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TESIS DOCTORAL

Essays on liquidity

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To my family and Sergio!

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Abstract

This thesis presents three theoretical models in which to coach the debate regarding the liquidity problems that arose during the US financial crisis and the European sovereign debt crisis. Chapter one analyzes the interplay between optimal capital and liquidity requirements in a risk-shifting framework. Chapter 2 shows why holding risky liquid assets might reduce the bank's illiquidity risk. Finally, Chapter 3 studies to what extent an increment in non-performing loans and the phenomenon of deposit flight Will have different impacts on a bank's access to external funding through their different impact on the balance sheet.

Resumen

Esta tesis presenta tres modelos teóricos que permiten debatir y analizar el rol que jugó el nivel de liquidez de los bancos tanto en la crisis financiera en USA como en la crisis de deuda soberana europea. El primer capítulo estudia la interdependencia entre requerimientos de capital y liquidez en un modelo en el cual los bancos tienen incentivos a asumir un nivel de riesgo mayor al socialmente óptimo. El segundo capítulo muestra por qué mantener activos lquidos riesgosos como reservas liquidas puede ayudar a disminuir el riesgo de iliquidez bancario. Finalmente, el tercer capítulo analiza en qué grado el incremento de la morosidad de los préstamos y el fenómeno de *deposit fly* afectaron el acceso a financiacin externa de los bancos durante la crisis financiera europea.

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Chapter 1

Introduction

Banks are key to economic growth and stability due to their role as financial intermediaries. Banks reduce the asymmetric information between savers and borrowers because they have the technology not only to assess the quality of borrowers but also to monitor that they meet their commitments. As banks ameliorate this asymmetric information problem, they enhance the flow of funds from savers to borrowers improving the allocation of resources in the economy and thus social welfare.

However, the last financial crisis in US and the European sovereign debt crisis showed that banks may also increase financial instability because they do not fully internalize the effects of their policies on the financial system. In the process of transforming short term deposits into longer term lending, banks may take an excessive illiquidity risk and reduce the quality of their loans in the pursuit of greater profitability. Such behavior may induce bank failures with harmful effects on employment and growth due to the existing interdependence between the banking system and the real economy.

Banks have played a key role in the occurrence and propagation of the last financial crisis. The crisis that began in 2007-2008 is considered, by many economists, as one of the more dramatic financial turmoils of recent decades and its causes have highlighted new concerns for regulators, academics, investors and policy makers. The crisis pointed out the significant role that banks' liquidity plays in determining the magnitude and effects of financial turmoil on the real economy.

The US financial crisis was born during a period of abundant liquidity in the banking sector due to the savings glut of emerging economies. Given apparently healthy macroeconomic conditions, banks considered subprime mortgages less risky than they really were leading to a high supply of mortgage lending.

The excessive creation of private loans and money implicitly derived in a deterioration of the banks' balance sheets. This credit expansion was funded through mortgage-backed securities and collateralized debt obligations under the assumption of a continuing home price appreciation.

The liquidity of the market allows banks to partition and sell their loans to a plenitude of investors increasing their credit supply capacity. Such mechanism allowed borrowers with questionable credit histories to have access to loans, which deteriorated the quality of the mortgages.

Once the housing market collapsed, subprime default started. Many banks experienced a significant increase in their non-performing loans, which boosted uncertainty and insolvency. Since it was difficult to assess the quality of banks' assets, the liquidity in the interbank market dried up reflecting the lack of trust each other. Banks were not able to rollover their short-term debt, and thus the lack of liquidity produced fire sales of illiquid loans, higher interest rates and a reduction in loans affecting many sectors of the economy.

Given the high exposure of many European banks to US financial markets, the potential for contagion seemed evident. However, unlike the subprime mortgage crisis in the US, many European banks experienced both an increase in non-performing loans and the phenomenon of deposits flight. Consequently, European banks suffered a reduction of retail and institutional deposits at an unprecedented rate. Moreover, European banks invested heavily in peripheral sovereign debt, which made banks solvency and liquidity depended on government conduct. When government structural deficits involved successive sovereign bond downgrades, peripheral sovereign bond became illiquid and the lack of confidence grew up leading the interbank funding liquidity to dry up. Thus, the performance of several European governments determined to what extent the liquidity of sovereign bonds fluctuated over time exacerbating the consequences of US financial crisis on the Euro zone and boosting the banks' illiquidity risk.

Moreover, from a regulatory perspective, there was no formal standard regulating liquidity. Before the financial crisis, the regulation focused on capital requirements to ameliorate solvency risk but clearly it was not sufficient to control the impact of liquidity problems on financial stability. Thus, the crisis exposed the need for joint capital and liquidity regulation. As a response to this situation, the Basel committee has introduced liquidity requirements and increased capital standards to improve financial stability. However, Basel III liquidity and capital regulation have followed a silo approach (independent committees with different objectives) and thus it is unclear which effects liquidity requirements will have and how they should relate to capital requirements.

Throughout this thesis I address many of the aforementioned issues providing a theoretical framework in which to discuss liquidity issues that arose during the last financial crisis.

In the first chapter entitled "Joint liquidity and capital regulation in a risk-shifting framework" (coauthored with Sergio Vicente) I analyze the problem of a regulator that sets both capital and liquidity requirements to maximize social welfare in a framework in which a bank decides its level of solvency risk facing a risk-return trade-off. Capital requirements reduce risk-shifting through a "skin-in-the-game" channel, but substituting deposits for capital is socially costly. Liquidity requirements mitigate short-term withdrawal risk, but aggravate risk-shifting because they reduce the banks returns. I find that liquidity and capital requirements complement each other when the cost of capital or the return on loans is high and offset each other otherwise, so that regulators should set liquidity and capital requirements jointly taking into consideration capital and liquidity feedback loops.

In the second chapter of my dissertation "Liquid assets quality and banks illiquidity risk" I develop a theoretical framework that analyses the consequences of holding liquid risky assets such as sovereign bonds on bank's liquidity risk. In the model, holding cash as a liquidity reserve has a high opportunity cost, hence banks reduce this type of reserves increasing liquidity risk. Using sovereign bonds as liquidity reserves is less expensive, but liquidity risk becomes significant if those assets become worthless. We show that diversification of the type of liquid assets mitigates liquidity risk by reducing both the opportunity cost of reserves and the risk when the value of sovereign bonds plunges. Furthermore, we show that even though the bank is protected by limited liability, it will choose a sufficiently diversified portfolio of liquid asset, such that it minimizes liquidity risk.

In the last chapter “Non performing loans, Deposits Flight and Interbank Market” I present a theoretical model to analyze the impact of liquidity shortages stemming from both sides of the bank’s balance sheet on the ability of the interbank market to provide liquidity in a moral hazard setting. The liquidity needs are originated by either a deposits flight shock (a contraction in the deposit supply) or a cash flow shock (an increase of non-performing loans). The model proposed suggests that the type of shock - rather than the amount of extra funds that banks needs to cover- is the main determinant of the banks’ decision to stop lending in the interbank market. The main conclusion of this paper is that an increase in non-performing loans leads to weak balance sheets with harmful effects on the interbank relationship.

Chapter 2

Joint liquidity and capital regulation in a risk-shifting framework

2.1. Introduction

Before the last financial crisis, most of the regulatory efforts were concentrated on capital requirements. Although concerns regarding illiquidity risk have been pervasive in the regulatory debate dating back to at least Bagehot (1873) celebrated essay, the emphasis on capital had relegated liquidity regulation to the background. However, during the 2007 early 'liquidity phase' of the financial crisis, several banking institutions experienced liquidity-driven difficulties despite their adequate capital levels (Gorton (2010b), Brunnermeier (2009), Banerjee and Mio (2015)). By September, Northern Rock suffered a bank run after seeking (and obtaining) liquidity support from the Bank of England. As response to the 2007-2009 financial crisis, the Basel Committee for Banking Supervision introduced the *Liquidity Coverage (LCR)* and the *Net Stable Funding (NSFR)* ratios within Basel III to reinforce the resilience of banks to illiquidity risk.[1]

[1] The LCR requires that banks hold a buffer of liquid assets so as to cover cash outflows during a 30-day window under liquidity distress. The NFSR, somehow redundantly, complements the former by requiring a long-term stable funding structure that protect banks against maturity mismatches. The British FSA issued the "Turner Review" in 2009, which included a set of guidances to regulate banks' liquidity in a similar spirit to the Basel Liquidity Coverage Ratio (see Turner et al. (2009)).

In general, capital and liquidity regulation have been analyzed in isolation. Classical treatments of bank capital regulation have emphasized the role of capital as a means of mitigating excessive risk-taking by banks (e.g., Furlong and Keeley (1989), Rochet (1992), Hellmann et al. (2000a), Repullo (2004)). On the other hand, liquidity regulation has typically been thought of as a way of dealing with issues of maturity mismatch or refinancing risk (e.g., Farhi et al. (2009a), Freixas et al. (2011), Perotti and Suarez (2011), Calomiris et al. (2015)). And it has not been until recently that the regulation of capital and liquidity has been addressed together (e.g., Vives (2014), De Nicol et al. (2014), Walther (2016)).[2]

In this paper we analyze joint liquidity and capital regulation in a bank risk-taking framework. Our model features a bank that is partially funded with deposits and protected by limited liability. Moreover, there is a deposit insurance scheme that insulates depositors against potential bank failures.[3] A consequence of these three features is that the bank does not internalize the losses that its potential failure inflicts on the deposit insurance scheme, leading to excessive risk-shifting. This market failure can be partially mitigated by a social welfare maximizer banking regulator, who can employ two regulatory tools: a capital and a liquidity requirement. The bank takes its capital structure as given (through capital requirements) and chooses its (unobservable) *insolvency risk* profile facing a standard risk-return trade-off.[4] Capital requirements increase the bank's skin in the game by reducing its deposit liabilities, therefore mitigating the bank's incentives to risk-shift. Additionally, the bank is subject to an uncertain (exogenous) level of early deposit withdrawals, which generates *illiquidity risk*.[5] Liquidity requirements reduce the bank's exposure

[2]Although their focus is on the provision of liquidity aid by a Lender of Last Resort, Rochet and Vives (2004) address the convenience of establishing a liquidity and a capital ratio in combination with LoLR interventions.

[3]Since deposits are insured, the rates of return on deposits do not reflect the riskiness of the bank's portfolio.

[4]In our framework, capital is used to mitigate the bank's risk-shifting incentives. If the bank's risk profile were verifiable, the first-best policy intervention would consist of a zero capital requirement.

[5]We want to emphasize that we are taking a rather extreme vision of solvency and liquidity risks as independently generated pheonomena. Arguably, one may expect that the risk of facing a large amount of early withdrawals be positively correlated with the risk profile of the bank, at least if withdrawing at an early stage may be influenced to a certain extent by the bank's fundamentals and not only by depositors' idiosyncratic motives. For instance, one may reasonably expect that liquidity shocks may be originated by signals of solvency problems. Moreover, a situation of liquidity distress may well lead to solvency troubles. However, we want to abstract from potential feedback loops between the generation of liquidity and solvency risk in the basic model to highlight that capital and liquidity requirements feed back into each other *even if* liquidity and solvency risks are generated indepedently. Nevertheless, in

to liquidity crises. We show that the effectiveness of capital (respectively, liquidity) requirements depends on the bank's level of liquidity (respectively, capital). Hence, optimal liquidity and capital requirements need be determined jointly.

We assume that substituting deposits for equity entails a social cost, so that capital requirements weigh the marginal return of capital–through its impact on reducing insolvency risk–and its social cost.[6] The effect of liquidity on capital requirements is that the share of liquid assets held by the bank affects the marginal return of capital. For instance, if the bank's liquidity is low, the likelihood that the bank survives a liquidity crisis is low. Hence, the expected return of reducing insolvency risk is low as well, because the bank may nevertheless fail due to liquidity driven troubles. Consequently, the marginal return of capital is low when liquidity is low. As liquidity increases, so does the probability of surviving a liquidity crisis and therefore the return of reducing insolvency risk. Hence, liquidity requirements are a *complementary tool* to capital regulation when liquidity requirements are small, for the marginal return of capital increases with liquidity when the liquidity level is low. However, liquidity holdings reduce the social loss in the event of a bank failure, because liquidity holdings constitute a cash-in-hand asset that can be used to reduce the deposit insurance toll in that event. When liquidity levels are sufficiently high, the positive effect of reducing illiquidity risk is outweighed by the reduction of the social loss in the event of a bank failure. Hence, liquidity requirements constitute an *offsetting tool* to capital requirement when these are high.

On the other hand, the rationale for liquidity regulation in this paper is to reduce illiquidity risk beyond the bank's will. In a laissez-faire economy, the bank would choose a certain level of

an extension we conduct an exploration of a model in which early withdrawals are relatively more likely the higher the level of solvency risk. In this setup, reducing solvency risk has a double effect and, consequently, capital requirements are larger than in the base model. However, the nature of the feedback effects of capital and liquidity remains.

[6]Previous research has emphasized that substituting deposits for equity is socially costly. For instance, Van den Heuvel (2008) and Gorton and Winton (2014) state that capital requirements are socially costly because substituting capital for deposits entails eliminating a valuable source of liquidity for consumers. Myers and Majluf (1984) argue that issuing new equity could be costly when old shareholders and managers have access to information that new investors do not have. Bolton and Freixas (2006) points out that the presence of asymmetric information makes equity capital more costly than other sources of bank

funding. Capital may also be costlier than deposits due to its relative scarcity (Martinez-Miera and Suarez (2014)) or because it carries a higher return than deposits in segmented markets (Allen et al. (2015)).

liquidity to balance out the positive effect of reducing illiquidity risk with the opportunity cost of liquidity, which is given by the foregone investments in more profitable assets. The regulator faces the same liquidity trade-off as the bank in nature, but liquidity is more valuable for the regulator than for the bank for two reasons. First, a bank failure is more costly for society than for the bank, because the bank does not internalize the loss that its failure inflicts on the deposit insurance fund. Second, liquid assets are valuable in the event of the bank failure. This is a source of value of liquidity that the bank neglects, because the bank does not take into account the value of its assets in the event of a failure. Hence, the opportunity cost of liquidity in terms of foregone investment opportunities is smaller for society than for the bank. As a result, the regulator establishes liquidity requirements in excess of the level that the bank would choose in the absence of regulation. An immediate implication of binding liquidity requirements is that they harm the bank's expected profits. Hence, liquidity requirements harm the bank's incentives to reduce insolvency risk.

The effectiveness of liquidity requirements does in turn depend on the level of bank's equity capital. On the one hand, increasing capital reduces the negative effect of a binding liquidity requirement on the bank's choice of insolvency risk. The reason is that the wedge between the bank's and society's objective function narrows down as capital increase–recall that this wedge is driven by the fact that the bank is partially funded with non-risk-priced deposits and is insulated by limited liability. Hence, by reducing the negative effect of liquidity on the bank's incentives to curtail insolvency risk, raising capital leads to an increase of the marginal social return of liquidity requirements. On the other hand, an increase in capital leads to a reduction of insolvency risk. Hence, as capital increases, the value of the bank's upside payoff carries a higher weight in the regulator's objective function, because the solvent state is more likely to occur. The higher likelihood that the bank's loan investments turn out successfully imply that the value of surviving a liquidity crisis is higher, but also that the opportunity cost of holding liquidity instead of granting loans increases as well. For a sufficiently high liquidity level, so that the bank is resilient enough to liquidity shocks, the effect of the increased opportunity cost of holding liquidity dominates. Hence, capital requirements are a complementary tool to liquidity requirements when they are low, while they constitute an offsetting tool to liquidity requirements when they are high.

Summing up, we have two regions for liquidity and capital. In the 'complementary tools' region, increasing the level of one of the regulatory variables leads to an increase of the effectiveness of the other. The optimal capital and liquidity requirements fall in that region when the cost of capital and the opportunity cost of liquidity are large, so that the level of optimal capital and liquidity are small. In the 'offsetting tools' region, on the contrary, increasing the level of one regulatory tool offsets the effectiveness of the other tool. The optimal capital and liquidity requirement fall in that region when the cost of capital and the opportunity cost of liquidity are small, so that the optimal requirements are large.

There are several important ingredients that we abstract from in the main analysis in order to keep the analysis tractable. We extend the analysis to allow for *asset liquidation at (exogenous) fire sales prices* in order to overcome a liquidity shock exceeding the bank's cash reserves. Although the main feedback effects between liquidity and capital prevail, namely that capital and liquidity are complementary or offsetting tools depending on the level of both variables, there are some changes that are worth noting. First, the magnitude of optimal liquidity requirements is unambiguously smaller than in the absence of asset liquidation. The reason is that the bank is now able to overcome larger liquidity shocks and therefore the marginal return of liquidity requirements through reducing illiquidity risk diminishes. It is worth remarking that, although liquidity requirements are reduced, illiquidity risk unambiguously reduces–the combined effect of lower liquidity and asset liquidation allows the bank to overcome higher liquidity shocks. On the other hand, capital requirements are unambiguously larger. Since illiquidity risk is smaller, the value of reducing insolvency risk–and therefore, of capital requirements–is larger.

The rest of the paper is organized as follows. In the next section we review related research. Section 2.3 contains the description of the basic model. We analyze the bank's asset choices and the main effects of both liquidity and capital on the bank's incentives to reduce insolvency risk in Section 2.4. The main analysis is contained in Section 2.5. We conduct several extensions in Section 2.6, where we analyze the implications of enriching the baseline model. We conclude in Section 2.7.

2.2. Related Literature

Several papers analyze the role of equity capital as a way to reduce excessive bank risk-taking. Furlong and Keeley (1989) shows that capital enhances the incentives of banks to reduce risk-shifting through increasing shareholders' losses in case of default. Hellmann et al. (2000a) notes that although capital may reduce risk-shifting by putting banks' equity capital at risk, they may also induce higher risk-taking because they erode banks' charter value. Their analysis calls for a combination of capital requirements and caps on deposit rates to avoid the negative effect of deleveraging on franchise values. Repullo (2004) reexamines the relationship between capital requirements and risk-taking behavior in a setup in which banks imperfectly compete for depositors. This paper shows that capital requirements reduces bank's risk-taking because the extra cost of capital born by regulated banks is passed onto depositors through reduced deposit rates, so that capital requirements do not erode banks' charter values. Our paper draws from these insights to address the effect of capital on risk-taking. In our model, capital reduces the bank's reliance on deposits, therefore increasing the bank's 'skin in the game'. As a consequence, capital requirements reduce risk-shifting incentives. The main novelty of our analysis is that we combine a liquidity requirement with a capital requirement. We find that the effectiveness of capital requirement depends on the level of liquidity requirements.

There is a handful of papers looking into the joint regulation of capital and liquidity requirements. In an early treatment focusing on the Lender of Last Resort (LoLR), Rochet and Vives (2004) analyze the optimal LoLR intervention policy in a model in which investors may face a coordination failure to roll-over credit to a solvent but illiquid bank. In this model, solvency and liquidity requirements could prevent the coordination failure, but they may entail a large cost in terms of foregone returns, which the LoLR intervention can mitigate. Calomiris et al. (2015) argues that liquidity, as a verifiable and riskless asset, can be used as a commitment device to engage in efficient risk management. The paper shows that in the presence of idiosyncratic liquidity shocks, a liquidity insurance scheme with liquidity requirements is optimal, as it diversifies away individual shocks while avoiding a common pool resource problem. Moreover, since liquidity enhances risk management, liquidity requirements lead to book equity being a more precise indicator of equity market values, thus identifying a channel by which liquidity enhances the value of equity requirements. In our paper, in contrast, liquidity leads to an increase in insolvency risk because it erodes the bank upside payoff by foregoing more profitable investments. As noted by König (2015), which studies the effect of liquidity regulation on banks' overall risk, liquidity requirements harm the bank's *upside payoff* by reducing the share of assets invested in loans. De Nicol et al. (2014) find out that while (mild) capital requirements lead to a higher volume of lending and a lower default probability than in the case of unregulated banks, these positive effects disappear when combined with liquidity requirements. In our model, in contrast, we find that liquidity requirements complement capital requirements by increasing the likelihood of surviving a liquidity crisis and, consequently, increasing the value of capital a tool to reduce insolvency risk. Vives (2014) analyze a global game in which investors' decisions to roll-over debt are strategic complements and solvency refers to the financial intermediary fundamentals to cope with coordinated actions. The analysis relates liquidity–to deal with liquidity issues–and solvency ratios–to cope with insolvency–to the level of transparency, as agents' information is a main driver of coordinated actions. The paper results calls for a joint determination of prudential and disclosure policies. In a recent treatment, Walther (2016) analyzes the problem of a set of banks that have incentives to induce excessive systemic risk through leverage. While the liquidity ratios in Basel III can be used to reduce the advent of fire sales, capital requirements can be used to avoid individual bank failures. In our model, in contrast, although capital and liquidity requirements are set to reduce solvency and illiquidity risk, respectively, capital and liquidity feed back into each other because the effectiveness of one instrument depends on the level of the alternative tool.

2.3. The basic setup

In this section, we describe the basic aspects of our model. Some of the assumptions are discussed further below. We consider a four-period economy, $t \in \{-1, 0, 1, 2\}$, where all agents are risk-neutral and there is a zero discount factor. In this economy there is a social welfare maximizer regulator that sets both liquidity and capital requirements (l^*, k^*) at $t = -1$. The capital structure of the bank at time $t = 0$ consists of an amount d of common deposits, which are available upon demand both at $t = 1$ and at $t = 2$; an amount b of long-term deposits, which are cashable at $t = 2$ only; and an amount k of equity capital, which is required to either meet or exceed the capital requirement, that is, $k \geq k^*$. We normalize the bank funds to 1, so that we have that $d + b + k = 1$. Consequently, we interpret the capital requirement as a fraction of the bank's assets. We fix the amount of deposits d exogenously and let the regulator set a minimum capital requirement $k^* \in [0, 1 - d]$.

We assume that common deposits constitute a substantial share of the bank's funds. In particular, we assume that $d \geq \underline{d}$, for some $\underline{d} > 0$, whose value we define precisely on Definition A.6 in Appendix B.1. For simplicity, we assume that all deposits are completely insured by a deposit insurance scheme, so that we can normalize (excess) interest rates on common deposits to 0. Long-term deposits, on the contrary, carry an (excess) interest rate $r \geq 0$ which, for simplicity, we take as exogenous.[7] Moreover, we assume that equity capital is costlier than deposits, both from a social perspective and to the bank. In particular, we assume that each unit of equity capital carry a(n excess) shadow cost of ρ, with $\rho > r$. For ease of exposition, we assume that the cost of raising a unit of equity capital to the bank is the same as the social shadow cost.[8] We assume that

[7]Although we assume that long-term deposits are insured, depositors may require a higher return than on common deposits because common deposits are available upon demand, so that they constitute a source of liquidity for consumers. Consumers may forgo some liquidity in exchance for an extra return of r.

[8]Nonetheless, this assumption does not play any role in our analysis, as banks will not be willing to hold equity capital in excess of capital requirements, that is, capital will effectively *not* be a choice variable for the bank.

the bank is protected by *limited liability*, signifying that the bank is not required to meet its deposit obligations in the event that its loan does not perform.

At time $t = 0$, the bank decides which fraction l of its resources to store as a liquid asset (i.e. cash) and which amount $1 - l$ to invest in a profitable project (i.e. a loan). The loan is used to finance a long term project, which delivers its returns at $t = 2$. If successful, the loan returns M units per unit of investment, which we refer to as the *project profitability* (or *profitability*, for short). The investment returns 0 if it fails. The probability of a success θ is an endogenous choice for the bank. Following Dell'Ariccia and Marquez (2006) and Allen et al. (2011), we let the bank choose an unobservable *solvency level* $\theta \in [0, 1]$, which determines the probability of success of its lending portfolio.[9] Choosing a solvency level θ carries a cost $\frac{c}{2}\theta^2$, for some $c > 0$.[10] This assumption gives rise to a standard risk-return trade-off, as higher risk (i.e. lower θ) leads to a higher return. We refer to *insolvency risk* as to the value $1 - \theta$.

We model illiquidity risk assuming that some depositors have liquidity needs at $t = 1$. As in Diamond and Dybvig (1983), we assume that a fraction β withdraws its deposits at $t = 1$, but we add aggregate uncertainty in the actual fraction β of early withdrawals. We assume that β is drawn from a common knowledge random variable with support $[0, d]$ and an associated c.d.f. $F(\cdot)$ and a well-defined density $f(\cdot)$. We assume that this distribution is log-concave and that the density function is continuous and positive everywhere in the interior of its support. For any given level of liquidity l held by the bank, *illiquidity risk* is given by $1 - F(l)$. For simplicity, we assume that the bank will be liquidated if the deposit withdrawals exceeds its cash reserves, that is, if $\beta > l$. We relax this assumption in Section 2.6.1, where we allow early partial asset liquidation at discounted prices.

Finally, we make technical assumptions, Assumptions A.1-A.4, which we outline and discuss in Appendix B.1.

[9] For instance, a bank may invest resources into developing a credit score model to improve the quality of its creditworthiness appraisals.

[10] See also Besanko and Kanatas (1996) and Dewatripont and Tirole (1994) for analogous arguments of costly actions that enhance banks' expected returns operating through a reduction of the probability of a project failure.

2.3.1. Discussion of assumptions

Some of the ingredients of the model deserve further explanation. On the one hand, we assume that the bank is protected by *limited liability*. The immediate implication of this assumption is that the bank does not internalize the losses that its default inflicts on society. Limited liability induces a wedge between the bank's objective function and social welfare, so that there is room for welfare-improving regulation.

Also, we assume that *deposits are insured*, so that they carry either a zero interest rate (common deposits) or an interest rate of r (long-term deposits). The results that we obtain do not strictly depend on the assumption that deposits are insured, but on the underlying assumption that deposit rates are independent of the bank's choice of risk θ and the regulator's choices of liquidity and capital requirements (l^*, k^*). If, on the contrary, deposits were not insured *and* the choice of risk were observable, deposit interest rates would be proportional to the inverse of the bank's choice of risk $1/\theta$. In this case, the bank would internalize the effect of its choice of insolvency risk on depositors and the bank would act as if not insulated by limited liability.

We assume that some fraction of the bank's liabilities is levied in the form of *long-term deposits*. In the absence of long-term deposits, any increase in the capital requirement would necessarily require either a reduction of the amount of common deposits (since, in that case, we would have that $d + k = 1$) or an enlargement of the bank's balance sheet at $t = 0$. Hence, capital requirements would interfere with either the common deposit base–the one that is subject to early withdrawals–or with the bank's operating scale, which would also affect the proportion of common deposits over the bank's balance sheet at $t = 0$.[11] In order to clean out the effect of capital requirements without interfering with the common deposit base or the asset size, we assume that the bank can always raise an amount b of long-term deposits so that $b + k = 1 - d$.

[11]Nonetheless, the results that we obtain generalize to a model in which capital requirements lead to an expansion of the bank's balance sheet at $t = 0$, insofar as deposits constitute a certain (considerable) share of the assets–If capital were too large a fraction of the bank's funds, the bank would choose to hedge against *any* potential early withdrawal, so that we would not have liquidity risk. An extended version of this paper with an analysis of this case is available from the authors upon request.

Assumption A.1 in Appendix B.1 on the minimum value for the social cost of capital ensures that the optimal capital requirement never exceeds $1 - d$.

We assume that equity capital is raised from *outside shareholders*. Essentially, as in Bolton and Freixas (2000) and Repullo (2013), we assume that the bank must allocate a fraction α of its ownership profits to reward its outside equityholders in exchange for their provision k of outside equity capital at an exogenously given competitive rate ρ, so that:

$$\alpha \cdot \Pi_B (l, \theta, k) = (1 + \rho) \cdot k,$$

where $\Pi_B (l, \theta, k)$ stands for the bank's profits and is defined precisely below, in equation (2.1) on Section 2.4.

Regarding the *early withdrawal phase*, we assume that the amount of early withdrawals is a random variable with an exogenously given distribution function. Unlike in Diamond and Dybvig (1983), we need aggregate uncertainty in the amount of early withdrawals in order to induce illiquidity risk. If the actual amount were known to the bank, it would store exactly as much liquidity as that amount so as to meet its early demand for deposits. The log-concavity assumption implies that the *reverse hazard rate* $g(\cdot)$, which is given by the ratio $g(l) \equiv \frac{f(l)}{F(l)}$ for all $l \in (0, d)$, is strictly decreasing and has an asymptote at $l = 0$, i.e., $\lim_{l \to 0} g(l) = +\infty$ (Bagnoli and Bergstrom (2005)), a property which we will draw from in our main result. This assumption provides an Inada-type condition for liquidity, since any increase in liquidity at $l = 0$ yields an infinite return (from 0 to positive profits). Moreover, the log-concavity assumption on the distribution of liquidity schocks implies that there are decreasing returns to liquidity since, as we show below, the marginal return of liquidity will be decreasing. The family of log-concave distributions with compact support includes the uniform distribution and all beta distributions with shape parameters at least as large as one, which encompass a large variety of shapes.

Moreover, we assume that the distribution function of early withdrawals is independent of the (solvency) risk profile adopted by the bank. Arguably, one may expect that the amount of early withdrawals be positively correlated with the risk profile of the bank, at least if withdrawing at an early stage may be influenced to a certain extent by the bank's fundamentals and not only by depositors' idiosyncratic motives. While we acknowledge that this correlation may be very relevant, the exercise that we carry out here assumes that illiquidity risk is originated independently of insolvency risk, because we want to highlight that liquidity and insolvency risk are interrelated through the bank's decisions even when originated independently. Nonetheless, in Section 2.6.2, we relax this assumption assuming that, for any choice of solvency θ, the distribution of early withdrawals $F(\cdot;\theta)$ is such that if $\theta_1 < \theta_2$, then $F(\cdot;\theta_1)$ first-order stochastically dominates $F(\cdot;\theta_2)$. Hence, higher levels of insolvency risk induce a distribution of "larger" early withdrawals in a first-order stochastic sense.

Finally, notice that our framework does not include any negative externality from a banking failure. Including an externality in the form of, for instance, a positive shadow cost associated to the bank liquidation or deposit insurance provision would reinforce our case for regulation, but would not add further insight.

2.4. Bank's asset choices

Before addressing the optimal regulation of capital and liquidity, we first analyze the problem of a bank that takes both its capital structure and the share of resources that must be stored as liquid assets as exogenously given and decides how much to invest in reducing the risk associated to its loan portfolio. This section allows us to assess the roles of liquidity and capital on the bank's incentives to reduce risk.

We focus on three aspects. First, in Section 2.4.1, we find the liquidity level that maximizes the bank's profits for any predetermined level of capital. This choice will serve as a benchmark to assess how the optimal liquidity requirement differs from the value that maximizes the bank's profits. Second, in Section 2.4.2, we analyze the interaction between liquidity and insolvency risk. Finally, in Sections 2.4.3 and 2.4.4 we assess the role of capital in shaping the bank's choice of risk.

The bank's problem consists of choosing a fraction of cash l_B and a screening level θ_B so as to solve

$$\left.\begin{array}{ll} \max_{\theta\in[0,1]} & \Pi_B(\theta,l,k) \\ \\ s.t. & \Pi_B(\theta,l,k)\geq 0 \end{array}\right\}, \tag{B}$$

where the bank's objective function is given by:

$$\Pi_B(\theta,l,k)\equiv\int_0^l \theta\cdot\left[\overbrace{\underbrace{\underbrace{M\cdot(1-l)}_{Loan\ Value}+\underbrace{l-\beta}_{Cash}}_{Assets}-\underbrace{(1-\beta+b\cdot r-k)}_{Deposit\ Liabilities}}^{Upside\ payoff}\right]\cdot f(\beta)\,d\beta-\underbrace{\frac{c}{2}\cdot\theta^2}_{Scr.Cost}-\underbrace{(1+\rho)\cdot k}_{Cost\ of\ Equity}. \tag{2.1}$$

The upper bound of the integral is given by the fact that the bank can only survive the early withdrawal phase if it holds enough cash so as to meet the demand for deposits at $t=1$, that is, as long as $l\geq\beta$. Moreover, the bank can only make positive profits if its investment is successful, which occurs with probability θ. Upon successful completion of the project, the bank will obtain an amount $M\cdot(1-l)$ from its loan investment, as well as an amount of cash $l-\beta$. Its deposit liabilities amount to $1-\beta+b\cdot r-k$.[12] Finally, we include the constraint that the bank makes non-negative profits.[13]

In order to rewrite the bank's profit maximizing problem, we define the *bank's upside payoff* as:

$$\pi(l,k)\equiv\underbrace{M\cdot(1-l)}_{Loan\ Value}+\underbrace{l}_{Leftover\ Cash}-\underbrace{(1+b\cdot r-k)}_{Deposit\ Liabilities}. \tag{2.2}$$

This expression stands for the bank's payoff conditional on a successful project completion once

[12] Notice that the amount β of early withdrawals does not affect bank's profits, as long as the bank holds enough cash at time $t=1$ so as to survive the early withdrawal phase. We show that early withdrawals indeed do affect banks' profits in Section 2.6.1, where we allow for asset liquidation to cope with excessive withdrawals. Nonetheless, the main insights about the effect of liquidity on solvency risk generalizes to the framework in which asset liquidation is possible.

[13] Below, we solve the problem of a regulator that maximizes social welfare. Hence, the non-negative-profit constraint for the bank will be harder to meet. In Appendix A we impose conditions on M, c and ρ so that the constraint does not bind in that problem. Consequently, it will not bind for this problem either. We henceforth ignore this constraint.

the cost of equity and the cost of reducing risk have been deducted. Observe that, for any level of equity capital k, $\pi(l, \cdot)$ is a strictly decreasing function of cash holdings, reflecting the fact that each unit of cash holdings carries an opportunity cost in terms of foregone loan opportunities. Also, for any liquidity level l, $\pi(\cdot, k)$ is a strictly increasing function of capital. While capital is costlier than deposits and, consequently, any additional unit of capital hurts the bank's profits, equity capital increases the bank's upside payoff. This feature reflects a central element of our analysis, namely that capital increases the bank's skin-in-the-game by reducing its deposit liabilities.[14]

Substituting out equation (2.2) into equation (2.1), we can write the bank's objective function as:

$$\Pi_B(\theta, l, k) = \theta \cdot F(l) \cdot \pi(l, k) - \frac{c}{2} \cdot \theta^2 - (1+\rho) \cdot k. \tag{2.3}$$

2.4.1. The bank's profit-maximizing liquidity level

We first analyze the profit-maximizing liquidity level for a predetermined level of capital k, which will serve as a benchmark to assess optimal liquidity requirements. We write $l_B(k)$ for the bank's profit maximizing liquidity level to highlight its dependence on the level of equity capital k. The first order condition for an interior bank's profit-maximizing level of liquidity is given by:

$$[l] \qquad \underbrace{g(l_B(k)) \cdot \pi(l_B(k), k)}_{Mg.\ Value\ of\ \ Surviving\ Early\ Withdrawals} \quad - \quad \underbrace{(M-1)}_{Opportunity\ Cost\ of\ Liquidity} \quad = 0 \ , \tag{2.4}$$

where $g(l_B(k)) \equiv \frac{f(l_B(k))}{F(l_B(k))}$ stands for the reverse hazard rate of the distribution of early withdrawals, evaluated at the optimum $l_B(k)$.

Observe from equation (2.3) that the bank's choice of liquidity $l_B(k)$ is given by the liquidity level that maximizes $F(l) \cdot \pi(l, k)$. The marginal effect of liquidity on bank's profits, which is given by equation (2.4), can be split into two components pushing in opposite directions. On the one

[14]Observe that the role of capital in this model is simply to reduce the amount of deposit liabilities. Hence, capital serves the purpose of increasing the bank's skin-in-the-game. As we shall see below, capital enhances the bank's incentives to reduce its risk level.

hand, liquidity increases the probability of surviving the early withdrawal phase. On the other hand, there is an opportunity cost of liquidity: the bank foregoes a loan return of M in exchange for 1 unit of stored cash. Hence, $F(l) \cdot \pi(l,k)$ is hump-shaped and $l_B(k)$ is its maximizer. The assumption that the distribution of early withdrawals is log-concave ensures that $g(\cdot)$ is strictly decreasing with an asymptote at $l = 0$ (Bagnoli and Bergstrom (2005)): when liquidity is very small, the likelihood of surviving the early withdrawal phase is very small and, consequently, the marginal value of holding liquidity exceeds the opportunity cost of liquidity in terms of foregone loan investments. Finally, observe that the profit-maximizing liquidity level $l_B(k)$ is independent of the bank's choice of insolvency risk. The reason for this is that the bank chooses its optimal liquidity level conditioning on the event that the project is successful, as it obtains no rents at the interim period $t = 1$ or in case the project fails.

We have argued above that equity capital increases the bank's upside payoff. Hence, the marginal value surviving early withdrawals increases with equity capital. On the contrary, the opportunity cost of liquidity is independent of capital. Hence, the optimal level of liquidity increases with capital. We summarize this discussion in the following instrumental result, which is straightforward to derive.

Lemma 2.1 (Bank's profit-maximizing liquidity level)**.** *For any level of capital k, we have that:*

(i) There exists a unique level of liquidity $l_B(k)$ that maximizes the bank's profits, which is given by the liquidity level that maximizes the (hump-shaped) expected bank's upside payoff $F(l) \cdot \pi(l,k)$.

(ii) The bank's profit-maximizing liquidity value $l_B(k)$ is strictly increasing in capital.

Figure 2.1 depicts the effect of capital on the bank's choice of liquidity, which is represented on the horizontal axis. The solid and dashed decreasing lines correspond to the marginal value of surviving the early withdrawal phase $g(l)\cdot\pi(l,k)$ for two different levels of equity capital $k_2 > k_1$, respectively. The marginal value $g(l)\cdot\pi(l,k)$ of surviving the early withdrawal phase corresponds to the product of two positive strictly decreasing functions and is therefore strictly decreasing. The marginal survival value for k_2 consists of an upward shift of the marginal survival value for k_1, reflecting the fact that $\pi(l, k_2) > \pi(l, k_1)$. The marginal value of liquidity $M - 1$ is independent of both liquidity and capital. It is depicted as a horizontal solid line. The profit-maximizing liquidity level $l_B(k)$ is given by the intersection between $g(l) \cdot \pi(l, k)$ and $M - 1$. Hence, we have that $l_B(k_2) > l_B(k_1)$.

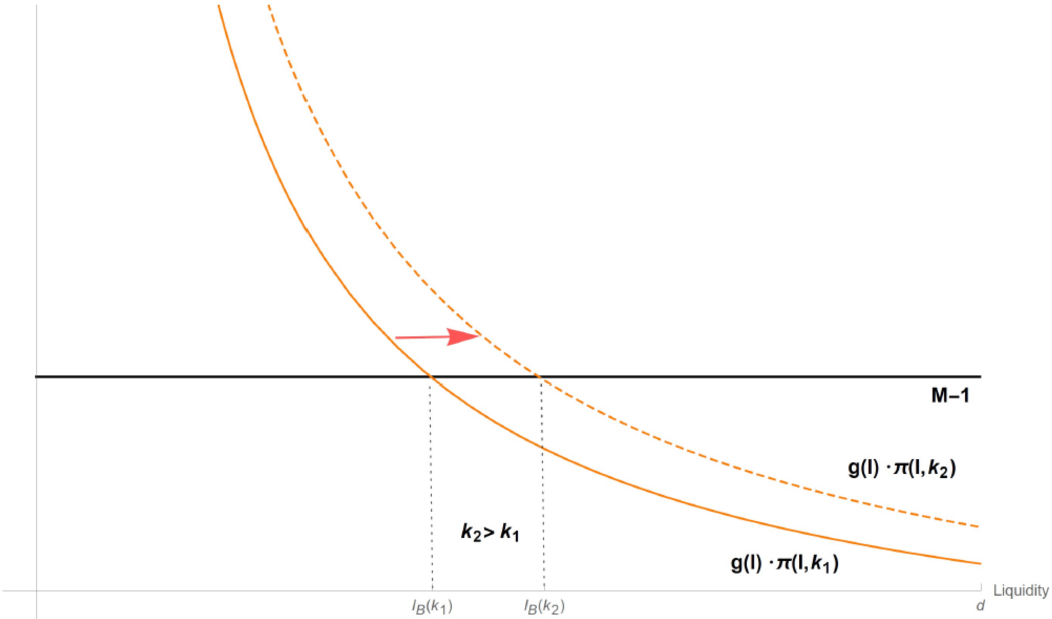

Figure 2.1: The effect of capital on bank's liquidity choice

2.4.2. The effect of liquidity on insolvency risk

We analyze now the interaction between liquidity and the bank's choice of insolvency risk. The first order condition for an interior solution to the bank's insolvency risk choice θ_B is given by:[15]

$$[\theta] \quad \theta_B(l, k) = \frac{1}{c} \cdot F(l) \cdot \pi(l, k) \enspace , \tag{2.5}$$

where $\theta_B(l, k)$ stands for the solvency level chosen by a bank with liquidity l and equity capital k.

[15] In order to ensure that $\theta_B(\cdot, \cdot)$ is interior, so that the regulation problem is interesting, we make an assumption on the range of c (see Appendix Appendix B.1).

Observe that the marginal return of reducing insolvency risk is given by the bank's upside payoff $F(l) \cdot \pi(l,k)$, weighted by the factor c. Intuitively, the bank obtains an amount $\pi(l,k)$ if it ends up being solvent. But a precondition for the bank to be solvent is that it survives the early withdrawal phase, whose likelihood is given by $F(l)$. Consequently, the bank chooses a higher level of solvency the higher its upside payoff. Since $l_B(k)$ maximizes the bank's upside payoff $F(l) \cdot \pi(l,k)$, it follows that $\theta_B(l,k)$ is also maximized at $l_B(k)$. The following result is immediate in the light of Statement (i) on Lemma 2.1.

Proposition 2.1 (Liquidity and insolvency risk)**.** *For any given level of equity capital k, the bank's choice of solvency $\theta_B(l,k)$ is a hump-shaped function of its liquidity holdings l, and is maximized at the bank's profit-maximizing liquidity level $l_B(k)$.*

Figure 2.2 illustrates Proposition 2.1. The dashed line corresponds to the bank's upside payoff $F(l) \cdot \pi(l,k)$ for a given level of capital k. As argued in Lemma 2.1, this function is hump-shaped in liquidity. The solid line represents the optimal solvency level, which is a scaled (down, by c) transformation of the bank's upside payoff.

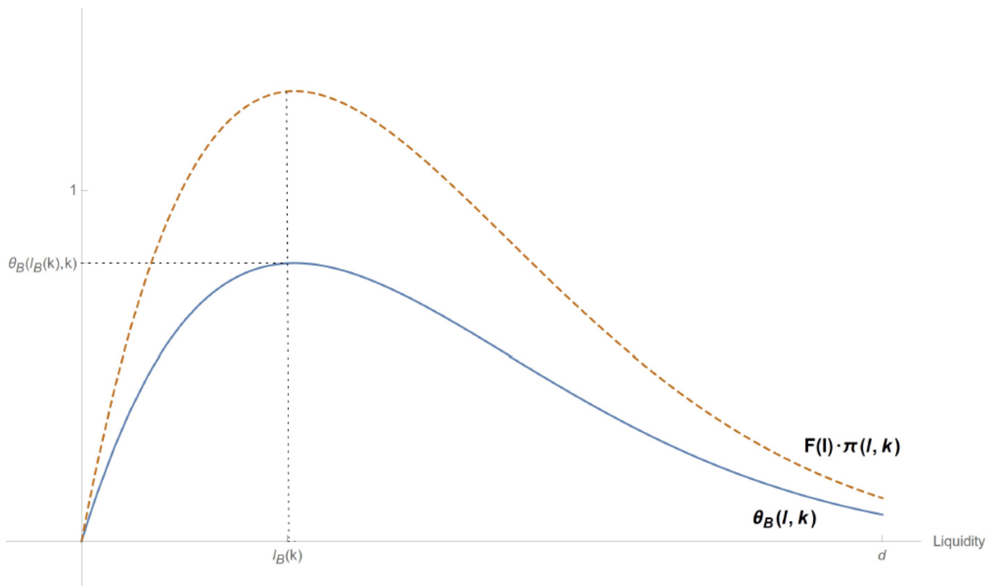

Figure 2.2: Bank's profit-maximizing liquidity and insolvency risk

We shall see below that liquidity requirements exceed the bank's profit maximizing level, which is the level that a bank would choose in the absence of liquidity regulation. Hence, liquidity requirements will induce a higher level of insolvency risk. The following corollary highlights the fact that an excessive value of liquidity harms solvency.[16]

Corollary 2.1 (Binding liquidity requirements increase insolvency risk)**.** *Any level of bank's liquidity in excess of the bank's profit-maximizing liquidity level $l_B(k)$ induces a higher bank's choice of insolvency risk than in the absence of liquidity regulation. Formally, $\frac{\partial \theta_B(l,k)}{\partial l} < 0$ for any $l > l_B(k)$.*

2.4.3. The effect of equity capital on insolvency risk

We now turn into the analysis of the effect of capital on the bank's choice of solvency. We first state the result, which follows immediately from differentiating equation (2.5), and then provide intuition for it.

Proposition 2.2 (Equity capital and insolvency risk)**.** *For any level of liquidity l, solvency $\theta_B(l,k)$ is a strictly increasing function of equity capital k.*

As argued above, equity capital increases the bank's upside payoff $\pi(l,k)$. Hence, the bank's marginal return of increasing its level of solvency, which is given by $F(l) \cdot \pi(l,k)$, is larger the higher the amount of equity capital. As a consequence, an elevation of equity capital leads to an increase in the solvency level.

2.4.4. Capital and the effect of liquidity on insolvency risk

We have seen above (Corollary 2.1) that liquidity in excess of the bank's profit-maximizing level of liquidity $l_B(k)$ leads to a reduction of solvency, because it harms the bank's profits. We shall see below that liquidity requirements will be set in excess of the bank's profit-maximizing liquidity level. Hence, optimal liquidity requirements will harm solvency. The following result

[16] This result is reminiscent of the result found by König (2015), who shows that liquidity may harm insolvency risk.

states that capital mitigates this effect. This results plays an essential role in the determination of the joint optimal capital and liquidity requirements.

Proposition 2.3 (Impact of equity capital on effect of liquidity on solvency)**.** *Equity capital reduces the (negative) effect of excessive liquidity on solvency. That is, for any $l > l_B(k)$, $\left|\frac{\partial \theta_B(l,k)}{\partial l}\right|$ is strictly decreasing in equity capital k.*

In order to provide intuition for this result, consider the effect of liquidity on solvency, which is proportional to the first order condition of the bank's problem with respect to liquidity, as stated in equation (2.4):

$$\frac{\partial \theta_B(l,k)}{\partial l} \propto \underbrace{g(l) \cdot \pi(l,k)}_{Mg.\ Value\ of\ Surviving\ Early\ Withdrawals} - \underbrace{(M-1)}_{Opportunity\ Cost\ of\ Liquidity} < 0 \text{ for } l > l_B(k).$$

Observe that equity capital increases the marginal value of surviving early withdrawals without affecting the opportunity cost of liquidity. Hence, increasing capital makes the (negative) effect of excess liquidity on solvency less negative.

Figure 2.4 depicts the bank's choice of solvency as a function of its liquidity holdings for two levels of capital (the upper curve for a higher level of capital than the lower curve, i.e., $k_2 > k_1$). The picture illustrates the three effects of equity capital on illiquidity and insolvency risk stated above. First, an increase in equity capital shifts the bank's profit-maximizing level of liquidity to the right (Lemma 2.1, Statement (ii)), that is, $l_B(k_2) > l_B(k_1)$. Second, an increase in capital leads to an upward shift of the solvency curve (Proposition 2.2), that is, $\theta_B(l,k_2) > \theta_B(l,k_1)$ for any l. Finally, the upper solvency curve is flatter than the lower solvency curve to the right of the profit-maximizing level of capital (Proposition 2.3), that is, $\left|\frac{\partial \theta_B(l,k_2)}{\partial l}\right| < \left|\frac{\partial \theta_B(l,k_1)}{\partial l}\right|$ for all $l > l(k_1)$.

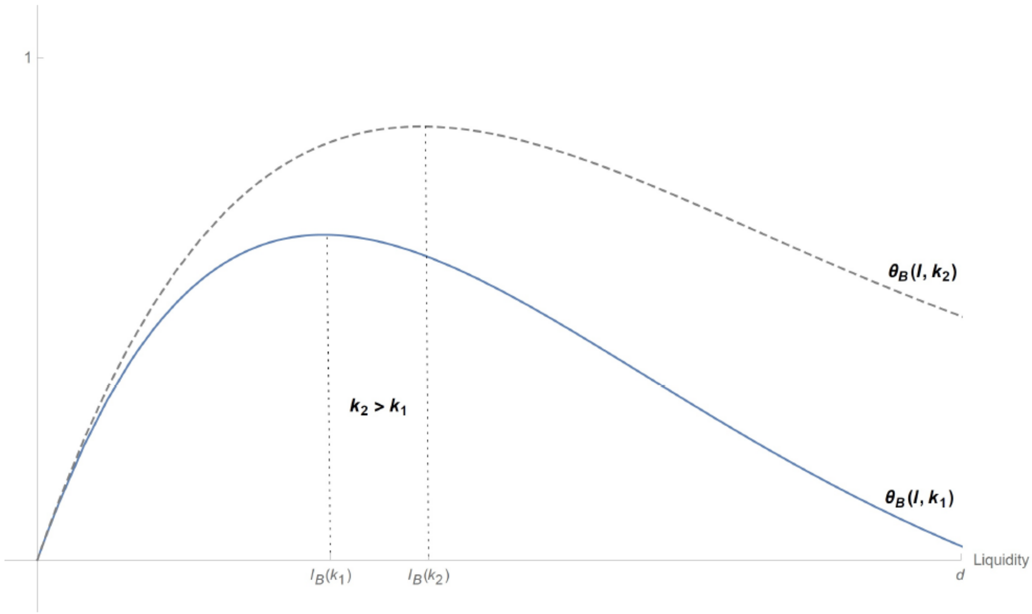

Figure 2.3: The effect of capital on solvency

2.5. Regulation of liquidity and capital

We now proceed to the analysis of the optimal regulation of capital and liquidity. In particular, we analyze the problem of a regulator that sets capital and liquidity requirements at $t=-1$ so as to maximize social welfare. More precisely, the regulator sets up a minimum liquidity and capital requirement pair (l^*, k^*) so as to solve the following problem:

$$\left.\begin{array}{ll} \max_{l\in[0,d],k\in[0,1-d]} & \Pi_R\left(l,k\right) \\ s.t. & b+k=1-d \\ & \Pi_B\left(\theta_B\left(l,k\right),l,k\right)\geq 0 \end{array}\right\}, \tag{R}$$

where the regulator's objective function is given by:

$$\Pi_R\left(l,k\right)\equiv\int_0^l \theta_B\left(l,k\right)\cdot M\cdot\left(1-l\right)\cdot f\left(\beta\right)d\beta+l-\left(1+b\cdot r-k\right)-\frac{c}{2}\cdot\theta_B^2\left(l,k\right)-\left(1+\rho\right)\cdot k. \tag{2.6}$$

First, observe that the level of solvency is not verifiable by the regulator and is therefore chosen by the bank. Hence, the regulator must take the bank's self-enforcing reaction to liquidity and capital requirements $\theta_B(l,k)$ as given. The first constraint follows from our assumption that the regulator cannot change the scale of the bank's resources, which we have normalized to 1. Hence, b is determined by the choice of k. The second constraint is the participation constraint for the bank, which has to obtain non-negative profits. Assumption A.2, laid out in Appendix B.1, guarantees that the bank has non-negative profits. Hence, we can prescind of the bank's participation constraint, which we omit henceforth.

2.5.1. The role of regulation

For the remainder of the paper it will be useful to write the bank's *(net) deposit liabilities*, which play a central role in the design of the optimal regulatory scheme, as follows:

$$D(l,k) \equiv 1 + b \cdot r - k - l. \tag{2.7}$$

The bank's liabilities consist of an amount $1-b-k$ of common deposits, as well as an amount $b \cdot (1+r)$ of long-term deposits. Hence, $D(l,k)$ stands for the amount of loan earnings that the bank will have to allocate to paying its deposit liabilities in case it succeeds–observe that the amount of liquid assets l that the bank uses to meet early withdrawals can also be used at $t=2$ and is therefore fully deducted from its deposit liabilities.

There is a natural alternative interpretation for $D(l,k)$ as the deposit insurance loss in case of a bank failure: $D(l,k)$ represents the depositors' claims that the bank cannot meet if it fails. This expression is core in our analysis. Indeed, combining expressions (2.1) and (2.6), the former evaluated at the bank's optimal choice of solvency, we can write the regulator's objective function as

$$\Pi_R(l,k) = \underbrace{\Pi_B(l,\theta_B(l,k),k)}_{Bank} - \underbrace{[1-\theta_B(l,k)\cdot F(l)]\cdot D(l,k)}_{Externality}. \tag{2.8}$$

Regulation is needed because the bank does not internalize the harm that its potential failure inflicts on the deposit insurance scheme, which is captured by the second term in expression (2.8). Put differently, since the bank is protected by limited liability, it does not face the downside of a failure. On the contrary, society experiences a welfare loss of $D(l,k)$ when the bank cannot meet its deposit obligations, an event which occurs with probability $1-\theta_B(l,k)\cdot F(l)$.

In a (first-best) world in which the bank's choice of insolvency risk were verifiable, the regulator would set a solvency requirement in excess of the bank's laissez-faire choice precisely because the bank does not internalize the social loss following a failure. In order to see that elevating the level of solvency is welfare-enhancing, we can differentiate expression (2.8) and write the effect of the solvency level on the regulator's objective function as:

$$\frac{\partial \Pi_R(l,k_R(l))}{\partial \theta_B}=\underbrace{\frac{\partial \Pi_B(l,\theta_B(l,k),k)}{\partial \theta_B}}_{Bank's\ FOC:=0}+F(l)\cdot D(l,k)>0. \tag{2.9}$$

The first term in expression (2.9) captures the effect of increasing solvency on the bank's profit. This term is zero, because it corresponds to the bank's first order condition for an optimal level of solvency. Loosely speaking, the bank takes care of this portion of the effect. The second term in expression (2.9) is positive, reflecting the fact that the bank chooses too low a solvency level because it does not internalize the social loss inflicted on the deposit insurance scheme when it fails.

Capital requirements would not play any role in this framework and would be set to zero. However, in a (second-best) world in which the choice of insolvency risk is not verifiable, capital requirements induce a reduction of insolvency risk through increasing the bank's skin in the game, as seen above (Proposition 2.2). The purpose of capital regulation, which entails the substitution of a valuable part of the depositors' base, is therefore to reduce the level of insolvency risk beyond the bank's will.

Liquidity requirements increase the bank's resilience against the advent of excessive early withdrawals. In the absence of regulation, the bank would hold the (the profit-maximizing) liquidity level $l_B(k)$ derived above (Lemma 2.1 (i)). In trading off illiquidity risk and investment in loans, the bank's laissez-faire choice $l_B(k)$ ignores the expected (social) cost of a potential bank failure captured by the second term in expression (2.8). Liquidity requirements are set taking this additional effect into account. The reason why liquidity and capital must be set jointly is because the effectiveness of one regulatory tool depends on the use of the other.

In what follows, we solve Problem (R). In order to get some insight on the effect of liquidity on the effectiveness of capital, in the following subsection we construct the *optimal capital response curve* $k_R(l)$, which establishes a scheme of optimal capital requirements for any given level of liquidity l. In Subsection 2.5.3 we assess the effect of capital on the effectiveness of liquidity. With this purpose, we construct the *optimal liquidity response curve* $l_R(k)$, which maps each level of capital k to its corresponding optimal liquidity requirement. In Subsection 2.5.2 we put them together to establish the joint optimal *liquidity and capital requirements* (l^*, k^*), which is the unique pair satisfying $l^* = l_R(k^*)$ and $k^* = k_R(l^*)$.

2.5.2. Optimal capital response curve

In this section, we construct the optimal capital response curve $k_R(l)$, which maps each liquidity level l to an optimal capital requirement. Differentiating the regulator's objective function with respect to capital, we have that an interior optimal capital response curve must satisfy:[17]

$$\frac{d\Pi_R(l, k_R(l))}{dk} = \underbrace{\underbrace{\frac{\partial \Pi_R(l, k_R(l))}{\partial \theta_B}}_{>0} \cdot \underbrace{\frac{\partial \theta_B(l, k_R(l))}{\partial k}}_{>0}}_{Indirect\ Effect:>0} - \underbrace{(\rho - r)}_{Direct\ Effect:<0} = 0. \tag{2.10}$$

On the one hand, there is a (negative) *direct effect of capital* of imposing capital requirements, which is given by the shadow cost difference $\rho - r > 0$ of substituting deposits by equity capital. In addition, there is a (positive) *indirect effect of capital* on social welfare, which follows from the fact that capital increases the level of solvency beyond the inefficiently low level that the bank

[17] Below, we find the conditions under which the optimal capital requirement is not interior.

would set in the absence of a capital requirement.

Recall from Expression (2.9) that $\frac{\partial \Pi_R(l,k_R(l))}{\partial \theta_B} > 0$. Combining this observation with Proposition 2.2, which states that equity capital induces a higher level of solvency, that is, $\frac{\partial \theta_B(l,k_R(l))}{\partial k} > 0$, we have that the indirect effect of capital on social welfare is positive.

Liquidity does not interfere with the direct effect, but it does have an impact on the indirect effect. Hence, the optimal capital response depends on the liquidity level. The following proposition characterizes the optimal capital response curve $k_R(l)$.

Proposition 2.4 (Optimal capital response curve)**.** *There exists a cost of capital $\rho_{\max}$ (whose value is determined on Definition A.3 in Appendix B.1) such that:*

(i) (Zero capital requirement when capital too costly) For any $\rho \geq \rho_{\max}$, the optimal capital response is $k_R(l) = 0$ for any l.

(ii) (Capital response curve hat-shaped) For any $\rho < \rho_{\max}$, there exist liquidity levels $0 < l_1(\rho) < \hat{l}(\rho) < l_2(\rho) \leq d$ such that the optimal capital response curve is hump-shaped for $l \in (l_1(\rho), l_2(\rho))$, attaining a maximum at $\hat{l}(\rho)$, and zero otherwise, that is:

$$k_R(l) : \begin{cases} = 0 & \text{for } l \leq l_1(\rho) \\ Increases & \text{for } l_1(\rho) < l < \hat{l}(\rho) \\ Decreases & \text{for } \hat{l}(\rho) < l \leq l_2(\rho) \\ = 0 & \text{for } l > l_2(\rho) \end{cases}.$$

(iii) (Capital response independent of M and "larger" as ρ decreases) As ρ decreases, we have that $l_1(\rho)$ shifts to the left, $l_2(\rho)$ shifts to the right (as long as $l_2(\rho) < d$) and $k_R(l)$ larger for any $l \in (l_1(\rho), l_2(\rho))$. Moreover, $k_R(l)$ is invariant in M.

Proof. See Appendix A. □

The intuition behind this result is as follows. For notational simplicity, we write the indirect effect of capital referred to in Expression (2.10) as follows:

$$IEK\,(l,k) \equiv \frac{\partial \Pi_R\,(l,k_R\,(l))}{\partial \theta_B} \cdot \frac{\partial \theta_B\,(l,k_R\,(l))}{\partial k} = F^2\,(l) \cdot D\,(l,k) \cdot \frac{1+r}{c}.$$

Capital increases social welfare because it induces the bank to adopt a higher level of solvency which, in turn, increases social welfare. In particular, the optimal capital requirement must be set at the level that equalizes the indirect marginal return of capital $IEK\,(l,k)$ to the shadow cost $\rho - r$ of substituting depositors by equityholders. When the cost of capital is very high ($\rho \geq \rho_{\max}$) the indirect marginal return of capital lies below the shadow cost of capital, so that the optimal capital response is zero for all liquidity levels.

Now consider the case in which capital is not too costly, that is, consider a value of the cost of capital such that $\rho < \rho_{\max}$. First, notice that $D\,(l,k)$ is strictly decreasing in capital: substituting depositholders by equityholders reduces the bank's net deposit liabilities.[18] Consequently, $IEK\,(l,k)$ is strictly decreasing in capital as well. Therefore, the marginal return of capital is maximized when $k = 0$. Now, observe that when liquidity is low, $IEK\,(l,0)$ is small: when the probability of surviving the early withdrawal phase $F\,(l)$ is small, the expected return of increasing solvency is small and therefore the return of capital is small as well. On the other extreme, when liquidity is high, the term $IEK\,(l,0)$ is also small: when the value of the deposit insurance loss in case of failure $D\,(l,0)$ is small, the wedge between the bank and the regulator's objective functions is small as well. Consequently, we have that when liquidity is either low or high (formally, either when $l \leq l_1\,(\rho)$ or when $l \geq l_2\,(\rho)$), the optimal capital response is zero, because the marginal social return of the first unit of capital is smaller than the shadow cost of substituting depositors by equityholders.

For intermediate values of liquidity, for which the return of the first unit of capital exceeds its opportunity cost, it is optimal to set a positive capital response. In this case, in the liquidity range

[18]Notice that we can write $D\,(l,k) = D\,(l,0) - k$.

$l \in (l_1(\rho), l_2(\rho))$, the optimal capital requirement is given by:

$$IEK(l,k) = \rho - r. \tag{2.11}$$

The capital response curve is hump-shaped in $l \in (l_1(\rho), l_2(\rho))$. Intuitively, $IEK(l,k)$ is hump-shaped because there are two effects of liquidity in the return of capital pushing in opposite directions. As liquidity increases, the probability of surviving excessive early withdrawals increase. Hence, the probability of reaching the loan maturity phase–which is the time at which increasing solvency matters– increases. However, as liquidity increases, the wedge between the bank and the regulator's objective functions shrinks, making regulation less effective. Our assumption that $F(\cdot)$ is log-concave ensures that when liquidity is low the first effect dominates, while when liquidity is large the second one is the predominant one.

So far, we have conducted this analysis fixing the cost of capital ρ. As ρ decreases, the incremental cost of capital $\rho - r$ is reduced. A glance at equation (2.11) reveals that cheaper capital leads to an upward shift of the optimal capital response curve in the range in which is positive: since the right-hand-side diminishes, the capital response must be increased so as to reduce the left-hand-side as well. Moreover, the minimum liquidity level $l_1(\rho)$ for which the marginal return of the first unit of capital exceeds the incremental cost of capital, which satisfies $IEK(l_1(\rho), k) = \frac{(\rho-r)\cdot c}{1+r}$ decreases as well. If $l_2(\rho)$ is interior, then it satisfies the same condition as $l_1(\rho)$, that is, $IEK(l_2(\rho), k) = \frac{(\rho-r)\cdot c}{1+r}$. Then, a reduction of the cost of capital leads to an increase in the maximum level of liquidity $l_2(\rho)$ for which there is a positive capital requirement. If, on the contrary, we have that $IEK(d,k) = \frac{(\rho-r)\cdot c}{1+r}$, then the capital requirement is positive for the largest possible value of liquidity, that is, $l_2(\rho) = d$. In this case, $l_2(\rho)$ does not change as ρ decreases. Finally, $\hat{l}(\rho)$ is decreasing in ρ because the marginal social return of capital $IEK(l,k)$ is decreasing in capital. Hence, as capital increases, the maximum of the marginal social return of capital decreases.

2.5.3. Optimal liquidity response curve

In this section, we construct the optimal liquidity response curve $l_R(k)$, which maps each capital level k to an optimal liquidity requirement. Differentiating the regulator's objective function with respect to liquidity, we have that an interior liquidity requirement satisfies:[19]

$$\frac{d\Pi_R(l_R(k),k)}{dl} = \underbrace{\frac{\partial\Pi_R(l_R(k),k)}{\partial l}}_{Direct\ Effect>0} + \underbrace{\underbrace{\frac{\partial\Pi_R(l_R(k),k)}{\partial\theta_B}}_{>0} \cdot \underbrace{\frac{\partial\theta_B(l_R(k),k)}{\partial l}}_{<0}}_{Indirect\ Effect<0} = 0. \tag{2.12}$$

We can split the effect of liquidity on social welfare into a direct and an indirect effect. The *direct effect of liquidity* captures the effect of liquidity on social welfare ignoring the effect of liquidity on the bank's choice of insolvency risk. The *indirect effect of liquidity* accounts for the (negative) impact of liquidity on the objective function through its influence on insolvency risk. As we shall see, capital influences both effects. Hence, the optimal liquidity requirement depends on the level of capital. The following proposition characterizes the optimal liquidity response curve $l_R(k)$.

Proposition 2.5 (Optimal liquidity response curve)**.** *(i) (Liquidity requirements binding) The liquidity requirement is binding for the bank for any level of capital, that is,* $l_R(k) > l_B(k) > 0$ *for all* k*.*

(ii) (Liquidity response either hump-shaped or increasing) There exists $\bar{M} > \underline{M}$ *such that:*

(ii.1) If $M < \bar{M}$*, the liquidity response* $l_R(k)$ *is a hump-shaped function of capital, that is, there exists a capital threshold* $\hat{k}(M) < 1 - d$ *such that:*

$$l_R(k) : \begin{cases} Increases & \text{for } k < \hat{k}(M) \\ Decreases & \text{for } k > \hat{k}(M) \end{cases}.$$

(ii.2) If $M \geq \bar{M}$*, the liquidity response* $l_R(k)$ *is strictly increasing in capital.*

(iii) (Liquidity response independent of ρ *and "smaller" as* M *increases) For any* $M' > M$*, we have that* $l_R(k)|_{M'} < l_R(k)|_M$*. Moreover,* $l_R(k)$ *is invariant in* ρ*.*

[19] As we shall see below, the optimal liquidity requirement is always interior.

Proof. See Appendix A.3. □

In order to provide some intuition for this result, we first construct the *"direct-effect liquidity response"* $l_D(k)$. The liquidity scheme $l_D(k)$ corresponds to the liquidity response curve that would be set by a regulator that ignored the (negative) indirect effect of liquidity on solvency level. Hence, for any given value of capital k, the "direct-effect liquidity response" curve $l_D(k)$ satisfies:

$$\underbrace{\frac{\partial \Pi_R(l_D(k), k)}{\partial l}}_{Direct\ effect} = \left(\begin{array}{c} + \underbrace{1}_{Cash-in-hand} \\ \underbrace{\theta_B(l_D(k), k) \cdot M \cdot F(l_D(k)) \cdot [g(l_D(k))(1 - l_D(k)) - 1]}_{Opportunity\ Cost} \end{array} \right) = 0. \quad (2.13)$$

The direct effect of liquidity on the social value is given by two factors. On the one hand, liquidity constitutes a cash-in-hand asset that reduces the bank's net deposit liabilities $D(l, k)$ regardless of the loan performance. The marginal value of cash-in-hand is therefore 1. On the other hand, storing cash has an opportunity cost in terms of foregone loans. Observe that an "expected loan maximizer bank" (ELVB) would choose a liquidity level l_{ELVB} satisfying $g(l_{ELVB})(1 - l_{ELVB}) = 1$.[20] Hence, any liquidity level in excess of l_{ELVB} contributes negatively to the expected loan value.

We now argue how capital affects the "direct-effect liquidity response" curve $l_D(k)$. Capital increases the probability of a loan success by the skin-in-the-game effect on the bank's choice of solvency (first term of Equation (2.13)). However, it does not affect the cash-in-hand effect (second term of Equation (2.13)). Hence, capital increases the relative weight of the expected loan value versus the cash-in-hand effect: the more resilient the banking system, the higher the opportunity cost of storing liquidity. Hence, as capital increases, $l_D(k)$ comes closer to l_{ELVB}. Consequently, the "direct-effect liquidity response" curve $l_D(k)$ is strictly decreasing in capital k.

[20] An "expected loan maximizer bank" (ELVB) would choose l so as to maximize the expected value of its loan portfolio, that is: $l_{ELVB} = \arg\max_{l \in [0,d]} \theta \cdot F(l) \cdot M \cdot (1 - l)$. Observe that the bank that we are modelling maximizes its expected profits which, in addition to loans, include its liabilities, that is: $l_B(k) = \arg\max_{l \in [0,d]} \theta \cdot F(l) \cdot \pi(l, k)$.

Nonetheless, for any given k, the "direct-effect liquidity requirement" $l_D(k)$ lies above the bank's profit-maximizing liquidity level $l_B(k)$, that is, $l_D(k) > l_B(k)$. The reason is that the bank does only value the cash-in-hand asset in the event of a success. Hence, the cash-in-hand factor in the bank's optimality condition is weighted by $\theta \cdot F(l_B(k)) < 1$.

Consider now the *indirect effect* of liquidity, which we can write as:

$$IEL(l,k) \equiv \frac{\partial \Pi_R(l,k)}{\partial \theta_B} \cdot \frac{\partial \theta_B(l,k)}{\partial l} = [F(l) \cdot D(l,k)] \cdot \frac{\partial \theta_B(l,k)}{\partial l} < 0 \text{ iff } l > l_B(k). \tag{2.14}$$

From Lemma 2.1, insolvency risk is minimized at the bank's profit-maximizing liquidity level $l_B(k)$. Hence, we have that $\frac{\partial \theta_B(l,k)}{\partial l} < 0$ for any $l > l_B(k)$ and, in particular, for $l_D(k)$. Moreover, recall from Equation (2.9) that an increase in solvency is welfare enhancing. Hence, the indirect effect of liquidity on social welfare is negative, reflecting the fact that liquidity requirements above the bank's profit-maximizing liquidity level reduce the bank's loan investments and therefore harm its incentives to reduce insolvency risk. As a consequence, the optimal liquidity requirement must be smaller than the a "direct-effect liquidity requirement" would be, that is, $l_R(k) < l_D(k)$.

Nonetheless, the indirect effect does not completely offset the direct effect. In order to see why, observe that the indirect effect diminishes as liquidity reduces and it completely vanishes at the bank's profit-maximizing liquidity level $l_B(k)$, since $\frac{\partial \theta_B(l_B(k),k)}{\partial l} = 0$. Hence, when $l = l_B(k)$ the indirect effect is zero, while the direct effect is positive. Therefore, we have that the optimal liquidity response does always constitute a binding requirement for the bank, that is:

$$l_B(k) < l_R(k) < l_D(k). \tag{2.15}$$

Capital does therefore exert two opposing effects over the optimal liquidity response curve $l_R(k)$. As capital increases, the negative effect of liquidity on welfare diminishes. The reason is twofold. First, because $D(l,k)$ gets reduced, so that the effect of the level of solvency on welfare diminishes. Second, because the negative effect of liquidity on solvency also reduces ("slope effect" of Proposition 2.1 displayed in Figure 2.4). Hence, as capital increases, liquidity requirements must increase. However, as these effects vanish out, an aditional effect pushing in the opposite direction emerges. As capital increases, the bank becomes more solvent (”level effect” of Proposition 2.2 displayed in Figure 2.4). Consequently, the opportunity cost of liquidity in terms of foregone loans increases. If this ”opportunity cost” effect gets to dominate, the optimal liquidity response curve $l_R(k)$ eventually decreases. However, the ”opportunity cost” effect becomes strong enough so as to eventually dominate only if the project profitability M is not too large.

The effect of the project profitability M on the ”opportunity cost” effect can be seen in Equation (2.13). The project profitability M constitutes a weighting factor of the expected loan value versus cash-in-hand–the higher the project profitability, the higher the opportunity cost of liquidity. As the project profitability M increases, liquidity requirements approach the liquidity level l_{ELVB} that an ”expected loan maximizer bank” (ELVB) would choose for *any* level of capital. Hence, the first term on Equation (2.13) becomes less responsive to an elevation of capital as M increases. When M is sufficiently large the optimal liquidity response function $l_R(k)$ is always increasing in k .

Figure 2.1 depicts the relationship between several liquidity measures for $M < \bar{M}$. On the one hand, we have the bank's profit maximizing liquidity level $l_B(k)$ which, as stated in Lemma 2.1 is strictly increasing in capital. The upper line corresponds to the direct-effect optimal liquidity response $l_D(k)$, which corresponds to the optimal liquidity response curve that would be set by a regulator that ignored the indirect effect of liquidity on the solvency level. As argued above, this liquidity measures is strictly decreasing in capital. As stated in Proposition 2.5, the regulator's liquidity response $l_R(k)$ is a hump-shaped function of capital with a maximum at some capital level $\hat{k}(M)$. Also, as specified by the set of inequalities 2.15, $l_R(k)$ lies above $l_B(k)$ (i.e., the liquidity requirement will bind) and below $l_D(k)$ (i.e., the indirect effect of liquidity pushes the optimal amount of liquidity requirements down). The indirect effect of liquidity (through its negative effect on solvency) is simply the difference between the direct effect and the overall effect of liquidity. From 2.3 we have that the negative effect of liquidity on solvency is mitigated as capital increases. Hence, $l_D(k) - l_R(k)$ is strictly decreasing in capital.

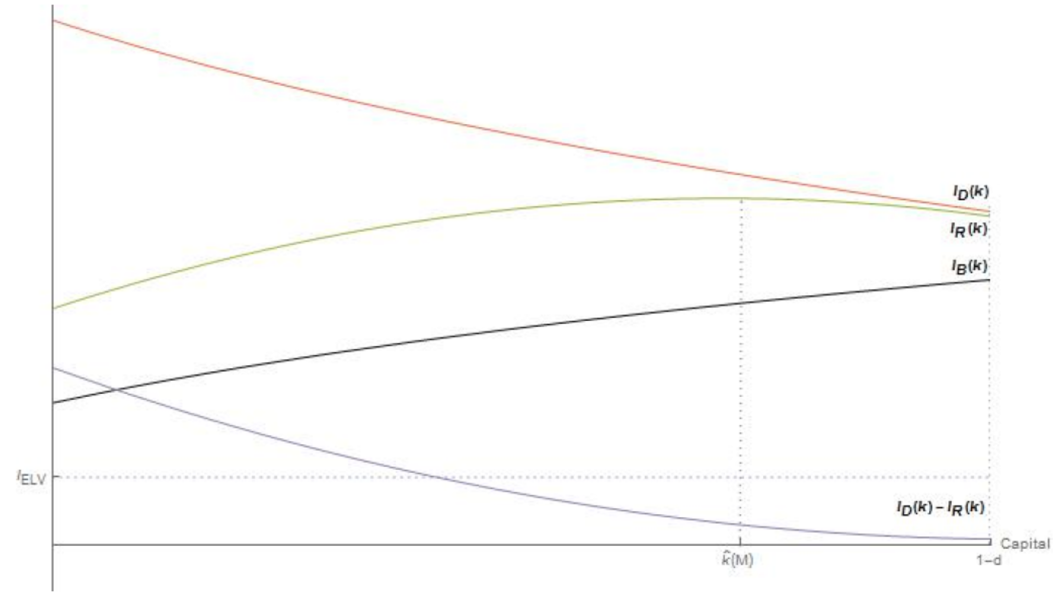

Figure 2.4: The effect of capital on the direct and indirect effect of liquidity

2.5.4. Joint capital and liquidity regulation

We are left with determining how capital and liquidity should be set together. In this section, we show that there is a unique pair of liquidity and capital requirements (l^*, k^*) such that the liquidity requirement is optimal given the capital requirement, i.e., $l^* = l_R(k^*)$ and, conversely, the capital requirement is optimal given the liquidity requirement, that is, $k^* = k_R(l^*)$. Moreover, we address the issue of whether capital and liquidity requirements are complements or substitutes depending on the cost of capital and the bank's profitability. We state the main result of the paper in the following proposition and provide intuition for this result below. Abusing notation, we let $l^*(\rho, M)$ and $k^*(\rho, M)$ stand for the optimal liquidity and capital requirement, respectively, for a given cost of capital ρ and return to investment M.

Proposition 2.6 (Optimal joint liquidity and capital requirements)**.** *For any given cost of capital ρ and return to investment (opportunity cost of liquidity) M, we have that:*

(i) (Uniqueness) There exists a unique pair of liquidity and capital requirements (l^*, k^*).

(ii) (Comparative statics)

(a) (Zero capital requirements for high cost of capital) There exists $\bar{\rho}(M) < \rho_{\max}$ *such that for any* $\rho > \bar{\rho}(M)$*, the capital requirement is zero.*

(b) (Raising the cost of one factor reduces the requirement of that factor) The optimal liquidity requirement $l^*(\rho, M)$ *is a strictly decreasing function of* M*. The optimal capital requirement* $k^*(\rho, M)$ *is a decreasing function of* ρ *(strictly decreasing if and only if* $\rho < \bar{\rho}(M)$*).*

(c) (Capital and liquidity complementary tools for low cost of capital and offsetting tools for high cost of capital)There exists a threshold $\hat{\rho}(M) < \bar{\rho}(M)$ *for the cost of capital such that:*

If $\rho \in (\hat{\rho}(M), \bar{\rho}(M))$*, then capital and liquidity requirements are complementary tools: a raise of the cost of capital* ρ *leads to an elevation of the liquidity requirement* l^**; and a raise of the opportunity cost of liquidity* M *leads to an elevation of the capital requirement* k^**.*

If $\rho \in (\rho_{\min}, \hat{\rho}(M))$*, then capital and liquidity requirements are offsetting tools: a raise of the cost of capital* ρ *leads to a reduction of the liquidity requirement* l^**; and a raise of the opportunity cost of liquidity* M *leads to a reduction of the capital requirement* k^**.*

(d) (Capital and liquidity are always complements when the return to investment is high) The threshold $\hat{\rho}(M)$ *that determines whether capital and liquidity are complementary or offsetting tools is strictly decreasing on the return to investment* M*. Moreover, there exists a threshold* $\bar{M}$ *such that* $\hat{\rho}\left(\bar{M}\right) = \rho_{\min}$*. Consequently, for any* $M > \bar{M}$*, capital and liquidity are complements for any cost of capital* $\rho < \bar{\rho}(M)$*.*

(e) (Zero capital requirements for high return to investment) The capital cost threshold $\bar{\rho}(M)$ *that establishes the maximum cost of capital for which capital requirements are positive decreases as the return to investment enlarges, that is,* $\bar{\rho}(M)$ *is strictly decreasing in* M*.*

Proof. See Appendix A.3. □

Figure 2.5 may help understand the intuition behind Proposition 2.6. In these figures, liquidity is depicted in the horizontal axis and the optimal capital response curve is drawn against the horizontal axis (that is, as functions are typically depicted). Capital is represented on the vertical axis. We draw the optimal liquidity response curve as a function that maps a certain capital level to a unique optimal liquidity requirement, that is, we invert the axes and plot the curve as a function of the variable represented on the vertical axis. From the perspective of the horizontal axis, the optimal liquidity response curve is a correspondence, mapping each liquidity level to the (potentially two) capital level(s) for which that particular liquidity level constitute an optimal liquidity response. Consider first the liquidity requirement curve $l_R(k)|_{M_{Low}}$ for a given (low) opportunity cost of liquidity M_{Low}. This curve intersects the capital requirement curves $k_R(l)|_{\rho}$, which represent the optimal capital response for several values $\rho_1 > \rho_2 > \hat{\rho}(M_{Low}) > \rho_3$ of the cost of capital. The jointly determined capital and liquidity requirement for a given cost of capital ρ is given by the intersection of the liquidity requirement curve $l_R(k)|_{M_{low}}$ and the corresponding capital requirement curve $k_R(l)|_{\rho}$. Notice that an increase in the cost of capital ρ would shift the capital requirement curve $k_R(l)|_{\rho}$ down-and-rightwards, as seen in Proposition 2.4. However, the cost of capital does not affect the liquidity requirement curve $l_R(k)|_M$. On the contrary, an increase in the opportunity cost of liquidity M leads to a downward (leftward, in the picture) shift of the liquidity requirement curve $l_R(k)|_M$, leaving the capital requirement curve $k_R(l)|_{\rho}$ unchanged. Consequently, increasing the cost of capital leads to a reduction of a positive capital requirement. Also, an increase in the opportunity cost of liquidity leads to a reduction of the liquidity requirement.

When the cost of capital is large ($\rho_1 > \bar{\rho}$), the curve $l_R(k)|_{M_{low}}$ intersects $k_R(l)|_{\rho_1}$ at its (zero) flat portion. Hence, the capital requirement $k^*(\rho_1, M_{Low})$ is zero. Moreover, an increase of the cost of capital ρ would leave capital requirements unchanged at zero, as the liquidity and capital requirement curves would continue to intersect at the flat segment of the capital response curve. The next pair of capital requirements corresponds to a cost of capital $\rho_2 \in (\hat{\rho}(M_{Low}), \bar{\rho})$, which is smaller than ρ_1. The optimal capital an liquidity response curves intersect at a point where both curves are upward slopping. In this range, capital and liquidity are complements: an increase of either the cost of capital ρ or of the opportunity cost of liquidity M would lead to a decrease of

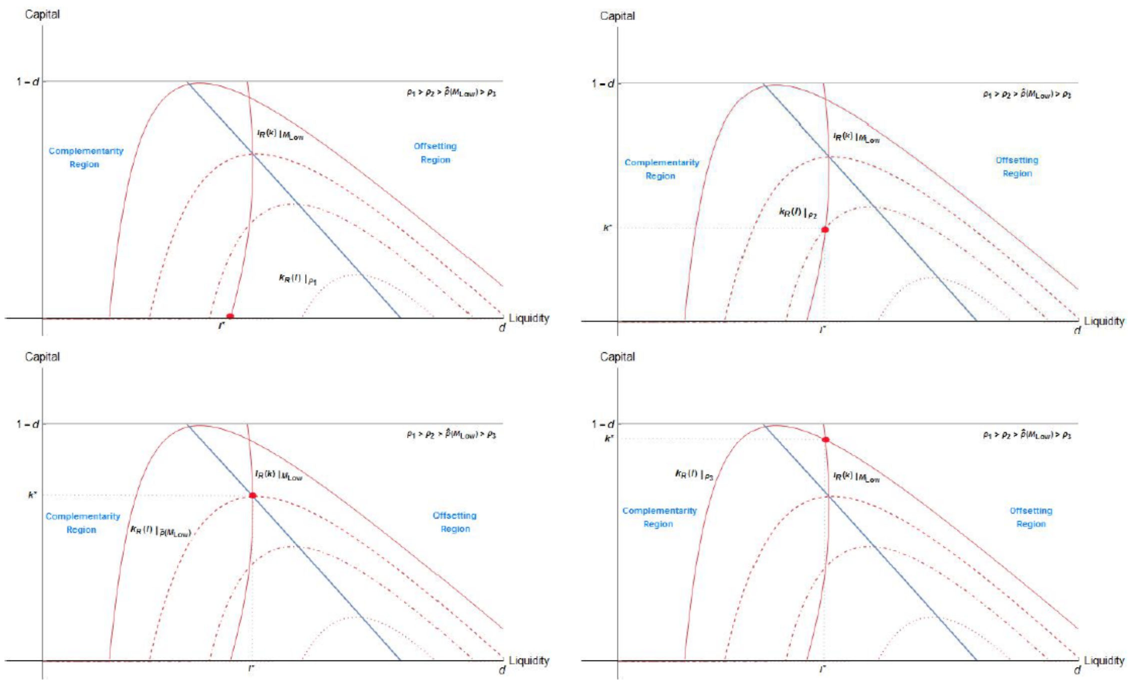

Figure 2.5: The effect of ρ on the equilibrium

both the capital and the liquidity requirement. The cost of capital that determines whether capital and liquidity are complements or substitutes is $\hat{\rho}\left(M_{Low}\right)$, which corresponds to the cost of capital for which both curves intersect at their respective peaks. The last capital response curve, for $\rho_3 < \hat{\rho}\left(M_{Low}\right)$, depicts a case in which capital and liquidity are substitutes: any change in the cost of one of the factors leads to opposite movements of the optimal capital and liquidity requirements.

The curve $l_R\left(M_{High}\right)$ in Figure 2.6.BIS corresponds to the optimal liquidity response curve for a higher return to investment. This curve lies below the curve $l_R\left(M_{Low}\right)$–to the left, in the picture–representing the fact that the optimal liquidity response is strictly smaller for any level of capital the higher the return to investment, which represents the opportunity cost of liquidity. Moreover, we have depicted this curve for a value of M exceeding $\bar{M}$, which is defined as $\hat{\rho}\left(\bar{M}\right) = \rho_{\min}$. In this case, the optimal liquidity response is increasing for all capital levels. Formally, we have that the capital level for which the optimal liquidity response achieves a maximum is beyond the maximum possible capital requirement, that is, $\hat{k}\left(M_{High}\right) > 1 - d$. This capital level is never

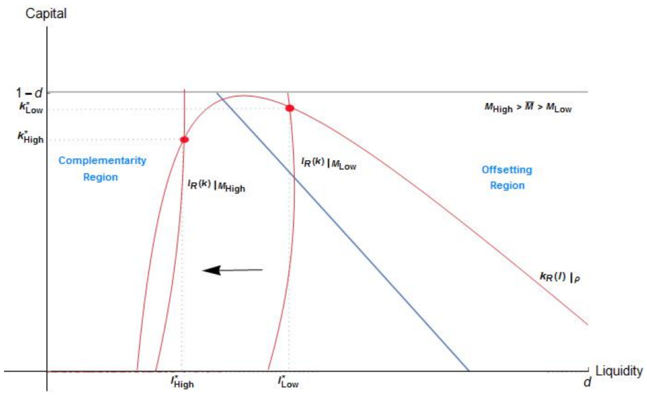

Figure 2.6: The effect of M on the equilibrium

an optimal capital requirement due to our assumption that the minimum cost of capital is at least as large as $\rho_{\min}$, which is precisely set at the value for which the optimal capital requirement hits the boundary, that is, $k^*|_{\rho_{\min}} = 1 - d$. When the return to investment is sufficiently large ($M > \bar{M}$), capital and liquidity are always complements, as long as the cost of capital is low enough so as to have a positive capital requirement (i.e., $\rho < \bar{\rho}(M)$). The reason is that the optimal liquidity response for any given capital level is lower the higher the opportunity cost of liquidity. Therefore, as the opportunity cost of liquidity increases, the optimal liquidity response gets closer to the liquidity level value that maximizes the expected loan value, so that the weight of the first term of expression (2.13)–which measures the effect of the loan value on social welfare and drives liquidity down as it increases–, gets smaller. Hence, the capital level $\hat{k}(M)$ for which the optimal liquidity response attains a maximum is increasing in M.

Finally, the return to investment does also affect the range for which the optimal capital requirement is positive. In order to see why, consider a given opportunity cost of liquidity M. The range for which the optimal capital response curve is positive is given by the liquidity segment $(l_1(\rho), l_2(\rho))$, which depends on the cost of capital ρ. The capital requirement is positive if and only if the optimal liquidity response when capital is zero exceeds the minimum liquidity level for which the optimal capital response is positive, that is, if and only if $l_R(0)|_M > l_1(\rho)$. Otherwise, if $l_R(0)|_M \leq l_1(\rho)$, we have that the optimal requirements are given by $l^* = l_R(0)|_M$ and $k^* = 0$.

Therefore, since $l_1(\rho)$ decreases in ρ, we have that for each M there exists $\bar{\rho}(M)$ such that the capital requirement is positive if and only if $\rho < \bar{\rho}(M)$.

2.6. Extensions

So far we have analyzed a simplified model in which several important ingredients are absent. In this section, we enrich the basic model and assess the validity of the predictions of the basic setup.

2.6.1. Asset liquidation

The bank's problem with asset liquidation

In the basic model the bank fails if the demand for deposits at the interim stage exceeds the bank's liquidity reserves. In this section we allow for asset liquidation at a discounted price if early withdrawals go beyond the bank's liquidity level, so that the bank can survive larger liquidity shocks. In this setup, the bank's problem reads:

$$\Pi_B^{AL}(\theta, l, k) \equiv \Pi_B(\theta, l, k) + \int_l^{l+\gamma} \theta \cdot \left[M \cdot \left(\overbrace{1 - l - \underbrace{\frac{\beta - l}{\psi}}_{Liquidation}}^{Leftover\ Investment} \right) - \underbrace{D(\beta, k)}_{Deposit\ Liabilities} \right] \cdot f(\beta)\, d\beta. \quad (2.16)$$

The first term corresponds to the bank's expected profit in the event that early withdrawals fall below the bank's liquidity reserves, which is equivalent to the bank's problem of the basic model defined in equation (B). The second term stands for the bank's expected profit in the event that the bank has to liquidate assets in order to overcome a large liquidity shock. For any withdrawal level $\beta > l$ in excess of the bank's liquidity reserves, the bank needs to liquidate an amount $\frac{\beta - l}{\psi}$ so as to obtain an amount $\beta - l$ of liquid assets, where $\psi < 1$ represents the (fire sales) price per unit of loan.[21] Hence, after a partial liquidation occurs, the bank is left with an amount $1 - l - \frac{\beta - l}{\psi}$ of loans,

[21] Clearly, if $\psi \geq 1$ the bank could obtain a higher amount of liquidity by liquidating assets in the interim period than by storing cash, in which case the optimal amount of liquidity reserves would be zero.

yielding a return of M per unit if successful. The bank's deposit liabilities after satisfying an early withdrawal of β amount to $D(\beta,k)$, as defined in equation (2.7). The upper bound of the integral is given by the maximum amount of extractions that the bank can bear, where γ represents the maximum amount of cash that the bank can obtain by liquidating assets and continue operating with a nonnegative continuation expected payoff.[22]

Differentiating equation (2.16) with respect to θ, it follows that:

$$\theta_B^{AL}(l,k) = \theta_B(l,k) + \underbrace{\frac{1}{c}\int_l^{l+\gamma}\left[M\cdot\left(1-l-\frac{\beta-l}{\psi}\right)-D(\beta,k)\right]\cdot f(\beta)\,d\beta}_{>0}. \tag{2.17}$$

Observe that the second term in the previous expression is positive given that asset liquidation only occurs if it enhances the bank's continuation payoff. Consequently, insolvency risk is strictly smaller than in the absence of asset liquidation: the possibility of liquidating assets to survive larger liquidity shocks makes it more valuable to reduce insolvency risk.

Binding liquidity requirements harms insolvency risk, as the following Proposition states.

Proposition 2.7 (Liquidity and solvency risk with asset liquidation)**.** *(i) The level of solvency $\theta_B^{AL}(l,k)$ is a hump-shaped function of liquidity which, for any given level of equity capital k, is maximized at the bank's profit-maximizing liquidity level $l_B^{AL}(k)$. Consequently, any liquidity requirement $l^* > l_B^{AL}(k)$ in excess of the bank's profit maximizing level induces a lower level of solvency.*

(ii) The bank's profit-maximizing liquidity level with asset liquidation is strictly smaller than in the basic model but the corresponding overall illiquidity risk , which includes the range of asset liquidation, is smaller. Formally:

$$l_B^{AL}(k) < l_B(k) < l_B^{AL}(k) + \gamma.$$

[22] Sensu stricto, γ should be given by the solution to the following system of equations: $M\cdot\left(1-l-\frac{\beta-l}{\psi}\right)-D(\beta,k) = 0$ and $\gamma = \beta - l$. However, for tractability, we treat γ as an exogenous variable and impose (sufficient) conditions for the value of γ so that the continuation value of the bank is nonnegative in the case in which the advent of early withdrawls is given by γ. Alternatively, one could argue that, even if the bank could put up with an amount of early withdrawals larger than γ by liquidating at a (constant, independent of the liquidating volume) rate of ψ per loan, exceeding the amount γ would lead to a disorderly process of liquidation that would make the bank's continuation non-viable.

Proof. See Appendix A. □

This proposition is the counterpart of Proposition 2.1 when we allow for asset liquidation. The reason why the profit-maximizing liquidity level does also maximize solvency is the same as in the basic model: the returns of reducing insolvency risk are given by the expected upside payoff. Therefore, the profit-maximizing liquidity level–which does also maximize the upside payoff–is the value that maximizes the return of reducing insolvency risk.

Proposition 2.7 also shows that the bank's incentives to keep liquid assets is reduced. This is due to two effects pushing in the same direction. On the one hand, the possibility of liquidating assets increases the likelihood of surviving the early withdrawal phase, thus increasing the opportunity cost of holding liquidity instead of investing in loans. On the other hand, the value of liquidity in terms of increasing the likelihood of surviving the early withdrawal phase is smaller with asset liquidation, because the possibility of liquidating assets expands the range of liquidity shocks that the bank can meet.[23] Finally, the fact that overall illiquidity risk is smaller follows immediately from the fact that the marginal value of liquidity is reduced when asset liquidation is possible, which implies that the likelihood of surviving the early withdrawal phase must be larger.

Capital and liquidity regulation with asset liquidation

We can write the wedge between the bank and the regulator's problem as:

[23] Notice that the marginal contribution of liquidity to increasing the likelihood of surviving the early withdrawal phase is strictly decreasing in liquidity, because $g(\cdot)$ is strictly decreasing, which implies that $g(l+\gamma) < g(l)$.

$$\begin{aligned}\Pi_B^{AL}\left(\theta_B^{AL}\left(l,k\right),l,k\right)-\Pi_R^{AL}\left(l,k\right) &= \underbrace{\left(1-\theta_B^{AL}\left(l,k\right)\right)\cdot\left[\overbrace{F(l)\cdot D(l,k)}^{No\ liquidation}+\overbrace{\int_l^{l+\gamma} D(\beta,k)\cdot f(\beta)\,d\beta}^{Partial\ liquidation}\right]}_{Solvency\ failure} \\ &\quad \underbrace{+\left(1-F\left(l+\gamma\right)\right)\cdot\left[D(l,k)-\gamma\right]}_{Liquidity\ failure: full\ liquidation}. \end{aligned} \tag{2.18}$$

The payoff discrepancy between the regulator and the bank occur when the bank fails, since the bank is protected by limited liability. There are three potential sources for the bank failure. First, the bank may overcome a liquidity shock and then become insolvent. In this case the deposit insurance fund liabilities amount to $D(l,k) = 1 + b \cdot r - l - k$. Second, the bank may undergo a partial liquidation at the interim stage and then become insolvent. In these situations, the social toll following the bank failure is given by $D(\beta,k) = 1 + b \cdot r - \beta - k$, as part of the deposit liabilities are paid back at the early withdrawal phase making use of the liquidation proceeds. Finally, if the bank fails as the result of a liquidity crisis, the deposit insurance fund uses the bank full liquidation value to reduce its financial obligations. Hence, beyond expanding the size of early withdrawals that the bank can survive, asset liquidation reduce the social toll following a bank failure.

We can now proceed analogously as in the basic model and find the optimal capital and liquidity responses, respectively. As compared to the basic model, we can highlight the following features:

(i) *For any given level of the bank's liquidity holdings, capital requirements have a higher impact in reducing insolvency risk, that is,* $\frac{\partial \theta_B^{AL}(l,k)}{\partial k} > \frac{\partial \theta_B(l,k)}{\partial k}$.

Since the likelihood of surviving a liquidity shock is higher with the possibility of incurring in asset liquidation, raising capital has a higher impact on the bank's expected upside payoff. As a consequence, the returns of reducing insolvency risk are higher.

(ii) *If the capital response function in the basic model entails a positive capital level, then the capital response function with asset liquidation attains a higher value. Formally, if $k_R(l) > 0$, then we have that $k_R^{AL}(l) > k_R(l)$.*

There are two effects that push in the direction of the raising the capital response as compared to the baseline model. On the one hand, as argued in the previous statement, capital is more effective in reducing insolvency risk–the 'substitution effect' is larger than the 'income effect'. On the other hand, the social value of reducing insolvency risk is higher than in the basic model, because the likelihood of surviving the early withdrawal phase is higher and therefore the likelihood of reaching the stage in which being solvent matters.

(iii) *The liquidity response function with asset liquidation is strictly lower than in the basic model, that is, $l_R^{AL}(k) < l_R(k)$.*

There are two effects making the liquidity response function smaller than in the baseline model. One of them have already been outlined in the bank's problem: the effectiveness of liquidity in reducing illiquidity risk is diminished. On the other hand, the possibility of liquidating assets raises the opportunity cost of liquidity. As in the case of the bank, one reason for this is the larger likelihood of surviving a liquidity shock and therefore an elevation of the expected value of investing in loans. Additionally, the value of liquidity as a cash-in-hand asset to reduce the social toll in case of failure is reduced, because the proceeds of liquidation can now serve that purpose.

We end this section by stating existence and uniqueness of liquidity and capital requirements.

Proposition 2.8 (Liquidity and capital requirements with asset liquidation)**.** *For any given cost of capital ρ and of loan returns M, there exists a unique pair (l_{AL}^*, k_{AL}^*) of liquidity and capital requirements given by $l_{AL}^* = l_R^{AL}(k_{AL}^*)$ and $k_{AL}^* = k_R^{AL}(l_{AL}^*)$.*

2.6.2. Solvency-driven liquidity shocks

So far, we have assumed that illiquidity and insolvency risk are generated independently and yet shown that capital and liquidity should be jointly determined. Our assumption, although extreme in nature, makes sense if, for instance, a large fraction of withdrawals is due to purely idiosyncratic consumption motives. However, it seems reasonable to assume that liquidity risk is influenced by the bank's asset quality, as in Jacklin and Bhattacharya (1988), where banks' fundamentals may trigger an information-based bank run. In this section, we show that our main predictions in terms of optimal regulation are qualitatively robust to the introduction of information-based runs.

In particular, consider the baseline model outlined in Section 2.3, in which the early withdrawals function with solvency-driven shocks $F^T(\cdot)$ is related to the level of insolvency risk as follows:

$$P[\beta \leq l] = F^T(l, \theta, T) = \begin{cases} \theta^T \cdot F(l) & \text{if } l < d \\ 1 & \text{if } l = d \end{cases},$$

where $F(\cdot)$ is the c.d.f of early withdrawals in the baseline model and $T \in (0, \bar{T})$ for some $\bar{T} < 1$. The c.d.f $F^T(\cdot)$ can be interpreted as follows. In the baseline model, for any given level of liquidity l held by the bank, $F(l)$ stands for the probability that the amount of early withdrawals is at most as high as l. The c.d.f $F^T(\cdot)$ simply weights this probability by a factor θ^T and places a mass of $1 - \theta^T$ on the (run-triggered) event of all depositors withdrawing early. Observe that for any $\theta' < \theta''$ it follows that $F^T(l, \theta', T) < F^T(l, \theta'', T)$. Hence, the level of solvency θ induces a first-order stochastic dominance ordering in the family $\{F^T(l, \theta, T)\}_{\theta \in [0,1]}$ whereby the larger the solvency level θ the "less likely"–in a first-order stochastic dominance sense–early withdrawals. For instance, an insolvent bank (one with $\theta = 0$) would experience a run for certain. On the other side of the spectrum, an absolutely solvent bank (i.e., a bank with $\theta = 1$) would face the smooth early withdrawals c.d.f function $F(\cdot)$ described in the baseline model.

The parameter T stands for the elasticity of the c.d.f function $F^T(\cdot)$ with respect to the solvency level θ. Accordingly, the higher T, the more sensitive $F^{T}(\cdot)$ to the solvency level θ. Clearly, $T = 0$ corresponds to the baseline model. The upper threshold $\bar{T}$ for the elasticity parameter is set so as to avoid the following pathological behavior. In the baseline model, increasing the bank's liquidity reserves leads to an unambiguous reduction of illiquidity risk. In this setup, in addition to this effect of liquidity on illiquidity risk, increasing the bank's liquidity reserves beyond the bank's profit-maximizing liquidity level leads to a reduction of the equilibrium choice of solvency θ, which increases the likelihood of a bank run. The upper threshold $\bar{T}$ ensures that increasing liquidity, regardless of any optimality consideration, unambiguously reduces illiquidity risk in equilibrium.

Before closing the description of the model, let us carry out the following thought experiment. Consider an *exogenously given* pair of liquidity and capital (l, k). Suppose that we increase T and we ask whether the solvency level $\theta(l, k)$ that the bank would choose as a function of the pair (l, k) increases or decreases. On the one hand, increasing T makes the effect of solvency on illiquidity risk more important. Hence, the bank would be willing to increase its level of solvency. However, increasing T does also reduce the bank's expected profits for any given pair (l, k) of liquidity and capital, because illiquidity risk is higher. This effect would push the bank's choice of solvency in the opposite direction. In order to clean out the effect of solvency-driven bank runs through the effect of solvency on liquidity exclusively, we can "level the playing field" by adding a fictitious transfer $H(T)$ to be added to the bank's upside payoff, so that the bank's expected profit *in equilibrium* does not change as we change T. Specifically, we could compute the bank's solvency function $\theta^{T}(l, k)$, as well as the optimal liquidity and capital requirements $\left(l^{T}, k^{T}\right)$ for any given T and compute $H(T)$ so that:

$$\Pi_{B}\left(\theta\left(l^{*}, k^{*}\right), l^{*}, k^{*}\right)-\Pi_{B}^{T}\left(\theta^{T}\left(l^{T}, k^{T}\right), l^{T}, k^{T}\right) = \tag{2.19}$$

$$\theta\left(l^{*}, k^{*}\right) \cdot F\left(l^{*}\right) \cdot \pi\left(l^{*}, k^{*}\right)-\theta^{T}\left(l^{T}, k^{T}\right) \cdot F\left(l^{T}\right) \cdot\left[\pi\left({}^{T}, k^{T}\right)+H(T)\right] = 0. \tag{2.20}$$

Clearly, $H(T)$ is strictly increasing, capturing the negative effect of increasing runs on the bank's expected profits. The following proposition establishes the pattern of optimal liquidity and capital responses as T varies.

Proposition 2.9 (Liquidity and capital with solvency-driven liquidity shocks)**.** *(i) Let $H(T)$ be as defined in equation (2.19). Then, if a liquid and solvent bank is compensated with a transfer $H(T)$, the optimal liquidity response function $l_R^T(k)$ is strictly decreasing in T and the optimal capital response function $k_R^T(l)$ is strictly increasing in T. Moreover, the bank's choice of solvency $\theta^T(l,k)$ is strictly increasing in T.*

(ii) Let $H(T) = 0$. Then, the optimal liquidity response function $l_R^T(k)$ is strictly decreasing in T. Additionally, there exist thresholds $\underline{l}$ and $\bar{l}$, to the left and right of the bank's profit maximizing level of liquidity, respectively, such that both the optimal capital response function $k_R^T(l)$ and the bank's choice of solvency $\theta^T(l,k)$ are strictly increasing in T if and only if $l \in (\underline{l}, \bar{l})$.

(iii) The triplet $\left(\theta^T(l,k), l_R^T(k), k_R^T(k)\right)$ converges to $(\theta(l,k), l_R(k), k_R(k))$ as T goes to zero.

Consider first the case in which the negative effect of solvency-driven shocks on the bank's expected profits is eliminated through the fictitious transfer $H(T)$. On the one hand, increasing the solvency level has a positive effect on illiquidity risk. Therefore, the bank's choice of solvency is larger as T increases. Moreover, the social return of capital increases. As a consequence, the optimal capital response is also larger as T increases. On the other hand, binding liquidity requirements have a negative effect on the choice of solvency. Hence, the negative effect of liquidity on solvency has a larger impact as solvency becomes more important, that is, as T increases. Consequently, the optimal liquidity response is smaller as T increases.

Additionally, solvency-driven shocks harm the bank's expected profit overall. As a consequence, the bank's choice of solvency, which is higher the larger the bank's upside payoff, may decreases as T increases. For the same reason, the optimal capital response may also be decreasing in T. In order to see why, consider a small value of liquidity. When liquidity is small, illiquidity risk is large regardless of the choice of solvency. Consequently, the effect of solvency on illiquidity risk will also be small. In this case, the negative effect of solvency-driven liquidity shocks on the bank's profits will dominate, so that the bank's choice of solvency will be smaller as T increases. Nonetheless, as liquidity approaches the bank's profit maximizing value of liquidity (and hence its expected profits increase), the negative effect on profits is dominated by the positive effect of solvency on illiquidity risk. Consequently, for liquidity values around the bank's profit maximizing one, the choice of solvency increase as T increases.

Our baseline model can be interpreted as the limiting model as T tends to zero. While the optimal liquidity and capital requirements quantitatively change as T varies, the qualitative results of both models are analogous.

2.7. Conclusion

This paper addresses optimal joint regulation of equity capital and liquidity from a microprudential perspective. We find that liquidity and capital should be jointly regulated, for the level of either regulatory tools affects the effectiveness of the other regulatory tool. More particularly, we find that liquidity complements capital when liquidity is low, because raising liquidity increases the likelihood of surviving a liquidity crisis and therefore the return of capital in reducing insolvency risk. Capital does also complement liquidity when capital is low, because raising capital reduces the negative effect of liquidity on solvency, so that the return of liquidity increases. In the region where this occurs, capital and liquidity are complementary regulatory tools. On the contrary, when liquidity is high, further liquidity raises lead to a diminished return of capital, because a high liquidity value reduces the loss inflicted on the deposit insurance scheme in case of a bank failure and therefore the social value of reducing insolvency risk. Moreover, when the level of capital is high, increasing capital further offsets the return of liquidity. The reason is that when capital is high, insolvency risk is low and therefore the opportunity cost of liquidity–in terms of foregone opportunities to invest in loans–is high as well. In this region, capital and liquidity offset each other as regulatory tools.

Whether optimal capital and liquidity requirements fall in the region where capital and liquidity are complementary tools or offset each other depends on the cost of capital and on the opportunity cost of liquidity. When the cost of capital and the return on loans–which determines the opportunity cost of liquidity–are large, capital and liquidity requirements are low. Hence high cost of capital and liquidity make this regulatory tools complementary. Any exogenous decrease in the price of one of the regulatory tools should therefore lead to an elevation of *both* requirements.

The results of this paper have an important regulatory implication. Although capital and liquidity requirements should be designed to cope with issues of different nature, namely solvency and illiquidity risk, and even if these are originated independently, regulation should take into account the feedback loops between these two regulatory instruments. This fact suggests that capital and liquidity committees should work together and take into account the cross effects of either regulatory tool on the effectiveness of the other.

Chapter 3

Liquid assets quality and bank's illiquidity risk

3.1. Introduction

Liquidity had played a relatively small if not insignificant role in the regulations approved by the Basel committee prior to the 2007-2008 financial crisis. However, after this crisis the Basel Committee introduced several liquidity-related regulations, and in particular, the establishment of the Liquidity Coverage Ratio (LCR): a requirement of banks to hold high-quality liquid assets (HQLA) sufficient to cover "their liquidity needs for a 30 calendar day liquidity stress scenario".[1] The establishment of the LCR reflects the formal regulation associated with the first of the seventeen "Sound Principles" for liquidity risk management established by the Committee in september 2008.[2]

The definition of high-quality liquid assets includes cash, government bonds and other liquid securities. We will use sovereign debt as a running example of a high-quality liquid asset. Indeed,

[1]Paragraphs 1 and 2 of the "Basel III: The Liquidity Coverage Ratio and liquidity risk monitoring tools", Basel Committee on Banking Supervision , January 2013

[2] "Principles for Sound Liquidity Risk Management and Supervision", Basel Committee on Banking Supervision, September 2008

regulators consider investment in sovereign debt as both liquid and risk free, in the sense that banks do not need to set capital against sovereign debt (Lenarčič et al., 2016). Our choice of sovereign debt as representative of a high-quality liquid asset is motivated by the performance of such assets during the period 2007-2013 that illustrates how the liquidity characteristics of an asset can change over time. In particular, during the said period banks increased their exposure to peripheral sovereign debt (Greece, Ireland, Portugal, Spain, and Italy), (Acharya and Steffen, 2015), and as the market conditions of those assets changed, the interplay of those assets and banks' balance sheets altered and generated a situation where governments became dependent on bank financing, while banks solvency and liquidity depended on the governments' good performance (Wallace, 2016). The importance of this interplay is reflected in the role it played in amplifying the magnitude of the European sovereign debt and financial crisis. Mink and De Haan (2013) state that the increment in sovereign risk weakened the banks' balance sheets and reduced the value of public bonds. According to the Governor of the Bank of Greece Mr Yannis Stournaras

"The Greek banking system suffered the consequences of successive sovereign downgrades, followed by bank downgrades that forced Greek banks out of the global financial markets." [3]

That the sovereign debt downgrades had a fundamental effect on bank solvency is well-established. However, our interest is on its liquidity dimension, as these assets also affected the banks' position through their role as liquid assets. After Greek debt were cataloged as "junk" , the ECB revoked the use of Greek bonds as collateral in 2015 (Blackstone, 2015), any effect that would have been magnified had the LCR been in place at the time. The effect was that Greek banks were driven to borrow from the Bank of Greece through more expensive emergency loans, increasing the already heavy pressure on their solvency.

Given the recent interest on sovereign debt and bank stability, we consider that it is important to improve our understanding on these matters. Within this context, the purpose of this paper is

[3]Speech at the Standing Committee on Economic Affairs of the Hellenic Parliament, Athens, 11 July 2016.

to dig deeper into the consequences of holding potentially risky liquid assets (such as sovereign bonds) on the bank's illiquidity risk. A better understanding of this question will allow the regulators to improve the design of their regulatory policies.

Several papers have analyzed the role of liquid assets on bank's illiquidity risk focusing on asymmetric information problems. There is a large literature, which has a significant impact on policy regulation , that provides "dissimilar" results on the relationship between liquid assets and financial stability.

On the one hand, liquid assets may increase financial instability. Myers and Rajan (1998) state that a high fraction of liquidity asset may harm the ability of the bank to commit a specific policy. So, under some circumstances, a high level of liquidity may harm the capability of the bank to attract external funds. Similarly, Morgan (2002) also points out that liquidity may induce banks to engage in trading activities. These activities allow the bank to assume riskier positions because they are harder to monitor. Wagner (2007) develops a model where credit derivative instruments allow the bank to improve the liquidity of its assets. Since this reduces the cost of liquidity shocks in terms of asset liquidation, then the bank has incentives to assume a risker profile harming financing stability. Malherbe (2014) shows that liquidity hoarding may have negative externalities on the market by exacerbating adverse selection problems resulting in a potential freeze on liquidity markets. Finally it is worth mentioning the ideas formalized in Diamond and Rajan (2011). Whenever healthy (i.e. liquid) banks expect future fire sales, they will hold back lending, since the return for those liquid assets goes up during distress. While troubled banks have incentives to keep illiquid assets even when selling part of them could save the bank. This is because selling the asset today, sacrifices potential revenues if the bank recovers. These bank behaviors coupled together may lead to situations in which the liquidity market freezes.

On the other hand, there is a huge literature that points out the goodness of liquid assets on financial stability. Calomiris and Kahn (1991) argue that short-term debt acts as a liquidation threat for bank managers, and therefore depositors are able to induce discipline to bankers. The authors also point out that liquidity reserves are useful to avoid undesirable liquidation of assets by uninformed depositors, who withdrawal their funds in "good" states of the world. Allen and Gale (2000) shows that financial contagion depends on the extent of completeness of the markets, and liquidity reserves may be an effective method to help reach financial stability. Rochet (2008) states that the use of liquidity requirements can be implemented not only to reduce the probability of an individual bank failure but also the intervention of the Central Bank during crisis. Farhi et al. (2009b) consider unobservable trades on private markets under a Diamond-Dybvig set up. The authors show that by imposing liquidity requirements on intermediaries the regulator is able to manage aggregate liquidity. As a consequence, by reducing the market interest rate a regulator may diminish the incentives of late consumers' deviation, thus improving the allocation reached thought the market. Perotti and Suarez (2011) analyze the effectiveness of a Pigovian tax on short-term funding and quantity based tools to control the externalities that arise from bank's short term funding. They find that if banks diverge from their capacity to lend profitably then Pigovian tax is the optimal tool. While if the gambling incentives among banks are very dissimilar then quantity based tools are more effective to control risk shifting incentives. Using the heterogeneous implementation of liquidity regulatory standards in the UK during 2010, Banerjee and Mio (2014) empirically analyze the effect of requirements on liquid assets (similar to the LCR). They find that banks which did not qualify for the regulatory waiver (i.e. banks under this new regulation), had no adverse effect on the total lending to the non-financial sector, nor on the size of their balance sheets. They only changed the composition of their balance sheet. Requirements implied a reduction of short term loans to other banks, while increasing HQLA (in the form of Central Bank's liabilities). Additionally the authors find that requirements reduced banks' dependence on wholesale funding while increasing their reliance on non-financial deposits. A recent contribution Calomiris et al. (2015) highlights the fact that cash is both verifiable and riskless, which encourages banks to reduce risk taking and thus ameliorates the conflict of interest with outside investors. As a result, they show that cash reserves do not only mitigate exogenous but also endogenous liquidity risk by reducing the bank risk taking incentives, which increases the availability of external financing.

Our paper is most closely related to the literature that highlights the goodness of liquidity reserves but remarks the negative side of cash reserves. Unlike Farhi and Tirole (2012), who state that banks may have excessive incentives to hoard toxic liquid assets harming financial stability, our analysis points out the role of risky liquid assets in improving financial stability by reducing the opportunity cost of liquid reserves.

We present a model in which there is a bank protected by limited liability and funded with insured deposits. Limited liability and insured deposits introduce a market failure because the bank does not internalize the cost of a failure, which leads bank to take a level of risk higher than the social optimum. [4]

The bank is subject to early deposit withdrawal shocks 'a la Diamond and Dybvig (1983) with aggregate uncertainty. If the bank does not have sufficient liquid funds to meet early withdrawals it is liquidated, which generates illiquidity risk. So as to avoid premature liquidation from this withdrawal shock, the bank can hoard liquid reserves. These liquid reserves have an opportunity cost for the bank because augmenting the liquidity of its portfolio implies a reduction of the fraction invested in more profitable assets.

We assume that the bank portfolio of liquid reserves is composed of cash and a risky sovereign bond. Cash dominates the sovereign bond in the sense that the former is the safest asset, while the sovereign bond is preferred to cash because it has a positive expected return. Since there is a chance that the sovereign bond becomes worthless, we say that the higher the fraction of cash, the higher the quality of liquid reserves. Accordingly, the bank can choose any degree of diversification between these assets determining the risk and return of liquid reserves.

Under this set up, we identify the mechanisms that drive the bank's choice not only of the fraction of its liquid asset but also the quality of these assets. We show that although cash is the most liquid and safest asset it does not mean that by holding cash is the optimal way of reducing illiquidity risk.

[4] Since deposit are insured, then the deposit rate is independent of the bank's risk, which induces moral hazard. See e.g. Furlong and Keeley (1989)

We show that even though the bank is protected by limited liability, it chooses the level of liquid reserves quality that minimizes its liquidity risk. The diversification of liquid assets allows the bank to alleviate the opportunity cost of its liquid holdings, which induces it to hold a higher fraction of liquid assets in its portfolio. This increase in liquid reserves can more than offset the intrinsic risk of the sovereign bond. Hence, we find that reducing the quality of liquid reserves leads the bank to assume lower liquidity risk.

Finally, we also show that the return on bonds plays an important role in determining bank's illiquidity risk. A low return on bonds leads the bank to set a small degree of diversification. Most of the liquid reserves are held in the form of cash, which raises the opportunity cost of liquid reserves. As a result, the illiquidity risk increases as the return on bonds decreases.

The rest of the paper is organized as follows. In the next section, we describe the general setting and the time line of the model. Section 3 presents the bank's maximization problem. Section 4 analyses the factors that drive the bank's decision to invest in either cash or a risky bond. In Section 5, we study the impact of liquid asset diversification on bank liquidity risk. We conclude in Section 6. Proofs are in the Appendix B.

3.2. Choosing Liquid Assets

We start with a benchmark version of the model, which includes several simplifying assumptions in order to identify the factors that drive the bank's choice of liquid assets. Consider a three period economy, where a bank gathers one unit of cash from depositors at $t = 0$ with the promise of payback D at $t = 2$. We model a bank's "stress scenario" by having a fraction β of depositors who demand their funds at the intermediate period $t = 1$. We assume that β is random on $[0, 1]$, with cumulative distribution $F(\beta)$ independent of other model parameters. We will use the standard uniform distribution as an example of F.

The bank's liquidity decision has two dimensions: the bank chooses the fraction of assets to set aside for liquidity purposes, α, and also chooses which assets to use for this purposes (which

HQLA to use). In addition investing in liquid assets, the bank invests the remainder, $1 - \alpha$, in illiquid loans. Illiquid loans provide a random return $\tilde{R}$ at $t = 2$, which for simplicity we will assume is given by: $\tilde{R} = R$ with probability P_R and zero otherwise. The illiquidity of these assets is simply modeled by setting the loans' liquidation value at t=1 equal to zero.

The bank has access to liquid assets indexed by i, with gross return $\tilde{\rho}_i$ at $t = 2$, which can be liquidated at $t = 1$ with a return M_i. For simplicity we consider only two assets: cash, l, with $M_l = 1$ and $\tilde{\rho}_l = 1$, and a risky asset denoted by A, where A provides a constant return equal to $\tilde{\rho}_A = (1 + r)$, with $r > 0$, at $t = 2$ and a liquid value of $M_A = 1$, both with probability $1 - \varepsilon$. However, with probability ε, asset A becomes completely useless at date $t = 1$ and it is worthless both as a liquid asset at $t = 1$ and as a source of revenue at $t = 2$, $M_A = 0$ and $\tilde{\rho}_A = 0$. We also assume that $\tilde{R}$ and $\tilde{\rho}_i$ are independent.

Let θ denote the fraction of liquid assets invested in cash so that $M = \theta M_l + (1-\theta)M_A$ denote the total value of HQLA at $t = 1$ and let ρ denote the return of the bank's assets after meeting early withdrawal requests.[5] Thus, the bank invests a fraction α in HQLA with a liquidity value of M and a return ρ. The bank can only meet demands from depositors who want to withdraw early using HQLA. If A is not valueless, then the bank can meet the demand for early withdrawals (β) if $\beta \leq \alpha$. If the banks does not hold enough of these assets the bank is liquidated. If the bank is liquidated, bank owners are protected by limited liability. If the bank is not liquidated, then at $t = 2$ the bank revenue is the sum of its loans, $(1 - \alpha)\tilde{R}$, and that of the remaining liquid assets $(\alpha - \beta)\rho$. The model variables are depicted in Figure 3.1 and the timing of the model is the following:

[5] It is immediate that the bank would want to meet liquidity demands with cash first, and use asset A only if $\beta > \alpha\theta$. Thus, $\rho(\alpha - \beta) = (\theta\alpha - \beta) + (1 - \theta)\alpha(1 + r)$ if $\beta \leq \theta\alpha$, and $\rho = (1 + r)$ if $\beta \in (\theta\alpha, \alpha)$.

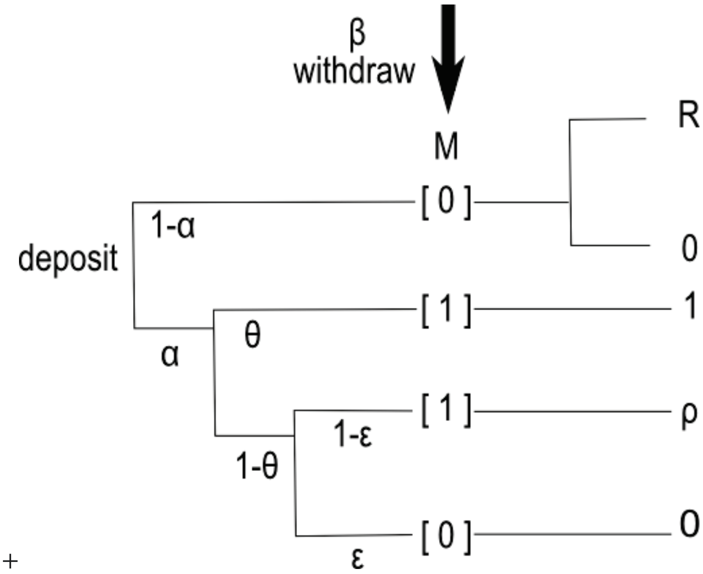

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Figure 3.1: The structure of the model

Time line of the model

$t = 0$	$t = 1$	$t = 2$
Banks decide both its asset structure α and its investment in cash or risky asset	Asset A is valueless with probability ε. Liquidity shock = β. The bank pays early withdrawals or is liquidated.	With probability $1 - \varepsilon$, A returns (1+r) The bank pays back debt or fails

3.3. The Bank's Optimal Asset Choice of HQLA

The bank will generate profits only if it is not liquidated or in bankruptcy, an event which we denote by $\mathcal{S}$, where $\mathcal{S} = \{\beta \leq \alpha M, (1-\alpha)\tilde{R} + (\alpha - \beta)\rho \geq (1-\beta)D\}$. Let $P_L(\theta)$ denote the probability that the bank is liquidated, so that $P_L(\theta) = \Pr\{\beta > \alpha M\}$. Thus, the bank's expected profit, $\Pi(\alpha, \theta)$ is:

$$\begin{aligned}\Pi(\alpha,\theta) &= \mathbb{E}\left[(1-\alpha)\tilde{R} + (\alpha-\beta)\rho - (1-\beta)D\right] \\ &= \Pr\{\mathcal{S}\}\mathbb{E}\left[\left((1-\alpha)\tilde{R} + (\alpha-\beta)\rho\right) - (1-\beta)D \,\middle|\, \mathcal{S}\right] \\ &= \Pr\{\mathcal{S}\}\mathbb{E}\left[\left((1-\beta)(\tilde{R}-D) - (\alpha-\beta)(\tilde{R}-\rho)\right) \,\middle|\, \mathcal{S}\right] \end{aligned} \tag{3.1}$$

The effects of changing α are

- An increase in the probability of positive profits

$$\frac{\partial}{\partial\alpha}\left(\Pr\{\mathcal{S}\}\right)\mathbb{E}\left[\left((1-\beta)(\tilde{R}-D) - (\alpha-\beta)(\tilde{R}-\rho)\right) \,\middle|\, \mathcal{S}\right]$$

- A change (reduction)in the support of the conditional expectation

$$\Pr\{\mathcal{S}\}\frac{\partial\left(\mathbb{E}\left[\left((1-\beta)(\tilde{R}-D) - (\alpha-\beta)(\tilde{R}-\rho)\right) \,\middle|\, \mathcal{S}\right]\right)}{\partial\alpha}.$$

which has two components: a decrease in revenue from bank's loans plus a change in the support of $\mathcal{S}$.

If we now consider that $\tilde{R}$ can only take two values and when it is zero the bank's profits are zero, then, when $\tilde{R} = R$ we have that the object inside the expectation in Equation 3.1 is:

$$K(\beta) = (1-\beta)(R-D) - (\alpha-\beta)(R-\rho) = (1-\alpha)R - D + \beta(D-\rho) + \alpha\rho.$$

where β is random and ranges over the values $\beta \in [0, \alpha M]$. As $\min(\rho) = 1$, conditional on $\tilde{R} = R$, the lowest possible value that K can take is at $\beta = 0$ and it is positive for sufficiently low values of α:

$$K_{min} = (1-\alpha)R - D + \beta(D-1) + \alpha = R - D - \alpha(R-1)$$

$$K_{min} > 0 \iff \alpha < \frac{R-D}{R-1}. \tag{3.2}$$

By assuming Condition 3.2 holds, we ensure that K is positive for all α. Nevertheless, one would expect under normal circumstances that for a bank α (the proportion invested in liquid assets) be small, as debt financing is the bank's main line of business, so assuming Condition 3.2 holds is reasonable. Also, the condition $K_{min} > 0$ is convenient, because it ensures that the event $\mathcal{S}$ is equal to the event $\beta \leq \alpha M$, so that $P_R\{S\} = P_R F(\alpha M)$. This simplifies the analysis as we can write the bank's charter value as:

$$\Pi_B(\alpha, \theta) = \mathbb{E}_A\Big[P_R F(\alpha M)\,[(1-\alpha)R - D + \mathbb{E}[\beta(D-\rho) + \alpha\rho|\beta \leq \alpha M]]\Big],$$

where $\mathbb{E}_A$ is the expectation of A. Recall that with probability $1-\varepsilon$ the asset A is as liquid as cash and hence $M = 1$, otherwise $M = \theta$. According to this, we have that

$$\begin{aligned}\Pi_B(\alpha, \theta) &= P_R\Big[(1-\varepsilon)F(\alpha)\,[(1-\alpha)R - D + \mathbb{E}[\beta(D-\rho) + \alpha\rho|\beta \leq \alpha]\\ &+ \varepsilon F(\alpha\theta)\,\Big[(1-\alpha)R - D + \mathbb{E}[\beta(D-\rho) + \alpha\rho|\beta \leq \alpha\theta]\Big]\end{aligned}$$

The choice of θ affects both M and ρ. It increases M as it increases the fraction of riskless liqud assets (more cash, less A–M goes from $1-\epsilon$ to 1) but it decreases ρ (lower expected return when the firm is liquid–ρ goes from $\theta + (1-\theta)\rho_A$ to ρ_A as β goes from zero to $\alpha\theta$).

Considering the optimal choice of α we look at the FOC:

$$\frac{d\Pi_B(\alpha)}{d\alpha} = P_R\mathbb{E}_A\left[\left(\frac{dF(\alpha M)}{d\alpha}\mathbb{E}[K(\beta)|\beta < \alpha M] + F(\alpha M)\frac{d\mathbb{E}[K(\beta)|\beta < \alpha M]}{d\alpha}\right)\right] = 0,$$

which can be written as

$$\begin{aligned}\frac{d\Pi_B(\alpha)}{d\alpha} =& P_R\Bigg[(1-\varepsilon)\left(\frac{dF(\alpha)}{d\alpha}\mathbb{E}[K(\beta)|\beta<\alpha]+F(\alpha)\frac{d\mathbb{E}[K(\beta)|\beta<\alpha]}{d\alpha}\right)+\\ &\varepsilon\left(\frac{dF(\alpha\theta)}{d\alpha}\mathbb{E}[K(\beta)|\beta<\alpha\theta]+F(\alpha\theta)\frac{d\mathbb{E}[K(\beta)|\beta<\alpha\theta]}{d\alpha}\right)\Bigg]=0.\end{aligned}$$

Using the fact that $\frac{dF(\alpha M)}{d\alpha} = f(\alpha M)M$ and

$$\frac{\partial}{\partial\alpha}\mathbb{E}[K(\beta)|\beta\le\alpha] \;=\; \frac{f(\alpha M)}{F(\alpha M)}MK(\alpha M) - M\frac{f(\alpha M)}{F(\alpha M)}\mathbb{E}[K(\beta)|\beta\le\alpha M],$$

the FOC with respect to α becomes

$$\begin{aligned}&\underbrace{\varepsilon[\theta g(\alpha\theta)\left(R(1-\alpha)+\alpha\theta-1\right)]+(1-\varepsilon)\Big[\frac{F(\alpha)g(\alpha)\left((R-1)(1-\alpha)\right)}{F(\alpha\theta)}\Big]}_{\text{Marginal value of liquid assets}}=\\ &\qquad\qquad\underbrace{[R-\theta]+(1-\varepsilon)\Big[\frac{F(\alpha)}{F(\alpha\theta)}-1\Big](R-1-r)}_{\text{Opportunity cost of liquid assets}},\end{aligned}\tag{3.3}$$

where $g(\cdot)=\frac{f(\cdot)}{F(\cdot)}$.

The intuition behind this FOC is as follows. As we have mentioned above, α measures the fraction of liquid assets in the bank's portfolio, which is composed of a fraction θ of cash and the remainder $1-\theta$ in risky bonds.

The left hand side (LHS) of the previous equation measures the marginal value of liquid assets. This marginal value has two components and stands for the increase in the bank's payoff because of a reduction in the bank's illiquidity risk. Note that with probability ε only cash is useful to deal with early withdrawals. While with probability $1-\varepsilon$ the amount of liquid assets to meet the demand of early withdrawals is equal to α. Hence, the first term represents the marginal value of liquid assets when A is worthless, while the second one is the marginal value of α when A is a valuable asset.

Finally, the right hand side (RHS) measures the expected opportunity cost of increasing liquid assets in terms of foregone loan opportunities. As A returns r with probability $1-\varepsilon$, holding risky liquid asset diminishes the opportunity cost of α. Note that the higher r, the lower the second term of the RHS, which encourages hoarding liquid assets. We now turn into the analysis of the optimal fraction of cash on the total liquid assets. Differentiating the bank's charter value with respect to θ, we get the following FOC

$$\varepsilon[\alpha g(\alpha\theta)\left(R(1-\alpha)+\alpha\theta-1\right)] = (1-\varepsilon)r. \tag{3.4}$$

Recall that the choice of θ determines the quality of the liquid assets. This quality can be measured through $\mathbb{E}_{\mathbb{A}}(M)$ because the higher $\mathbb{E}_{\mathbb{A}}(M)$, the higher the expected value of liquid assets αM at $t=1$. Note that if liquid reserves are composed of cash, then $\mathbb{E}_{\mathbb{A}}(M)=1$, and thus α reaches its highest quality level. However, if the bank invests a fraction $1-\theta$ in the risky liquid asset, we have that $\mathbb{E}_{\mathbb{A}}(M) = \theta + (1-\theta)(1-\varepsilon) < 1$. That is, $\alpha\mathbb{E}_{\mathbb{A}}(M) < \alpha$ during the "early withdrawal" phase. The left hand side of Condition 3.4 represents the marginal benefit of improving the liquid assets' quality. This benefit comes from the fact that as the fraction of cash increases, the bank's illiquidity risk in the state in which A is worthless decreases.

However, improving the quality of liquid assets has an opportunity cost in terms of the foregone interest rate r, which is represented by the right hand side of Condition 3.4. As a result, the bank trades-off these two effects in order to set the quality of its liquid assets.

3.4. No diversification case: $\theta \in \{0, 1\}$

In order to comprehend the mechanisms that drive the bank's decision about what liquid asset to invest in, we solve a simplified version of the model by assuming that $\theta \in \{0, 1\}$. In other words, we assume that there is no liquid asset diversification. Hence, the bank invests either in cash or in the risky liquid asset but not in both simultaneously.

According to this, M may take two values $\{M_l, M_A\}$. If the bank invests in cash, then $\mathbb{E}_{\mathbb{A}}(M) = 1$ and therefore the fraction of liquid assets α has maximal quality. However, if the bank invests in the risky liquid asset, we have that $\mathbb{E}_{\mathbb{A}}(M) = 1 - \varepsilon < 1$, which implies that α has minimal quality.

Consider first the case in which the bank decides to keep its liquid reserves in the form of cash. In this case, the bank goes bankrupt at $t = 1$ only if the fraction of liquid assets α is lower than the amount of early withdrawals β. According to this, the bank's problem consists of choosing a fraction of cash α so as to solve

$$\max_{\alpha \in [0,1]} \quad \Pi_B\left(\alpha; 1\right) ,$$

where

$$\Pi_B(\alpha, 1) \quad \equiv \quad \underbrace{\underbrace{P_L F(\alpha)}_{\text{Probability of success}} \quad \underbrace{\Big[(1-\alpha)R + \mathbb{E}[(1-\beta)D + \alpha - \beta | \beta \leq \alpha]\Big]}_{\text{Expected bank's upside payoff}}}_{\text{Bank's charter value with maximal liquid asset quality}}, \tag{3.5}$$

represents bank's charter value under cash. Notice that $\Pi_B(\alpha, 1)$ is the product between the probability of success and the expected bank's upside payoff, which is the expected value of the bank's pay-off conditional on surviving the "early withdrawal" phase.

The first order condition for an interior bank's profit-maximizing level of liquid assets α is given by:

$$\frac{\partial \Pi_B(\alpha_l, 1)}{\partial \alpha} \quad = \quad \underbrace{g(\alpha_l)\Big[(R-D)(1-\alpha_l)\Big]}_{\text{Marginal value of surviving early withdrawals}} \quad - \quad \underbrace{(R-1)}_{\text{Opportunity cost of liquid asset}} \quad = 0, \tag{3.6}$$

The first term of Condition 3.6 stands for the marginal value of surviving the "early withdrawal" phase, where $g(\alpha) = \frac{f(\alpha)}{F(\alpha)}$ is the reverse hazard rate of the distribution of early withdrawals evaluated at α. Since β follows an uniform distribution, then $g(\cdot)$ is strictly decreasing in α. The interpretation of the marginal value of surviving early withdrawal is clear. This term represents the increment in the bank's charter value by reducing the liquidity risk.

However, hoarding liquid reserves in the form of cash has an opportunity cost of $R - 1$ for the bank, which reflects the foregone loan opportunities.

Based on this, Condition 3.6 states that the bank faces a risk-return trade-off. The higher the chance of surviving the "early withdrawal" phase, the lower the expected bank's upside pay-off. Hence, the optimal level of cash α_l must equalize the marginal value of surviving early withdraws to its opportunity cost $R - 1$.

Using the fact that $g(\alpha) = \frac{1}{\alpha}$ for the uniform distribution, Condition 3.6 can be rewritten as follows

$$\frac{\partial \Pi_B(\alpha_l, 1)}{\partial \alpha} = \underbrace{\frac{(R-D)(1-\alpha_l)}{\alpha_l}}_{\text{Marginal value of surviving early withdrawals}} -(R-1) = 0,$$

so that the bank's profit-maximizing level of liquid assets is given by

$$\alpha_l = \frac{R-D}{R-D+\underbrace{R-1}_{\text{Opportunity cost}}}.$$

We now turn into the analysis of the case in which $\theta = 0$. That is, the bank only invests in the risky liquid asset which, with probability $1 - \varepsilon$, it is as liquid as cash is and also provides a gross return $\tilde{\rho}_A = 1 + r$. Based on this, the bank's problem consists of determining a fraction of risky liquid asset α so as to solve

$$\max_{\alpha \in [0,1]} \quad \Pi_B(\alpha; 0) \ ,$$

where

$$\Pi_B(\alpha,0) = \underbrace{\underbrace{(1-\varepsilon)P_R F(\alpha)}_{\text{Probability of success}} \underbrace{\Big[(1-\alpha)R + \mathbb{E}[(1-\beta)D + (\alpha-\beta)(1+r)|\beta \leq \alpha]\Big]}_{\text{Expected bank's upside payoff}}}_{\text{Bank's charter value with minimal liquid asset}}. \quad (3.7)$$

Note that, for a given value of α, $\Pi_B(\alpha,0)$ differs from $\Pi_B(\alpha,1)$ for two reasons. First, since A has an intrinsic risk ε the probability of surviving is lower than in the previous case. Second, if successful, A provides a return of r, which implies that the expected bank's upside pay-off is higher than in the case of $\theta = 1$.

The first order condition with respect to α can also be written in terms of the inverse hazard ratio as follows

$$\frac{\partial \Pi_B(\alpha_A,0)}{\partial \alpha} = \underbrace{g(\alpha_A)\Big[(R-D)(1-\alpha_A)\Big]}_{\text{Marginal value of surviving early withdrawals}} - \underbrace{(R-1-r)}_{\text{Opportunity cost of risky liquid asset}} = 0. \quad (3.8)$$

First of all, notice that Condition 3.8 does not depend on ε because if A is worthless the bank goes bankrupt. Thus, the bank trades off the marginal value of surviving early withdrawals against the opportunity cost of the risky asset only when A is a valuable asset. As a result, the marginal value of surviving early withdrawals is the same as in the previous case. However, the return of A reduces the opportunity cost of holding liquid assets, which implies that the bank's profit-maximizing level of liquid assets is given by

$$\alpha_A = \frac{R-D}{R-D+\underbrace{R-1-r}_{\text{Opportunity cost}}}.$$

The fact that A has a lower opportunity cost than cash leads the bank to choose a higher fraction of liquid assets in its portfolio than when $\theta = 1$ (cash only). This result is formalized in the following remark.

Lemma 3.1. *The bank invests a higher fraction of liquid assets α when it holds its liquid reserves in the form of risky liquid assets instead of cash. That is, $\alpha_A > \alpha_l$.*

Proof. See Appendix B $\square$

The decision of the bank on what asset to invest in will depend on the bank's charter value in each case. That is, whether

$$\Pi_B(\alpha_l, 1) \gtreqless \Pi_B(\alpha_A, 0)$$

$$\Leftrightarrow$$

$$F(\alpha_l)\Big[(1{-}\alpha_l)R{+}\mathbb{E}[(1{-}\beta)D{+}\alpha_l{-}\beta|\beta \leq \alpha_l]\Big] \gtreqless (1{-}\varepsilon)F(\alpha_A)\Big[(1{-}\alpha_A)R{+}\mathbb{E}[(1{-}\beta)D{+}(\alpha_A{-}\beta)(1{+}r)|\beta \leq \alpha_A$$

Since α_A is independent of ε, it is possible to find a threshold for the intrinsic risk of A, denoted by $\overline{\varepsilon}$, such that

$$\Pi_B(\alpha_l, 0) = \Pi_B(\alpha_A, 1).$$

In this way, the threshold $\overline{\varepsilon}$ of the intrinsic risk of A determines whether the bank invests either in cash or in the asset A, is given by the following condition

$$\overline{\varepsilon} = 1 - \frac{\Pi_B(\alpha_l, 0)}{F(\alpha_A)\Big[(1-\alpha_A)R + \mathbb{E}[(1-\beta)D + (\alpha_A - \beta)(1+r)|\beta \leq \alpha_A]\Big]}. \tag{3.9}$$

Hence, if $\varepsilon = \overline{\varepsilon}$ the bank is indifferent between investing in cash and the risky liquid assets, while the bank will strictly prefer the risky liquid asset to cash if and only if $\varepsilon < \overline{\varepsilon}$. Otherwise, the bank keeps its liquid reserves in cash. [6]

[6] Observe that $\overline{\varepsilon}$ represents the highest intrinsic risk of the asset A that the bank is willing to accept because whether $\varepsilon > \overline{\varepsilon}$ the bank holds its liquid reserves in the form of cash. Hence, if $\varepsilon = \overline{\varepsilon}$ the bank reaches the highest level of liquidity risk in case of investing in A.

An interesting question that arises from this problem is how the bank's decision of investing in these assets affects its level of liquidity risk. Note that the result presented in Lemma 3.1 implies that the answer to this question is not straightforward.

In order to shed light on this question, we compare the bank's liquidity risk under both alternative investments for $\varepsilon = \overline{\varepsilon}$.

On the one hand, if the bank keeps its liquidity reserves in the form of cash, it is straightforward to see that the likelihood of surviving the "early withdrawal" phase is $F(\alpha_l)$. Since β follows an uniform distribution, we have that

$$F(\alpha_l) = \alpha_l = \frac{R - D}{R - D + R - 1}.$$

On the other hand, when bank's liquid assets are held in the form of risky assets, the probability of surviving the "early withdrawal" phase has two components.

The bank survives the liquidity phase only if both A is a valuable asset and the fraction of early withdrawals is lower than α_A. According with these facts, the probability of surviving the "early withdrawal" phase can be written as

$$(1 - \overline{\varepsilon})F(\alpha_A) = (1 - \overline{\varepsilon})\frac{R - D}{R - D + R - 1 - r}.$$

We would expect that keeping liquid reserves in assets with the highest extent of quality involves the lowest liquidity risk. However, this result is not is not necessarily true because investing in cash leads to a lower liquidity risk than investing in A only if

$$F(\alpha_l) > (1 - \overline{\varepsilon})F(\alpha_A).$$

We formalize this result in the following proposition and then provide intuition for it.

Proposition 3.1. *If the bank invests in the risky liquid asset, then the bank liquidity risk is equal or lower than if the bank had kept its liquid reserves in the form of cash; i.e. if $\varepsilon < \overline{\varepsilon}$, then $F(\alpha_l) < (1-\varepsilon)F(\alpha_A)$. While if $\varepsilon = \overline{\varepsilon}$ the bank's liquidity risk is the same regardless whether the bank invests either in cash or in the Asset A.*

Proof. See Appendix B □

Proposition 3.1 presents a counter-intuitive result because one tend to think that the bank's liquidity risk should be mitigated by keeping liquid reserves in the form of cash, which is the most liquid and safe asset. The intuition behind this result is as follows. Investing in the risky liquid asset has a dual effect on the bank's liquidity risk. On one hand, A has an intrinsic risk given by ε, which increases the financial instability of the bank. Since there is no diversification among liquid assets, then the bank goes bankrupt when A becomes worthless.

On the other hand, reducing the quality of the liquid reserves from $\mathbb{E}_{\mathbb{A}}(M) = 1$ to $\mathbb{E}_{\mathbb{A}}(M) = 1 - \varepsilon < 1$ entails a lower opportunity cost for the bank.

Hence, this fact leads the bank to choose a portfolio with a higher fraction of liquid assets than in the case of cash. That is $\alpha_A > \alpha_l$. As a result, if A is a valuable asset, then the bank may face a lower liquidity risk than if it had invested in cash.

Hence, when A is a valuable asset, the reduction in the bank's liquidity risk exactly compensates its intrinsic risk. Recall, that $\overline{\varepsilon}$ is the highest level of the intrinsic risk that the bank is willing to assume when investing in asset A.

Therefore, if $\varepsilon < \overline{\varepsilon}$ the bank prefers investing in A because the lower liquidity risk when A is valuable more than offsets the intrinsic risk of A.

3.5. Diversification case: $\theta \in [0, 1]$

Now, let us analyze how the diversification of the quality of the liquid assets impacts the bank's liquidity risk. As we have seen before, the quality of the bank's liquid assets is determined by the fraction of the total liquid reserves θ invested in cash. Recall that a larger θ increases the expected value of liquid reserves $\alpha\mathbb{E}_{\mathbb{A}}[M]$ during the liquidity phase. Accordingly, the bank that chooses, not only the fraction of liquid asset in the portfolio α, but also its quality by solving the following problem:

$$\max_{\alpha\in[0,1],\theta\in[0,1]} \quad \Pi_B\left(\alpha;\theta\right)\ ,$$

where

$$\begin{aligned}\Pi_B(\alpha,\theta) &= P_R\Bigg\{\Big[(1-\varepsilon)\alpha\left[(1-\alpha)R - D + \mathbb{E}[\beta(D-\rho)+\alpha\rho|\beta\leq\alpha M]\right.\\ &+ \varepsilon\left(\alpha\theta\right)\left[(1-\alpha)R - D + \mathbb{E}[\beta(D-\rho)+\alpha\rho|\beta\leq\alpha\theta]\right]\Bigg\}.\end{aligned}$$

Notice that we have two terms inside brackets, which represent the bank's charter value conditional on loans being successful. The first term stands for the bank's charter value when A is a valuable asset. In that case, the total liquid assets available to meet the demand of early withdrawals is given by α and they have a return of ρ. The second term represents the bank's charter value when the risky liquid asset is worthless. As a result, the bank only has $\alpha\theta$ units of cash to overcome the "early withdrawal" phase. From now on and without loss of generality, we are going to assume that $D = 1$.

From the first order condition with respect to α, we construct the optimal fraction of liquid assets in the bank's portfolio $\alpha(\theta)$ as a function of the quality level θ . This curve maps each quality level to an optimal fraction of liquid assets given by

$$\alpha(\theta) = \frac{(R-1)(1-\varepsilon(1-\theta))}{2R(1-\varepsilon(1-\theta)) + (r-(2+r)\varepsilon)\theta^2 - (2+r)(1-\varepsilon)}. \tag{3.10}$$

Lemma 3.2. *The fraction invested in liquid assets $\alpha(\theta)$ is a decreasing function of θ*

Proof. See Appendix B □

Lemma 3.2 states an interesting result: The larger the quality of the liquid assets, the lower the liquid reserves. From Condition 3.3, this negative relation between α and θ can be explained by two facts. First, we know that an improvement in quality of the liquid reserves is coupled with a higher opportunity cost. Note that the return of liquid assets ρ decreases with θ, which increases the opportunity cost of hoarding liquid assets in term of more profitable investments.

Second, if A is worthless the reversed hazard rate $g(\alpha\theta)$ is a decreasing function of $\alpha\theta$. Hence, we have that as θ increases the marginal value of liquid assets $\alpha\theta$ decreases as well. So, both effects induce a negative relationship between the optimal fraction invested in liquid assets α and the fraction of cash in the portfolio θ.

We now turn to the analysis of the optimal level of θ as a function of α. From the first order condition respect to θ, we obtain that

$$\theta(\alpha) = Min[Max[\frac{[R(1-\alpha)-1]\,\varepsilon}{\alpha((1-\varepsilon)r-2\varepsilon)}, 0], 1]. \tag{3.11}$$

Lemma 3.3. *The optimal quality of liquid assets chosen by the bank $\theta(\alpha)$ is a decreasing function of α.*

Proof. See Appendix B □

The intuition of this result is straightforward. Improving the quality of the liquid reserves only has a payment for the bank if A becomes worthless. Otherwise, the total amount of liquid asset is αM, where $M = 1$ regardless the value of θ. Nevertheless, if A is worthless $M = \theta$ and the total amount of liquid reserves $\alpha\theta$ increases with θ. As we have seen before, the marginal value of liquid assets presents decreasing returns to scale. Hence, the higher α, the higher the amount of liquid assets $\alpha\theta$, and therefore the marginal value of increasing θ decreases. Based on this, we have that $\theta(\alpha)$ is a decreasing function of α.

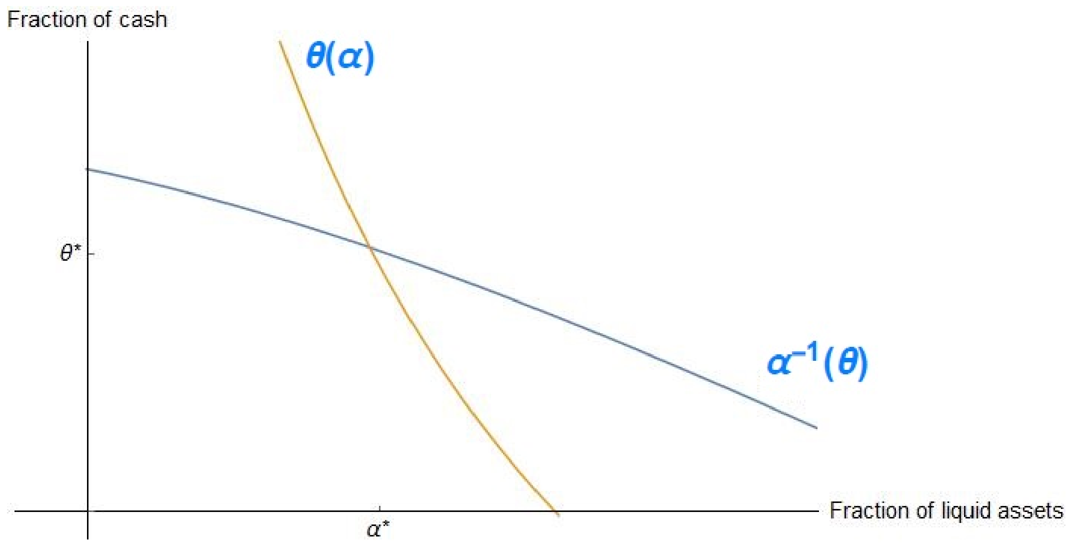


Figure 3.2: Bank's optimal liquid assets

Figure 3.2 depicts the bank's choice of the fraction of liquid asset and its extent of liquidity for $\varepsilon = \bar{\varepsilon}$. The curve $\theta(\alpha)$ represents the optimal proportion of cash on the total liquid assets α. The curve $\alpha^{-1}(\theta)$ is the inverse of $\alpha(\theta)$. Hence, $\alpha^{-1}(\theta)$ maps, for a given value of α, the value of θ for which α is the optimal fraction of liquid assets chosen by the bank. The intersection of these curves (α^*, θ^*) represents the proportion of liquid assets held by the bank and its level of quality, which is given by $\mathbb{E}[M] = \theta^* + (1 - \theta^*)(1 - \bar{\varepsilon})$.

So far, we have analyzed the mechanisms that determine the liquidity of the portfolio chosen by the bank. We now turn into the analysis of the impact of the quality of liquid assets on the bank's illiquidity risk. To do so, we define the function

$$R(\theta, \alpha(\theta)) \equiv R(\theta) = 1 - \underbrace{[(1-\varepsilon)F(\alpha(\theta)) + \varepsilon F(\alpha(\theta) \cdot \theta)]}_{\text{Prob of surviving the "early withdrawal" phase}}, \tag{3.12}$$

which represents the bank's illiquidity risk as a function of θ and the second term of $R(\theta)$ stands for the probability of surviving the "early withdrawal" phase. Note that, with probability $1 - \varepsilon$, A is a valuable asset and hence the probability of surviving the "early withdrawal" phase is $F(\alpha(\theta))$. However, if A is worthless the chance of overcoming the liquidity phase is reduced to $F(\alpha(\theta) \cdot \theta)$. The following proposition characterizes the bank's illiquidity risk as a function of θ.

Proposition 3.2. *For any $\varepsilon \leq \bar{\varepsilon}$, the bank's illiquidity risk $R(\theta)$ is a U-shaped function of θ and reaches its minimum value at $\theta = \hat{\theta}$. Moreover, if $\varepsilon = \bar{\varepsilon}$ we have that $R(1) = R(0)$.*

Proof. See Appendix B □

Proposition 3.2 has an important implication, which is presented in the following corollary. We provide intuition for this result below.

Corollary 3.1. *For any $\varepsilon \leq \bar{\varepsilon}$, a bank increases its resilience to liquidity shocks by reducing the quality of its liquid assets with respect to cash. That is, liquid asset diversification allows the bank to mitigate its illiquidity risk.*

Figure 3.3 helps us to understand the intuition behind Proposition 3.2 and its corollary. This figure depicts two curves when $\varepsilon = \bar{\varepsilon}$. The straight line shows the bank's liquidity risk when $\theta \in \{0, 1\}$, which means that the bank has to choose between the highest and the lowest level of quality for its liquid assets. Moreover, recall from Proposition 3.1 that when $\varepsilon = \bar{\varepsilon}$ the bank's illiquidity risk is the same regardless of whether it invests either in cash or in the risky liquid asset. That is $F(\alpha_l) = (1 - \bar{\varepsilon})F(\alpha_A)$ and hence we have that $R(0) = R(1) = 1 - F(\alpha_A)$.

The U-shaped curve $R(\theta)$ represents the bank's illiquidity risk when the bank is able to choose among different levels of quality of the liquid assets. As we have explained before, the quality of liquid assets is determined by the fraction of the total liquid assets θ invested in cash. The higher θ is, the higher the expected value of the liquid assets $\mathbb{E}_{\mathbb{A}}[M]$ is. However, increasing θ has a direct and an indirect effect on the bank's illiquidity risk.

On the one hand, the direct effect leads to lower illiquidity risk: Increasing θ ameliorates the bank's illiquidity risk by augmenting the chance of surviving the liquidity phase when the risky liquid assets is worthless.

On the other hand, increasing the fraction of cash in the bank's portfolio has an undesirable indirect effect, which induces the bank to assume a higher liquidity risk: as the liquid asset quality increases, the lower the fraction of liquid assets in the portfolio. From Lemma 3.2, we know that liquid reserves $\alpha(\theta)$ decrease with θ because their opportunity cost increases while the marginal value of liquid assets decreases with θ. The fact that the β follows uniform distribution, which is log-concave, ensures that the first effect predominates for small values of θ, while the second one dominates for the high ones.

Let us analyze the curve $R(\theta)$ by starting with $\theta = 0$. In this case, the bank only invests in the risky liquid assets and therefore $R(0) = R(1) = 1 - F(\alpha_A)$. As a consequence, improving the quality of liquid assets has a high return for the bank. If $\theta = 0$ and A is worthless, then the bank goes bankrupt. From Equation 3.4, we can observe that if θ is small, the reversed hazard rate $g(\alpha\theta)$ is relatively high, which implies that increasing θ has a huge and positive impact on the bank's liquidity risk. Hence, the direct effect predominates on the indirect effect and thus $R(\theta)$ decreases with θ.

However, as the fraction of cash increases, not only the bank's illiquidity risk decreases when A becomes worthless but also the reversed hazard rate $g(\alpha\theta)$, which implies that the marginal return to cash decreases as well. Hence, for large values of θ, the indirect effect exceeds the direct effect and thus the bank's illiquidity risk increases with θ.

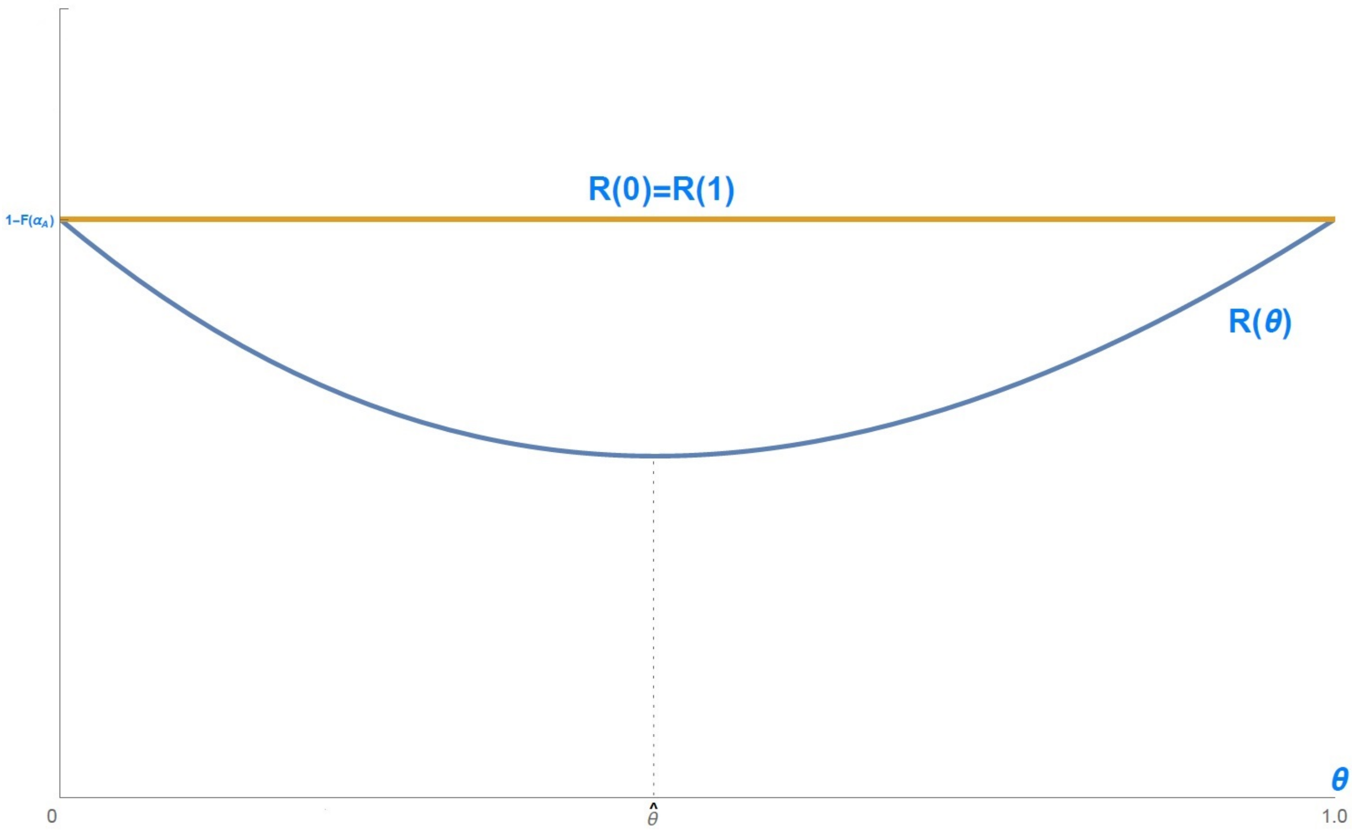


Figure 3.3: The effect of liquid asset diversification on the bank's illiquidity risk

So far, we have seen that if we allow the bank to manage the quality of its liquid assets, then the bank's illiquidity risk is never higher than the case in which the bank only invests in cash. Figure 3.3 illustrates this result. We can observe that if the bank only invests either in cash ($\theta = 1$) or in the risky liquid asset ($\theta = 0$), the level of bank's risk is the same in both cases $R(0) = R(1)$. However, liquid asset diversification helps the bank to mitigate its illiquidity risk by allaying the opportunity cost of keeping liquid reserves. The positive effect of that diversification on the bank's risk is reflected in the fact that $R(\theta) < R(0) = R(1)$ for any $\theta \in (0, 1)$.

From Corollary 3.1, we now know that liquid asset diversification allows the bank to mitigate its liquidity risk. However, the bank may choose a level of diversification different from $\hat{\theta}$ and thus assuming a level of risk higher than $R(\hat{\theta})$. Therefore, the following question arises: How does the bank manage the extent of diversification of its liquid assets Proposition 3.3 helps to clarify that question.

Proposition 3.3. *The bank's optimal diversification level is the one that minimizes its illiquidity risk $R(\theta)$. Formally, we have that $\theta^* = \hat{\theta}$.*

Proof. See Appendix B □

Proposition 3.3 states that even though the bank is protected by limited liability and deposit are insured, it diversifies its liquidity reserves in order to minimizes its illiquidity risk. The intuition behind this result is as follows. As said before, θ has a direct and an indirect effect on the bank illiquidity risk $R(\theta)$. We also know that these effects are equal for $\theta = \hat{\theta}$, which means that the positive impact of increasing the quality of liquid assets exactly offsets the increment in the bank's illiquidity risk by reducing $\alpha(\theta)$. Assume that the bank chooses a level of quality $\theta_B < \hat{\theta}$, which implies that the bank's illiquidity risk is relatively high when A is worthless. Accordingly, inducing a higher level of quality θ considerably reduces the illiquidity risk in that state of the nature. This fact is reflected in the bank's charter value, which increases with θ for any $\theta < \hat{\theta}$. In the contrary case, if $\theta_B > \hat{\theta}$, improving the quality of liquid assets has a poor impact on the liquidity risk if A becomes worthless. However, it increases the opportunity cost of liquid reserves. This higher opportunity cost induces a lower value of α, which raises the bank's illiquidity risk and harms the bank's charter value. As a result, the bank maximizes its charter value by choosing a level of quality $\theta^* = \hat{\theta}$.

Through the curves $\frac{\partial R(\theta)}{\partial \theta}$ and $\frac{\partial \Pi_B(\alpha(\theta),\theta)}{\partial \theta}$, Figure 3.4 illustrates the impact of the trade-off between quality and opportunity cost of liquid assets on both the bank's illiquidity risk and the bank's charter value respectively. In that figure, we can observe that the optimal level of liquid asset diversification is the one which minimizes the bank's illiquidity risk. It is worth mentioning that this result does not mean that the bank chooses the social optimal level of illiquidity risk. Since the bank is protected by limited liability it has incentives to choose a

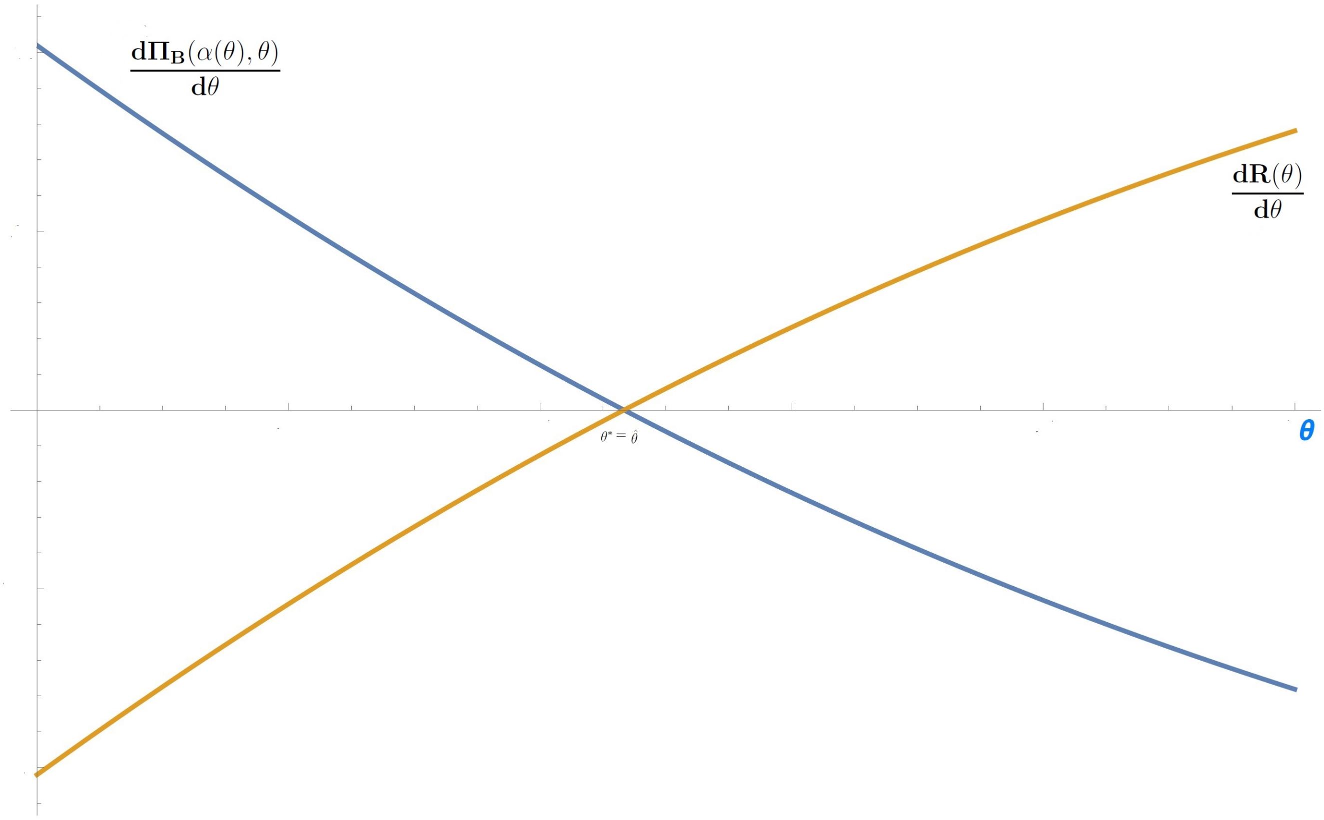


Figure 3.4: Optimal level of liquid asset diversification

fraction of liquid asset lower than the socially optimal level. Nevertheless, Proposition 3.3 states that the bank optimally sets the level of diversification that minimizes its illiquidity risk given its socially subotpimal choice of liquid assets α.

Finally, it is important to mention the impact that a low liquid asset return r has on the bank's illiquidity risk. By differentiating Condition 3.12 with respect to r, we get that

$$\frac{\partial R(\theta)}{\partial r} = -\frac{(R-1)(\varepsilon-1)(1+\varepsilon(\theta-1))^2(1-\theta)^2}{(2+r)(\varepsilon-1)+2M(1+\varepsilon(\theta-1))+(r-(2+r)\varepsilon)\theta^2)^2} < 0,$$

for any value of θ. That is, as r decreases, the bank's illiquidity risk increases. This fact suggests that when the interest rate of liquid assets are relatively low, the bank tends to hold its liquid reserves in the form cash, which improves the quality of liquid assets. However, since cash has a high opportunity cost the bank optimally reacts by reducing the fraction of liquid asset in its portfolio. The final effect of a reduction in the interest rate r is an increment of the bank's illiquidity risk.

3.6. Conclusions

The performance of Greece's sovereign debt during the period 2007-2013 showed how an asset liquidity can fluctuate. Motivated by this fact and the extended use of sovereign debt as liquidity reserves, we examine the consequences of holding liquid risky assets on bank's illiquid risk.

We show, contrary to the widespread belief, that holding liquid reserves in the form of cash might induce a relatively high illiquidity risk. On the one hand, cash is the safest but also the costliest liquid asset in term of foregone loan opportunities. Accordingly, the bank's liquid reserves might be low if they must be held as cash. On the other hand, sovereign bonds have an intrinsic risk but also provides a return to the bank, which allays the opportunity cost of liquid reserves. In this case, by holding liquid assets in the form of sovereign bonds might generate a high illiquidity risk when these assets become illiquid.

We prove that liquid asset diversification allows the bank to mitigate its illiquidity risk. The reason is that by diversifying the bank reduces both the opportunity cost of liquid reserves and the illiquidity risk when the sovereign bond becomes illiquid.

We also show that, even though the bank is protected by limited liability and deposits are insured, its optimal diversification level is the one that minimizes its illiquidity risk. Finally, we highlight the role of the return of risky liquid assets to ameliorate illiquidity risk. We prove that if the return of the risky liquid asset is relatively low, then the bank has incentives to hold liquid reserves in the form of cash. As a consequence, the level of diversification and liquid reserves are relatively low increasing the illiquidity risk.

Chapter 4

Non performing loans, deposits flight and interbank market freeze

4.1. Introduction

During the subprime mortgage crisis that took place in the US from 2007 to 2009 the banks had serious difficulties to fund their operations through the interbank market. Thanks to the existence of deposit insurance there was no “classic” bank run (by depositors). However, banks suffered fund withdrawal by financial institutions due to the freeze in the market for the rollover of short-term debt (Brunnermeier, 2008; Uhlig, 2010; Gorton, 2010a). In particular, banks with a higher proportion of non-performing loans experienced a dramatic reduction in their access to interbank funds (Huang and Center, 2009; Afonso et al., 2011; Purnanandam, 2011).

Unlike the subprime mortgage crisis in the US, Europe experienced both an increase in non-performing loans and the phenomenon of deposit flight during the financial crisis (Shin, 2009; Van Rixtel and Gasperini, 2013; Moro, 2013; Connor and O’Kelly, 2013; Malliaropulos, 2014; Whelan, 2014). Countries such as Greece, Ireland, Portugal and Spain were affected by a reduction of retail and institutional deposits at an unprecedented rate [1]. Merler and Pisani-Ferry (2012) state that, as a result of deposit flight, banks found it difficult to obtain external financing. Thus, both liquidity needs were not fulfiled by the market, which revealed the fact that European banks did not trust each other.[2] Hence, the banks had to raise the interest rate with the goal of keeping and attracting deposits which, in turn, significantly eroded bank profitability [3].

We argue that moral hazard plays a central role in the provision of liquidity, and this is consistent with a large body of literature (Holmstrom and Tirole, 1998; Holmström and Tirole, 2000; Hellmann et al., 2000b). In particular, we argue that liquidity shocks affect the net worth of the bank and, therefore, in the middle of the crisis banks are reluctant to remove bad loans from their portfolios for several reasons. First, because banks face reputational cost and a bad reputation worsens its funding conditions in the market (Gabrieli, 2010). Second, because banks restructuring their portfolios may have to sell their assets at fire sale prices and this implies considerable losses for their shareholders (Diamond and Rajan, 2009). Finally, because banks might be requested to increase their capital buffers by the financial regulator which, in turn, is reflected in a higher cost for the bank, especially during turbulent times (Kashyap et al., 2008). The fact that banks may be reluctant to remove bad loans from their portfolios implies that the banks' portfolio are not fully pledgeable. As a result, banks may not be able to cover their liquidity needs through the interbank financing due to moral hazard.

This paper adopts a moral hazard setting in order to examine the impact of liquidity shortages on the ability of the interbank market to provide liquidity. In particular, this model attempts to clarify to what extent the decrease in the value of the assets and the

[1] Evans-Pritchard, Ambrose. Deposit Flight from Spanish Banks Smashes Record in July. http://www.telegraph.co.uk/. Aug 28, 2012.

[2] Onaran, Yalman. Deposit Flight at European Banks Means Risk Piling Up at ECB. http://www.bloomberg.com/. Sep 14, 2011.

[3] Onaran, Yalman. Deposit Flight From Europe Banks Eroding Common Currency. http://www.bloomberg.com/. Sep 18, 2012.

phenomenon of deposits flight that characterized the European financial crisis led impaired banks to limit their access to external funds.

A major suggestion of our model is that the reason for which banks need funds is a crucial factor in order to explain the interbank market freeze, in the sense that there is no interest rate at which interbank lending will occur (Acharya and Skeie, 2011). Our analysis reveals that the mechanism behind the phenomenon of deposit flight and the increase in non-performing loans has a different effect on the bank's balance sheet. As a result, shocks affect borrower's incentive to restructure their portfolios in a different way. Consequently, there is a considerable variation among banks' ability to meet liquidity needs, and this ability is determined by the type of shock that the bank is facing, instead of the amount of cash that it needs to cover. Thus, when regulating a bank's liquidity one not only has to look at potential liquidity shortfalls but also one has to consider the reasons behind those shortfalls.

Based on the events that took place during the financial crisis, we study two sources of liquidity shocks. The first shock is defined as a "deposit flight". We assume that it arises from the liability side of the banks' balance sheet when depositors require a higher interest rate in order to keep their savings in the bank. The second shock is called a "cash flow" shock. We intend to capture the increase in the non-performing loans of the bank's portfolio by reducing the cash flow obtained by the banks investment [4]. As a result of this shock, banks need a higher quantity of external funds in order to satisfy its financial duties.

We assume that shocks generate the same liquidity needs in order to assess their effect on the interbank market's liquidity provision. At first sight, if both shocks generate the same liquidity needs, it could be anticipated that they would have the same effects because, at the end of the day, the bank would need the same extra quantity of money in order to continue with its operations. Nevertheless, we find that banks are more vulnerable to liquidity shocks

[4]PFS Advisory Team, DELOITTE CONSULTING. High Levels of Problem Loans in Southeast Europe and Eurasia: The Silent Killer of Economic Growth. http://www.rciproject.com/. Sep 14, 2011.

when financing needs come from the asset side of the balance sheet. Our main result shows that a cash flow shock is more severe than a deposit shock because it produces not only a higher financing cost but also reduces the "cash in hand" of the bank. Based on this, we argue that under a deposit shock (rather than a cash flow shock) the interbank market is able redistribute funds across banks. This means that there are situations in which the interbank market may provide funds efficiently only if the liquidity shortfall is produced by a deposit shock. Under an increase of non-performing loans, the moral hazard problem is exacerbated and banks are constrained by the more expensive local deposit supply. This could lead to a problem of insolvency for some banks with positive net present value that may not be able to obtain funds from the overnight market or deposits over a short period of time.

A considerable literature deals with the inefficient provision of liquidity by the market. In these works, the interbank market freezing has been depicted as a consequence of information problems. Acharya et al. (2012) stated that the breakdown of liquidity's provision was due to reduction in the ability to use assets as collateral. In their model, the structure of the information together with the frequency of refinancing debt determines the value of the assets used as collateral. Boissay et al. (2013) used a DSGE model to analyze the effect of interbank market freezes on macroeconomic variables. The sudden freeze is produced by a problem of moral hazard and asymmetric information in the banking sector. Bruche and Suarez (2010) show that deposit insurance may lead to the drying up of the trade of interbank loans due to a rise in counterparty risk. Heider et al. (2009) emphasize how asymmetric information is able to amplify the consequences of counterparty risk. This fact may impair the role of the interbank market as liquidity provider and inefficient liquidation may arise. In Allen et al. (2009) banks hoard liquidity in order to face aggregate liquidity shocks, which may lead banks not to provide liquidity in the market. Acharya et al. (2011) study the lending relationship among banks in a model characterized by moral hazard and asymmetric information. They show that banks with liquidity surplus enjoy a significant degree of market power. This results in banks not providing enough liquidity, which leads to inefficient asset sales.

Many of these works analyze liquidity shortages as stemming from the bank's assets side of the balance sheet. The novelty of this paper consists in defining two types of liquidity shocks that are thought to represent the events taking place during the European financial crisis. Our analysis allows us to establish that the relationship between liquidity needs and a sudden freeze of the overnight market is determined by the source of the liquidity needs rather than just by the amount of funds that the banks need. We show that the combination of a maturity mismatch problem and the possibility that banks undertake projects with low probability of success produces coordination failures in the interbank market. Therefore, we argue the interbank market freezes may be primarily explained by the decrease in the value of the assets rather than by the phenomenon of deposits flight that characterized the European financial crisis. These results may lead to a better understanding of the mechanisms of liquidity redistribution among financial institutions as well as of the framework in which financial institutions operate.

The paper is structured in the following way. Section 2 introduces the model. In Section 3, the timing is described and, in Section 4 the analytical problem of the agents is solved. The equilibria of the model are defined in Section 5 and the main results are presented in Section 6. We conclude with an analysis of the policy implication in Section 7. Proofs are contained in the Appendix C.

4.2. The Model

Consider a three period model t=0,1,2 with two banks, Bank A [5] and Bank B that are monopolies in their respective short-term deposit markets. Additionally, each market is populated by a unit mass continuum of identical agents. For expositional simplicity, we will assume that there is no discounting between periods.

[5] For the sake of simplicity, we assume that only Bank A is affected by shocks.

In this economy, banks can invest in a long term portfolio of loans that takes I units of deposits at date 0 and produces $C_f < I$ and $R > I$ units of consumption at dates 1 and 2, respectively.

We assume that banks do not have any income of their own to make their investment. Consequently, they use short term deposits in order to carry out their projects, and thus a maturity mismatch among assets and liabilities arises.

Once investment is sunk, Bank A can suffer either a deposit or a cash flow shock. Since shocks do not hit Bank B, banks become heterogeneous with respect to their intermediation ability, which creates the incentive to trade interbank loans.

However, a classic moral hazard problem arises in the market. In the interim stage, Bank A has the opportunity to remove its bad loans. This action increases the probability of project success but is unobservable for Bank B.

Thus, the interbank trade may be affected by the asymmetric information problem. Moral hazard introduces the possibility of obtaining a market freeze equilibrium, where banks do not lend to each other. This collapse of trading volume produces an inefficient allocation of funds within the banking system given that the reallocation of funds is not possible.

It is important to notice that, in our setting, the moral hazard problem cannot be solved through contracts, since outside financiers cannot observe the choice of the bank. On the other hand, limited liability prevents Bank A to be punished when the return on the long-term investment turns out to be zero.

In order to understand how the model operates, below we present the main features banks, as well as the banks and consumers' decision problem.

4.2.1. Households

We consider a representative depositor that lives for three periods. He receives an income W in periods 0 and 1. However, his only source for consumption in period 2 is from savings. The depositor, in order to save, lends funds to banks as short term deposits, which are held from t to t+1. Thanks to his deposits, that are protected by deposit insurance, the agent receives gross interest rates R_0 and R_1 in periods 0 and 1 respectively.

We assume that individual preferences are captured by the following constant relative risk aversion utility function:

$$U = \frac{C_0^\theta}{\theta} + \frac{C_1^\theta}{\theta} + \frac{C_2^\theta}{\theta}, \tag{4.1}$$

where $0 < \theta < 1$. As we will see later on, this parameter will play an important role in our model for two reasons. First, it determines the intertemporal elasticity of substitution between consumption in any two periods [6]. Second, θ will be the main determinant for the shape of the inverse deposit supply curve. This function shows the minimum gross interest rate such that depositors are willing to receive at the next period in order to supply a determined amount of deposit today. The inverse deposit supply will be denoted by Γ_t.

4.2.2. Banks

We assume that both Bank A and Bank B carry out the same long term investment, which is, at date 0, only funded through retail deposits. The investment yields an income C_f at date 1 and either $R > 0$ or no income at date 2. The probability of producing an income R depends on the behavior of banks in the interim stage. They can make one of two following unobservable decisions. First, to keep its original portfolio and get a private benefit "B" regardless of the investment's outcome. The probability of repayment in this case is P_L. The second option is to restructure their portfolio in order to improve the quality of its loans. Thus, the probability of success will be increased to P_H, but banks will not get a private benefit.

We assume that if there is no liquidity shock, banks always has incentive to remove its bad loans in period 1, and thus the investment has a positive net present value.

Moreover, in the interim stage, Bank A faces a liquidity shortage that may derive from one

[6] Notice that the substitution elasticity between consumption is equal to $\zeta = \dfrac{1}{1-\theta}$

of two mutually exclusive and equi-probable shocks: a) a drop in the deposit supply or b) an increase in the bank's non-performing loans. These shocks diminish the present value of the bank but they behave differently. In the case of an increase of non-performing loans, the cash of the project is reduced by an amount α, and the bank receives $C_f - \alpha$ instead of C_f. In the case of a deposit shock, Bank A will suffer a contraction of its deposits and, consequently, an increase of its financing cost. Notice that, in both cases, shocks do affect neither the payment R nor the probability of the project success (P_L,P_H). However, the occurrence of these liquidity shocks might affect the Bank A's incentive to remove bad loans, and thus bank's investment could be riskier.

4.2.3. Interbank Market

In our model, banks are able to trade short term loans, which are denoted by X. The Interbank market helps the banking system to achieve a better allocation of liquidity when the problem of asymmetric information is not severe. This market is modeled as competitive, which means that the interbank interest rate, R_I, is given for both banks.

Notice that at date 0, trading funds between banks does not produce any benefit because both banks and deposit markets are symmetric. However, when shocks are present, the reallocation of funds through this market allows Bank A to dampen the effect of the shocks. Additionally, given that Bank B is not affected by any shocks, it will act as the lending bank.

4.3. Timing

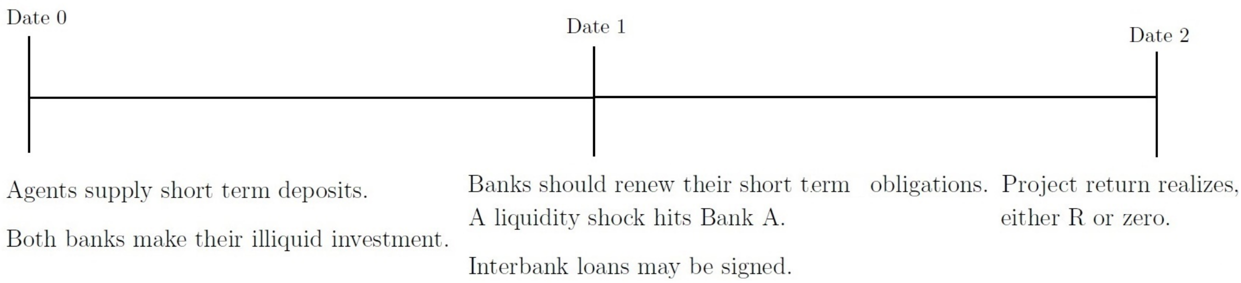
In this section, we present the timing of the model. See figure 1 for an illustration. In period 0, agents receive an income W, which they allocate between consumption and saving. Households' savings are supplied to the bank as short term deposits and they receive as payment the gross interest rate R_0.

Both Bank A and Bank B demand these liquid deposits in order to finance an illiquid loan portfolio, which requires an initial investment of I.

At the beginning of date 1, banks do not have enough cash relative to the maturity of deposits ($C_f < R_0 I$), and thus, they face an asset-liability mismatch. In order to meet current obligations, banks should offer agents an interest rate high enough to renew their short term deposits or, eventually, liquidate their investment.

On the same date, Bank A is affected by a liquidity shock that will deteriorate its financial situation. This fact will result in a financial situation for Bank A which is worse than that of Bank B and incentivize both banks to engage in interbank loans. After the shock, Bank A must decide either to keep its original portfolio or to manage its loans diligently in order to get a repayment with higher probability. As bank A's decision is unobservable for bank B, if the liquidity shock is strong, the interbank market could get disrupted. If this happens, an inefficient allocation of funds may arise.

At time 2, banks' projects are liquidated (paying R or zero) and the proceeds are split up according to the commitments made at date 1.



At date 1, banks make their investment. Then, in period 2, Bank A is affected by a liquidity shortfall and banks may trade funds each other. In case the trade is not possible, the impaired bank should get funds from its deposit market which could imply the insolvency of the bank. At the end, the project pays R or zero, and assets are worthless.

Figure 4.1: Timeline

4.4. Decision Problem

We devote this section to the analysis of the optimal decisions of depositors and banks, and then apply market-clearing conditions in order to obtain the equilibrium of the model. In particular, we will illustrate how liquidity shocks operate under asymmetric information affecting the behavior of the different agents.

The model has two states of nature indexed by $j = \{C, D\}$, which refer to cash flow and deposit flight, respectively.

In state D, individuals suffer a reduction of their income and withdraw their deposits to cover consumption needs. This leads to a lower volume of funds supplied by agents, while the bank's cash flow is not influenced. On the other hand, if the realized state is C, Bank A suffers a drop in its cash flow, whereas individual's income is not affected.

Thus, we derive the inverse deposit supply and the optimal mix of interbank loans and deposit that Bank A demand. Additionally, we present the problem of lending bank, and describe our criterion for making both types of shocks comparable.

4.4.1. The Household's Problem

Now, we will describe the household's problem keeping in mind that there are two independent deposit markets, labeled A and B. Each market contains a continuum of identical depositors with an endowment of W in period 0 and 1, respectively. They obtain utility from the intertemporal consumption and their preferences are represented by (1).

The only difference between markets is that depositors of market A will face two equally likely states, at date 1. In state D, they are affected by an income shock of size ρ, whereas in state C , their incomes are not affected. According to this, depositors of market A will receive, in period 1, $W_D = W - \rho$ and $W_C = W$. [7] Based on this, the agent should choose the optimal level of consumption and saving for each state of nature, as follows:

[7] We assume that W is large enough in order to obtain an interior solution.

$$\underset{C_{tj},S_{tj}}{\text{Maximize}} \quad E_0\big[U\big] = E_0\left[\frac{c_0^\theta}{\theta} + \frac{c_1^\theta}{\theta} + \frac{c_2^\theta}{\theta}\right] \tag{4.2}$$

$$s.t = \begin{cases} c_0 + s_0 = W \\ c_{1j} + s_{1j} = W_j + s_0 R_0 \\ c_{2j} = s_{1j} R_{1j} \end{cases}$$

Then, from this problem, it is possible to obtain the functions of consumption and saving for period 1. This functions will be denoted by $C_{1,j}(R_{1j}, W_j)$ and $S_{1,j}(R_{1j}, W_j)$, respectively. Additionally, the inverse deposit supply that arises from the agent's problem at $t = 0$ is denoted as $\Gamma_0(s; W)$. In particular, the inverse deposit supply at $t = 1$ after a shock of type $J = \{C, D\}$ is

$$\Gamma_{1j}(s; W_j) = \left[\frac{(W_j + s_0 R_0 - s)}{s}\right]^{\dfrac{\theta - 1}{\theta}}. \tag{4.3}$$

Note that the endowment of depositors in B is not affected by any type of shocks. As a consequence, the inverse deposit supply in this market at t=1 is $\Gamma_1^B = \Gamma_{1C}$ for any state of nature. Note also that Γ_{1C} is the inverse deposit supply when no shocks affect depositors.

4.4.2. Bank A's Problem

The goal of banks is to maximize their profits. In order to do this, they make a long term investment through their retail deposit market, at date 0. Thus, with the purpose of keeping a volume of deposits equal to I from t_0 to t_1, they pay the gross interest rate $R_0 = \Gamma_0(I; W)$.

At date 1, banks need to renew their obligations by an amount of $L_A = R_0 I - C_f$ to be able to continue with their on-going operations.[8] However, in contrast to Bank B, Bank A faces two mutually exclusive liquidity shocks. In the case of an increase in non-performing loans, the cash of the investment is reduced to $C_f - \alpha$. Thus, bank's obligations will rise by an amount α: $L_C = R_0 I - C_f - \alpha$. If the bank does not have access to the interbank market, the cost of renewing Bank A's obligation from deposit market is given by

$$\Gamma_{1C}(L_C; W_C) L_C.$$

Under a deposit shock, the total amount of obligations that should be renewed is $L_D = L_A$. However, the effect to this shock is to increase the cost of obtaining new financing from the deposit market, which is reflected in the inverse deposit supply Γ_{1D}. Thus, the total cost under autarky is given by

$$\Gamma_{1D}(L_D; W_D) L_D.$$

Assume now that Bank A is able to borrow in the interbank short-term funds market to substitute retail deposits. Therefore, after a shock of type $j \in \{C, D\}$, Bank A should choose the optimal mix between deposit and interbank funds X_j^A to minimize its funding cost, as follows:

$$\underset{X_j^A \geq 0}{\text{Minimize}} \quad C^A(X_j^A) \equiv \Gamma_{1j}(L_j - X_j^A; W_j)[L_j - X_j^A] + R_I X_j^A$$

Notice that, the first term represents the total cost to obtain deposit by $L_j - X_j^A$, while the second one is the cost of borrowing an amount of X_j^A funds from the interbank market, where the interbank interest rate (R_I) is taken as given.

[8] Notice that it is optimal for banks use all their cash flow to cancel deposits.

Then the First Order Conditions of Bank A's problem is:

$$X_j^A(R_I) = \begin{cases} 0 & \text{if } R_I > \Gamma'_{1j}(L_j; W_j)L_j + \Gamma_{1j}(L_j; W_j) \\ 0 < X_j^A & \text{if } R_I = \Gamma'_{1j}(L_j - X_j^A; W_j)[L_j - X_j^A] + \Gamma_{1j}(L_j - X_j^A; W_j) \end{cases}$$

Thus, $X_j^A(R_I)$ represents the demand of interbank funds at the interest rate R_I in state j. $X_j^A(R_I)$ is zero when the interbank interest rate is higher than the marginal cost of renewing Bank A's obligation through deposit market.

4.4.3. Equivalence Between Shocks

In order to compare the effect of the shocks on the interbank market's liquidity provision, we want them to produce the same liquidity needs for Bank A, under autarky. Therefore, we first need to determine the liquidity shortfall produced by a shock of size ρ in state D, and then we equalize it to the drop in cash flow.
Let define $\hat{R}_1$ be the deposit rate that Bank A would offer depositors to obtain the amount $L_A = R_0 I - C_f$ when there is no shock; i.e. $\hat{R}_1 = \Gamma_{1C}(L_A)$. If Bank A offers the rate $\hat{R}_1$ when there is a deposit flight shock, the amount of liquidity generated by ρ is given by

$$\alpha(\rho) = L_A - S_{1D}(\hat{R}_1), \tag{4.4}$$

where $S_{1D}(\hat{R}_1)$ is the volume of deposit supplied by consumers at the rate $\hat{R}_1$ in state D. According to this, the liquidity need for Bank A is the difference between the obligations that should be renewed and the amount of deposits obtained at the interest rate $\hat{R}_1$.
Then, if we define the cash flow shock equal to $\alpha(\rho)$, the liquidity needs for Bank A are the same in both states. Notice that in state C, Bank A should renew an amount of obligations

given by $L_C = L_A - \alpha(\rho)$. However, Bank A can only obtain $S_{1C}(\hat{R}_1) = L_A$ at the deposit rate $\hat{R}_1$. Therefore, the liquidity shortfall is $\alpha(\rho)$.

4.4.4. Interbank Supply

The ability of banks to withstand liquidity shocks crucially depends on the alternative sources of funds that they have access to. The interbank market appears as the main alternative source of liquidity for banks. This market is able provide banks with large amounts of short term funds to help them dampen the impact of a liquidity shortage.

In our model, Bank B acts as lending bank because it is not affected by any shocks. Thus, Bank B is able to get cheaper funds than Bank A through its own market for deposit, which creates the incentives to trade funds.

However, the trade of interbank funds might be affected by the moral hazard problem. This fact is due to that the Bank A may be reluctant to remove bad loans from its portfolio affecting its pledgeable income.

We assume that, if Bank A restructures its loans, the project has a positive net present value in any state of nature:

$$P_H R - \Gamma_{1j}(L_j; W_j) L_j > 0.$$

The first term is the expected income when the probability of success is high, and the second one is the total cost of the project under autarky. In case Bank A does not restructure its loan portfolio, the net present value of its investment and the private benefits will be negative regardless of the occurrence of shocks. This assumption can be written as follows,

$$P_L R - \Gamma_{1C}(L_A; W_C) L_A + B < 0.$$

where Γ_{1C} and L_A represent the inverse deposit supply and the obligations that should be renewed in period 1 when Bank A is not affected by any shocks. Therefore when a shock occurs, if Bank A decides to "take a private benefit", lenders would lose money in expectation, and the trade of interbank funds would be zero.

We also assume that Bank B can observe the type of shock that occurs, but it is not able to see the behavior of Bank A. Therefore in state j, Bank B lends to Bank A an amount of X_B only if the following incentive compatibility constrain is satisfied:

$$P_H\big(R-\Gamma_{1j}(L_j;W_j)[L_j-X^B]-R_IX^B\big) \geq P_L\big(R-\Gamma_1(L_j;W_j)[L_j-X^B]-R_IX^B\big)+B \quad (4.5)$$

As a result of the unobservable actions of Bank A, there exists an upper bound to the level of funds that Bank B would be willing to lend through the interbank market. This upper bound denoted as $\bar{X}^B(\rho)$ is determined by the highest income that can be pledged to the lenders for any ρ. Then in the state j, $\bar{X}^B(\rho)$ should satisfy the following equality

$$R_I\bar{X}_j^B(\rho) = R-\Gamma_{1j}(L_j-\bar{X}_j^B(\rho);W_j)[L_j-\bar{X}_j^B(\rho)]-\frac{B}{\Delta P}, \quad (4.6)$$

Condition (4.6) is important because it allows us to interpret our results in a clear way, as will be discussed later.

We also assume that Bank B will only provide funds in the interbank market after that it has renewed all the obligations incurred by its investment, $L_A = R_0I - C_f$. According to this, the total cost of providing X^B interbank loans can be written as follows:

$$C_I(X^B) = \Gamma_1^B(L_A+X^B;W_C)[L_A+X^B] - \Gamma_1^B(L_A;W_C)L_A,$$

where the first term is the cost of obtaining an amount of $L_A + X^B$ deposits from Market B. The second term represents the cost of the obligations that Bank B should cover in order to carry its investment. Hence, the difference between these terms is the cost of obtaining an extra amount of X^B deposits to be supplied in the interbank market. Furthermore, remember that the inverse deposit supply in Market B is given by $\Gamma_1^B = \Gamma_{1C}$ as Bank B's depositors are not affected by any shock.

According to this, the volume of funds supplied at interest rate R_I is obtained from the following maximization problem:

$$\underset{X^B \geq 0}{\text{Maximize}} \quad \pi^B(X^B) \equiv P_H R_I X^B - C_I(X^B)$$

Then, the First Order Conditions for X^B is:

$$X^B = \begin{cases} 0 & \text{if } R_I < \Gamma_1^{B\prime}(L_A; W_C)L_A + \Gamma_1^B(L_A; W_C) \\ 0 < X^B & \text{if } P_H R_I = \Gamma_1^{B\prime}(L_A + X^B; W_C)[L_A + X^B] + \Gamma_1^B(L_A + X^B; W_C). \end{cases}$$

However, Bank B will provide an amount of $X^B(R_I)$ interbank funds only if inequality (4.5) is satisfied. Notice also that as neither depositors in Market B nor Bank B are affected by shocks, which implies that the availability of funds is the same under both types of shocks.

4.5. Equilibrium

In this section, we analyze the market allocations that may arise in this economy in which banks finance their illiquid assets with both short term deposits and interbank funds. [9] Moral hazard introduces the possibility of obtaining three different equilibria. We show that which of these arises will depend on both the type of shock, and the Bank's opportunity cost of restructuring its portfolio.

The first equilibrium is denoted as "Active Interbank Market Equilibrium " (AIE). In that case, Bank A is able to meet the necessary funds from the interbank market to dampen the negative effects of liquidity shortages. Formally, we define this equilibrium as follows.

[9] It is important to say that only a symmetric equilibrium is analyzed in this paper.

Active Interbank Market Equilibrium: It is said that an AIE exists in this economy, if given the vector $(R_0^*, R_{1j}^*, R_{Ij}^*)$, there is a volume of interbank $X_j^* > 0$ such that:

1. $X_j^A(R_{Ij}^*) = X^B(R_{Ij}^*) = X_j^*$

2. $P_H\Big(R - \Gamma_{1j}[L_j - X^B(R_{Ij}^*)] - R_{Ij}^* X^B(R_{Ij}^*)\Big) \geq P_L\Big(R - \Gamma_{1j}[L_j - X^B(R_{Ij}^*)] - R_{Ij}^* X^B(R_{Ij}^*)\Big) + B$

Then, banks and consumers maximize their profits and utility, respectively, and all markets clear, for J={ C,D }. Under this equilibrium, funds are allocated efficiently and the impaired bank can partially reduce its financial cost . This allocation is possible because the pledgeable income is enough for Bank B.
Moreover, the model may produce a "market freeze equilibrium" (MFE) in which trade is not possible. Thus, banks are not able to get funding from external sources because there is no R_I at which interbank lending occurs.

Market Freeze Equilibrium: It is said that a MFE exists in this economy, if given a liquidity shortfall of size α , there is no interbank interest rate R_{Ij}^* (for J={ C,D }) , such that, both conditions (1) and (2) in Definition 1 are satisfied simultaneously.

In our analysis, the phenomenon of a market freeze occurs when the borrowing debt capacity is impaired due to the asymmetric information among banks. This fact leads to a possible failure of solvent banks that are forced to liquidate assets with a positive present value.

We say that a MFE arises when it is optimal for the lending bank to supply $X^B = 0,$ at the competitive interbank interest rate R_I^*. Therefore in this case (regardless of whether Bank A is hurt by either a deposit flight or a cash flow shock) the impaired bank should fund all its obligations through its own deposit market, which is a costly source of funding. A third possible equilibrium is denoted as Mixed Equilibrium .

Mixed Equilibrium: It is said that a Mixed Equilibrium exists in this economy, if at the interbank interest rate R^*_{Ij} (for J={ C,D }) , both conditions (1) and (2) in Definition 1 are satisfied, though only for one type of shock. The Mixed Equilibrium arises when for the same liquidity shortage the lending bank is willing to provide funds under one type of shocks, while for the other one the optimal amount of interbank loan supply is zero. The intuition of these outcomes will be explained in the following section.

4.6. Analysis of Equilibra

In the preceding sections, we have defined the type of equilibria that will arise in our model. In what follows, we present our findings in order to allow the reader to easily understand the mechanisms through which our model operates. Additionally, we derive the necessary conditions under which these equilibria are reached. In particular, we focus on the implications that liquidity shocks have for banks' financing cost and balance sheets, as well as for trading in the interbank funds.

4.6.1. Autarky

Our first analysis deals with the impact that each shock has on the financial costs of banks, when banks fund all their liquidity needs exclusively using the deposits market.

Proposition 4.1. *If banks suffer a liquidity shock of size $\alpha(\rho)$, such that $\frac{dS_1}{dW} \geq \frac{dC_1}{dW}$ and $\rho \in [0, \bar{\rho}]$, then a drop in the bank's cash flow generates always greater financial costs than those generated by a deposit flight shock. Mathematically, $\Gamma_{1C}(L_C; W_C)L_C \geq \Gamma_{1D}(L_D; W_D)L_D$.*

Proof. See Appendix C. □

Proposition 4.1 imposes two sufficient conditions such that a cash flow shock generates greater financial costs than a deposit flight. First, it requires that $\frac{dS_1}{dW} \geq \frac{dC_1}{dW}$, which is satisfied by our utility function for any value of θ. This requirement implies that consumption should be less sensitive than savings to income shocks. The second requirement is an upper bound for ρ.[10] As we show in the Appendix C, for each value of θ, it is possible to find a $\bar{\rho}(\theta)$ such that for every ρ in $[0, \bar{\rho}(\theta)]$ an increase in non-performing loans produces higher costs than a deposit shock. We also show that $\bar{\rho}(\theta)$ is an increasing function of θ. This requirement is necessary because when θ tends to zero the saving elasticity with respect to the deposit rate goes to zero. Consequently, for all $\rho > \bar{\rho}(\theta)$ Proposition 4.1 does not hold because income shocks produce strong movements in the interest rate, which result in significant increases in the banks' financing costs.

The main implication of Proposition 4.1 is that, despite the fact that Bank A has the same liquidity needs under both shocks, their effect on the bank's financing costs is different. In order to analyze the effect of shocks on bank's costs, we assume that cash flows are big enough to cover the interest accrued at date 1, i.e. $(R_0 I - 1)I < C_f$. This assumption implies that liabilities will not increase under deposit shocks, allowing us to easily explain our findings without changing the results of the model. Based on this, the mechanism behind each shock can be described as follows. First, in the case of a deposit flight, depositors need to be compensated by a higher interest rate if they are going to keep their money in the bank for another period. Consequently, banks have to offer a higher interest rate to the former, which lowers the banks' overall profitability. The increase in the interest rate caused by this shock will harm the equity of the bank as deposits will be renewed at higher cost. In this case and according to our model, the total cost under autarky can be written as

$$\Gamma_{1D}(L_D; W_D)L_D = \Gamma_{1C}(L_A; W_C)L_A + \Delta_{EQ_D}, \tag{4.7}$$

where the first term on the right side is the total cost of renewing the banks obligations when there is no shock, and the second one is the decrease in equity due to the increase of interest

[10] Given the assumptions of our model, this condition will only be binding for small values of θ

paid. Thus, Δ_{EQ_D} is defined as

$$\Delta_{EQ_D} = L_D\Big(\Gamma_{1D}(L_A; W_D) - \hat{R}_1\Big) > 0, \tag{4.8}$$

where the first term is the amount of deposit that should be renewed in the interim stage, and the second one is the increase in the rate produced by the shock.

Suppose now that the bank suffers an increase in its non-performing loans. In that case, the cash flows of the bank is reduced, generating a decrease of α in the value of the bank's assets. This drop in the cash should be compensated by an increase in banks' liabilities. Attempting cover this funding shortage will force Bank A to raise the interest rate on deposits in order to attract more depositors. As in the previous case, the total cost can be written as the sum of the costs in the case of no shocks, the increment in interest paid (Δ_{EQ_C}), and an extra term α that represents the increment in liabilities due to the shock.

$$\Gamma_{1C}(L_C; W_C)L_C = \Gamma_{1C}(L_A; W_C)L_A + \Delta_{EQ_C} + \alpha > 0, \tag{4.9}$$

where the total amount of interest paid under a cash flow shock is

$$\Delta_{EQ_C} = L_C\Big(\Gamma_{1C}(L_C; W_C) - \hat{R}_1\Big) + \alpha\Big(\hat{R}_1 - 1\Big). \tag{4.10}$$

Proposition 4.1 states that $\Gamma_{1C}(L_C; W_C)L_C \geq \Gamma_{1D}(L_D; W_D)L_D$, therefore the sum of the variation of liabilities and bank's equity given by α and equation (4.10) respectively, overcome the change in bank's equity produced by the deposit flight given by (4.8). As a result, banks that face an increase in their non-performing loans will have weaker balance sheets than those affected by a deposit flight.

4.6.2. Interbank Market

We have assumed that the impaired bank only had access to one source of funding (the deposit market). Now, we will allow Bank A to supplement traditional retail deposits with interbank loans as alternative source of funding.

The role of interbank markets as distributor of liquidity is well recognized in the literature. If the problem of asymmetric information is not severe, then the trading in the interbank market allows banking system to reach a better allocation of funds than autarky. The following lemma states the conditions to obtain an AIE.

Lemma 4.1. *There is an upper bound ρ_C such that $\forall \rho \in [0, \rho_C]$ both conditions (1) and (2) in Definition 1 are satisfied simultaneously, and thus banking system reaches an Active Interbank Market Equilibrium.*

Proof. See Appendix C. □

Before explaining the intuition of Lemma 4.1, we show the following result.

Lemma 4.2. *For all value of ρ, it is possible to find an R^*_{Ij} such that $X^A_j(R^*_{Ij}) = X^B(R^*_{Ij}) = X^*_j$, and thus the first condition in Definition 1 is always satisfied.*

In order to prove this, we use Bolzano's theorem and define the function λ as the difference between the FOC of Bank A and Bank B, as follows:

$$\begin{aligned}\lambda(X)_j = {} & \Gamma'_{1j}(L_j; W_j)[L_j - X] + \Gamma_{1j}(L_j - X; W_j) - \Gamma^B_1(L_A + X; W_C)'[L_A + X]\frac{1}{P_H} \\ & - \Gamma^B_1(L_A + X; W_C)\frac{1}{P_H}\end{aligned} \tag{4.11}$$

Since $\lambda_j(0) > 0$ and $\lambda_j(L_A) < 0$, then $\forall \rho \in [0, \bar{\rho}]$ there is an $X^*_j \in (0, L_A)$, such that $\lambda_j(X^*_j) =$

0. Hence, if we define interbank interest rate as

$$R^*_{Ij} = \left(\Gamma_1^{B\prime}(L_A + X^*_j; W_C)[L_A + X^*_j] - \Gamma_1(L_A + X^*_j; W_C)\right)\frac{1}{P_H},$$

the allocation $X^A_j(R^*_{Ij}) = X^B(R^*_{Ij}) = X^*_j$ may be reached in equilibrium if (X^*_j, R_{Ij}) satisfies Condition 4.5.

According to this result, the intuition behind Lemma 4.1 is straight forward. As long as $\rho \in [0, \rho_C]$, the requirement (2) in Definition 1 is satisfied because liquidity shocks does not produce a high cost for the impaired bank, and thus an AIE is reached. In that case, the moral hazard problem does not arise and liabilities can be transferred from one bank to another. This leads to the amount of deposits that Bank A gets from its market to be lower, but, on the other hand, it will also increase its obligations with Bank B in order to dampen the effect of the shocks. Moreover, the trade of funds helps ameliorate the asymmetric information problem because when Bank A borrows interbank funds to supplement deposits the compatibility constrain in (4.5) is relaxed. Thus, the lending bank internalizes some gains from trade because the pledgeable income increases. As a result, the market reaches a competitive equilibrium and the impaired bank is able to dampen the effects of liquidity shocks .

We now turn to the case where the interbank market reaches a mixed equilibrium. Lemma 2 allows us to understand how the trade is conducted under different shocks and reveals that the origin of the shock is a fundamental factor in explaining a market freeze.

Lemma 4.3. *There is a value of $\rho = \rho_D$ such that $\forall \rho \in [\rho_C, \rho_D]$ the banking system reaches a Mixed Equilibrium. In this equilibrium, the interbank market only provides funds when the liquidity shortage is caused by a deposit shock, while if a cash flow shock occurs, the market freezes. Moreover, if $\rho > \rho_D$ the interbank market freezes regardless of the type of shock.*

Proof. See Appendix C. □

In our model, liquidity needs for Bank A can be caused by either a deposit flight or an increase in non-performing loans, and both of them impair the present value of the bank. We have shown that cash flow shocks are especially harmful when banks have only one source of financing. Nonetheless, if banks are able to access the interbank market, it might be the case that a deposit shock exceeds the cost of an increase in non-performing loans. However, under the conditions of Proposition 4.1, we rule out that possibility. Thus a cash flow shock is also more costly when Bank A is able to access the interbank market, as we formally show in the Appendix C. This fact implies that for all ρ in $(\rho_C, \rho_D]$

$$\frac{B}{\Delta P} < R - \Gamma_{1D}(L_D - X_D^*; W_D)[L_D - X_D^*] - R_I^* X_D^*, \tag{4.12}$$

and

$$\frac{B}{\Delta P} > R - \Gamma_{1C}(L_C - X_C^*; W_C)[L_C - X_C^*] - R_I^* X_C^*. \tag{4.13}$$

Conditions (4.12) and (4.13) say that Bank A only has an incentive to restructure its loan portfolio under a deposit shock, while it takes private benefits under a cash flow shock. As a consequence, Bank B only provides funds in state D. The fact that Bank B supplies $X_B = 0$ under a cash flow shock can be explained by its effect on the bank's balance sheet. An increase in non-performing loans generates two issues. First, Bank A should pay a higher interest rate to its depositors. Second, there is a drop in the "cash on hand" of the banks results in more leveraged institutions.

These two effects result in a weaker balance sheet in comparison with the deposit shock case and further exacerbate the moral hazard problem. In addition, unlike an increase in the interest rate produced by a deposit flight, the new obligations incurred by the drop in the cash flow (α) cannot be compensated by the interbank market. This damages the ability of banks to reduce their financial costs, and thus, the borrowers' capacity to repay becomes progressively affected.

In other words, the increase of the interest generated by a deposit shock can be reduced through an alternative source of funds. However, the increases in bank's obligations (α) produced by a cash flow shock cannot be dampened through the interbank market.

Additionally, the second statement of Lemma 4.3 says that a MFE is reached for all $\rho > \rho_D$. This result is due to the fact that the second condition in Definition 1 is not satisfied regardless of the shock structure.

To illustrate our results, we look at a numerical example. Assume the following parameter values: the agent's endowment is $W = 4$, the initial investment is $I = 2.25$, $\theta = 0.5$. The probability of project success if Bank A removes bad loans or does nothing are $P_H = 0.9$ and $P_L = 0.4$, respectively. Additionally, bank's investment yield $C_f = 1$ in period 1 and $R = 3$ in period 2.

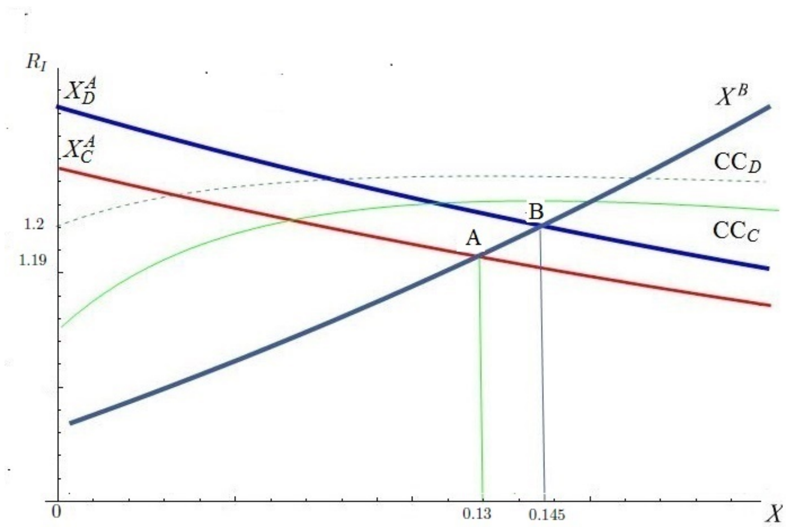


Figure 4.2: Active interbank market equilibrium

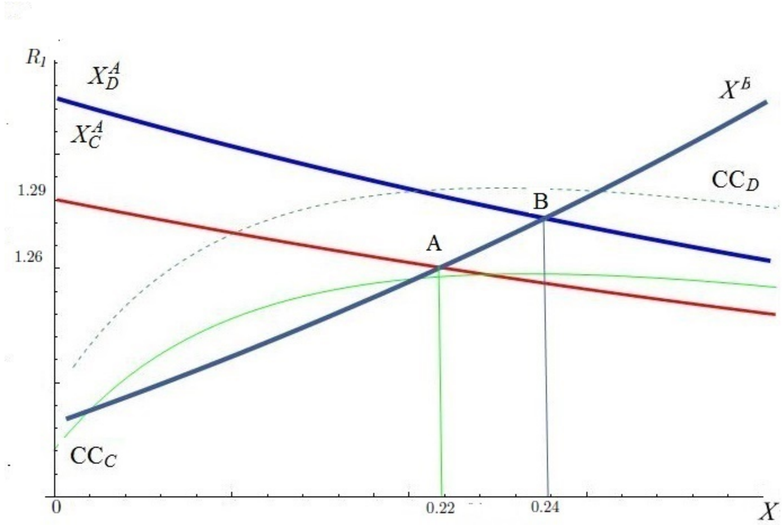


Figure 4.3: Mixed interbank market equilibrium

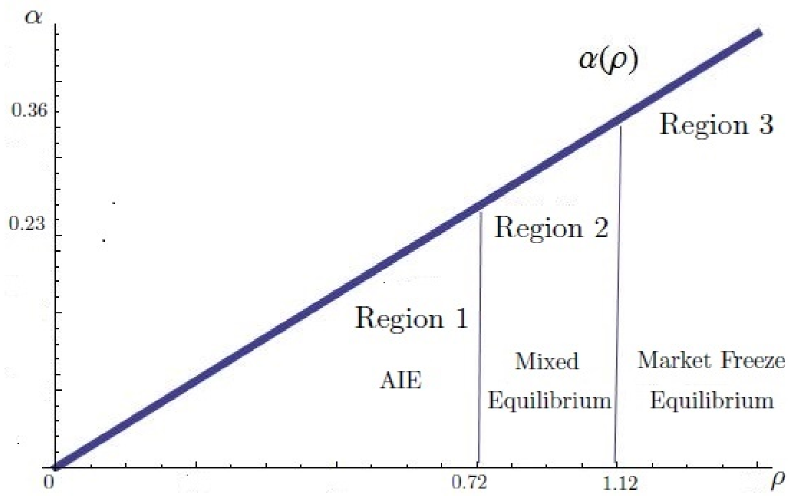


Figure 4.4: Regions

Figures 4.2 and 4.3 depict the interbank market equilibrium. The curves X_J^A and CC_J represent the demand of interbank funds and the compatibility constraint for states of nature $J = \{C, D\}$, and supply of funds is given by X_B. All of the mentioned curves are functions of the volume of interbank funds (X).

In this example, we solve the model in order to compare the market equilibrium for $\rho = 0.5$ and $\rho = 1.12$. Figure 4.2 shows an AIE that arises when depositors might suffer an income shock of $\rho = 0.5$. According to equation (4.4), a value of $\rho = 0.5$ leads to a liquidity need of $\alpha = 0.16$ for Bank A. Since the points $A = (X_C^*, R_{IC}^*)$ and $B = (X_D^*, R_{ID}^*)$ are below both compatibility constraints, both conditions in Definition 1 are satisfied, and thus we have an AIE. Figure 4.3 depicts the market allocation when $\rho = 1.12$ and the liquidity need for Bank A is $\alpha = 0.33$. We can observe that $B = (X_D^*, R_{ID}^*)$ is below $CC_D(X_D^*)$, while $A = (X_C^*, R_{IC}^*)$ is above $CC_C(X_C^*)$. This implies that only under a deposit flight shock the market provides liquidity to Bank A. Therefore, if a cash flow shock of size $\alpha = 0.33$ occurs, the market freezes and the impaired Bank must cover its needs for liquidity through the deposit market.

On the other hand, figure 4.4 depicts three regions. Each of them contains the values of ρ such that the model leads to an AIE, MFE, or a mixed equilibrium. The curve $\alpha(\rho)$ represents the Bank A's liquidity needs with respect to ρ. Then given previous parameters, we obtain an AIE for $\rho \in [0, 0.72]$, and thus liquidity shocks are satisfied by the market. In the range $\rho \in [0.72, 1.12]$, Bank B only provides liquidity when a deposit shock occurs and for $\rho > 1.12$ the market freezes.

Summing up, our analysis highlights the importance of moral hazard on the trading in the funds. When either the opportunity cost of removing bad loans is not high or shocks are not strong, interbank lending market allows the banking system to reach the optimal allocation of funds. However, when the impaired banks has incentives to take a private benefits (B) instead of removing its bad loans banks stop lending each other.

This example shows that the cash flow shock can be more harmful than a deposit flight even though both of them produce the same liquidity needs for Bank A. An increase in non-performing loans leads to weak balance sheets with negative effects for the interbank relationships. In that case, financial institutions face a higher liquidity risk because banks may not be able to meet their liquidity needs through the interbank market at a reasonable cost. As a result, we can conclude that liquidity needs originated by deposit shocks are more easily absorbed by the market and, therefore, they are less damaging for banks.

4.7. Policy Implications

A full discussion of the policy and welfare implications is beyond the scope at this paper. Instead, we will analyze the more modest question of the effectiveness of capital requirements in order to ameliorate the moral hazard problem analyzed here. Since the bank industry is a highly leveraged business, regulators must dictate certain requirements, restrictions and guidelines with the purpose of avoiding excessive risk taking by banks. Minimal capital levels are required in order to improve the solvency of the banking system. In this section, we analyze the impact of capital requirements to induce prudent behavior of banks and achieve a better allocation of resources in this economy.

Regulators require banks to maintain an amount of capital K from period 0. We assume that the opportunity cost of capital is denoted by ψ and it is higher than the deposit rate. Then, if the investment is successful the bank will lose ψK, while in case of default, banks' shareholders lose their capital. Capital requirements modifies equation (4.5) as follows

$$R - \Gamma_{1j}(L_j; W_j)[L_j - X^B] - R_I X^B + K(1-\psi) \geq \frac{B}{\Delta P} \tag{4.14}$$

where the only difference with respect to the case without regulation is the term $K(1-\psi)$. This term captures the increase in Bank's A incentives to restructure its bad loans when bank's shareholders can lose their capital.

From now on, we assume that ρ is uniformly distributed over $[0, \bar{\rho}]$. In order to improve the market allocation, the regulator should determine the optimal level of capital to reduce the moral hazard problem. First, we need to define the highest liquidity shortfalls withstood by the interbank market for each value of K. These values are denoted by $\hat{\alpha}(\rho_{CK})$ and $\hat{\alpha}(\rho_{DK})$, for a cash flow and a deposit shock, respectively. Notice that when $K = 0$, we have the result without intervention and these values are denoted as ρ_C and ρ_D, respectively. We present the following corollary which is a straightforward implication of Lemma 2.

Corollary 4.1. *For any level of capital requirement, the interbank market is able to withstand higher liquidity shortfalls under a deposit flight. Mathematically, $\hat{\alpha}(\rho_{CK}) < \hat{\alpha}(\rho_{DK})$, which implies that $\rho_{CK} < \rho_{DK}$.*

With the purpose of solving the problem of the regulator, we define the function BS_j as the benefits of the capital requirements after a shock of type $J = \{C, D\}$ as follows

$$BS_j(K) = \int_{\rho_j}^{\rho_{jK}} \left(P_H R - \Gamma_{1j}(L_j - X_j^*; W_j)[L_j - X_j^*] - R_I^* X_j^* - P_L R - \Gamma_{1j}(L_j; W_j)[L_j] + B\right) \cdot d\rho$$
$$- \psi K. \tag{4.15}$$

In order to describe how the function $BS_j(K)$ captures the benefits of capital requirements, we analyze the capital requirement in state D [11]. According to Corollary 1, the limit of integration for (BS_j) can be dived into two parts:

- $[0, \rho_D] \cup [\rho_{DK}, \bar{\rho}]$ For these values of ρ, capital requirements produce a net cost equal to ψK because the regulation is not effective due to two different reasons. First, for all ρ in $[0, \rho_D]$ shocks are too small, so the interbank market is able to allocate funds efficiently. Second, for all ρ in $[\rho_{DK}, \bar{\rho}]$, banks face strong shocks and capital requirements are not enough to satisfy (4.14) and, therefore, the market freezes.
- $[\rho_D, \rho_{DK}]$ In this interval, the benefits of capital requirements are given by the reduction in the moral hazard problem. As a result, Bank A chooses extract a private benefits and its investment has a probability of success P_H. This incentivizes Bank B to provide

[11] The same reasoning can be applied at cash flow shock.

interbank funds reducing Bank A's costs.

$$P_H R - \Gamma_{1D}(L_D - X_D^*; W_D)[L_D - X_D^*] - R_I^* X_D^* - P_L R + \Gamma_{1D}(L_D; W_D) L_D - B$$

Notice that the effectiveness of capital is determined by the size of the interval $[\rho_D, \rho_{DK}]$, where $\dfrac{d\rho_{DK}}{dK} \geq 0$.

Then, the regulator should choose K in order to maximize the expected value of $BS_j(K)$, which is denoted as R_K.

$$\underset{K \geq 0}{\text{Maximize}} \quad R_K \equiv E_0\big[BS_C(K) + BS_D(K)\big]$$

Where, the First Order Conditions for K is:

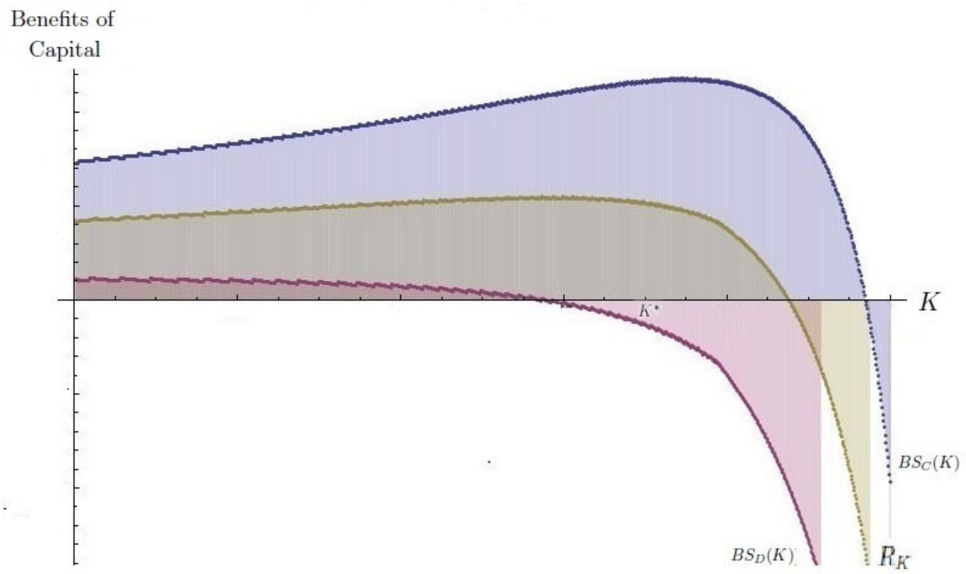
$$\begin{aligned} 2\psi = &\frac{d}{dK}\Big(\int_{\rho_D}^{\rho_{DK}} \big(P_H R - \Gamma_{1D}(L_D - X_D^*; W_D)[L_D - X_D^*] - R_I^* X_D^* - P_L R - \Gamma_{1D}(L_D; W_D) L_D + B\big) \cdot d\rho\Big) \\ &+ \frac{d}{dK}\Big(\int_{\rho_C}^{\rho_{CK}} \big(P_H R - \Gamma_{1C}(L_C - X_C^*; W_C)[L_C - X_C^*] - R_I^* X_C^* - P_L R - \Gamma_{1C}(L_C; W_C) L_C) + B\big) \cdot d\rho\Big) \end{aligned}$$

We give now the following lemma that contains an important result in order to determine de optimal level of capital requirements.

Lemma 4.4. *If the conditions in Proposition 4.1 are satisfied, then the benefits of capital requirements are higher when banks are affected by an increase in their non-performing loans. Mathematically,* $BS_C(K) > BS_D(K)\ \forall K$.

Proof. See Appendix C. □

The intuition behind Lemma 4.4 is as follows. As we explained earlier, the interbank market satisfies higher liquidity shortfalls under a deposit flight. Therefore, for all ρ in $[\rho_C, \rho_D]$ a mixed equilibrium is reached and the market only provides funds under a deposit shock. This fact leads to capital requirements only producing a benefits under a cash flow shock, because when a deposit flight occurs liquidity needs are fulfilled using the interbank market. Consequently, for ρ in $[\rho_C, \rho_D]$ capital requirements lead to a net cost in the case of a deposit shock, which results in $BS_C(K) > BS_D(K)$. To illustrate Lemma 4.4, we can apply the previous analysis in order to compute the optimal level of capital. We have simulated the responses to shocks for $BS_C(K)$ and $BS_D(K)$. The parameters used in this example are: $I = 1$, $\theta = 0.5$, $P_H = 0.9$, $P_L = 0.4$, $B = 1.5$, $R = 5$, $C_f = 0.5$ and $\psi = 0.7$. In addition, we assume that ρ is uniformly distributed over $[0, 1]$. Figure 4.5 depicts the social benefits of capital requirements with respect to K under both a deposit shock ($BS_D(K)$) and a cash flow shock ($BS_C(K)$). Additionally, the curve R_K is the expected value of $BS_j(K)$. Based on Lemma 4.4, the function $BS_D(K)$ is always above $BS_C(K)$, which results in R_K being in between. In particular, this example shows us a striking result. The optimal level of capital, K^*, produces a positive benefit in a cash flow state, but a net cost under a deposit shock. This fact arises because in state D capital requirements have a low effectiveness to reduce the problem of moral hazard. It is due to the interbank market is able to withstand higher liquidity shocks under deposit shocks than under a cash flow shock. On the other hand, when a cash flow shock occurs, the lending bank faces a more severe moral hazard problem and, hence, the regulation becomes more effective to restore trade. As a consequence, there is a trade-off between the cost and benefits of capital in both states of nature that should be taken into account when choosing the optimal level of capital requirements.



This figure shows the expected social benefits of capital requirements. In this case, it is possible to see that the optimal level of capital has negative net benefits under a deposit shock.

Figure 4.5: The social benefits of capital requirements

4.8. Conclusion

The focus of this paper is to explain the fundamental relationship between the type of liquidity shock and the ability of the interbank market to provide liquidity to banks. We employ a standard moral hazard model with limited liability in order to study the mechanism through which liquidity shocks operate. A key element in the model is that banks are monopolists in their deposit markets, therefore these markets are isolated. However, when a shock occurs banks may have incentives to trade funds in the interbank market. Based on events that took place during the European financial crisis, a bank may face a liquidity shortfall because it can suffer a deposit flight or an increase in its non-performing loans. After a shock, the impaired bank can access interbank funds in order to dampen the negative effect of a shock. However, a problem of moral hazard arises because banks may be reluctant to remove bad loans from their portfolio when dealing with a crisis. Financial restructuring tries to restore solvency by increasing the likelihood of loan repayment. Notwithstanding, banks may be unwilling to restructure their portfolio for several reasons: first, they can be forced to sell bad assets at a fire-sale price; second, financial institutions may suffer reputational costs; and, finally, the government may require an increase in the capital buffer. Since the decision of removing bad loans is unobservable, the risk of bank's investment is private information. This fact can break down the interbank market exacerbating the cost of liquidity shortages.

The model analyzed the strength of the balance sheet as a determinant of market freeze, when banks face liquidity shocks. Our model suggests that the effect on banks' balance sheets is larger in the case of "non-performing loans" shocks than in the case of "deposit shocks". Unlike a deposit flight shock, an increase in non-performing loans reduce the "cash in hand" of the bank and, thus, expands bank's liabilities. Therefore an increase in non-performing loans leads to weaker balance sheets than when there is a problem of deposit flight. Then, even with the existence of the interbank market, the effect on financing cost of cash flow shocks is larger than the effect of deposit shocks. Therefore banks' financial performance and the bank's pledgable income are always higher when subjected to deposit shocks. These results show that the type of shock is a fundamental factor in order to explain the interbank market freeze.

During the last European financial crisis liquidity shortfalls were caused by the phenomenon of deposit flight and the increase in non-performing loans, which led interbank lending market to freeze. Our model also suggests that the phenomenon of deposit flight may not be an important factor in order to explain why banks stopped lending to other banks during the last crisis. In contrast, it points out that under deposit shocks the market reaches a better performance allowing banks to withstand higher liquidity shortfalls.

Therefore, we argue that the interbank market freeze may be primarily explained by the decrease in the value of assets rather than by the phenomenon of deposits flight that characterized the European financial crisis. These results may lead to a better understanding of the mechanisms of liquidity redistribution among financial institutions as well as of the framework in which financial institutions operate.

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Appendix A

Appendix of Chapter 2

A.1. Technical assumptions

We start off by defining the level of liquidity $l_\Phi(k)$ that maximizes the marginal return of capital for any given value of k, which is implicitly defined by:

Definition A.1 (Capital return maximizing level of liquidity)**.**

$$g\left(l_\Phi\left(k\right)\right) \cdot D(l_\Phi\left(k\right), k) \equiv \frac{1}{2}.$$

Next, we define thresholds for the maximum and minimum values of the *social cost of capital*.

Definition A.2 (Upper threshold social cost of capital)**.**

$$\rho_{\max} \equiv r + F^2\left(l_\Phi\left(0\right)\right) \cdot D\left(l_\Phi\left(0\right), 0\right) \cdot \frac{1+r}{c}.$$

Definition A.3 (Lower threshold social cost of capital)**.**

$$\rho_{\min} \equiv r + F^2\left(l_\Phi\left(1-d\right)\right) \cdot D\left(l_\Phi\left(1-d\right), 1-d\right) \cdot \frac{1+r}{c}.$$

In the main text we show that for values $\rho \geq \rho_{\max}$ the optimal capital response is zero for any given value of liquidity. In addition, in order to limit the number of cases to discuss and avoid a capital requirement in excess of the maximum level of capital that the bank may hold, we assume that the social cost of capital is sufficiently high so as to have an interior solution. Formally:

Assumption A.1.

$$\rho > \rho_{\min}.$$

We now define a (lower) threshold for the project profitability as follows:

Definition A.4 (Lower threshold project profitability)**.**

$$\underline{M} \equiv \frac{D(l_\Phi\left(0\right), 1-d)}{1-l_\Phi\left(0\right)} + \left\{ \frac{c \cdot \left[\begin{array}{c} \left(\frac{F^2(l_\Phi(0)) \cdot (1-l_\Phi(0)) \cdot D(l_\Phi(0), 1-d)}{c}\right)^2 + \\ 2\frac{F(l_\Phi(0)) \cdot (1-l_\Phi(0))^2}{c} \cdot \left[(1+\rho_{\max}) \cdot (1-d) + D(l_\Phi\left(0\right), 1-d)\right] \end{array} \right]^{1/2}}{F^2(l_\Phi\left(0\right)) \cdot \left(1-l_\Phi\left(0\right)\right)^2} \right\}.$$

The following assumption is a sufficient condition that guarantees that the project profitability is sufficiently high so that the optimal regulatory policy does not entail shutting down the bank.[1]

[1]This is a strong sufficient condition to guarantee that the social value of the bank is not negative, but that we do otherwise not use to prove our results. Given our choice of $\underline{M}$, in equilibrium we have that the social value generated by the bank is positive. We could therefore relax this assumption so that the social value produced by the bank is exactly zero. In order to do so, on Definition A.4 we could replace the maximum possible equilibrium level of liquidity $l_\Phi\left(0\right)$ by the actual equilibrium value of liquidity l^*, as well as the maximum level of capital $1-d$ by the equilibrium value k^*. This replacement would reduce the lower threshold for the minimum level of profitability $\underline{M}$, so that the range of profitability would be enlarged. However, l^* and k^* are determined endogenously in equilibrium, whereas both $l_\Phi\left(0\right)$ and $1-d$ are determined exogenously by the model parametric assumptions. By focusing on these upper thresholds we are able to provide a explicit expression that depends exclusively on the model parameters.

Assumption A.2.

$$M \geq \underline{M}.$$

In order to ensure an interior solution for the level solvency we need to impose a condition on the cost c of reducing insolvency risk. Define the (lower) threshold for the cost of reducing insolvency risk as:

Definition A.5 (Lower threshold insolvency risk reduction cost)**.**

$$\underline{c} \equiv F\left(l_B\left(1-d\right)\right) \cdot \pi\left(l_B\left(1-d\right), 1-d\right).$$

The following asssumption guarantees that insolvency risk is not totally eliminated in equilibrium.

Assumption A.3.

$$c \geq \underline{c}.$$

We also need to impose a condition so as to ensure that the bank is a ”proper bank” in the sense that deposits constitute a non-negligible source of funds for the bank. Otherwise, if the bank relies on too large an amount of non-deposit liabilities, a 100% liquidity requirement may be optimal whenever eliminating illiquidity risk entailed a negligible cost in terms of foregone loans because of a small deposit base. In this situation, deposits would simply be stored as liquid assets and would not play affect the bank’s profits. We define the (lower) threshold for the deposit base as:

Definition A.6 (Lower threshold deposits)**.**

$$g\left(\underline{d}\right) \cdot \left(1-\underline{d}\right) \equiv \frac{1}{2\left(1+r\right)}.$$

Notice that log-concavity of $F(\cdot)$ implies that $\lim_{l\to 0} g(l) = +\infty$, so that we can make $g(l) \cdot (1-l)$ arbitrarily large as l approaches 0. Moreover, log-concavity requires that $g(d) < +\infty$, so that $g(l)\cdot(1-l)$ can be made arbitrarily close to 0 by choosing values of l sufficiently close to d. In addition, observe that $g(l) \cdot (1-l)$ is a strictly decreasing function of l for all $l < d$, since both $g(l)$ and $(1-l)$ are positive strictly decreasing functions of liquidity l. Hence, $\underline{d}$ is well-defined, unique and such that $\underline{d} < 1$. The following asssumption ensures an interior liquidity requirement, so that liquidity risk is never eliminatd in equilibrium.

Assumption A.4.

$$d \geq \underline{d}.$$

Finally, we need a condition on the maximum value that the interest rate r on long-term deposits can achieve, so that deposit liabilities do not explode.

Definition A.7 (Upper threshold long-term deposits interest rate)**.**

$$\bar{r} \equiv \frac{1}{1-d}.$$

We then have that:

Assumption A.5.

$$r \in [0, \bar{r}].$$

A.2. Additional technical assumptions for the asset liquidation case

As the base model, let be $\tilde{l}_{\Phi_{AL}}(k)$ the level of liquidity that maximizes the indirect effect of capital under asset liquidation for a given value of k, which is implicitly defined as:

$$g(\gamma + \tilde{l}_{\Phi_{AL}}(k))(D(\tilde{l}_{\Phi_{AL}}(k), k)) = \frac{1}{2}.$$

Then, we set both an upper and a lower bound for capital cost as follows

$$\rho_{max} = r + F(\gamma + \tilde{l}_{\Phi_{AL}}(0))^2 \left(D(\tilde{l}_{\Phi_{AL}}(0), 0) - \frac{\int_{\tilde{l}_{\Phi_{AL}}(0)}^{\gamma + \tilde{l}_{\Phi_{AL}}(0)} (\beta - \tilde{l}_{\Phi_{AL}}(0)) f(\beta)}{F(\gamma + \tilde{l}_{\Phi_{AL}}(0))} \right) \cdot \frac{1+r}{c}$$

and

$$\rho_{min} = r + F^2(\gamma + \tilde{l}_{\Phi}(1-d)) D\left(\tilde{l}_{\Phi_{AL}}(1-d), 0\right) \cdot \frac{1+r}{c}.$$

The lower bound ρ_{min} ensures that capital requirements never exceed $1-d$, while for any $\rho \geq \rho_{max}$ the optimal capital response is zero for any l.

According to this, we assume **(Assumptions 1-AL)**:

$$\rho_{\min} < \rho < \rho_{\max}.$$

Moreover, we impose a minimum cost for the cost of solvency to ensure an interior solvency level denoted by

$$c_1 = M \cdot F(\gamma + l_{ELV_{AL}}) \cdot (1 - l_{ELV_{AL}}),$$

where $l_{ELV_{AL}}$ is implicitly defined as $g(\gamma + l_{ELV_{AL}})(1 - l_{ELV_{AL}}) = 1$.

Moreover, in order to guarantee that $g(\gamma + l_{R_{AL}}(k))(1 - l_{R_{AL}}(k)) - 1 + \frac{\chi(l)}{\psi} < 0$, we define

$$c_2 = \frac{1 + r(1-d)}{\frac{1+F(l_{ELV_{AL}})}{F(l_{ELV_{AL}})} + g(\gamma + l_{ELV_{AL}})\gamma - 1}.$$

Then, we assume **(Assumptions 2-AL)**:

$$c \geq \underline{c} = Max[c_1, c_2].$$

In order to ensure that the bank always has enough asset in place to liquidate at least a fraction $\frac{\gamma}{\psi}$, we set

$$\gamma_1 = (d - \tilde{l}_{\Phi_{AL}}(0))\psi.$$

On the other hand, we implicitly define γ_2

$$g(\gamma_2)\pi(0,0) = M - 1$$

to ensure that $\theta_{B_{AL}}(l,k)$ is a hump-shaped function of l.

Then, we define $\overline{\gamma}$ as

$$\overline{\gamma} = Min[\gamma_1, \gamma_2],$$

and assume that **(Assumptions 3-AL)**:

$$\gamma \leq \overline{\gamma}.$$

Finally, we implicitly define $\underline{d}$ as

$$g(\gamma + l_{ELV_{AL}})\,(\underline{d} - l_{ELV_{AL}}) = \frac{1}{2}.$$

Then, we assume **(Assumptions 4-AL)**:

$$d > \underline{d}.$$

A.3. Omitted Proofs

Before proceeding to the proof of the Propositions in the main body of the text, we show the following instrumental result, which will be useful in the proofs of Propositions 2.4 and 2.5.

[Properties of $\Phi(l,k)$ curve] Define $\Phi(l,k) \equiv 2g(l) \cdot D(l,k) - 1$, where $g(l) \equiv \frac{f(l)}{F(l)}$ and $D(l,k) \equiv 1 + (1-d) \cdot r - (1+r) \cdot k - l$, as defined in Sections 2.3.1 and 2.3, respectively. Let $l_\Phi(k)$ be implicitly defined as $\Phi(l_\Phi(k), k) = 0$. Then, the functions $\Phi(l,k)$ and $l_\Phi(k)$ satisfy the following properties:

(i) For all $k \in [0, 1-d]$, $\lim_{l\to 0} \Phi(l,k) = +\infty$ and $\Phi(d,k) < 0$.

(ii) For all $k \in [0, 1-d]$, $\Phi(l,k)$ is strictly decreasing in l.

(iii) For all $l \in (0, d]$, $\Phi(l,k)$ is strictly decreasing in k.

(iv) The function $l_\Phi(k)$ is strictly decreasing. Moreover, $l_\Phi(0) \in (0, d)$.

Proof of Fact A.3. (i) The first property, $\lim_{l\to 0} \Phi(l,k) = +\infty$, follows from the fact that $F(\cdot)$ is log-concave, so that $\lim_{l\to 0} g(l) = +\infty$. The second property, namely that $\Phi(d,k) = 2g(d) \cdot [(1+r) \cdot (1-d-k)] - 1 < 0$, follows directly from Assumption A.4 on Section B.1.

(ii) Both $g(l)$ and $D(l,k)$ are positive and strictly decreasing functions, so that the product $g(l) \cdot D(l,k)$ is also positive and strictly decreasing. The result is then immediate.

(iii) This property follows immediately from $\frac{\partial D(l,k)}{\partial} = -(1+r) < 0$.

(iv) The locus $l_\Phi(k)$ satisfies $\Phi(l_\Phi(k), k) = 0$. Implicitly differentiating this expression with respect to k yields $\frac{\partial l_\Phi(k)}{\partial k} = -\frac{\partial \Phi(l,k)}{\partial k} / \frac{\partial \Phi(l,k)}{\partial l} < 0$, the last inequality following from properties (ii) and (iii). We now show that $l_\Phi(0) < d$. From from Property (ii) in Fact A.3, we have that $\Phi(l,0)$ is strictly decreasing in its first argument. Also, $\Phi(l,0)$ is continuous in its first argument as well. Moreover, $\lim_{l\to 0} \Phi(l,0) = +\infty$. In addition, it follows from item (i), just proved, that $\Phi(d,0) < 0$. Hence, it follows that there exists a unique $l_\Phi(0) \in (0,d)$ that solves $\Phi(l_\Phi(k), k) = 0$. $\square$

[Properties of the regulator's objective function] The regulator's objective function $\Pi_R(l,k)$ satisfies:

(i) For any given $l \in [0,d]$, $\Pi_R(l,k)$ is strictly concave in k in all of its domain.

(ii) For any given $k \in [0,]$, $\Pi_R(l,k)$ is strictly quasiconcave in l in the domain $[l_B(k), l_\Phi(k)]$.

Remark A.1. For a family of log-concave distribution functions, the function $\Pi_R(l,k)$ is strictly convex in l around zero. Hence, $\Pi_R(l,k)$ is *not* concave in l in all of its domain. However, $\Pi_R(l,k)$ is quasiconcave in l in all of its domain. We restrict ourselves to showing that $\Pi_R(l,k)$ is quasiconcave in l in the domain $[l_B(k), l_\Phi(0)]$ because this weaker statement suffices for our purposes and the proof is (much) less involved.

Proof of Fact A.3. (i) For any given $l \in [0,d]$, we have that

$$\frac{\partial^2 \Pi_R(l,k)}{\partial k^2} = -\frac{F^2(l) \cdot (1+r)^2}{c} < 0.$$

(ii) We prove this property through a succession of steps through which we show that, for any given k, there exists a unique value of liquidity $l_R(k) \in (l_B(k), l_\Phi(0))$ such that $\frac{d\Pi_R(l_R(k),k)}{dl} = 0$. Consider an arbitrary $k \in [0, 1-d]$ and, for the sake of notational simplicity, rename the first derivative of the bank and the regulator's objective functions, respectively, as follows:

$$H_B(l,\theta,k) \equiv \frac{d\Pi_B(l,\theta,k)}{dl},$$

and

$$H_R(l,k) \equiv \frac{d\Pi_R(l,k)}{dl}.$$

Proof. $H_R(l_B(k), k) > 0$, where $l_B(k)$ is the unique value satisfying $H_B(l_B(k), k) = 0$.

We can write the regulator's objective function in terms of the bank's as follows:

$$\Pi_R(l,k) = \Pi_B(l, \theta_B(l,k), k) - (1 - \theta_B(l,k) \cdot F(l)) \cdot D(l,k).$$

We can therefore write the regulator's first derivative w.r.t. liquidity as:

$$H_R(l,k) = H_B(l,\theta_B(l,k),k) - \frac{d}{dl}\left[(1-\theta_B(l,k)\cdot F(l))\cdot D(l,k)\right].$$

From the bank's FOC w.r.t. to liquidity, we have that $H_B(l_B(k),\theta_B(l_B(k),k),k) = 0$. Moreover, from Proposition 2.1, it follows that $\frac{\partial}{\partial l}\theta_B(l_B(k),k) = 0$. Hence, we can write

$$H_R(l_B(k),k) = 1 + \theta_B(l_B(k),k)\cdot F(l_B(k))\cdot[g(l_B(k))\cdot D(l_B(k),k) - 1]$$

Notice that $g(l_B(k))\cdot D(l_B(k),k) \geq 0$. Hence $g(l_B(k))\cdot D(l_B(k),k) - 1 \geq -1$. Moreover, $\theta_B(l_B(k),k)\cdot F(l_B(k)) < 1$ (both $\theta_B(\cdot,\cdot)$ and $F(\cdot)$ are smaller or equal to 1, the former always strictly smaller). Hence, we have that $H_R(l_B(k),k) > 0$.

$H_R(l_\Phi(0),k) < 0$ where, given $\Phi(l,k) = 2g(l)\cdot D(l,k) - 1$ as defined in Fact A.3, $l_\Phi(k)$ is the unique value satisfying $\Phi(l_\Phi(k),k) = 0$.

Straightforward algebra leads to:

$$H_R(l_\Phi(0),k) = \frac{F(l_\Phi(0))^2}{c}\cdot\left(\begin{array}{c} M\cdot\pi(l_\Phi(0),k)\cdot[g(l_\Phi(0))\cdot(1-l_\Phi(0))-1] \\ +D(l_\Phi(0),k)\cdot[g(l_\Phi(0))\cdot\pi(l_\Phi(0),k)-M+1] \end{array}\right) + 1.$$

Recall that $\pi(l,k) \equiv M\cdot(1-l) - D(l,k)$. Abusing notation, we have that $\frac{\partial\pi(l_\Phi(0),k)}{\partial M} = 1 - l_\Phi(0)$. Recognizing that $H_R(l_\Phi(0),k)$ depends on M both directly and through $\pi(l_\Phi(0),k)$, we can then derive the following expression:

$$\frac{dH_R(l_\Phi(0),k)}{dM} = 2M\cdot\frac{F(l_\Phi(0))^2}{c}\cdot(1-l_\Phi(0))\cdot[g(l_\Phi(0))\cdot(1-l_\Phi(0))-1].$$

By construction of $l_\Phi(0)$, we have that $g(l_\Phi(0))\cdot D(l_\Phi(0),k) = 1/2 < 1$. Hence, it follows that $g(l_\Phi(0))\cdot(1-l_\Phi(0)) - 1 < 0$. Hence, we have that $\frac{\partial H_R(l_\Phi(0),k)}{\partial M} < 0$. Hence, there exists $\underline{\underline{M}}$ such that $H_R(l_\Phi(0),k)\,|_M < 0$ for all $M > \underline{\underline{M}}$. Tedious but straightforward algebra shows that $\underline{\underline{M}} < \underline{M}$, as defined in Definition A.4.

$H_R\left(\hat{l}, k\right) = 0$ for some $\hat{l} \in (l_B(k), l_\Phi(0))$.

It follows directly from Steps A.3 and A.3 and the continuity of $H_R(l,k)$ in l.

For any $\hat{l} \in (l_B(k), l_\Phi(0))$ such that $H_R\left(\hat{l}, k\right) = 0$, it follows that $\frac{\partial H_R(\hat{l},k)}{\partial l} < 0$.

Define the following function:

$$\chi(l) \equiv M \cdot \pi(l,k) \cdot [g(l) \cdot (1-l) - 1] + D(l,k) \cdot [g(l) \cdot \pi(l,k) - M + 1].$$

We can then write the regulator's first derivative w.r.t. liquidity as follows:

$$H_R(l,k) = \frac{F(l)^2}{c} \cdot \chi(l) + 1.$$

By construction of $\hat{l}$ (Step A.3), we have that:

$$H_R\left(\hat{l}, k\right) = \frac{F(\hat{l})^2}{c} \cdot \chi\left(\hat{l}\right) + 1 = 0.$$

Hence, it follows that $\chi\left(\hat{l}\right) < 0$.

The first derivative of $H_R(l,k)$ w.r.t. l can be written, for $l > 0$, as:

$$\frac{\partial H_R(l,k)}{\partial l} = \frac{F(l)^2}{c} \cdot \left(2g(l) \cdot \chi(l) + \frac{\partial \chi(l)}{\partial l}\right).$$

Since $\chi\left(\hat{l}\right) < 0$, recognizing that $F(l) > 0$ and $g(l) > 0$ for all l, it suffices to show that $\frac{\partial \chi(\hat{l})}{\partial l} < 0$. We can write:

$$\begin{aligned}
\frac{\partial \chi(l)}{\partial l} &= M^2 \cdot [1 - 2g(l) \cdot (1-l)] + \left[M^2 \cdot (1-l) - D(l,k)\right] \cdot (1-l) \cdot \frac{\partial g(l)}{\partial l} \\
&\quad + [g(l) \cdot (1-l) - 1] + [D(l,k) \cdot g(l)] \leq \\
&\leq \left[\left(M^2 - 1\right) \cdot (1 - 2g(l) \cdot (1-l))\right] + \left[M^2 \cdot (1-l) - D(l,k)\right] \cdot (1-l) \cdot \frac{\partial g(l)}{\partial l}
\end{aligned}$$

By definition of $l_\Phi(0)$, we know that $1 - 2g(l_\Phi(0)) \cdot (1 - l_\Phi(0)) = 0$. Recognizing that $g(l) \cdot (1-l)$ is strictly decreasing in l (recall that both $1-l$ and $g(l)$ are strictly decreasing and positive functions, the latter because of log-concavity of the distribution function), it follows that $1 - 2g\left(\hat{l}\right) \cdot \left(1 - \hat{l}\right) < 0$. Clearly, $M^2 - 1 > 0$. Moreover, since $M > 1$, we have that $M^2 \cdot (1-l) - D(l,k) > M \cdot (1-l) - D(l,k) = \pi(l,k) > 0$. From log-concavity of the distribution function, we have that $\frac{\partial g(l)}{\partial l} < 0$ for all l. Hence, it follows that $\frac{\partial \chi(\hat{l})}{\partial l} < 0$.

There exists a unique $\hat{l} \in (l_B(k), l_\Phi(0))$, which we dub $l_R(k)$, such that $H_R(l_R(k), k) = 0$.

Assume, for the sake of contradiction, that there is more than one $\hat{l} \in (l_B(k), l_\Phi(0))$ such that $H_R\left(\hat{l}, k\right) = 0$. Let $\hat{l}_1, \hat{l}_2$, with $\hat{l}_1 < \hat{l}_2$ be the first two values satisfying $H_R\left(\hat{l}_1, k\right) = H_R\left(\hat{l}_2, k\right) = 0$. From continuity of $H_R(l,k)$ with respect to l, it follows that $\frac{\partial \chi(\hat{l}_2)}{\partial l} > 0$, which contradicts the statement shown in Step A.3. □

□

Proof of Proposition 2.4. Since, for any given l, $\Pi_R(l,k)$ is strictly concave in k in all of its domain, the first order necessary condition for an interior optimal capital requirement is also sufficient. Consider the first order condition of the social welfare objective function with respect to capital:

$$\frac{d\Pi_R(l,k)}{dk} = \frac{\partial \Pi_R(l,k)}{\partial \theta} \cdot \frac{\partial \theta_B(l,k)}{\partial k} + \frac{\partial \Pi_R(l,k)}{\partial k},$$

which we can write as follows:

$$\frac{d\Pi_R(l,k)}{dk} = \frac{1+r}{c} \cdot F^2(l) \cdot D(l,k) - (\rho - r).$$

Take any given set of parameters c, r and ρ. Fix any $l \in [0,d]$. Then, $k_R(l) > 0$ only if $\frac{d\Pi_R(l,k_R(l))}{dk} = 0$. Hence, if $k_R(l) > 0$ we have that $\Upsilon(l, k_R(l)) = \frac{(\rho - r)\cdot c}{1+r}$. Moreover, if $\frac{d\Pi_R(l,k)}{dk} < 0$ for all $k \in [0, 1-d]$, then $k_R(l) = 0$. The proof consists of comparing $\Upsilon(l,k) \equiv F^2(l) \cdot$

$D(l,k)$ with $\frac{(\rho-r)\cdot c}{1+r}$. In order to do this comparison, we first show that $\Upsilon(l,k)$ satisfies the following properties.

The function $\Upsilon(l,k) \equiv F^2(l) \cdot D(l,k)$ satisfies the following properties.

(a) For any given $k \in [0, 1-d]$, $\Upsilon(l,k)$ is strictly increasing in l for $l < l_\Phi(k)$ and strictly decreasing in l for $l > l_\Phi(k)$, where $l_\Phi(k)$ was implicitly defined as $\Phi(l_\Phi(k), k) = 0$ in Fact A.3.

(b) For any given $l \in (0, d]$, $\Upsilon(l,k)$ is strictly decreasing in k.

(c) $\Upsilon(l_\Phi(k), k)$ is strictly decreasing in k.

Proof of Fact A.3. (a) Fix k, and take the first derivative of $\Upsilon(l,k)$ with respect to l to get $\frac{\partial \Upsilon(l,k)}{\partial l} = F^2(l) \cdot \Phi(l,k)$, where $\Phi(l,k)$ is as defined on Fact A.3. Since $F^2(l) > 0$, the sign of $\frac{\partial \Upsilon(l,k)}{\partial l}$ is determined by $\Phi(l,k)$. From Property (i) in Fact A.3 and the definition of $l_\Phi(k)$, we have that $\frac{\partial \Upsilon(l,k)}{\partial l} > 0$ for all $l \in (0, l_\Phi(k))$. Also, from Property (i) in Fact A.3, we have that $\frac{\partial \Upsilon(d,k)}{\partial l} < 0$. Moreover, from Property (ii) in Fact A.3, it follows that $\frac{\partial \Upsilon(l,k)}{\partial l} < 0$ for all $l \in (l_\Phi(k), d)$. Hence, the result follows.

(b) This statement follows directly from first differentiating $\Upsilon(l,k)$ with respect to k, to obtain $\frac{\partial \Upsilon(l,k)}{\partial k} = -F^2(l) \cdot (1+r) < 0$.

(c) Take any two $k_1 < k_2$. From (b) it follows that, for any given $l \in (0, d]$, $\Upsilon(l, k_1) > \Upsilon(l, k_2)$. Also, by definition of $l_\Phi(k)$, we have that $\Upsilon(l_\Phi(k_1), k_1) \geq \Upsilon(l, k_1)$ for all $l \in (0, d]$. Combining these two inequalities, we have that $\Upsilon(l_\Phi(k_1), k_1) > \Upsilon(l, k_2)$ for any given $l \in (0, d]$. In particular, the last inequality holds true for $l = l_\Phi(k_2)$. Hence, we can write $\Upsilon(l_\Phi(k_1), k_1) > \Upsilon(l_\Phi(k_2), k_2)$. □

Now we show the Proposition statements.

(i) Let $\rho_{\max}$ be the unique value satisfying $\Upsilon(l_\Phi(0), 0) = \frac{(\rho_{\max}-r)\cdot c}{1+r}$. Observe that, for any given k, $\Upsilon(l,k)$ is maximized at $l_\Phi(k)$ (Property (a)); also, for any given l, $\Upsilon(l,k)$ is maximized at $k = 0$ (Property (b)). Hence, $\Upsilon(l,k)$ is jointly maximized for $(l,k) = (l_\Phi(0), 0)$. Hence, by construction of $\rho_{\max}$, it follows that $\Upsilon(l,k) < \frac{(\rho-r)\cdot c}{1+r}$ for all (l,k), as we wanted to show.

(ii) Now, let $\rho < \rho_{\max}$. We prove this result in a succession of steps.

The optimal capital response at $l = 0$ is zero, i.e., $k_R(0) = 0$.

Observe that $\Upsilon(0,k) = 0 < \frac{(\rho-r)\cdot c}{1+r}$ for all $k \in [0, 1-d]$. Hence, we have that $k_R(0) = 0$.

There exists a value $l_1(\rho) \in (0,d)$ such that $\Upsilon(l_1(\rho), 0) = \frac{(\rho-r)\cdot c}{1+r}$. Moreover, $\Upsilon(l,0) < \frac{(\rho-r)\cdot c}{1+r}$ for $l < l_1(\rho)$ and $\Upsilon(l,0) > \frac{(\rho-r)\cdot c}{1+r}$ for $l > l_1(\rho)$.

First, recall from the previous statement that $\Upsilon(0,0) = 0 < \frac{(\rho-r)\cdot c}{1+r}$. Moreover, by construction of $\rho_{\max}$, which is such that $\Upsilon(l_\Phi(0), 0) > \frac{(\rho_{\max}-r)\cdot c}{1+r}$, we have that $\Upsilon(l_\Phi(0), 0) > \frac{(\rho-r)\cdot c}{1+r}$, where $l_\Phi(0) \in (0,d)$ (From Property (iv) in Fact A.3). Moreover, it follows from Property (a) in Fact A.3, that $\Upsilon(l,k)$ is strictly increasing in l for $l < l_\Phi(0)$. Hence, by continuity of $\Upsilon(l,k)$ in its first argument, it follows that there exists a unique value for liquidity, which we dub $l_1(\rho)$, such that $\Upsilon(l_1(\rho), 0) = \frac{(\rho-r)\cdot c}{1+r}$. It also follows that $\Upsilon(l,0) < 0$ for $l < l_1(\rho)$ and $\Upsilon(l,0) > 0$ for $l > l_1(\rho)$.

The optimal response for $l \le l_1(\rho)$ is zero, i.e., $k_R(l) = 0$ for any $l \le l_1(\rho)$.

We have that $\Upsilon(l,k)$ is strictly decreasing in k for all $l > 0$ (Property (b) in Fact A.3). Also, we know from Step A.3 that $\Upsilon(l,0) < 0$ for $l < l_1(\rho)$. Hence, it follows that $\Upsilon(l,k) < 0$ for any $l < l_1(\rho)$ and any $k \in [0, 1-d]$.

There exists a unique liquidity maximizer $\hat{l}(\rho) \in (l_1(\rho), d)$ of the capital response function, that is, $k_R(\hat{l}(\rho)) > k_R(l)$ for all $l \neq \hat{l}(\rho)$. Moreover, $\hat{l}(\rho)$ and $k_R(\hat{l}(\rho)$ are defined by $\Phi\left(\hat{l}(\rho), k_R(\hat{l}(\rho)\right) = 0$.

By construction of $\rho_{\max}$, we have that $\Upsilon(l_\Phi(0), 0) > \frac{(\rho-r)\cdot c}{1+r}$. From Property (c) in Fact A.3, it follows that there exists a unique $k' > 0$ such that $\Upsilon(l_\Phi(k'), k') = \frac{(\rho-r)\cdot c}{1+r}$. Let $\hat{l}(\rho) = l_\Phi(k')$. Hence, we have that $k_R\left(\hat{l}(\rho)\right) = k'$. Now, consider $k'' > k'$. Then, using again Property

(c) in Fact A.3, it follows that $\Upsilon\left(l_{\Phi}\left(k'\right),k''\right) < \frac{(\rho-r)\cdot c}{1+r}$. Now, from Property (iv) in Fact A.3, we have that $l_{\Phi}\left(k\right)$ is strictly decreasing in k. Hence, since $k'' > k'$, we have that $l_{\Phi}\left(k''\right) < l_{\Phi}\left(k'\right)$. Also, from Property (a) in Fact A.3, we know that $\Upsilon\left(l,k\right)$ is strictly increasing in l for $l < l_{\Phi}\left(k\right)$. Hence, since $l_{\Phi}\left(k''\right) < l_{\Phi}\left(k'\right)$, it follows that $\Upsilon\left(l_{\Phi}\left(k''\right),k''\right) < \Upsilon\left(l_{\Phi}\left(k'\right),k''\right)$. Consequently, $\Upsilon\left(l_{\Phi}\left(k''\right),k''\right) < \frac{(\rho-r)\cdot c}{1+r}$. Hence, $k_R\left(\hat{l}\left(\rho\right)\right) = k'$ is indeed the maximum capital response, which is achieved at $\hat{l}\left(\rho\right) = l_{\Phi}\left(k'\right)$. Finally, from construction of $l_{\Phi}\left(k'\right)$, it follows that $\Phi\left(\hat{l}\left(\rho\right),k_R(\hat{l}\left(\rho\right)\right) = 0$.

The function $\Phi\left(l,k_R\left(l\right)\right)$ is such that $\Phi\left(l,k_R\left(l\right)\right) > 0$ if $l < \hat{l}\left(\rho\right)$.

From Step A.3, we know that $\Phi\left(\hat{l}\left(\rho\right),k_R(\hat{l}\left(\rho\right)\right) = 0$. Consider $l < \hat{l}\left(\rho\right)$. We also know from Step A.3 that $k_R(\hat{l}\left(\rho\right)) > k_R(l)$. From Properties (ii) and (iii) in Fact A.3, we know that $\Phi\left(l,k\right)$ is strictly decreasing in k and in l. Hence, it follows that $\Phi\left(l,k_R\left(l\right)\right) > \Phi\left(\hat{l}\left(\rho\right),k_R(\hat{l}\left(\rho\right)\right) = 0$ for $l < \hat{l}\left(\rho\right)$.

For any $l \in \left(l_1\left(\rho\right),\hat{l}\left(\rho\right)\right)$, we have that the optimal capital response increases in l, that is, $k_R\left(l\right)$ strictly increasing in l.

From Step A.3, we have that $\Upsilon\left(l,0\right) > \frac{(\rho-r)\cdot c}{1+r}$ for $l > l_1\left(\rho\right)$. Hence, we have a interior solution, that is, $k_R\left(l\right)$ is such that $\Upsilon\left(l,k_R\left(l\right)\right) = \frac{(\rho-r)\cdot c}{1+r}$. From Property (b) in Fact A.3, we know that $\Upsilon\left(l,k\right)$ is strictly decreasing in k. Hence, there exists a unique capital level, which we label $k_R\left(l\right)$, such that $\Upsilon\left(l,k_R\left(l\right)\right) = \frac{(\rho-r)\cdot c}{1+r}$. Now, we show that $k_R\left(l\right)$ is strictly increasing by showing that $\frac{\partial k_R(l)}{\partial l} > 0$. Implicitly differentiating $\Upsilon\left(l,k_R\left(l\right)\right) = \frac{(\rho-r)\cdot c}{1+r}$ with respect to l it follows that $\frac{\partial k_R(l)}{\partial l} > 0$ if and only $\frac{\partial \Upsilon(l,k_R(l))}{\partial l} > 0$, that is, if and only if $\Phi\left(l,k_R\left(l\right)\right) > 0$. Hence, it follows from Step A.3 that if $l < \hat{l}\left(\rho\right)$ we have $\frac{\partial k_R(l)}{\partial l} > 0$.

For any $l \in \left(\hat{l}\left(\rho\right),l_2\left(\rho\right)\right)$, we have that the optimal capital response decreases in l, that is, $k_R\left(l\right)$ strictly decreasing in l.

From construction of $k_R\left(l\right)$, we have that $\Upsilon\left(\hat{l}\left(\rho\right),k_R\left(\hat{l}\left(\rho\right)\right)\right) = \frac{(\rho-r)\cdot c}{1+r}$. Moreover, since $\Upsilon\left(l,k\right)$ is strictly decreasing in k, it follows that $\Upsilon\left(\hat{l}\left(\rho\right),0\right) > \frac{(\rho-r)\cdot c}{1+r}$. By continuity of $\Upsilon\left(l,k\right)$ with respect to its first argument, it follows that there exists $\tilde{l}_2\left(\rho\right) > \hat{l}\left(\rho\right)$ such that $\Upsilon(l,0) > \frac{(\rho-r)\cdot c}{1+r}$ for all $l \in \left(\hat{l}(\rho), \tilde{l}_2(\rho)\right)$. Hence, for all $l \in \left(\hat{l}(\rho), \tilde{l}_2(\rho)\right)$, we have that $k_R(l) > 0$. Hence, for all $l \in \left(\hat{l}(\rho), \tilde{l}_2(\rho)\right)$, $k_R(l)$ is given by $\Upsilon(l, k_R(l)) = \frac{(\rho-r)\cdot c}{1+r}$. Consider $l \in \left(\hat{l}(\rho), \tilde{l}_2(\rho)\right)$ and let $k_R(l)$ be such that $\Upsilon(l, k_R(l)) = \frac{(\rho-r)\cdot c}{1+r}$. Recall that $\Upsilon\left(\hat{l}(\rho), k_R(\hat{l}(\rho)\right) = \frac{(\rho-r)\cdot c}{1+r}$. From Property (a) in Fact A.3, we have that $\Upsilon\left(l, k_R(\hat{l}(\rho)\right) < \Upsilon\left(\hat{l}(\rho), k_R(\hat{l}(\rho)\right) = \frac{(\rho-r)\cdot c}{1+r}$ for all $l \in \left(\hat{l}(\rho), \tilde{l}_2(\rho)\right)$. Now, since $\Upsilon(l,k)$ strictly decreasing in k, it follows that $k_R(l) < k_R\left(\hat{l}(\rho)\right)$. Hence, we have shown that $k_R(l) < k_R\left(\hat{l}(\rho)\right)$ for $l \in \left(\hat{l}(\rho), \tilde{l}_2(\rho)\right)$. Now, consider any $l' \in \left(\hat{l}(\rho), \tilde{l}_2(\rho)\right)$ such that $l' > l$. Then, by the same token, we have that $k_R(l') < k_R(l)$.

Finally, to complete this statement of the proof, consider the following two cases. First, assume that $\Upsilon(d,0) \geq \frac{(\rho-r)\cdot c}{1+r}$. Then, it follows that $d = \tilde{l}_2(\rho)$. In this case, we have that $l_2(\rho) = d$. Now suppose, on the contrary, that $\Upsilon(d,0) < \frac{(\rho-r)\cdot c}{1+r}$. In this case, we have that there exists $l_2(\rho) < d$ such that $\Upsilon(l_2(\rho), 0) = \frac{(\rho-r)\cdot c}{1+r}$. Then, for any $l \in (l_2(\rho), d)$, we have that $\Upsilon(l,0) < \frac{(\rho-r)\cdot c}{1+r}$, in which case $k_R(l) = 0$.

(iii) For the remainder of the proof, let $\rho' < \rho$. Recall from Step A.3 above that $l_1(\rho)$ is defined as $\Upsilon(l_1(\rho), 0) = \frac{(\rho-r)\cdot c}{1+r}$. Then, we have that $\Upsilon(l_1(\rho), 0) > \frac{(\rho'-r)\cdot c}{1+r} = \Upsilon(l_1(\rho'), 0)$, where the first inequality follows from $\rho' < \rho$ and the last inequality follows from definition of $l_1(\rho')$. Since $\Upsilon(l,0)$ is strictly increasing in l for any $l < l_\Phi(0)$ (Property (a) in Fact A.3) and $l_1(\rho) < l_\Phi(0)$, the result follows.

Now, if $l_2(\rho) = d$, we have argued in the last statement of the proof of item (ii) that $\Upsilon(d,0) \geq \frac{(\rho-r)\cdot c}{1+r}$. Hence, it follows that $\Upsilon(d,0) > \frac{(\rho'-r)\cdot c}{1+r}$. Hence, $l_2(\rho) = d$. Now, suppose that $l_2(\rho) < d$, in which case $\Upsilon(l_2(\rho), 0) = \frac{(\rho-r)\cdot c}{1+r}$. It then follows that $\Upsilon(l_2(\rho), 0) > \frac{(\rho'-r)\cdot c}{1+r}$. Since $\Upsilon(l,0)$ is strictly decreasing in l for any $l > l_\Phi(0)$ (Property (a) in Fact A.3) and $l_2(\rho) > l_\Phi(0)$, the result follows.

Finally, consider $l \in (l_1(\rho), l_2(\rho))$. Then, we have that $\Upsilon(l, k_R(l)) = \frac{(\rho-r)\cdot c}{1+r} > \frac{(\rho'-r)\cdot c}{1+r}$. Hence, since $\Upsilon(l,k)$ is strictly decreasing in k, it follows that $k_R(l)$ must increase so as to meet the last inequality □

Proof of Proposition 2.5. Since, for any given k, $\Pi_R(l,k)$ is strictly quasiconcave in l in the domain $[l_B(k), l_\Phi(0)]$, the first order necessary condition for an interior optimal liquidity requirement is also sufficient.

(i) Any liquidity requirement such that $l \leq l_B(k)$ is non-binding, so that the bank would choose $l_B(k)$ if the liquidity requirement is smaller. This requirement is not optimal, since $\frac{d\Pi_R(l_B(k),k)}{dl} > 0$ (Step A.3 in the Proof of Fact A.3). Moreover, we have that there exists a unique value of liquidity $l_R(k) \in (l_B(k), l_\Phi(0))$ such that $\frac{d\Pi_R(l_R(k),k)}{dl} = 0$ (Step A.3 in the Proof of Fact A.3). Hence, the liquidity requirement $l_R(k)$ is such that $l_R(k) > l_B(k)$.

(ii) In order to prove this result, we show that there exists a value of capital $\hat{k}(M)$ such that, for any $k < \hat{k}(M)$, it follows that $\frac{\partial l_R(k)}{\partial k} > 0$, while for any $k > \hat{k}(M)$, we have that $\frac{\partial l_R(k)}{\partial k} < 0$. We start by calculating the slope of $l_R(k)$, which is given by:

$$\frac{\partial l_R(k)}{\partial k} = -\frac{\frac{\partial^2 \Pi_R(l,k)}{\partial k \partial l}}{\frac{\partial^2 \Pi_R(l,k)}{\partial l^2}}.$$

From the regulator's problem second order condition with respect to l, we know that $\frac{\partial^2 \Pi_R(l,k)}{\partial l^2} < 0$, so that the sign of $\dfrac{\partial l_R(k)}{\partial k}$ is given by the second order cross derivative, which in turn is given by:

$$\frac{\partial^2 \Pi_R(l,k)}{\partial k \partial l} = \frac{(1+r)F(l)^2}{c}\left[2g(l)D(l,k) - 1\right].$$

Note that the term in brackets determines the sign of $\frac{\partial^2 \Pi_R(l,k)}{\partial k \partial l}$, and therefore the sign of $\dfrac{\partial l_R(k)}{\partial k}$. Particularly, if the second order cross derivative equals zero, then we have that $\dfrac{\partial l_R(k)}{\partial k} = 0$. Let us now define:

$$\Phi(l,k) \equiv [2g(l)D(l,k) - 1] = 0, \tag{A.1}$$

as the loci of k and l such that $\frac{\partial^2 \Pi_R(l,k)}{\partial k \partial l} = 0$. Note that the term inside brackets decreases with capital and liquidity. Hence, if the the pair $(l_R(k), k)$ lies below the curve $\Phi(l,k)$ evaluated at

$(l_R(k), k)$, then we have that $2g(l_R(k))D(l_R(k), k) - 1 > 0$ and that $\dfrac{\partial l_R(k)}{\partial k} > 0$, as otherwise $\dfrac{\partial l_R(k)}{\partial k} < 0$. Since, for $k = 0$, we have that $\Phi(l_R(0), 0) > 0$, while for $k = 1 - d$ we have that $\Phi(tl_R(1-d), 1-d) < 0$, the continuity of $l_R(k)$ guarantees that there exists at least a value of equity capital $\hat{k}(M)$ such that $\Phi\left(l_R(\hat{k}(M)), \hat{k}(M)\right) = 0$.

We now prove that $\hat{k}(M)$ is unique. We prove this statement by showing that the curve $l_R(k)$ never crosses the curve $\Phi(l, k)$ from above. Hence, $\frac{\partial l_R(k)}{\partial k}$ cannot change its sing from negative to positive. By the sake of contradiction, assume that there exists a value $\hat{k}' > \hat{k}(M)$ such that $l_R(k)$ crosses $\Phi(l, k)$ at $(l_R(\hat{k}'), \hat{k}')$ from above. This implies that $\frac{\partial l_R(\hat{k}')}{\partial l} < 0$. However, since $\Phi(l_R(\hat{k}'), \hat{k}') = 0$, the slope of l_R at $\hat{k}'$ must be $\frac{\partial l_R(\hat{k}')}{\partial k} = 0$. Then, there is no value of $k > \hat{k}(M)$ such that $\frac{\partial l_R(k)}{\partial k} > 0$. We thus conclude that $l_R(k)$ increases for $k < \hat{k}(M)$ and decreases for $k > \hat{k}(M)$. Hence, $l_R(k)$ is a hump-shaped function of capital. $\square$

Proof of Proposition 2.6. (i) (Uniqueness) First, we show that the regulator's maximization problem satisfies the condition for the Theorem of the Maximum (Berge (1963)) for quasiconcave objective functions over convex-valued sets to apply. Let M take a fixed value. Define the set of possible joint liquidity and capital optimal requirements $\Phi = [0, d] \times [0, 1-d]$ and the set of all possible values of the cost of capital $\Gamma = [\rho_{\min}, \rho_{\max}]$. Define the real-valued function $\tilde{\Pi}_R : \Phi \times \Gamma \to \mathbf{R}$ as $\tilde{\Pi}_R(l, k, \rho) \equiv \Pi_R(l, k)$, as defined in equation (2.6), where $\tilde{\Pi}_R(l, k, \rho)$ is simply a relabeling of the regulator's objective function to let it depend not only on k and l, but also on the cost of capital parameter ρ. Moreover, define the mapping $\Phi(\rho) : \Gamma \to \Phi$ of possible joint liquidity and capital optimal requirements for each value of the cost of capital $\rho \in \Gamma$. That is, we have that $\Phi(\rho) = [0, d] \times [0, 1-d]$ for all $\rho \in \Gamma$. Let the pair $(l^*(\rho, M), k^*(\rho, M)) \in \arg\max_{(l,k) \in \Phi(\rho)} \tilde{\Pi}_R(l, k, \rho)$ be the optimal liquidity and capital requirements, for a given fixed M, and let $\tilde{\Pi}_R^*(\rho) = \tilde{\Pi}_R(l^*(\rho, M), k^*(\rho, M), \rho)$ be regulator's value for each $\rho \in \Gamma$. First, we have that $\tilde{\Pi}_R(l, k, \rho)$ is jointly continuous in all its arguments. Moreover, $\tilde{\Pi}_R(l, k, \rho)$ is strictly quasiconcave in (l, k) for each $\rho \in \Gamma$ (we have shown above that $\Pi_R(l, k)$ is strictly concave in k and single-peaked–hence quasiconcave–in l; proving that $\Pi_R(l,k)$ is quasiconcave in (l,k) entails computing the bordered Hessian of $\Pi_R(l,k)$, which is tedious but straightforward). Also, $\Phi(\rho)$ is a compact-valued, convex-valued and continuous correspondence for all $\rho \in \Gamma$, because it is an invariant compact and convex set for all $\rho \in \Gamma$. Hence, by the Theorem of the Maximum for quasiconcave functions over convex-valued sets, it follows that $\tilde{\Pi}_R^*(\rho)$ is continuous in ρ and $(l^*(\rho,M), k^*(\rho,M))$ is a (single-valued) continuous function for all $\rho \in \Gamma$, for any given M.

(ii.a) (Zero capital requirements for high cost of capital) From Proposition 2.4, we have that for all $\rho \geq \rho_{\max}$ the optimal capital response curve is zero for all liquidity levels. In this case, we have that the optimal liquidity and capital requirements are given by $l^* = l_R(0)$ and $k^* = 0$.

For any $\rho < \rho_{\max}$, the optimal capital response curve is positive in a range $(l_1(\rho), l_2(\rho))$ and zero elsewhere, i.e., $k_R(\rho) > 0$ if and only if $l \in (l_1(\rho), l_2(\rho))$. Moreover, we also know that $l_1(\rho)$ is continuous and decreasing in ρ. Hence, $l_1(\rho)$ attains a maximum at $l_1(\rho_{\max})$ and a minimum at $l_1(\rho_{\min})$. Also, from Proposition 2.5, we know that $l(k)|_M$ is continuous and decreasing in M for all k. Hence, $l(0)|_M$ attains a maximum at $l(0)|_{\underline{M}}$ for each k. Moreover, we know that $\inf_{M \in (\underline{M},\infty)} \{l(0)|_M\} = l_{ELV}$ and also that $l(0)|_M > l_{ELV}$ for all $M < \infty$. Let $\bar{\rho}(M)$ be implicitly defined as $l(0)|_M = l_1(\bar{\rho}(M))$. We distinguish two cases. If $l_1(\rho_{\min}) > l_{ELV}$, then there exists $\tilde{M} > \underline{M}$ such that for any $M \geq \tilde{M}$ the optimal liquidity and capital response curves do only intersect in the range in which the optimal capital response curve is flat (case 1). However, if either $l_1(\rho_{\min}) > l_{ELV}$ and $M < \tilde{M}$, or if $l_1(\rho_{\min}) \leq l_{ELV}$, then the curves do intersect for some M at some point in which the optimal capital response is positive (case 2). In case 1, we have that the optimal liquidity and capital requirements are given by $l^* = l_R(0)$ and $k^* = 0$. We focus henceforth on case 2.

Let $\rho \geq \bar{\rho}(M)$. We show that the optimal liquidity and capital requirements are given by $l^* = l_R(0)$ and $k^* = 0$, respectively, for any $\rho \geq \bar{\rho}(M)$. Fix M and let $\tilde{\rho}$ be supremum of the set of values of the cost of capital for which the curves intersect at some level for which capital is positive, that is, $\tilde{\rho} = \sup\{\rho : k^* (\rho, M) > 0\}$. If $\tilde{\rho} \leq \bar{\rho}(M)$, then we are done. Suppose now that $\tilde{\rho} > \bar{\rho}(M)$. Then, by construction of $\tilde{\rho}$, we have that the curves intersect in the range in which the optimal capital response is zero for $\rho > \tilde{\rho}$ and for some level of capital bounded away from zero, $\tilde{k} > 0$, for $\rho = \tilde{\rho}$. Hence, it follows that the capital requirement would be $k^* (\rho, M) = 0$ for $\rho > \tilde{\rho}$. We now show that and $k^* (\tilde{\rho}, M) = 0$ as well. Define the sequences $\rho_n \equiv \tilde{\rho} + \frac{1}{n}$ and $k_n^* \equiv k^* (\rho_n, M)$ for all $n \in \mathbf{N}$. On the one hand, we have that $\lim_{n\to\infty} \rho_n = \tilde{\rho}$. On the other hand, we have that $k_n^* = 0$ for all $n \in \mathbf{N}$, so that it follows that $\lim_{n\to\infty} k_n^* = 0$. Therefore, since $(l^* (\rho, M), k^* (\rho, M))$ is continuous, it follows that $k^* (\lim_{n\to\infty} \rho_n, M) = k^* (\tilde{\rho}, M) = 0$.

From now on, we consider the case $\rho < \bar{\rho}(M)$. First, observe that by construction of $\bar{\rho}(M)$, we have that $l_1 (\rho) < l (0) |_M$.

(ii.b) (Raising the cost of one factor reduces the requirement of that factor) For any given ρ and M, we have that $k_R (l) |_\rho$ is strictly decreasing in ρ for any $l \in (l_1 (\rho), l_2 (\rho))$ and $l_R (M)$ is strictly decreasing in M. The result follows immediately.

(ii.c) (Capital and liquidity complements for low cost of capital and substitutes for high cost of capital)

First, we fix M and analyze the impact of changing ρ on the optimal liquidity requirements. Observe that the optimal liquidity response curve $l_R (k)$ is increasing in the complementarity region and decreasing in the substitutability region. Moreover, $l_R (k)$ is independent of ρ. Hence, since M is fixed, $l_R (k)$ does not change with changes in ρ. Now, consider the map of optimal capital responses $\{k_R (l) |_\rho\}_{\rho \in (\rho_{\min}, \bar{\rho}(M))}$. This map consists of a collection of hat-shaped functions that increase in the complementarity region and decrease in the substitutability region. Suppose that, for any given ρ, $(l^* (\rho, M), k^* (\rho, M))$ belongs to the complementarity region. Then, an increase in ρ shifts the optimal capital response curve down. Since the optimal liquidity response function does not change, the optimal requirements are given by moving down along the optimal liquidity response curve, where both curves intersect. Since $l_R(k)$ is increasing (as $(l^*(\rho, M), k^*(\rho, M))$ is in the complementarity region), it follows that the optimal liquidity requirement $l^*(\rho, M)$ decreases. By the same token, suppose that for any given ρ, $(l^*(\rho, M), k^*(\rho, M))$ belongs to the substitutability region. Then, an increase in ρ also shifts the optimal capital response curve down. Since the optimal liquidity response function does not change, the optimal requirements are given by moving down along the optimal liquidity response curve, where both curves intersect. Since $l_R(k)$ is decreasing, it follows that the optimal liquidity requirement $l^*(\rho, M)$ increases. Finally, we need to show that there exists a threshold $\hat{\rho}(M)$ such that the optimal liquidity and capital requirements belong to the complementarity region if $\rho > \hat{\rho}(M)$ and to the substitutability region if $\rho < \hat{\rho}(M)$. Observe that for $\bar{\rho}(M)$, the optimal liquidity requirement $l^*(\bar{\rho}(M), M)$ belongs to the complementarity region. As ρ decreases, the optimal liquidity requirement $l^*(\rho, M)$ moves up along the optimal liquidity response curve $l_R(k)$ until the curve reaches its maximum and crosses to the substitutability region, which occurs for a given $\hat{\rho}(M)$. Then, $l^*(\rho, M)$ decreases along the optimal liquidity response curve $l_R(k)$ within the substitutability region for $\rho < \hat{\rho}(M)$.

Now, we fix $\rho < \bar{\rho}(M)$ and analyze the impact of changing M on the optimal capital requirements. Observe that the optimal capital response curve $k_R(l)$ is increasing in the complementarity region and decreasing in the substitutability region. Now, consider the map of optimal liquidity responses $\{l_R(k)\,|_M\}_{\rho \in (\underline{M}, \infty)}$. This map consists of a collection of hump-shaped functions that increase in the complementarity region and decrease in the substitutability region. Moreover, $k_R(l)$ is independent of M. Hence, since ρ is fixed, $k_R(l)$ does not change with changes in M. Suppose that, for any given M, $(l^*(\rho, M), k^*(\rho, M))$ belongs to the substitutability region. Then, an increase in M shifts the optimal liquidity response curve down (or to the left, if one looks at Figure 2.6). Since the optimal capital response function does not change, the optimal requirements are given by moving up along the optimal capital response curve, where both curves intersect. Since $k_R(l)$ is decreasing (as $(l^*(\rho, M), k^*(\rho, M))$ is in the substitutability region), it follows that the optimal capital requirement $k^*(\rho, M)$ increases. By the same token, if $(l^*(\rho, M), k^*(\rho, M))$ belongs to the complementarity region, an elevation of M leads to an increase of the optimal capital requirement $k^*(\rho, M)$. Finally, define $\hat{M}$ implicitly so that the chosen cost of capital ρ constitutes the threshold that establishes whether $(l^*(\rho, M), k^*(\rho, M))$ are in the complementarity or in the susbtitutability region, that is, $\rho = \hat{\rho}\left(\hat{M}\right)$. Then, for any $M > \hat{M}$ it follows that $\rho > \hat{\rho}(M)$. In this case, the optimal liquidity and capital requirements $(l^*(\rho, M), k^*(\rho, M))$ belong to the substitutability region. On the contrary, if $M < \hat{M}$ then $\rho > \hat{\rho}(M)$, so that the optimal requirements $(l^*(\rho, M), k^*(\rho, M))$ belong to the complementarity region.

(ii.d) (Capital and liquidity are always complements when the return to investment is high) It follows directly from Proposition 2.5, statement (ii.b).

(ii.e) (Zero capital requirements for high return to investment) The cost of capital threshold $\bar{\rho}(M)$ that determines whether capital requirements are positive or not is defined as $l(0)|_M = l_1(\bar{\rho}(M))$. Since $l_1(\rho)$ is invariant with M and $l(0)|_M$ is strictly decreasing in M, the result follows immediately. $\square$

Appendix B

Appendix of Chapter 3

B.1. Technical assumptions

Assumption B.1. In order to avoid economic nonsense cases, we assume $R > 2$.

Assumption B.2. In order to guarantee the uniqueness of the bank's solution (α^*, θ^*), we assume that $\varepsilon \leq \varepsilon_1$, where

$$\varepsilon_1 = \frac{r}{R+r}.$$

Assumption B.3. We assume that

$$D < R - (1+r),$$

which ensures that

$$\varepsilon_1 > \overline{\varepsilon}.$$

In other words, this assumptions guarantees the existence of $\overline{\varepsilon}$.

B.2. Omitted Proofs

Proof of Lemma 3.1. From the bank's maximization problem, we know that $\alpha_A > \alpha_l$. Since there is an one-to-one relationship between θ and the asset quality $\mathbb{E}_{\mathbb{A}}(M)$, we conclude that the fraction of liquid asset α decreases with $\mathbb{E}_{\mathbb{A}}(M)$.

□

Proof of Proposition 3.1. Now, we proves that if $\varepsilon = \bar{\varepsilon}$, then the bank's illiquidity risk is the same regardless the bank invest in cash or in the risk asset. First of all, we have that $\Pi_B(\alpha_l, 0) = \frac{(R-D)^2}{2(2R-D-1)}$ and $\Pi_B(\alpha_A, 1) = \frac{(R-D)^2}{2(2R-D-1-r)}$. Replacing these values in Condition 3.9, we get that

$$\bar{\varepsilon} = \frac{r}{2R-1-D}.$$

Then, by simple algebraic manipulation, we have that the probability of surviving the "early withdrawal" phase when $\theta = 0$ is

$$(1-\bar{\varepsilon})\alpha_A = \left(1 - \frac{r}{2R-1-D}\right)\left(\frac{R-D}{2R-1-D-r}\right) = \frac{R-D}{2R-1-D} = \alpha_l.$$

Therefore, if $\varepsilon = \bar{\varepsilon}$ the bank's illiquidity risk is the same under both $\theta = 0$ and $\theta = 1$. Note also that α_A is independent of ε. Hence, for any $\varepsilon < \bar{\varepsilon}$ investing in the risky liquid asset leads to a lower illiquidity risk than cash. □

Proof of Lemma 3.2. By differentiating $\alpha(\theta)$ respect to θ, we have that

$$\frac{d\alpha(\theta)}{d\theta} = -\frac{r(R-1)(R(2+r)+2(R-2)R\theta+(R-2)r\theta^2)}{(R(2(R-1)-r)+2Rr\theta+(R-2)r\theta^2)^2} < 0.$$

This result follows from Assumption B.1 □

Proof of Lemma 3.3. For any $\alpha \leq \frac{(R-1)\varepsilon}{r+\varepsilon(R-2-r)}$, we have that $\theta(\alpha) = 1$. While $\theta(\alpha) = 0$ if $\alpha \geq \frac{R-1}{R}$ Finally, if $\alpha \in \left(\frac{(R-1)\varepsilon}{r+\varepsilon(R-2-r)}, \frac{R-1}{R}\right)$, we have that

$$\frac{\partial\theta(\alpha)}{\partial\alpha} = \frac{\varepsilon(R-1)}{\alpha^2\left(2\varepsilon - r(1-\varepsilon)\right)} < 0. \tag{B.1}$$

This result follows from Assumption B.2. □

Proof of Proposition 3.2. In order to prove that $R(\theta)$ is a U-shaped function, we differentiate it with respect θ:

$$\frac{dR(\theta)}{d\theta} = -\frac{2(R-1)(1+\varepsilon(1-\theta))\left(\varepsilon(R-r-2(1-\theta(1+r)) - r\theta - \varepsilon^2(1-\theta)(R-2-r)\right)}{\left(2R(1-\varepsilon(1-\theta)) + (r-\varepsilon\theta^2(2+r))\right)^2}.$$

Note that the sign of $\frac{dR(\theta)}{d\theta}$ is driven by the numerator because of the denominator is always positive. Also, the numerator increases with θ, it is negative for $\theta = 0$ and positive if $\theta = 1$. Hence, there exist a value

$$\hat{\theta} = \frac{-\varepsilon\left(R-r-2\right)}{r-\varepsilon\left(2(1+r-\varepsilon^2) + \varepsilon^2(R-1)\right)},$$

which is positive under our assumptions, such that $\frac{dR(\hat{\theta})}{d\theta} = 0$. As a resutl, we can conclude that $R(\theta)$ is a U-shaped function.

Finally, the fact that $R(0) = R(1)$ when $\varepsilon = \bar{\varepsilon}$ follows from Proposition 3.1.

□

Proof of Proposition 3.3. In order to prove that $\theta^* = \hat{\theta}$, we use Condition 3.10 to write the bank's charter value as a function of θ. Then, we maximize $\Pi_B(\alpha(\theta), \theta)$ with respect to θ

and get that

$$\theta^* = \frac{-\varepsilon\left(R - r - 2\right)}{r - \varepsilon\left(2(1 + r - \varepsilon^2) + \varepsilon^2(R - 1)\right)} = \hat{\theta}.$$

As a result, the bank's optimal diversification level is the one that minimizes its illiquidity risk $R(\theta)$.

$\square$

Appendix C

Appendix of Chapter 4

C.1. Omitted Proofs

Proof of Proposition 4.1. This proof is divided into two parts. First, we show that a cash flow shock produces a higher cost than a deposit withdrawal for a small values of ρ. Second, we find the highest income shock, $\bar{\rho}$, such that proposition 1 holds.

a) Assume that the representative consumer has a concave utility function and C_t is a normal good. In addition, let be Γ_1 the inverse deposit supply of this agent in period 1, where W is the parameter of interest. Γ_1 is expressed as follows

$$\Gamma_1(S; W). \tag{C.1}$$

If no shocks occurs, the total financing cost of obtaining an amount of deposit S is given by:

$$\Gamma_1(S; W)S. \tag{C.2}$$

An income shock does not affect the level of deposit than bank needs but it reduces the income of the agent, and thus the deposit supply. We denote the consumer's income, after a shock, as $W(\rho) = W - \rho$, and the total cost in is given by:

$$\Gamma_1(S; W(\rho))S. \tag{C.3}$$

On the other hand, a cash flow shock does not affect the deposit supply but it increases the need for deposit of the bank. Following equation (4.4) a cash shock is defined as:

$$\alpha(\rho) = \hat{R}_1 - S(\hat{R}_1; W(\rho)). \tag{C.4}$$

For small values of ρ, the new requirement of deposit is:

$$S(\rho) = S + \alpha(\rho) \approx S + \frac{dS(\hat{R}_1; W(0))}{dW}\rho, \tag{C.5}$$

Hence the total cost under a cash flow shock is given by:

$$\Gamma_1(S(\rho); W(0))\Big(S + \frac{dS(\hat{R}_1; W(0))}{dW}\rho\Big) \tag{C.6}$$

We represent the differences between financial cost produced by each type of shock through function $\gamma(\rho)$, as follows:

$$\gamma(\rho) = \Gamma_1(S; W(\rho))S - \Gamma_1(S(\rho); W)S(\rho), \tag{C.7}$$

where $\gamma(0) = 0$.

We now want to know if this function is increasing or decreasing in the neighborhood of the point $\rho = 0$.

$$\gamma'(0) = \frac{d(\Gamma_1(W(0), S)S)}{dW}\frac{dW}{d\rho}S - \frac{d(\Gamma_1(W, S(0))S(0))}{dS}\frac{dS}{d\rho}S - \Gamma_1(W, S(0))S(0)),$$

but remember that $\dfrac{dW}{d\rho} = -1$. So, $\gamma'(0)$ will be negative if only if

$$0 < S\left[\frac{d\Gamma_1}{dW} + \frac{d\Gamma_1}{dS}\frac{dS}{d\rho}\right] + \Gamma_1\frac{dS}{d\rho}.$$

We can write, $\dfrac{d\Gamma_1}{dS} = \dfrac{1}{\dfrac{dS}{d\Gamma_1}}$ and it is known from the consumer's problem that $\dfrac{dS}{d\Gamma_1} = -\dfrac{dC}{d\Gamma_1}$. Thus,

$$0 < S\left[\frac{d\Gamma_1}{dW} - \frac{\dfrac{dS}{d\rho}}{\dfrac{dC}{d\Gamma_1}}\right] + \Gamma_1\frac{dS}{d\rho}.$$

From (C.5),

$$\frac{dS}{d\rho} = \frac{dS}{dW},$$

and given that C_t is a normal good,

$$\frac{d\Gamma_1}{dW} = \frac{\dfrac{dC}{dW}}{\dfrac{dC}{d\Gamma}} < 0.$$

Then, if the inequality $\dfrac{dC}{dW} < \dfrac{dS}{dW}$ is satisfied, it is possible to conclude that:

$$0 < S\left[\frac{d\Gamma_1}{dW} - \frac{\dfrac{dS}{d\rho}}{\dfrac{dC}{d\Gamma_1}}\right] + \Gamma_1\frac{dS}{d\rho}.$$

Therefore, $\gamma'(0)$ is negative, and thus a cash flow shock produces a higher total cost than a deposit flight for small values of ρ.

b) In the particular case of the utility function (1), we find a value of $\bar{\rho}$, such that:

$$\gamma(\bar{\rho}) = \Gamma_1(S; W(\bar{\rho}))S - \Gamma_1(S(\bar{\rho}); W)S(\bar{\rho}) = 0. \tag{C.8}$$

Then, if we substitute Γ_1 with equation (4), and S with $IR_0 - C_f$, we have

$$(R_0I - C_f)\left(\frac{W - \bar{\rho} + C_f}{R_0I - C_f}\right)^{\left(\frac{\theta-1}{\theta}\right)} - (R_0I - C_f + \alpha(\bar{\rho}))\left(\frac{W - \alpha(\bar{\rho}) + C_f}{R_0I - C_f + \alpha(\bar{\rho})}\right)^{\left(\frac{\theta-1}{\theta}\right)} = 0, \tag{C.9}$$

where $\alpha = \dfrac{\rho(R_0I - C_f)}{R_0I + W}$.

Operating, we obtain θ as a function of $\bar{\rho}$ as follows:

$$\theta(\bar{\rho}) = \frac{ln(R_0I - C_f + \alpha(\bar{\rho})) - ln(R_0I - C_f)}{ln(W - \bar{\rho} + C_f) - ln(W - \alpha(\bar{\rho}) + C_f)} + 1 \tag{C.10}$$

Condition (C.10) implies that there is an interval $[0, \bar{\rho}]$ such that Proposition 1 holds.

Additionally, we now show that $\theta(\bar{\rho})$ is increasing in $(\bar{\rho})$. For the sake of simplicity, we write:

$$A = ln(R_0I - C_f + \alpha(\rho)) - ln(R_0I - C_f) > 0 \tag{C.11}$$

$$B = ln(W - \bar{\rho} + C_f) - ln(W - \alpha(\bar{\rho}) + C_f) < 0$$

$$\frac{d\theta(\bar{\rho})}{d\bar{\rho}} = \frac{\dfrac{\alpha'}{R_0I - C_f - \alpha(\bar{\rho})}B - A\left[\dfrac{-1}{W - \bar{\rho} + C_f} - \dfrac{\alpha'}{W - \alpha(\bar{\rho}) + C_f}\right]}{B^2} \tag{C.12}$$

Then, $\theta'(\bar{\rho})$ is positive only if,

$$\frac{\alpha'}{R_0I - C_f - \alpha(\bar{\rho})}B < A\left[\frac{1}{W - \bar{\rho} + C_f} - \frac{\alpha'}{W - \alpha(\bar{\rho}) + C_f}\right]$$

$$\frac{\dfrac{\alpha'}{R_0I - C_f - \alpha(\bar{\rho})}}{\left[\dfrac{1}{W - \bar{\rho} + C_f} - \dfrac{\alpha'}{W - \alpha(\bar{\rho}) + C_f}\right]} > \frac{A}{B} = \theta(\bar{\rho}) - 1$$

But, the left hand is positive because $0 < \alpha' < 1$ and $\bar{\rho} > \alpha(\bar{\rho})$. Thus based on a) and b), we conclude that for all ρ in $[0, \bar{\rho}]$ a cash flow shock produces higher cost than a deposit flight when bank only has access to its deposit market.

□

Proof of Lemma 4.1 and Lemma 4.3 . In order to demonstrate Lemma 4.1, we follow several steps. We first show that if an equilibrium lies on the left of a value $\tilde{X}$, then $X_D^* > X_C^*$ whereas on the right hand of $\tilde{X}$, the reverse occurs. The following step is to prove that the result of Proposition 1 holds when the bank gets the same amount of interbank loans in both states of nature. Then, we use this fact to demonstrate that for all X_j^* such that $X_j^B = X_j^A = X_j^*$, the cash flow shock does more damage than a deposit flight. In the last point, we show the conditions for the existence of different equilibriums.

a) There is an $\tilde{X}$ such that demands of interbank loans for each type of shock cross each other.

We now show that there is a value of $\tilde{X}$ such that: to the left of $\tilde{X}$, the amount demanded under a deposit shock is higher than when a cash flow occurs ($X_D^* > X_C^*$), while the opposite will be the case at the right of $\tilde{X}$ ($X_C^* > X_D^*$). Remember that X_j^* was defined as the volume of interbank loans such that $X_j^A(R_{Ij}^*) = X^B(R_{Ij}^*)$. However, the existence of

X_j^* is a necessary, but not sufficient, condition for having a positive volume of trade in the market.

We define the function $G(X)$ as the difference between the Bank A's FOC for $J = \{C, D\}$. The goal is to show that $G(0) < 0$ and $G(R_0I - Cf) > 0$, and then the existence of $\tilde{X}$ is given by the continuous function theorem. Thus,

$$G(X) = \Gamma'_{1C}(R_0I - C_f + \alpha - X) + \Gamma_{1C} - \Gamma'_{1D}(R_0I - C_f - X_D^A) - \Gamma_{1D}, \tag{C.13}$$

if we substitute Γ_{1C} and Γ_{1D} by their respective values, we have:

$$G(X) = \left[\Big(\frac{W - \alpha + C_f + X}{R_0I - C_f - X + \alpha}\Big)^{\left(\frac{\theta-1}{\theta}\right)} - \frac{\theta - 1}{\theta}\Big(\frac{W - \alpha + C_f + X}{R_0I - C_f - X + \alpha}\Big)^{\left(\frac{-1}{\theta}\right)} \frac{R_0I + W}{R_0I - C_f - X + \alpha}\right]$$

$$-\left[\Big(\frac{W - \rho + C_f + X}{R_0I - C_f - X}\Big)^{\left(\frac{\theta-1}{\theta}\right)} - \frac{\theta - 1}{\theta}\Big(\frac{W - \rho + C_f + X}{R_0I - C_f - X}\Big)^{\left(\frac{-1}{\theta}\right)} \frac{R_0I + W - \rho}{R_0I - C_f - X}\right],$$

where $\alpha = \dfrac{\rho(R_0I - C_f)}{R_0I + W}$.

i- We want to see that $G(0) < 0$.

Since $\Gamma_{1C} < \Gamma_{1D}$, it will be enough to show that:

$$\Big(\frac{W - \alpha + C_f + X}{R_0I - C_f - X + \alpha}\Big)^{\left(\frac{-1}{\theta}\right)} \frac{R_0I + W}{R_0I - C_f - X + \alpha} \le \Big(\frac{W - \rho + C_f + X}{R_0I - C_f - X}\Big)^{\left(\frac{-1}{\theta}\right)} \frac{R_0I + W - \rho}{R_0I - C_f - X}$$

After some calculations, and considering that $X = 0$, we express the inequality (**??**) as the function $\tilde{G}(\rho)$ as follows,

$$\tilde{G}(\rho) = 2ln(R_0I + W) - ln((R_0I + W)^2 - \rho^2) - \frac{1}{\theta}\Big[ln(R_0IW + R_0IC_f + W^2 + WC_f - \rho R_0I + \rho C_f)$$
$$- ln(R_0IW + R_0IC_f + W^2 + WC_f - \rho R_0I + \rho C_f - \rho^2)\Big]$$

Notice that $\tilde{G}(0) = 0$ and $\frac{d\tilde{G}(0)}{d\rho} < 0$. Therefore, this fact leads to $G(0) < 0$.

ii- In the case of $X = R_0I - CF$, it is straightforward to see that $G(R_0I - CF) > 0$ due to the second term of this function is zero.

Then by the continuity of $G(.)$, there is an $\tilde{X}$ such that the FOC are equal for both types of shocks.

b) If at the rate R_I the bank gets the same amount of interbank loans X under both states, then a cash flow shock produces higher cost than a deposit flight.

If Bank A substitutes deposits for interbank loans, the total cost under each shock is given by $C^A(X_D)$ and $C^A(X_C)$. We define the function $\Psi(X)$ as the difference of the Bank A's cost as follows:

$$\Psi(X) = C^A(X_D) - C^A(X_C).$$

$$\Psi(X) = \Gamma_{1D}[R_0I - C_f - X] + R_IX - \Gamma_{1C}[R_0I - C_f - X + \alpha] - R_IX.$$

If we define $\tilde{C}_f = C_f + X$ and operates as in equation (C.10), we obtain:

$$\theta(\bar{\rho}; \tilde{C}_f) = \frac{ln(R_0I - \tilde{C}_f + \alpha(\bar{\rho})) - ln(R_0I - \tilde{C}_f)}{ln(W - \bar{\rho} + \tilde{C}_f) - ln(W - \alpha(\bar{\rho}) + \tilde{C}_f)} + 1. \quad \text{(C.14)}$$

Then,

$$\frac{d\theta}{d\tilde{C}_f}\Big|_{(\bar{\rho},C_f)} = \left[\frac{-1}{R_0 I - \ C_f + \bar{\rho}}\right] B - A\Big[\frac{1}{W-\bar{\rho}+C_f} - \frac{1}{W-\bar{\rho}+C_f}\Big] < 0. \tag{C.15}$$

The sign of this derivative is negative. Thus, if the bank is able to access to interbank market, the upper bound value of rho ($\bar{\rho}$) is relaxed. It means that for each value of θ, Proposition 4.1 holds for a higher value of ρ. Accordingly, for all ρ in $[0,\bar{\rho}]$ a cash flow shock continue to produce higher costs than a deposit flight shock when Bank A substitutes deposits for interbank loans .

c) For all ρ there is an X_j^* such that $X_j^B = X_j^A = X_j^*$, and X_j^* is increasing in the size of the shock.

The first part of this statement was proved above, we now show, using the implicit function theorem, the second part:

$$\frac{dX^*}{d\rho} = -\frac{\left(-\dfrac{Cf+W+X^*-\rho}{Cf-IR_0+X^*}\right)^{\dfrac{\theta+1}{\theta}}\left(\dfrac{Cf+W-X^*}{-Cf+IR_0+X^*}\right)^{-\dfrac{\theta+1}{\theta}}(IR_0+W)^2(Cf-IR_0+X^*)^3}{(IR_0-Cf+X^*)^3(IR_0+W-\rho)^2 P_H} > 0.$$

d) If $\rho \in [0,\bar{\rho}]$ and $X_j^B = X_j^A = X_j^*$ for $J=\{C,D\}$, then a cash flow shock produces higher cost than a deposit shock.

Based on (a), the points (X_D^*, R_{ID}^*) and (X_C^*, R_{IC}^*) can be either on the left side or on the right side of $\tilde{X}$.

i- Assume that (X^*_D, R^*_{ID}) and (X^*_C, R^*_{IC}) are on the left side of $\tilde{X}$. Then, we know by (b) that:

$$\Psi(X^*_C, R^*_{IC}) = \Gamma_{1D}[R_0 I - C_f - (X^*_C] + R^*_{IC} X^*_C - \Gamma_{1C}[R_0 I - C_f - (X^*_C + \alpha] - R^*_{IC} X^*_C < 0$$

Notice that (X^*_C, R^*_{IC}) is affordable when a deposit flight occurs. However, Bank A choses (X^*_D, R^*_{ID}) because this allocation produces a lower cost than (X^*_C, R^*_{IC}). Therefore, a cash flow shock become more expensive for the bank.

ii- Assume that (X^*_D, R^*_{ID}) and (X^*_C, R^*_{IC}) are on the right side of $\tilde{X}$. Let $X_C(R^*_{ID})$ the amount of interbank loans demanded at the rate R^*_{ID} when Bank A suffers a cash flow shock. We know by (b) that:

$$\Psi(X^*_C, R^*_{ID}) = \Gamma_{1D}[R_0 I - C_f - X^*_C] + R^*_{ID} X^*_C - \Gamma_{1C}[R_0 I - C_f - (X^*_C + \alpha] - R^*_{ID} X^*_C < 0.$$

Given that X^*_D minimize the Bank A's cost at the interest rate R^*_{ID}, we have:

$$\Psi(X^*_C, R^*_{ID}) > \Gamma_{1D}[R_0 I - C_f - X^*_D] + R^*_{ID} X^*_D - \Gamma_{1C}[R_0 I - C_f - X^*_C + \alpha] - R^*_{ID} X^*_C.$$

On the other hand, on the right side of $\tilde{X}$, we know that $X^*_D < X^*_C$ and therefore $R^*_{IC} > R^*_{ID}$. Since the Bank A's cost is increasing in R_I,

$$\Gamma_{1C}[R_0 I - C_f - X^*_C + \alpha] - R^*_{ID} X^*_C < \Gamma_{1C}[R_0 I - C_f - X^*_C + \alpha] - R^*_{IC} X^*_C,$$

Therefore, we can conclude that:

$$C^A(X^*_D(R^*_{ID})) < C^A(X^*_C(R^*_{IC}))$$

$$\Gamma_{1D}(R_0 I - C_f - (X^*_D) - R^*_{ID} X^*_D < \Gamma_{1C}(R_0 I - C_f - (X^*_C + \alpha) - R^*_{IC} X^*_C.$$

Thus based on i) and ii), we say that in equilibrium a credit shock produce higher cost than a deposit flight.

e) Existence of Equilibriums

i- Active Interbank Market Equilibrium

From (c), we know that for all ρ in $[0, \bar{\rho}]$ there is an X_j^* such that $X_j^B = X_j^A = X_j^*$. In particular considering a value ρ_C, such that satisfies the following compatibility constrain:

$$\frac{B}{\Delta P} = R - \Gamma_{1C}[R_0 I - C_f - X_C^* + \alpha(\rho_C)] - R_I^* X_C^* \tag{C.16}$$

Then by (d), we have the following compatibility constrain if a deposit shock occurs:

$$\frac{B}{\Delta P} < R - \Gamma_{1D}[R_0 I - C_f - X_D^*] - R_I^* X_D^*. \tag{C.17}$$

Therefore, both requirements in Definition 1 are satisfied for ρ in $[0, \rho_C]$ and banking system reaches an AIE.

ii- Mixed Equilibriums

A Mixed equilibrium arises when the market provides for liquidity to banks only for one type of shock. We now show that a Mixed equilibrium where the market provides liquidity under a cash flow shock it is not possible. Assume that there is a $\rho < \rho_C$ such that Bank A can get interbank loans from Bank B only under a cash flow shock. Thus, if a deposit flight occurs the market freezes, which imply that:

$$\frac{B}{\Delta P} > R - \Gamma_{1D}[R_0 I - C_f - X_D^*] - R_I^* X_D^*,$$

and

$$\frac{B}{\Delta P} < R - \Gamma_{1C}[R_0 I - C_f - X_C^* + \alpha(\rho_C)] - R_I^* X_C^*$$

But this fact contradicts the statements (d). Then, a Mixed equilibrium such that the market provide funds under a cash flow shock is not possible.

On the other hand, if $\rho \in (\rho_C, \rho_D]$ the banking system reaches a Mixed equilibrium where the market freezes when a cash flow shock occurs. Moreover, ρ_D is defined as the size of shock such that:

$$\frac{B}{\Delta P} = R - \Gamma_{1D}[R_0 I - C_f - X_D^*] - R_I^* X_D^*,$$

Consider $\rho_\varepsilon = \rho_C + \varepsilon$, where is a small positive number. From (c), there is a $X_j^A = X_j^B = X_j^*$ for $J = \{D, C\}$. However, by statement (d) and the continuity of both (A19) and (A20), we have that under ρ_ε,

$$\frac{B}{\Delta P} < R - \Gamma_{1D}[R_0 I - C_f - X_D^*] - R_I^* X_D^*,$$

and

$$\frac{B}{\Delta P} < R - \Gamma_{1C}[R_0 I - C_f - X_C^* + \alpha(\rho_\varepsilon)] - R_I^* X_C^*.$$

Therefore, if $\rho \in (\rho_C, \rho_D]$ Bank B only provides liquidity under a deposit shock.

iii- Market Freeze equilibrium

If $\rho > \rho_D$, then the market freezes under both types of shock. The proof of this is straight forward of i) and ii).

C.2. Policy Implications.

Given a level of capital K, the values of ρ_{CK} and ρ_{CK} are determined by the following equations:

$$\frac{B}{\Delta P} = R - \left(\frac{W - \rho_{DK} + C_f}{R_0 I - C_f - X_D^*(\rho_{DK})} \right)^{\left(\frac{\theta - 1}{\theta}\right)} \left[R_0 I - C_f - X_D^*(\rho_{DK}) \right] - R_I^* X_D^*(\rho_{DK}) + K(1 - \psi),$$

$$\begin{aligned} \frac{B}{\Delta P} =& R - \left(\frac{W - \hat{\alpha}(\rho_{CK}) + C_f}{R_0 I - C_f + \hat{\alpha}(\rho_{CK}) - X_C^*(\rho_{CK})} \right)^{\left(\frac{\theta - 1}{\theta}\right)} \left[R_0 I - C_f - X_C^*(\rho_{CK}) + \hat{\alpha}(\rho_{CK}) \right] \\ & - R_I^* X_C^*(\rho_{CK}) + K(1 - \psi). \end{aligned}$$

Notice that X_j^* is the higher liquidity provided by the market in equilibrium when the capital requirements are K. Then, the grader liquidity shortfall in each state will be $\hat{\alpha}(\rho_{CK})$ and $\hat{\alpha}(\rho_{DK})$.