Input cost, capacity utilization and substitution in the short-run[*]

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Abstract. This article studies the behavior of input cost shares in an environment where labor is costly to adjust, materials can be adjusted at no cost and capital is fixed. A model relating cost shares with relative prices and adjustment costs is proposed, allowing joint estimation of the elasticity of substitution and the adjustment cost function, which is an unknown function of the capacity utilization. Based on a panel of more than 700 manufacturing firms, we find evidence of strong input share variations according to the degree of capacity utilization. The estimated shapes of adjustment costs curves of labor are in agreement with our theoretical model, and we obtain sensible elasticities of substitution estimates. Based on such estimates, we find evidence of a negative (positive) bias in downturns (recoveries) in conventional productivity growth measures.

1 Introduction

This article studies the short run behavior of input cost shares. In particular, we study how shares of normally considered variable inputs (materials and labor) can be affected by short run decisions of firms in the presence of exogenous demand shocks. And we argue that the shares' behavior will follow a pattern closely related to the relative adjustment costs and possibility of substitution among the inputs. The problem is well motivated in practice, since cost shares

are widely used in applied analysis (for example, for estimating elasticities of substitution between inputs or to measure elasticities of the output with respect to the inputs). However, such applications can be quite misleading when shares' short-run behavior is not taken into account.

This work can be seen as an application in the tradition of temporary equilibrium models. Following Berndt and Fuss (1986a), temporary equilibrium can be defined as "ocurring whenever the shadow value of any input and/or output differs from its market price". In production applications, authors begin by assuming that in the short-run some inputs are variable and others are costly to adjust, and that firms will minimize short-run variable costs, which may include some costs of adjustment. This is the approach followed, for example, in Berndt and Fuss (1986b), Morrison (1986), Slade (1986) and Schankerman and Nadiri (1986), to mention only a few. The marginal products of the incompletely adjusted inputs will differ from their market rental prices. That is, they will have "shadow" prices or costs that will not be equal to the observed prices. Several procedures have been proposed to test for these situations, to retrieve or approximate the shadow prices, and to use them to properly compute the growth of productivity or the patterns of substitution among inputs.

In this paper, firstly, we build a theoretical framework to explain the relationships between short-run decisions and the observed cost shares in a technological environment where cost shares are independent from output in the long-run. We will assume that firms minimize short-run costs conditional on the level of available capital, considering labor an input costly to adjust and materials freely variable. The degree of adjustment turns out to be related to the degree of utilization of capacity, that is, to the ratio of the output to be produced to the potential output given the installed capital. Labor will be "hoarded" in downturns to avoid incurring on high costs of adjustment, and therefore the marginal cost of labor will be low and all the available possibilities of substitution exploited.

Secondly, we develop an econometric model to simultaneously assess the degree of substitutability between labor and materials and the impact of capacity utilization in the relative shares. The model, which embodies a highly non-linear unknown function of adjustment costs, is estimated using alternative parametric and semiparametric techniques.

Thirdly, we apply the framework to study the consequences of the short- run shares' behavior on the nonparametric analysis of productivity. We argue that short-run behavior can imply mismeasurement of the elasticities when conventional measures are used. We compare in theory and practice the Solow residuals computed with the observed shares, corrected shares, and also, to check a common practice, value added shares.

We use a micropanel of more than 700 Spanish manufacturing firms, observed during a five-year period (1990-1994) that has something of a "natural experiment." The period was characterized by the development of a strong recession that peaked in 1993, followed by one year of recovery as shown in Fig. 1. The firms' data allow us to compute rather precisely the materials, labor and capital

changes and cost shares and, unlike other industrial panels, the use of individual price change indexes and capacity utilization assessments.
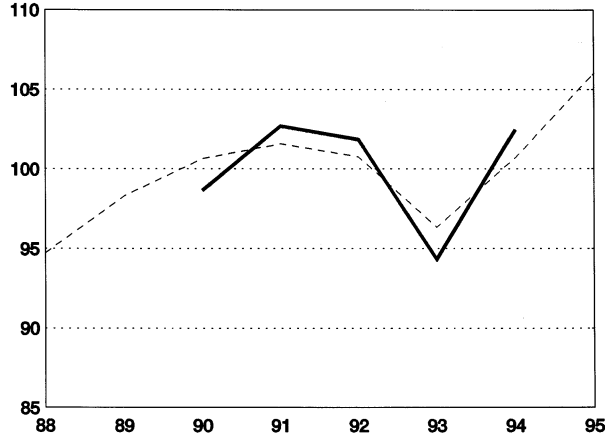
Fig. 1. Manufacturing vs. sample outputs. *Dashed line*: Manufacturing value added index (National Accounts). *Solid line*: Sample average production index

We find a series of interesting empirical results. On the one hand, we find strong evidence in short-run adjustments, and the estimated adjustment cost funtions show plausible values and a nice convex shape. However, the consideration of the adjustment costs does not seriously affect the estimates of the elasticities of substitution between labor and materials, for which we obtain values that are comparable to the scarce existing evidence (see Hamermesh 1993). But, when the estimated marginal costs of labor are used to correct the observed input cost shares, we find strong evidence of biases in the conventionally-computed productivity growth measures. We conclude that the (observed shares) Solow residual can understate productivity growth in downturns and overstate it in recoveries, and that the true (production) productivity growth is badly approximated by the value-added measurements. The first conclusion is close to what one would expect from Slade (1986) Monte Carlo experiments, but both conclusions question the invariance properties of the Solow residual stressed by Hall (1990).

2 Theoretical framework

Assume a firm that minimizes its variable costs conditional on the installed capital, with materials and labor as inputs. Materials are freely variable but labor is subject to adjustment costs. That is, the short-run behavior of the firm can be seen as

$$Min \quad \bar{w}Le^{AC\left(\ln \frac{L}{L^*}\right)} + \bar{p}M, \quad s.t. \quad F\left(L, M, K^*\right) = Q, \tag{1}$$

where L and M represent respectively the quantities of labor and materials, $\bar{w}$ and $\bar{p}$ their market prices, and K^* stands for the disposable capital. $F(.)$ is the production function, $AC(.)$ gives the (proportional) adjustment costs of labor

when the firm deviates from L^*, the optimal labor demand given capital and market prices. Notice that, with fixed capital and no constraints on production, the firm would produce the optimal output level given capital and the market prices of labor and materials, $Q^* = Q(\overline{w}, \overline{p}, K^*)$. So, demand for materials and labor would be $M^* = M(\overline{w}, \overline{p}, K^*)$ and $L^* = L(\overline{w}, \overline{p}, K^*)$. The adjustment costs function summarizes all the factors that can increase the unit cost of labor when the labor input is outside its equilibrium level L^* given capital. We assume that $AC(0) = 0$, and that $\partial AC(x)/\partial |x| > 0$ and $\partial^2 AC(x)/\partial |x|^2 \geq 0$ for all $x \neq 0$.

Our specification of adjustment costs is not the standard one, in which changes in the labor input, instead of proportional deviations from equilibrium, are considered the argument of the adjustment cost function. It is aimed at picking up the short-run adjustment costs and admits at least three different interpretations. Firstly, firms can be simply assumed to adjust labor without friction from period to period to its long-run desired demand L^* (according to the changes in capital) and take the adjustment costs as the intraperiod costs of deviating from the latter equilibrium reached. This interpretation is in the spirit of the standard specification. Secondly, adjustment costs can alternatively be seen as the costs of not adjusting labor to the planned long-run values. These costs can mostly be understood as stemming from the short-run productivity losses of maintaining a size of the labour force different from that planned for the available capital. Thirdly, our specification can be taken as an approximation of the fully dynamic adjustment costs derived from unexpected and transitory shocks affecting firms involved (or not) along long-run paths of adjustment. Firms for which L^* is the current long-run objective, subject to unexpected shocks that they take as transitory, if they bear high costs of adjusting permanent workers and possess other dimensions of the labor input available to be adjusted (hours, effort, temporary workers...), are likely to carry out almost the entire adjustment inside the affected period. Hamermesh (1993) mentions this result referred to working hours. In Appendix A we develop a simple model in which the labor input consists of workers and hours, we provide an explanation of the likely content of adjustment costs under the two first interpretations, and we formally show Hamermesh result.

Suppose that F is homothetic and, at the same time, weakly homothetically separable in the variable inputs. That is, F is homothetic and can also be written as $\widetilde{F}(f(L, M), K)$ where $f(L, M)$ is a homothetic subfunction – see, for example, Chambers (1988) –. These two assumptions together imply that F can be written as $\widehat{F}(G(f(L, M), K))$ where G is linear homogeneous. The assumption of homotheticity imply that, in long-run equilibrium, all the input/ouput ratios and the cost shares would be independent of the output level[1]. Given these assumptions

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[1] Homotheticity is a critical assumption for the derivation of the model and for identification in the empirical exercise. We will derive reasons for input shares to change in the presence of homotheticity. Alternatively, shares' movemements could be attributed to the non-homotheticity of the production function. However, this has never been considered too realistic. Virtually any empirical study on factor substitution with flexible functional forms, the only type that does not impose this restriction, assumes it from the beginning (see for example the list of selected studies in Chung 1994, Table 12.1). In fact, our exercise stresses a phenomenon that could be taken erroneously as an effect of the non-homotheticity of the underlying function.

we have

$$Min \left\{ wL + \overline{p}M \left| \widehat{F} \left(G \left(f(L,M), K^* \right) \right) \leq Q \right. \right\}$$

$$= Min \left\{ wL + \overline{p}M \left| G \left(\frac{f(L,M)}{K^*}, 1 \right) K^* \leq \widehat{F}^{-1}(Q) \right. \right\}$$

$$= Min \left\{ wL + \overline{p}M \left| f(L,M) \leq T(Q,K^*) = y \right. \right\},$$

and the short-run objective of the firm can be written as

$$Min \ \overline{w}Le^{AC(\ln \frac{L}{L^*})} + \overline{p}M \qquad s.t \ f(L,M) = y, \qquad (2)$$

where y represents the level of the intermediate aggregated input (a mix of labor and materials) associated to the production of Q given K^*.

Define $AC'(x) = \partial AC(x)/\partial x$ and $AC''(x) = \partial^2 AC(x)/\partial x^2$. From the first order conditions in (2) we obtain that

$$\frac{\partial f(L,M)/\partial M}{\partial f(L,M)/\partial L} = \frac{\overline{p}}{w_l(1 + AC'(\ln l))} \equiv \frac{\overline{p}}{w_l z_l}, \qquad (3)$$

where $l = \ln \frac{L}{L^*}$, $w_l = \overline{w}e^{AC(\ln l)}$ is the cost of unit of labor and $z_l = (1 + AC'(\ln l))$ represents the ratio of the marginal cost of labor to its unit cost. Interestingly enough, $z_l \lesseqgtr 1$ if $L \lesseqgtr L^*$.

Given the homotheticity of $f(.)$, (3) can be rewritten as

$$\frac{M}{L} = v \left(\frac{\overline{p}}{w_l z_l} \right) \equiv v(\omega), \qquad (4)$$

where ω is an abbreviation for the relative marginal cost, that is, the ratio of the price of materials to the marginal cost of labor, and $\partial v(\omega)/\partial \omega < 0$.

Equation (4) expresses the important short-run link between the ratio of the amounts of the variable inputs held by the firm and their relative marginal cost. This relative marginal cost depends, in addition to market prices, on the adjustment costs of labor.

In order to study the effect of exogenously induced variations in output on the ratio of materials to labor we proceed as follows. By homotheticity, $f(L,M) = y$ can be written as $\widehat{f}(g(L,M)) = y$, where $g(\cdot)$ is a linear homogeneous function. Therefore $L = \widehat{f}^{-1}(y)/g(1, v(\omega)) = h(y)c(\omega)$, where $h(\cdot)$ and $c(\cdot)$ are functions with positive first derivatives. Hence, we can write the proportional deviation of equilibrium labor as

$$\ln \ell = \ln \frac{h(uy^*)}{h(y^*)} + \ln \frac{c(\omega)}{c(\omega^*)}, \qquad (5)$$

where y^* represents equilibrium output, $u = y/y^*$ stands for the utilization of capacity implied by the output to be produced in the short-run (i.e. the ratio of the produced output to the optimal output, the standard definition of capacity utilization; see, for example, Berndt and Morrison (1981)), but in terms of the aggregated intermediate input y, and ω^* are the relative prices observed in the

market. If F is homothetic, then $y/y^* = T(Q, K^*)/T(Q^*, K^*)$; if F is linearly homogeneous ($F = G$), this relationship specializes to $y/y^* = T(Q/K^*)/T(Q^*/K^*)$ with $T = G^{-1}$; and if F is a constant returns Cobb-Douglas, $y/y^* = (Q/Q^*)^{\frac{1}{1-\varepsilon_k}}$ where ε_k is the output elasticity of capital. In general $\ln u$ can be seen as approximately proportional to $\ln \frac{Q}{Q^*}$ and, in the empirical exercise, we will use this fact to replace u with the utilization of capacity in terms of output.

Since ω is a function of the labor adjustment costs, (5) defines only implicitly the employed labor as a function of the capacity utilization given the market prices. But the impact of the utilization of capacity on the adjustment in labor can be computed by implicit differentiation as

$$\frac{\partial \ln \ell}{\partial \ln u} = \frac{1}{\epsilon + \epsilon_M \sigma \lambda(\ell)} > 0, \tag{6}$$

where ϵ is the scale elasticity of $f(.)$, ϵ_M the elasticity of output with respect to materials, σ the elasticity of substitution between materials and labor, and $\lambda(\ell) = AC'(\ln \ell) + AC''(\ln \ell)/(1 + AC'(\ln \ell))$ is a measure of the slope and curvature of the adjustment costs function. The elasticity of M/L with respect to the utilization capacity is obtained, using (4) and (6), as

$$\frac{\partial \ln M/L}{\partial \ln u} = \sigma \lambda(\ell) \frac{\partial \ln \ell}{\partial \ln u} = \frac{1}{\epsilon_M + \frac{\epsilon}{\sigma \lambda(\ell)}} > 0. \tag{7}$$

Expression (7) implies that the ratio of materials to labor will be invariant to the utilization of capacity if there is no possibility of substitution ($\sigma \to 0$) or/and if there are no adjustment costs ($\lambda(\ell) \to 0, \forall \ell$). Expressions (6) and (7) make clear that if $\sigma > 0$ and there are some adjustment costs, we can expect labor in the downturns to be "hoarded" to save costs, using all the available possibilities to substitute materials by labor-intensive processes. This seems in agreement with common sense and casual observation.

3 A model for cost shares

Let s_m be the observed cost share of materials. Also define the relative share of materials as $m = s_m/(1 - s_m)$. We can write

$$m = \frac{\bar{p}M}{w_\ell L} = \frac{\bar{p}}{w_\ell} v\left(\frac{\bar{p}}{w_\ell z_\ell}\right) = z_\ell \omega v(\omega). \tag{8}$$

The last equality provides an expression for the relative share in terms of the ratios of the marginal cost of labor to its unit cost (z_ℓ), the relative marginal costs (ω) and the ratio of materials to labor as a function of these costs ($v(\omega)$). When $z_\ell = 1$ (i.e. when $AC'(\ln \ell) = 0$), the above expression collapses to the conventional explanation of costs shares in terms of market prices $m = \omega^* v(\omega^*)$.

Differentiating (8) we obtain

$$\frac{dm}{m} = \frac{dz_\ell}{z_\ell} + (1 - \sigma)\frac{d\omega}{\omega},$$

which implies that

$$\frac{dm}{m} = \sigma \frac{dz_\ell}{z_\ell} + (1 - \sigma) \left(\frac{d\bar{p}}{\bar{p}} - \frac{dw_\ell}{w_\ell} \right). \qquad (9)$$

Expression (9) splits the change in the materials share in two components. One component, given by the second term of the right hand, is the change related to the variation in the observable relative unit costs of materials and labor. The other component is the change associated with the variation in the unobservable ratio of the marginal cost of labor to its unit cost.

Using the relationship established in Sect. 2 between the change in the labor input and the utilization of capacity, we can specify the adjustment costs as a function of capacity utilization, that is $AC(\ln l) = \widetilde{AC}(\ln u)^2$. This variable has the double advantage of being more easily observable and exogenously determined. Then, equation (9) can be approximated, in discrete terms, by

$$\Delta \ln m = \sigma \Delta \widetilde{AC}'(\ln u) + (1 - \sigma) \Delta \ln \frac{\bar{p}}{w_l}. \qquad (10)$$

Model (10) forms a basis of an estimable econometric model. We are interested in estimating the unknown parameter σ, representing the elasticity of substitution, and the unknown function $\widetilde{AC}(.)$, reflecting the labor adjustment costs. Hence, we face a partially linear semiparametric model, which can provide empirical evidence on the theoretical framework developed in the previous section.

The model to be estimated can also be written as

$$\Delta \ln m_{it} = \Delta \theta(\ln u_{it}) + \beta \Delta \ln \frac{\bar{p}_{it}}{w_{lit}} + \varepsilon_{it}, \ i = 1...N \quad \text{and} \quad t = 1...T, \qquad (11)$$

where $\beta = 1 - \sigma$, $\theta(\ln u_{it}) \equiv (1 - \beta)\widetilde{AC}'(\ln u_{it})$ is an unknown function, and ε_{it} is a disturbance term that we will assume uncorrelated with the explanatory variables.

Estimators of models like (11) have been proposed by Robinson (1988) and Speckman (1988) among others. Noticing that u_{it} and ε_{it} are uncorrelated,

$$\triangle \ln m_{it} - E\left(\triangle \ln m_{it} \mid u_{it}\right) = \beta \left\{ \triangle \ln \frac{\bar{p}_{it}}{w_{\ell it}} - E\left(\triangle \ln \frac{\bar{p}_{it}}{w_{\ell it}} \mid u_{it}\right) \right\} + \varepsilon_{it}, \quad (12)$$

the semiparametric estimator of β is the ordinary least squares estimator (OLS) after substituting the conditional expectation functions in (12) by some non-parametric estimate. Since u_{it} takes only discrete values (between 1 and 100 in percentual terms), we can employ any smoothing method for estimating the conditional expectations, and the resulting OLS robust standard errors are valid (see

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[2] The $\widetilde{AC}(\cdot)$ function can also be seen to include the scale effect derived from the replacement of u by the utilization of capacity in terms of output.

Delgado and Mora 1995a, b). Even a mere average of the values of the dependent variable with the same u_{it} value, which will be called the "nonsmoothing" estimator, can be employed as an estimator of the conditional expectation. For the sake of comparison, we will use β estimates based on kernel estimates and "nonsmoothing" estimates of the conditional expectations in (12).

Since the unknown function $\theta(.)$ depends on only one argument, we can also approximate it by some polynomial expansion, e.g. a three-order Taylor expansion of the type

$$\theta(x) = \alpha_0 + \alpha_1 x + \alpha_2 x^2 + \alpha_3 x^3, \tag{13}$$

and β can be estimated by OLS in (11), where $\theta(.)$ is replaced by the given parameterization. The goodness of such parameterization can be tested by comparing the resulting estimates with those obtained applying the semiparametric method.

4 Estimating the elasticity of substitution and adjustment costs

Our estimations are based on a 5-year balanced panel (1990-1994) of 719 Spanish manufacturing firms. This sample comes from a broader stratified sample of Spanish manufacturing, in which firms above a given size (200 workers) are over-represented. Our subsample consists of the set of firms for which the data required in this exercise were available.

The data richness is very unusual. On one hand, firms report overall materials and labor costs, an estimate of the average yearly change in the price of the materials that they buy, and the data needed to compute total effective hours of work (normal hours+overtime-lost hours). From this data we compute the materials and labor cost shares and the change in the relative unit costs of materials to labor. Firms also supply an assessment of their average utilization of the installed capacity during the year. On the other hand, from the accounting figures on assets we can compute the firm's capital (in equipment, excluding building), and from individual information on the interest rates payed by financing, we are able to estimate individual user's costs of capital. In estimations, we split the sample in 10 industry subsamples to take into account the industries' heterogeneity (see below).

Finally, the period under study is also somewhat exceptional, providing an interesting "natural experiment". Our sample data range from the end of a boom to the beginning of a new recovery, including a sharp downturn (see Table 1 and Fig. 1). By the years 1990 and 1991 production became stagnant, though investment and capital were increasing at high rates until the latest year. The utilization of capacity already decreased this year, and production and used capacity fell sharply during the following two years, 1992 and 1993. In 1994 it started a strong recovery that affected production and the used capacity.

All this shows a strong impact on the cost shares of the inputs (see Table 1 and Fig. 2). The firms' average materials share decreases sharply during the recession,

Table 1. Output, input prices, capacity utilization and input cost shares

|	1990	1991	1992	1993	1994
Manufacturing					
Value added[a] (rate of change in real terms)	2.4	0 9	−0.8	−4.4	4.5
Investment[b] (rate of change)	21	7	−8	−17	11
Sample averages					
Production (rate of change in real terms)	−	4.1	−0.8	−7.4	8.6
User cost of capital (percentage points)	22.2	22.4	22.9	21.5	19.9
Relative price of materials to labour (rate of change)	−	−9.5	−7.8	−2.6	5.7
Stock of capital (rate of change in real terms)	−	18.4	8.1	3.1	3.3
Utilization rate (percentage points)	82.4	80.1	77.7	74.8	78.5
Capital share in total costs (percentage points)	5.2	5.6	6.0	6.0	5.3
Labour share in total costs (percentage points)	30.1	30.6	31.5	33.0	30.5
Materials share in total costs (percentage points)	64.7	63.8	62.5	61.0	64.2

Notes
[a] National Accounts (National Institute of Statistics)
[b] Industrial Investment Survey (Ministry of Industry)

while the shares of labor and capital tend to increase. A simple calculation with the share values reveals that the ratio of materials costs to capital costs fell by about 15-20% in the worst years, while the ratio of labor costs to capital costs fell only by about 5-8%. Strikingly, both ratios tended to recover their original values at the end of the period.

The figures clearly suggest a downward short-run adjustment on the part of the firms to a lower demand. This adjustment is mainly based on the materials and labor inputs, while the accumulation of capacity is simply lessened. But the adjustment seems to affect materials and labor differently, as expected. As long as the relative market price of materials to labor is also changing during the period (see Table 1), the impact of the adjustment on the materials-labor ratio cannot be disentangled and assessed straightfowardly.

Table 2 reports the results of the estimation of model (11). Firstly, the table reports the parametric estimates under the restrictions $\alpha_1 = \alpha_2 = \alpha_3 = 0, \alpha_2 = \alpha_3 = 0$, $\alpha_3 = 0$, and without restricting the α coefficients of equation (13). Next, the table reports the semiparametric estimates.

For the parametric specification of $\theta(.)$ we observe that, by looking at the joint significance of the polynomial terms, the linear specification ($\alpha_2 = \alpha_3 = 0$) is in general not adequate. It is worth mentioning that as we introduce powers of $\ln u$, the OLS estimates of β do not vary sensitively, possibly due to the near independence of u and $\bar{p}/w_\ell$. Perhaps this fact can be attributed to the non-monetary character of most of the adjustment costs, which mitigates the dependence between the observed unit costs and the unobserved marginal costs picked up by the capacity utilization. However, if the parameterization of $\theta(.)$ is incorrect, the OLS estimators are inconsistent. In general, the semiparametric and the parametric estimates are quite similar. The semiparametric estimates based on kernels do not show significant variation for the different bandwidth choices. Also, the kernel estimates and "nonsmoothing" estimates are fairly similar.
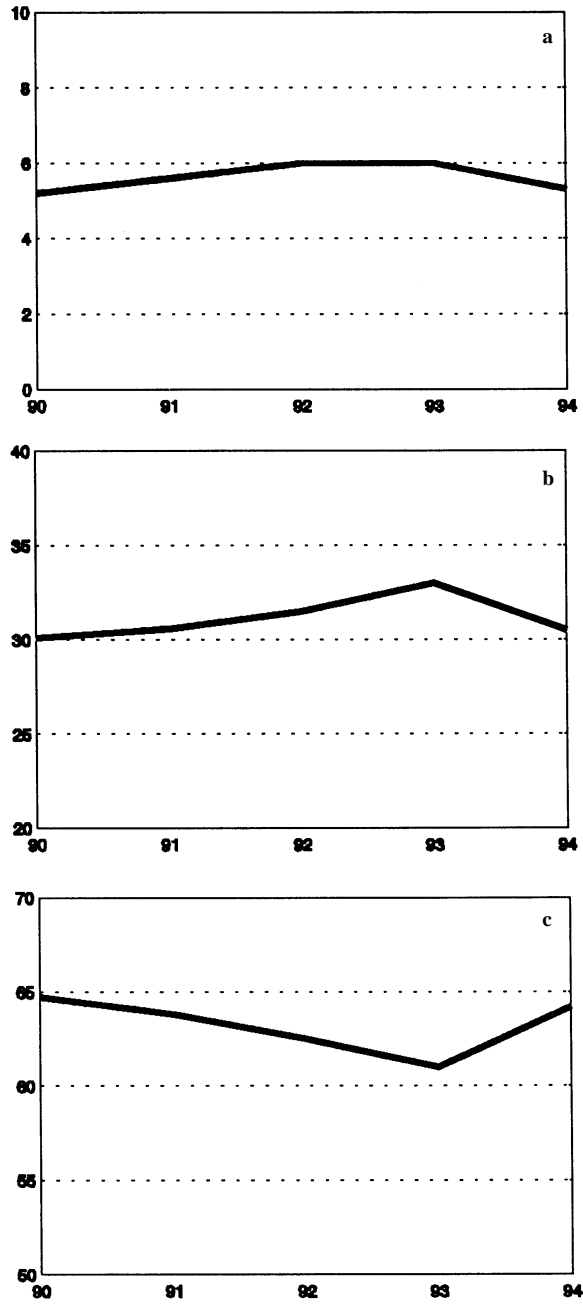
Fig. 2. Input cost shares evolution (percentage points). **a** Capital share in total costs; **b** labor share in total costs; **c** materials share in total costs

Table 2. Estimations of the model $\Delta \ln m = \Delta \Theta(u) + \beta \Delta \ln \frac{\bar{p}}{w_1} + \epsilon$

(Total number of firms=719; sample period: 1990-1994)

Sector	N⁰ of firms (observations)	Coefficients	Parametric estimates $\Theta(u) = \alpha_0 + \alpha_1 \ln u + \alpha_2(\ln u)^2 + \alpha_3(\ln u)^3$				Semiparametric estimates			
							Kernels			Non-smoothing
			$\alpha_i = 0$	$\alpha_2 = \alpha_3 = 0$	$\alpha_3 = 0$	$\alpha_i \neq 0$	$c = 0.5$	$c = 1$	$c = 2$	
1. Ferrous and non-ferrous metals + metal products	89 (356)	β	0.50 (4.25)	0.49 (4.43) [0.002]	0.48 (4.42) [0.005]	0.49 (4.53) [0.008]	0.52 (4.40)	0.53 (4.53)	0.52 (4.50)	0.50 (3.93)
		$\Delta\Theta(u)$								
		R^2	0.10	0.13	0.13	0.13	0.12	0.12	0.11	0.13
2. Non-metallic minerals	55 (220)	β	0.59 (5.67)	0.59 (5.71) [0.45]	0.59 (5.42) [0.68]	0.59 (5.22) [0.83]	0.59 (4.57)	0.62 (5.07)	0.60 (5.46)	0.61 (4.11)
		$\Delta\Theta(u)$								
		R^2	0.15	0.16	0.16	0.16	0.14	0.15	0.15	0.14
3. Chemical and pharmaceutical products + rubber and plastic products	96 (384)	β	0.04 (0.51)	0.04 (0.49) [0.06]	0.03 (0.42) [0.07]	0.04 (0.50) [0.0001]	0.03 (0.57)	0.05 (0.91)	0.06 (1.01)	−0.00 (−0.09)
		$\Delta\Theta(u)$								
		R^2	−0.00	0.03	0.05	0.06	0.00	0.00	0.00	0.00
4. Industrial and agricultural machinery	49 (196)	β	0.33 (1.97)	0.37 (2.62) [0.0002]	0.36 (2.60) [0.0004]	0.37 (2.65) [0.0002]	0.57 (4.46)	0.49 (3.68)	0.42 (3.00)	0.45 (1.64)
		$\Delta\Theta(u)$								
		R^2	0.04	0.14	0.14	0.14	0.12	0.10	0.08	0.03
5. Office and data processing machines + electrical and electronic goods	63 (252)	β	0.43 (2.36)	0.41 (2.24) [0.20]	0.37 (2.11) [0.009]	0.38 (2.15) [0.02]	0.42 (5.95)	0.41 (5.39)	0.41 (5.25)	0.30 (5.40)
		$\Delta\Theta(u)$								
		R^2	0.11	0.12	0.15	0.15	0.12	0.11	0.11	0.08

Table 2. (continued)

Sector	N⁰ of firms (observations)	Coefficients	Parametric estimates $\Theta(u) = \alpha_0 + \alpha_1 \ln u + \alpha_2(\ln u)^2 + \alpha_3(\ln u)^3$				Semiparametric estimates			
			$\alpha_i = 0$	$\alpha_2 = \alpha_3 = 0$	$\alpha_3 = 0$	$\alpha_i \neq 0$	Kernels $c = 0.5$	$c = 1$	$c = 2$	Non-smoothing
6. Transport equipment	53 (212)	β	0.51 (3.72)	0.30 (1.91) [0.00005]	0.33 (2.07) [0.00002]	0.31 (2.02) [0.00002]	0.51 (3.74)	0.50 (3.45)	0.45 (2.90)	0.49 (2.21)
		$\Delta\Theta(u)$								
		R^2	0.12	0.22	0.22	0.23	0.10	0.09	0.07	0.09
7. Food, beverages and tobacco	100 (400)	β	0.20 (2.10)	0.21 (2.10) [0.58]	0.21 (2.11) [0.83]	0.21 (2.13) [0.94]	0.29 (3.87)	0.27 (3.63)	0.27 (2.33)	0.28 (3.12)
		$\Delta\Theta(u)$								
		R^2	0.04	0.04	0.04	0.04	0.07	0.07	0.05	0.06
8. Textiles, leather and clothing	111 (444)	β	0.42 (3.22)	0.40 (3.14) [0.035]	0.40 (3.11) [0.04]	0.40 (3.10) [0.08]	0.39 (3.52)	0.43 (3.37)	0.41 (3.52)	0.36 (3.89)
		$\Delta\Theta(u)$								
		R^2	0.08	0.09	0.09	0.09	0.07	0.08	0.07	0.06
9. Timber and furniture	50 (200)	β	0.35 (2.95)	0.33 (2.67) [0.15]	0.33 (2.50) [0.32]	0.35 (2.62) [0.35]	0.31 (2.78)	0.31 (3.38)	0.27 (3.06)	0.32 (2.41)
		$\Delta\Theta(u)$								
		R^2	0.08	0.09	0.10	0.11	0.05	0.06	0.05	0.05
10. Paper and printing products	53 (212)	β	0.78 (4.86)	0.79 (4.79) [0.60]	0.78 (4.79) [0.77]	0.76 (4.90) [0.006]	0.82 (7.27)	0.73 (7.10)	0.75 (7.08)	0.97 (5.33)
		$\Delta\Theta(u)$								
		R^2	0.27	0.27	0.27	0.28	0.24	0.22	0.24	0.30

Notes The numbers in parentheses are t-ratios computed from a robust estimator of the variance matrix that takes into account the correlation over time of the individuals' residuals. The numbers in brackets are the p-values of the joint significance of the coefficients of the polynomials. The bandwidth chosen for the Kernel estimates is $h = cn^{-\frac{1}{3}}$ for $c = 0.5$, 1 and 2.

Some comments on the elasticity of substitution estimates are in order. Remarkably, half of the sectors show elasticities of substitution that range in a tight interval of values (from 0.6 to 0.7). The other half can be divided in two high elasticity of substitution sectors (Food, beverages and tobacco, with an elasticity about 0.8, and Chemical, rubber and plastic products, with an elasticity near unity), and three low elasticity of substitution activities (Metals and metal products, with an elasticity about 0.5, Non-metallic minerals, with about 0.4, and Paper products, with the lowest value 0.2).

Our estimates tend to range among the highest estimates for the sparse available evidence on elasticities of substitution between materials and labor (see Hamermesh (1993), Table 3.6). This seems reasonable given the disaggregated character of our data. On the other hand, materials are an aggregate of different intermediate inputs, and many of them can be bought by firms under different degrees of elaboration. The expansion of subcontracting has recently given to manufacturing firms a flexible way to replace labor services by more finished materials and vice versa. This is a likely source of high elasticities of substitution labor materials.

Unfortunately, there is not to our knowledge, a similar industries estimation to compare our ranking of elasticies of substitution, and it is difficult to say anything on a priori grounds. However, it can be checked below that our results on elasticities of substitution are consistent with the results on adjustment costs.

The similarity between the several β estimates among the different estimation procedures suggests that the parametric specification of $\theta(.)$ is correct. In Fig. 3 we report plots of the function $\theta(\ln u)$, and of the integral of this function, which provides an estimate of $(1 - \beta)\widetilde{AC}(\ln u)$ based on its polynomial specification. In all sectors the estimates of the adjustment cost function (escaled by the factor $1 - \beta$), show the right slope and curvature. The estimates also seem to confirm that the capacity utilization reported by firms is really a properly scaled measure of the use of their installations, the fact that we do not observe values above 1 probably being the consequence of the specificity of the period covered.

The estimated adjustment costs function, and hence the impact of these costs on the input shares, is significant in 7 of the 10 sectors. Two of the three exceptions coincide with the sectors with the lowest elasticities of substitution (Paper products and Non-metallic minerals), as could be expected given the theoretical conditions developed in Sect. 2. The third sector (Food, beverages and tobacco) constitutes a surprise, because it is a sector with a high elasticity of substitution. Perhaps this can be rationalized by noticing that it has been always considered a sector with low adjustment costs. When the function is significant (and also for two of the three exceptions), the estimated marginal cost always has the correct sign and the integral of the function has a nice convex shape with slight exceptions at some extreme values (see Fig. 3).

In addition, when we value the adjustment costs using the integral of the estimated function and the elasticity of substitution estimates, we obtain sensible values that agree with casual knowledge. Recall that, given our adjustment cost function specification, adjustment costs can be read as measured in percentage

Plot of $\Theta(\ln u)$ Plot of $(1 - \beta)\bar{AC}(\ln u)$

SECTOR 1

SECTOR 2

SECTOR 3
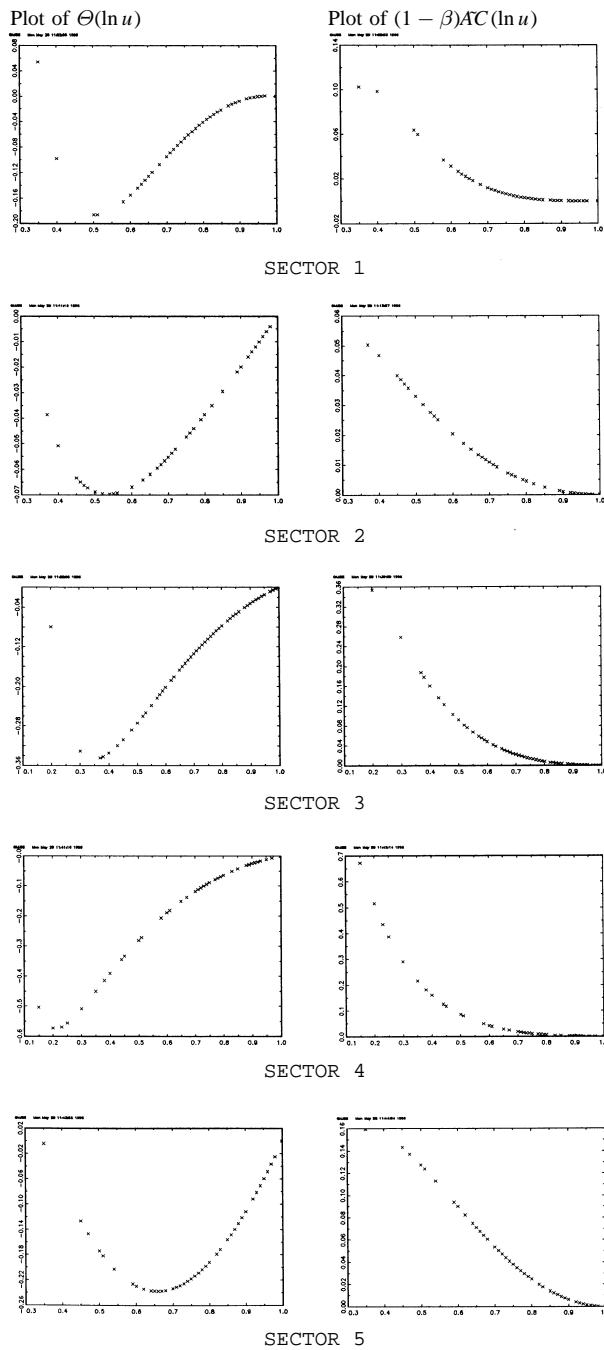
SECTOR 4

SECTOR 5

Fig. 3. The AC(.) function. The first column graphs the values of the function $\Theta(\ln u)$, estimated with a cubic polynomial, against u. The second column graphs the integral of the function against u. Sector 10 is excluded because the obtained estimates are meaningless

Plot of $\Theta(\ln u)$ Plot of $(1-\beta)\widehat{AC}(\ln u)$

SECTOR 6

SECTOR 7
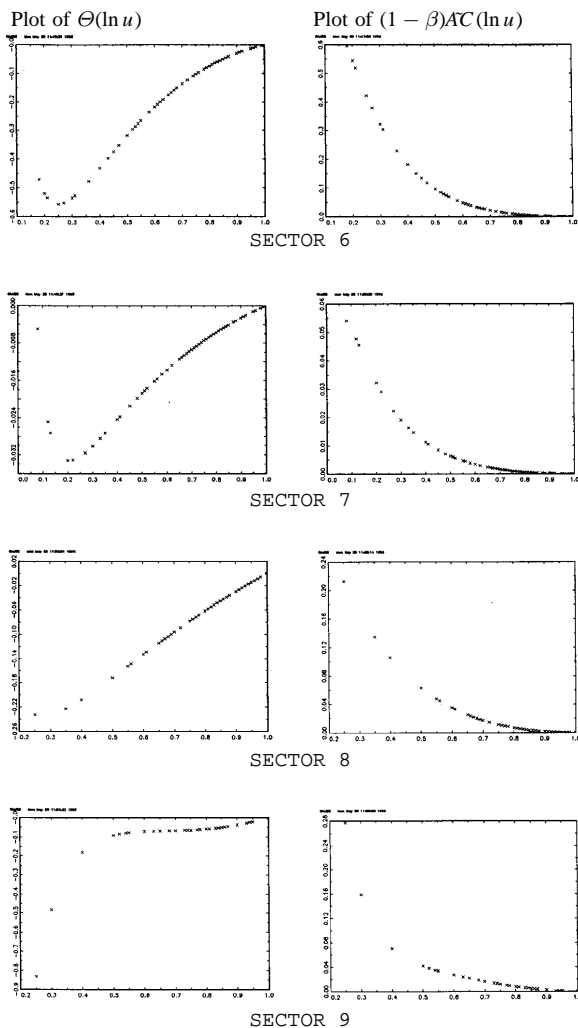
SECTOR 8

SECTOR 9

Fig. 3. (continued)

points of the standard wage bill. Therefore, adjustment costs for a given u value, 0.5 say, can be simply obtained by dividing the corresponding ordinate in the second column of Fig. 3 by the elasticity of substitution of the sector. With capacity utilization at 50%, the adjustment cost of the 7 sectors with significant polynomials range from 6% to 19% of the wage bill. Timber and furniture, and Textile and clothing, are the sectors with lowest costs (6% and 10% respectively); Industrial and agricultural machinery, and Office machines and electrical and electronical goods are the sectors with the highest (16% and 19% respectively).

5 An application to productivity analysis

Production shares have been used since Solow (1957) in the non-parametric analysis of productivity growth based on the fact that, under perfect competition and constant returns to scale, input shares in output and cost coincide and must be equal to the output production elasticities. Under market power, input shares in output and cost shares do not coincide anymore, but the cost shares remain equal to the elasticities and, if the returns to scale are not constant, these cost shares must simply be multiplied by the elasticity of scale (see e.g. Hall 1990). However, the use of observed cost shares is based on the assumption that firms are in a long-run equilibrium. If demand is subject to exogenous shocks, and some of the inputs are costly to adjust in the short run, the observed input shares are no longer an adequate measure of the output elasticities of the inputs.

Let us define a firm's production function similar to the one used in (1), $Q = F(X, K^*)$, where now $X = \{X_i\}_{i=1}^{m}$ is a set of m variable inputs, and K^* represents an input fixed in the short-run. The Solow residual is defined as $S = q - \sum_{i=1}^{m} \varepsilon_i x_i - \varepsilon_k k^*$, where $q = dQ/Q$ is the output rate of change, $x_i = dX_i/X_i$ represents the rate of change of the i-th input and ε_i its corresponding elasticity[3]. In order to compute S in practice, these unobserved elasticities must be replaced by some estimate computed from observed data.

Under the assumption that firms minimize costs and they are operating in a long-run equilibrium, $\lambda \frac{\partial F}{\partial X_i} = \overline{w}_i$, where $\overline{w}_i$ is the market rental price of the i-th input and λ is the Lagrange multiplier of the problem or marginal cost. Marginal cost can be computed, for example, as $\lambda = \frac{\sum \overline{w}_i X_i}{Q \sum \frac{\partial F}{\partial X_i} \frac{X_i}{Q}} = \frac{c}{\varepsilon}$, where c is the unit variable cost and $\varepsilon = \varepsilon_F - \varepsilon_K$ the elasticity of scale of the production function less the elasticity of the fixed factor. Then, we can write

$$\varepsilon_i = \varepsilon \frac{\overline{w}_i X_i}{cQ} = \varepsilon \, s_i. \tag{14}$$

That is, the observed variable input cost shares s_i corrected by a scale factor are a proper measure of the elasticities, which is robust to the type of competition.

In a context where adjustment costs are present, $\lambda \frac{\partial F}{\partial X_i} = w_i z_i \neq \overline{w}_i$, where w_i is the unit cost of the i-th input and z_i represents the ratio of its marginal cost to the unit cost. Hence, the observed input cost shares will not be a good measure of the elasticities.

Let us examine the relevant costs and shares, the bias induced by the observed shares, and its possible correction, in a framework which generalizes the model in Sect. 2. Suppose that the firm minimizes the cost of the set of costly variable inputs X given the quantity of the fixed input K^*. That is

$$Min \ \sum_{i=1}^{m} \overline{w}_i X_i \ \exp\left\{AC_i\left(\ln \frac{X_i}{X_i^*}\right)\right\} \qquad s.t. \ F(X, K^*) = Q, \tag{15}$$

---

[3] We include the changes in the fixed factor K to account for the displacements of the long-run equilibrium.

where X_i^* is the optimal level of the i-th input given K^* and market prices, and the adjustment cost $AC_i(.)$ are supposedly input specific.

From the first order conditions of the above problem we obtain $\varepsilon_i = \varepsilon\widehat{s_i}$, where $\widehat{s_i} = s_i z_i / \sum_{i=1}^m s_i z_i$ with $z_i = 1 + AC_i'$. In a long run equilibrium, $z_i = 1 \ \forall i$ and the elasticities will match the observed shares. However, considering the simplest case where $z_j < 1$ but $z_i = 1 \ \forall i \neq j$, it can be easily shown that $\widehat{s_j} < s_j$ and $\widehat{s_i} > s_i \ \forall i$.

Let us analyze the bias implied by the conventional calculation of the Solow residual. Consider, to simplify, $k^* = 0$. The true productivity change becomes $S = q - \varepsilon \sum_{i=1}^m \widehat{s_i} x_i$. The residual in terms of observed shares is $S^1 = q - \varepsilon \sum_{i=1}^m s_i x_i$, and therefore it implies a bias given by $S^1 - S = \varepsilon \sum_{i=1}^m (\widehat{s_i} - s_i) x_i$. For illustrative purposes, consider the case of two inputs such that

$$(\widehat{s_1} - s_1) = -(\widehat{s_2} - s_2) = \Delta s > 0$$

that is, the input 2 is being hoarded. Then, $S^1 - S = \varepsilon \, \Delta s \, (x_1 - x_2)$. If there is a downturn in which $x_1 < 0$ and $x_2 < 0$ but $|x_2| < |x_1|$, then $\Delta s \, (x_1 - x_2) < 0$ and S^1 will understate the true productivity change. If a recovery begins and $x_1 > 0$ and $x_2 > 0$ but $x_1 > x_2$, then $\Delta s \, (x_1 - x_2) > 0$ and S^1 will overstate the true productivity change. Because the described situations are likely, the biases are also the most likely to emerge in the conventional computation of the Solow residual.

Sometimes, the Solow residual is computed from value-added data. Assume for simplicity that X contains only one (composite) intermediate component X_M, materials, say. The Solow value-added residual is defined in this case as $S^2 = g - \sum_{i \neq M} \widetilde{s_i} x_i$, where $g = dG/G \, \big|_{p=\bar{p}}$ measures the change in real value-added by a Divisia index, and $\widetilde{s_i}$ represents the i-th input share in non-intermediate costs. No correction for scale is tried. This residual will coincide with the productivity increase as measured by S^1, up to a proportionality factor, under constant returns to scale and perfect competition. If this is not the case,

$$S^2 = \frac{pQ}{G} S^1 - \frac{pQ}{G}(1 - \varepsilon) \sum_{i \neq M} \widetilde{s_i} x_i + \varepsilon \, s_M \pi \sum_{i \neq M} \widetilde{s_i}(x_M - x_i), \qquad (16)$$

where s_M is the share of materials in total costs and π represents the ratio of pure profit to value-added (this is the generalization of a formula in Hall 1990, page 79). Therefore, the residual computed from value-added data can be a bad approximation of the true productivity growth, especially in the presence of a varying ratio of materials to the rest of the inputs.

To assess the practical importance of the biases, we have computed the S^1 conventional (observed cost shares) Solow residual, the true S (corrected cost shares) residual, and the S^2 value-added residual, using the data on materials, labor and capital, for our whole sample. However, to make the alternatives fully comparable, we have dropped from the sample 33 firms with negative value-added in some year (value-added calculations are meaningless in this circumstance).

The underlying production function is always assumed linearly homogeneous and the elasticity of capital is approximated by its current share in total costs. Therefore, the estimation of $\widehat{\varepsilon} = (1 - \widehat{\varepsilon}_k)$ corresponds to the current joint share of labor and materials in total costs. Several alternatives were tried but, given the low weight of the capital share, they virtually did not change the results.

In computations we use the usual Torquinst-Divisia approximation for discrete changes, which averages the observed shares of the years from which we measure the change. To ensure the exact accomplishment of formula (16) in this context, we have computed a somewhat special Divisia value-added index that uses Torquinst-type weights of the real output and materials changes[4]. We use the corresponding ratio of value-added to production, γ, to scale the value-added residual to make it fully comparable in dimension to the other residuals.

The correction of the shares to compute S is based on the estimation of z_l as $\widehat{z}_l = 1 + \frac{\widehat{\alpha_1}}{1-\widehat{\beta}} ln \ u$ (see equation (11)). More complex estimations, taking into account the whole polinomials, gave very similar results.

Table 3 reports the simple averages of the computed indices for every year and for the total sample, for the quartils according to the distribution of capacity utilization in 1993, and for two selected sectors (Industrial and agricultural machinery and transport equipment).

As can be seen from the table, the conventional Solow residual tends to understate the true productivity change in the downturn (1993) and overstate it in the upturn (1994). This is as expected, but the average bias is not too big. However, the bias in productivity growth measurement is very important for the firms with acute underutilization of capacity and the selected sectors.

The value-added Solow residual turns out to be more unpredictable, presenting rather important differences with S in almost every year and subsample. But it shows a systematic understatement of the productivity increase in the downturn, which is independent of the subsample considered, with an average bias bigger than that attributable to the conventional production residual.

6 Conclusions

The data analyzed, corresponding to firms immersed in a period of acute recession followed by one year of recovery in Spanish manufacturing, provides strong support for a model of short-run adjustments, with materials taken as an input that can be adjusted freely and labor as an input costly to adjust, minimizing short-run costs conditional on the installed capital. Modelling the input cost shares, we have found evidence of significant elasticities of substitution between materials and labor and, at the same time, convex cost of adjusting labor out

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[4] We use the formula $g_t = \frac{1}{\frac{G_{t-1}}{R_{t-1}} + \frac{G_t}{R_t}} q_t - \frac{\frac{TC_{t-1}}{R_{t-1}} + \frac{TC_t}{R_t}}{\frac{G_{t-1}}{R_{t-1}} + \frac{G_t}{R_t}} x_{Mt}$ where G = value-added, R = total revernue, TC = materials cost and q_t, x_{Mt} are the real rates of change in production and intermediate consumption.

Table 3. Biases in productivity growth measurement

Samples	Years	Solow residual (alternative estimates)					
		S^1 (cost shares)	S (corrected cost shares)	$S^1 - S$	γS^2 (scaled value added)	$\gamma S^2 - S^1$	$\gamma S^2 - S$
	1991	2.73	2.66	0.07	2.89	0.16	0.23
Total number	1992	1.87	1.83	0.04	1.99	0.12	0.16
of firms = 686	1993	1.96	2.20	−0.24	1.68	−0.28	−0.52
	1994	3.11	2.95	0.16	3.03	−0.08	0.08
	1991	2.74	2.66	0.08	2.76	0.02	0.10
1st Quartil	1992	0.85	0.78	0.07	1.07	0.22	0.29
$U \le 65$	1993	−1.18	−0.30	−0.88	−0.93	0.25	−0.63
	1994	4.05	3.70	0.35	3.63	−0.42	−0.07
	1991	4.74	4.68	0.06	4.51	−0.23	−0.17
2nd Quartil	1992	0.90	0.87	0.03	1.31	0.41	0.44
$65 < u \le 75$	1993	1.81	1.86	−0.05	1.31	−0.50	−0.55
	1994	4.04	3.91	0.13	3.89	−0.15	−0.02
	1991	1.59	1.51	0.08	2.05	0.46	0.54
3rd Quartil	1992	2.83	2.81	0.02	2.84	0.01	0.03
$75 < u \le 87$	1993	3.27	3.27	0.00	2.79	−0.48	−0.48
	1994	3.26	3.15	0.11	3.37	0.11	0.22
	1991	1.96	1.90	0.06	2.31	0.35	0.41
4th Quartil	1992	2.93	2.91	0.02	2.78	−0.15	−0.13
$87 < u \le 100$	1993	4.11	4.12	−0.01	3.68	−0.43	−0.44
	1994	1.07	1.04	0.03	1.25	0.18	0.21
	1991	2.18	2.12	0.06	2.31	0.13	0.19
Sector 4	1992	3.20	3.20	0.00	3.48	0.28	0.28
	1993	2.01	3.66	−1.64	2.20	0.19	−1.45
	1994	1.10	0.09	1.01	0.86	−0.24	0.77
	1991	4.48	4.47	0.01	4.71	0.24	0.25
Sector 6	1992	−0.51	−0.61	0.10	−0.57	−0.06	0.04
	1993	−3.28	−2.08	−1.20	−2.42	0.86	−0.34
	1994	7.49	6.62	0.87	6.24	−1.25	−0.38

Notes U represents capacity utilization in 1993

of its equilibrium level given capital, which can explain the fluctuations in cost shares related with the capacity utilization.

As a result, we have obtained estimates of the marginal cost or the shadow price of labor, which is the right price to assess the true elasticities of the output with respect to the variable inputs in a situation where the observed input shares in cost are misleading. The use of the shadow price estimates for correcting the input shares in cost and the comparison of the alternative productivity measures have provided relevant empirical insights. Conventional computations of productivity growth turn out to be prone to understate it in the downturns and overstate it in the upturns. In addition, value-added based measures have been shown to give seriously biased results.

On the other hand, while the estimation of the elasticities of substitution has proved to be relatively robust to the control by the utilization of capacity, it seems clear that a source of cyclical short-run movements on shares has been detected. This casts some doubts on the right interpretation of the output effects

obtained in the share equations often used to estimate the parameters of translog production functions, when applied to short-run observations without any correction for capacity utilization. In our view, these are consequences that deserve future research.

Appendix A

Labor adjustment costs

For the sake of simplicity, in what follows we will develop a model with working hours and workers in quadratic terms. Similar models can be found, for example, in Nickell (1986) or Bils (1987). See also Hamermesh and Pfann (1996) for a recent survey on adjustment costs.

Assume that the labor input consists of the total hours of work according to the relationship $L = \bar{h}e^s N$, where $\bar{h}$ represents normal hours of work, s the proportional deviation of effective hours of work from the normal hours and N the number of workers. We will consider normal hours of work exogenously determined, and deviations are understood as measuring changes in "efficiency" hours. That is, the time of work may be either actually reduced or simply show a loss of intensity through a decrease in effort. Given capital and prices, there is an equilibrium number of workers that we will denote by N^*, corresponding to the equilibrium level of the labor input $L^* = \bar{h}N^*$.

The wage rate for the normal hour of work is $\bar{w}$ but the proportional deviation of effective hours of work will imply a non-proportional deviation of the cost per worker, $\bar{w}\bar{h}$, according to the relationship $\bar{w}\bar{h} \exp\left\{s + bs^2/2\right\}$. This can be interpreted as the result of the eventual application of compulsory part-time work schemes, the operation of premium schedules for work-intensity and overtime, etc. At the same time, changes in employment will lead to costs. Assume, for the moment, that the change in employment is intraperiod. Thus, we will specify the costs or firing $(N^* - N)$ workers, and then hiring the same number again, as $\exp\left\{c\left(\ln N/N^*\right)^2/2\right\}$.

Firms, confronted with adjusting labor to a level outside the equilibrium level, given capital, consider the choice of varying either hours or workers according to the subproblem

$$Min \quad W = \bar{w}\bar{h}N \exp\left\{\left(s + \frac{b}{2}s^2\right) + \frac{c}{2}\left(\ln \frac{N}{N^*}\right)^2\right\}, \quad s.t. \ \ L = \bar{h}e^s N \ .$$

Equating the marginal cost at the optimum of changing the labor input through changing either hours or workers we have

$$s = \frac{c}{b}\ln\frac{N}{N^*},$$

which shows that the firm will deviate from normal hours of work only if employment is also adjusted and that, in this case, the deviation will be higher, the

greater are the adjusting costs of employment are, represented by parameter c, for a given value of b.

In order to obtain the same labor input outside of equilibrium without incurring on hourly overcosts, the firm would desire an employment $N^0 = L/\bar{h}$. Therefore, s can also be written as $s = \ln N^0/N$. Combining the two expressions for s, it is easy to see that the change in employment will represent only a proportion of the change desired in labor input (the usual partial adjustment mechanism), i.e.

$$\ln \frac{N}{N^*} = \frac{b}{b+c} \ln \frac{N^0}{N^*}.$$

In addition, it follows that

$$\ln \frac{N}{N^*} = \frac{b}{b+c} \ln \frac{L}{L^*} \text{ and } s = \frac{c}{b+c} \ln \frac{L}{L^*} .$$

Therefore, the change in the requirement of the labor input is accomplished by modifying hours and workers in a given proportion.

Replacing s and $\ln N/N^*$ in the objective function by their optimal values, we can obtain

$$W = \bar{w}L \exp \left\{ \frac{a}{2} \left(\ln \frac{L}{L^*} \right)^2 \right\}, \text{ where } a = \frac{bc}{b+c}.$$

Therefore, the adjustment costs in (1) can be seen as the costs resulting from the election of the aggregate labor input (total hours of work) with an implicit optimal assignment of its components (working hours and workers).

Alternatively, the employment adjustment costs can be seen to stem from a loss in employment efficiency when it is outside of its equilibrium level given capital. That is, the problem may be set as

$$Min \ \ W' = \bar{w}\bar{h}N \exp \left\{ s + \frac{b}{2}s^2 \right\}, \quad s.t. \ \ L = \bar{h}N \exp \left\{ s - \frac{c}{2} \left(\ln \frac{N}{N^*} \right)^2 \right\},$$

providing approximately the same solution as the previous one. By linearizing the marginal condition of this problem, we obtain $s \simeq \frac{c}{b} \ln \frac{N}{N^*}$. On the other hand, $\ln L/L^* \simeq s + \ln N/N^*$. From these equalities the same solution as in the previous problem follows.

Assume now that firms make their decisions on the employees-hours mix, minimizing the present value of the expected stream of labor input requirements. This problem is

$$Min_{N_t,s_t} \ E_t \left\{ \sum_{\tau=t}^{\infty} \delta^{\tau-t} \overline{w}_\tau \overline{h} N_\tau \ \exp \left[\left(s_\tau + \frac{b}{2}s_\tau^2 \right) + \frac{c}{2} \left(\ln \frac{N_\tau}{N_{\tau-1}} \right)^2 \right] \right\}$$

$$s.t \ E_t(\bar{h}e^{S_\tau} N_\tau - L_\tau) = 0, \text{ for } \tau = t, t+1...,$$

where E_t is the expectations operator and δ represents the discount factor.

The (dynamic) marginal cost condition is now

$$1 + c \ln \frac{N_t}{N_{t-1}} - \frac{\delta W_{t+1}^e}{W_t} c \ln \frac{N_{t+1}^e}{N_t} = 1 + bs_t,$$

where W_t represents the total wage bill at time t, and the e superscript indicates planned values. If we assume that $\delta W_{t+1}^e / W_t \simeq 1$, the condition may well be approximated by the considerably simpler formula

$$s_t = \frac{c}{b} \left(\ln \frac{N_t}{N_{t-1}} - \ln \frac{N_{t+1}^e}{N_t} \right).$$

Noticing that $s_t = \ln N_t^0 / N_t$, we have the following differential equation

$$\ln N_{t+1} - \left(2 + \frac{b}{c} \right) \ln N_t + \ln N_{t-1} = -\frac{b}{c} \ln N_t^0$$

from which we can obtain the usual employment path solution

$$\ln N_t - \lambda \ln N_{t-1} = (1 - \lambda)^2 \sum_{s=0}^{\infty} \lambda^s \ln N_{t+s}^0, \qquad (17)$$

where $\ln N_{t+s}^0$ represents the expected employment requirements at t+s to provide the labor input requierements without incurring on hourly overcosts, and λ is a function of the b/c ratio with $\partial \lambda / \partial (b/c) < 0$.

Assume that a firm is producing at equilibrium, i.e. the input quantity is L^* and employment N^*, and suddenly experiences at time t a fall in the input and employment requirements of L^0 and N^0. The firm expects this new situation will last for k periods, and that in period $t+k$ the input and employment requirements will again be L^* and N^*. Using (17) it is easy to check that employment at t will be adjusted to the value

$$\ln N_t = \ln N^* + (1 - \lambda)^2 (\ln N^0 - \ln N^*) \qquad (18)$$

if, for simplicity, we consider $k = 1$. The deviation at time t from normal hours will then be

$$s_t = \ln N^0 - \ln N_t = [1 - (1 - \lambda)^2](\ln N^0 - \ln N^*) \qquad (19)$$

In the following periods, employment will be given by

$$\ln N_{t+s} = \ln N^* + (1 - \lambda)^2 \lambda^s (\ln N^0 - \ln N^*) \qquad (20)$$

and the deviation from normal hours, given that the input requirements are already the equilibrium levels, will simply be

$$s_{t+s} = \ln N^* - \ln N_{t+s} = -(1 - \lambda)^2 \lambda^s (\ln N^0 - \ln N^*). \qquad (21)$$

From formulas (18) to (21) it is clear that the adjustment will more or less affect the employment according to the value of λ (employment will be less

N = log employment

ln N^X

ln N^O

time

s = deviation from
normal hours

0

ln $\frac{N^O}{N^X}$

time
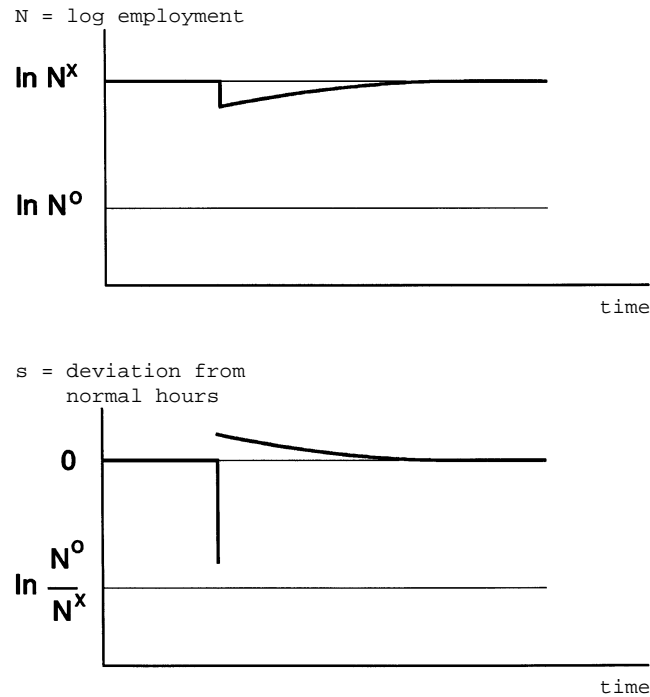
Fig. 4. The adjustment of workers and hours to an unexpected transitory shock

adjusted the higher λ is, i.e., the higher the relative firing and hiring costs are). But only changes in employment are going to persist. Therefore, as the formulas and Fig. 4 make clear, if λ is high enough and shocks are transitory, the adjustment costs function used in (1) can be a good approximation to the adjustment costs derived from a full dynamic problem.

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