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Production Complementarities and Flexibility in a Model of Entrepreneurship

Online Appendix

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This appendix has six parts. Section A provides details on the data, going over variable definitions, sample selection and summary statistics. Section B gives further analytical characterizations in solving the model. Section C explains the identification of the parameter governing the degree of working hour complementarities in the benchmark calibration using numerical results from alternative models. Section D discusses the robustness of the main results of the paper to allowing for progressive taxation in the model. Section E solves an extension of the model with heterogeneities in the value of leisure volatility. Finally, Section F provides additional figures that were excluded in the main body for the sake of brevity.

A Data details

This section describes the important aspects of the data used in this paper.

Hours and income. Information on hours in the data is the one of the usual hours worked over the reference period (last four months). Income, however, corresponds to the last month only. To allow for matching the income with hours, I assume that the income in the last month is the same with the average over the last four months.

Due to the high volume of explanatory variables, the results of the regressions to control for various characteristics affecting hours and hourly income are omitted. They are available

Moment	**Excluding** ≥ 100	**Excluding** < 100	**All**
Fraction of entr	0.088	0.001	0.089
Ratio of mean income (entr/worker)	0.935	1.788	0.946
Ratio of median income (entr/worker)	0.858	1.289	0.860
Ratio of mean hourly income (entr/worker)	0.950	1.914	0.963
Ratio of median hourly income (entr/worker)	0.811	0.811	0.811
Hours (entr, mean)	0.243	0.260	0.244
Hours (worker, mean)	0.239	0.239	0.239
Sd of log-inc (entr)	0.658	0.822	0.663
Sd of log-inc (worker)	0.336	0.336	0.336
Sd of log-hours (entr)	0.405	0.381	0.404
Sd of log-hours (worker)	0.234	0.234	0.234
Autocorr of log-inc (entr)	0.746	0.648	0.747
Autocorr of log-inc (worker)	0.811	0.811	0.811
Autocorr of log-hours (entr)	0.694	0.769	0.695
Autocorr of log-hours (worker)	0.661	0.661	0.661
Sample size	1,400,196	1,278,477	1,401,653

Table A1: Summary statistics

Note: Second (third) column gives the statistics in a sample that excludes entrepreneurs employing at least (less than) 100 employees.

upon request. The R-squared statistics in hours and hourly income regressions are 0.12 and 0.06 respectively.

Sample selection. As mentioned in the paper, the sample does not include entrepreneurs with firms larger than 100 employees. It also excludes casual businesses and all individuals in agriculture. It pools the panels 1996, 2001, 2004 and 2008 together to form a dataset of school-leavers of ages 18 to 65. Table A1 provides the summary statistics for the sample used in the paper, and contrasts it with the one using entrepreneurs with big businesses.

Classification into salaried work and entrepreneurship. Individuals are labeled as entrepreneurs if they have had at least one business in the reference period and they worked more hours in their business than in their paid jobs (if they had any). Workers are people who have had at least one paid job in the reference period and worked every week in the last month. Casual businesses are those that report that they have not withdrawn more than $2500 in the last 12 months from their businesses, or that they have no expectation to do so in the next 12 months.

Industry categories to study hours-wages relationship. The industry categorization in the SIPP changes substantially from the panel 2001 to the panel 2004. Accordingly, computations with industry categorizations only consider 2004 and 2008 panels. The results are similar using the sample based on 1996 and 2001 panels only. In 2004 and 2008, workers are grouped into 265 different industries, while in 1996 and 2001, this number is 238.

In the SIPP, the industry categorization is much more detailed for workers than for entrepreneurs. In contrast to 268 different values of the industry code for workers (*ejbind1*), entrepreneurs are grouped into only 15 categories (*ebsind1*). In order to be able to use the same categories for both occupational groups, the categories for workers are collapsed into 15. To be specific, I assign *ejbind1* values of 170 to 290 as Agriculture; 370 to 690 as Mining; 770 as Construction, 1070 to 3990 as Manufacturing, 4070 to 4590 as Wholesale trade, 4670 to 5790 as Retail trade, 6070 to 6390 as Transportation, warehousing and utilities, 6470 to 6780 as Information, 6870 to 7190 as Finance, insurance, real estate, and rental and leasing, 7270 to 7790 as Professional, scientific, management, administrative, and waste management services, 7869 to 8290 as Educational services, and health care and social assistance, 8370 to 8690 as Arts, entertainment, recreation, accommodation, and food services, 8770 to 9290 as Other services, 9370 to 9590 as Public administration, and 9890 as Military. (Agriculture is excluded in the figures as is the case for the main sample in this paper, and military is dropped for obvious reasons.) Then the figures use the hours in difference to the means from an individual's industry categorization. The top 5 percent of the overall hourly income distribution is dropped to clean extremely high values of hourly income associated to extremely low reported hours with sizable income, and the bottom 5 percent is dropped for symmetry.

B Theoretical results

This section gives the theoretical results regarding the solution of the equilibrium. The solutions to the firms' problem are followed by their implication on the equilibrium wages.

Solving non-entrepreneurial sector's problem. Maximization with respect to K_b gives:

$$\alpha \frac{Y_b}{K_b} = r + \delta \tag{1}$$

Then the FOC with respect to the number of workers to be hired with skill level y and hours l is:

$$(1-\alpha) y \frac{Y_b}{L_b} E_{\mu_b,y}(l^\rho)^{\frac{1}{\rho}} \left[\frac{1}{\rho} \frac{l^\rho}{E_{\mu_b,y}(l^\rho)} + (1 - \frac{1}{\rho}) \right] = w(l,y)l, \tag{2}$$

if $\mu_b(l,y) > 0$, where

$$E_{\mu_b,y}(l^\rho) = \frac{\int_0^1 \mu_b(l,y) l^\rho dl}{\int_0^1 \mu_b(l,y) dl}.$$

Solving entrepreneurs' problem. Here, the focus is on the labor and capital demand of entrepreneurs, taking as given the hours they supply to their firms. The profit maximization problem for firm n is:

$$\begin{aligned} \max_{K_n, \{\mu_n(l,y)\}} \quad & Y_n - K_n(r+\delta) - \int_0^\infty \int_0^1 w(l,y)\mu_n(l,y) l dl dy \\ & s.t. \, Y_n = A_n h_n^\omega \left(K_n^\alpha \left[\left(\int_0^1 \mu_n(l,y) l^\rho dl \right)^{\frac{1}{\rho}} \left(\int_0^1 \mu_n(l,y) dl \right)^{1-\frac{1}{\rho}} \right]^{1-\alpha} \right)^\eta \\ & K_n \geq 0, \mu_n(l,y) \geq 0 \, \forall \, l \in [0,1]. \end{aligned}$$

The maximization with respect to K_n gives:

$$\alpha\eta \frac{Y_n}{K_n} = r + \delta. \tag{3}$$

The maximization with respect to $\mu_n(l,y)$ for $l \in [0,1]$ gives:

$$(1-\alpha)\eta y \frac{Y_n}{L_n} E_{\mu_n,y}(l^\rho)^{\frac{1}{\rho}} \left[\frac{1}{\rho}\frac{l^\rho}{E_{\mu_n,y}(l^\rho)} + (1-\frac{1}{\rho})\right] = w(l,y)l \tag{4}$$

if $\mu_n(l,y) > 0$ where

$$E_{\mu_n,y}(l^\rho) = \frac{\int_0^1 \mu_n(l,y) l^\rho dl}{\int_0^1 \mu_n(l,y) dl}.$$

The following claim gives an implication of competitive markets that will be useful in what follows to characterize the wage functions in equilibrium.

Claim: Suppose K_b and $\{\mu_b(l,y)\}$ solve the maximization problem of a non-entrepreneurial firm b. Then for every entrepreneurial firm n, $K_n \equiv m_n K_b$ and $\mu_n \equiv m_n \mu_b$ solve the profit maximization problem, where

$$m_n \equiv \left(\frac{\eta A_n h_n^\omega}{B^\eta}\right)^{\frac{1}{1-\eta}} \frac{1}{Y_b}.$$

Proof: We need to show that $K_n \equiv m_n K_b$ and $\mu_n \equiv m_n \mu_b$ satisfy the first order conditions of an entrepreneurial firm n. First, notice that this implies:

$$L_n = m_n L_b$$

$$E_{\mu_n,y}(l^\rho) = E_{\mu_b,y}(l^\rho)$$

$$Y_n = A_n h_n^\omega m_n^\eta (K_b^\alpha L_b^{1-\alpha})^\eta.$$

Since K_b and μ_b solve the (1) and (2):

$$\begin{aligned}
\alpha\eta\frac{Y_n}{K_n} &= \alpha\eta\frac{A_n h_n^\omega m_n^\eta (K_b^\alpha L_b^{1-\alpha})^\eta}{m_n K_b} \\
&= \alpha\eta A_n h_n^\omega m_n^{\eta-1}\frac{(K_b^\alpha L_b^{1-\alpha})^\eta}{Y_b}\frac{Y_b}{K_b} \\
&= \alpha\eta A_n h_n^\omega m_n^{\eta-1}\frac{1}{B^\eta Y_b^{1-\eta}}\frac{r+\delta}{\alpha} \\
&= r+\delta.
\end{aligned}$$

Hence, the first order condition for capital is satisfied. To see the same result for the labor demand notice that:

$$\begin{aligned}
&(1-\alpha)\eta y\tfrac{Y_n}{L_n}E_{\mu,y}(l^\rho)^{\frac{1}{\rho}}\left[\tfrac{1}{\rho}\tfrac{l^\rho}{E_{\mu,y}(l^\rho)}+(1-\tfrac{1}{\rho})\right] \\
&= (1-\alpha)\eta y\tfrac{A_n h_n^\omega m_n^\eta (K_b^\alpha L_b^{1-\alpha})^\eta}{m_n L_b}E_{\mu_b,y}(l^\rho)^{\frac{1}{\rho}}\left[\tfrac{1}{\rho}\tfrac{l^\rho}{E_{\mu_b,y}(l^\rho)}+(1-\tfrac{1}{\rho})\right] \\
&= (1-\alpha)\eta y\tfrac{A_n h_n^\omega m_n^{\eta-1} (K_b^\alpha L_b^{1-\alpha})^\eta}{Y_b}\tfrac{Y_b}{L_b}E_{\mu_b,y}(l^\rho)^{\frac{1}{\rho}}\left[\tfrac{1}{\rho}\tfrac{l^\rho}{E_{\mu_b,y}(l^\rho)}+(1-\tfrac{1}{\rho})\right] \\
&= (1-\alpha)\eta y A_n h_n^\omega m_n^{\eta-1}\tfrac{1}{B^\eta Y_b^{1-\eta}}\tfrac{Y_b}{L_b}E_{\mu_b,y}(l^\rho)^{\frac{1}{\rho}}\left[\tfrac{1}{\rho}\tfrac{l^\rho}{E_{\mu_b,y}(l^\rho)}+(1-\tfrac{1}{\rho})\right] = w(l,y)l.
\end{aligned}$$

This establishes that the proportional allocation solves the optimality conditions for entrepreneur n. ■

The implication of the above result is that in equilibrium, ratios of factors of production and relative size of labor aggregation units across firms is the same with that in the overall economy. Denote by $\mathbf{M}_A$ and $\mathbf{M}_B$ the set of entrepreneurial and non-entrepreneurial firms in the economy. Moreover, define:

$$\bar{K} \equiv \int_{n\in\mathbf{M}_A} K_n dn + \int_{b\in\mathbf{M}_B} K_b db$$

$$\bar{x}(l,y) \equiv \int_{n\in\mathbf{M}_A} \mu_n(l,y)dn + \int_{b\in\mathbf{M}_B} \mu_b(l,y)db$$

$$\bar{L} \equiv \int_{n\in\mathbf{M}_A} L_n dn + \int_{b\in\mathbf{M}_B} L_b db.$$

It also follows that for all $n \in \mathbf{M}_A$ and $b \in \mathbf{M}_B$:

$$E_{\bar{x}2,y}(l^\rho) = E_{\mu_n,y}(l^\rho) = E_{\mu_b,y}(l^\rho). \tag{5}$$

Equilibrium wages. Next, I will describe the equilibrium wage function w. Equation (1) gives that for any non-entrepreneurial firm b:

$$\frac{K_b}{L_b} = \left(\frac{\alpha}{r+\delta}\right)^{\frac{1}{1-\alpha}}.$$

Equation (2) implies that in an equilibrium where $\mu_b(l,y) > 0$, wages satisfy:

$$\begin{aligned} w(l,y) &= (1-\alpha)y\frac{Y_b}{L_b}E_{\mu_b,y}(l^\rho)^{\frac{1}{\rho}}\left[\frac{1}{\rho}\frac{l^{\rho-1}}{E_{\mu_b,y}(l^\rho)} + (1-\frac{1}{\rho})l^{-1}\right] \\ &= (1-\alpha)\left(\frac{\alpha}{r+\delta}\right)^{-\frac{\alpha}{1-\alpha}} yE_{\mu_b,y}(l^\rho)^{\frac{1}{\rho}}\left[\frac{1}{\rho}\frac{l^{\rho-1}}{E_{\mu_b,y}(l^\rho)} + (1-\frac{1}{\rho})l^{-1}\right]. \end{aligned}$$

Using (5), one can write the price of hourly labor in terms aggregate variables:

$$w(l,y) = (1-\alpha)\left(\frac{\alpha}{r+\delta}\right)^{-\frac{\alpha}{1-\alpha}} yE_{\bar{x},y}(l^\rho)^{\frac{1}{\rho}}\left[\frac{1}{\rho}\frac{l^{\rho-1}}{E_{\bar{x},y}(l^\rho)} + (1-\frac{1}{\rho})l^{-1}\right].$$

C Identification of ρ

This section has two objectives. First is to illustrate that the hours dispersion for workers implied by the model is steeply increasing in the assumed degree of substitutability, ρ. Since this particular moment is the target when setting ρ in the calibration, results here indicate that higher (lower) values for ρ in the benchmark would lead to an hours dispersion for workers that is too large (small) compared to the data. Second objective is to demonstrate that it is possible to fit the dispersion in workers' hours without complementarities, but only by sacrificing the model performance in entrepreneurs' hours dispersion. In turn, this exercise highlights how the dispersion in entrepreneurial hours discipline the value of leisure shocks.

Targeting the dispersion in workers' hours, the benchmark calibration sets ρ equal to 0.02. Here, this exercise is repeated by setting ρ equal -1, 0.5 and 1, while targeting the same moments as in Section 4 of the paper with the remaining parameters. Note that the setting with $\rho = 1$ is precisely the "No complementarities" model studied in Section 6. The parameters for these models are summarized in Table A2 and the results are given in Table A3.

Table A3 assures that the hours dispersion for workers generated by the model increases with the assumed degree of substitutability, ρ. For instance, when ρ increases from -1 to 0.5, the implied dispersion for workers' hours increases from 0.17 to 0.28. As discussed in Section 6.1, the setting with ρ equal to 1 generates a dispersion in workers' hours that is 50 percent higher than that in the data.

Can we help the "No complementarities" model match the observed hours dispersion of workers by reducing the size of the value of leisure shocks? In order to answer this question, I continue with the assumption without complementarities and set σ_v equal to 75 and 50 of the level in the "No complementarities" calibration, and finish with the "Stable preferences" setting of Section 6.2, i.e. $\sigma_v = 0$. For each setting, the rest of the model parameters are calibrated to match the remaining targets. The last three columns of Table A3 give the results.

It is evident from this exercise that the implied dispersion for both occupational groups are strictly increasing in the assumed size of the value of leisure shocks. In particular, a model featuring perfect substitutability between workers' hours can actually match the dispersion in hours of workers, for some value of σ_v between 75 percent and 50 percent of the value in the original "No complementarities" case. However, such calibration would fall short of generating the dispersion in entrepreneurs' hours. Specifically, between 20-30 percent of the dispersion in entrepreneurs' hours would remain unexplained if one follows that approach. To put it differently, once disciplined by the entrepreneurial hours dispersion, the complementarities help the model account for the low dispersion in workers' hours.

The motive in recalibrating the entire set of parameters in each alternative above is to see what would be the consequences if a modeler, (i) were to believe the actual ρ was different from the calibrated value in the benchmark and run the benchmark exercise with alternative values ρ, (ii) were to assume away the complementarities and try the calibration exercise by reducing the size of the value of leisure shocks, σ_v. Nevertheless, it is still informative to deviate from the reference models (Benchmark (BM) for the first set, "No complementarities" (NC) model for the second) by only altering the parameters of interest.[1] Table A4 first reports the results varying ρ from its value in the benchmark calibration, without recalibrating rest of the parameters (i.e. take them as in Section 4). In particular, using the benchmark calibration for rest of the parameters and only increasing ρ gradually from -1 to 1 increases the implied dispersion in workers' hours in a monotone way from 0.17 to 0.34. These values are actually very similar to the changes found in Table A2, which shows the corresponding moments from recalibrated models.

The last three columns of Table A4 summarize the exercise of reducing the size of the leisure shocks in the "No complementarities" economy keeping the rest of the parameters as they were in that model (i.e. as given in Section 6.1). The results are consistent with those in Table A2, showing that decreasing the size of the leisure shocks decreases the hours of entrepreneurs below the level observed in the data. Accordingly, the qualitative remarks resulting from the recalibration exercises in the paper go through in case of only changing the parameters of focus.

[1]This exercise also serves as a counterfactual, experimenting changes in the relevant parameters.

Parameter	**Degree of substitutability**				**Val. of leisure volatility**		
	ρ				σ_v (normalized, NC 1)		
	-1	0.02 (BM)	0.5	1 (NC)	0.75	0.50	0 (SP)
A_0	2.540	2.627	2.705	2.617	2.651	2.543	2.389
ρ	-1.000	0.020	0.500	1.000	1.000	1.000	1.000
σ_v	0.527	0.533	0.537	0.541	0.406	0.271	0.000
σ_A	0.103	0.105	0.106	0.128	0.127	0.140	0.163
σ_y	0.053	0.043	0.032	0.000	0.040	0.053	0.060
π_v	0.631	0.641	0.647	0.682	0.629	0.533	NA
π_A	0.794	0.786	0.789	0.855	0.775	0.773	0.760
v_0	74	78	80	81	76	72	69
ω	0.557	0.557	0.552	0.548	0.552	0.552	0.530
s	0.010	0.010	0.010	0.008	0.010	0.010	0.014

Table A2: Calibrated parameters with alternative models

Note: The parameters not reported in this table are kept as in the benchmark calibration. The alternative models are those included in Section 6 of the paper.

Moment	**Data**	**Degree of substitutability**				**Val. of leisure vol.**		
		ρ				σ_v (normalized, NC 1)		
		-1	0.02 (BM)	0.5	1 (NC)	0.75	0.50	0 (SP)
Panel A: Targeted								
Fraction of entr	0.088	0.089	0.088	0.087	0.088	0.088	0.088	0.087
Hours (entr, mean)	0.243	0.243	0.242	0.243	0.244	0.244	0.242	0.243
Hours (worker, mean)	0.239	0.239	0.239	0.239	0.239	0.239	0.239	0.239
Sd of log-inc (entr)	0.658	0.655	0.655	0.657	0.659	0.657	0.657	0.656
Sd of log-inc (worker)	0.336	0.336	0.336	0.336	0.360	0.336	0.335	0.336
Sd of log-hours (entr)	0.405	0.402	0.404	0.405	0.405	0.333	0.268	0.217
Sd of log-hours (worker)	0.234	0.171	0.232	0.278	0.360	0.263	0.178	0.097
Autocorr of log-inc (entr)	0.746	0.744	0.746	0.746	0.747	0.746	0.750	0.742
Autocorr of log-hours (entr)	0.694	0.691	0.694	0.693	0.694	0.696	0.697	0.771
Population switching to worker	0.005	0.004	0.005	0.005	0.005	0.005	0.005	0.005
Panel B: Untargeted								
Ratio of mean income (e/w)	0.935	0.833	0.943	1.035	1.192	1.003	0.911	0.901
Ratio of median income (e/w)	0.858	0.736	0.838	0.919	1.000	0.880	0.727	0.526
Autocorr of log-inc (w)	0.811	0.885	0.808	0.742	0.689	0.790	0.895	0.989
Autocorr of log-hours (w)	0.661	0.658	0.660	0.661	0.688	0.663	0.657	0.997
Earn more than med. worker (e)	0.364	0.312	0.404	0.462	0.501	0.434	0.357	0.368
Absolute change in hours (e)	0.037	0.053	0.053	0.053	0.051	0.044	0.036	0.016
Absolute change in hours (w)	0.018	0.022	0.030	0.036	0.043	0.034	0.025	0.000

Table A3: Summary of results with alternative models

Moment	Data	*(Around BM calibration)* **Degree of substitutability** ρ				*(Around NC calibration)* **Val. of leisure vol.** σ_v (normalized, NC 1)		
		-1	0.02 (BM)	0.5	1	0.75	0.50	0
Panel A: Targeted								
Fraction of entr	0.088	0.190	0.088	0.074	0.051	0.091	0.088	0.039
Hours (entr, mean)	0.243	0.246	0.242	0.246	0.256	0.244	0.244	0.252
Hours (worker, mean)	0.239	0.237	0.239	0.240	0.239	0.235	0.233	0.236
Sd of log-inc (entr)	0.658	0.668	0.655	0.676	0.597	0.612	0.547	0.519
Sd of log-inc (worker)	0.336	0.291	0.336	0.368	0.412	0.277	0.198	0.206
Sd of log-hours (entr)	0.405	0.392	0.404	0.408	0.389	0.325	0.247	0.185
Sd of log-hours (worker)	0.234	0.173	0.232	0.275	0.341	0.277	0.198	0.206
Autocorr of log-inc (entr)	0.746	0.726	0.746	0.739	0.694	0.759	0.775	0.795
Autocorr of log-hours (entr)	0.694	0.668	0.694	0.694	0.673	0.708	0.730	0.843
Population switching to worker	0.005	0.004	0.005	0.004	0.004	0.005	0.005	0.002
Panel B: Untargeted								
Ratio of mean income (e/w)	0.935	0.835	0.943	0.915	0.960	1.156	1.221	1.350
Ratio of median income (e/w)	0.858	0.701	0.838	0.818	0.917	0.905	0.886	1.667
Autocorr of log-inc (w)	0.811	0.847	0.808	0.786	0.759	0.706	0.749	1.000
Autocorr of log-hours (w)	0.661	0.660	0.660	0.660	0.656	0.706	0.749	1.000
Earn more than med. worker (e)	0.364	0.301	0.404	0.386	0.442	0.439	0.444	0.529
Absolute change in hours (e)	0.037	0.054	0.053	0.053	0.055	0.040	0.029	0.009
Absolute change in hours (w)	0.018	0.022	0.030	0.036	0.044	0.032	0.021	0.000

Table A4: Summary of results with alternative models, no recalibration

Note: The table repeats the exercises of Table A3, without recalibrating the parameters to match the targets. For the first group of models ρ varies, while the rest of the parameters remain as in the benchmark. For the second group of models, σ_v varies while the rest of the parameters remain as in the "No complementarities" calibration.

D Robustness check: Progressive taxation

The benchmark model and the alternative computations of the paper assume a flat rate in the income taxation. The rationale behind the flat rate, instead of a progressive tax regime, is as follows. The agents in the model are not as disperse in their incomes as a crude look at the US data would reflect. There is no unemployment in the model, and no source of permanent variation across individuals, such as human capital, gender, or race. In fact, it is worth reminding that the target moments to calibrate the model parameters are conditional on observable characteristics such as education. Accordingly, the progressive component of taxation would not have the same role in the model as it does in the data, as there is not the same variation in income across model agents. Moreover, the ranking of individuals in their residual incomes would be different from that in the original income levels, hence might lead to assigning them a tax rate different from what they face in reality. For these two main reasons, the benchmark calibration abstracts away from the progressive taxation.

However, US tax policy is progressive, hence it is worthwhile to study the robustness of the main results of the paper to introducing such a feature. The quantitative analysis in this section solves the benchmark model and the two main alternatives with progressive taxation using a commonly-used functional form relating the tax rate to the taxable income. In particular, it follows Heathcote et al. (2014), and assumes that the income taxes are given as: $T(x) = x - \lambda x^{1-\kappa}$, where x is the total income, including labor and capital earnings. Here, κ determines the degree of progressivity in the tax regime.[2] Using this specification, the analysis includes two different sets of computations following two studies estimating the progressivity parameter κ, with considerable discrepancy between each other. First estimation is that of Guner et al. (2014) using $\kappa = 0.036$. Second is the own estimation of Heathcote et al. (2014) equal to $\kappa = 0.151$. As the latter paper discusses, the main reason between the difference in the two values is their data, as Guner et al. (2014) do not use the government transfers in their estimation. In any case, the difference in the two estimations serves well for my purpose of showing the robustness of the results to a possible progression in the taxes. In these computations, λ is an endogenous variable, being adjusted to keep the

[2] Notice that $\kappa = 0$ implies a flat rate –as is the case in the main body of the paper with $\lambda = 1 - \tau$. $\kappa = 1$ implies uniform after-tax income.

government's budget balanced.

For each of these specifications, I redo the calibration exercises for the benchmark (that of Section 4) and the alternatives with no complementarities and stable preferences (those of Section 6). The calibrated parameters are summarized in Table A5 and the implied moments are given in Table A6.

Regardless of the degree of progressivity in the tax scheme, the implied moments are similar to those of the computations with a flat rate. In turn, this exercise reassures the robustness of the results of the paper to assuming the, more realistic, progressive income taxes. Hence, in line with the aforementioned motives for the flat rate, the analysis in the main body of the paper uses the simpler flat rate tax policy.

Parameter	**Low progressivity** [Guner et al. (2014)]			**High progressivity** [Heathcote et al. (2014)]		
	BM	NC	SP	BM	NC	SP
A_0	2.643	2.650	2.393	2.596	2.624	2.366
ρ	0.005	1.000	1.000	-0.10	1.00	1.00
σ_v	0.545	0.550	0.000	0.578	0.585	0.000
σ_A	0.107	0.130	0.167	0.111	0.134	0.170
σ_y	0.044	0.000	0.062	0.044	0.000	0.060
π_v	0.643	0.687	NA	0.651	0.691	NA
π_A	0.785	0.853	0.759	0.780	0.852	0.754
v_0	86	89	76	75	78	67
ω	0.556	0.549	0.527	0.554	0.550	0.550
s	0.010	0.008	0.015	0.010	0.008	0.014

Table A5: Calibrated parameters with alternative taxation schemes

Note: The table summarizes the calibrated parameters for the benchmark and the two alternatives with "No complementarities" (NC) and "Stable preferences" (SP), in case of introducing progressive taxation instead of the flat rate. Progressive income taxes are introduced following the formulation in Heathcote et al. (2014). In the first set of computations, the progressivity parameter, τ, is set at 0.036 following Guner et al. (2014). In the second set, τ is 0.151 following Heathcote et al. (2014).

Moment	Data	Low progressivity [Guner et al. (2014)]			High progressivity [Heathcote et al. (2014)]		
		BM	NC	SP	BM	NC	SP
Panel A: Targeted							
Fraction of entr	0.088	0.089	0.088	0.088	0.088	0.088	0.088
Hours (entr, mean)	0.243	0.242	0.243	0.243	0.243	0.243	0.244
Hours (worker, mean)	0.239	0.239	0.239	0.239	0.239	0.239	0.239
Sd of log-inc (entr)	0.658	0.657	0.659	0.656	0.658	0.658	0.657
Sd of log-inc (worker)	0.336	0.336	0.361	0.336	0.336	0.369	0.334
Sd of log-hours (entr)	0.405	0.405	0.405	0.219	0.405	0.406	0.200
Sd of log-hours (worker)	0.234	0.234	0.361	0.100	0.234	0.369	0.093
Autocorr of log-inc (entr)	0.746	0.748	0.746	0.740	0.746	0.745	0.752
Autocorr of log-hours (entr)	0.694	0.695	0.695	0.790	0.694	0.697	0.795
Population switching to worker	0.005	0.005	0.005	0.005	0.005	0.005	0.005
Panel B: Untargeted							
Ratio of mean income (e/w)	0.935	0.939	1.192	0.889	0.949	1.202	0.896
Ratio of median income (e/w)	0.858	0.839	1.005	0.520	0.844	1.039	0.535
Autocorr of log-inc (w)	0.811	0.804	0.693	0.989	0.805	0.693	0.989
Autocorr of log-hours (w)	0.661	0.662	0.692	0.998	0.667	0.693	0.998
Earn more than med. worker (e)	0.364	0.395	0.508	0.351	0.408	0.522	0.356
Absolute change in hours (e)	0.037	0.052	0.050	0.015	0.052	0.050	0.014
Absolute change in hours (w)	0.018	0.030	0.043	0.000	0.030	0.043	0.000

Table A6: Moments with alternative taxation schemes

Note: The table documents the moments implied by the benchmark and the two alternatives with "No complementarities" (NC) and "Stable preferences" (SP), in case of introducing progressive taxation instead of the flat rate. Progressive income taxes are introduced following the formulation in Heathcote et al. (2014). In the first set of computations, the progressivity parameter, τ, is set at 0.036 following Guner et al. (2014). In the second set, τ is 0.151 following Heathcote et al. (2014).

E Extension: Heterogeneity in value of leisure volatility

The benchmark model does not feature any household-level heterogeneities. This allows a simpler analysis with considerable success in replicating many features of the data, but it might be a rather naive assumption in the context of entrepreneurship. For instance, Levine and Rubinstein (2017) show that even though the income differentials pointed out in Hamilton (2000) exist, there is a subpopulation within the self-employed that (i) earn more than salaried workers, (ii) earn more than other self-employed and (iii) form the smaller portion of the self-employed population.

This section extends the benchmark model to allow for heterogeneities in the volatility in the value of leisure. In particular, the model here assumes that a $1-\lambda$ fraction of consumers have their value of leisure constant across time and between each other. The rest of the population is subject to the process described in the benchmark model of the paper for their v. The calibration strategy is the same as in the benchmark other than setting λ equal to 0.6 and 0.8. The calibrated parameters of these settings are given in Table A7, and the implied moments are given in Table A8. Overall, the implied moments in case of such heterogeneity are similar to the benchmark.[3]

Parameter	**Unstable group, λ**	
	0.8	0.6
A_0	2.593	2.615
ρ	0.150	0.320
σ_v	0.575	0.622
σ_A	0.110	0.107
σ_y	0.042	0.040
π_v	0.636	0.648
π_A	0.789	0.789
v_0	78	78
ω	0.549	0.549
s	0.011	0.010

Table A7: Calibrated parameters with heterogeneous value of leisure volatility
Note: The parameters not reported in this table are kept as in the benchmark calibration.

[3]These more sophisticated models are also similar to the benchmark model in the qualitative implications of shutting down the complementarities. For instance, setting ρ equal to 1 in the set up with $\lambda = 0.6$ leads to over-predicting the dispersion in workers' hours by around 35 percent.

Moment	Data	Size of unstable group, λ		
		1 (BM)	**0.8**	**0.6**
Panel A: Targeted				
Fraction of entr	0.088	0.088	0.087	0.087
Hours (entr, mean)	0.243	0.242	0.243	0.241
Hours (worker, mean)	0.239	0.239	0.239	0.239
Sd of log-inc (entr)	0.658	0.655	0.661	0.657
Sd of log-inc (worker)	0.336	0.336	0.337	0.331
Sd of log-hours (entr)	0.405	0.404	0.406	0.408
Sd of log-hours (worker)	0.234	0.232	0.233	0.234
Autocorr of log-inc (entr)	0.746	0.746	0.749	0.754
Autocorr of log-hours (entr)	0.694	0.694	0.695	0.698
Population switching to worker	0.005	0.005	0.005	0.005
Panel B: Untargeted				
Ratio of mean income (entr/worker)	0.935	0.943	0.966	0.951
Ratio of median income (entr/worker)	0.858	0.838	0.851	0.809
Autocorr of log-inc (worker)	0.811	0.808	0.801	0.798
Autocorr of log-hours (worker)	0.661	0.660	0.657	0.674
Earning more than median worker (entr)	0.364	0.404	0.414	0.389
Earning more than median worker (entr, hourly)	0.300	0.348	0.383	0.367
Absolute change in hours (entr)	0.037	0.053	0.050	0.046
Absolute change in hours (worker)	0.018	0.030	0.027	0.023

Table A8: Summary of results with heterogeneous value of leisure volatility

Differently from the benchmark, however, such an extension allows comparison between different possible groups within the population. Table A9 shows how the implied hours, income and number of entrepreneurs change across types (different in their volatility of leisure) in these two alternative calibration exercises. Overall pattern is clear: (i) entrepreneurship is more common in the group with volatile value of leisure, (ii) an entrepreneur of the stable type works much more on average, and (iii) the income of an average entrepreneur of the stable type, relative to that of a worker of the same type is much higher than the corresponding ratio for for the unstable type.

Discussion on the inflexibility premium. The differences in the entrepreneurial income between the subgroups can be interpreted as the inflexibility premium for salaried

Moment	Size of unstable group, λ						
	1 (BM)	**0.8**			**0.6**		
	All	**All**	**Unstable**	**Stable**	**All**	**Unstable**	**Stable**
Fraction of entr	0.088	0.087	0.095	0.059	0.087	0.110	0.054
Hours (entr, mean)	0.242	0.243	0.241	0.254	0.241	0.236	0.254
Ratio of mean income (e/w)	0.943	0.966	0.955	1.040	0.951	0.919	1.052

Table A9: Summary of results with heterogeneous value of leisure volatility

work. One type of households do not have preference-related need for flexibility, hence the relative income of entrepreneurs in this population can measure the counterfactual income level of entrepreneurs had the value of leisure shocks were shut down. The rest of the population, however, has the additional flexibility motive for entrepreneurship. Accordingly, the comparison of average income of entrepreneurs in this subgroup can be a measure the aforementioned premium. Differences in the relative income between the entrepreneurs from the two groups documented in Table A9 show that the implied inflexibility premium for salaried work is around 9 percent and 13 percent depending on the assumed size of the stable group.

F Supplementary figures

There are a few figures hinted briefly in the main body of the paper that are omitted. This section goes over each of these separately.

Repeating Figure 2 of the paper without conditioning on education groups. Figure 2 of the paper shows the hours and hourly income relationship for workers and entrepreneurs, separately for high school graduates and college graduates. The logic behind this is that individuals with similar skills would more likely work in similar tasks and production units, hence might have more complementarities towards each other.

In order to show that the patterns in Figure 2 are robust to merging these skill groups, Figure A1 replicates the former figure without conditioning on the educational groups. Both panels of the latter show that the findings are similar in this unconditional exercise.

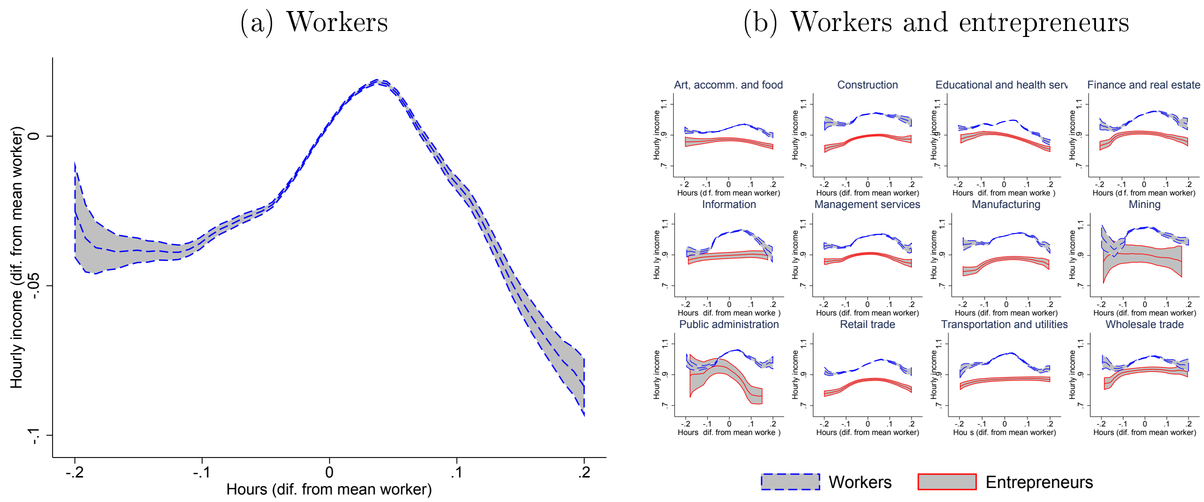

Figure A1: Working hours and wages within each sector, not conditioning on education groups

Note: Figures in Panel A plot the Kernel-weighted local polynomial smoothing of hourly income against the levels of hours (both differences from sector mean). Figures in Panel B plot that of hourly income across the levels of difference in hours from the mean worker in each sector. Plotted around the curves are the confidence intervals at 95 percent level. The initial levels of hourly income are normalized to have an overall average of 1 for workers.

Model counterpart of Panel B of Figure 1 of the paper. Section 5 of the paper mentions that the model generates the patterns highlighted in the data regarding the distribution of changes in hours, and residual hours conditional on hourly income. Figure A2 is included here to illustrate this finding.

Hours and hourly income relationship in the model without complemantarities. The empirical evidence in Section 2 of the paper documents the relationship between hours and hourly income for different sectors and skill groups within workers and entrepreneurs. In particular, Figure 2 shows that hourly income peaks at a level of working hours close to the sector mean for workers, and that for entrepreneurs it is rather monotone for most sectors and skills –either increasing or decreasing.

In addition, Section 5 of the paper shows the implications on the benchmark model in this aspect. Figure 3 in the paper illustrates the implications of model complementarities on the hourly income and hours relationship, particularly that the wages follow a hump-shaped pattern with hours as is the case in the data. The exercise here highlights how the corresponding implications of the model of Section 6.1 without complementarities contrast

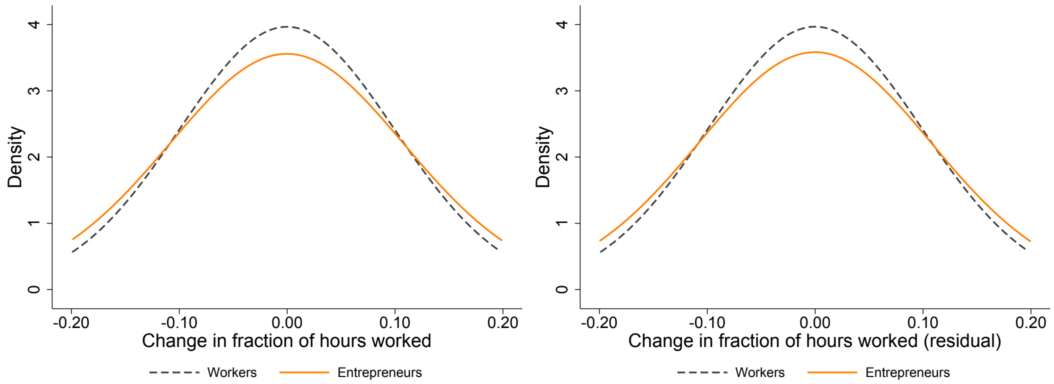

Figure A2: Distribution of changes in hours (model)

Note: For the left panel, the variable in the x-axis is the change in the fraction of hours worked between two model periods. For the right panel, the x-axis is the change in the fraction of residual hours (hours conditional on hourly income) worked between two consecutive periods. The sample only includes individuals that keep the same occupation (worker or entrepreneur) for the two periods.

those from the data and the benchmark model. Figure A3 reminds that the hours of workers in this model does not affect their wages, particularly that there is not a hump-shaped relationship. This is the trivial implication of having a linear aggregation of working hours within each labor unit. Meanwhile, the hourly income of entrepreneurs are increasing in their hours, in a fashion similar to the benchmark model and some of the sectors in the data.

(a) Low productivity (b) High productivity

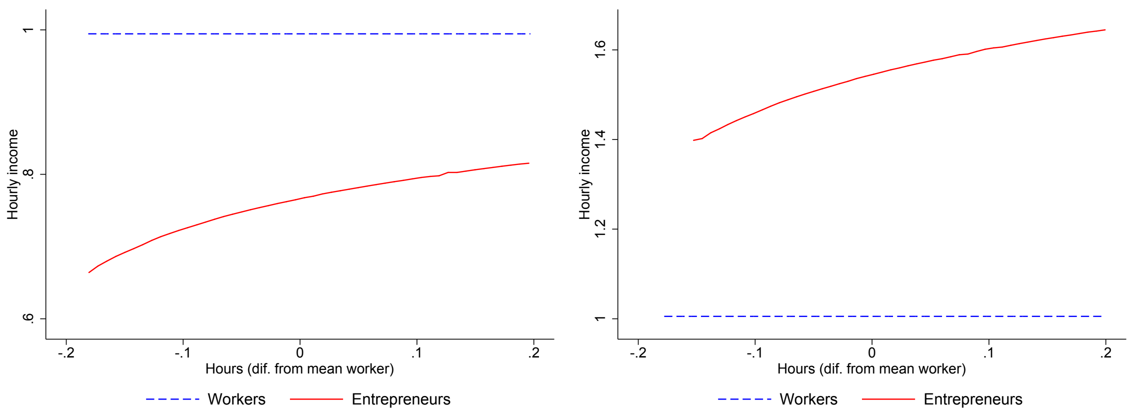

Figure A3: Working hours and wages across productivity levels ("No complementarities" model, $\rho = 1$)

Note: The initial levels of hourly income are normalized to have an overall average of 1 for workers. The left (right) figure uses workers with lower (higher) worker productivity and entrepreneurs with middle (higher) entrepreneurial productivity. Left (right) figure plots the Kernel-weighted local polynomial smoothing of hourly income across the levels of difference in hours from the mean worker with low (high) productivity. Confidence intervals are omitted as there is no further variation in hourly incomes given hours and productivity levels.

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