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División de Economía
Universidad Carlos III de Madrid
Calle Madrid, 126
28903 Getafe (Spain)

THE q THEORY OF INVESTMENT UNDER UNIT ROOT TESTS

David de la Croix and Omar Licandro*

Abstract

We test a q investment model for Belgium using a multivariate cointegration approach. The introduction of the degree of capacity utilization *duc*, in addition to investment and average q, is necessary to determine the cointegration space. This supports the idea that marginal q differs from average q by a factor which is a function of *duc*, as suggested by Licandro (1992).

Key Words
Cointegration Tests; Degree of Capacity Utilization; Monopolistic Competition; q Investment Theory; Quantity Rationing Model

* David de la Croix, Catholic University of Louvain. Omar Licandro, Universidad Carlos III de Madrid. We are grateful to Frédéric Aprahamian and to Luc Bauwens for their useful comments. Paul Olbretchts kindly provided the statistical material

Average q and marginal q

According to the q theory of investment, in the presence of convex adjustment costs for capital input, investment depends upon the ratio between the discounted value of all expected future profits generated by the installation of an additonal unit of capital and the purchase price of investment goods. This ratio, called marginal q, is equal to the average q under the conditions derived by Hayashi (1982). In that case, the discounted value of future profits is equal to the average stock market value of capital. The main interest of this approach is that expectations about future profits are contained in the q term.

Various authors have tried to test empirically the influence of stock market value on firms' investment (cf. Abel and Blanchard (1986)). Average q turns out to be highly significant in explaining investment, although it is unsatisfactory for various reasons: Lagged values of q also have significant coefficients (while the theory suggests that current q includes all available information about future profits); moreover, the residuals are autocorrelated. These elements suggest that marginal q differs from average q and that this difference is a function of missing variables (see Galeotti and Schiantarelli (1991)).

Following an idea initially proposed by Malinvaud (1987), Licandro (1992) has shown that the difference between marginal q and average q could be explained by the ratio of production to capacities, called the degree of capacity utilization and denoted *duc*. The idea is that if firms face demand uncertainty at the time of their input (and possibly also price) decision, the predetermined capacity can be underutilized. Together with average profitability, the intensity of this underutilization determines the marginal value of the firm. In this case, average q differs from marginal q by a factor which depends on the sequence of all expected future *duc*. In the long run, investment depends on average q and *duc*. This framework can be compared to another recent contribution by Schiantarelli and Georgoutsos (1990) who stress the role of imperfect competition: Contrary to Licandro's (1992) model, their firm can always adjust employment in order to fully utilize its capacity. However, monopolistic competition allows them to include output per unit of capital as an additional explanatory variable in investment equation.

In order to investigate the relevance of Licandro's (1992) contribution, we test the existence of two different cointegration relations. The first relation is derived from the usual classical q investment model, with the simplifying assumption that the adjustment cost function is exponential. In this case, the investment rate α is simply a function of the logarithm of q, which is the average value of the firm (measured by the stock market index) divided by the price of investment. This equation assumes the equality between average q and marginal q:

$$\alpha = b_0 + b_1 \, ln \, q \tag{1}$$

The second specification we want to test is taken from Licandro (1992). It is direclty comparable to the classical model and takes the following long-run form:

$$\alpha = b_0 + b_1 \, ln \, q + b_2 \, ln \, duc \tag{2}$$

where b_2 is positive: the closer the firm is to the full utilization of capacity, the more it invests.

The data

The Belgian data we use are quarterly and extend from 1971:3 to 1990:2 (76 observations). The investment rate α is defined as i_t/k_{t-1} where i is real investment according to VAT declarations. The capital stock k has been built using the investment series with an annual depreciation rate of 10%. Average q is measured by the stock market index of industrial values divided by an index of the wholesale price of the investment goods sector. The *duc* comes from the business surveys of the Belgian National Bank. Before doing any multivariate analysis, we have to test the order of integration of the variables α, $\ln q$ and $\ln duc$. Note that the classical theory suggests that these variables are stationary since the endogenous response of investment should ensure that *duc* is driven to its "normal" level and q to unity in the long run. To test the level of integration of any variable y, we use a standard ADF test selecting carefully the number of lags p in the following regression:

$$\Delta y_t = \mu + a y_{t-1} + \sum_{i=1}^{p} \Delta y_{t-i} + \epsilon_t$$

The role of these lags is to cope with remaining autocorrelation in the residuals. However, if their number is too large the power of the test is weakened. Therefore, careful inspection of whether the residuals are white noise and whether the lags are significant, as well as the use of some information criterion, can give important guidance. The estimations are presented in Table 1. Together with the number of lags and the value of the ADF test (τ_a) we give the Akaike information criterion (AIC), the LM test for the autocorrelation of residuals up to order 4 (whose critical value at 95% is a $\chi^2_4 = 9.49$) and an LM test of heteroscedasticity regressing the squared residuals on the squared levels of the estimated dependent variable (whose critical value at 95% is a $\chi^2_1 = 3.84$). The best model with respect to these criteria is given in italics. From MacKinnon (1990), the critical value below which stationarity is rejected is -2.90 for τ_a, implying that our three variables are not stationary in levels. The optimal lag length is 4 for α and $\ln duc$ while the best representation for $\ln q$ is a pure random walk.

Table 1

ADF Unit Root Tests

	α				$\ln q$				$\ln duc$			
	τ_a	AIC	AR4	HET	τ_a	AIC	AR4	HET	τ_a	AIC	AR4	HET
DF	-2.69	-11.24	6.85	0.06	*-0.14*	*-4.97*	*5.18*	*1.82*	-1.72	-7.92	19.53	0.09
ADF1	-2.32	-11.20	10.4	0.49	-0.43	-4.95	3.5	4.81	-2.54	-8.07	7.21	4.08
ADF2	-1.15	-11.32	10.8	2.25	-0.77	-4.92	3.06	0.38	-2.74	-8.03	7.26	5.92
ADF3	-0.68	-11.32	7.07	3.42	-1.01	-4.88	2.65	0.13	-2.65	-7.98	7.05	5.56
ADF4	*-1.25*	*-11.34*	*6.32*	*0.01*	-0.92	-4.82	1.04	0.17	*-1.96*	*-8.00*	*1.77*	*1.81*
ADF5	-1.45	-11.31	6.78	0.01	-1.2	-4.79	3.52	0.79	-2.08	-7.95	5.53	2.21
ADF6	-1.16	-11.31	5.01	0.03	-1.13	-4.73	3.19	0.76	-2.42	-7.96	2.3	2.94
ADF7	-1.08	-11.25	6.3	0.03	-1.12	-4.67	15.5	0.91	-2.4	-7.90	4.94	2.86
ADF8	-1.58	-11.27	3	0.11	-0.68	-4.64	6.03	0.9	-2.32	-7.83	5.31	2.68

An argument against this result could be that there is very slow mean reversion in all three variables, which is not adequately captured by the ADF test. It would be too optimistic to hope that twenty years of data would reveal such slow mean reversion. To cope with this argument we have computed the non-parametric test due to Phillips and Perron (1988) which should allow for a larger class of error term distributions. Their statistic $Z(\tau_a)$ are presented in Table 2 for different values of the truncation lag parameter l (cf. the article of Phillips and Perron for details). The critical value is the same as in the parametric ADF test. This test raises some doubts about the robustness of the conclusion concerning investment rate, since for $l = 1$ the unit root hypothesis is rejected. However, since we have no good reason for selecting any particular value of l, we are not able to draw a definitive conclusion. Since for all other values of l the test does not reject the unit root hypothesis, we treat the investment rate as an integrated variable. Non-reported tests on the first differences of the variables show that the three variables are $I(0)$ in variations. We now try to find some cointegration relationship between these variables.

Table 2
Non-Parametric Unit Root Tests

	DF	l=1	l=4	l=7	l=10
α	-2.69	-3.11	-2.54	-2.57	-2.41
$\ln q$	-0.14	-0.13	-0.12	-0.11	-0.11
$\ln duc$	-1.72	-1.29	-1.39	-2.76	-2.17

The cointegration space

Using the methodology of Johansen (1988), we estimate a system composed of three autoregressive error correction equations. The determination of the dimension of the cointegration space can be found by a procedure which is a kind of multivariate generalization of the augmented Dickey-Fuller test. Once this rank is determined, standard asymptotic theory can be used for testing on the cointegrating vectors and analyzing the shape of the error correction terms. We are interested in identifying the cointegration space, if any, and testing whether $\ln duc$ is important in this identification. In order to avoid mispecification problems due to the omission of important lags, we have chosen a reasonably large number of lags (7) in the VAR with respect to the number of observations. The following model is tested:

$$\Delta x_t = \mu + \sum_{i=1}^{6} \gamma_i \Delta x_{t-i} + \xi \beta' x_{t-7} + \epsilon_t$$

where $x_t = [\alpha_t \quad \ln q_t \quad \ln duc_t]'$, μ is a 3×1 vector of constants, γ_i are a 3×3 matrices, ξ and β are $3 \times r$ matrices, r being the dimension of the cointegration space. Table 3 presents the cointegration tests used to determine r, the number of cointegration vectors. Two statistics are provided: the first is based on the maximal eigenvalue of Johansen's stochastic matrix; the second uses the trace of this matrix. The last column gives the threshold above which the null hypothesis is rejected.

Table 3
Dimension of the cointegration space

	null	alternative	statistics	95% threshold
(eigenvalue)	$r = 0$	$r = 1$	22.93	22.00
	$r \leq 1$	$r = 2$	10.68	15.67
(trace)	$r = 0$	$r \geq 1$	35.45	34.91
	$r \leq 1$	$r = 2$	12.52	19.96

The two statistics unanimously allow us to conclude that there is only one cointegration relationship in the model. This unique cointegration vector, normalized to have a unit weight for α, implies the following long-run relationship:

$$\alpha = 0.049 + 0.011\, ln\, q + 0.083\, ln\, duc$$

The signs are compatible with the theoretical model. This can be compared to what we would have obtained using the Engle and Granger (1987) estimates:

$$\alpha = 0.052 + 0.007\, ln\, q + 0.082\, ln\, duc$$

Johansen's procedure also allows to test the presence of cointegrating vectors in each equation of the system, using a likelihood ratio test whose critical value is distributed as a χ^2_1. This aims at analyzing whether the variations of the variables adjust to the long run of the model. This test in presented in Table 4: Only the investment rate and the degree of utilization of capacity adjust in order to satisfy the long run relationship.

Table 4

Absence of cointegrating vectors

equation	$\Delta\alpha$	$\Delta \ln q$	$\Delta \ln duc$
test	11.48	0.00	4.90

We now turn to our main question, and test whether *duc* is important in the determination of the cointegration space. If *duc* is of no importance in the long run, its weight in the cointegration relation should be zero. We therefore test the exclusion of *duc* from the cointegration space using a likelihood ratio test whose critical value is distributed as a χ^2_1. The result is presented in Table 5 together with the exclusion tests for the other variables. From this, it is clear that *duc* helps significantly to determine the cointegration space.

Table 5

Exclusion from the cointegration space

variable	α	$\ln q$	$\ln duc$
test	10.29	8.86	10.07

Moreover, if we try to specify the system without $\ln duc$ by considering only two equations (for α and $\ln q$), Johansen's test implies that the dimension of the cointegration space is zero (Table 6).

Table 6

Dimension of the cointegration space without $\ln duc$

	null	alternative	statistics	95% treshold
(eigenvalue)	$r = 0$	$r = 1$	13.51	15.67
(trace)	$r = 0$	$r = 1$	15.96	19.96

This leads to the conclusion that $\ln duc$ not only improves significantly the determination of the cointegration space, but moreover seems necessary to locate some long-run relationship involving investment and average profitability. These results give some support to the idea that, in the long run, average q diverges from marginal q and that this difference can be explained by the ratio of effective output to total capacity.

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