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Modeling, evaluation and scale on artificial pedestrians: a literature review

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Modeling pedestrian dynamics and their implementation in a computer are challenging and important issues in the knowledge areas of transportation and computer simulation. The aim of this paper is to provide a bibliographic outlook so that the reader could have a quick access to the most relevant works related with this problem. We have used three main axes to organise the paper contents: pedestrian models, validation techniques and multiscale approaches. The backbone of the paper is the classification of existing pedestrian models; we have organised the works in the literature under five categories, according to the techniques used for implementing the operational level in each pedestrian model. Then, the main existing validation methods, oriented to evaluate the behavioural quality of the simulation systems, are reviewed. Furthermore, we review the key issues that arise when facing multiscale pedestrian modeling, where we firstly focus on the behavioural scale (combinations of micro and macro pedestrian models) and secondly, on the scale size (from individuals to crowds). The paper begins introducing the main characteristics of walking dynamics and its analysis tools and concludes with a discussion about the contributions that different knowledge fields can do in a near future to this exciting area.

Categories and Subject Descriptors: I.6.1 [**Model Classification**]: Simulation Theory; I.6.8 [**Types of Simulation**]: Combined; D.2.8 [**Metrics**]: Performance measures

General Terms: Design, Algorithms, Performance, Verification

Additional Key Words and Phrases: Pedestrian modeling and simulation, validation methods, crowds, multiscale simulation, macroscopic and microscopic pedestrian models

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1. INTRODUCTION AND OUTLINE

The study of pedestrian groups began with the works of the psychologist Gustave Le Bon in the 19th century, who studied crowds and multitudes from a psychological point of view. He stated the fact that the individual personality in a crowd is submerged and a collective crowd mind dominates (*La Psychologie des Foules*, 1896). During the 20th century, the first pedestrian flow studies focused on different social situations where people congregate in crowds, mainly in urban areas [Hankin and Wright 1958; Canetti 1962; Hall 1963; Oeding 1963; Older 1968; Navin and Wheeler 1969] and, in the early

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seventies a keystone work by Fruin [Fruin 1971a] proposes the concept of level-of-service in public spaces. Different problems, such as evacuation of buildings [Pauls 1977], the relationship between pedestrians and architectural spaces [Templer 1974; Pushkarev and Zupan 1975; Okazaki 1978] and the derivation of analytical formulas from empirical data [Predtechenskii and Milinskii 1978], were among the main motivations for research in the seventies. In the eighties, the studies on pedestrians took two different directions: firstly studies aided by the use of new technologies (mainly with computer vision algorithms); and secondly, the design of algorithmic models to generate simulations for computer graphics applications [Gipps and Marksjo 1985; Borgers and Timmermans 1986]. In this last direction, models have evolved from early computational simulations with raw numerical data outputs into complex virtual 3D populated environments. At present, pedestrian research is imbricated into several knowledge areas which have their own specific discussion forums and conferences. Three main periodical events in this field are the Pedestrian and Evacuation Dynamics Conferences (PED), the Traffic and Granular Flow Conferences (TGF) and the Transportation Research Board (TRB) meetings. Several reviews have been published focusing on crowd modeling and simulation which are listed in Section 5.

When reviewing the works done in pedestrian modeling and simulation, the main difficulty we came across with was the heterogeneity in the published works, due to three main factors. Firstly, as a consequence of the aforementioned transversality of research on pedestrians, we find publications in very different areas such as applied physics, statistics, operations research, computer science, transportation, wireless communications, civil engineering, social sciences and anthropology, leading to a variety of methodological approaches. Secondly, different works can be motivated by very different goals. Some works aim to reproduce observed properties of pedestrian dynamics, or focus on the emergence of collective phenomena and self-organization while others want to study specific dangerous situations such as evacuations or pedestrian-induced vibrations in bridges. There are works which evaluate the influence of social variables on a group or crowd organization and others focus on creating believable graphic simulations for games, virtual reality or movies. Finally, the scale of representation (macroscopic, mesoscopic and microscopic) [1] conditions both the techniques used and the achievable results. Moreover, many pedestrian models overlap over research areas, or propose approaches that operate in different levels (local, tactical, strategic) creating outputs at different scales (individuals, groups, crowds). As a result, the definition of criteria capable of creating a taxonomy with well separated groups is not an effective task.

This survey has been structured in three main axes: models, validation and scale. Firstly we focus on pedestrian models in the literature, which have been organized under five different categories. We describe the common characteristics of each category and we try to show the big picture from the seminal works to recent ones. Then, we review the validation approaches and techniques appeared in the literature to evaluate the pedestrian models. We also propose three different categories: real-data based, pedestrian dynamics characteristics-based, perception-based. Finally, we have also included a literature review for the multi-scale problem, where we have considered the behavioural point of view (micro+macro models combinations) and the scale size (number of simulated pedestrians, from individuals to crowds).

The rest of the paper is organized as follows: Section 2 summarizes the main characteristics of pedestrian dynamics and their analysis tools. Section 3 reviews the most important models that have been used in this area proposing a classification by categories. Section 4 presents the analysis techniques used to evaluate the quality or

[1] Described in Section 3

soundness of a simulation. Section 5 reviews the works that have considered multi-scale issues. The paper concludes with a discussion about knowledge areas that can increase their influence in pedestrian simulation in the near future.

2. CHARACTERIZATION OF PEDESTRIAN DYNAMICS

Pedestrian dynamics is difficult to characterize because, contrary to other displacement models, walking is not associated with a vehicle on a lane, and the underlying infrastructure is highly heterogeneous (sidewalks, stairs, elevators, crossings, shopping malls,. . .). Moreover, walking alone is completely different to walking inside a group. The spatial presence of others affects the walking speed and collective movement of pedestrians is highly influenced by psychological facts and cultural conventions [Sobel and Lillith 1975]. As a consequence, the best way to characterize pedestrian dynamics is highly dependant on the goal of the model and on the particular situation to be described. In this section, we define the values of some of the characteristic parameters most commonly used to describe walking and introduce the tools for analyzing pedestrian dynamics.

Pedestrians' speed and space. Pedestrian's speed is influenced by factors such as urgency for arriving, bad weather conditions, relaxing walk or density of the facility. This generates differences in the statistic values obtained in different experiments. According to empirical data in planar facilities under normal walking conditions [Weidmann 1993], velocity follows a Normal distribution $N(1.34, 0.26^2)$ measured in m/s. Since, from a mechanical point of view, walking is modeled as an inverted pendulum, pedestrians need more space when walking than when standing. The work by [Meister 2006] indicates that the minimum space needed for a standing pedestrian is $0.15m^2$ resulting in a maximum pedestrian density of $6.4Ped/m^2$, although the work by [Predtechenskii and Milinskii 1978] sets a value for minimum space of $0.1m^2$, which permits densities up to $9.0Ped/m^2$. Empirical studies indicate that there is no autonomous individual movement with densities larger than $5.4Ped/m^2$ [Nitzsche 2013]. In Figure 1, the curves that show densities higher than $6.5Ped/m^2$ (Helbing and Predtechenskii's works) display data derived from evacuations from buildings (Predtechenskii) and the pilgrimage in Makkah (Helbing). In these high density scenarios, people suffer compression forces and are not capable of voluntary movements.

Fruin's levels of service. Fruin defined different comfort levels for the pedestrian movements based on macroscopic magnitudes [Fruin 1971a; 1971b]. Upon the comfort levels, the concept of level of service (LOS) was defined as a criterion for safety in public places. For a given facility, LOS relate different flow qualities to maximum capacity ratios. The *capacity* is the maximum sustainable flow rate at which people can be expected to traverse a point on a lane during a specified time period, usually in individuals per hour. The criteria to determine the LOS for a pedestrian are based on objective parameters (like the speed and the average space available) and subjective parameters (like the pedestrian's ability to cross a pedestrian stream). Table I describes the LOS for pedestrians in normal walking expressed in macroscopic magnitudes. These LOS can vary in other situations (e.g. evacuations).

Fruin applied his calculations to urban environments like city streets in normal conditions. Young [2007] compared Fruin's free-flow walking speed data with data from the corridors inside an airport, concluding that there were no significant differences. However, in other environments, Fruin's data do not adequately describe the reality. For example, in crowded environments like the observations taken at the exits of the Wembley Stadium, densities higher than those of Fruin's data were observed in which pedestrians moved without restrictions [Still 2000]. The works by [Petritsch et al. 2005; Petritsch et al. 2008] aimed to develop LOS models for urban streets and

Table I. Fruin's Levels of Service for pedestrians

Level of service	Space (m^2/ped)	Average speed (m/s)	Flow (ped/min/m)
A = Free Flowing	≥ 12.077	≥ 1.321	≤ 6.562
B = Minor Conficts	≥ 3.716	≥ 1.270	≤ 22.966
C = Some Restrictions to Speed	≥ 2.230	≥ 1.219	≤ 32.808
D = Restricted Movement for Most	≥ 1.394	≥ 1.143	≤ 49.213
E = Restricted Movement for all	≥ 0.557	≥ 0.762	≤ 82.021
F = Shuffling Movements for all	≥ 0.557	≥ 0.762	variable

signalized crossings. A recent related review of pedestrian LOS is the work by Kadali and Vedagiri [2016].

The fundamental diagram of pedestrian dynamics. The fundamental diagram shows the relationship between flow J (pedestrians crossing a surface per unit time) and density ρ (pedestrians per unit area). From a macroscopic perspective, the hydrodynamic equation of fluid dynamics gives a method for flow measurement $J_s = \rho v$ where J_s is the flow per unit of width (also named specific flow) and v is the mean velocity. This results in the total flow $J = \rho v b$, where b is the width of the facility [Schadschneider et al. 2008]. Using this relationship, the fundamental diagram can be presented in two other equivalent forms; $v(\rho)$ and $v(J_s)$.

Density can be measured empirically using different technologies (e.g. video tracking [Teknomo et al. 2000], Wi-Fi and Bluetooth [Schauer et al. 2014]). At a given point, it is usually estimated by weighting the influence of each pedestrian, e.g. using a Gaussian function point [Helbing et al. 2007] or a bi-linear interpolation function [Narain et al. 2009]. In general, empirical density measurements show a high variability and lack of consistency. To mitigate this, other methods have been proposed based on space partition using Voronoi diagrams [Steffen and Seyfried 2010; Nikolic et al. 2015; Nikolic et al. 2016].

The fundamental diagram is a basic tool in the analysis of real pedestrian flows, the design of facilities and the assessment of infrastructures like arenas or stadiums [Nelson and Mowrer 2002; Schadscheneider and Seyfried 2009]. Furthermore, it is used for modeling [Narang et al. 2015] and evaluating models [Helbing and Molnár 1995; Fang et al. 2012] and is a cornerstone test to decide whether a model generates pedestrian streams with fidelity [Hoogendoorn et al. 2001; Steiner et al. 2007].

Predtechenskii and Milinskii [1978] show the descriptive capability of the fundamental diagram on different scenarios (horizontal paths, stairs and openings) and under different circumstances (emergency, normal and comfortable conditions), and demonstrate that the averaged speed of the pedestrians' flow is not only a function of the density but also of the type of path. The most comprehensive empirical work about free walking (pedestrians walking in a space without restrictions) is that of Weidmann [1993] who used 25 different studies of pedestrians under normal conditions (including uni and bi-directional flows, and non planar facilities such as stairs) to compose his general fundamental diagram. This work is used as a baseline reference for urban design and for empirical studies (e.g. [Seyfried et al. 2005]). Recently, it has also been shown that different traffic flows (vehicles, bicycles and pedestrians) exhibit common characteristics [Seyfried et al. 2014]. Figure 1 shows the shape of the fundamental diagram obtained in different empirical studies in planar facilities used as references in planning guidelines.

Kladek [1966] proposed an analytical expression for road traffic, used by Weidmann [1993] to describe uni-directional pedestrian flows. It can be formulated as:

$$v_d(D) = v_f(1 - e^{-\gamma(\frac{1}{D} - \frac{1}{D_{max}})}) \qquad (1)$$

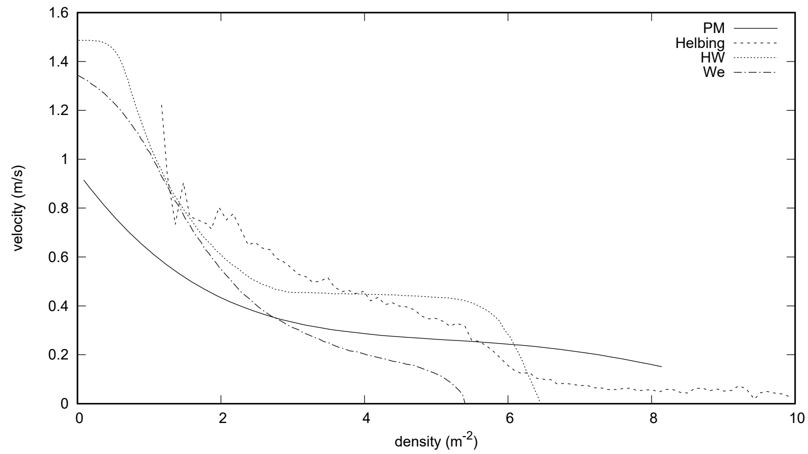

Fig. 1. Empirical fundamental diagrams for pedestrians in planar facilities. Plotted data are available at the web http://www.asim.uni-wuppertal.de/datenbank (PM [Predtechenskii and Milinskii 1978], Helbing [Helbing et al. 2007], HW [Hankin and Wright 1958], We [Weidmann 1993]).

where v_f (m/s) is the speed at free flow, D $(1/m^2)$ the current density, D_{max} $(1/m^2)$ the maximum density at which flow can take place and γ $(1/m^2)$ a free parameter. Different values for the parameters have shown to be valid under specific experimental conditions [Lämmel et al. 2009]. Next, we describe some of the main characteristics of the fundamental diagram for pedestrians in planar facilities:

— In the $v(\rho)$ formulation, speed decreases with growing density, although the relationship shows a non-lineal form [Schadschneider et al. 2008].
— From the diagram, the following characteristics can be directly obtained [Daamen 2004]: the capacity is the maximum of the flow/density curve; the free speed is the mean maximum speed; the critical density is the lower bound for unconstrained free walking; the jam density is the point where speed and flow are null.
— The fundamental diagram can vary significantly for $\rho < 0.2\ m^{-2}$ and $\rho \geq 4\ m^{-2}$: in low densities, the pedestrians are free to choose their speed; in high densities, jams and crowds appear, and the flow can be turbulent [van den Berg and Bouvy 1994].

Beyond these common properties, empirical studies with real pedestrians, performed in different conditions, reveal different shapes of the fundamental diagram, as it is reflected in Figure 1 where all the curves describe the dynamics of real pedestrians walking on a planar surface. Several explanations have been suggested: differences in the measuring methods [Seyfried and Schadschneider 2008], heterogeneity of the flow in congested areas [Daamen et al. 2005], uni-directional and multi-directional flow dissimilarities [Navin and Wheeler 1969; Lam et al. 2003; Kretz et al. 2006; Zhang et al. 2012], population and cultural differences [Morrall et al. 1991; Johansson et al. 2007; Chattaraj et al. 2013] or psychological factors [Predtechenskii and Milinskii 1978]. Differences have also appeared when comparing different facilities such as walkways, crosswalks or stairwalks, indicating that specific diagrams should be adopted for different facilities [Lam et al. 1995]. Moreover, empirical fundamental diagrams show results that are not completely explained. The work by Seyfried et al. [2013] studies empirical data in pedestrian bottlenecks to conclude that "maximal flow values measured at bottlenecks can exceed the maximum of empirical fundamental diagrams significantly". The work by [Johansson 2009] focuses on discovering other characteristics of pedestrian dynamics that homogenizes this situation, proposing the net-time headway $\hat{T}$. It is defined as the safety time necessary to avoid collisions with the neighbor pedestrians and has the following analytic expression:

$$\hat{T} = \frac{\hat{D}}{v} = \frac{1}{v}\left(\frac{1}{\sqrt{D}} - \frac{1}{\sqrt{D_{max}}}\right) \quad (2)$$

where D is the average density, D_{max} is the largest density and v is the pedestrian's speed. Johansson demonstrates empirically that several data sets that show divergences in the fundamental diagram have similar $\hat{T}$ *vs.* ρ curves.

Mobility Models. A mobility model is a description of mobility patterns based on the traces followed by pedestrians and vehicles, to determine their impact on the performance of a mobile or wireless network [Markoulidakis et al. 1997; Camp et al. 2002]. The resulting behavior can be described through variables such as pause times, frequency of contacts with other network users or distribution of waypoints (visited locations) [Lee et al. 2012]. Vehicular and traffic models are specially relevant in this context, since the mobility patterns are affected by speed and travel distance.

Movement traces can be either captured from real users or simulated with artificial pedestrians models [Mota et al. 2014; Treurniet 2014]. Simulated traces are often generated with an stochastic description of the movement, where pedestrians take random decisions after every simulation step. Examples are the Random Walk or Gauss-Markov models [Bai and Helmy 2006] or the Random Waypoint model [Johnson and Maltz 1996]. Other models, such as Column, Pursue or Reference Point Group mobility models, also take into account the interaction between pedestrians [Sánchez and Manzoni 2001], or use deterministic models such as steering or rule-based models [Legendre et al. 2006]. It has been observed that considering social interaction or time evolving behaviors are important factors to get realistic models [Musolesi and Mascolo 2009; Boldrini and Passarella 2010; Zhu et al. 2012; Kosta et al. 2014].

3. PEDESTRIAN MODELING

In pedestrian modeling, several taxonomic criteria have been proposed: space representation, population representation, behavior representation or model purpose [Kretz 2007]. The population representation criterion, that divides the models in *macroscopic*, *mesoscopic* and *microscopic*, is a classic characterization of pedestrian dynamics [May 1990; Duives et al. 2013]. In macroscopic models, individuals have no autonomy neither to change their kinematic state nor to control their interactions. Common variables of these models are mean velocity density or flow, and the main outputs are maximum passenger flow, occupancy, capacity or characteristic speed. They focus on groups and crowds. Mesoscopic models consider individuals but not individual interactions, still focusing on groups but providing more detailed information about each pedestrian. The goal is to keep some control over the individual (e.g. the permanence of a specific pedestrian in an area) but to move the group as a collective, avoiding local interactions. Microscopic models consider that each individual can control her own dynamics and can recreate with accuracy specific local interactions (collisions, overtaking) or model individual interests or preferences. The population representation criterion, however, is rather general and clusters under the same group many models which are very different in nature and, moreover, it differs among disciplines. I.e. in mobility models, those interested in the movement of individuals are considered as microscopic, while those that focus on the number of users occupying different regions of the domain are considered as macroscopic [Bai and Helmy 2006].

An alternative is to use a constructive criterion that arises from the standard division of task levels in pedestrian modeling. There are three levels of modeling the pedestrian behavior: operational, tactical and strategic [Hoogendoorn and Bovy 2001]. In the operational level, the velocity control is considered. In macroscopic models the problem is to control the mean velocity of the group, in microscopic models it is more complex since the system has to take into account collisions with other pedestrians or objects. In the tactical level, short sequences of individual decisions are scheduled as part of a planning. Route choice is the typical scheduling task common to microscopic and macroscopic models at this level. In the strategic level planning tasks are performed, considered as the capability of organizing activities. Many models, focused on pedestrian dynamics, do not consider planning, whereas others, such as those that simulate urban environments with complex activities like shopping, need it.

In this paper, *a classification based on the operational level of modeling the pedestrian dynamics is proposed*. We shall review different pedestrian models and modeling methodologies, and organize them taking into account the properties of their operational level model. This criterion offers more precise borders than the mentioned population criterion. For clarity, we present the works that belong to a model grouped by similarity instead of using a chronological criterion. We have divided pedestrian models in the following categories:

Mechanics based models. Includes models that inspire their operational level in continuum mechanics or force models. Most of these models are formulated by means of differential equations to describe pedestrian dynamics. We have also included optimization based models, since many of them are based in energy considerations.

Cellular Automata models. Cellular Automata (CA) is a computational paradigm which has given important results in natural processes modeling. Their main characteristics are the use of a regular spatial discretization and finite automata to describe time evolution.

Stochastic models. In this group, approaches that use stochastic processes or random utility theory as the basis to model pedestrian flows are included.

Agency models. Following the autonomous agents paradigm [Wooldridge and Jennings 1995; Wooldridge 2003], pedestrians are represented as agents that sense the environment and take autonomous decisions about their current state and goal.

Data-driven models. In this group, models are based on real pedestrian data. This category assumes that data contain the complete information for modeling pedestrians behavior. The work is focused on extracting the information from the raw data and apply it to build or set up a model or to steer individuals in simulations.

Figure 2 displays the different categories and models discussed. Core problems in pedestrian studies like routing or sensing are not operational-level problems. Therefore, the use of widespread techniques such as navigation fields for wayfinding or vision-based techniques in the case of sensing are considered features of the models, not models *per se*. Following this criterion, the rest of subsections present the different categories of the pedestrian modeling problem.

3.1. Mechanics based models

This category is inspired by mathematical descriptions of physical processes. The intention or need of a pedestrian to chose a direction of movement is somehow identified with a pressure or force in some mechanical system. Using this analogy, models for pedestrian dynamics are built using formulations coming from gas and fluid dynamics or classical point mechanics. In addition, we enclose in this category a family of models that are formulated as optimization problems since, in many cases, either the motivation or the formulation are comparable.

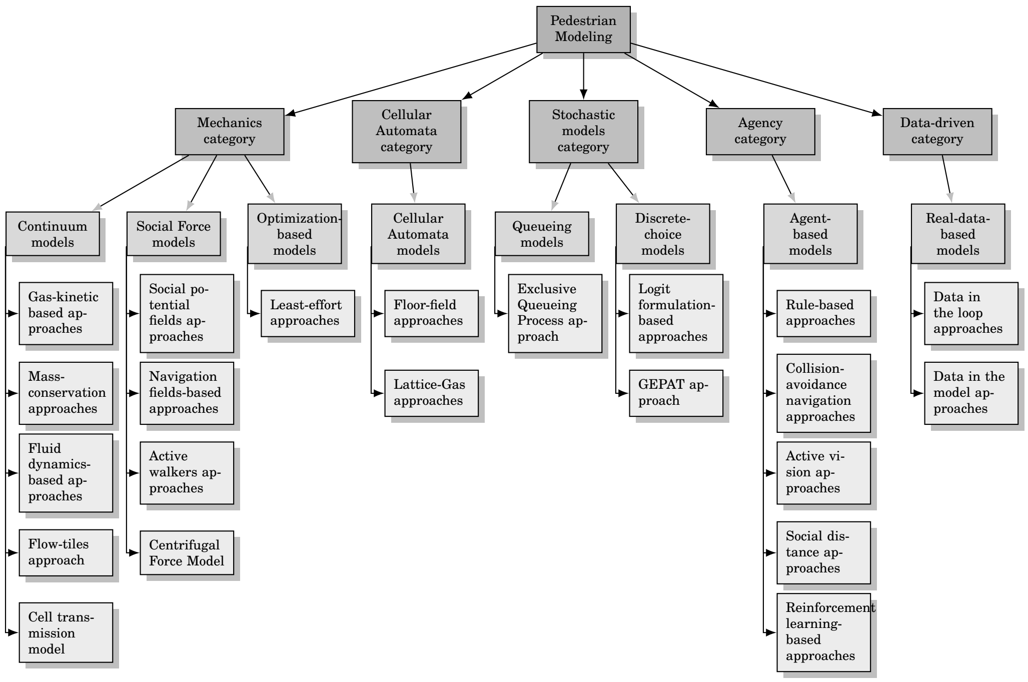

Fig. 2. The categories and models discussed in this Section. The lists of approaches (in light gray) are not complete since many of the approaches in the models discussed below have no specific denomination.

3.1.1. Continuum models. Pedestrians in a dense crowd, when seen from a macroscopic perspective, can be described by means of flow (or velocity) and local density. Therefore, fluid dynamics can provide useful models for this situation. Henderson realized that empirical data fitted the Maxwell-Boltzman distribution of speeds in ideal gases [Henderson 1974] and proposed the first fluid dynamics model for pedestrians. He assumed an homogeneous crowd, in the sense that every particle (pedestrian) has the same mass and probability density function for velocity [Sahaleh et al. 2012]. The work by Helbing [1992a] proposes an alternative formulation to consider anisotropies of pedestrian interactions and a preferred direction of motion. The model is posed as a system of differential equations on spatial density, mean velocity and velocity variance. A drawback of these models is the assumption of momentum and energy conservation that is not applicable to pedestrians.

Hughes proposed a model that combines a continuity equation with a fundamental diagram and a potential field to indicate walkers' direction [Hughes 2002; 2003]. The model describes crowds as an homogeneous flow with a common goal where pedestrians try to minimize their expected walking time, and assumes that a pedestrian's velocity is influenced by density in its surroundings. Suitable for problems involving high density crowds, it is able to reproduce a particular case of the Braess' paradox. Treuille et al.'s continuum crowds [Treuille et al. 2006] is a direct derivation of the Hughes' model. The authors present a common-goal crowd simulation framework capable of emergent collective behaviors such as lanes and group crossings. A reformulation of the Hughes' model is proposed by Huang et al. [2009] who relates the potential field with the immediate cost to reach the goal, satisfying the reactive dynamic user equilibrium principle. A discussion about the Hughes' model with emphasis on existence theorems can be found in [di Francesco et al. 2011]. The gas-kinetic and fluid-dynamics models can include forces to represent pedestrian's intentions by changing the pedestrian's type of motion. They are guided by stochastic laws, parametrized to represent demand for commodities, location of stores or city center entry points [Helbing 1992a]. A hybrid model of this kind is presented in [Helbing 1992b].

While previous works are based mainly on momentum conservation, an alternative approach is to use the mass conservation equation. The works by Colombo and Rosini [2005] and Coscia and Canavesio [2008] use the mass conservation equation, together with boundary conditions to model pedestrian strategies and panic conditions. In the same line, the work by Bellomo et al. [2015] uses a parametrized mean velocity to reproduce different behaviours and the work by Hoogendoorn et al. [2014] relates the macroscopic mass conservation equation with the microscopic social forces model. Other family of approaches that use these principles, combined with the fundamental diagram, is the cell transmission model (CTM). CTM is a derivation of the first-order flow theory proposed firstly in the work by Daganzo [1994] to describe vehicular traffic. In the model for pedestrians, space is divided in cells where the principle of mass conservation prevails (in terms of individuals). The pedestrian flow between two adjacent cells is governed by the fundamental diagram. The work by Asano et al. [2007] adapts the original formulation to the multi-directional flows situations, such as crossings. The work by Guo et al. [2011] proposes the creation of a cell potential to generate the flow between adjacent cells. The approach demonstrates good results in evacuation situations. The work by Hänseler et al. [2014] combines cell potential with a logit-based path choice and is able to reproduce behaviors from organized situations, such as queueing, or others that reflect impatience, such as pushing up.

Fluid dynamics models have been applied in computer graphics to generate dense populated scenarios. For instance, the *flow tiles* model [Chenney 2004] is based on the design of velocity fields on small, confined areas, that are combined to reproduce flows in large scenarios. A drawback of this model is that crossing situations cannot be simulated, since the created streams do not cross. The work by Narain et al. [2009] presents a mixed representation (discrete and continuous) for the crowd with a collision avoidance model adequate for high density crowds. In the same line, the work by Golas et al. [2014b] develops a collision avoidance model capable of working in the full range of densities. Combined with a pressure field to include inter-personal stress and individual discomfort, the model can simulate crowd turbulence [Golas et al. 2014a].

One of the key limitations of these models is the continuum assumption. As Bellomo pointed out [Bellomo and Dogbe 2011], continuum models "assume the validity of the paradigms of continuum mechanics" which are statistical results of the interactions of microscopic particles. However, not even in dense crowds, the relative pedestrians-crowd scale is comparable with the particle-fluid scale, where the formulae of continuum mechanics are meaningful.

3.1.2. Social force model. In the *Social Force Model* (SFM) [Hirai and Tarui 1975; Helbing and Molnár 1995] each individual moves as a consequence of the action of several forces (external and internal) on the pedestrian. External forces arise from the physical interactions with the environment, whereas internal forces come from the need of the pedestrian to act towards her goals. Using the notation in [Helbing and Johansson 2009], each individual α is influenced by a sum of forces that represent her tendency to go towards a desired direction, and the need to avoid other pedestrians and obstacles. The total force which governs the dynamics of each individual, $\vec{F_\alpha}$, is defined by

$$\vec{F_\alpha} = \frac{1}{\tau_\alpha}(v^0_\alpha \vec{e}^0_\alpha) + \sum_{\omega(\neq\alpha)} \vec{R}_{\alpha\omega}(t) + \sum_k \vec{R}_{\alpha k}(t) \tag{3}$$

where the first term accounts for intention to move in one privileged direction $\vec{e}_\alpha^0$ with a duration defined by the reaction time τ_α. The second and third terms represent repulsive forces to avoid other pedestrians ω and obstacles k. Repulsive forces are modeled as gradients of a potential field which decreases with the distance from de pedestrian. That is, if $\vec{p}_{\alpha,\omega}$ is the relative position of pedestrian ω to from pedestrian α, then

$$\vec{R}_{\alpha,\omega}(\vec{p}_{\alpha,\omega}) = -\nabla V_{\alpha\omega}(b(\vec{p}_{\alpha,\omega})) \tag{4}$$

where $b(\vec{p}_{\alpha,\omega})$ is some function which is decreasing on $|\vec{p}_{\alpha,\omega}|$. A similar formulation is used for modeling the repulsion between a pedestrian and an object k.

The SFM can be considered as a cornerstone in microscopic pedestrian modeling, leading to many derived works. Heïgeas et al. [2003] use a particle system to simulate a crowd where interactions among individuals are modeled as physical forces obtained from a damper-spring model. Moreover, SFM supports the use of additional forces that can represent a variety of social or psychological motivations (e.g. the tendency to keep away from danger or the attractive effect of a stage) [Helbing et al. 2005; Ward 2007], leading to so called *Social Potential Fields* (SPF). The work by Pelechano et al. [2007] presents HiDAC, a layered behavioral architecture where social forces are combined with psychological and geometrical rules. HiDAC is extended to OCEAN [Durupinar et al. 2008], which provides pedestrians with individual personalities based on the Ocean psychological model to study its influence in the crowd. SFM has been also adapted to study problems such as panic in escapes [Helbing et al. 2000] and evacuations [Yan 2010; Wagoum et al. 2010], and specific situations such as *freezing by heating* [Helbing and Johansson 2009] or stop-and-go waves [Chraibi et al. 2015].

The SFM can be combined with navigation fields for efficient path finding, like in the Dynamic Navigation Field (DNF) by Gilman et al. [2005], where a class of algorithms for planning dynamically generate vector fields according to the spatial situation of the particles. DNFs follow a case-based reasoning strategy in order to reuse calculated paths. This requires a continuous update and adaptation of stored solutions which is computationally expensive. Another approach that models navigation using potential fields is the active walker model [Freimuth and Lam 1992]. In this model two entities (the environment and the walker) are mutually influenced. The environment has a potential field dependent on position and time, modified by the action of the walkers by increasing its value in regions more frequently used. On the other hand, the walkers move following a desired direction that depends of the destination point and the ground potential. This model has been used to reproduce the evolution of trails created by pedestrians on the ground [Helbing et al. 1997c; Helbing et al. 1997a].

The SFM can be found as a part of a more complex behavioral controller [Pelechano and Badler 2006; Zainuddin and Shuaib 2010], as a prediction technique for video tracking of individuals inside crowds [Ali and Shah 2008] or used to identify unusual situations in videos of crowds [Mehran et al. 2009]. SFM is also used in hybrid models with CA [Ji et al. 2016] and in other areas such as robotics [Gayle et al. 2009].

Despite their popularity, the social force models and, in general, force-based models have problems derived from their Newtonian formulation. Chraibi et al. [2011] identify two main issues. The first one is the fact that, in real pedestrians, collisions do not follow the Newton's third law, since normally interaction is not conservative (e.g. pedestrians jostling in a queue). The second problem comes from the assumption that forces on a pedestrian are additive according to the superposition principle of forces. This can lead to undesired effects (e.g. in the form of high velocities) in situations of high density. Further problems appear due to inertia, leading to overlapping and oscillations. To address this, several modifications have been proposed. Dietrich and Köster [2014] propose the gradient navigation model, where a set of gradients of the distance function modify the velocity, avoiding second order derivatives. This model avoids oscillations since no forces are present. The Centrifugal Force Model [Yu et al. 2005; Chraibi et al. 2010] proposes a new expression for the repulsive force between pedestrians. Given two pedestrians α, ω, and their relative position $\vec{p}_{\alpha,\omega}$ and speed $v_{\alpha,\omega}$, the definition of the repulsive force is:

$$\vec{R}_{\alpha,\omega} = -m_\alpha k_{\alpha,\omega} \frac{v_{\alpha,\omega}^2}{|\vec{p}_{\alpha,\omega}|} \vec{e}_{\alpha,\omega} \tag{5}$$

where $\vec{e}_{\alpha,\omega} = \vec{p}_{\alpha,\omega}/|\vec{p}_{\alpha,\omega}|$. According to Chraibi et al. [2011], the coefficient $k_{\alpha,\omega}$ reduces the force to a range of influence of $180°$, modeling with this anisotropy the dependence of the reaction behavior with vision. Other important problem is that empirical studies have reported unrealistic results for SFM in low or moderate density scenarios [Lakoba et al. 2005; Saboia and Goldenstein 2011]. The work by Lakoba et al. [2005] analyzes pedestrians overlapping and proposes three main modifications: dependence of repulsion forces on the crowd's density, orientation dependence of the social forces (face to face, back to face,...) and awareness of the direction where the goal is placed.

3.1.3. Optimization-based models. Many disciplines use formulations based on optimization. Classical mechanics is a well known example, where the solution to a problem is described as the one that minimizes some function, which is often related to work or energy. The principle of least effort [Zipf 1949] applied to the pedestrian context proposes the idea that pedestrians' trajectories are the result of an optimization process on the energy. E. g., it is known that pedestrians minimize metabolic energy when walking at roughly 1.33 m/s [Henderson 1971]. Next, we consider the models that represent the goal or behavior to achieve as an utility function that has to be optimized. The range of the model (macroscopic or microscopic) is determined by the type of utility function (global in the macroscopic case or individual in the microscopic case).

Hoogendoorn and Bovy [2003] use a predictive controller that minimizes the cost of walking by means of an optimal control problem, subject to pedestrians' kinematics constraints. Each individual is aware of, and can predict, other walker's trajectories. Walking is considered as a differential game where pedestrians cooperate-compete with other individuals. In the work by Hoogendoorn and Bovy [2004b], the pedestrians' velocities are computed to optimize an utility function that reflects the expected cost of walking from the instantaneous position to the destination area. The NOMAD commercial pedestrian simulator has been built with the Hoogendoorn's model. The work by Ramming [2002] uses simulation to predict route utilities for the utility function. The route with the highest utility is added to the choice set.

The work by Arechavaleta et al. [2008] focuses on human walking. Assuming that real walking trajectories are optimal, the authors propose an inverse optimal control problem to discover the optimal criterion that creates human walk. The generated optimal trajectories are compared to those in a database of real trajectories for validation. The work by Curtis and Manocha [2014] uses a geometric optimization method originally devised for robots, that computes collision-free velocity in the velocity space, giving rise to self-organization phenomena.

In Still [2000] constraints on the pedestrians' speed distribution are introduced. The requirement of visiting certain places or sub-regions is also imposed as a constraint, as part of the route plan of an individual. The cost function is related to the path length, total time or total effort. The optimization process uses a type of simulated annealing over a set of allowed paths, randomly varying them and selecting the cheaper candidates in an iterative scheme. The commercial tool Legion is based on Still's work.

The work by Guy et al. [2010] uses the Zipf's principle of least effort to model pedestrian dynamics. For a given trajectory τ, the energy expanded by a person depends on the squared velocity and can be modeled as

$$E(\tau) = m \int_{\tau} (e_w |\vec{v}|^2 + e_s)\, dt \tag{6}$$

where e_w indicates caloric efficiency and e_s is a reference rate of energy consumption. The minimization of Equation 6 under certain velocity constraints yields a least-effort trajectory that avoids collisions among pedestrians in a large crowd. In Guy et al. [2012a], the authors develop a global navigation system that avoids collisions based on this formulation. Several emergent collective behaviors such as lanes or arching congestion around a door can be generated. In the work by Fu et al. [2014] the least effort principle is extended to support route choice. It is tested in an scenario with two exits, where exit selection and herding behaviors are clearly demonstrated.

Optimization-based models have a drawback in the fact that the parameters of the function to optimize in many occasions are not directly related with the generated movement. Therefore, it can be a non-intuitive task to control the movements by varying the values of the parameters as well as simulate behaviors such as panic.

3.2. Cellular Automata models

A cellular automaton is a discrete computational model that consists of a grid of cells with computation capabilities, usually a finite state machine. In pedestrian dynamics, Cellular Automata (CA) discretize the space as a lattice of cells whose states includes information about presence and direction of individuals, environmental obstacles and relevant objects. In many applications, it is very common to introduce probabilities and stochastic choice models in the transition rules; each pedestrian defines her movements using transition probabilities to neighbor cells. This use of probability functions tightly connects CA with the stochastic category. However, the CA paradigm is so important in many pedestrian application fields, such as traffic, and has generated so much research that, in our opinion, it must constitute a category *per se*. Two main properties for computational issues of CA are locality (each cell only communicates with its neighbors) and modularity (each cell is an independent process), favouring parallelization [Margolus and Toffoli 1987].

3.2.1. Cellular Automata-based models. CA are widely used in microscopic traffic simulation since the 90's of the last century. Since CA models for vehicle traffic were basically one-dimensional, a new rule set was proposed by Blue and Adler [1998] for pedestrian traffic, extending it to bi-directional flows in Blue and Adler [2001]. These two works are considered the seminal references for CA applied to pedestrian dynamics. Other examples of adaptation of CA models for studying uni and bi-directional pedestrian flow can be found in the works by Meyer-König et al. [2001], Nowak and Schadschneider [2013] and Bandini et al. [2014]. The evacuation problem has been extensively studied with this model in different scenarios: in buildings [Yang et al. 2005], with obstacles [Varas et al. 2007] or inside corridors [Yu and Song 2007].

A fruitful CA approach is the floor field model [Burstedde et al. 2001; Schadschneider 2002], where each tile stores the value of the probability of transition from the current tile to a specific neighbor tile. Probability values can be calculated using different criteria, such as the minimum distance to the goal or the expected travel time derived from the Eikonal equation [Hartmann 2010]. This approach was improved by Kirchner et al. [2003b] introducing the effects of friction and clogging, demonstrating their importance for the adequate reproduction of the dynamics in CA models, e.g. to distinguish between competitive and cooperative movement [Kirchner et al. 2003a]. The simplest approach in the floor field model consists on using one or more static floor fields in which the cells contain the same information along the entire simulation Kretz

[2009]. Seitz and Köster [2012] introduces the Optimal Steps Model in which a pedestrian decides the next step based on the optimization of a function constituted by a superposition of the different layers that are considered as utility functions of specific navigational problems. However, the behaviors obtained using static fields can produce unrealistic effects (e.g. pedestrians are not aware of a congestion until they are very close to it). In Schadschneider and Seyfried [2009] the probabilities of movement are modified by both a dynamic and a static floor field. The static field represents the influence of the facility whereas the dynamic field logs the pedestrians' movements and causes the broadening and weakening of these traces with time. The same idea is presented in Kirchner and Schadscheneider [2002] for simulating evacuations. On the other hand, the type of tessellation selected to discretize the floor is important in floor field approaches. In Leng et al. [2014], a hexagonal tessellation with weights, to compensate the anisotropy of the hexagon in the orthogonal is proposed.

A variant of the CA model is the Lattice Gas model (LG) [Muramatsu et al. 1999; Muramatsu and Nagatani 2000] that was proposed initially to reproduce the pedestrian counter flow in a channel, although it has been used to study other dynamics (e.g. evacuations[Helbing et al. 2003]). In LG, the pedestrians are treated as biased random walkers. The transition probabilities have a drift parameter D that implement the intensity of the bias toward a preferential direction, that depends of the topological configuration at time t. When $D = 0$ the probabilities are similar to those of CA.

CA models have also been mixed with other models: forces [Wei-Guo et al. 2006], multi-agent systems [Dijkstra et al. 2001] and with proxemic rules (see Section 3.4) [Wąs et al. 2006]. The pioneering approach by Gipps and Marksjo [1985] presented a hybrid model that combines force models with a CA where the transition rules follow a cost-benefit criterium, whereas the influence of the other neighbors is represented by repulsive forces.

Criticism to CA models derives mainly from their discrete nature. In addition, a lattice is too regular (symmetric) to define realistic movements for pedestrians, and the finite number of states and rules can generate non-natural homogeneous behaviors [Bierlaire et al. 2003]. There are approaches aimed to overcome these limitations. The work by Lubaś et al. [2016] proposes a non-homogeneous asynchronous CA for modeling crowds where the transition rules are cell-dependent. On the other hand, the CA paradigm has become a popular test platform for studying pedestrian dynamics because of its computational simplicity.

3.3. Stochastic models

Under this category we group together works in which the use of stochastic processes and random decisions play a key role. It includes a subgroup of research works that are based on queuing theory, and another including discrete choice models.

3.3.1. Queueing models. Queueing theory is a branch of stochastic processes inside of operations research field [Stoyan and Daley 1983]. In the pedestrian modeling context, queuing theory describes the facilities as graphs, where each node represents a region. A queue appears in a node when a positive difference exists between the service demanded and the service provided by the node [Rahman et al. 2013]. Different probabilistic models can be used to describe the arrival intensity to the queue and the service mechanism. The theory tries to set up a model for the dynamics of the queues that represent a pedestrian flow in a lane. Following the work by Rahman et al. [2013]: "The basic entities which characterize a queueing model are: i) the arrival date, ii) the service mechanism iii) the queue strategy (e.g. first come first served) and iv) the number of service nodes".

One pioneering work in pedestrian dynamics was the model proposed by Yuhaski and Smith [1989] which uses queueing theory to model circulation in the levels of a building. Representing the rooms with a planar graph, the queueing network is calculated as its dual graph. The facilities are then described as a hierarchical graph while the flow is described using a transition matrix among its nodes. In the work by Lovas [1994], different pedestrian facilities were modeled creating a network where queueing processes model pedestrian dynamics in terms of flow. The network models the environment, where the nodes can represent doors, rooms or intersections and the links, corridors or other facilities. The stochastic transitions between nodes are modeled as markov processes. As this model is concerned with flow control, it can be considered within the macroscopic type. A close work to these ideas is that of Mitchell and MacGregor [2001]. It uses Erlang's formulae (designed to calculate the probabilities of waiting in phone lines) in the context of pedestrian planning and performance analysis of a facility.

In the classic approach, queue theory focuses on the queue dynamics and does not consider external influences. However, more complex models have been developed. That of Okazaki and Matsushita [1993] takes into account other pedestrian behaviors outside the queue like people approaching to queues and getting out of them. Li and Han [2011] proposed a discrete queue system which takes into account physiological and psychological aspects. It was capable of reproducing the traffic shock wave phenomenon effectively. The queueing theory is able to calculate and predict the number of waiting people and waiting time in the queue spaces. The work by Tomoeda et al. [2013] reveals the existence of an optimal density in a queue of pedestrians that plays an important role in crowd dynamics. Other work with a different approach in queueing theory is Arita and Schadschneider [2014], where the authors propose the exclusive queueing process (EQP) which considers a queue from a microscopic level. EQP has a richer phase diagram than the classic modeling respect to the arrival probability and service probability parameters.

One main criticism for queueing models is that queues are essentially unidimensional structures. Being pedestrian dynamics bi-dimensional, these models cannot capture the complex movements of pedestrians such as merging and intersecting. Other common objection to the approach is that queueing models can not deal with high density scenarios [Okazaki and Matsushita 1993].

3.3.2. Discrete choice models. Discrete Choice Models (DCMs) are a family of macroscopic models that have been applied in the context of travel decisions [Ben-Akiva and Lerman 1985]. In the context of pedestrian modeling, DCMs use random utility (RU) theory as a foundation [McFadden 1981]. Following the description made in the the work by [Bierlaire and Robin 2009], they consider a decision-maker which is performing a choice among a set of alternatives $C \subset J$. The decision-maker associates an utility U_i with each alternative c_i and selects the alternative associated to the highest utility. The utility is modeled as a random variable to account for uncertainty due to various issues such as unobserved variables and measurement errors. The utility is decomposed into a deterministic term and a probabilistic error term, ε, so that

$$U_i = V_i + \varepsilon_i. \tag{7}$$

The specification of V_i includes the selection of the attributes of i relevant to the decision-maker, as well as its socioeconomic characteristics. The complexity of the model comes from the assumptions about the probability distribution of the random variable ε_i. In the Logit model [Train 2003] the independence and indentical distribution of ε_i across i for all decision-makers is assumed, leading to a simple and tractable formulation. The set of choices a decision-maker has to consider covers different levels of the pedestrian behavior. In the following list, we have ordered them from the lower level (operational) to the highest level (strategic). Following our taxonomy, only the models that decide at the operational level should belong properly to this category.

(1) Choices of speed and next step. This type of choice focuses on deciding the direction and speed of pedestrians at a given time. Many variables can be considered in the decision model. Among the macroscopic variables commonly used we find flow and density [Lam and Cheung 2000] or the type of environment, such as crosswalks [Knoblauch et al. 2007] or airport terminal corridors [Young 2007]. Among the microscopic variables, overtaking, age, trip purpose or internal friction and crashes are also relevant to make the decision. The work by Antonini et al. [2006] uses an utility function to decide among 33 different alternatives of combined speed and direction changes. The utility function combines the influence of five behavioral patterns taking into account collision-avoidance and permanence in a direction towards destination. This work is extended in [Robin et al. 2009].
(2) Route choice. Itinerary choice is a critical dimension of the pedestrian behavior and is based in the assumption that pedestrian displacements are purpose-based. Route choice models are traditionally based on a network structure [Borgers and Timmermans 1986] favoring studies with different facilities (ramps,stairs,...) such as the work by Cheung and Lam [1998] that focuses on real data of escalators and stairways in Hong Kong railway stations. However, this structure has been criticized by several works: Hoogendoorn et al. [2003] expose the limitations of the network models and propose a continuous time and space assignment; the work by Kretz et al. [2013] concludes that bi-dimensional nature of pedestrian dynamics cannot be adequately represented by a network. Therefore, other strategies have been developed. In the work by Hoogendoorn and Bovy [2004a] the authors design a heuristic user-optimal dynamic assignment method for route choice based on the pedestrians' accumulated experience. Okada and Asami [2007] incorporate utility at nodes in a pedestrian flow model, and route choice probabilities are derived using an aggregate logic model while Guo and Huang [2010] use the Logit model to assess the exit choice in a closed room in evacuation conditions. In communication networks, route choice of pedestrians can have great impact on routing protocols. The Random Waypoint model [Johnson and Maltz 1996] is commonly used to select the next place visited by a simulated individual. Kosta et al. [2014] define location preferences considering social and psychological factors, such as proximity or popularity, while Hsu et al. [2007] consider a set of preferences that change along time. The pedestrian route choice problem is reviewed in [Papadimitriou et al. 2009].
(3) Activity choice. This choice focuses on what to do next. For pedestrians, the work by Hoogendoorn and Bovy [2004b] distinguishes between the choice of an activity pattern performed at the strategic level of decision and an activity scheduling performed at the tactical level; Borgers and Timmermans [1986] considers that the choice activity is not planned but triggered by stimuli in the environment. The selection of the activities by network users is also considered in the field of mobility modeling [Zhu et al. 2012].
(4) Mode choice. This is the most traditional discrete choice model. It focuses on the election of transportation mode, where walking is one of the alternatives (e.g. the work by Ewing et al. [2007] on travel decision of students going to school), or on the choice among stairways, escalators, or elevators while walking.

According to [Ben-Akiva and Lerman 1985; Antonini 2006], the maximization of an utility function through discrete choices imposes a rational behavior to the individuals in terms of consistency (similar decisions under similar circumstances) and transitive preferences (in the choice over different alternatives). However, this formulation can be too rigid in some problems. The fact that RU maximizes the utility function is not adequate where decisions made by real pedestrians are not the best possible but good ones (e.g. shopping activities). The work by Zhu and Timmermans [2007] presents a hybrid heuristic model named GEPAT where RU is combined with a genetic algorithm for exploration of solutions. In this model harder decisions use more complex rules generating suboptimal solutions, imitating the bounded rational behaviors of real pedestrians. BIOGEME [Bierlaire 2003] is a software tool to work with DCMs, which allows parameter estimation for several models, including nonlinear utility functions.

3.4. Agency models

Agency theory defines the concept of agent as a piece of software that can autonomously make decisions and interact with the environment to satisfy her design objectives. The proactive nature of agents, together with their autonomy, make them suitable to conceptualise human activities (physical situations or mental states).

Agent-based models (ABM) of pedestrians are based on the Multi-agent systems (MAS) computational paradigm [Wooldridge 2013]. Agents can represent pedestrians bringing together different levels of operation (reasoning, reactive and proactive behavior) and different levels of interaction (coordination, negotiation,. . .) exploiting them in an autonomous way. The survey by Papadimitriou et al. [2009] notices the adequacy of MAS for microscopic pedestrian modeling due to the capability of considering local and physical interactions, and the possibility of incorporating individual decision-making and learning processes. Other works highlight the adequacy of these models to generate collective self-organization in crowds [Cristiani et al. 2015] or to simulate structured spatial activities in enhanced geographical systems [Batty 2003].

One important group inside ABM is the rule-based models where the agents are directed by decisions which are triggered by rules. Rules can control different behavioral levels, from physical maneuvers to social or cognitive behaviors. The most important seminal example of rule-based microscopic model is the Reynold's work [Reynolds 1987]. His system is based on a particle model, where each individual (called a *boid*) autonomously decides its own orientation. Using simple rules which describe how to maneuver depending on the kinematic situation of its neighbors (separation, alignment and cohesion), Reynolds simulated groups of animals (flocks, herds) with a realistic appearance in terms of group navigation. The same author applied a similar rule-based approach to the specific problem of steering behaviors [Reynolds 1999] separating the steering problem from the locomotion problem. These works are the starting point for other type of models like those based on collision avoidance techniques described below. Rule-based systems are used in different fields, such as mobility modeling where they are used to generate pedestrians traces [Legendre et al. 2006; Medina et al. 2010]. Rules that reflect social behaviors, such as joining or abandoning groups inside a crowd, have also been proposed [Musse and Thalmann 1997; Gu et al. 2011]. Rule systems can be included as a part of a more complex decision making module. For example, the work by [Shao and Terzopoulos 2005] uses a rule-based system for the reactive part of their hierarchical cognitive model, while planning strategies are used at the navigational level taking into account the physiological and psychological needs of the individual.

A kind of agent-based approaches is constituted by the collision avoidance models, which focus on computing collision-free paths and can be combined with other high-level approaches such as layered behavioral architectures. A key aspect of collision avoidance problem is the anticipation capability. Feurtey [2000] use a cost function for evaluating the cost of deviating in the future from a specific trajectory, while Paris et al. [2007] propose a reactive method for individual interaction in a crowd, where the system calculates a trajectory solution for each individual that anticipates collisions. Vukadinovic et al. [2014] combine a graph search algorithm with a local collision avoidance strategy to evaluate *ad-hoc* network protocols in leisure parks. In [Pettré et al. 2009; Olivier et al. 2012], a metric named *minimal predicted distance* is defined to detect a critical distance in order to avoid collisions between walkers. With few parameters, the model is able to adapt motion to manage interactions. The work by Karamouzas et al. [2009] tries to minimize the number of interactions with other pedestrians, as well as the energy used in them. Using the definition of a comfort space around each virtual pedestrian, collisions are predicted when several comfort areas intersect in a given temporal window. Velocity-based approaches are an important family of solutions to the collision avoidance problem. A seminal work is the velocity obstacles (VO) approach [Fiorini and Shiller 1998]. It builds, for each agent and each discrete time t, a cone of velocities that will result in collision with an object B in a time $T > t$. Given N agents and M objects, this is a combinatorial optimization problem hard to compute. Coming from the robot motion planning field, it has been fruitfully exploited in crowd simulation. The next step in the VO concept is the reciprocal velocity obstacle (RVO) formulation proposed by van den Berg et al. [2008a]. In this approach, each pedestrian assumes that the rest of the group is also aware of collisions and develop a similar strategy. Compared to the VO approach, RVO guarantees oscillation-free navigation in a crowd (agents do not fall in a loop where two velocities are selected alternatively), but it only guarantees collision avoidance under specific conditions. A different approach within this family is ClearPath [Guy et al. 2009]. ClearPath exploits parallel computation techniques to solve the derived convex optimization trajectory problems generated by the VO method although it can not guarantee that a collision-free solution is found for all situations. A refinement that overcomes this difficulty is ORCA [van den Berg et al. 2011]. ORCA uses RVO to calculate the candidate sets of velocities but imposes that the sets have to be as close as possible to the current agents' velocities. ORCA guarantees collision-free navigation for multiple agents.

Since interaction with the environment is a key property of virtual agents, sensing the environment is crucial in agent-based models. The field of active vision [Gibson 1979; Aloimonos et al. 1988] considers the set of problems in vision under the assumption that the observer is *active*. An observer is called *active* when engaged in some kind of activity whose purpose is to control himself or something. One of the first computer applications that used active vision in artificial animals is [Terzopoulos and Rabie 1997], where artificial fishes are autonomous embodied agents with active perception systems that control their eyes and actuated body. Concerning pedestrians, the work by Renault et al. [1990] proposes synthetic vision-based pedestrians which avoid obstacles inside a narrow corridor. This proposal is also used by Noser et al. [1995] who extend the idea, combining synthetic vision with a dynamic octree as a visual memory of the environment. Based on the ideas of this work, Peters and O' Sullivan [2003] have developed a system for providing virtual humans with the capability of being aware of their neighborhood. The authors call this capability as "Bottom-up visual attention" to remark the fact that the awareness is triggered by the events of the environment without involving high level decision routines. The work by Kuffner [1999] combines path planning techniques, path following strategies and 3D visual perception to guide the virtual humans inside the virtual environment. Graphics rendering hardware is used to build the synthetic vision system which feeds the internal navigation module to re-plan the trajectories if new obstacles are perceived. In the approach proposed by Ondrej et al. [2010], the virtual pedestrians are capable of detecting collision and dangerous scenarios through synthetic vision. Another different approach based on the idea of a synthetic vision is the work by Penn and Turner [2001] based on the space syntax theory. In this approach, pedestrians move using the information stored in a visibility graph, where visibility lines (straight lines of free space) are represented as nodes and the intersections between lines are represented as edges. This graph gives the pedestrians a sense of vision that is used to navigate between nodes that represent free space. The correlation of the simulation results with real data reveals the importance of the position and the point of view in the future decisions about the direction of the march as well as the influence of spatial linear arrangements in the movement patterns.

The existence of relationships among the components of a group influences in their mobility behaviors. The consideration of social relationships among individuals and of certain mobility preferences have proven to be relevant in the traces used to test communication networks [Musolesi and Mascolo 2007; Mei and Stefa 2009; Boldrini and Passarella 2010; Borrel et al. 2009]. Modeling of individual desires or interests can generate realistic community behaviors. For instance, Pluchino et al. [2014] study the dynamics of groups of embodied agents which move along the different rooms of a museum and are attracted by the different artworks. Each agent has a field of vision to detect the goals as well as the obstacles and moves trying to maximize her satisfaction function that depends on the individual patience and interest. Another strategy is used by Batty et al. [2003] to simulate a street parade. In a first step, a set of agents discover the paths to the targets by exploration. Then, the group follows these paths like the insects behaviors based on pheromones. In a second step, congestion is controlled when individuals apply simple group rules, like flocking, reaching a collective steady state.

Other indication of the generality of the model is the capability to articulate interdisciplinary frameworks. The social distances model (SDM) uses the works by E.T. Hall [Hall 1963; 1966] to define proxemics-inspired models. Proxemics studies the groups of individuals in terms of inter-personal distances, imposed unconsciously by cultural and social rules, which generate behavioral patterns. Hall based his studies mainly on observation and developed a notation system for proxemic behavior. The work by Manenti and Manzoni [2011] inserts proxemic considerations in the pedestrian behaviors of a crowd simulation system and study the derived implications. The work by He et al. [2016] presents an algorithm based on ORCA collision avoidance model which includes macro-level driving behaviors based on proxemic rules to simulate coherent groups that follow a leader inside high density scenarios, and Wąs [2008] presents a multi-agent framework for the SDM.

Machine learning can be applied to different steps of the process of pedestrian modeling and simulation. Their use in pedestrian modeling connect with the idea that walking is the result of a learning process in human beings. In their survey, Helbing and Johansson [2009] suggest the existence of an unconscious learning process in the human march to avoid delays and collisions which explains, according to the authors, the emergence of collective behaviors in crowds. Reinforcement Learning (RL) [Sutton and Barto 1998] is a subfield of machine learning suitable to get control modules for different purposes of pedestrian simulation systems. In RL, the agents learn a controller by interacting with the environment. The idea of including the optimization process inside the simulation loop differentiates this approach to those revised in the continuum section and it was also discussed in the work by Batty et al. [2003]. The use of RL with virtual agents is considered by Torrey [2010], Martinez-Gil et al. [2010], Casadiego and Pelechano [2015] in discrete spaces. The works by Martinez-Gil et al. [2014] and Martinez-Gil et al. [2015] describe a multi-agent system (MARL-Ped) in which calibrated embodied agents that simulate pedestrians learn to move inside a continuous virtual environment. The individual behaviors are learned using RL by interacting with the environment. The authors demonstrate empirically that the system is capable of generating plausible pedestrian behaviors with multi-level decision making (at both, the operational and tactical levels).

The main problem with agent-based models is their scalability due to the per-individual computational costs. The processes of environment awareness and decision-making can be computationally expensive when the simulation has many agents. Other problems derived from the individual control are the possible generation of oscillations (alternative selection of the same dynamic configurations) and behavioral artifacts due to model-specific problems such as the inadequate selection of a rule, fails in perception or generalization of a situation, unsolvable configurations or not well-learned situations.

3.5. Data-driven models

The works that use real data can be classified in two main groups. First, works that organize real data, from individual or group behavior, into a collection and use them directly to simulate pedestrian behaviors, which we call "data in the loop" approaches. Second, works that use real data to adjust parameters of an existing model that we call "data in the model" approaches.

In the first group we highlight several works. In the work by Lerner et al. [2007] examples of trajectories are extracted form video sequences and, later, they are used to generate natural pedestrian behaviors in virtual environments. In [Porzycki et al. 2013], a simulation system is synchronized with a flow of real data provided by the sensors of the Microsoft Kinect device. The detected individuals are used to initialize embodied agents in the simulation system. The work by Ju et al. [2010] goes a step forward; different crowd formations and individual trajectories are extracted from real and synthetic examples and, then, new crowds are generated by selecting formation-trajectory pairs that are blended to create interpolated crowd organizations. All these works have a drawback. The interpolation process between different situations can cause artifacts or non realistic behaviors, and they are in general not adequate for high density crowds.

In the second group a variety of approaches are used. The work by Lemercier et al. [2012] calibrates a pedestrian following model to be used in queues simulation. They use real data to fit the parameters of the Aw-Rascle traffic model. In the work by Kim et al. [2016], trajectories from videos are extracted to learn the characteristics of pedestrian dynamics to compute collision-free trajectories that are responsive to external events and environment changes. This approach is capable of simulating low and middle size crowds at interactive rates. In [Lee et al. 2007], individual trajectories of pedestrians in a crowd are obtained from a camera recording and an agent model is learnt using a regression-based method. Other work that uses video tracking to train a model of a crowd is Pellegrini et al. [2009]. Several works use machine learning techniques to calculate parameters of pedestrian models from real data. The work by Kretzschmar et al. [2014] learns collective pedestrian navigation from demonstrations, using a maximum entropy method to model collective navigation from observed trajectories. In the work by Bera et al. [2016] the parameters of the RVO crowd navigation model are adjusted using the feedback of an online tracking system. Using an iterative schema, the predicted trajectories with the RVO model are compared with the real tracked trajectories, and then, the parameters are further refined using a genetic algorithm. The main difficulty that faces this second group of works is data extraction, which can be time-consuming and challenging, specially in crowds. Two surveys in crowded scene analysis using video recording are [Zhan et al. 2008] and [Li et al. 2015]. Mobility models for network evaluation are built and adjusted from users' traces which, in many cases, are captured from a real scenario, using a "data in the model" approach. The survey by Pirozmand et al. [2014] provides a review of the subject from this perspective.

3.6. Summary of modeling categories

As a summary of the different modelling methodologies reviewed in Section 3, Table II presents some of the most important characteristics in each group of the proposed taxonomy.

In order to provide a better insight on the scientific publications of pedestrian models, we have classified journals into groups. In the last column of the table we present the groups in which each modeling methodology appears. The groups with more publications are marked with an asterisk (*). The groups of journals (using the WOS Journal Title Abbreviations) are:

Group A. Comput Graph Forum, Comput Graph Int, Comput Animat Virtual Worlds, ACM Trans. Graph., Comput Animat, Comput Graph, IEEE Comput Graph Appl, IEEE Trans Vis Comput Graph
Group B. Transportation Research Procedia, Transport Res A-B-C-F, Journal of Transportation Engineering, Transport Res Rec: Journal of the TRB, Highway Research Records, Transport Sci, Transportation Research Circular (TRB), J Transp Eng, Traffic Engineering, Transport Plan Techn, Traffic Engineering and Control
Group C. Int J Rob Research, Auton Agent Multi Agent Syst, IEEE Trans. Robot, Neurocomputing, Knowl Eng Rev
Group D. Siam Rev.,PLoS (ONE, Comput. Biology), Nature, PNAS
Group E. Geogr Anal, Trans Arch Inst, Build Environ, Int J Geogr Inf Sci
Group F. Computer Vision and Pattern Recognition, IEEE Trans Pattern Anal Mach Intell, Comput Vis Image Und, ACM Trans Appl Percept, Journal of Computer Vision Research
Group G. Cellular Automaton, Cybern Syst
Group H. Procedia Soc Behav Sci, J Artif Societies Soc Simul, J Soc Psychol
Group I. Phys. Rev. E, Physica A: Statistical Mech Appl, New J Phys, Annu Rev Fluid Mech, Phys Rev E, J Stat Mech, Am J Phys
Group J. Math Models Methods Appl Sci, J Differ Equ, SIAM J Appl Math, Optim Control Appl Methods
Group K. Math. Comp. Simul., ACM Trans Model Comput Simul, Simulation, Simulation Modelling Practice and Theory,Queueing Syst Theor Appl
Group L. Complex Syst, Evolution Nat Struct, Adv Complex Syst

From the data shown in Table II we can extract two main conclusions: the prevalence of the microscopic models respect to the macroscopic ones and the concentration of works in two bibliographic areas: on the one hand research publications related to Computer Graphics and, more specifically, Computer Animation (Group A); on the other hand, publications related to Transportation Research. This indicates that fundamental studies have made way to research in fields where the models are applied.

In Table III we propose a classification of works according to the type of problem they consider. We annotated works classified into one model, not considering theoretic or pure empirical studies.

4. PEDESTRIANS BEHAVIOR VALIDATION

This section reviews techniques used to validate artificial pedestrian behaviors. In the research on this topic three main streams can be identified: the use of pedestrian dynamics descriptors, such as the fundamental diagram; the use of real data to validate the output of the models or simulation systems; the use of visual realism and the way a human observer perceives a simulation as the validity criterion.

Table II. Characteristics and area of applications of Pedestrian Models. *(Predominant) Type:* mac = macroscopic mic = microscopic. *Size:* in= individuals, g = groups (dozens of individuals), c = crowds (hundreds of ind.) lc = large crowds (thousands of indiv.) *Field:* Scientific fields in which the models appear. *Simulators:* Research and/or commercial tools based on this model.

Model	**Type**	**Size**	**Space**	**Field**	**Simulators**	**Journal groups**
Continuum	mac	lc	cont	Traffic, Engineering	PedRoute	A*,B*,J, L
Social Force	mic	g, c, lc	cont.	Planning , Engineering, Simulation	OCEAN, Ped-Sim, SimWalk, Golaem, VISSIM, JWalkerS, JuPedSim	A, B*, D, F, I*, K, L
Optimal control based	mac-mic	in, g, c	cont.	Infrastructure design, Civil eng., Animation	Legion, NOMAD	A, B*, C, J
Cellular Automata	mic	in, g, c, lc	disc.	Research	HERMES	B*, E, G, I*
Agent-based	mic	in, g, c, lc	cont.	Animation, Simulation	PEDFLOW, Massive, AnyLogic	A*, B, C, E, H, K
Queue-based	mac-mic	g, c	cont., disc.	Civil engineering, Architecture design, Safety		B*, D, E, K
Discrete choice	mic	g, c	cont.	Transport analysis and planning, research		B
Data-driven	mic	in, g, c	cont.	Simulation, Evacuation		A*, B

4.1. Validation techniques based on pedestrian dynamics chraracteristics

At the macroscopic level, flow and, therefore, the fundamental diagram is the main tool used in validation. The research group of Schadscheneider and Seyfried have used it to validate the social-force model [Chaibri et al. 2009], CA models [Schadscheneider and Seyfried 2009] and the centrifugal-force model [Chraibi et al. 2010]. It has also been used to validate simulation systems such as Hermes [Zhang et al. 2010; Schadschneider et al. 2013] and PEDFLOW [Zhang et al. 2014].

Unlike at the macroscopic level, there is not a consensus on the microscopic validation parameters. One of the first proposals was the use of flow performance, used in the TRANSYT software [Vincent et al. 1980]. Following this idea Helbing et al. [1997b] proposes a flow performance based on efficiency and discomfort. Both measures are used as evaluation parameters to optimize pedestrian facilities and describe the interaction of pedestrians and their locality. The work by Teknomo [2002] defines efficiency, $\tilde{E}$, as the ratio of the mean velocity in the desired direction to the desired speed, and discomfort, $\tilde{U}$, as a measure of sudden velocity changes due to crashes or avoidance maneuvers. The work by Campanella et al. [2014] proposes two kinds of validation tests: quantitative and qualitative. Quantitative tests measure the average travel time, the fundamental diagram and the bottleneck capacity. Qualitative tests assess different kinds of pedestrian flows: unidirectional, bidirectional and flows in narrows corridors. They have been used in the calibration and validation of the NOMAD simulator.

4.2. Validation techniques based directly on acquired data

The validation with real data is the most direct way of validating models [Pettré et al. 2009] and simulation systems such as NOMAD [Daamen et al. 2013] have used it. In [Paris et al. 2007] and [Karamouzas and Overmars 2012], calibration and validation is carried out using videos of real crowds. In other works, data are extracted to be compared to those generated by the model. Daamen et al. [2014] extract characteristics from real trajectories using non-linear regression techniques and use them to validate a social force model while Guy et al. [2012b] define the Entropy Metric evaluation method. This last metric has two steps; a probability distribution is estimated

Table III. Classification of works inside a model attending the type of problems considered on it.

Type of problem	Category	Model	Reference
Evacuations	Mechanics	Continuum	[Guo et al. 2011]
		Social force	[Yan 2010; Wagoum et al. 2010]
	Stochastic	Discrete choice	[Guo and Huang 2010]
	CA	CA	[Yang et al. 2005; Varas et al. 2007]
			[Meyer-König et al. 2001; Kirchner et al. 2003a]
Panic situations	Mechanics	Continuum	[Colombo and Rosini 2005; Coscia and Canavesio 2008]
		Social Forces	[Helbing et al. 2000]
Historical events	Mechanics	Continuum	[Hughes 2003]
		Social Forces	[Heïgeas et al. 2003]
	Agency	Agent-based	[Shao and Terzopoulos 2005]
Crowd disasters	Mechanics	Continuum	[Johansson et al. 2007; Golas et al. 2014b]
Emergent behaviors (Lanes, bottlenecks, traffic waves, zipper effect. . .)	Mechanics	Continuum	[Hughes 2003; Treuille et al. 2006; Hänseler et al. 2014]
		Social forces	[Helbing et al. 2005; Helbing and Johansson 2009; Chraibi et al. 2015]
		Optim.-based	[Hoogendoorn and Bovy 2003; Guy et al. 2012a; Fu et al. 2014]
	Stochastic	Queue models	[Li and Han 2011]
	CA	CA	[Burstedde et al. 2001; Schadschneider 2002; Kirchner et al. 2003a; Kretz 2009; Nowak and Schadschneider 2013]
	Agency	Agent-based	[Reynolds 1987; 1999; Ondrej et al. 2010; Martinez-Gil et al. 2014]
Complex behaviors (psychologic, social & cultural aspects)	Mechanics	Continuum	[Bellomo et al. 2015]
		Social Forces	[Pelechano et al. 2007; Durupinar et al. 2008]
	Agency	Agent-based	[Shao and Terzopoulos 2005; Manenti and Manzoni 2011]
Collision avoidance problems	Agency	Agent-based	[van den Berg et al. 2008a; Pettré et al. 2009; Karamouzas et al. 2009; Guy et al. 2009; van den Berg et al. 2011; Olivier et al. 2012; He et al. 2016]
Path finding and route choice	Mechanics	Social force	[Freimuth and Lam 1992; Helbing et al. 1997c; Helbing et al. 1997a; Gilman et al. 2005]
	Stochastic	Discrete choice	[Borgers and Timmermans 1986; Hoogendoorn and Bovy 2004a]
	Agency	Agent-based	[Penn and Turner 2001; Bruneau and Pettré 2015]
Specific scenarios (crossings, walkways, corridors . . .)	Mechanics	Continuum	[Helbing 1992a; Asano et al. 2007]
		Optim.-based	[Curtis and Manocha 2014]
	CA	CA	[Blue and Adler 1998; 2001]
	Stochastic	Discrete choice	[Lam and Cheung 2000]
	Agency	Agent-based	[Batty et al. 2003]
Graphics simulation and virtual environments	Mechanics	Continuum	[Chenney 2004; Treuille et al. 2006; Narain et al. 2009; Golas et al. 2014b]
	Agency	Agent-based	[Kuffner 1999]
	Data-driven	Data-driven	[Lerner et al. 2007; Lee et al. 2007; Ju et al. 2010; Lemercier et al. 2012; Kim et al. 2016; Bera et al. 2016]
	CA	CA	[Kneidl et al. 2013]
	Agency	Agent-based	[van den Berg et al. 2008b]
Models of real environments (Airports, parades, marathon . . .)	Mechanics	Continuum	[Hughes 2002]
		Social Force	[Ward 2007]
	Stochastic	Discrete choice	[Young 2007]
	Agency	Agent-based	[Batty et al. 2003; Yilmaz et al. 2009; Pluchino et al. 2014]

for states which best represent the collected data and, then, the simulation system makes predictions from these states. Finally, the work by Singh et al. [2009] proposes a benchmark for evaluating steering behaviors. The authors select a set of metrics of evaluation such as collision, distance, turning based metrics and a set of test cases (e.g. crossings, bottlenecks, one way and two way corridors), although they do not offer data of real pedestrians as a baseline.

4.3. Validation techniques based on perception

The GV2 research group of the Trinity College Dublin has conducted wide research on the perception factors that create believability in simulations with virtual pedestrians and groups. The work by Peters and Ennis [2009] uses data from video and perceptual experiments to measure viewer's perception and model plausible dynamic crowd scenarios. In Hoyet et al. [2013], data captured from thirty actors in different gaits such as walking, jogging and dancing are used. They apply this motion to the same virtual character (one male and one female) to explore whether characteristic motion features transfer across an individual's different gaits. The work by Ennis and O'Sullivan [2012] studies conversing groups. Using combinations of motions, previously identified in real pedestrian conversational groups, and behavior theories from anthropology, the authors simulate plausible groups of conversing pedestrians. The motions are collected from real humans with a motion capture system and the plausibility of the simulations is evaluated by humans with a questionnaire. Several works evaluate how the perception of specific aspects of a simulation affect its quality: Pražák et al. [2011] focus on the footskate artifact, McDonnell et al. [2009] examine the influence of body aspect and motion in the sex perception of walking models and Hoyet et al. [2016] study the influence of pedestrians' shoulder motion in the visual quality of the perceived crowd.

There are strategies that relate qualitative aspects with collected information. For instance, in [Kretzschmar et al. 2014] the Turing test metaphor is used to obtain statistics about qualitative metrics perceived from the simulations. The work by Pelechano et al. [2008] uses a different set of quality metrics based on the sense of *presence*. The evaluation mainly depends on the user perceptions when immersed into a virtual scenario with artificial pedestrians. The work by Olivier et al. [2014] studies the requirements of a virtual reality system, in order to validate the behavior of a human integrated in a virtual crowd, without the difficulties of real world acquisition systems.

A work that combines several approaches is that of Wolinski et al. [2014]. They calibrated five pedestrian models [Reynolds 1987; Helbing and Molnár 1995; van den Berg et al. 2011; Pettré et al. 2009; Ondrej et al. 2010] using three different souces of data: real data acquired in several standard pedestrian situations, fundamental diagrams and human animation sketchs. They use optimization techniques to adjust the parameters for each model and then, a benchmarking analysis is carried out.

5. MULTISCALE SIMULATION

In pedestrian simulation, multiscale issues appear when considering the problem of assembling microscopic and macroscopic approaches. The work by Seyfried et al. [2006] demonstrates that a macroscopic model that reproduces adequately the density-velocity relationship does not necessarily represent correctly the microscopic situation, discarding simple solutions to this problem. Moreover, as Hughes [2003] points out, it is important to understand the transition from the continuum to the microscopic behavior in high density states, since turbulences frequently appear in this interface which are a cause of crowd disasters. Next, we discuss the models that combine both descriptions and some issues that appear when considering scaling up to large crowds.

5.1. Combining microscopic and macroscopic perspectives

A common approach to combine macroscopic and microscopic perspectives models is the use of several layered models that share information. It is important to distinguish this from the layered-behavior based approach; there, layers are behavioral levels of abstraction, while, here, layers describe the movement features of the crowd at different scales. In [Kneidl et al. 2013], a CA model is combined with a floor field and a navigation graph. On the small-scale layer, the space is discretized in cells and pedestrians move from one cell to another using the potential-based CA, managing local interactions. On the large-scale, the navigation graph is used for global path planning.

Other approaches are based on the observation of specific characteristics of the dynamics. Narain et al. [2009] observe that, at high densities, the interpersonal distance is a constraint that reduces the capacity of pedestrian movement. In the microscopic level, a global planner determines the preferred velocity for each pedestrian whereas, at the macro-level, the crowd is considered as a fluid that use a collision avoidance technique based on the idea of unilateral incompressibility constraint. The work by Cristiani et al. [2011] couples microscopic and macroscopic scales in a rigorous mathematical framework. The authors reinterpret the conservation of mass in terms of an abstract mass measure, which behaves as discrete at the microscopic level and as continuous at the macroscopic level. This approach is able to introduce microscopic effects, such as the *follow the leader* behaviour or the *zipper effect*, in a macroscopic context. This work is complemented in [Tosin 2014] with a theory of well-posedness for initial-value problems. Another work that treats the problem from a formal perspective is [Bellomo et al. 2013], who demonstrate how local interactions at the micro-scale are shifted to the dynamics of macro-scale.

5.2. Scaling in number of individuals

Several works have focused on developing techniques to get interactive rates in simulation of crowds. The work by Musse and Thalmann [2001] classifies the problems using the size of the group distiguishing among individuals, groups and crowds. They use different simulation techniques depending on the group size, from programmed behaviors, through behavioral rules to external control.

When the simulation problem concerns groups or low density crowds, path planning algorithms provide good solutions by calculating the individual paths for all the components of the group. Several individual path planning methods can be considered as graph-based navigation techniques. Specifically roadmaps [Kavraki et al. 1996; Sud et al. 2007], navigation meshes [Snook 2000; Oliva and Pelechano 2015], the corridor map method [Geraerts and Overmars 2007] and those based on vector fields, reviewed in [LaValle 2006]. The combination of microscopic and macroscopic solutions is not efficient when thousands of individuals are involved. Navigation fields and, particularly, force fields are commonly combined with collision avoidance techniques as an alternative to path planning computation [Patil et al. 2011]. An alternative to the navigation field is the navigation graph [Yersin et al. 2005; Pettré et al. 2006; Pettré et al. 2007] which represents a set of navigable areas as the nodes of a graph. Navigation is possible only between connected areas. The velocity fields are computed based on the environment description given by the navigation graph. Other approach for steering dense crowds are centralized rule systems [Seer et al. 2010] which use macroscopic rule systems to control the flow.

Top-down decompositions of the problem have also been proposed for crowd simulations. In [Stylianou et al. 2004], the authors use a level-of-detail-based control of pedestrians that populate a virtual environment. If the user maintains a global view, the movement of the pedestrians are computed globally maintaining flux and densities. When the user focuses on a smaller region, the system adopt a detailed and more realistic movement control for pedestrians in that region. Curtis et al. [2016] propose a modular framework in which the whole problem is decomposed into subproblems, that can be solved in separate modules using different techniques or models.

For a broader study on specific crowd problems, important surveys are: [Duives et al. 2013] on emergent behaviors, [Bellomo and Dogbe 2011] with a mathematical/physical perspective, [Zheng et al. 2009] focused on models for building evacuation simulations and [Schadschneider et al. 2009; Zhou et al. 2010]. Books devoted to crowds study are [Pelechano et al. 2008; Pelechano et al. 2016; Thalmann and Musse 2012].

6. CONCLUSIONS

When the reader addresses the existing literature on artificial pedestrians, she has to deal with a large dispersion of publications across different areas and a heterogeneity in the applied methodologies. We have proposed an organization of the literature using three main axes: a categorization of modeling methodologies, an analysis of validation techniques and a review of issues related to scale.

From the literature review conducted in this paper, we detect several evolution lines that mark the current trends in pedestrian simulation. Research has evolved from fundamental papers, devoted to understand pedestrian dynamics, to applied works in which well established models are used. This evolution from analysis to application leads to an increasing importance of computing, which nowadays plays a key role with simulation being an ubiquitous tool in pedestrian dynamics research. The current expansion of the multiprocessor architectures makes possible the implementation of complex microscopic models and systems. An example is the use of parallel real time computation strategies for simulating evacuations [Kemloh et al. 2013]. Multi-core parallel architectures allow to consider each simulated pedestrian capable of managing simultaneously and autonomously different computational processes corresponding with different levels of decision making (operational, tactical or strategic) influenced by inner mental processes (stress, impatience, panic), bring actors closer to real pedestrian behaviors.

Data acquisition is also a rapidly evolving aspect in this field. Tracking systems available for pedestrians detection are capable of sampling not only positions and speeds but also poses and interactions. In addition, the use of smartphones and portable devices are producing a revolution in the way data is captured. Papers devoted to crowdsensing methods are increasing their presence in pedestrian data acquisition area. This will make possible an important advance in those research lines which need accurate data to develop their work. Despite the advance in data acquisition, assessment and validation processes have not experienced a similar progression and more precise and well-established metrics are necessary to evaluate the performance of the simulations. In this sense research on perceptual cues, relevant for creating plausible simulations, is neccesary. In the same direction, new ways of interacting with simulations such as immersive virtual environment technology or teleoperation can provide new methodologies for evaluating the behavioral outcome of a simulation.

Several fields could gain influence in the future to face with new problems in pedestrian modeling and simulation. Complex systems is a research area that has not been sufficiently exploited. Pedestrian groups and crowds are typically complex systems where local interactions display emergent collective behaviors that cannot be explained as the sum of individual interactions. Several theories could help to understand instabilities and phase changes in crowds [Moussaïd et al. 2012], discovering power-laws for scaling behaviors [Newman 2011; Karamouzas et al. 2014] or proposing mathematical structures for generating multiscale dynamical effects [Bellomo et al. 2012]. Information theory can also provide insights in this subject. Complexity theories could be used to classify patterns of movements and find similarities between maneuvers and local interactions or to find new quality metrics (e.g. the work by Guy et al. [2012b] uses the notion of entropy to define a quality metric for simulators) or new forms of representation. Game theory is becoming important for modelling multiobjective situations in pedestrian scenarios. In a multiobjective scenario, pedestrians have to achieve two or more competing goals. These problems do not have a unique solution, but require a trade-off between goals by means of Pareto-solutions. Finally, machine learning, and more specifically the use of learning techniques to build decision making modules for microscopic simulators, is a promising area.

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