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Head starts in dynamic tournaments?[☆]

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HIGHLIGHTS

- We study head starts in dynamic two-player tournaments a la Lazear and Rosen (1981).
- A principal values aggregate effort and the highest effort exerted by the players.
- It is always optimal to bias the tournament by awarding a head start.
- A small head start increases the highest effort without decreasing aggregate effort.

ABSTRACT

In promotion contests or other tournament-like situations, a principal may attach some value to the highest effort expended by an agent. We show that whenever agents interact over multiple periods, awarding a head start to one of them is optimal even with completely symmetric agents. Awarding a small head start increases maximum individual effort without decreasing aggregate effort.

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D82
L12

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1. Introduction

Awarding a head start is typically considered an affirmative action policy which serves to level the playing field between *ex-ante unequal* players. A small literature on contests and tournaments analyzes the effect of such affirmative action on asymmetric competitors' performances, see e.g. Franke (2012) and Kirkegaard (2012) We show that a principal may also wish to award a head start with *completely identical* competitors – i.e. to 'unlevel' the playing field – when he attaches some value to the highest individual effort. Situations where this might be an appealing goal include for example sports competition, where only the felt enjoyment of the spectators matters.

The intuition underlying our results is the following. Fierceness of competition in the final period diminishes with the advantage an agent has achieved over the other. An agent that has been given a head start early on thus has an incentive to work harder than her opponent to retain and increase her lead. The reward for her extra effort is reduced competition and thus lower cost of effort in the future. We show that the introduction of a small head start increases the favored individual's effort at the beginning of the tournament (and reduces the effort of the disadvantaged individual) while having only second-order effects on effort at the end of the tournament.[1]

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[1] Other literature analyzing head starts in tournaments are Lazear and Rosen (1981), Konrad (2009), Klein and Schmutzler (forthcoming), Siegel (2014), Seel and

The paper is closely related to Drugov and Ryvkin (forthcoming) and Meyer (1992). Drugov and Ryvkin (forthcoming) study a one-shot contest with a general technology (contest success function) and provide conditions under which biasing a contest with symmetric players is beneficial for a principal with goals such as the maximization of aggregate effort, the highest effort, or the highest winning effort. In contrast, we study a dynamic two-stage game with observation errors between symmetric players. We show that with the introduction of dynamics, a small head start *always* benefits the principal, regardless of the relative importance of highest and aggregate effort (but assuming the weight on aggregate effort is smaller than 1), even when no head-start would be strictly optimal in the static or one-shot game. Thus, our paper complements their analysis by showing how dynamic considerations may increase the value of biasing a contest for a principal. Meyer (1992) also considers a dynamic setting where two identical contestants compete in two subsequent contests. She finds that a principal maximizing aggregate effort should introduce a bias in the second contest in favor of the first contest winner. This increases the value of winning in contest one and thus efforts in the first period at the expense of efforts in the second period. In contrast to our setting, the bias is endogenously determined by first period outcomes. We show that introducing a bias that is completely exogenous also may benefit the principal.

The model is closely related to those in Denter and Sisak (2015), Aoyagi (2010), or Ederer (2010), all of which study dynamic (two-stage) tournament models à la Lazear and Rosen (1981) or Nalebuff and Stiglitz (1983). The distinctive feature of the current model is to focus on head starts in tournaments.

2. Model

Two agents, $i = A, B$, compete for a prize of common value $v = 1$ (e.g. a bonus or promotion) over two stages, $t = 1, 2$. In each stage, each agent may exert costly effort $x_i^t \geq 0$ to increase her chances of receiving the prize. At the end of stage 2, the difference in observed cumulative effort determines the chance of success.

The principal values both the agents' aggregate efforts as well as the greatest individual effort exerted.[2] Hence, if X denotes aggregate effort and $\tilde{x}$ the greatest individual cumulative effort,[3] the principal seeks to maximize

$$\pi = \lambda X + (1-\lambda)\tilde{x}.$$

$\lambda \in [0, 1)$ is a weight the principal assigns to aggregate effort.[4] The principal need not put a positive weight on X for our results to hold. We only need $\lambda < 1$.

The principal may bias the tournament in favor of one agent by awarding a head start. Such a head start increases an agent's cumulative effort and through this her chance of success, but does not enter π. Denote the head start by $h \geq 0$. Without loss of generality assume the head start is awarded to agent A. Agent i's effort at t, x_i^t, comes at costs $C(x_i^t)$, where we assume $C'(x) > 0$ for all $x > 0$, $C'(0) = 0$, and $C''(x) > 0$. Throughout we assume that the cost function is sufficiently convex, i.e. that $C''(x)$ is sufficiently large. This ensures both existence and uniqueness of equilibrium, see for example Denter and Sisak (2015). In Assumption 1 in Appendix A.1 we specify an exact condition guaranteeing uniqueness.

Define

$$X_A^1 = h + x_A^1, \qquad X_A^2 = h + x_A^1 + x_A^2$$
$$X_B^1 = x_B^1, \qquad X_B^2 = x_B^1 + x_B^2.$$

Thus, $X = X_A^2 + X_B^2 - h$. X_A^1 and X_B^1 are observed by the agents before their choice of effort in stage two x_i^2, $i = A, B$, for example because they work alongside each other. The principal awards the prize after observing X_A^2 and X_B^2 to the agent with the higher aggregate effort (including head start). However, his evaluation is noisy and disturbed by an additive observation error ϵ with density $f(\epsilon)$ (and cdf $F(\epsilon)$) that is differentiable, quasi-concave and symmetric around zero.[5] Hence, the principal observes $X_A^2 - X_B^2 - \epsilon$ rather than the true value $X_A^2 - X_B^2$. Thus in contrast to the agents the principal only imperfectly observes effort.

Given the set-up, agent A receives the prize with probability

$$Pr[X_A^2 - X_B^2 > \epsilon] = F(X_A^2 - X_B^2).$$

3. Optimal head start

We solve the two-stage game by backward induction. At the beginning of stage 2, agents observe and condition their decision on

$$\Delta^2 = h + x_A^1 - x_B^1 \qquad (1)$$

as solely the difference in efforts is relevant for the principal's decision of whom to award the prize. Individual payoffs in the second stage are

$$\pi_A^2 = F(\Delta^2 + x_A^2 - x_B^2) - C(x_A^2)$$
$$\pi_B^2 = 1 - F(\Delta^2 + x_A^2 - x_B^2) - C(x_B^2).$$

Assuming the variance of the noise term ϵ is sufficiently large or the cost function is sufficiently convex, the agents' payoff functions are strictly concave.[6]The unique pure-strategy Nash equilibrium is symmetric and given by

$$x_A^2 = x_B^2 = x^2(\Delta^2) = C'^{-1}\left(f(\Delta^2)\right),$$

see for example Denter and Sisak (2015). The intuition for this result is as follows: The marginal increase in the probability to come out ahead is identical for both and equal to $f(\Delta^2 + x_A^2 - x_B^2)$. Given identical and strictly increasing marginal cost functions, this implies $x_A^2 = x_B^2 = x^2(\Delta^2)$ in the unique equilibrium. Because $C'(0) = 0, x^2(\Delta^2) > 0$ whenever $f(\Delta^2) > 0$.

By quasi-concavity of the density function f and symmetry around zero, individual and aggregate effort is always maximal when both agents are evenly matched in the second period ($\Delta^2 = 0$). Thus, considering only the second period, a head start always reduces both individual and aggregate effort.

Given the stage 2 subgame equilibrium, expected equilibrium payoffs from that stage are

$$\pi_A^2 = F(\Delta^2) - C(x^2(\Delta^2))$$
$$\pi_B^2 = 1 - F(\Delta^2) - C(x^2(\Delta^2))$$

Wasser (2014), and Stracke (2013). Kawamura and de Barreda (2014) also highlight the desirability of head starts with (ex-ante) symmetric players though in their framework the principal is concerned with selection while in ours he is interested in effort.

2 The principal may as well try to maximize the expected effort of the winner. However, it can be shown that in out framework this does not change results at all and we hence abstain from modelling this goal explicitly.

3 Formally, $\tilde{x} = \max\left\{x_A^1 + x_A^2, x_B^1 + x_B^2\right\}$.

4 Technically speaking, a concern for the greatest individual effort may reflect a best-shot production technology. This arises naturally if agents work on individual projects and only the best of these is implemented by the principal.

5 Note that we could equivalently let the principal choose a biased observation error.

6 For example, if $C(x) = \frac{c}{2}x^2$ and noise is normal with variance σ^2, we need that $c\,\sigma^2 > (2e\pi)^{-1/2}$.

yielding expected stage 1 payoffs corresponding to

$$\pi_A^1 = F(\Delta^2) - C(x^2(\Delta^2)) - C(x_A^1)$$
$$\pi_B^1 = 1 - F(\Delta^2) - C(x^2(\Delta^2)) - C(x_B^1).$$

The focus of the paper is to show that a positive bias is optimal if the principal values the greatest individual effort ($\lambda < 1$). To show this we proceed as follows. First, we assume the head start to be zero, $h = 0$. We then solve the first stage and determine the equilibrium. Then we determine comparative statics around $h = 0$.

Taking the first order conditions with respect to an agent's own effort yields

$$\begin{aligned} \frac{\partial \pi_A^1}{\partial x_A^1} &= f(\Delta^2) - C'(x^2(\Delta^2))\frac{dx^2(\Delta^2)}{d\Delta^2} - C'(x_A^1) \overset{!}{=} 0 \\ \frac{\partial \pi_B^1}{\partial x_B^1} &= f(\Delta^2) + C'(x^2(\Delta^2))\frac{dx^2(\Delta^2)}{d\Delta^2} - C'(x_B^1) \overset{!}{=} 0 \end{aligned} \quad (2)$$

where we already made use of $\frac{\partial \Delta^2}{\partial x_A^1} = 1 = -\frac{\partial \Delta^2}{\partial x_B^1}$ using the definition of Δ^2 in (1). Under Assumption 1 (Appendix A.1) there exists a unique equilibrium in pure strategies which is symmetric and where $x_A^1 = x_B^1 = x^1 = C'^{-1}(f(0))$. To see this, observe that $\Delta^2 = 0$ in the symmetric equilibrium with $h = 0$, and thus $\frac{dx^2(\Delta^2)}{d\Delta^2}|_{\Delta^2=0} = \frac{1}{C''(C'^{-1}(f(0)))}f'(0) = 0$, as $f'(0) = 0$ by differentiability and symmetry around zero. Note that in this situation both players choose identical strategies in *both* stages and those efforts are also equal across stages, $x^1 = x^2(0)$. Further note that the derivative of effort in stage 2 with respect to Δ^2 is zero at this point. Hence, we need to show that at least one agent's effort increases in stage 1 when the principal increases h from zero. Using Cramer's rule it is easy to show that $\frac{\partial x_A^1}{\partial h}\Big|_{h=0} = -\frac{\partial x_B^1}{\partial h}\Big|_{h=0}$. It hence remains to be shown that this derivative is non-zero.

Lemma 1.

$$\frac{\partial x_A^1}{\partial h}\bigg|_{h=0} > 0.$$

Proof. See Appendix. □

We are now able to state our main result:

Proposition 1. *The optimal head start is strictly positive, $h > 0$.*

Proof. The principal's objective function is $\pi(h) = \lambda X(h) + (1 - \lambda)\tilde{x}(h)$. Since with $h = 0$ all efforts are identical, we can take $\tilde{x}(h) = x_A^1(h) + x_A^2(\Delta^2(h))$, thus focussing on player A. Aggregate effort is $X(h) = x_A^1(h) + x_B^1(h) + 2x^2(\Delta^2(h))$. Taking the derivative of $\pi(h)$ with respect to h yields:

$$\begin{aligned} \frac{\partial \pi(h)}{\partial h} &= \lambda\left(\frac{\partial x_A^1(h)}{\partial h} + \frac{\partial x_B^1(h)}{\partial h} + \frac{2f'(\Delta^2)}{C''(C'^{-1}(f(\Delta^2)))}\right) \\ &\quad + (1-\lambda)\left(\frac{\partial x_A^1(h)}{\partial h} + \frac{f'(\Delta^2)}{C''(C'^{-1}(f(\Delta^2)))}\right). \end{aligned}$$

At $h = 0$ we know that $\frac{\partial x_A^1(h)}{\partial h} = -\frac{\partial x_B^1(h)}{\partial h}$ and that $\Delta^2 = 0$. Since $f'(0) = 0$ the derivative simplifies to

$$\frac{\partial \pi(h)}{\partial h}\bigg|_{h=0} = (1-\lambda)\frac{\partial x_A^1}{\partial h}\bigg|_{h=0} = -\frac{(1-\lambda)f(0)f''(0)}{[C''(x^1)]^2 + 2f(0)f''(0)},$$

which is strictly positive by Lemma 1 if $\lambda < 1$. Hence, the principal's payoff strictly increases in h when $h = 0$ and the maximum is attained for some $h > 0$. □

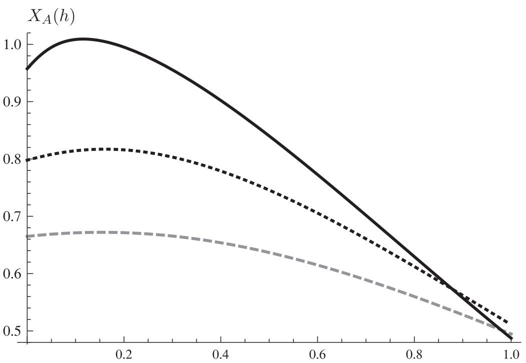

Fig. 1. Agent A's total effort $X_A(h) = x_A^1(h) + x_A^2(h)$ as a function of h. Observation noise is assumed Gaussian with zero mean and standard deviation σ, cost is quadratic, $C(x) = \frac{1}{2}x^2$. The solid curve corresponds to $\sigma = 5/6$, the dotted to $\sigma = 1$, and the dashed to $\sigma = 6/5$.

Giving a head start to player A has no effect on aggregate effort at the margin, but increases the greatest effort. Hence, it is always optimal to set a positive head start. This is illustrated in Fig. 1.

4. Discussion

To which degree do our results depend on the specific cost structure? We assume additive separability of costs over time. In many situation this is appropriate, such as choice of work effort in a firm where outlays are not monetary but effort that has to be exerted every day, month or year. In other situations, however, a different cost structure may be more appropriate. Let us consider the other extreme and assume that costs depend only on the sum of efforts over time, i.e. $C(x^1+x^2)$. In this case, second stage effort in an interior solution will always be chosen such that total effort by both agents is identical, $x_A^1+x_A^2 = x_B^1+x_B^2 = X$. To see this, note that from the first-order conditions we get $f(\Delta^2+x_A^2-x_B^2) = C'(x_i^1+x_i^2)$ for both in an interior equilibrium, implying marginal cost in stage 2 must be identical for both. In this type of equilibrium, aggregate effort is determined uniquely through $f(h) = C'(X^*) \Leftrightarrow X^* = C'^{-1}(f(h))$, which is maximized when $h = 0$. This logic implies that there always exists an equilibrium, where both individuals invest first period efforts $x_A^1 = x_B^1 = X^*$ and do not add any effort in stage two. Effectively in this equilibrium, the game reduces to a one-shot game. Hence, the cost structure, and therewith the details of the problem studied, plays an important role for the optimality of a head start.

Appendix. Mathematical appendix

A.1. Uniqueness of equilibrium and the cost function

In games like the one we are studying, it is well known that the cost function needs to be sufficiently convex to guarantee concavity of the players' optimization problems (see, e.g., Denter and Sisak, 2015). We assume this to be the case throughout. Still, we need another assumption that guarantees uniqueness of equilibrium:

Assumption 1. Throughout, we assume that for all $(x^1, x^2, \Delta^2) \in [0, C^{-1}(1)]^2 \times \mathbb{R}$

$$1 > \frac{K(x^2, \Delta^2)}{K(x^2, \Delta^2) + C''(x_i^1)[C''(x^2)]^3} > -1,$$

where

$$\begin{aligned} K(x^2, \Delta^2) &:= [C''(x^2)]^2\left([f'(\Delta^2)]^2 + f''(\Delta^2)f(\Delta^2)\right) \\ &\quad - C'''(x^2)f(\Delta^2)[f'(\Delta^2)]^2 - [C''(x^2)]^3 f'(\Delta^2). \end{aligned}$$

The assumption assures that the slope of players' best response functions is never greater than 1 in absolute value. For example, if we assume noise is standard normal and $C(x) = \frac{1}{2}x^2$, the condition holds (and payoffs are also strictly concave).

A.2. Proof of Lemma 1

In Eq. (2) we already derived the players' first period FOCs. If $h = 0$ and there is sufficient noise, i.e. if $E[\epsilon^2]$ is sufficiently large, or if $C''(x)$ is sufficiently large, there is a unique symmetric equilibrium in which $x_A^1(0) = x_B^1(0) = x^1 = C'^{-1}(f(0)) = x^2(0)$. Thus, in this situation $\Delta^2 = 0$.

Now we totally differentiate the set of FOCs to obtain comparative statics results:

$$\left.\frac{\partial^2 \pi_A^1}{\partial (x_A^1)^2}\right|_{h=0 \wedge x_A^1 = x_B^1 = x^1} = -\frac{f(0)f''(0)}{C''(x^1)} - C''(x^1)$$

$$\left.\frac{\partial^2 \pi_A^1}{\partial x_A^1 \partial x_B^1}\right|_{h=0 \wedge x_A^1 = x_B^1 = x^1} = \frac{f(0)f''(0)}{C''(x^1)}$$

$$\left.\frac{\partial^2 \pi_A^1}{\partial x_A^1 \partial h}\right|_{h=0 \wedge x_A^1 = x_B^1 = x^1} = \frac{f(0)f''(0)}{C''(x^1)}$$

and

$$\left.\frac{\partial^2 \pi_B^1}{\partial x_A^1 \partial x_B^1}\right|_{h=0 \wedge x_A^1 = x_B^1 = x^1} = \frac{f(0)f''(0)}{C''(x^1)}$$

$$\left.\frac{\partial^2 \pi_B^1}{\partial (x_B^1)^2}\right|_{h=0 \wedge x_A^1 = x_B^1 = x^1} = -\frac{f(0)f''(0)}{C''(x^1)} - C''(x^1)$$

$$\left.\frac{\partial^2 \pi_B^1}{\partial x_B^1 \partial h}\right|_{h=0 \wedge x_A^1 = x_B^1 = x^1} = -\frac{f(0)f''(0)}{C''(x^1)}$$

where we used that $x^2(0) = x^1 = C'^{-1}(f(0))$ and $f'(0) = 0$. Now define

$$A = \begin{pmatrix} \frac{\partial^2 \pi_A^1}{\partial (x_A^1)^2} & \frac{\partial^2 \pi_A^1}{\partial x_A^1 \partial x_B^1} \\ \frac{\partial^2 \pi_B^1}{\partial x_A^1 \partial x_B^1} & \frac{\partial^2 \pi_B^1}{\partial (x_B^1)^2} \end{pmatrix},$$

$$A^{h,A} = \begin{pmatrix} -\frac{\partial^2 \pi_A^1}{\partial x_A^1 \partial h} & \frac{\partial^2 \pi_A^1}{\partial x_A^1 \partial x_B^1} \\ -\frac{\partial^2 \pi_B^1}{\partial x_B^1 \partial h} & \frac{\partial^2 \pi_B^1}{\partial (x_B^1)^2} \end{pmatrix},$$

and

$$A^{h,B} = \begin{pmatrix} \frac{\partial^2 \pi_A^1}{\partial (x_A^1)^2} & -\frac{\partial^2 \pi_A^1}{\partial x_A^1 \partial h} \\ \frac{\partial^2 \pi_B^1}{\partial x_A^1 \partial x_B^1} & -\frac{\partial^2 \pi_B^1}{\partial x_B^1 \partial h} \end{pmatrix}.$$

Comparative statics are then $\left.\frac{dx_A^1}{dh}\right|_{h=0 \wedge x_A^1 = x_B^1 = x^1} = \frac{|A^{h,A}|}{|A|}$ and $\left.\frac{dx_B^1}{dh}\right|_{h=0 \wedge x_A^1 = x_B^1 = x^1} = \frac{|A^{h,B}|}{|A|}$. Using the above expressions and simplifying yields

$$\left.\frac{dx_A^1}{dh}\right|_{h=0 \wedge x_A^1 = x_B^1 = x^1} = -\left.\frac{dx_B^1}{dh}\right|_{h=0 \wedge x_A^1 = x_B^1 = x^1} = -\frac{f(0)f''(0)}{C''(x^1)^2 + 2f(0)f''(0)} > 0.$$

To see this, note that Assumption 1 reduces to

$$1 > \frac{f(0)f''(0)}{[C''(x^1)]^2 + f(0)f''(0)} > -1$$

when $h = 0$, $x^1(0) = x^2(0)$ and $\Delta^2 = 0$, which implies the inequality.

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