$$\beta_H^f(\langle EU_L, y_H\rangle - \langle EU_H, y_H\rangle) = 0, \tag{5.36}$$

$$p_L(z) = \beta_L^f(EU_H(z) - EU_L(z)) + q^f r_L(z) \quad \text{if } y_L(z) > 0, \tag{5.37}$$

$$p_H(z) = \beta_H^f(EU_L(z) - EU_H(z)) + q^f r_H(z) \quad \text{if } y_H(z) > 0. \tag{5.38}$$

Conditions (5.34)–(5.36) state that the optimal shadow prices q^f and (β_L^f, β_H^f) are complementary multipliers for the technological constraint (5.29) and the incentive–compatibility constraints (5.27)–(5.28), respectively. Conditions (5.37) and (5.38) state that the optimal measures y_L and y_H are complementary multiplier vectors for the primal constraint systems (5.31) and (5.32), respectively.

The expressions on the righthand side of (5.31) and (5.32),

$$p_i(z) - q^f r_i(z) - \beta_i^f(EU_j(z) - EU_i(z)), \quad j \neq i, \quad i, j = L, H. \tag{5.39}$$

represent the *average producer surplus* from a deterministic contract that specifies effort e_i and net trade z. Suppose the contract specifies high effort. The price of the contract is $p_H(z)$. The shadow cost of the contract is

$$q^f r_H(z) + \beta_H^f(EU_L(z) - EU_H(z)). \tag{5.40}$$

The first term in (5.40) is an economic cost. Specifically, $r_H(z)$ is the average amount of the consumption good that the firm transfers to its customers under the contract, and $qr_H(z)$ is the shadow value of the transfer. The second term in (5.40) is an incentive cost (benefit). If the net trade z is such that the customers prefer shirk, the term reflects an incentive cost which is proportional to the utility gain from shirking. If the net trade z is such that the customers prefer not to shirk, the term reflects an incentive benefit which is proportional to the utility loss from shirking. If the customers are indifferent between conforming to the specification and shirking, the

[21]If no optimal solution for problem (D_f) exists, the convention is to set the value of problem (D_f) equal to $-\infty$. Likewise, if no feasible solution for problem (P_f) exists, the value of problem (P_f) is equal to $+\infty$. See Krabs [17].

term is zero (so there is no incentive cost or benefit). A similar interpretation applies to deterministic contracts that specify low effort.[22]

The primal constraint systems (5.31) and (5.32) imply that the average producer surplus from any deterministic contract is bounded above by zero. The average producer surplus from a random contract may be calculated from expression (5.39) by integrating, and it is also bounded above by zero.[23] On the other hand, the complementary slackness conditions (5.37) and (5.38) state that the optimal production plan (y_L, y_H) puts all the weight on pairs of effort and net trade for which the average producer surplus is equal to zero. It thus follows that the optimal production plan (y_L, y_H) *puts all the weight on pairs of effort and net trade that maximize the average producer surplus.* The maximal average producer surplus is zero.

We define the maximal average producer surplus:

$$\pi(p, \beta_L^f, \beta_H^f, q^f) = \sup_{(e_i,z)\in E\times Z} \Big\{ p_i(z) - q^f r_i(z) - \beta_i^f (EU_j(z) - EU_i(z)) \Big\} \tag{5.41}$$

The complementary slackness conditions (5.37) and (5.38) can then be restated as

$$0 = \pi(p, \beta_L^f, \beta_H^f, q^f) = p_i(z) - q^f r_i(z) - \beta_i^f (EU_j(z) - EU_i(z)) \text{ if } y_i(z) > 0,\ i = L, H. \tag{5.42}$$

Remark: The complementary slackness conditions (5.34)–(5.38)) that characterize an optimal production plan can be derived using standard Lagrangian analysis. The Lagrangian function associated with problem (D_f) is

$$\begin{aligned} \mathcal{L}(y, p, \beta_L^f, \beta_H^f, q^f) &= \sum_{i=L,H} \langle p_i, y_i \rangle - q^f \sum_{i=L,H} \langle r_i, y_i \rangle - \sum_{i=L,H;\, j\neq i} \beta_i^f \langle EU_j - EU_i, y_i \rangle \\ &= \sum_{i=L,H;\, j\neq i} \int_Z \Big(p_i(z) - q^f r_i(z) - \beta_i^f (EU_j(z) - EU_i(z)) \Big)\, dy_i(z). \end{aligned} \tag{5.43}$$

The Lagrangian function (5.43) represents the total producer surplus under production plan y.

[22]The first welfare theorem (see Section 6) implies that the incentive–compatibility constraint with low effort will not bind in a competitive equilibrium. In equilibrium, the shadow cost of a low–effort contract will then be the standard economic cost: $q^f r_L(z)$. On the other hand, the incentive–compatibility constraint with high effort will bind with $\beta_H^f > 0$. Hence, the shadow cost a high–effort contract, as specified in (5.40), will not be a convex function of z.

[23]The average producer surplus from a random contract (x_L, x_H) is

$$\sum_{i\in\{L,H\}} \int_Z \Big(p_i(z) - q^f r_i(z) - \beta_i^f (EU_j(z) - EU_i(z) \Big)\, dx_i.$$

A feasible production plan y is optimal if and only if there exists there exists a triple $(\beta_L^f, \beta_H^f, q^f) \in \mathbf{R}^3_+$ such that β_L^f, β_H^f and q^f are respective complementary multipliers for constraints (5.27), (5.28) and (5.29), and y maximizes the Lagrangian function on L. Note that the Lagrangian function in (5.43) is maximized when y puts all the weight on pairs of effort and net trade that maximize the function inside the integral, and thus attain the maximal average producer surplus.

Optimal consumption plans

The consumer chooses $x = (x_L, x_H) \in L$ to solve the following linear programming problem:

(D_c) $\quad \sup \quad \langle EU_L, x_L\rangle + \langle EU_H, x_H\rangle$

s.t.

$$\begin{aligned} \langle \mathcal{I}, x_L\rangle + \langle \mathcal{I}, x_H\rangle &= 1, & (5.44)\\ \langle p_L, x_L\rangle + \langle p_H, x_L\rangle &\leq 0, & (5.45)\\ x_L, x_H &\geq 0. & (5.46)\end{aligned}$$

The budget constraint (5.45) says that the value the contract chosen by the consumer must be non–positive. Since contracts are lotteries over pairs of effort and net trades, constraint (5.45) is analogous to the full–information budget constraint, according to which the value of the consumer's net trade must be non–positive.

The primal problem (P_c) consists of finding a pair $(\alpha^c, \lambda) \in \mathbf{R}^2$ to solve

(P_c) $\quad \inf \quad \alpha^c$

s.t.

$$\begin{aligned} \alpha^c &\geq EU_L(z) - \lambda p_L(z) \quad \forall z \in Z, & (5.47)\\ \alpha^c &\geq EU_H(z) - \lambda p_H(z) \quad \forall z \in Z, & (5.48)\\ \lambda &\geq 0, & (5.49)\end{aligned}$$

where α^c and λ are the shadow prices of the adding–up constraint (5.44) and the budget constraint (5.45).

Throughout the section we assume that optimal solutions for problems (D_c) and (P_c) exist and that the optimal values of these problems are identical. An analogous argument to the one used in Jerez [14] implies that problems (D_c) and (P_c) have these properties if the price system p satisfies certain conditions. The competitive equilibrium price system derived at the end of this section satisfies these conditions.

By the complementary slackness theorem, feasible solutions (x_L, x_H) and (α^c, λ) for problems (D_c) and (P_c), respectively, are optimal if and only if they satisfy the complementary slackness conditions:

$$\lambda(\langle p_L, x_L\rangle + \langle p_H, x_H\rangle) = 0, \tag{5.50}$$

$$\alpha^c = EU_L(z) - \lambda p_L(z) \quad \text{if } x_L(z) > 0, \tag{5.51}$$

$$\alpha^c = EU_H(z) - \lambda p_H(z) \quad \text{if } x_H(z) > 0. \tag{5.52}$$

Condition (5.50) states that the optimal shadow price λ is a complementary multiplier for the budget constraint (5.45). The monotonicity of preferences implies that λ is positive, so the budget constraint holds with strict equality.[24] Conditions (5.51) and (5.52) state that the optimal measures x_L and x_H are complementary multiplier vectors for the primal constraint systems (5.47) and (5.48). The expression on the righthand side of (5.47) and (4.18),

$$EU_i\,(z) - \lambda p_i\,(z)\,, \tag{5.53}$$

represents the *expected consumer surplus* from a deterministic contract that specifies effort e_i and net trade z. The primal constraint systems (5.47) and (5.48) imply that the expected consumer surplus from any deterministic contract is bounded above by α_c. The expected consumer surplus from a random contract may be calculated from expression (5.39) by integrating, and it is also bounded above by α_c. The complementary slackness conditions (5.51) and (5.52) state that the consumption plan (x_L, x_H) puts all the probability weight on pairs of effort and net trade for which the expected consumer surplus is equal to α_c. It thus follows that an optimal consumption plan (x_L, x_H) *puts all the probability weight on pairs of effort and net trade that attain the maximal expected consumer surplus.* This maximal surplus is equal to α_c.

We define the maximal expected consumer surplus:

$$v(p, \lambda) = \sup_{(e_i, z) \in E \times Z} \left\{EU_i(z) - \lambda p_i(z)\right\}. \tag{5.54}$$

The complementary slackness conditions (5.51) and (5.52) can then be expressed as

$$\alpha_c = v(p, \lambda) = EU_i(z) - \lambda p_i(z) \text{ if } x_i(z) > 0,\ i = L, H. \tag{5.55}$$

[24]Suppose λ were zero. If U_i is unbounded on Z for some $i \in \{L, H\}$, the righthand side of the corresponding primal constraint system is unbounded on Z, so α_c cannot be finite. But then problems (D_c) and (P_c) cannot have the same optimal value (since the value of problem (D_c) is finite). If U_i is bounded for $i = L, H$, the corresponding primal constraint system cannot hold with strict equality for any $z \in Z$, so the support of x_L and x_H is empty (remember that U_i is strictly increasing). But then problem (D_c) cannot have an optimal solution.

Since the values of problems (D_c) and (P_c) are identical, the consumer's indirect utility is equal to the maximal expected consumer surplus α_c.

Remark: The complementary slackness conditions (5.50)–(5.52) can also be derived using Lagrangian analysis. The Lagrangian function associated with problem (D_c) is

$$\begin{aligned}\mathcal{L}(x,\lambda) &= \sum_{i=L,H} \langle EU_i, x_i\rangle - \lambda \sum_{i=L,H} \langle p_i, x_i\rangle + \alpha_c(1 - \sum_{i=L,H} \langle \mathcal{I}, x_i\rangle) \\ &= \sum_{i=L,H} \int_Z (EU_i(z) - \lambda p_i(z))\, dx_i(z) + \alpha_c \left(1 - \sum_{i=L,H} \int_Z dx_i(z)\right) \quad (5.56)\end{aligned}$$

Any contract $x \in X$ satisfies the adding–up condition, so the last term in (5.56) always vanishes. For any $x \in X$, the Lagrangian function (5.56) represents the expected consumer surplus under contract x.

A budget feasible contract x is optimal if and only if there exists $\lambda \in \mathbf{R}_+$ such that λ is a complementary multiplier for the budget constraint (5.45), and x maximizes the Lagrangian function on L. Note that the Lagrangian function in (5.56) is maximized when x puts all the probability weight on pairs of effort and net trade that maximize the expected consumer surplus.

Competitive Equilibrium Prices and Allocations

The competitive equilibrium prices and allocations can be characterized by combining the complementary slackness conditions for the problems of the firm and the consumer, (5.34)–(5.38) and (5.50)–(5.52), and the market clearing condition.

The complementary slackness conditions for the firm's problem imply that the price of the contract offered by the firm is equal to the shadow cost of the contract. This result is analogous to the standard constant–returns condition that the price of a good is equal to its marginal cost of production. Let x^* be the contract traded in a competitive equilibrium. The complementary slackness conditions (5.37) and (5.38) for the firm's problem together with the market clearing condition imply that, for any pair (e_i, z) specified with positive probability by contract x^*,

$$p_i^*(z) = q^{f*} r_i(z) + \beta_i^{f*}(EU_j(z) - EU_i(z)). \quad (5.57)$$

Here, q^{f*}, β_L^{f*} and β_L^{f*} are the optimal shadow prices in the firm's problem in a competitive equilibrium. Hence, the price of contract x^* is

$$\langle p^*, x^*\rangle = \langle p_L^*, x_L^*\rangle + \langle p_H^*, x_H^*\rangle, \quad (5.58)$$

where

$$\langle p_i^*, x_i^*\rangle = \langle q^{f*} r_i - \beta_i^{f*}(EU_j - EU_i), x_i^*\rangle. \quad (5.59)$$

That is, the price of the contract traded in a competitive equilibrium is equal to its shadow cost. Equation (5.59), combined with the complementary slackness conditions (5.34)–(5.36) for the firm's problem and the market clearing condition, moreover implies that

$$\langle p^*, x^* \rangle = 0. \tag{5.60}$$

That is, the price of the contract traded in a competitive equilibrium is equal to zero (i.e., the value of the expected net trade implied by the contract is zero).[25]

Unlike the standard Arrow–Debreu model, the competitive equilibrium price system is not fully determined under moral hazard. This is a standard feature of models with a continuum of commodities, where the prices of commodities that are not traded in equilibrium are indeterminate.[26] In these infinite dimensional models, there are many price systems that support a competitive equilibrium allocation. We have already noted that, for any pair $(e_i, z) \in E \times Z$:

$$p_i^*(z) \leq q^{f*} r_i(z) + \beta_i^{f*}(EU_j(z) - EU_i(z)). \tag{5.61}$$

That is, the producer surplus is non–positive. The price of a contract x that is not traded in a competitive equilibrium must satisfy

$$\langle p^*, x \rangle = \langle p_L^*, x_L \rangle + \langle p_H^*, x_H \rangle, \tag{5.62}$$

where

$$\langle p_i^*, x_i \rangle \leq \langle q^{f*} r_i - \beta_i^{f*}(EU_j - EU_i), x_i \rangle. \tag{5.63}$$

The price of contracts that are not traded in a competitive equilibrium are then *lower* that the shadow cost of these contracts. If this was not the case, the firm could make infinite profits by supplying an infinite amount of one of these contracts.

A competitive equilibrium price system may be selected by taking the supremum over the set of prices $p \in P$ that satisfy conditions (5.57) and (5.61). The selected price system is[27]

$$p_i^*(z) = q^{f*} r_i(z) + \beta_{i*}^{f}(EU_j(z) - EU_i(z)) \quad \forall z \in Z,\, i = L, H.$$

Under this price selection criterion, the price of a contract (whether traded or not) is equal to its shadow cost. This ensures that no small perturbation of an optimal production plan yields negative profits to the firm.

[25]Note that this result also follows from the zero profit result in Lemma 5.1 and the market clearing condition.

[26]See Mas–Colell and Zame [19].

[27]Under this price selection criterion, the equilibrium prices lie in a subset of the price space P that is isomorphic to $\Re^3$. An equilibrium price system is then fully characterized by a triple $(\beta_L^{f*}, \beta_H^{f*}, q^{f*}) \in \Re_+^3$.

Crucially, in addition to the competitive equilibrium prices, we may characterize a competitive equilibrium allocation. The characterization is obtained by combining the complementary slackness conditions for the firm's problem (5.34)–(5.38), with the complementary slackness conditions for the consumer's problem (5.50)–(5.52), and the market clearing condition.

Proposition 5.1 *The allocation (x^*, y^*) is a competitive equilibrium allocation if and only if (x^*, y^*) is a feasible, and there exist $(\alpha_c^*, \beta_L^{f*}, \beta_H^{f*}, q^{f*}) \in \mathbf{R}_+^4$ and $\lambda^* > 0$ such that*

$$q^{f*}(\langle r_L, x_L^* \rangle + \langle r_H, x_H^* \rangle) = 0; \tag{5.64}$$

$$\beta_L^{f*}(\langle EU_H, x_L^* \rangle - \langle EU_L, x_L^* \rangle) = 0, \tag{5.65}$$

$$\beta_H^*(\langle EU_L, x_H^* \rangle - \langle EU_H, x_H^* \rangle) = 0, \tag{5.66}$$

$$\alpha_c^* \geq EU_L(z) - \lambda^* q^{f*} r_L(z) - \lambda^* \beta_L^{f*}(EU_H(z) - EU_L(z)) \quad \forall z \in Z, \quad \textit{with equality if } x_L^*(z) > 0, \tag{5.67}$$

$$\alpha_c^* \geq EU_H(z) - \lambda^* q^{f*} r_H(z) - \lambda^* \beta_H^{f*}(EU_L(z) - EU_H(z)) \quad \forall z \in Z, \quad \textit{with equality if } x_H^*(z) > 0. \tag{5.68}$$

Substituting the market clearing condition $y^* = x^*$ into the firm's complementary slackness conditions (5.34)–(5.36) gives (5.64)–(5.66). Substituting the remaining complementary slackness conditions in the firm's problem, (5.37)–(5.38), into the consumer's complementary slackness conditions (5.51)–(5.52) gives (5.67)–(5.68). Note that the complementary slackness condition (5.50), which is associated to the consumer's budget constraint, is not included in the characterization in the competitive equilibrium allocation. The reason is that condition (5.50) is redundant. We have already shown that the price of the contract traded in a competitive equilibrium is zero (equation (5.60)). This result implies that the budget constraint binds. The proposition states that conditions (5.64)–(5.68) are not only necessary but also sufficient for x^* to be the contract traded in a competitive equilibrium. The proof of the "if" statement is in the Appendix.

The key result of this paper is that the characterization of a competitive equilibrium allocation is equivalent to the characterization of an incentive–efficient allocation in Section 4. Since the marginal utility of money λ^* is strictly positive, the conditions that characterize a competitive equilibrium allocation, (5.64)–(5.68), can be restated as

$$q^*(\langle r_L, x_L^*\rangle + \langle r_H, x_H^*\rangle) = 0; \tag{5.69}$$

$$\beta_L^*(\langle EU_H, x_L^*\rangle - \langle EU_L, x_L^*\rangle) = 0, \tag{5.70}$$

$$\beta_H^*(\langle EU_L, x_H\rangle - \langle EU_H, x_H\rangle) = 0, \tag{5.71}$$

$$\alpha_c^* \geq EU_L(z) - q^* r_L(z) - \beta_L^*(EU_H(z) - EU_L(z)) \quad \forall z \in Z, \\ \text{with equality if } x_L^*(z) > 0, \tag{5.72}$$

$$\alpha_c^* \geq EU_H(z) - q^* r_H(z) - \beta_H^*(EU_L(z) - EU_H(z)) \quad \forall z \in Z, \\ \text{with equality if } x_H^*(z) > 0. \tag{5.73}$$

where $q^* = \lambda^* q^{f*}$, $\beta_L^* = \lambda^* \beta_L^{f*}$, and $\beta_H^* = \lambda^* \beta_H^{f*}$.[28] Crucially, conditions (5.69)–(5.73) are the complementary slackness conditions that characterize an incentive efficient allocations, (4.20)–(4.24). This implies that an allocation is a competitive equilibrium allocation if and only if it is incentive efficient. Thus, the two welfare theorems hold.

6 Welfare Theorems and Existence

The welfare theorems stem directly from Propositions 4.1 and 5.1.

Theorem 6.1

(i) (First Welfare Theorem) Suppose (x^*, y^*, p^*) is a competitive equilibrium. Let $(\beta_L^{f*}, \beta_H^{f*}, q^{f*})$ and (α_c^*, λ^*) be the optimal shadow prices in the problem of the firm and the consumer, respectively. Since λ^* represents the marginal utility of money, the optimal shadow prices in the firm's problem can be measured in utils:

$$(\beta_L^*, \beta_H^*, q^*) = \lambda^* \left(\beta_L^{f*}, \beta_H^{f*}, q^{f*}\right).$$

Then x^* and $(\alpha_c^*, \beta_L^*, \beta_H^*, q^*)$ are optimal solutions for problems (D) and (P), respectively.

(ii) (Second Welfare Theorem) Suppose x and $(\alpha, \beta_L, \beta_H, q)$ are optimal solutions for problems (D) and (P), respectively. Let $y = x$ and define $p = (p_L, p_H)$ with

$$p_i(z) = q r_i(z) + \beta_i(EU_j(z) - EU_i(z)) \quad \forall z \in Z; \quad i = L, H.$$

Then (x, y, p) is a competitive equilibrium. Also, α is the consumers' indirect utility in a competitive equilibrium.

[28] Equivalently, we may normalize utilities so that the marginal utility of money is one, and set $\lambda^* = 1$.

The first part of the theorem says that the competitive equilibrium allocation solves the planner's problem (D). It also says that there is an equivalence between the optimal shadow prices in the problem of the firm and the consumer and the solutions to the primal problem (P). Since the price of a traded contract is equal to the shadow cost of the contract, this implies that there is an equivalence between the competitive equilibrium price system and the optimal solutions to the primal problem (P). The second part of the theorem says that any optimal solution to the planner's problem (D) can be decentralized as a competitive equilibrium allocation. It also says that there is an equivalence between the optimal solutions to the primal problem (P) and the price system that decentralizes the incentive–efficient allocation. In particular, with the price selection criterion in Section 5, there is a one–to–one correspondence between the prices that decentralize the incentive–efficient allocation and the optimal solutions to the problem (P).

The result in Theorem 6.1 is parallel to the general result obtained by Makowski and Ostroy's [18] for large economies with full information. Makowski and Ostroy use the measure–theoretic description of the full–information economy to show that efficient allocations solve a linear programming problem. They then establish an equivalence between the competitive equilibrium allocations and prices, on the one hand, and the solutions to the primal and dual problems, on the other hand. Theorem 6.1 extends Makowski and Ostroy's result to an economy with moral hazard.

The first welfare theorem can also be derived using standard arguments. We provide the proof for completeness.

Theorem 6.2 *(First Welfare Theorem) A competitive equilibrium allocation is incentive efficient.*

Proof. Suppose the contrary. That is, there is $(x', y') \in L^2$, with $x' \in X, y' \in Y,\ x' = y'$, and $\langle EU, x' \rangle > \langle EU, x^* \rangle$. The consumer's optimization condition (i) in Definition 5.1 then implies $\langle p^*, x' \rangle > \langle p^*, x^* \rangle$. By feasibility, this is equivalent to $\langle p^*, y' \rangle > \langle p^*, y^* \rangle = 0$, which contradicts the firm's optimization condition (ii). □

The existence of optimal solutions to the primal and dual problems directly implies the existence of a competitive equilibrium. In Jerez [14], we have established the existence of optimal solutions to the primal and dual problems.

Theorem 6.3 *A competitive equilibrium exists.*

7 Comparison with Prescott and Townsend

In this section, we compare our analysis to that of Prescott and Townsend [20]. The key modeling choice of Prescott and Townsend is to impose the incentive–compatibility constraints on the consumers, and not on the firms. This modeling choice highly simplifies the analysis because consumers are the informed agents. The presence of the incentive–compatibility constraints on the set of admissible consumption plans is interpreted as a restriction on the set of contracts that can be traded in equilibrium.[29]

Suppose that we impose the incentive–compatibility constraints on the consumers, and not on the firms. In this case, the production set is the set of production plans that are technologically feasible:

$$Y \;=\; \Big\{(y_L, y_H) \in L^+ : \langle r_L, y_L\rangle + \langle r_H, y_H\rangle \leq 0\Big\}. \tag{7.74}$$

The production plans that are technologically feasible are those that are (at least) actuarially fair. This is similar to the full–information benchmark, where the state–contingent plans that are technologically feasible are those that are (at least) actuarially fair. The main difference with respect to the full information benchmark is that firms do not sell state–contingent goods separately. They sell packages, or contracts. A second difference is that contracts may be random.

As in Prescott and Townsend [20], we assume that the consumption set under full information, and thus the net trade set Z, is a finite set. With this simplifying assumption, the commodity space and the price space are given by the Euclidean space. The firm's problem is then a standard linear programming problem, since all the integrals can be replaced with finite sums.[30] The firm chooses a production plan $y \in Y$ to maximize profits. The first–order conditions for the firm's problem are:

$$p_i(z) = q^f r_i(z) \quad \forall z \in Z,\ i = L, H, \tag{7.75}$$

where q^f is the optimal shadow price of the technological constraint. Condition (7.75) implies that *the price of a contract must be equal to its economic cost.* Following Prescott and Townsend, we normalize prices by setting $q^f = 1$:

$$p_i^*(z) = r_i(z) \quad \forall z \in Z,\ i = L, H. \tag{7.76}$$

The only candidate for a competitive equilibrium price system (up to some arbitrary normalization) is then the "actuarially fair" price system.[31]

[29] See Kehoe, Levine and Prescott [16].

[30] The same is true for the problem of the planner and the consumer.

[31] If the net trade set Z is not finite, the price of a *traded* contract must be actuarially fair. The

When the incentive–compatibility constraints are imposed on the consumers, the consumer's problem becomes:

$$\sup_{(x_L,x_H)\in L} \quad \langle EU_L, x_L\rangle + \langle EU_H, x_H\rangle$$

s.t.

$$\langle \mathcal{I}, x_L\rangle + \langle \mathcal{I}, x_H\rangle \;=\; 1, \tag{7.77}$$

$$-\langle EU_L, x_L\rangle + \langle EU_H, x_L\rangle \;\leq\; 0, \tag{7.78}$$

$$\langle EU_L, x_H\rangle - \langle EU_H, x_H\rangle \;\leq\; 0, \tag{7.79}$$

$$\langle p_L, x_L\rangle + \langle p_H, x_H\rangle \;\leq\; 0, \tag{7.80}$$

$$x_L, x_H \;\geq\; 0. \tag{7.81}$$

That is, the consumer chooses a contract that is incentive compatible and budget feasible in order to maximize utility. Crucially, when the equilibrium price system (7.76) is substituted in the consumer's problem, this problem coincides with the planner's problem. The consumers choose from the actuarially fair insurance plans under the incentive–compatibility constraint, so the second best is attained. Thus, the first welfare theorem holds. Note that any incentive–efficient allocation can be decentralized as a competitive equilibrium allocation at the actuarially fair prices in (7.76). Thus, the second welfare theorem also holds.

The main difference between our competitive equilibrium notion and that of Prescott and Townsend is that, in our model, all the relevant information is conveyed through the price system. The equilibrium prices reflects all the relevant information, including the welfare costs arising from the incentive–compatibility constraints, because we impose these constraints on the firms. In a competitive equilibrium, the price of a contract that induces the consumers to shirk (not to shirk) is above (below) the actuarially fair price. The price of a contract that makes the consumers indifferent between providing the high effort and shirking is actuarially fair. This is the case of the contract traded in a competitive equilibrium. This contract is incentive efficient and it provides only partial insurance. Note that the amount of insurance is then unaffected by our assumption to impose the incentive–compatibility

price of *non–traded* contract is indeterminate and cannot be higher than the actuarially fair price. In a similar environment with ex post private information, Kehoe, Levine and Prescott [16] show that prices can be restricted without loss of generality to be linear on Z. With this restriction, prices are always actuarially fair, as in the full–information benchmark. This restriction is equivalent to our price selection criterion in the case where the firm does not face the incentive–compatibility constraints (i.e., the supremum of the set of competitive equilibrium prices is the actuarially fair price system in (7.76).

constraints on the firms. That is, the equilibrium allocations are the same as when the incentive–compatibility constraints are imposed on the consumers.

There is also a formal difference between our paper and that of Prescott and Townsend [20]. With their approach, the competitive equilibrium prices are the same as in the full–information benchmark, so they are linear on the agents' net trade sets. With our approach, the competitive equilibrium prices are not the same as in the full–information benchmark. Instead, they are non-linear on the agents' net trade sets. The reason is that they must internalize the welfare costs arising from the incentive–compatibility constraints, and the welfare cost of incentives are non-linear and may even be non-convex. Note that this feature of our model is perfectly consistent with standard general equilibrium analysis, because prices remain linear in the commodities, the insurance contracts.

Our results are related to recent work on adverse selection. Bisin and Gottardi [4] study a simple adverse selection economy. Their key insight is to model the economy as one with externalities in consumption. This is possible because the consumption plan of each risk type affects the set of admissible plans of the other types. Following Prescott and Townsend [20], the authors impose the incentive–compatibility constraints on the consumers. They show that a competitive equilibrium exists and the two welfare theorems hold if there is an appropriate enlarged set of markets which includes markets for consumption rights. They also show that a competitive equilibrium exists but need not be incentive efficient if markets for consumption rights are not available. Rustichini and Siconolfi [22] study a class of economies with adverse selection. They analyze two notions of competitive equilibrium: strong and weak equilibrium. In a strong equilibrium, the incentive–compatibility constraints are imposed on the firms. In a weak equilibrium, the incentive–compatibility constraints are feasibility constraints on the allocation (just like the standard resource constraints), and are neither imposed on the consumers nor on the firms. Rustichini and Siconolfi show that strong equilibria are incentive efficient but may fail to exist. In independent work (Jerez [13]), we have studied a similar notion of equilibrium with these same properties. By contrast, Rustichini and Siconolfi also show that weak equilibria always exist but they are indeterminate and need not be incentive efficient. They also show that the set of equilibrium allocations (both weak and strong) does not change if the incentive–compatibility constraints are also imposed on the consumers.

8 Conclusion

We have studied a simple insurance economy with moral hazard, in which random contracts overcome the non–convexities generated by the incentive–compatibility constraints. The novelty is that we have used *linear programming and duality theory* to study the relation between incentive compatibility and pricing. Using linear programming has the advantage that we have been able to impose the incentive–compatibility constraints on the agents that face the informational asymmetries (the firms). This is in contrast to the seminal paper of Prescott and Townsend [20] who imposed them on the informed agents (the consumers). We have derived the two welfare theorems, established the existence of a competitive equilibrium, and characterized the equilibrium prices and allocations. Our competitive equilibrium has two key properties: (i) the equilibrium prices reflect all the relevant information, including the welfare costs arising from the incentive–compatibility constraints; (ii) the equilibrium allocations are the same as when the incentive–compatibility constraints are imposed on the consumers.

In closely related work, Bennardo and Chiappori [2] also study an insurance economy with moral hazard. The main difference between their paper and our paper is that we follow a general equilibrium approach and describe the trading process as a Walrasian market. In contrast, they follow a game–theoretic approach and describe the trading process as a two–stage game. In the first stage, firms simultaneously offer insurance contracts under the incentive–compatibility constraints. In the second stage, consumers choose from the set of contracts offered by the firms. Bennardo and Chiappori define a "Bertrand equilibrium" as a Nash equilibrium of the two–stage game. They also show that a Bertrand equilibrium exists and is incentive efficient. One may therefore think of our notion of competitive equilibrium as being a reduced form of Bennardo and Chiappori's Bertrand equilibrium. Note that there is another difference between the two papers: while we only consider idiosyncratic risk, Bennardo and Chiappori [2] consider both idiosyncratic and aggregate risk. We could easily extend our results to an environment with both idiosyncratic and aggregate risk. Restricting our attention to idiosyncratic risk therefore is purely for convenience.

The question remains whether we may apply the approach of this paper also to economies with adverse selection. In these economies, the presence of *external* welfare costs arising from the incentive–compatibility constraints makes the decentralization of incentive–efficient allocations problematic. In a companion paper (Jerez [13]), we show that our approach can in principle be extended to economies with adverse selection. We study a simple economy with adverse selection and present a notion of competitive equilibrium that imposes the incentive–compatibility constraints on the firms (see also Rustichini and Siconolfi [22]). Applying the linear programming methodology of the present paper, we show that a competitive equilibrium is incentive efficient but may fail to exist. While Jerez [13] is just a first step towards applying the linear programming methodology to study economies with adverse selection, it suggests that this methodology can be helpful in understanding the problems which arise in decentralizing incentive–efficient allocations.

Appendix A

A.1 The Linear Semi–Infinite Programming Problems

In this section, we set up the primal LSIP problem and derive its dual. The LSIP problems in Sections 4 and 5 obtain as particular cases of the problems in this section by applying the definitions in Table I.

A.1.1 The Primal Problem

Let $1 \leq m \leq n$ and $\Re^n$ be equipped with the Euclidean norm and partially ordered by means of the cone

$$K^n_m = \{\, y \in \Re^n : \, y_j \geq 0, \, j = 1, ..., m\}.$$

Let $\omega \in \Re^2_+$ and define $Z = \{\, z \in \Re^2 : \, z \geq -\omega \,\}$. Let $C(Z)$ denote the vector space of continuous real–valued functions on Z, endowed with the topology of uniform convergence on compact sets and partially ordered by means of the cone

$$C_+(Z) = \{\, f \in C(Z) : \, f(z) \geq 0 \quad \forall z \in Z \,\}.$$

The *primal* problem is to find $y \in \Re^n$ to solve

$$\begin{array}{lll} (P) & \inf & c \cdot y \\ & \text{s.t.} & Ay \geq b, \\ & & y \in K^n_m, \end{array}$$

where $c \in \Re^n$, $b = (b_L, b_H) \in C(Z) \times C(Z)$, and $A : \Re^n \to C(Z) \times C(Z)$ is a continuous linear mapping. Problem (P) is linear and has n unknowns and infinitely many constraints.

A.1.2 The Dual Problem

Let $M_c(Z)$ denote the space of signed Borel measures on Z which have compact support and are finite on compact sets. This space is the topological dual space of $C(Z)$ (Hewitt [12]).

	The Planner	The Firm	The Consumer
(n, m)	$(4, 3)$	(3,3)	(2,1)
y	$(\beta_L, \beta_H, q, \alpha)$	$(\beta_L^f, \beta_H^f, q^f)$	(λ, α^c)
c	$(0, 0, 0, 1)$	$(0, 0, 0)$	$(0, 1)$
$b = (b_L, b_H)$	(EU_L, EU_H)	(p_L, p_H)	(EU_L, EU_H)
$f_1 = (f_{1L}, f_{1H})$	$(-EU_L + EU_H, 0)$	$(-EU_L + EU_H, 0)$	(p_L, p_H)
$f_2 = (f_{2L}, f_{2H})$	$(0, EU_L - EU_H)$	$(0, EU_L - EU_H)$	$(\mathcal{I}, \mathcal{I})$
$f_3 = (f_{3L}, f_{3H})$	(r_L, r_H)	(r_L, r_H)	——
$f_4 = (f_{4L}, f_{4H})$	$(\mathcal{I}, \mathcal{I})$	——	——

Table I: The Primal and Dual Problems in Sections 4 and 5

Let $C(Z) \times C(Z)$ be paired in duality with $M_c(Z) \times M_c(Z)$. The reflexive space $\Re^n$ is paired with itself. The two pairings are endowed with their natural bilinear forms (to highlight the dimensionality of the spaces in the pairing we use the dot product and bracket notation for finite and infinite dimensions, respectively):

$$\langle f, x \rangle = \int_Z f_L dx_L + \int_Z f_H dx_H, \qquad f = (f_L, f_H) \in C(Z) \times C(Z),$$
$$x = (x_L, x_H) \in M_c(Z) \times M_c(Z);$$
$$y \cdot z = \sum_{j=1}^{n} y_j z_j, \qquad y \in \Re^n,\ z \in \Re^n.$$

The adjoint of A, $A^* : M_c(Z) \times M_c(Z) \to \Re^n$, is defined by the relation

$$y \cdot (A^* x) = \langle Ay, x \rangle, \qquad \text{for all } y \in K_m^n,\ x \in M_{c+}(Z) \times M_{c+}(Z). \tag{A.1}$$

We may write $Ay = \sum_{j=1}^{n} y_j f_j$, where $f_j = (f_{jL}, f_{jH}) \in C(Z) \times C(Z)$ for $j = 1, \cdots, n$. Then (A.1) can be expressed as

$$y \cdot (A^* x) = \sum_{j=1}^{n} y_j \langle f_j, x \rangle, \qquad \text{for all } y \in K_m^n,\ x \in M_{c+}(Z) \times M_{c+}(Z). \tag{A.2}$$

Write $A^* x \leq c$ as

$$\sum_{j=1}^{n} y_j (\langle f_j, x \rangle - c_j) \leq 0, \quad \forall y \in K_m^n.$$

The *dual* problem is to find $x \in M_c(Z) \times M_c(Z)$ to solve

$$\begin{aligned} (D) \quad \sup \quad & \langle b, x \rangle \\ \text{s.t.} \quad & \langle f_j, x \rangle \leq c_j, \quad j = 1, ..., m, \\ & \langle f_j, x \rangle = c_j, \quad j = m+1, ..., n, \\ & x \geq 0. \end{aligned}$$

Problem (D) is a linear programming problem with infinitely many unknowns and n constraints.

A.2 Proof of Proposition 6.1

(a) *The "if" statement.*

Let (x^*, y^*) be a competitive equilibrium allocation. Since $x^* \in X$ and $y^* \in Y$ and $x^* = y^*$, the allocation is feasible. Let $(\beta_L^{f*}, \beta_H^{f*}, q^{f*})$ and (α_c^*, λ^*) denote the optimal shadow prices in the firm's problem and the consumer's problem, respectively. We now that $\beta_L^{f*}, \beta_H^{f*}, q^{f*}, \alpha_c^* \geq 0$ and $\lambda^* > 0$. Substituting $y^* = x^*$ into the firm's complementary slackness conditions (5.34)–(5.36) gives (5.64)–(5.66). Substituting the remaining complementary slackness conditions in the firm's problem as stated in (5.42) into the consumer's complementary slackness conditions as stated in (5.55) gives (5.67)–(5.68).

(b) *The "only if" statement.*

Suppose (x^*, y^*) is feasible and there exist $(\alpha_c^*, \beta_L^{f*}, \beta_H^{f*}, q^{f*}) \in \mathbf{R}_+^4$ and $\lambda^* > 0$ such that (5.64)–(5.68) hold. Since x^* is a feasible consumption plan, y^* is a feasible consumption plan, and $y^* = x^*$, it remains to show that contract x^* is both an optimal consumption plan and an optimal production plan for some choice of the price system. Let the price system be given by

$$p_i^*(z) = q^{f*} r_i(z) + \beta_i^{f*}(EU_i(z) - EU_i(z)), \quad i = L, H. \tag{A.3}$$

At these prices, $(\beta_L^{f*}, \beta_H^{f*}, q^{f*})$ is a feasible solution for problem (D_f). Moreover, conditions (A.3), and (5.64)–(5.66) are the complementary slackness conditions in the firm's problem. Thus, x^{f*} and $(\beta_L^{f*}, \beta_H^{f*}, q^{f*})$ are optimal solutions for problems (D_f) and (P_f), respectively. Note also that $\langle p^*, x^* \rangle = 0$.

Substituting the price system (A.3) in the consumer's problem implies that (α_c^*, λ^*) is feasible for (P_c) since (5.67) and (5.68) hold. Moreover, conditions (5.67) and (5.68) are the last two complementary slackness conditions in the consumer's problem. Also, since $\langle p^*, x^* \rangle = 0$, the first complementary slackness condition in the consumer's problem also holds (since the budget constraint binds). Thus, x^* and (α_c^*, λ^*) are optimal solutions for problems (D_c) and (P_c), respectively.

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