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Gloria González-Rivera, Javier Maldonado, Esther Ruiz

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Gloria González-Rivera*[1], Javier Maldonado[2], and Esther Ruiz†[2]

[1]Department of Economics, University of California, Riverside.

[2]Department of Statistics, Universidad Carlos III, Madrid.

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**Abstract**

We propose a new global risk index, Growth-in-Stress (GiS), that measures the expected fall in a country GDP as the global factors, which drive world growth, are subject to stressful conditions. Stress is measured as the 95% contours of the joint probability distribution of the factors. With GDP growth rates of a sample of 87 countries from 1985 to 2015, we extract three global factors: a first world growth factor driven mainly by all industrial and emerging countries; a second factor driven by "other developing" countries in Africa and America; and a third factor that is mostly related to East Asian economies. We find that the average GiS across industrialized, emerging and other developing countries has been going down from 1987. Post 2008 financial crisis, mainly from 2011 on, the world overall has fallen in a state-of-complacency with the

*Corresponding author.

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cross-sectional average GiS falling quite dramatically; in 2015 the average worst outcome seems to be no growth at the 95% probability factor stress. However, the cross-sectional dispersion within groups is quite wide. It is the smallest among industrialized countries and the largest among emerging and developing countries. We also measure the factor stress on different quantiles of the GDP growth distribution of each country. We calculate an Armageddon-type event as we seek to find the average 5% GiS quantile under the extreme 95% probability events of the factors and find that it can be as large as an annual 20% fall in GDP.

**Keywords:** Business Cycle, Dynamic Factor Model, Factor uncertainty, Predictive regressions, Principal Components, Quantile regressions, Stress Index, Value in Stress.

# 1 Introduction

There is a large evidence on the presence of cross-country links in macroeconomic fluctuations with world and regional business cycles having different effects on developing and developed economies. For example, Kose et al. (2003), Imbs (2010), Kose et al. (2012) and Bjornland et al. (2017) conclude that the world factor is more important in explaining fluctuations in developed stable economies, whereas country-specific factors are more important in developing, volatile economies. Similarly, Ozturk and Sheng (in press) show that some regional recession episodes are associated with higher uncertainty than global recession episodes. For instance, the peaks of uncertainty in Indonesia and South Korea are higher around the 1997 Asian financial crisis than around the recent global recession. The presence of world and regional business cycles leads to the possibility of exploring a macroeconomic global risk when these common cycles are subject to extreme negative scenarios. The related literature considering common factors and macroeconomic risk has considered the factors fixed at their (estimated) expected values. However, if factors are drivers of economic growth, the potential growth risk must naturally be a function of factor risk. Thus, we need to consider factors beyond their expected values and to explore their lower quantiles where stress is measured.

The proposed methodology is based on using predictive quantile regressions of output growth augmented with common factors as predictors. The factors are extracted using principal components (PC) from a large set of macroeconomic aggregates modeled using Dynamic Factor Models (DFMs), and their joint probability density is computed by the subsampling method proposed by Maldonado and Ruiz (2017). To construct the risk index for each country, we consider the Value-in-Stress (ViS) risk measure proposed by González-Rivera (2003) in the context of monitoring capital requirements to control market risk. Adapted to a macroeconometric context, the ViS, denoted as GiS for Growth-in-Stress, is defined as (minus) the lowest expected Gross Domestic Product (GDP) growth (or quantile of growth) in a given country when there is extreme stress in the macroeconomic common factors. We calculate the risk exposure of each country to extreme changes in the macroeconomic factors and the country's ability to withstand stressful scenarios, which may eventually generate economic crises. One important advantage of our approach is that, together with the calculation of

GiS, we are able to concurrently learn the magnitude of the factor stress; in other words, the stressful scenarios are endogenously determined, which is very different from the standard practice in stress testing where the stressful scenarios are chosen *a priori*. We also analyze whether the risk exposure is different across industrialized, emerging and other developing countries. We calculate the GiS of 87 countries using the annual data on macroeconomic growth from 1985 to 2015, obtained from the World Bank's World Development Indicators and supplemented with the International Monetary Fund's World Economic Outlook (WEO) data base.

The most recent literature in macroeconomic risk analyzes two different but related dimensions of risk. Some works focus on uncertainty indexes and some others on downside risk to economic growth. The main difference between uncertainty and risk indexes is that the former measure variances (uncertainty) while the latter measure the lower tail (risk) of growth. Though variances take into account deviations from the mean in both directions, a policy maker, who wishes to monitor downside risk, would be more interested in the lower quantiles of growth. Our work measures the effect of stressed factors not only on the average growth but also on different quantiles of growth.

The proposed macroeconomic risk index is related to the macroeconomic uncertainty indexes proposed by Jurado et al. (2015) who use augmented predictive regressions based on PC factors, and by Henzel and Rengel (2017) who implement two step Kalman filter factors. However, there are two main differences with our work. First, Jurado et al. (2015) and Henzel and Rengel (2017) construct uncertainty indexes based on weighted combinations of the uncertainty of the idiosyncratic components while we are concerned with the common factors instead of the idiosyncratic noises. Second, instead of focusing on conditional variances, we measure the risk in the tails of the factors' joint distribution, i.e. we consider multivariate quantiles instead of variances. Other uncertainty indexes are proposed by Rossi and Sekhposyan (2015) and Ozturk and Sheng (in press), which are based on survey data from the European Central Bank Survey of Professional Forecasters and the Consensus Forecasts, respectively; see Ozturk and Sheng (in press) for a detailed survey of the literature on economic uncertainty indexes.

More closely related to our proposal is the risk index proposed by Adrian et al. (in press) who model the full distribution of future real GDP growth as a function of current financial and economic conditions. They estimate a semi-parametric distribution of growth using quantile regressions. Risk is computed either as the expected shortfall of this distribution or using an entropy measure with respect to the unconditional distribution of growth that is time invariant and based on quantile regressions in which only the constant term is included. In this latter case, they quantify upside and downside vulnerability of future GDP growth as the "extra" probability mass that the conditional density assigns to extreme right and left tail outcomes relative to the probability of these outcomes under the unconditional density. There are three main differences between our proposed GiS index and that of Adrian et al. (in press). First, the GiS is based on stressed conditions of the common factors and their effects on growth while Adrian et al. (in press) consider that factors fixed at their estimated mean values. Second, the factors considered in this paper are world and regional factors while Adrian et al. (in press) focus on financial local factors. Finally, Adrian et al. (in press) focus their analysis on growth risk in USA, while we extend our analysis to 87 countries around the world. Our methodology is also related to that proposed by Giglio et al. (2016) who also fit factor augmented quantile regressions to evaluate the ability of various measures of systemic financial risk to predict real activity outcomes. In this case, there are also important differences with our paper. As in Adrian et al. (in press), they consider the effect of financial common factors, which are treated as observable. However, they are not proposing a proper risk measure for growth but just predicting it. In their empirical application, they consider US and European countries but not developing or emerging ones.

The rest of the paper is organized as follows. In section 2, we describe the GiS index. In section 3, we estimate the common factors and GiS index for a large number of industrialized, emerging and other developing countries. In section 4, we conclude. An on-line appendix provides detailed results of the estimation of the predictive and quantile regressions.

# 2 Growth-in-Stress Index

The choice of key macroeconomic variable(s) is crucial to describe the state of the economy. Following the standard choice in the related macroeconomic literature, we focus on GDP growth as representative of the business cycle. Let $GDP_{it}$ be the GDP of country $i$ at time $t$, and define the corresponding growth as $y_{it} \equiv \triangle log(GDP_{it})$. For each country, we forecast growth by the following single equation autoregressive model augmented with factors

$$y_{it+1} = \mu_i + \phi_i y_{it} + \sum_{k=1}^{r} \beta_{ik} F_{kt} + u_{it+1}, \tag{1}$$

where $F_{kt}$, for $k = 1, ..., r$ are the $r$ unobserved common factors, also known as diffusion indexes, that summarize the variations of the large cross-section of growths and $u_{it}$ is a white noise process; see Stock and Watson (1999) and Forni et al. (2000) for the introduction of factor-augmented predictive regressions. Factor augmented regressions as that in (1) have been considered by Jurado et al. (2015) to construct their uncertainty index.

If the interest is not only the center of the probability distribution of growth but also its lower or upper tails, we can consider a factor-augmented quantile regression model that estimates the $\tau$ quantile of $y_{it+1}$ conditional on $y_{it}$ and $F_t$; see Ando and Tsay (2011) for factor-augmented quantile regressions. In particular, we consider the following model

$$q_\tau(y_{it+1}|y_t, F_t) = \mu_i(\tau) + \phi_i(\tau) y_{it} + \sum_{k=1}^{r} \beta_{ik}(\tau) F_{kt} + v_{it+1}, \tag{2}$$

where $q_\tau(y_{it+1}|y_t, F_t)$ is the $\tau$th quantile of $y_{it+1}$ conditional on $y_{it}$ and $F_t = (F_{1t}, ..., F_{rt})'$, and $v_{it}$ is an uncorrelated sequence such that $q_\tau(v_{it+1}|y_t, F_t) = 0$. Quantile regressions with factors as explanatory variables have also been considered by Adrian et al. (in press) to compute their risk index and by Giglio et al. (2016) to evaluate the ability of various measures of systemic risk to predict real activity outcomes. The quantile approach is appropriate for evaluating the potentially asymmetric and nonlinear association between global and regional factors and economic growth.

The GiS index for country $i$ at time $t+1$ is defined as the minimum expected growth (or quantile of growth) of the country when the underlying factors are subject to $\alpha$-probability extreme scenarios, that is

$$GiS_{t+1}^{(i)} = -\min h(y_{i,t+1}) \tag{3}$$

$$s.t. \quad g(F_t, \alpha) = 0$$

and depending on whether the interest is in the average growth or in a quantile of growth, $h(y_{it+1})$ is given by the predicted $y_{it+1}$, as defined in equation (1), or by the predicted

$q_\tau(y_{it+1}|y_t, F_t)$, as defined in equation (2), respectively. Note that to easy the interpretation, we multiply the sign of $h(y_{it+1})$ by $-1$ so that larger values of GiS mean larger risk. The constraint in (3) requires to know the multivariate probability density of the factors, from which the function $g(F_t, \alpha) = 0$ is a contour. The function $g(F_t, \alpha) = 0$ is the ellipsoid that contains the true factor vector, $F_t$ with probability $\alpha$. For instance, if $\alpha = 95\%$, the ellipsoid will contain 95% of the factor events. Those values of $F_t$ on the boundary of the ellipsoid $g(F_t, \alpha) = 0$ are considered the extreme events. Therefore, if $\alpha = 0.95$, the GiS measures the minimum expected growth (or quantile of growth) at time $t + 1$ when the factors are on the boundary of the ellipsoid $g(F_t, 0.95) = 0$. In Figure 1, we illustrate graphically how to obtain the GiS for two different probability contours, $\alpha_1 < \alpha_2$, when the number of factors is two, i.e. $r = 2$. First, we plot the two ellipsoids, $g(F_t, \alpha_1) = 0$ and $g(F_t, \alpha_2) = 0$. Second, we plot the so called iso-growth curves. These are the combinations of $F_1$ and $F_2$ that produce the same predicted value of growth (or quantile of growth) $h(y_{i,t+1})$. For $\alpha_1$, the GiS is given by the predicted value of growth corresponding to the iso-growth curve that is tangent to $g(F_t, \alpha_1) = 0$, while for $\alpha_2$, the GiS is given by the predicted value of the iso-growth curve tangent to $g(F_t, \alpha_2) = 0$. Observe that, as result of the minimization exercise, we will obtain not only the GiS but also the combination of factors giving rise to this GiS. This combination corresponds to the point where the ellipsoid and the iso-growth curve are tangent. In Figure 1, $\text{GiS}_1$ is generated by the combination $(F_{11}, F_{21})$ while $\text{GiS}_2$ is generated by $(F_{12}, F_{22})$. This is an important advantage of our approach; once the $\alpha$-probability level is chosen, the stressful scenarios are endogenously determined, which is very different from the standard practice in stress testing exercises where the stressful scenarios are chosen *a priori*.

The factors to calculate the GiS in (3) are modeled using a dynamic factor model (DFM). The specification of the DFM follows common practice in the literature; see Jurado et al. (2015), Giglio et al. (2016) and Henzel and Rengel (2017), among others.[1] We consider the following DFM

$$Y_t = PF_t + \varepsilon_t, \tag{4}$$

[1] Note that our approach is different from other related DFM models as we do not specify *a priori* global and specific factors for industrialized, emerging and other developing countries as in Kose et al. (2012) or global and regional factors as in Aastveit et al. (2016) and Bjornland et al. (2017).

where $Y_t = (y_{1t}, ..., y_{Nt})'$ is the $N \times 1$ vector of growth rates observed at time $t$ for $t = 1, ..., T$; $P$ is the $N \times r$ matrix of factor loadings such that $P'P$ is a diagonal matrix with distinct entries arranged in decreasing order; $F_t$ is the vector of unobserved common factors; and $\varepsilon_t = (\varepsilon_{1t}, ..., \varepsilon_{Nt})'$ is the $N \times 1$ vector of idiosyncratic noises, which are assumed to be potentially weakly cross-correlated and heteroscedastic; see Bai (2003) for the assumptions on model (4) to guarantee the asymptotic validity of the Principal Components (PC) factor extraction procedure. The number of factors $r$ is assumed to be known.

We extract the factors using PC due to its well known computational simplicity and popularity; see Bai and Ng (2008a) for a review of PC factor extraction. For a unique identification of the factors, we assume $\frac{F'F}{T} = I_r$; see Bai and Ng (2013) for a discussion on identification issues in the context of PC factor extraction. The $r \times T$ matrix of extracted factors $\hat{F} = (\hat{F}_1, ..., \hat{F}_T)$ is given by $\sqrt{T}$ times the eigenvectors corresponding to the $r$ largest eigenvalues of the $T \times T$ matrix $Y'Y$ where $Y = (Y_1, ..., Y_T)$. The matrix of estimated factor loadings, $\hat{P}$, is computed by $\hat{P} = \frac{Y\hat{F}'}{T}$. Bai (2003) shows that, if $\frac{\sqrt{N}}{T} \to 0$ when $N, T \to \infty$, then $\hat{F}$ is a consistent estimator of the space spanned by the true factors. Finally, to obtain the joint probability density of the factors to compute $g(F_t, \alpha)$ in (3), we follow Maldonado and Ruiz (2017) who propose constructing ellipsoids based on the point-wise asymptotic normality of the PC estimated factors (Bai, 2003) with a covariance matrix computed by using a subsampling procedure, which is designed to measure parameter uncertainty associated with the factor estimation.[2]

The estimated factors are substituted either in equation (1) or in equation (2) depending on whether the interest is on the macroeconomic global risk affecting the center or one particular quantile of the growth distribution. In the former case, the estimated factors are substituted in equation (1) and the predictive regression parameters are estimated by Least Squares (LS) as in Stock and Watson (1999). When the interest is on a particular quantile of the growth distribution, the parameters of the quantile regressions in equation (2) can be

[2]Note that the bootstrap procedure implemented by Aastveit et al. (2016) to compute prediction intervals of the factors underestimates the uncertainty as they do not consider parameter uncertainty; see Maldonado and Ruiz (2017) who show that the subsampling correction of the covariance asymptotic matrix provides point-wise prediction regions for the factors with coverage very close to the nominal.

estimated as in Koenker and Bassett (1978); see Ando and Tsay (2011).[3] Recently, Ohno and Ando (2018) propose a shrinkage procedure to estimate the parameters of factor augmented predictive regressions, which can be implemented in both (1) and (2).

Finally, with the estimated ellipsoids containing the true factors, $g(F_t, \alpha) = 0$, and the estimated predictive regression or quantile regression augmented with the factors, $h(y_{t+1})$, it is possible to solve numerically the minimization problem in (3) by evaluating (1) or (2) in *all* points of the ellipsoid[4].

# 3 GiS indexes in industrialized, emerging and other developing countries

We compute the GiS of 87 countries.[5] The data consists of GDP measured at constant national prices and observed annually from 1985 to 2015 for $N = 87$ countries, obtained from the World Bank's World Development Indicators and supplemented with the International Monetary Fund's WEO data base. The same data base has been considered by Kose et al. (2012) for a larger number of countries (106) and variables (GDP, real private consumption and real fixed asset investment) over the period 1960-2008. Given the dramatic shift of the global landscape since the mid-1980s, we only consider the period starting in 1985, which is defined by Kose et al. (2012) as the wave of globalization. On the other hand, we extend the sample period with data observed after the 2008 global financial crisis. GDP is transformed to growth rates by taking the first differences of log of GDP. Consequently, the time series

[3]Stock and Watson (2002a) show the consistency of the LS estimator while Bai and Ng (2006) derive its asymptotic normality. Bai and Ng (2006, 2008b) show that when the generated regressors are the estimated factors, they can be plugged in as if they were observed as far as $\frac{\sqrt{T}}{N} \to 0$ for $N, T \to \infty$ in regression models or $\frac{T^{5/8}}{N} \to 0$ for $N, T \to \infty$ in quantile regressions, respectively.

[4]Note that this "brute force" approach of minimizing growth is only feasible when the number of factors is relatively small. When the number of factors is large, one needs to use optimization techniques, for example, second-order cone programming (SOCP); see Bertsimas et al. (2013) and the references there in. Alternatively, Chassein and Goerigk (2017) proposed using regret combinatorial optimization.

[5]The software to estimate the GiS has been developed by the third author in R programming language. It is available upon request.

length is $T = 30$.

## 3.1 Estimating the factors

Previous to factor extraction, the growth series are demeaned and standardized. Notice that the demeaning procedure eliminates differences in mean growth rates among countries. To identify the number of common factors, we implement the procedure proposed by Alessi et al. (2010), which selects $r = 3$. After extracting the factors using PC, we obtain the idiosyncratic residuals and identify outliers as those residuals exceeding six times the interquartile range[6]; see Marcellino et al. (2003), Artis et al. (2005), Stock and Watson (2002b) and Breitung and Eickmeier (2011) who also use the interquartile range to identify outliers in the context of DFM. We identify the following outliers due to exceptional events: i) the consumer response to the Mexican Peso crisis in 1994, which caused a fall in Mexican growth in 1995, see McKenzie (2006); ii) in 1994, Rwanda's growth fell due to the genocide against the Tutsi, see Lopez and Wodon (2005); iii) the political crisis of 2002 in Madagascar that seriously hampered economic growth, see Vaillant et al. (2014). As in Breitung and Eickmeier (2011), we substitute each outlying original growth by the median of the last previous five observations. From now on, the growth rates considered in the analysis, denoted by $y_{it}$, are the corresponding growth rates corrected by outliers.

After demeaning and standardizing the outlier-corrected growth series, $y_{it}$, Alessi et al. (2010) still selects $r = 3$ common factors explaining 42% of the total growth variability with the first factor accounting for 20%. These percentages are comparable to those found by other authors in related research. For example, Aastveit et al. (2016) find that global and regional factors explain around 30% and 20% respectively of the business cycle variation in four small open economies (Canada, New Zealand, Norway and United Kingdom). Kose et al. (2003) attribute up to 35% of the variance in GDP across G7 countries to one common international business cycle. Finally, Bjornland et al. (2017), who analyze quarterly real

[6] Kristensen (2014) analyzes the effects of outliers on PC factor extraction and predictive regressions. He proposes a robust factor extraction procedure based on Least Absolute Deviations (LAD). However, this robust procedure cannot be implemented in our context because of the lack of an asymptotic distribution, which is needed to obtain the probability ellipsoids containing the factors.

GDP growth from 1978 to 2011 for 33 countries covering four geographical regions and both developed and emerging economies, report that the common business cycle accounts for 5% to 45% of the total variability of growth depending on the particular region of the world and the period of time considered. Consequently, we extract three factors by PC and compute their confidence bounds as well as those for the corresponding weights, $\hat{P}$, using the subsampling procedure proposed by Maldonado and Ruiz (2017).[7] After visual inspection, the idiosyncratic components are considered approximately stationary.[8]

In Figures 2 to 4, we plot the estimated factors and weights corresponding to the DFM in equation (4) together with their 95% bounds. Following Kose et al. (2012), the countries are classified into three groups: i) Industrial whose weights are represented by red bars; ii) Emerging markets represented by blue bars; and iii) Other developing countries represented by gray bars. In Table 1, we report the classification of each country and we list the countries in the same order as their weights plotted in Figures 2 to 4. Consider the first factor plotted in Figure 2 together with its weights and corresponding 95% confidence intervals. This factor can be interpreted as a world growth factor with all industrial and emerging countries but Morocco, Peru and China having positive weights. In the case of Morocco, the weight is not significant while in Peru and China, the weights are negative although relatively small in magnitude. We also observe that the weights are negative and relatively small or non-significant in several "other developing countries", mainly in Africa. It is also remarkable that the weights of India and Indonesia, although positive, are relatively small. The dynamic profile of the estimated global factor is very similar to that found in Kose et al. (2012), Aasveit et al. (2016) and Bjornland et al. (2016), with declines in the early 1990s, in 2000/2001 during the bursting of the dot-com bubble, and in 2008-2009 during the Great

[7]Kose et al. (2003) and Kose et al. (2012) extract common factors of macroeconomic variables by implementing a data augmentation Bayesian procedure based on the spectral density matrix. Alternatively, Bjornland et al. (2017) implement Bayesian estimation of the corresponding state space model using Gibbs simulation. These procedures also provide predictive densities for the factors.

[8]We do not formally test for non-stationarity of the idiosyncratic noises because the temporal dimension is rather small and the lack of power of most popular nonstationarity tests is well known in this case; see, for example, Kwiatkowski et al. (1992). Banerjee et al. (2008) also point out related problems associated with cointegration tests in the context of non-stationary panels.

Recession, which is by far the most severe.

In Figure 3, we plot the second factor together with its weights. We observe that this factor is negative until the mid-1990s and then is positive with a relatively weak drop during the Great Recession. This factor has positive weights in most "other developing" countries in Africa and America. Furthermore, China's weight is not significant while India's is positive and large. As far as we know, this factor has not been identified before. Other related works, as in Aastveit et al. (2016), have not included African countries or developing countries in South America. Only Kose et al. (2012) extract factors using data from a similar set of countries as those considered in this paper. However, they specified *a priori* common factors associated with industrialized, emerging and other developing countries. According to our results, the factors are not exactly associated with these groups of countries but with a mixture of these groups and geographic regions.

Finally, the third factor, plotted in Figure 4 together with its weights, is not affected by the 2008 global crisis. Furthermore, its weights are negative for all industrialized countries but Japan (non-significant) and Germany (rather small positive weight). In America and Asia, the weights are positive for all emerging and other developing countries. In particular, China's weight is rather large. This factor is related to an East Asian common factor; compare with the factor estimated by Moneta and Ruffer (2009) for the period 1993-2005 based on quarterly growth from ten East Asian countries, and by Bjornland et al. (2017) for the period 1978-2011. This factor clearly reflects the Asian financial crisis, which affected output in 1998; see, for example, Radelet and Sachs (1998) and Cabalu (1999).

According to the interpretation of the factors above, the impressive growth performance of emerging market economies, such as China and India, seems not to be affected by the growth slowdown observed in the world factor. This conclusion is in agreement with Kose et al. (2012) who conclude that emerging markets have "decoupled" from industrial economies, meaning that their business cycle dynamics were no longer tightly linked to the business cycles of industrial countries.

As an illustration of the joint ellipsoids of the factors obtained by the subsampling procedure, we plot the 95% ellipsoids for 1998 and 2004 for USA (Figure 8) and China (Figure 9). The ellipsoid corresponding to 1998 has larger volume, meaning that the uncertainty of the underlying factors in 1998 is larger just around and after the Asian financial crisis. Furthermore, we observe that the increase in uncertainty is mainly due to the first and second factors.

### 3.2 Predictive regressions

For each country growth, we estimate the predictive regression (1) by LS using the estimated factors $\hat{F}_{1t}, \hat{F}_{2t}$ and $\hat{F}_{3t}$ as regressors. Note that the predictive regressions are estimated using the original growth rates without demeaning and standardizing so that we can recover information about the average growth. In Figure 5, we summarize the estimated parameters, $\hat{\beta}_{i1}, \hat{\beta}_{i2}$ and $\hat{\beta}_{i3}$, by plotting a histogram of their values across all countries (first row) and across countries in Africa (second row), America (third row), Asia (fourth row) and Europe/Oceania (fifth row). Across all countries (first row), there are not clear patterns either in the signs or magnitudes of the estimates. Their histograms are roughly centered around zero and have similar ranges going from -2.5 to 2.5 approximately. The marginal effect of the first factor (first column), $\hat{\beta}_{i1}$, to forecast growth is similar across Africa, America, and Asia with values roughly centered around zero but it tends to be mainly positive in the Europe/Oceania group. The marginal effect of the second factor (second column), $\hat{\beta}_{i2}$, tends to be positive in Africa and negative in Asia and virtually zero in Europe/Oceania, and the marginal effect of the third factor (third column), $\hat{\beta}_{i3}$ is mainly positive in America. It is interesting to observe the link of the American continent with the third factor, which is loading mostly in East Asian countries. We should mention that the factors are mildly significant and the estimated magnitudes are rather small.[9]

In Table 2, we report the coefficient of determination, $R^2$, for each factor augmented

[9] Note that the results in Bai and Ng (2006) require $\frac{\sqrt{T}}{N} \to 0$ for the asymptotic normality of the LS estimator. In our application, $\frac{\sqrt{30}}{87} = 0.06$. However, Goncalves and Perron (2014) show that the LS estimator of the parameters of the predictive regressions may be affected by negative biases. In addition, the contemporaneous correlation between growth and the estimated factors is rather large for some countries and multicolinearity could be severe. Therefore, we should be cautious about inference on the parameters of the predictive regressions. The estimated parameters together with their p-values and the Box-Ljung statistic for the joint significance of the first four autocorrelations of the residuals, $Q(4)$, of each predictive regression are reported in an online appendix.

predictive regression. Overall, we observe that half of the predictive regressions have $R^2$ larger than 30% and only 10% of the regressions have $R^2$ larger than 50%. The results above show that the effects of the factors on one-step-ahead average growth are very mild.

Next, we analyze the effect of the factors on different quantiles of growth by estimating the factor augmented quantile predictive regressions (2) with $\tau = 0.05, 0.5$ and $0.95$.[10] Note that when $\tau = 0.5$, the quantile regression reduces to the conditional median regression, which is more robust to outliers than the conditional mean regression (1); see Ando and Tsay (2011). In Figure 6, we plot the cross-sectional histograms of the estimated parameters $\hat{\beta}_{i1}(\tau), \hat{\beta}_{i2}(\tau)$ and $\hat{\beta}_{i3}(\tau)$ for the lower quantile $\tau = 0.05$.[11] The main difference with the results of the predicitive regression for expected growth is that the magnitude of the parameter estimates is much larger for all countries. Across all countries (first row), the histograms are roughly centered around zero with an approximate range from -5 to 5. The marginal effect of the first factor (first column), $\hat{\beta}_{i1}(\tau)$, to forecast the 0.05 quantile of growth tends to be mainly positive in the America and the Europe/Oceania group and negative in Asia. The marginal effect of the second factor (second column), $\hat{\beta}_{i2}(\tau)$, tends to be positive in Africa, and the marginal effect of the third factor (third column), $\hat{\beta}_{i3}(\tau)$ is mainly positive in America and negative in Europe/Oceania. In general, the joint effect of the three factors is more relevant to forecast the 0.05 quantile of growth than to forecast expected growth.

In Table 2, we report the goodness of fit measure proposed by Koenker and Machado (1999), denoted as $R^1$, which is the analogous counterpart to the coefficient of determination in regression models.[12] We observe that the fit of the median regression is in general lower than that of the average growth regression. However, the fit improves dramatically in the tail quantiles. For the lower tail, the 5% quantile, we find that about 30% of the regressions have $R^1$ coefficients larger than 50%. Therefore, it seems that the factors are more relevant to explain future tails than the center of the growth distribution. This conclusion is in agreement with the main findings of Giglio et al. (2016) and Adrian et al. (in press) who conclude that the estimated lower quantile of growth depends on financial conditions, while

[10]The estimator of the parameters is based on the algorithm by Koenker and d'Orey (1987). Results based on the shrinkage estimator proposed by Ohno and Ando (2018) are similar. They are available upon request.

[11]Histograms for $\tau = 0.5$ and 0.95 are available in the on-line appendix.

[12]The estimated parameters and their corresponding $p$-values are reported in the online appendix.

the upper quantiles are stable over time.[13]

Finally, following Adrian et al. (in press), we use the factor augmented quantile predictive regressions for different values of $\tau$ to compute the growth densities for each country and for each year.[14] In Figure 7, we plot these densities for two countries, namely China and USA. We observe that, in both countries, the densities are skewed to the left with the densities in China having the concentration of mass in values of growth larger than those in USA (less risk). Furthermore, the dispersion (uncertainty) of the densities in China is also smaller than that of the densities for USA. Finally, note that the effect of the global crisis in the USA densities is very obvious while there is not any clear effect on the densities in China.

### 3.3 Forecasting recession risk under stressed factors

To obtain the GiS corresponding for each country, we solve the optimization problem in (3) with $h(y_{it+1}) = \hat{y}_{it+1}$ being the predicted expected mean growth, which is calculated by plugging in the LS estimates of the parameters in (1). The ellipsoid $g(F_t, \alpha) = 0$ is estimated using the resampling procedure of Maldonado and Ruiz (2017). In Figures 8 and 9, we illustrate this optimization problem by plotting the 95%-probability ellipsoids $g(F_t, 95\%) = 0$ corresponding to 1998 and 2004 for USA and China, respectively. In the top left panel figure, we also plot the iso-growth surfaces corresponding to the predictive regressions for 1999 and 2005 that are tangent to the ellipsoids. We observe that the surfaces of the predictive regressions are rather different in shape and orientation in the two countries considered.

After estimating the GiS for each country and year, we observe that, in Africa, the country with the lowest GiS over time is Cameroon while the country with the largest GiS and, consequently, the highest risk of recession is Uganda.[15] These two countries also have the smallest and the largest risk among the developing countries. In America, the country with the lowest risk of recession is Guatemala while the country with the largest risk is Venezuela.

[13] Adrian et al. (in press) show that current economic conditions forecast the median of the distribution of growth, but do not contain information about the other quantiles of the distribution.

[14] Adrian et al. (in press) fit the skewed t-distribution proposed by Azzalini and Capitanio (2003) to obtain a density by smoothing the quantile function.

[15] Time series plots of the GiS estimated in each country appear in the online appendix.

For Asian countries, Syria and China have the largest and the smallest risk of recession, respectively. It is also important to note that among the countries classified as emerging, China has the lowest risk while Venezuela has the largest. Finally, in Europe/Oceania, the largest risk of recession corresponds to Iceland while Norway has the lowest. These two countries also have the largest and the lowest risks among the industrialized countries.

In Figure 10, we summarize the GiS results by plotting year-by-year the cross-sectional average GiS together with the cross-sectional bounds constructed as $\pm 2$ cross-sectional standard deviations of the GiS when countries are grouped by continent. In Figure 11, we plot the same quantities when countries are grouped by type. Several conclusions emerge from these figures. We observe that in all continents, the average risk has been slightly decreasing over time, with the Asian continent enjoying the smallest average GiS. The African and American continents offer very similar average risk profiles. Note that the decrease in average GiS is more pronounced among countries in Africa, America and Asia than among countries in Europe/Oceania. In this latter case, the GiS is more stable over time. This result is in contrast with other macroeconomic uncertainty indexes which conclude that risk has been increasing over time. There are two potential explanations for this apparent contradiction. First, note that most uncertainty indexes focus on industrialized countries while we consider growth in countries all over the world. As explained above, the decrease of the GiS is more pronounced in emerging and developing countries than in industrialized countries. Second, our index measures growth risk when the global and regional common factors are stressed while most alternative indexes focus on uncertainty. Even if the variance (uncertainty) of the distribution of growth increases, the expected growth under stressed factors can also increase and, consequently, the GiS decreases. The $\pm 2$ standard deviation bounds are also becoming narrower over time and have very similar profiles in the African, Asian, and American continents with a sharp jump in 1999 coinciding with the Asian financial crisis. The lower bound is rather stable when compared with the upper bound that is more volatile over time. This is because the standard deviations during the years with high recession risk are larger than the standard deviations when the risk is low. The plot for the European/Oceania continents is rather different from the other plots as the bounds are much narrower indicating that these countries are very similar in risk profile. We observe that post 2008 financial crisis, mainly from 2011 on, the world has fallen in a state-of-complacency with the average GiS falling quite dramatically to reach the lowest levels of risk, between 1 and 0% in 2015. In Figure 8, we summarize risk among developing, emerging and industrialized countries. We observe that the GiS plots of industrialized and emerging countries coincide with those of Europe/Oceania and Asia, respectively, and the plot corresponding to developing countries is very similar to that of African countries.

In addition to analyzing the effects of stressed factors on the average of growth, we also predict the GiS of each country for the $\tau = 0.05$, 0.5 and 0.95 quantiles of the country growth distribution. We solve the minimization problem in (3) with $h(y_{it+1}) = \hat{q}_\tau(y_{it+1}|y_t, F_t)$ and compute $\hat{q}_\tau(y_{it+1}|y_t, F_t)$ as in equation (2) by plugging in the parameter estimates. As an illustration, in Figures 8 and 9, we plot the 95% ellipsoids for the factors in 1998 and 2004 together with the tangent iso-growth surfaces for one-step-ahead (1999 and 2005) growth quantiles ($\tau = 0.05, 0.5$ and 0.95) obtained from the estimated factor augmented predictive regressions for USA and China, respectively. We observe that the tangent surfaces based on the mean and those based on the median growth are rather similar. However, the tangent surfaces for the 5% and/or 95% growth quantiles can be very different in shape and orientation from the mean and median surfaces as we show in the case of China. In summary, the effect of stressed factors can be rather different depending on the specific quantile of the growth distribution being considered.

In Figure 12, we plot a summary of the $\tau$-quantile GiS. As before, we plot the cross-country average and $\pm 2$ times the standard deviations of the predicted $\tau$-quantile GiS for all industrialized, emerging and other developing countries. First, compare the GiS results for $\tau = 0.5$ with those plotted in Figure 11 where GiS is predicted for the mean growth. The plots in both cases are almost identical for industrialized and emerging countries. However, for developing countries, the bounds become narrower mainly because the upper bound has coming down substantially. For the $\tau = 0.05$ quantile of growth, we are looking at catastrophic outcomes. For the three groups, the cross-country average of the predicted 5% quantile GiS is rather high at 20% (or slightly below 20%) and it does not decrease much over time. Obviously, these are the worst outcomes. Extreme events in the three world factors could wipe out one-fifth of GDP in those countries that are already going through deep recessions. On the contrary, when a country is in its 95% growth quantile, it could withstand extreme events in the world factors as the predicted average GiS for this quantile is close to 0%, that is, no growth on average, and with bounds becoming narrower over time.

# 4 Conclusions

The existence of world business cycles raises the question on the vulnerability of individual country economies when they face extreme scenarios in those factors that drive world growth. With this objective in mind, we have proposed a new global risk index, Growth-in-Stress (GiS), that measures the expected fall in a country GDP when the global factors are subject to stressful conditions. There are three components to this measure: the existence of global factors, the definition of stress, and the choice of the objective function.

We have extracted three global factors out of a sample of GDP growth of 87 countries, classified as industrialized, emerging, and other developing, over the period 1985-2015. The first factor, which accounts for 20% of the total variability of growth, is driven by all industrial and emerging countries and it is considered a world growth factor; the second factor is driven by other developing countries in Africa and America; and the third factor is mainly related to East Asian economies. All three factors account for 42% of the total growth variability. To our knowledge, the African/American factor has not been reported in the literature yet. We have defined stressful events in the factors by considering the extreme multivariate quantiles of the joint distribution of the three factors. We have constructed 95% probability ellipsoids that contain the true factors so that the extreme events are those seating on the boundary of the ellipsoid. Obviously, it is up to the researcher to choose the level of risk or stress desired. It is this approach of considering stress directly on the factors that makes our index a risk index instead of an uncertainty index. Finally, we have estimated country-specific predictive regressions augmented with the three factors to predict (i) the one-step-ahead average growth, and (ii) the one-step-ahead $\tau$-quantile growth in each country. With these three elements in place (factors, stress, and objective function), we proceed to compute GiS as the predicted minimum growth and minimum $\tau$-quantile generated by the point of tangency between the 95% probability ellipsoid and the properly oriented surfaces based on the predictive regressions.

Our results confirm that global risk has been decreasing over time. Not only the cross-sectional average GiS has been going down but also the $\pm 2$ standard deviation bounds have become narrower over time. The cross-sectional average GiS was about 5% in 1987 and between 0-1% in 2015 considering the 87 countries in Africa, America, Asia and Europe/Oceania. However, there is a lot of heterogeneity across countries and continents. Several countries in Africa and America are exposed to very high risks with GiS larger than 10%. The countries in the Europe/Oceania group are more homogeneous as the bounds around the cross-sectional average GiS are the tightest of all continents. From 2011 on, all continents have entered in a state-of-complacency and by 2015 the average worst outcome seem to be no growth at the 95% factor stress. We also measure the factor stress on different quantiles ($\tau = 0.05, 0.5$ and $0.95$) of the DGP growth distribution of each country. Overall, the 50% quantile GiS and the average GiS are quite similar. For those countries that are already or approaching recession i.e., those in the 5% quantile of the growth distribution, an extreme event in the factors has catastrophic consequences as we have calculated that GDP may experience a 20% drop.

The exercise that we have described is predictive but it has been conducted in-sample. The time series is too short to implement an out-of-sample exercise though it would be possible to increase the frequency of the series so that we have a larger sample size. The methodology that we propose is general enough to be applicable to any other macroeconomic aggregate beyond GDP growth. Moreover, the factors could also be extracted from systems of macroeconomic/financial variables instead of extracting them from the system of growths.

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Table 1: List of Countries

| Conuntry | Group | Code |
|---|---|---|
| Algeria | Other | DZA |
| Benin | Other | BEN |
| Botswana | Other | BWA |
| Burkina Faso | Other | BFA |
| Cameroon | Other | CMR |
| Congo, Rep. | Other | COG |
| Egypt, Arab Rep. | Emerging | EGY |
| Gabon | Other | GAB |
| Gambia, The | Other | GMB |
| Ghana | Other | GHA |
| Kenya | Other | KEN |
| Lesotho | Other | LSO |
| Madagascar | Other | MDG |
| Mali | Other | MLI |
| Mauritania | Other | MRT |
| Mauritius | Other | MUS |
| Morocco | Emerging | MAR |
| Mozambique | Other | MOZ |
| Nigeria | Other | NGA |
| Rwanda | Other | RWA |
| Senegal | Other | SEN |
| Seychelles | Other | SYC |
| South Africa | Emerging | ZAF |
| Tanzania | Other | TZA |
| Togo | Other | TGO |
| Tunisia | Other | TUN |
| Uganda | Other | UGA |
| Zimbabwe | Other | ZWE |
| Argentina | Emerging | ARG |
| Bolivia | Other | BOL |
| Brazil | Emerging | BRA |
| Canada | Industrialized | CAN |
| Chile | Emerging | CHL |
| Colombia | Emerging | COL |
| Costa Rica | Other | CRI |
| Dominican Republic | Other | DOM |
| Ecuador | Other | ECU |
| El Salvador | Other | SLV |
| Guatemala | Other | GTM |
| Honduras | Other | HND |
| Mexico | Emerging | MEX |
| Nicaragua | Other | NIC |
| Panama | Other | PAN |
| Paraguay | Other | PRY |
| Peru | Emerging | PER |
| Trinidad and Tobago | Other | TTO |
| United States | Industrialized | USA |
| Uruguay | Other | URY |
| Venezuela, RB | Emerging | VEN |
| Bangladesh | Other | BGD |
| China | Emerging | CHN |
| Hong Kong SAR, China | Emerging | HKG |
| India | Emerging | IND |
| Indonesia | Emerging | IDN |
| Iran, Islamic Rep. | Other | IRN |
| Israel | Emerging | ISR |
| Japan | Industrialized | JPN |
| Korea, Rep. | Emerging | KOR |
| Malaysia | Emerging | MYS |
| Nepal | Other | NPL |
| Pakistan | Emerging | PAK |
| Philippines | Emerging | PHL |
| Singapore | Emerging | SGP |
| Sri Lanka | Other | LKA |
| Syrian Arab Republic | Other | SYR |
| Thailand | Emerging | THA |
| Turkey | Emerging | TUR |
| Austria | Industrialized | AUT |
| Belgium | Industrialized | BEL |
| Denmark | Industrialized | DNK |
| Finland | Industrialized | FIN |
| France | Industrialized | FRA |
| Germany | Industrialized | DEU |
| Greece | Industrialized | GRC |
| Iceland | Industrialized | ISL |
| Ireland | Industrialized | IRL |
| Italy | Industrialized | ITA |
| Luxembourg | Industrialized | LUX |
| Netherlands | Industrialized | NLD |
| Norway | Industrialized | NOR |
| Portugal | Industrialized | PRT |
| Spain | Industrialized | ESP |
| Sweden | Industrialized | SWE |
| Switzerland | Industrialized | CHE |
| United Kingdom | Industrialized | GBR |
| Australia | Industrialized | AUS |
| New Zealand | Industrialized | NZL |

Table 2: Goodness of fit: $R^2$ of factor-augmented predictive regressions and $R^1_\tau$ of factor-augmented predictive quantile regression

**Africa**

| | DZA | BEN | BWA | BFA | CMR | COG | EGY | GAB | GMB | GHA | KEN | LSO | MDG | MLI | MRT | MUS | MAR | MOZ | NGA | RWA | SEN | SYC | ZAF | TZA | TGO | TUN | UGA | ZWE |
|---|---|---|---|---|---|---|---|---|---|---|---|---|---|---|---|---|---|---|---|---|---|---|---|---|---|---|---|---|
| $R^2$ | 0.39 | 0.14 | 0.11 | 0.43 | 0.64 | 0.13 | 0.23 | 0.08 | 0.13 | 0.29 | 0.32 | 0.34 | 0.40 | 0.29 | 0.11 | 0.14 | 0.49 | 0.14 | 0.17 | 0.46 | 0.39 | 0.19 | 0.31 | 0.55 | 0.06 | 0.06 | 0.10 | 0.26 |
| $R^1_{0.95}$ | 0.42 | 0.36 | 0.34 | 0.15 | 0.41 | 0.40 | 0.40 | 0.47 | 0.23 | 0.52 | 0.47 | 0.37 | 0.36 | 0.62 | 0.29 | 0.24 | 0.63 | 0.21 | 0.39 | 0.22 | 0.24 | 0.18 | 0.42 | 0.28 | 0.13 | 0.30 | 0.24 | 0.31 |
| $R^1_{0.50}$ | 0.20 | 0.15 | 0.06 | 0.31 | 0.37 | 0.20 | 0.17 | 0.17 | 0.16 | 0.20 | 0.29 | 0.26 | 0.29 | 0.23 | 0.16 | 0.19 | 0.35 | 0.14 | 0.18 | 0.29 | 0.31 | 0.22 | 0.18 | 0.38 | 0.12 | 0.18 | 0.21 | 0.25 |
| $R^1_{0.05}$ | 0.54 | 0.49 | 0.33 | 0.49 | 0.66 | 0.43 | 0.28 | 0.37 | 0.35 | 0.21 | 0.21 | 0.25 | 0.55 | 0.29 | 0.22 | 0.37 | 0.51 | 0.56 | 0.49 | 0.50 | 0.45 | 0.24 | 0.29 | 0.58 | 0.47 | 0.22 | 0.45 | 0.36 |

**America**

| | ARG | BOL | BRA | CAN | CHL | COL | CRI | DOM | ECU | SLV | GTM | HND | MEX | NIC | PAN | PRY | PER | TTO | USA | URY | VEN |
|---|---|---|---|---|---|---|---|---|---|---|---|---|---|---|---|---|---|---|---|---|---|
| $R^2$ | 0.20 | 0.35 | 0.07 | 0.24 | 0.41 | 0.11 | 0.08 | 0.09 | 0.16 | 0.38 | 0.12 | 0.07 | 0.09 | 0.34 | 0.27 | 0.21 | 0.45 | 0.53 | 0.44 | 0.41 | 0.20 |
| $R^1_{0.95}$ | 0.21 | 0.49 | 0.43 | 0.27 | 0.48 | 0.23 | 0.44 | 0.15 | 0.31 | 0.55 | 0.49 | 0.15 | 0.25 | 0.3 | 0.34 | 0.54 | 0.40 | 0.49 | 0.27 | 0.20 | 0.20 |
| $R^1_{0.50}$ | 0.21 | 0.44 | 0.1 | 0.12 | 0.23 | 0.09 | 0.08 | 0.11 | 0.11 | 0.24 | 0.21 | 0.09 | 0.23 | 0.36 | 0.23 | 0.09 | 0.26 | 0.41 | 0.28 | 0.27 | 0.16 |
| $R^1_{0.05}$ | 0.43 | 0.67 | 0.39 | 0.36 | 0.43 | 0.46 | 0.29 | 0.37 | 0.38 | 0.29 | 0.41 | 0.4 | 0.33 | 0.38 | 0.50 | 0.33 | 0.67 | 0.39 | 0.58 | 0.53 | 0.44 |

**Asia**

| | BGD | CHN | HKG | IND | IDN | IRN | ISR | JPN | KOR | MYS | NPL | PAK | PHL | SGP | LKA | SYR | THA | TUR |
|---|---|---|---|---|---|---|---|---|---|---|---|---|---|---|---|---|---|---|
| $R^2$ | 0.53 | 0.38 | 0.22 | 0.19 | 0.14 | 0.35 | 0.10 | 0.37 | 0.33 | 0.17 | 0.16 | 0.21 | 0.28 | 0.27 | 0.22 | 0.36 | 0.47 | 0.03 |
| $R^1_{0.95}$ | 0.39 | 0.36 | 0.57 | 0.18 | 0.39 | 0.53 | 0.27 | 0.50 | 0.48 | 0.23 | 0.43 | 0.40 | 0.26 | 0.47 | 0.35 | 0.54 | 0.48 | 0.1 |
| $R^1_{0.50}$ | 0.35 | 0.37 | 0.22 | 0.21 | 0.27 | 0.28 | 0.1 | 0.18 | 0.34 | 0.27 | 0.06 | 0.27 | 0.24 | 0.19 | 0.17 | 0.24 | 0.33 | 0.08 |
| $R^1_{0.05}$ | 0.56 | 0.46 | 0.34 | 0.3 | 0.12 | 0.47 | 0.23 | 0.44 | 0.23 | 0.24 | 0.43 | 0.17 | 0.27 | 0.37 | 0.39 | 0.63 | 0.51 | 0.12 |

**Europe and Oceania**

| | AUT | BEL | DNK | FIN | FRA | DEU | GRC | ISL | IRL | ITA | LUX | NLD | NOR | PRT | ESP | SWE | CHE | GBR | AUS | NZL |
|---|---|---|---|---|---|---|---|---|---|---|---|---|---|---|---|---|---|---|---|---|
| $R^2$ | 0.33 | 0.37 | 0.09 | 0.25 | 0.42 | 0.16 | 0.55 | 0.37 | 0.38 | 0.44 | 0.24 | 0.38 | 0.31 | 0.54 | 0.62 | 0.27 | 0.16 | 0.41 | 0.19 | 0.39 |
| $R^1_{0.95}$ | 0.35 | 0.4 | 0.22 | 0.22 | 0.47 | 0.27 | 0.24 | 0.31 | 0.43 | 0.37 | 0.30 | 0.46 | 0.32 | 0.59 | 0.39 | 0.33 | 0.29 | 0.48 | 0.34 | 0.21 |
| $R^1_{0.50}$ | 0.21 | 0.22 | 0.21 | 0.22 | 0.26 | 0.12 | 0.34 | 0.36 | 0.32 | 0.25 | 0.19 | 0.28 | 0.23 | 0.33 | 0.39 | 0.13 | 0.12 | 0.15 | 0.14 | 0.38 |
| $R^1_{0.05}$ | 0.43 | 0.44 | 0.34 | 0.43 | 0.49 | 0.44 | 0.62 | 0.36 | 0.46 | 0.48 | 0.43 | 0.54 | 0.44 | 0.55 | 0.54 | 0.47 | 0.42 | 0.51 | 0.50 | 0.39 |

Figure 1: Graphical illustration of computation of GiS when the number of common factors is two.

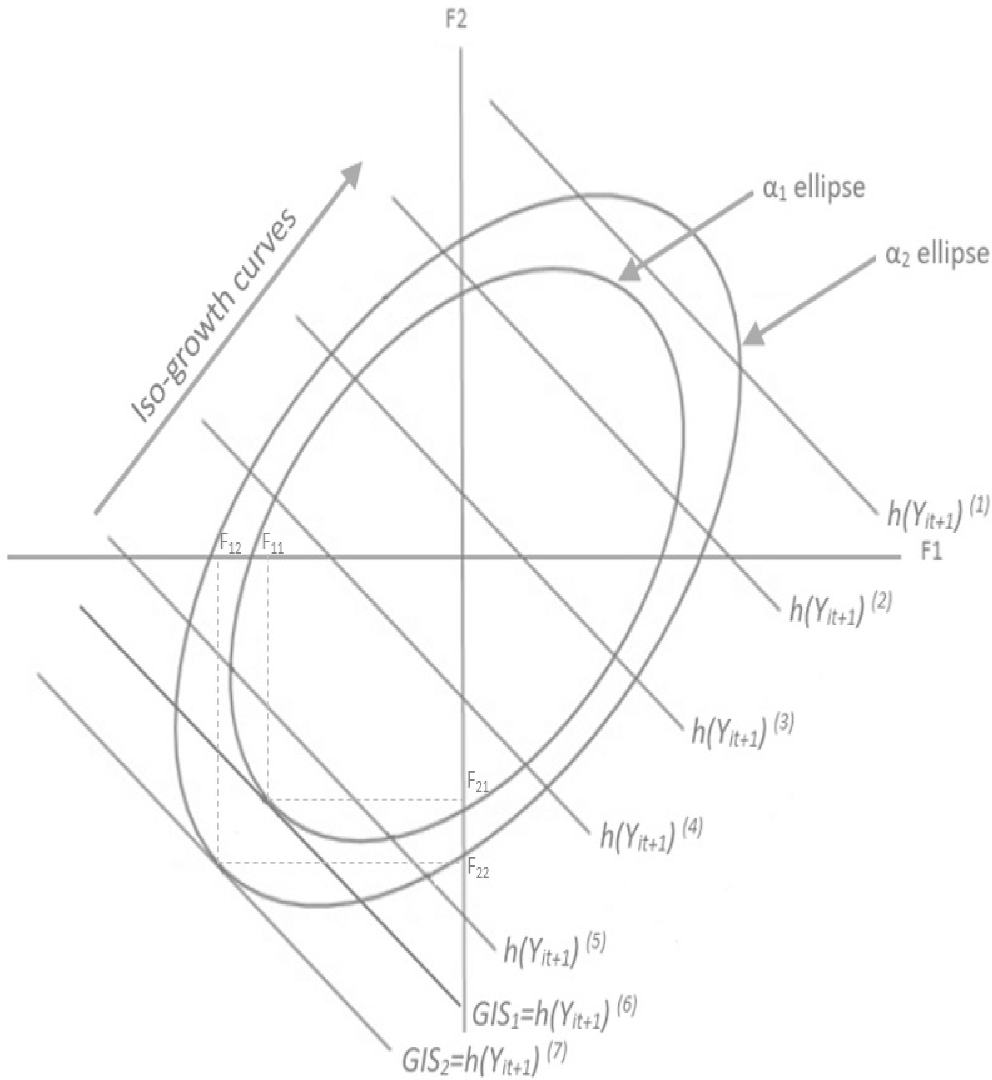

Figure 2: Top panel: First factor extracted using Principal Components from system of growths together with 95% prediction intervals (in red). Bottom panel: Estimated weights of the first factor for each country together with 95% confidence intervals. The bars in red, blue, and gray correspond to industrialized, emerging, and other developing countries, respectively. The countries from the lighter to darker gray areas correspond to African, American, Asian, European and Oceania countries, respectively. Within each continent, the countries appear in the same order as in Table 1.


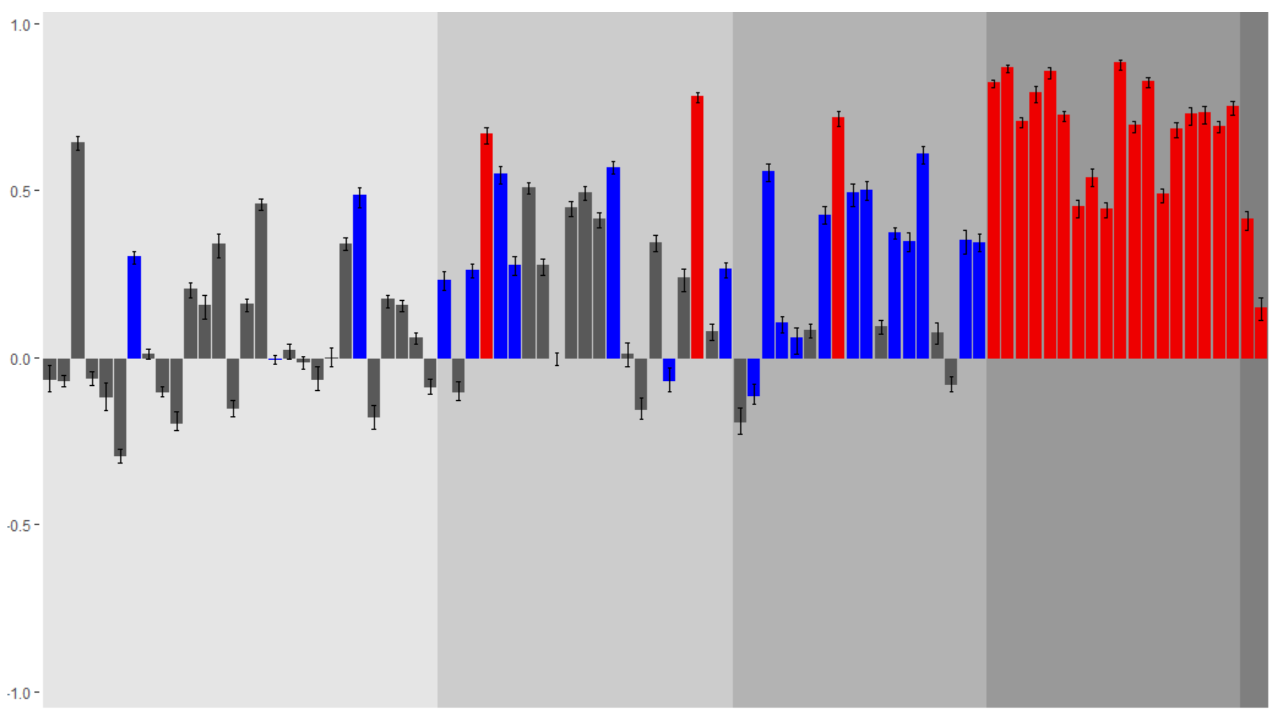

Figure 3: Top panel: Second factor extracted using Principal Components from system of growths together with 95% prediction intervals (in red). Bottom panel: Estimated weights of the second factor for each country together with 95% confidence intervals. The bars in red, blue, and gray correspond to industrialized, emerging, and other developing countries, respectively. The countries from the lighter to darker gray areas correspond to African, American, Asian, European and Oceania countries, respectively. Within each continent, the countries appear in the same order as in Table 1.

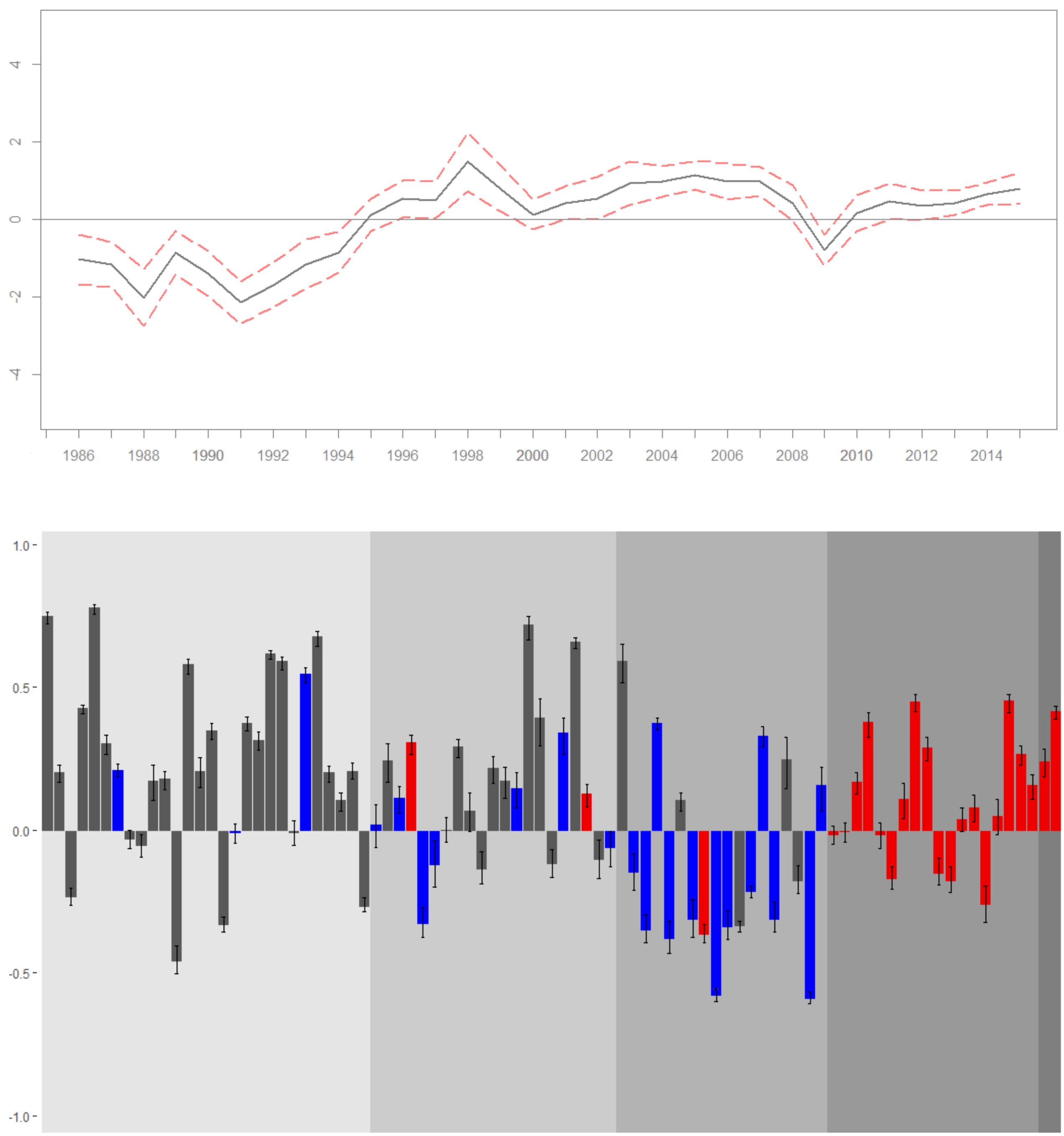

Figure 4: Top panel: Third factor extracted using Principal Components from system of growths together with 95% prediction intervals (in red). Bottom panel: Estimated weights of the third factor for each country together with 95% confidence intervals. The bars in red, blue, and gray correspond to industrialized, emerging, and other developing countries, respectively. The countries from the lighter to darker gray areas correspond to African, American, Asian, European and Oceania countries, respectively. Within each continent, the countries appear in the same order as in Table 1.

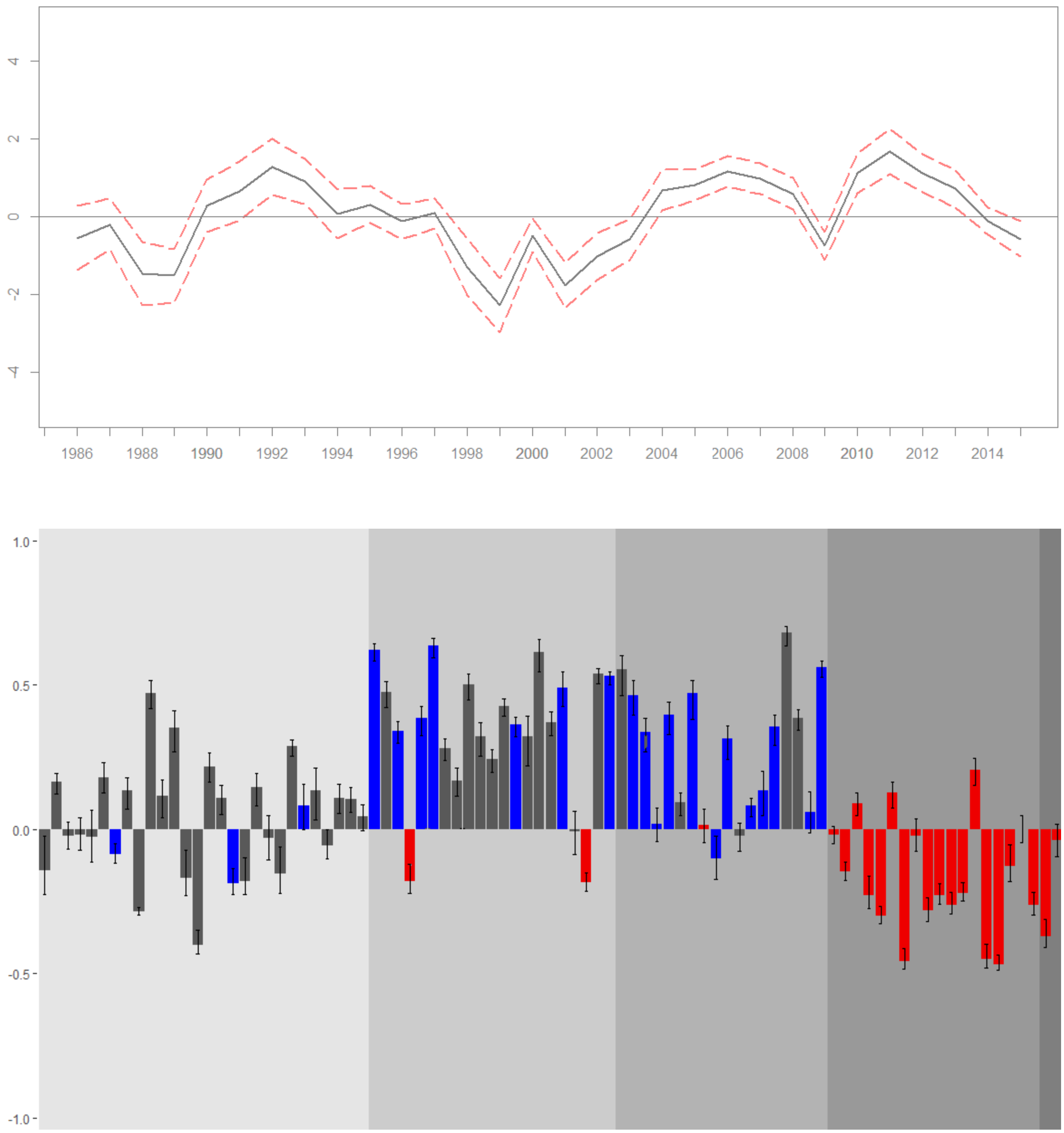

Figure 5: Cross-sectional histograms of estimated parameters of the factor augmented predictive regressions corresponding to factor 1 (first column), factor 2 (second column) and factor 3 (third column) computed through all countries (first row) and countries in Africa (second row), America (third row), Asia (fourth row) and Europe/Oceania (fifth row).

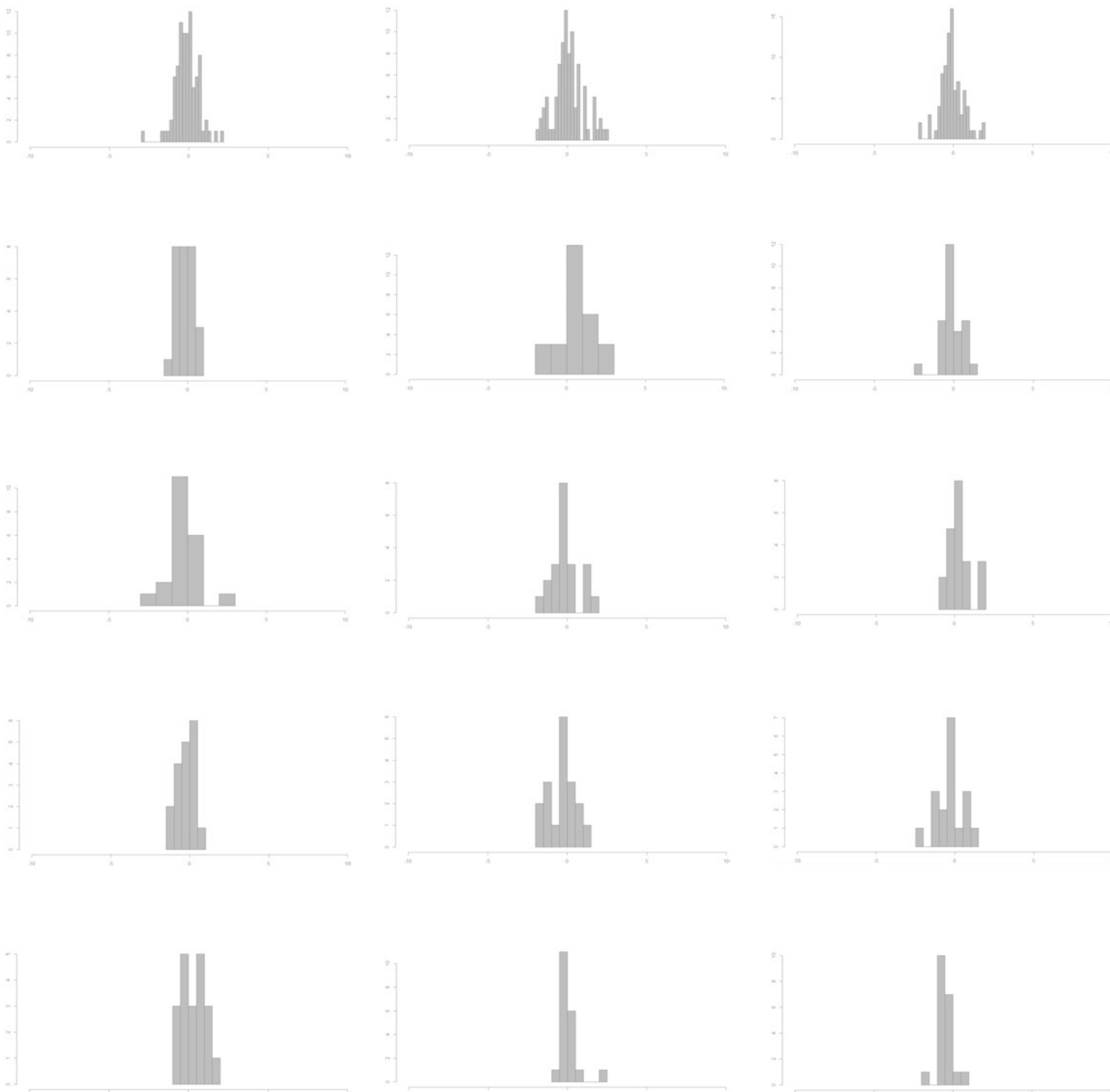

Figure 6: Cross-sectional histograms of estimated parameters of the factor augmented quantile predictive regressions for $\tau = 0.05$ corresponding to factor 1 (first column), factor 2 (second column) and factor 3 (third column) computed through all countries (first row) and countries in Africa (second row), America (third row), Asia (fourth row) and Europe/Oceania (fifth row).

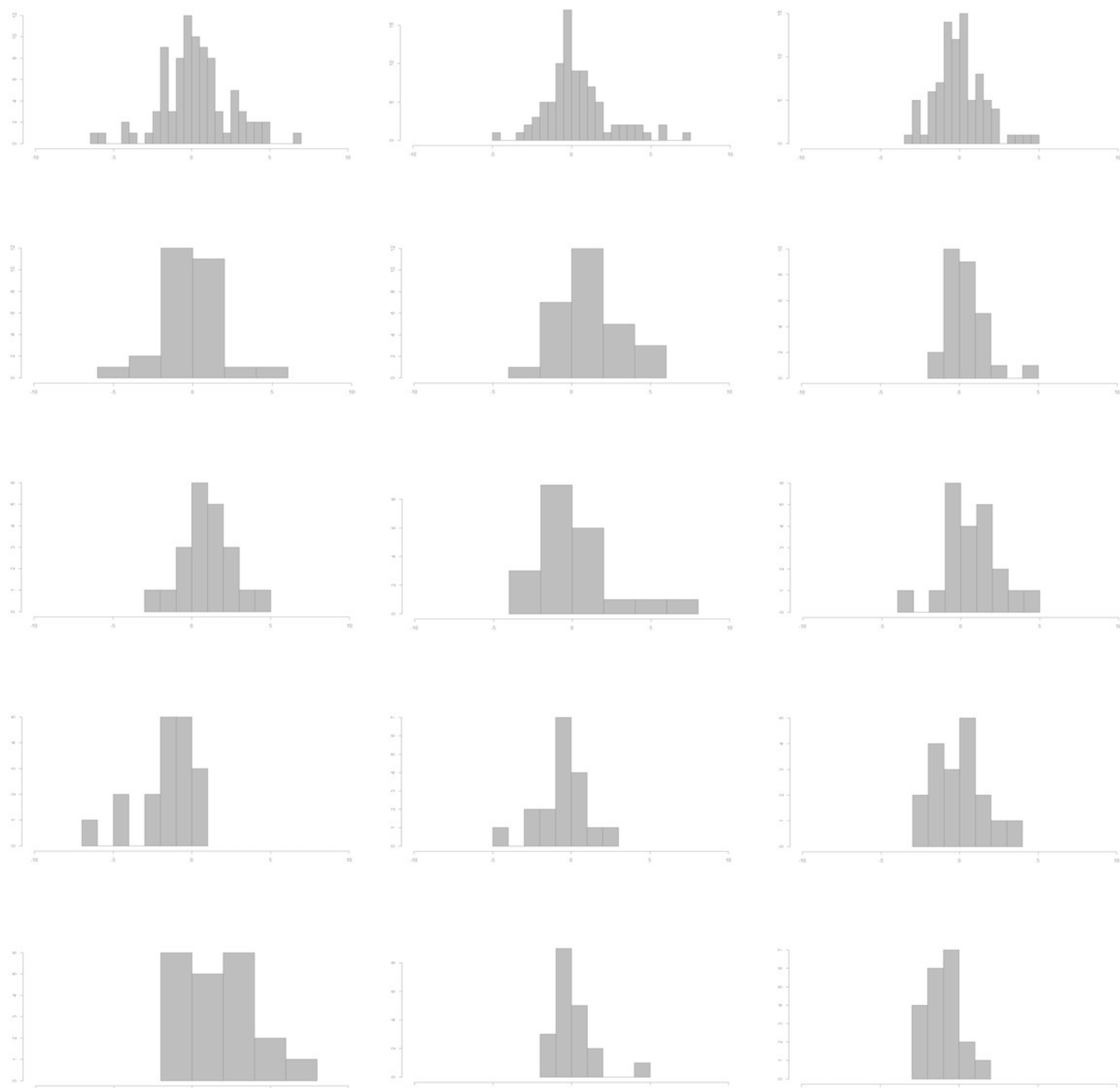

Figure 7: Estimated densities of growth for USA (top panel) and China (bottom panel) based on factor augmented quantile regressions.

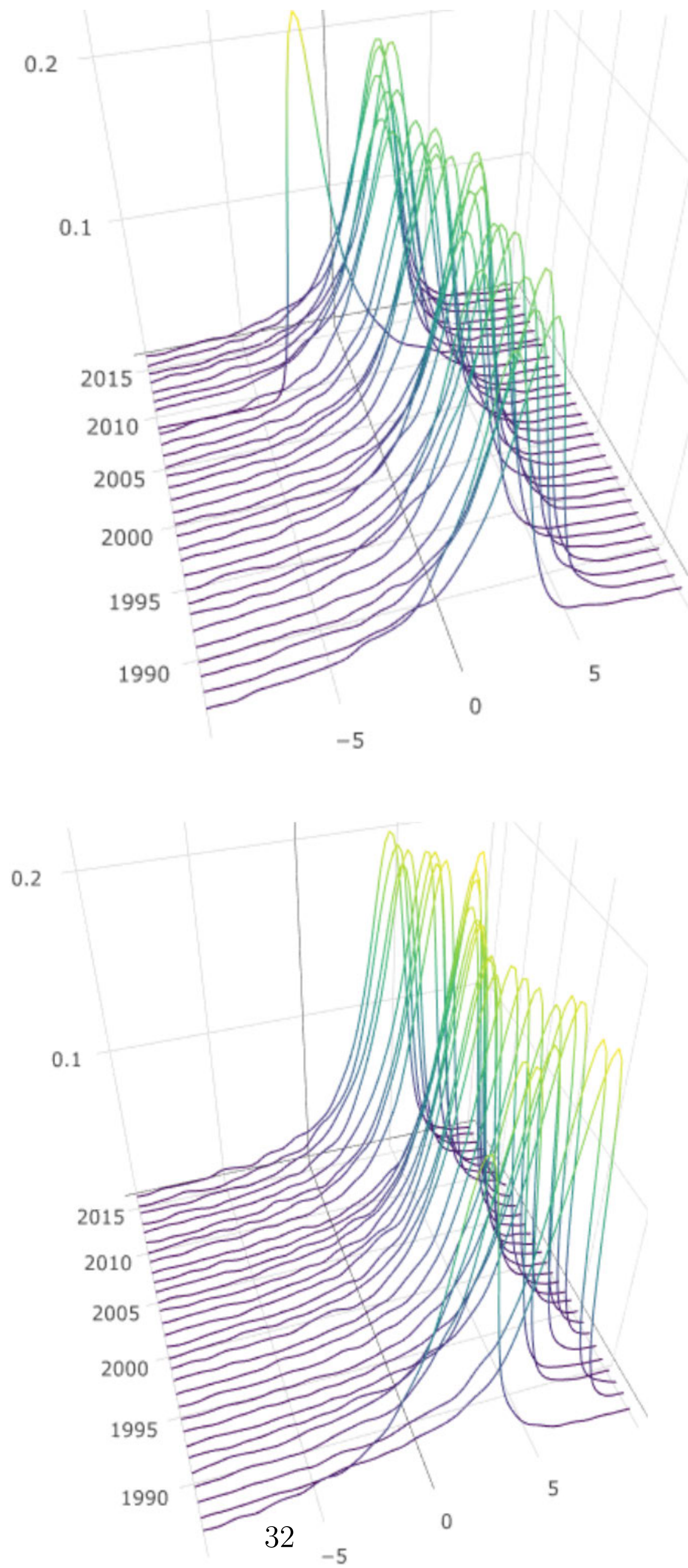

Figure 8: Resampling ellipsoids for the three factors in 1998 (blue) and 2004 (red). Predicted iso-growth surfaces in USA for 1999 and 2005 based on predictive regression (top left panel) and quantile regressions with $\tau = 0.05$ (bottom left panel), $\tau = 0.5$ (top right panel) and $\tau = 0.95$ (bottom right panel). For each year, the GiS is the tangency point between the ellipsoid and the corresponding surface.

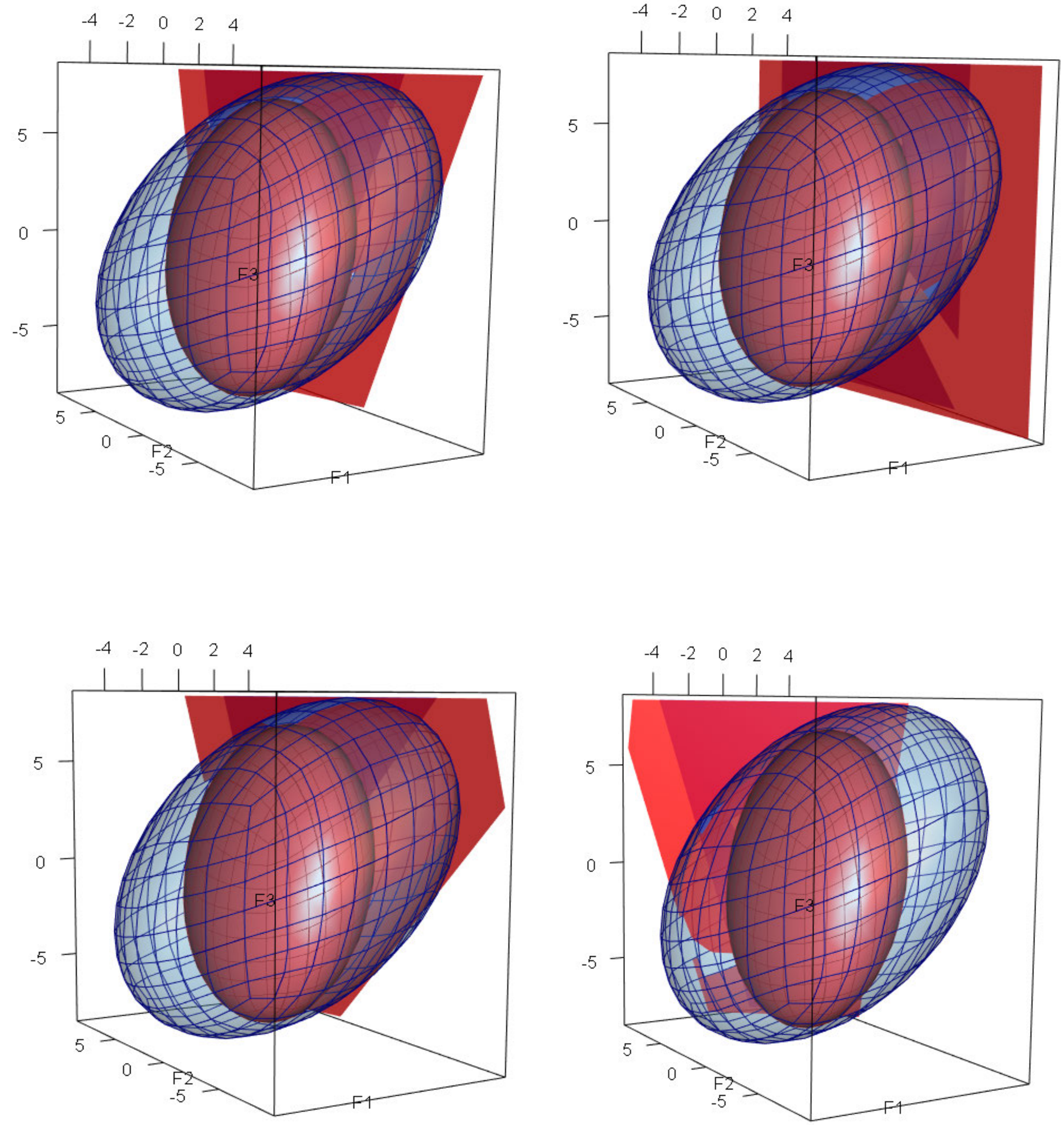

Figure 9: Resampling ellipsoids for the three factors in 1998 (blue) and 2004 (red). Predicted iso-growth surfaces in China for 1999 and 2005 basedn on predictive regression (top left panel) and quantile regressions with $\tau = 0.05$ (bottom left panel), $\tau = 0.5$ (top right panel) and $\tau = 0.95$ (bottom right panel). For each year, the GiS is the tangency point between the ellipsoid and the corresponding surface.

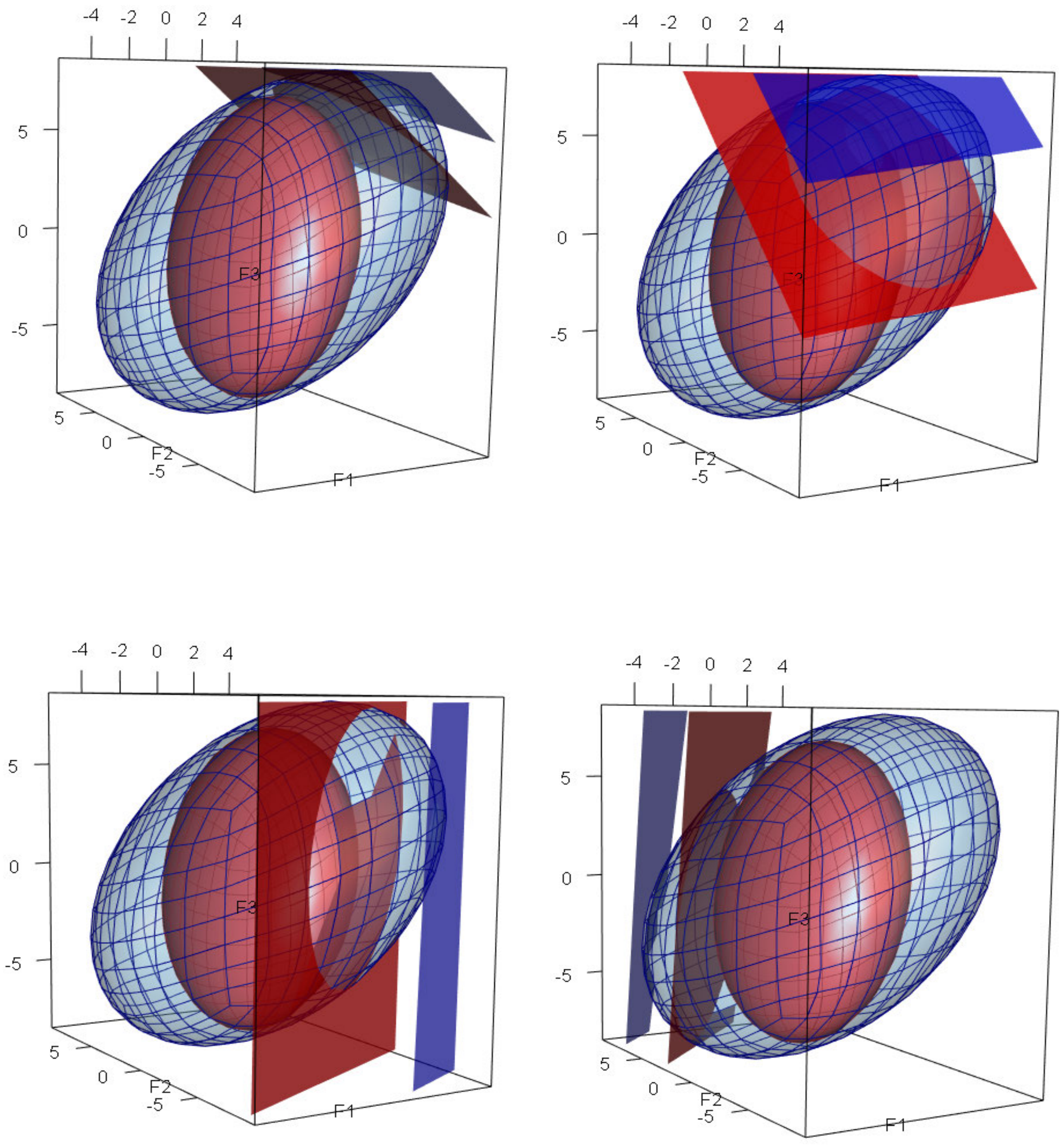

Figure 10: Cross-sectional average GiS (black line) and $\pm 2$ standard deviations (red lines) among countries in Africa (top left panel), America (top right panel), Asia (bottom left panel) and Europe and Oceania (bottom right panel).

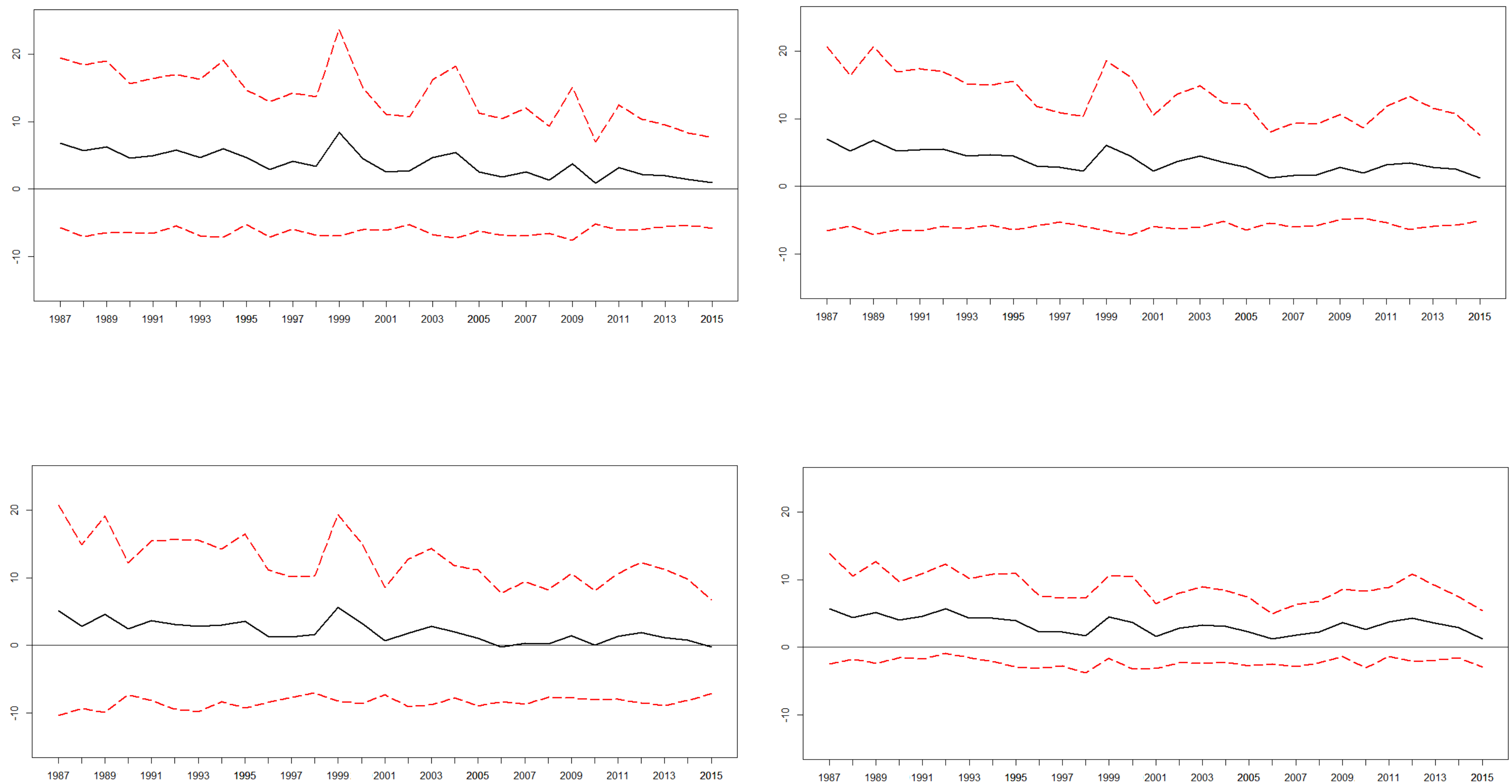

Figure 11: Cross-sectional average GiS (black line) and ±2 standard deviations (red lines) among other developing (top panel), emerging (middle panel) and industrialized (bottom panel) countries.

Figure 12: Cross-sectional average GiS (black line) and ±2 standard deviations (red lines) for $\tau = 0.05$ (first row), 0.5 (second row) and 0.95 quantiles of the growth distribution among industrialized (first column), emerging (second column) and other developing (third column) countries.