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MINIMUM CONSUMPTION AND TRANSITIONAL DYNAMICS IN WEALTH DISTRIBUTION *

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Abstract

This paper investigates the evolution of wealth distribution in a one sector growth model along its transition path. A key feature of the model is that a household´s consumption cannot fall below a positive level each period. This requirement introduces a positive association between the intertemporal elasticity of substitution and household wealth. Households only differ in their initial holdings of capital. The model is calibrated to match some key statistics of the US economy. The level of inequality in the wealth distribution of our artificial economy has a n inverted Ushape. The level of wealth inequality and its evolution resembles that of the US economy. However, our model illustrates that the existence of a Kuznets curve is very sensitive tothe sources of growth: whether it is driven by productivity growth or capital accumulation. Additionally, our model predicts an upsurge in wealth inequality following the productivity slowdown in the 1970´s.

Keywords:

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1 Introduction

The objective of this paper is to assess how much of the U.S. wealth inequality observed in the late XIXth and XXth centuries can be accounted for by the dynamics of a standard one sector growth model. The key feature of our model is that a household's consumption cannot fall below a positive level each period. Households only differ in their initial holdings of capital. We define wealth as the household net worth: value of total accumulated assets minus value of total liabilities. This definition coincides with accumulated capital in our model. There is no uncertainty of any source in this model. We assume that there exist perfect capital markets. This last assumption plus the existence of a minimum consumption requirement imply that the Engel curves are affine functions of the level of wealth. This property of the model ensures that the distribution of wealth does not affect the aggregate dynamics of the model —provided that no household's income falls below the consumption requirement—, whereas the wealth distribution does change along the transition path.

The model is calibrated to match some key aggregate statistics of the U.S. economy. Once done this, we calibrate the wealth distribution of the initial period, which we take to be 1870, so that it matches some of the U.S. historical data for that period. We find that the evolution of the wealth distribution and that of the level of wealth inequality in our artificial economy resemble quite well those of the U.S. economy. The level of wealth inequality in our artificial economy increases at low levels of per capita income and declines as the economy approaches the steady state, as the evolution of the U.S. inequality does.

The evolution of wealth inequality depends on several things. First of all, the existence of a minimum consumption requirement introduces a positive association between household wealth and the intertemporal elasticity of substitution (IES). Savings rates will be increasing with the level of household wealth. Thus, poor households will accumulate capital at a rate not smaller than that of wealthy households only if the aggregate level of income is high enough relative to the minimum consumption requirement. If income is low they cannot undertake any savings (or they have to borrow) to finance the minimum consumption requirement. This effect is enhanced by a large level of risk aversion. Thus, depending on the combination of two parameters, the risk aversion parameter and the minimum consumption level, the level of inequality will increase, decrease or display an inverted U shape along the transition path.

Secondly, our experiments suggest that the evolution of wealth inequality is very sensitive to the length of the transition path. The farther away is an economy from its steady state when it starts a process of sustained growth, the longer the period of increasing inequality and the higher its level. The length of the transition path depends on the contribution of Total Factor Productivity growth (TPF, hereafter) and the initial level of aggregate capital. The higher the level of productivity growth, the shorter the period of increasing inequality, if there is any. The reason for this is that the larger the level of TFP growth the sooner the economy attains a level of income above which poor households start accumulating capital faster than wealthy households. Thus, our experiments indicate that a Kuznets curve in wealth inequality cannot be considered as a feature of the development process, as many authors argue (see, for instance, Deininger and Squire 1998, **?**). We are aware that the ongoing discussion refers to income inequality rather than wealth inequality, but we want to make clear that we do not advocate the existence of a Kuznets curve. We rather point out in this paper that differences in Total Factor Productivity growth may help us to understand the different wealth inequality experiences observed across countries.

Finally, our model is able to predict an upsurge in wealth inequality following the productivity slowdown that started in the 1970's. The predicted rise in inequality is a of magnitude similar to that experienced by the US economy.

The only source of wealth inequality in our model is differences in savings rates across households. Other authors have explored the link between differences in savings rates and the dynamics of the wealth distribution. Chatterjee (1994) investigates the wealth distribution implications of a standard neoclassical model in which preferences are quasi-homothetic: Engel curves are affine functions of the level of wealth. The main difference between his approach and ours is the measure of wealth used. He investigates the evolution of the distribution of life-time wealth: i.e., present value of life-time earnings plus the value of accumulated capital, whereas we define wealth as the household net worth. Chatterjee proves that when a minimum consumption requirement exists, the distribution of life-time wealth becomes more unequal along the transition path. We show that the distribution of household net worth can either become more egalitarian or unequal along the transition path, depending on the region of the parameters space considered. The calibrated version of our model can account very well for the evolution of the U.S. wealth (net worth) distribution along the late XIXth and XXth centuries.

Caselli and Ventura (2000) study theoretically the evolution of wealth distribution in a framework similar to ours. They build a model in which households derive utility from a privately produced good and a public good. There are perfect capital markets. The public good in the utility function plays a similar role to a negative minimum consumption requirement. They find a similar result: The evolution of the wealth distribution exhibits an inverted U shape along the transition path. The main difference between their approach and ours is that they use a C.E.S. production function with a elasticity of substitution between capital and labor less than one, while we use the standard Cobb-Douglas technology. Thus, our model is able to replicate the standard long-run stylized facts.

Chatterjee and Ravikumar (1999) also study the link between capital accumulation and inequality along the transition path and introduce a minimum consumption requirement. They also generate a Kuznets curve for wealth inequality. As opposed to us, they use a linear technology and calibrate their model to match some estimates for the Indian economy.

The key departure of our model from the standard neoclassical framework is that we introduce a minimum consumption requirement. A minimum consumption requirement introduces a positive association between household wealth and the intertemporal elasticity of substitution (IES). Wealthier households have a higher IES and, therefore, a higher savings rate. Using panel data on Indian villagers, Atkeson and Ogaki (1996), and Atkeson and Ogaki (1997) find economically significant differences in the IES across rich and poor households. Rosenzweig and Wolpin (1993) also use Indian data and find a minimum consumption requirement to be statistically significant and to amount to a sizable fraction of consumption expenditures of the average household. Moreover, Rebelo (1992) and Ogaki, Ostry, and Reinhart (1996) argue that low savings rates and low elasticity of savings with respect to the interest rate point to the existence of a minimum consumption requirement. Thus, we think that a minimum consumption requirement is important to understand the process of capital accumulation and growth.

The rest of the paper is organized as follows: Section 2 describes the economic environment. Section 3 shows some theoretical results and establishes the connection between level of per capita income and wealth inequality. In section 4 we present the calibrated version of our model and the results of our simulations. In section 5 we study the sensitivity of the evolution of the wealth distribution with respect to the level of productivity growth. Section 6 concludes.

2 The model economy

The model economy is a discrete time infinite horizon economy, populated by a measure one of households that live forever. Each period, households obtain utility from consuming a commodity, C_t, that is produced using physical capital and labor. The representative firm uses a Cobb-Douglas technology to produce the consumption good,

$$F(K_t, N_t) = A_t K_t^{\theta} N_t^{1-\theta},\ \theta \in (0,1), \tag{1}$$

where K_t denotes aggregate capital, and N_t denotes aggregate labor. A_t is the exogenous technological progress factor that grows at rate γ. Capital depreciates at a constant rate, $\delta \in (0,1)$. There exist perfect capital markets: i.e., individuals are able to borrow and lend without any restriction at the market interest rate.

Households are endowed each period with one unit of labor. They do not value leisure and differ in their initial holdings of capital, k_0^i, where i is just an index to rank the types of individuals according to their level of initial wealth (physical capital). There are I types of households. We assume that all types have the same measure. K_0 denotes the initial aggregate capital of the economy. Population grows at the rate η. All households have identical preferences defined over consumption at every date,

$$U = \sum_{t=0}^{\infty} \beta^t (1+\eta)^t u(C_t),\ \beta \in (0,1). \tag{2}$$

The variable C_t denotes consumption per capita at period t. The one period utility function is the following:

$$u(C_t) = \frac{(C_t - \alpha_t)^{1-\sigma}}{1-\sigma},\ \sigma > 1,$$

where α_t denotes the subsistence level of per capita consumption at period t. We assume that the amount of consumption considered a “primary necessity” varies with income. The idea of the existence of a minimum level of consumption is often associated to the existence of a poverty line. The perception of this poverty line or, what amounts to the same, the level of consumption below which a person can be accounted as poor varies within and across countries.[1] We take the view that the minimum consumption level increases with income. However, it is unlikely that the minimum level changes rapidly with income within a country. We assume that the subsistence level of consumption grows at the balanced growth rate of per capita income,

$$\alpha_t = (1+g)^t \alpha.$$

Although this assumption simplifies the analysis, it is not strictly necessary to yield the result. What will be essential is that the minimum consumption grows at a rate not greater than that of the economy.

2.1 The firm's problem

The firm faces a series of static one-period profit maximization problems,

$$\max_{K_t, N_t} \; A(1+\gamma)^t K_t^{\theta} N_t^{1-\theta} - w_t N_t - r_t K_t \tag{3}$$

The wage rate, w_t, and the real rental price of capital, r_t, are equal to the marginal productivity of the production factors in equilibrium.

2.2 The household's problem

In this subsection we specify the modified model economy we are going to study throughout this paper. This economy exhibits a balanced growth path, along which the rate of growth of the per capita variables is $g = (1+\gamma)^{\frac{1}{1-\theta}} - 1$. We detrain all the variables to eliminate long run growth. Small letters denote the variables detrended. Once done this, the household i's problem can be written as

[1] On this regard, the 1990 World Bank Report (pp. 26-27) says,

> A consumption-based poverty line can be thought of as comprising two elements: the expenditure necessary to buy a minimum standard of nutrition and other basic necessities and a further amount that varies from country to country, reflecting the cost of participating in the everyday life of society. The first part is relatively straightforward. The cost of minimum adequate caloric intakes and other necessities can be calculated by looking at the prices of the foods that make up the diets of the poor. The second part is far more subjective; in some countries indoor plumbing is a luxury, but in others is a “necessity”.

$$
\max_{\left\{c_t^i, k_{t+1}^i\right\}_{t=0}^{\infty}} \qquad \sum_{t=0}^{\infty} \phi^t \frac{(c_t^i - \alpha)^{1-\sigma}}{1-\sigma}
$$

$$
\text{s.t.} \qquad c_t^i + (1+g)(1+\eta)k_{t+1}^i \ \square \ w_t + (1 + r_t - \delta)k_t^i, \tag{4}
$$

$$
k_0^i \text{ given}
$$

where $\phi = \beta(1+\eta)(1+g)^{1-\sigma}$. The factor $1+\eta$ is due to population growth and $(1+g)^{1-\sigma}$ is due to technological progress. This economy converges to a steady state without long run growth. The evolution of prices and wealth inequality here is identical to that of the original economy.

3 The transition path and the distribution of wealth

In this section we obtain the individuals' demand functions and investment and the law of motion of capital. Once done this, we turn to analyze the evolution of the wealth distribution. First of all, we need to specify our measure of household i's wealth at any period t. We use as a measure of wealth total capital accumulated by a household at a certain period, k_t^i. In this respect, we depart from Chatterjee (1994). His measure of wealth is present value of the total stream of income plus the accumulated wealth, i.e., life-time wealth:

$$
\sum_{s=t}^{\infty} \frac{p_s}{p_t} w_s + (1 + r_t - \delta)\, k_t^i.
$$

Using that definition he shows that wealth inequality worsens along the transition path when there exists a minimum consumption requirement. This definition of wealth is not very standard in the empirical studies. For instance, Wolff (1994) defines marketable wealth (or net worth) as the difference in value between total assets and total liabilities and debt. Since the purpose of this paper is to assess quantitatively the importance of capital accumulation to understand the evolution of wealth inequality, we prefer to use a measure of wealth that can be readily compared to the data. Hereafter the terms wealth, household wealth and capital holdings at any period t will be used with the same meaning. If we refer to life-time wealth we will do so by its name.

3.1 The demand function and the aggregate dynamics of the model

Here we estate some properties of the household's demand function. Solving the problem for household i, (4), the household i's demand function is

$$c_t^i = \alpha + \frac{1}{M_t}\left(\sum_{s=t}^{\infty}\frac{p_s}{p_t}(w_s-\alpha) + (1+r_t-\delta)\,k_t^i\right), \tag{5}$$

$$M_t = \sum_{s=t}^{\infty}\phi^{\frac{s-t}{\sigma}}\left(\frac{p_s}{p_t}\right)^{\frac{\sigma-1}{\sigma}}. \tag{6}$$

where p_t is the price of consumption good in period t in terms of consumption good in period 0. Function (5) says that the amount consumed above α at period t is the fraction $\frac{1}{M_t}$ of life-time wealth net of future needs of consumption.

The Engel curve is an affine function of the level of wealth. This ensures that the aggregate stock of capital next period does not depend on the distribution of wealth. This result is due to the specific utility function used, the existence of perfect capital markets and the assumption that individuals do not obtain utility from leisure. In this economy, as in Chatterjee (1994) or Caselli and Ventura (2000), growth affects the evolution of wealth inequality, but inequality does not affect growth. The dynamics of the aggregate variables does not depend on the initial level of inequality and is identical to the dynamics of the representative agent version of this economy. Nevertheless, the dynamics of the wealth distribution and the final distribution in the steady state will depend on the initial level of inequality. Next subsection discusses this point.

3.2 The evolution of the wealth distribution

Replacing the demand function shown in (5) in the household's budget constraint, we can obtain that the law of motion of household wealth follows

$$k_{t+1}^i = B_t + D_t k_t^i, \tag{7}$$

where

$$D_t = \frac{1}{(1+g)(1+\eta)}(1+r_t-\delta)\left(1-\frac{1}{M_t}\right), \tag{8}$$

$$B_t = \frac{1}{(1+g)(1+\eta)}\left[(w_t-\alpha) - \frac{1}{M_t}\sum_{s=t}^{\infty}\frac{p_s}{p_t}(w_s-\alpha)\right] \tag{9}$$

We can also express aggregate capital in terms of factors B_t and D_t, $k_{t+1} = B_t + D_t \cdot k_t$. The factors D_t and B_t have ready economic interpretations. Factor D_t is the fraction of current wealth after interest rate has been paid that is not consumed at period t and, therefore, is saved. Factor B_t is the amount of current labor income above α that is saved. Recall that $\frac{1}{M_t}\sum_{s=t}^{\infty}\frac{p_s}{p_t}(w_s-\alpha)$ is the fraction of present value of labor earnings above α that finances excess consumption, $c_t^i-\alpha$, at period t. Hence, if $B_t<0$, the households cannot finance consumption only with labor earnings and they need to either use capital earnings, to deplete their stock of capital or to borrow.

To study the evolution of wealth inequality, let us define X_t^i as the ratio of individual i's wealth to aggregate capital, $X_t^i=\frac{k_t^i}{k_t}$. Thus, the evolution of the ratio X_t^i respect to the average is given by

$$X_{t+1}^i-1=\frac{D_t\cdot k_t}{B_t+D_t\cdot k_t}(X_t^i-1) \tag{10}$$

Expression (10) shows that the share X_{t+1}^i gets closer to (further away from) the average when the factor $\frac{D_t\cdot k_t}{B_t+D_t\cdot k_t}$ is smaller (greater) than one. The value of that ratio depends on the signs of the factors B_t and D_t. It is easy to check in expression (6) that M_t is always greater than one. Therefore, the factor D_t is always positive. The factor B_t, however, can have either sign. Thus, we can already advance that the evolution of B_t governs the evolution of the wealth distribution in this economy. The value and the evolution of the factor B_t varies along the transition path of the particular economy we examine. For instance, in an economy very close to its steady state in which households have a low propensity to consume out of life-time wealth, $\frac{1}{M_t}$, we can expect B_t to be positive. On the contrary, if at some period t current wage is nearly the minimum consumption and $\frac{1}{M_t}$ is high B_t most surely will be negative. Hence, intuitively we can say that the value of the factor B_t at any period t and, therefore, the evolution of the wealth distribution, depend on how far an economy is from its steady state and its speed of convergence. We will discuss this more thoroughly in the following sections

To analyze the evolution of the wealth distribution we first introduce the notion of inequality. We give the definition of Lorenz-dominance in terms of our notation.

Definition 1 *Let all households be ordered according to their initial level of wealth. Let I be the number of types of households according to their level of wealth. $\frac{1}{I}X_t^i$ is the share of wealth held by group i. Then, the distribution of capital at period $t+1$ is more egalitarian than the distribution at period t if and only if it is satisfied that for* 1 □ J □ I

$$\sum_{i=1}^{J}\frac{1}{I}X_{t+1}^i\geq\sum_{i=1}^{J}\frac{1}{I}X_t^i. \tag{11}$$

The following Proposition relates the level of inequality, measured using the concept of Lorenz-dominance, with the aggregate dynamics of our model.

Proposition 1 *The distribution of capital at period $t+1$ is more egalitarian than the distribution of wealth at period t if and only if B_t is non negative.*

Proof. we can write expression (11) as

$$\sum_{i=1}^{J} \frac{1}{I} \left(1 - X_{t+1}^{i}\right) \square \sum_{i=1}^{J} \frac{1}{I} \left(1 - X_{t}^{i}\right), \tag{12}$$

then, substituting (10) in (12) we obtain

$$\frac{D_t \cdot k_t}{B_t + D_t \cdot k_t} \cdot \sum_{i=1}^{J} \frac{1}{I} \left(1 - X_{t}^{i}\right) \square \sum_{i=1}^{J} \frac{1}{I} \left(1 - X_{t}^{i}\right),$$

which holds true for B_t non negative. Recall that $\sum_{i=1}^{J} \frac{1}{I} X_t^i$ should be less than or equal to $\frac{J}{I}$, which is the fraction of aggregate wealth held by groups 1 to J assuming capital is evenly distributed across agents. ■

This Proposition states that in order to know the evolution of the wealth distribution over time we just need to study the evolution of B_t.

4 Quantitative implications of the model

Now we turn to analyze the quantitative predictions of the model about the level of wealth inequality along the transition path. To do so, we calibrate our model economy to match some key features of the U.S. economy. The linearity of the Engel curves will allow us to study this economy in two steps: we will analyze the evolution of prices and aggregate variables in the representative agent version of this model and we will turn afterwards to the full model to study the evolution of the wealth distribution.

4.1 Calibration

Our model is calibrated to replicate some statistics of the U.S. economy for the period 1949-1970. As Cooley and Prescott (1995), we assume that total capital is private fixed capital plus the stock of durable consumption goods, the stock of government capital and the stock of land. Consistently with this broad definition of capital, GDP includes the imputed services of consumption durable goods and the government capital stock. The share of capital consistent with these definitions of capital and GDP is 0.4. The implied capital-output ratio is equal to 3.32. The steady state ratio output to consumption is 1.33. The rate of population growth, η, is 1 percent per year and the rate of growth of real per capita output, g, is 3 percent. Furthermore, we set the initial level of technology A equal to 1. All these values are taken from Cooley and Prescott (1995).

Finally, we are left with the parameters σ and α. Atkeson and Ogaki (1996) estimate a value of the intertemporal elasticity of substitution for total consumption expenditures in the U.S. economy equal to 0.4 for the period 1968-1988. Thus we take 0.4 as the value of the intertemporal elasticity of substitution (IES, hereafter) in the steady state,

$$IES = \frac{c_{ss} - \alpha}{\sigma \cdot c_{ss}} = 0.4, \tag{13}$$

where c_{ss} denotes steady state per capita consumption. Since we have assumed α to be non negative, the equality (13) imposes an upper bound for σ: it has to be less than or equal to 2.5. We also believe reasonable to impose that α/c_{ss} should not be greater than 0.4. We have the notion that the minimum consumption α should be below 40 percent of c_{ss}. These considerations restrict the region of values of σ to $[1.5, 2.5]$. We try three different values for σ, 1.5, 2.1, and 2.5. The last case corresponds to the standard neoclassical model in which there is no minimum consumption requirement. In each case the value of α is chosen so that (13) holds.

We study the evolution of the model economy for these three cases. We summarize the calibrated parameters in the following Tables:

Preferences		Technology			
ϕ	η	A	θ	δ	g
0.9579	0.01	1	0.4	0.0344	0.03

Table 1

	σ	α	α/c_{ss}
Case 1	1.5	0.67	0.40
Case 2	2.1	0.27	0.16
Case 3	2.5	0.00	0.00

Table 2

Table 2 shows that the level of minimum consumption as a fraction of steady state consumption is lower the higher the value of σ. This is because we are matching α so that the Intertemporal Elasticity of Substitution is 0.4 in the steady state. Just to have an idea of what the fraction α/c_{ss} represents, we have computed the amount of food expenditures as a fraction of non durable consumption expenditures. We have used data reported in the National Income and Product Accounts for the period 1949-1970. This fraction decreases over time and it is 21.0 and 15.6 percent in 1949 and 1970, respectively.

4.2 Predictions on aggregate dynamics

We analyze in this subsection the properties of the dynamics of this model. First, we need to set the initial level of per capita capital. King and Rebelo (1993) study the transitional dynamics of the neoclassical growth model. They calibrate the initial level of capital so that capital accumulation explains one half of the sevenfold per capita output growth observed over the period 1870-1970. Cooley and Prescott (1995) estimate that the contribution of capital accumulation to total growth in that period is around to one third and that Total Factor Productivity explains the other two thirds. Then, we compromise between the two numbers and assume that the contribution of capital accumulation is 0.4. Then, k_0 should satisfy

$$\frac{F(k_{ss},1)}{F(k_0,1)} = 7^{0.4}. \tag{14}$$

Gordon (2000) reports that the average rate of growth of non-farm non-housing business GDP was 4.42 percent for the period 1870-1913 and decreased to be 3.12 for the subsequent period 1913-1970. We will use these numbers to assess the quantitative performance of the model.

Figures 1a and 1b show the evolution of the aggregate variables for the different cases considered. Figure 1a shows the evolution of output, consumption, and capital as a fraction of their steady state value. The transition path is longest when $\sigma = 1.5$ since the value of α used implies the highest level of minimum consumption: 40 percent of the steady state consumption. Nevertheless, in all the cases the transition is completed after 75 periods (this corresponds to the year 1944). The main differences appear in Figure 1b, that shows the evolution of the savings rate, the capital-output ratio, the growth rate and the factor B_t.

Notice that at the beginning of the transition, the growth rate of output is highest for $\sigma = 2.5$ and lowest for $\sigma = 1.5$. This is so because the minimum consumption requirement as a percentage of average consumption is much higher in the economy with $\sigma = 1.5$, which generates a much lower IES along the transition. The growth rate falls to 3.79, 3.60 and 3.57 percent after 30 periods, respectively for each calibration, and declines steadily to be 3 percent. The implied average growth rate for the period 1870-1913 is 4.69, 4.73, and 4.74 respectively for each case, being the highest rate for the case in which $\sigma = 2.5$. Gordon (2000) reports that the average growth rate for that period was 4.42. For the period 1913-1970 the average growth rate is 3.11, 3.10 and 3.09 for each case, whereas the rate reported by Gordon (2000) is 3.12.

The economy with the highest savings rate is the one in which $\sigma = 2.5$ and the lowest when $\sigma = 1.5$. The other case lies in between. Williamson (1991) reports that the gross savings rate was 23 percent in 1870 and rose to 28 percent at the turn of the century. The savings rate for each case in the initial period is 18.88, 26.42 and 29.38 and for all cases it is around 25.8 percent at the turn of the century.

Williamson also reports that the return to conventional reproducible assets was 6.6 percent at the turn of the century. The evolution of the real interest rate is very similar in the four cases considered. It starts around 35 percent and drops to around 10.5 percent after 30 periods (which corresponds to the year 1900 in our model). Thus, the interest rate is higher in the model than in the data. Nevertheless, we should keep in mind that the interest rate of our model economy has no counterpart in the U.S. economy, since we are assuming that aggregate capital includes government capital and consumer durables.

Thus, all the cases do not differ much in the aggregate evolution of all variables, but in one: The evolution of the factor B_t. Figure 1b shows that if $\sigma = 1.5$ the factor B_t is always negative and approaches zero as the economy gets closer to the steady state. This implies that in this economy the distribution of wealth becomes more unequal along the transition path. For $\sigma = 2.5$ the factor is always positive, therefore, inequality always decreases. For $\sigma = 2.1$, B_t first increases and afterwards decreases. In this case the evolution of the wealth distribution has an inverted U shape. Thus, the model is able to generate a Kuznets curve. The next point is how much variation in inequality this model generates along the transition path.

4.3 The evolution of inequality

In this subsection we analyze the size of the variation in inequality generated in the transition path and confront our results to the —some authors would say scant— evidence we have about the evolution of the U.S. wealth distribution. To do this, we divide the individuals in ten groups and we fix an initial distribution of wealth across individuals. The evolution of consumption, investment and capital across households are obtained using expressions (5), (7) and the constraint of the individual problem (4). The Gini coefficient is the index used to measure the inequality of wealth and income.

4.3.1 Historical evidence

There are no historical data on the evolution of the Gini coefficient of wealth for the entire period 1870-1970. The existing data is very fragmentary but it gives us some clue about how inequality evolved over time. The studies that most thoroughly analyze the data available for that period are Lindert (2000) and Williamson (1991).

The evolution of wealth inequality in the late XIXth and early XXth centuries appear to be controversial. Williamson (1991) reports that in 1870 the top 1 percent of all adults held 27 percent of total assets, whereas the top 10 percent held 70 percent. The associated Gini coefficient was 0.83. Since then until the years after World War II there is a gap that some authors have filled with fragmentary data. Table 3 contains data from Lindert (2000). This data, although not directly used in this paper, gives us some idea of the evolution of inequality. The table shows the net worth held by the top 1 percent of individuals for selected years within the period 1890-1989. This share was 25.8 percent in 1890 and peaked somewhere between 1913 and 1929, declining until 1976, year after which it started rising again. This behavior is in line with the documented rise in wealth inequality after the 1970's. (See, for instance, Wolff 1994)

There is a gap between 1890 and 1922. Lindert (2000) asserts that inequality in

America in 1929 was roughly the same as in England at that time. The same author reports that the share of marketable net worth held by the top 5 percent of all adults was 78.9 at that year. This data combined with the data for U.S. in 1870 tells us that wealth inequality should have risen within the period 1870-1929. We are aware of the fragility of the evidence handled for this period. We just can say that there is evidence for a increase of wealth inequality in that period and how much and exactly when it started it is still a hotly debated issue.

Net worth held by top 1% of households								
1890	1922	1929	1933	1939	1945	1949	1953	1962
25.8	36.7	44.2	33.3	36.4	29.8	27.1	31.2	31.8
1965	1969	1972	1976	1979	1981	1983	1986	1989
34.4	31.1	29.1	19.9	20.5	24.8	30.9	31.9	35.7

Table 3. Wealth inequality in the U.S., 1890-1989. Source: Lindert (2000).

4.3.2 Calibration of the initial wealth distribution

The initial distribution is chosen to match the data of wealth distribution for the U.S. economy in 1870. According to Williamson (1991), the Gini coefficient of wealth was 83.3 percent and the share of wealth held by the top 10 percent of all adults was 70 percent of the total amount of wealth and the share of the top 1 percent was 27 percent. For the rest of the deciles, there is no available information. We choose to divide the households in deciles and, therefore, we ignore the information about the share of the wealthiest 1 percent of the population.[2] Many distributions of capital match the remaining two statistics. We have conducted a number of experiments with different initial distributions reported in Table 4, and the results are not changed in a substantial way. We present in this paper those results obtained using distribution 1 shown in Table 4.

Decil	1st	2nd	3rd	4th	5th	6th	7th	8th	9th	10th
Distrib. 1	0.10	0.53	0.59	0.90	1.01	3.17	5.40	8.09	10.1	70.09
Distrib. 2	0.12	0.13	0.19	0.50	2.42	3.40	6.40	7.00	9.75	70.09
Distrib. 3	0.05	0.08	0.10	0.62	2.01	4.00	6.45	7.60	9.00	70.09
Distrib. 4	0.25	0.28	0.32	0.60	0.82	3.90	6.88	7.35	9.43	70.09

Table 4. Initial distributions of wealth.

[2] We are aware that the individual wealth distribution is different of the household wealth distribution, but we do not have other inequality data for that year. Thus, we take Williamson's data as an approximation to the level of household wealth inequality.

4.3.3 Comparing the cases to the data

Figure 1b already tells us that the model calibrated with $\sigma = 1.5$ predicts an ever increasing level of wealth inequality, which we have not observed in the data. Likewise, setting $\sigma = 2.5$ the model predicts that the level of inequality is monotonically decreasing over time. Figure 2a shows the evolution of the Gini coefficient of wealth in the period 1870-1940. The model economy reaches the steady state in 1940 and thereafter the level of inequality remains unchanged. Thus, we see that the model predicts an early surge in inequality and a steady reduction afterwards. Thus, we think of the case in which $\sigma = 2.1$ as the one that best resembles the evolution of wealth inequality in the U.S. economy among the four considered.

It can be argued that the data used to choose $\sigma = 2.1$ as the case that best matches the data, instead of $\sigma = 2.5$ is too fragmentary and not sufficient to make such a choice. After all, the most reliable evidence is that of the post World War II years and it shows that inequality kept decreasing during the entire period until the 1970's. That is exactly what the case in which $\sigma = 2.5$ implies: an ever reduction of inequality.

Tables 5 and 6 show different statistics that describe the household wealth distribution in 1962 as reported by Wolff (1994) and compares them to the statistics delivered by our model in the steady state.[3] The definition of wealth used by Wolff is net worth: total assets minus total liabilities. Notice how remarkably well the model predicts the distribution of wealth when σ is set to be equal to 2.1. Thus, we take this case as the one that best resembles the historical evidence we have on U.S. inequality. We will refer to this case hereafter and it will be our benchmark.

Year 1962	Percentage share of wealth held by quintile				
	Top	Second	Third	Fourth	Bottom
Data	81.00	13.40	5.40	1.00	-0.70
Model, $\sigma = 2.1$	80.90	13.42	4.00	1.28	0.004
Model, $\sigma = 2.5$	71.64	14.42	6.43	4.13	3.39

Table 5

Year 1962	Percentage share of wealth held by		
	Top 10%	Second 10%	Bottom 80%
Data	66.90	14.00	19.10
Model, $\sigma = 2.1$	71.78	10.12	19.10
Model, $\sigma = 2.5$	61.53	10.10	28.36

Table 6

[3] Our model economy reaches the steady state after 75 periods, which corresponds to the year 1944. Unfortunately, we have not found information on the wealth distribution across deciles prior to 1962. Thus, we take the distribution of 1962 as a proxy for that of previous years.

4.3.4 The evolution of wealth and income inequality

Figure 2a shows that the Gini coefficient of wealth is 83 percent in the first period, reaches 86.74 percent after 9 periods, and decreases to be 84.3 thereafter. The Gini coefficient of income is 33.3 percent in the first period, reaches 35 percent after 10 periods, a 8 percent increase, and decreases to its previous. The experiment also shows that after 60 periods there is little variation in the level of inequality. This accounts for the remarkable stability of wealth inequality in the post-war U.S. economy.

4.3.5 Inequality across agents

Figure 2b shows the evolution of the shares of capital for 3rd, 5th, 7th, and 10th deciles, respectively. The evolution of the shares mirrors that of the Gini coefficient of wealth: Until period 13 the shares of 3rd, 5th, and 7th decrease, and that of the top decile increases. After that period, the behavior is reversed. Figure 2c shows the savings rates for the aforementioned deciles. The first thing that calls our attention is the enormous difference of savings rates across deciles: The households in the 10th decile save in the steady state more than 50 percent of their income, whereas the saving rate of those in the 7th decile is around 16.5 percent. Most of the aggregate investment is comprised by the savings of the top decile: 67.25 percent of the investment in the steady state is due to savings of the wealthiest individuals, being this fraction 73.99 percent at the peak of the wealth inequality. These differences in savings rates across wealth deciles are consistent with Avery and Kennickell's (1991) findings. They report that almost all the net saving between 1983 and 1986 was made by the top decile of the 1986 wealth distribution.

4.3.6 Differences in wealth and income concentration

Díaz-Gimenez, Quadrini, and Ríos-Rull (1997) report that in 1992 the Gini coefficient for income was 0.57 and that for wealth 0.78. Atkinson (1997) finds that the Gini coefficient for income in 1970 was around 0.40. This model, therefore, underestimates the level of income inequality and overestimates that of wealth inequality. Two features of the model may account for this: Agents are identical in their human capital and they do not value leisure. The first assumption implies that the associated income distribution is always going to be more egalitarian in our artificial economy than that of the U.S. economy. The second assumption implies that labor supply does not vary with wealth. If labor supply were endogenous, given the level of human capital, wealthier agents would supply fewer hours in the market than the wealth poorer agents and, consequently, wealth inequality would be lower. Thus, our model points out that in order to account for the concentration of wealth relative to that of income, the wealth effect on the labor supply should be small.

4.4 Discussion of the results

In this subsection we want to discuss the key features of the model that deliver the result on the evolution of the level of inequality in the benchmark case.

At low levels of income the wage is too low so that it only suffices to finance the minimum consumption α or barely a little more (this corresponds to B_t negative). This implies that poor households have to get indebted to finance any amount of consumption above α. The amount of debt will be larger the smoother they want their consumption path to be. Wealthy households finance extra consumption using capital earnings and still save a positive amount. Thus, wealth inequality widens. As the economy grows, the wage grows to finance excess consumption, so poor households' level of debt decreases (B_t still negative but decreasing in absolute value). Still, inequality widens since wealthy households are increasing their stock of capital. There is a level of per capita income for which the factor B_t becomes positive. Current labor earnings more than finance current consumption and poor households start paying off their debt. After some periods, they start accumulating a positive amount of capital at a rate higher than that of wealthy households and, therefore, inequality starts declining.

Let us turn to the case in which $\sigma = 1.5$. In this case, α is 40 percent of the steady state consumption. This implies that at low levels of income poor individuals have to borrow an amount so heavy (or to deplete their stock of capital so much) so that their share of capital is always decreasing along the transition path. Another way of saying this is that households want very smooth consumption paths (they have a very low intertemporal elasticity of substitution) and therefore, incur in high levels of debt at low levels of income. The economy grows at a very slow rate and by the time labor earnings are high enough poor households are still paying off debt and, consequently, inequality is ever increasing along the transition path. The opposite occurs in the case in which $\sigma = 2.5$, case in which inequality decreases along the transition path.[4]

5 Wealth inequality and the sources of growth

In this paper we have focused our attention on wealth inequality. We have seen that once we introduce a minimum consumption requirement, the standard neoclassical growth model can account for the observed pattern of wealth inequality in the U.S. economy within the period 1870-1970. The level of wealth inequality in the model economy displays an inverted U shape, as the available data suggests. Hence, the model delivers a Kuznets curve for the wealth distribution, as well as for that of income. We prefer to remain silent on this last respect since we have assumed that all households have identical level of human capital and, hence, our model is not suited to analyze the evolution of income

[4] When $\sigma = 2.5$ the value of α is 0. Therefore this is the standard model with homothetic preferences. We do not mean to imply that with homothetic preferences inequality is always decreasing. For instance, we have computed that setting $\alpha = 0$ and σ greater than 4 inequality widens along the transition path, whereas it always decreases for lower values of σ. There exists a value of σ for which inequality remains constant along the transition path (results can be obtained from the authors upon request).

inequality. In this section we want to analyze the forces that drive the existence of such Kuznets curve. Intuitively, we can see that the length of the transition path is key for that pattern in the evolution of inequality to appear. The length of the transition path depends on the level of the Intertemporal Elasticity of Substitution and on the level of Total Factor Productivity (TFP hereafter) growth. Here we want to explore the relationship between the evolution of inequality and the level of TFP growth.

5.1 Total Factor Productivity growth or factor accumulation?

To analyze the importance of Total Factor Productivity growth in determining the evolution of inequality we conduct two exercises. We simulate the evolution of the U.S. economy within the period 1870-1970 assuming a TFP growth rate different from that used previously. We make two new calibrations of the model along the following lines: we change the value of the Total Factor Productivity growth rate in 20 percent of its original value, which was 1.79 percent to account for a growth rate of 3 percent in the balanced growth path. The change in productivity growth implies a balanced path growth rate of 2.40 percent, and 3.61 respectively in each exercise. The value of the rest of the parameters remains unchanged. Finally, to simulate the transition we need to choose an initial level of capital. We choose the initial level of capital so that total growth in the period 1870-1970 is identical to that of the benchmark economy:

$$(1+\widehat{g})^T \cdot \frac{F\left(\widehat{k}_T, 1\right)}{F\left(\widehat{k}_0, 1\right)} = (1+g)^T \cdot \frac{F\left(k_T, 1\right)}{F\left(k_0, 1\right)}, \ T = 99,$$

Values with hats denote values in the recalibrated model economy. k_T is the value of capital in the benchmark economy at period 99 (which corresponds to 1970). $\widehat{k}_T$ is obtained so that aggregate capital in period 99 amounts to the same fraction of steady state capital as in the benchmark economy.

Figure 3a shows the evolution of wealth inequality and capital in the case in which the per capita income growth rate is 2.4 percent. The transition path is simulated assuming a much lower level of initial wealth inequality; otherwise the poorer individuals' income would be below the minimum consumption requirement. The Gini coefficient of wealth is 0.60 at the initial period and increases for 13 periods, reaching a value of 0.8992. After 100 periods it decreases to 0.8260, being the percentage share of wealth held by the top decile 87.17 per cent in 1962, whereas it is 81.00 in the data. Hence, inequality increases further and for more periods. The key to understand the differences is the length of the transition. In this case, initial capital is 3.30 percent of its steady state value, whereas is 14.29 percent in the benchmark case. Thus, here the transition is longer.

The opposite occurs when the balanced growth path rate is assumed to be 3.61 percent. In this case, shown in Figure 3b, we have taken the initial wealth distribution identical to that of the benchmark case. Notice that inequality is always decreasing. Again, the key is the level of initial capital relative to its steady state value. In this case, initial capital is 61.85 percent of the steady state value, larger than in the benchmark case.

These two exercises suggest that the length of the transition path is key for the existence of a Kuznets curve in wealth inequality. The length of this transition depends on the contribution of TFP growth and the initial level of capital. For any given level of initial capital, the higher the rate of growth of TFP, the closer is the economy to its steady state and, therefore, the shorter the period, if any, of increasing inequality. In others words, the larger the contribution of capital accumulation to growth, the longer is the period of increasing inequality.

For instance, let us think of the Ak model. In this model total growth is driven solely by capital accumulation. It is easy to check from expression (9) that the factor B_t is always negative, regardless of the level of aggregate capital. Therefore, inequality is ever increasing: savings rates are increasing with capital holdings. Since the interest rate is constant, this implies that the rate of growth of the capital shares is greater for wealthier individuals. Hence, inequality is always increasing.

Thus, our results suggest that investigating this avenue could help us to understand the different evolution of wealth inequality observed across countries. Since Kuznets (1955) formulated his famous conjecture, there has been an ongoing discussion about whether the Kuznets curve is a feature of development. The researchers that investigate this issue focus their attention on income inequality, instead of wealth inequality. Deininger and Squire (1998) and Fields and Jakubson (1992) are among the authors that more forcefully reject the Kuznets curve as a feature of development. They argue that previous studies that support the existence of the Kuznets curve use cross section data, whereas the initial Kuznets' conjecture concerned the evolution of inequality within a country.[5] They also show that when using panel data the Kuznets curve cannot be accepted as a stylize fact of development: it appears as often as not for different countries.

Our results, referring to wealth inequality, are in line with their findings. First of all, our model predicts that in any two countries identical in all respects but the initial distribution of wealth, the initial difference in wealth inequality will persist over time.[6] Thus, cross section data will not be informative about the evolution of inequality over time for a given country. Secondly, the exercises performed in this section suggest that the evolution of wealth inequality within a country is very sensitive to the level of productivity growth. Therefore, the Kuznets curve can appear as often as not in time series data.

[5]Fields and Jakubson (1992) argue that the Kuznets curve is not rejected by cross sectional data because of the so called "Latin American effect". Latin American countries have the highest secular levels of inequality in the world and most of them are middle countries. Thus, the cross section Kuznets curve is served.

[6]It is easy to check that the Gini coefficient of wealth can be written as $G_{t+1} = \prod_{r=0}^{t} \left(\frac{D_r k_r}{B_r + D_r k_r} \right) \cdot G_0$. Therefore, any two countries that start a process of sustained growth with different level of inequality will experiment similar evolution in the dynamics of inequality, but the levels will remain different at each point in time.

5.2 The productivity slowdown and the upsurge of inequality

In the last two decades many developed countries, U.S. among them, have experienced an upsurge in their levels of income and wealth inequality. This phenomenon has been documented by, among others, Atkinson (1997), Gottschalk and Smeeding (1997), and Wolff (1994). These observations, more than any study, refute the existence of a Kuznets curve, in the sense that developed countries may experience, as any other poorer country, a rise in inequality. On the other hand, over the period 1970-1990 the U.S. economy experienced a slowdown in the growth rate of Total Factor Productivity, per capita income, capital, and consumption. The previous discussion leads us to think that the decrease in the economy growth rate may have affected the evolution of the wealth distribution.

Here we want to explore the predictions of our model when we assume a change in Total Factor Productivity growth rate as the one experienced in the U.S. economy during the period 1970-1990. Several authors have measured the changes in the growth rate of Total Factor Productivity. Gordon (2000), for instance, uses data for the period 1870-1996. He uses as a measure of output that of the non farm non-housing sector. He finds that the rate of growth of TFP for the period 1870-1913 is about the half of its value for the period 1913-1970 and much lower in the last period 1970-1996. We cannot use his estimates since he uses a different measure of output and a different share of capital in aggregate production. Thus, we have constructed a series for aggregate capital and GDP consistent with our calibration procedure and computed the rate of growth of TFP assuming that the contribution of labor is zero. This is also consistent with Cooley and Prescott's (1995) finding that the secular contribution of labor to growth is negligible. We have constructed these series using data from the National Income and Product Accounts and they are available upon request. Figure 4 shows the evolution of the growth rate of TFP in our sample. We have divided the whole sample in three periods, 1949-65, 1966-80 and 1981-99. The average for the second period is lower than those of the earlier and final period. Specifically, the average rate of growth in the period 1966-1980 is 26.83 percent of its previous value in the period 1949-1965. The average growth rate for 1981-1999 is 92.15 percent of its value in the first period. Figure 5 shows the evolution of the capital-output ratio in those periods. Notice that it starts increasing in 1965, peaks in 1980 and falls steadily thereafter to a level close to its level in 1965.

The exercise we conduct is the following: we assume that our artificial economy is in its steady state in 1965. From there on we assume that the TFP growth rate is 26.83 percent of its previous value (which is 1.79 percent to obtain a steady state growth rate of 3 percent) during the next 15 periods. This corresponds to the period 1966-80. After this, we assume that the TFP growth rate increases to be 92.15 percent of its value in the period 1949-65.

Thus, we proceed as if the economy moved from 1965 to 1980 to a new balanced growth path along which the growth rate were smaller than the previous one. After that year we proceed as if the economy moved to a new balanced growth path with a higher growth rate in the steady state. In both cases we assume that those changes in the TFP growth rates are totally unexpected and that households view them as permanent. Figure 6 shows the evolution of the Gini coefficient for wealth and income, respectively, the growth rate of output and the capital-output ratio. Notice that the evolution of the capital-output ratio in the model resembles closely that of the ratio in the data. Thus, our exercise predicts a reduction in inequality in the 1970's and an increase in the last period. These results are in line with the data: according to Wolff (1994), wealth inequality increased substantially during the 1980's after a period of intense reduction during the 1970's. Actually, wealth inequality started rising in the mid-1970's, but we should keep in mind that earnings inequality also started rising at that time and that the only source of wealth inequality in our model economy is differences in the savings rates.

The intuition of the results is the following: the decrease in the TFP growth rate in the period 1966-80 implies that the economy moves towards a new steady state in which the capital-output ratio is higher than previously. Thus, households increase their savings. Since the economy is fairly close to its new steady state, poorer households increase their savings rate in a higher proportion than wealthier households and inequality decreases along that path. In the period 1981-99 the rise in TFP growth rate makes the economy to move towards a new steady state in which the capital-output ratio is lower than in the previous steady state. A lower capital-output ratio implies a higher consumption-output ratio for all households, regardless of their level of wealth. Since households want to smooth consumption along the path, poorer households decrease their savings rate in a higher proportion than wealthier households and wealth inequality rises.

Our experiment predicts a fall in the aggregate savings rate during the 1980's, as it has been documented by Avery and Kennickell (1991) and Bosworth, Burtless, and Sabelhaus (1991), among others. Moreover, the model predicts that the fall in the savings rate is more severe among households in the low wealth deciles than among wealthier households. This is why wealth inequality increases during that period in our model. The evidence we have found about the behavior of the savings rates across deciles for that period is indirect. Bosworth, Burtless, and Sabelhaus (1991) found that the decline in the savings rate during the 1980's was actually smaller among bonds and stockholders than it was among households with no marketable financial assets. Wolff (1998) reports that more than 43 percent of the wealth of the richest 20 percent of the households takes the form of investment assets. In contrast, almost two thirds of the wealth of the bottom 80 percent of the households was invested in their own home. This evidence leads us to think that the decline in the savings rate of wealthy households was less severe than that of the poor households, as our model predicts. Thus, our experiment suggests that differences is savings rates have played an important role in the rise of wealth inequality during the 1980's.

6 Final comments

This paper has shown that a modified version of the neoclassical growth model can account very well for the observed evolution of wealth level and inequality in the U.S. economy in the last century. The only source of inequality in this model is the initial distribution of wealth. Households are identical in their levels of human capital and, thus, there is perfect equality in the earnings distribution. It could be argued that a model that allowed for earnings inequality could account better for the evolution of the wealth distribution in the U.S.. Nevertheless, we need to keep in mind that the correlation between earnings and wealth is extremely low (Díaz-Gimenez, Quadrini, and Ríos-Rull (1997) estimate a correlation of 0.23 using the 1992 Survey of Consumer Finances).

We have made a key assumption: there are perfect capital markets. If there were any borrowing restriction, inequality along the transition path would be lower and the level of growth would be higher. We think that perfect capital markets is not a too bad assumption. Poor individuals always have available informal markets or the family network to borrow or to get insurance (see Townsend (1994), Rosenzweig and Stark (1989)).

The model predicts that the level of wealth inequality first increases and decreases afterwards, remaining constant in the steady state. In other words, the evolution of wealth inequality shows a Kuznets curve. Nevertheless, we argue that the Kuznets curve is not a feature of the development process and that the relative contribution of each source — capital accumulation and productivity growth— is key for the existence of a Kuznets curve in wealth inequality along the transition path. Our experiments suggest that countries with lower Total Factor Productivity growth should experience higher levels of wealth inequality and for longer periods of time. We think that this path is worth investigating, since the evolution of inequality differs greatly across countries. Finally, this model is able also to account for the upsurge in wealth inequality that followed the slowdown in growth started in the 1970's.

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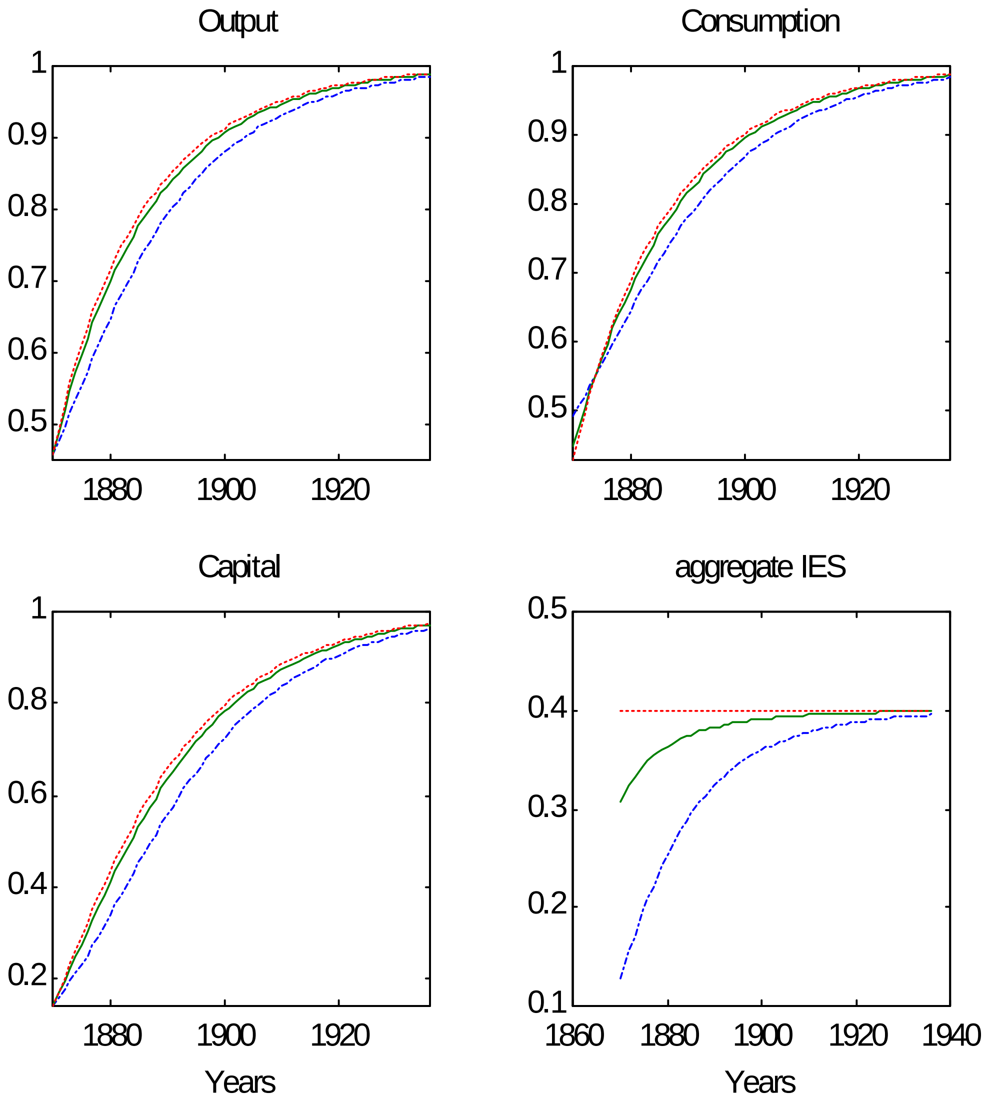

Figure 1a. Aggregate transitional dynamics

Key: Dashed dotted line, $\sigma = 1.5$; solid line, $\sigma = 2.1$; and dashed line, $\sigma = 2.5$.

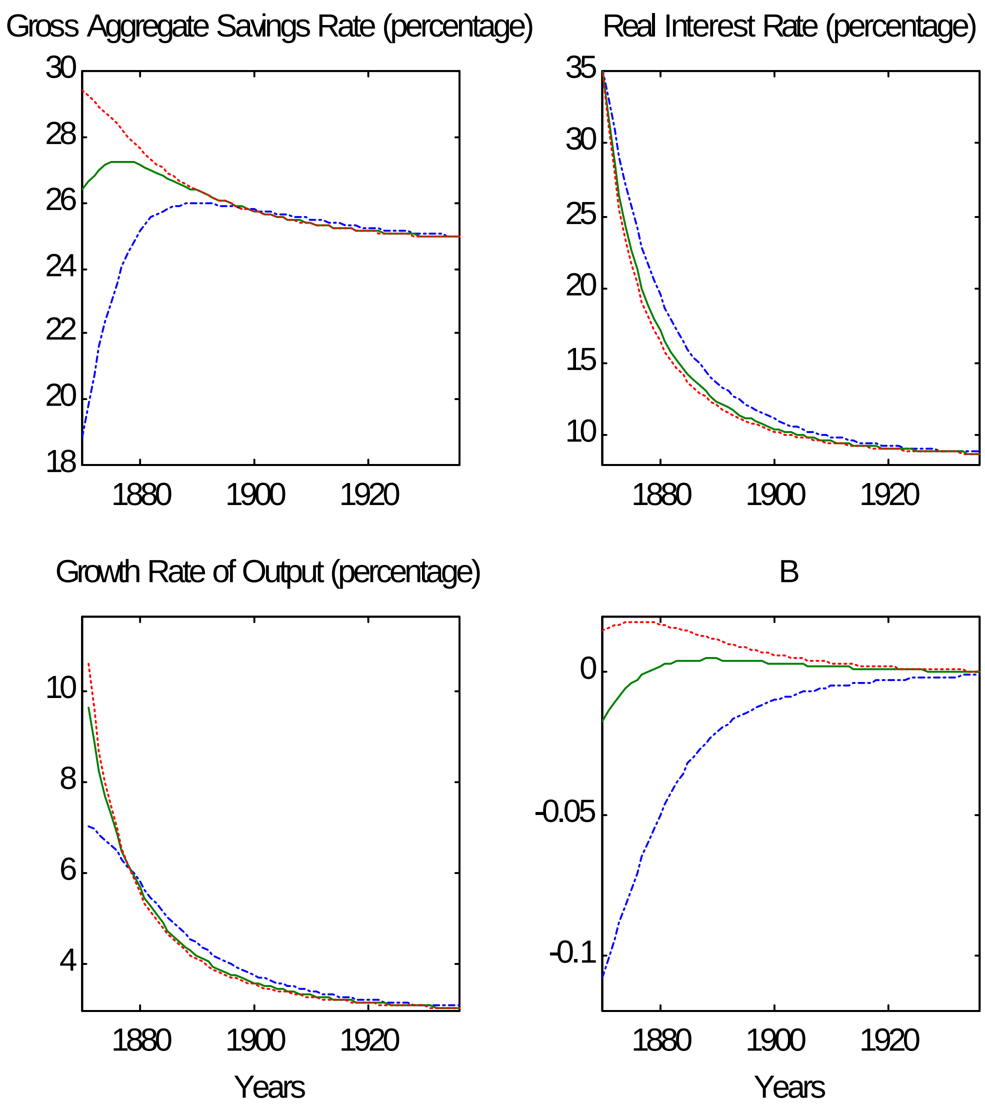

Figure 1b. Aggregate transitional dynamics

Key: Dashed dotted line, $\sigma = 1.5$; dashed line, $\sigma = 2$; solid line, $\sigma = 2.1$; and dotted line, $\sigma = 2.5$.

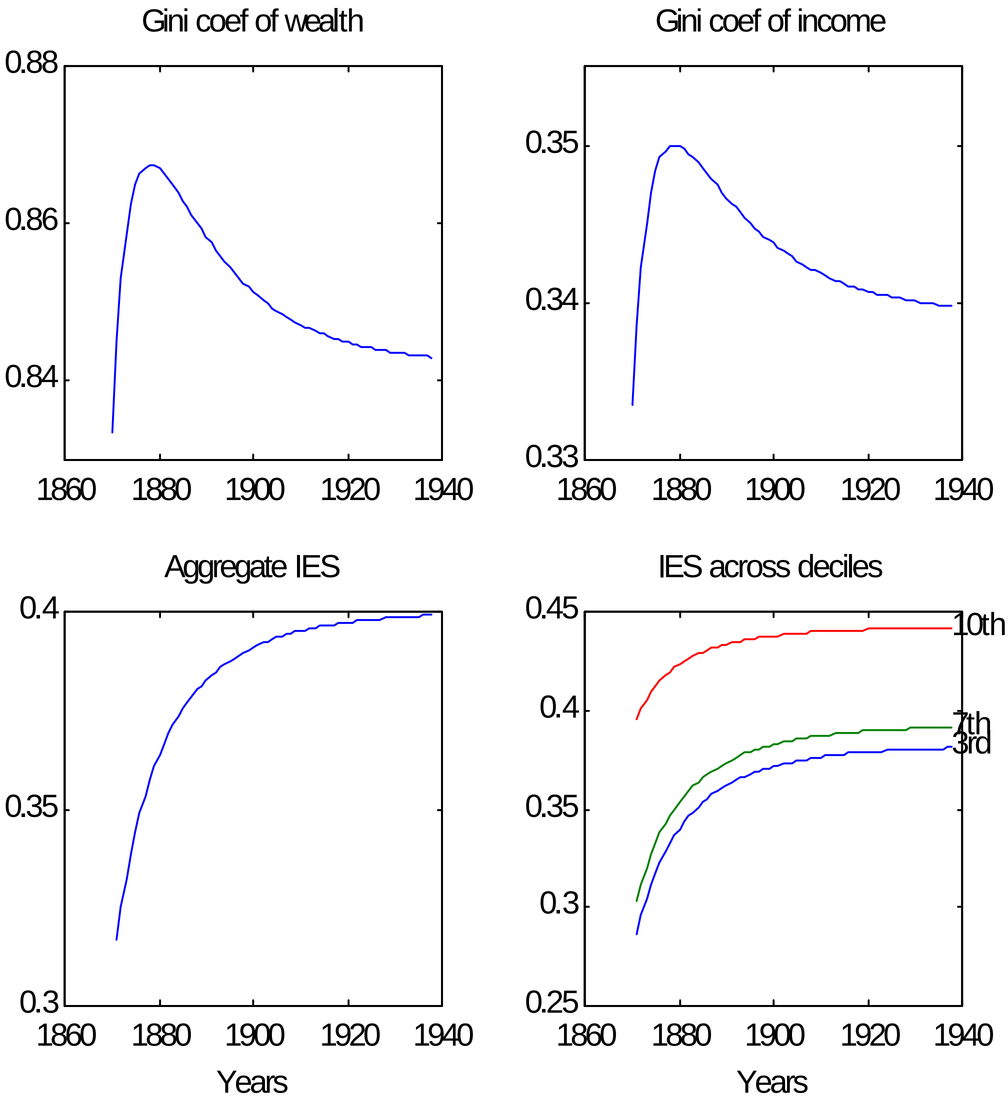

Figure2a. Evolution of inequality. $\sigma = 2.1$.

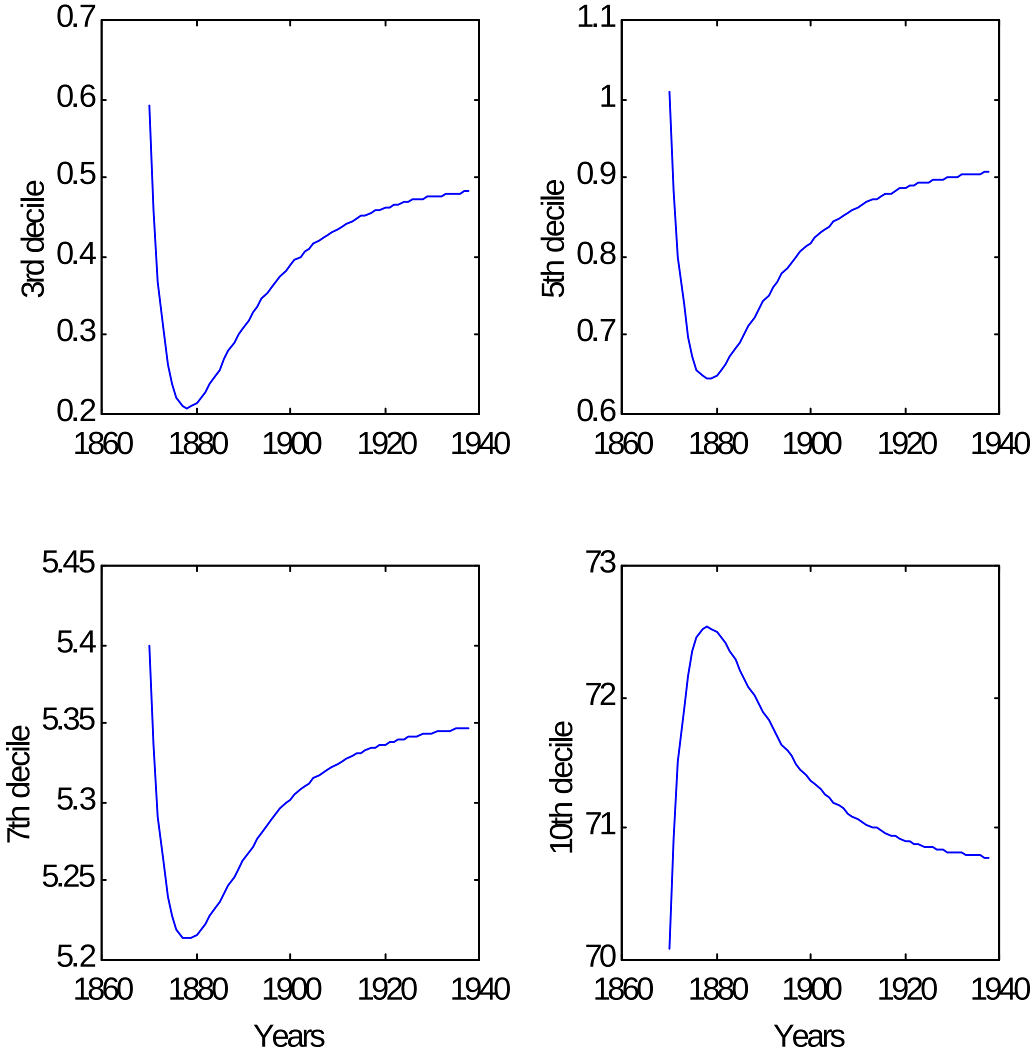

Figure 2b. Shares of wealth across deciles. $\sigma = 2.1$

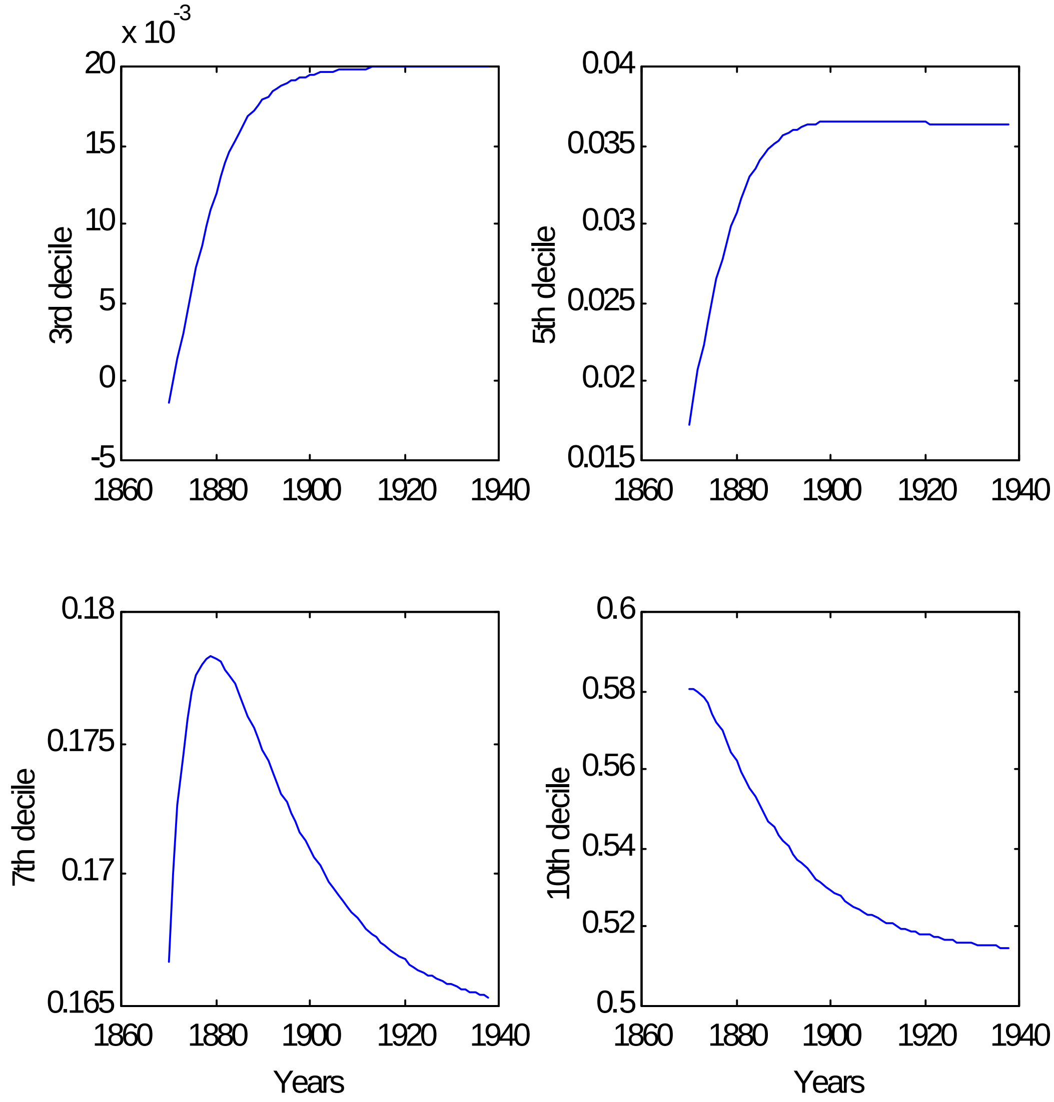

Figure 2c. Savings rates across deciles. $\sigma = 2.1$

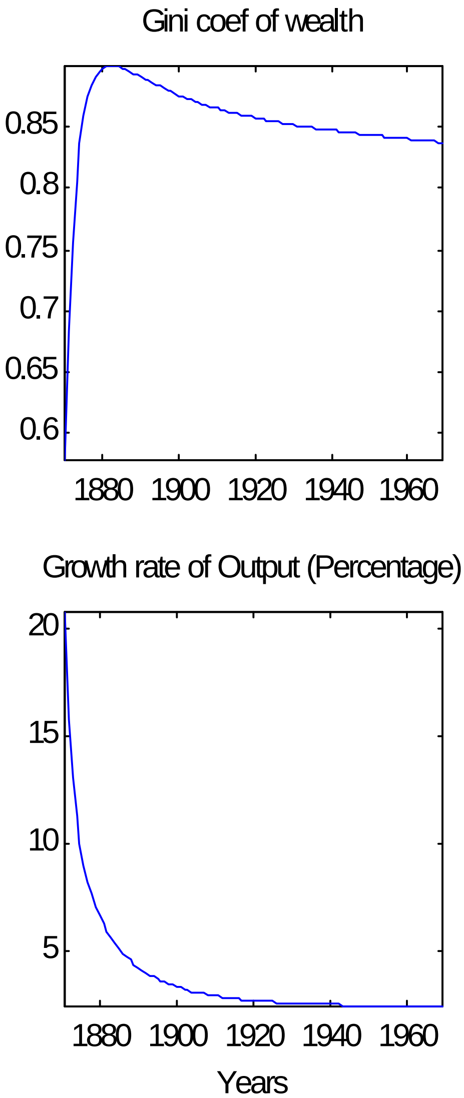

Figure 3a. Evolution of inequality when $g = 2.4$ percent.

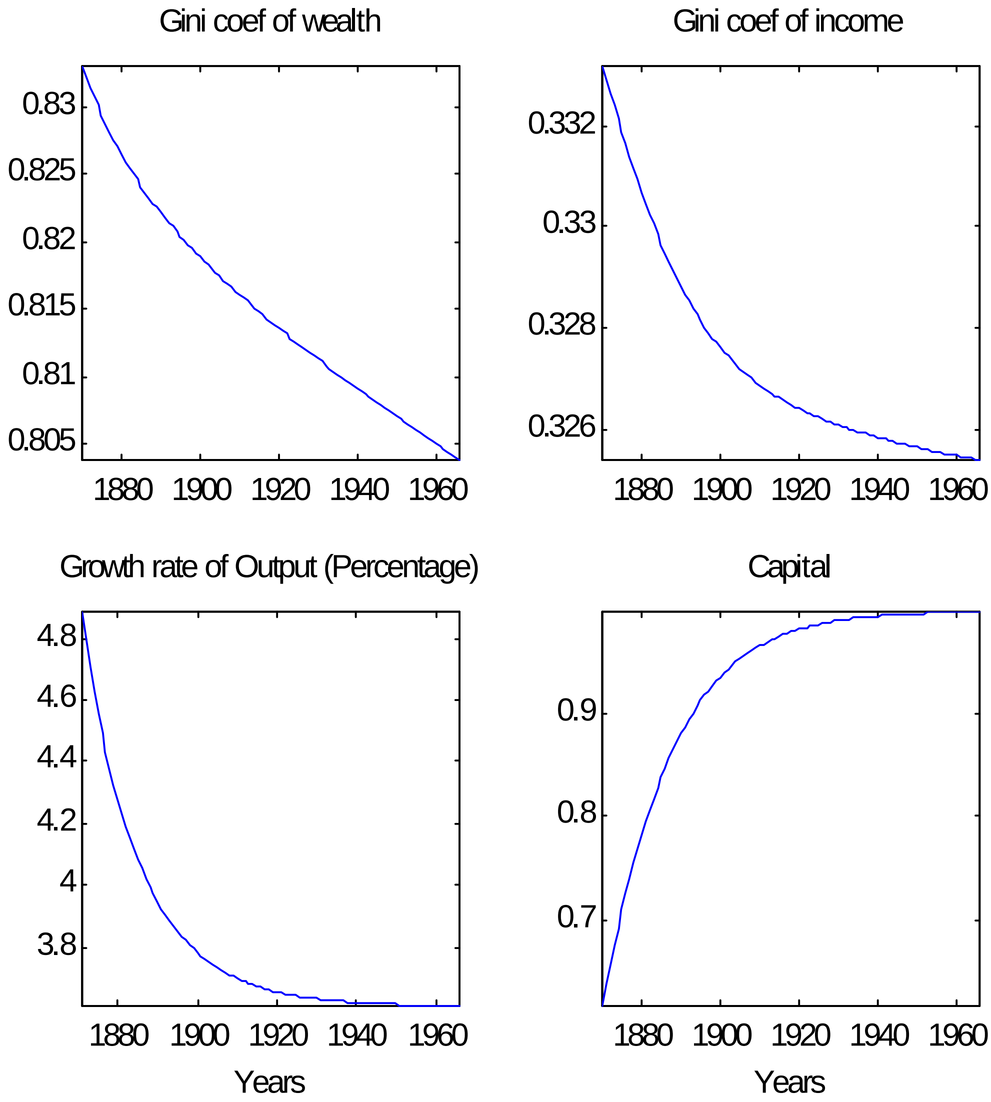

Figure 3b. Evolution of inequality when $g = 3.61$ percent.

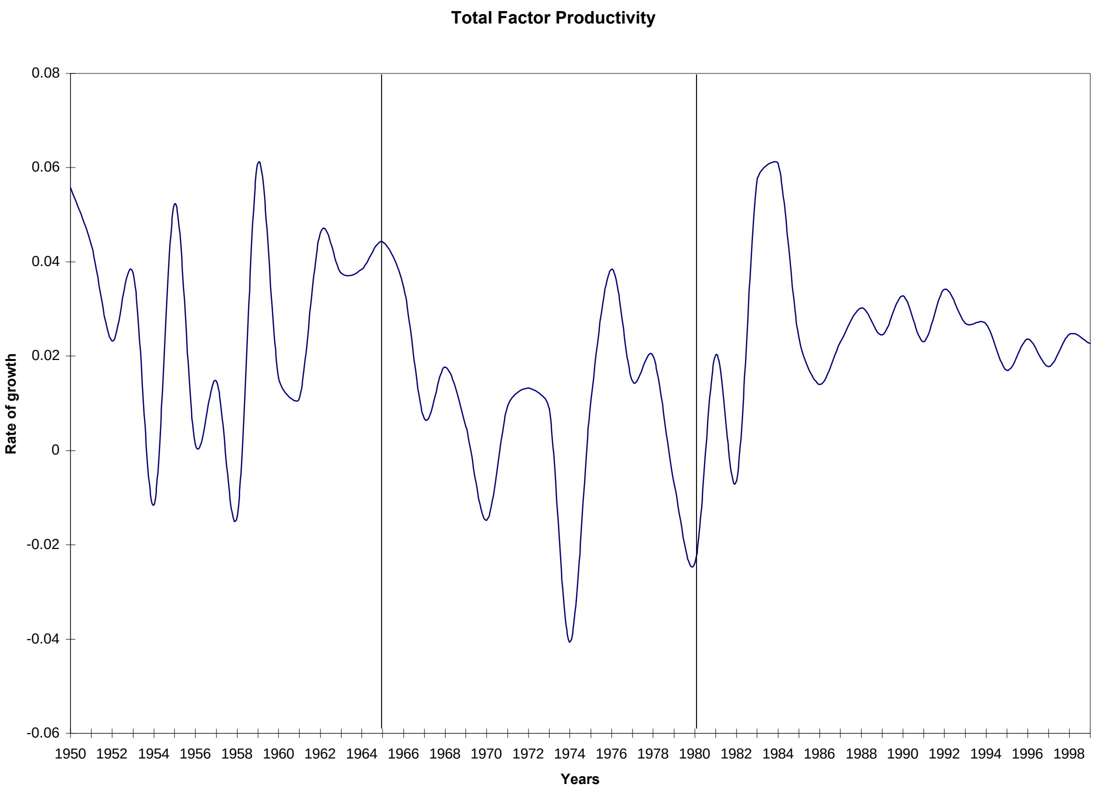

Figure 4. Rate of growth of Total Factor Productivity 1949-1999.

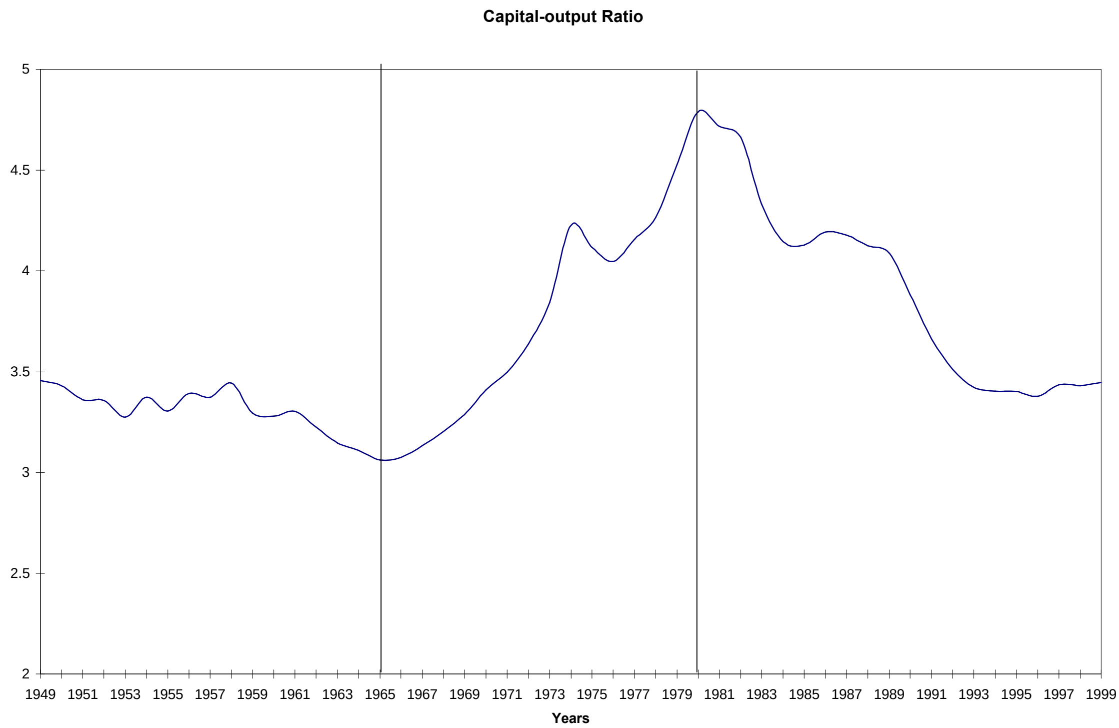

Figure 5. Capital-Output ratio 1949-1999.

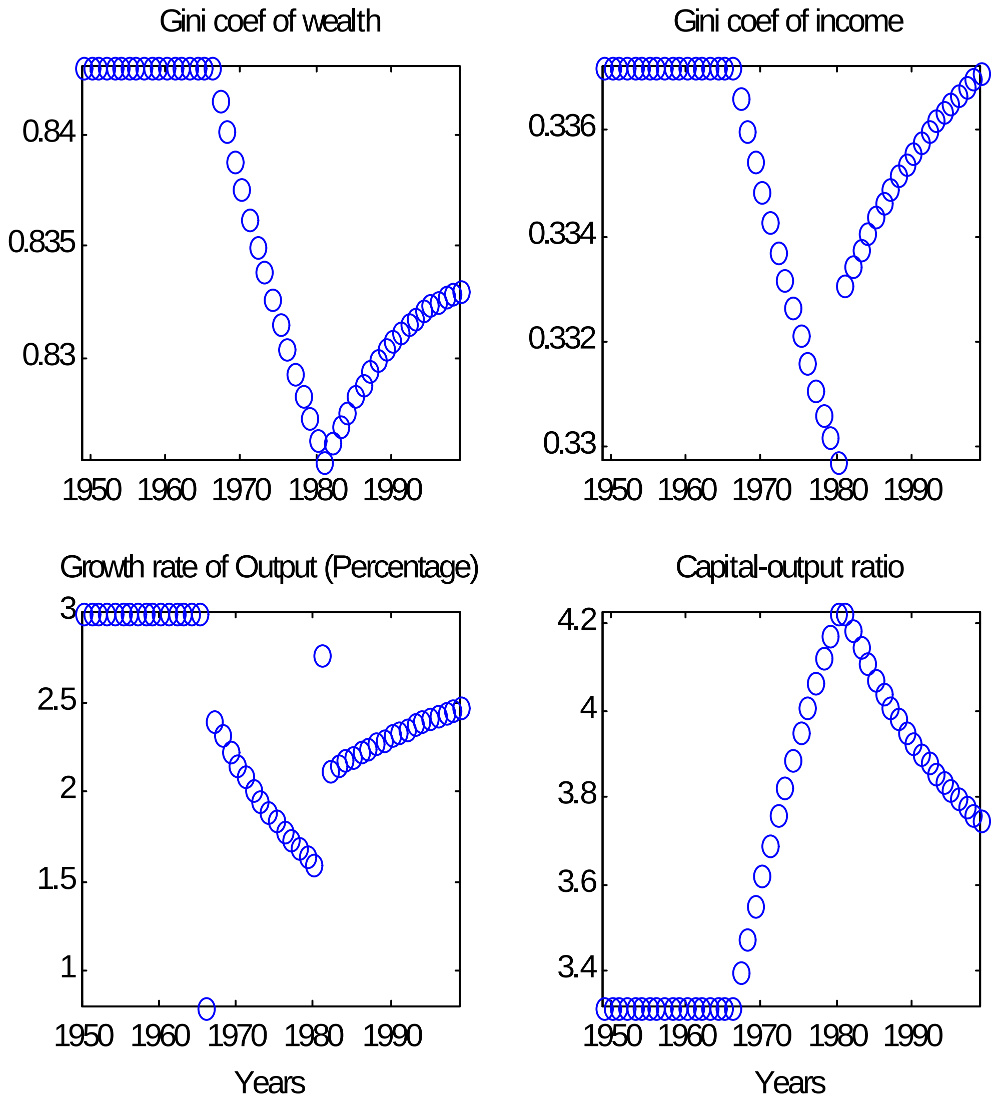

Figure 6. Evolution of inequality, 1965-1999