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Degree on Mechanical Engineering

Bachelor Thesis

**Natural Frequency Analysis of a Human Femur
Using Finite Elements Method**

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1 MOTIVATIONS AND GOALS

1.1 MOTIVATIONS

1.1.1 *Human femur selection*

Mechanical Engineering, especially at degree studies level, is basically focused on man-made components. Therefore, different mechanical components have been studied for the main engineering fields, such as energy or vehicle industries. All of those different components have been studied in detail, following their industrial process where their material and structural properties are modified to meet their specific requirements. However, to design and built all those components, human beings have always been firstly inspired by something that was already there. Therefore, a wide range of different nature features has been inspiring the different industrial inventions developed by humans.

As an engineer, instead of continuing analysing any mechanical feature industrially designed, this project aim was to study a component created by nature, analysing it from an engineering point of view. And there is when human femur appears, being one essential part of our body which also allows connecting engineering with the medical field. Moreover, the complex structure and properties that femur present can bring many inspiring ideas that, again, Nature shows to Engineering.

1.1.2 *Natural frequency analysis applied to human femur*

The increasing use and importance of vibrational analysis is a reality into the Engineering field. But engineering related activities are not the only ones at which vibrational properties can be helpful. In this project, the importance of the use of natural frequency analysis at the medical field has been investigated. Numerous medical applications have been found on natural frequency analysis. This project has been based on one of the main medical areas at which natural frequency analysis is focused, human skeleton, and more specifically, femur bone. In addition, natural frequency analysis is also used to characterize bone properties, and it can help in order to create bone models. Therefore, the two main ideas that inspired human femur natural frequency analysis lay into the medical and engineering fields.

I) Medical applications of natural frequency

During the research process previous to the actual project development, many different medical applications of natural frequency analysis on long bones were found. The main benefits that natural frequency analysis brings into the medical field regarding bones are the following:

Bone regeneration: Recent studies have been discovered the importance that natural frequency values could bring to the reparation of a damaged bone [5]. It has been also ensured that mechanical loading can be used in order to control and improve bone enhance and apposition. Moreover, the article introduces that it has been recently

studied that this bone regeneration can be improved when mechanical loading applies frequencies close to the actual bone natural frequency values. Therefore, it is obvious that, regarding these new investigations, obtaining an accurate value of bone natural frequencies can be critical to regenerate bone in a successful way.

Bone diagnosis: It is well established that natural frequencies have been widely used to diagnose a huge range of osseous problems [5], [8]. The main bone illnesses that can be predicted by the use of bone natural frequency analysis are both Osteoporosis and Osteoarthritis. Moreover, any other bone illness that affects their density or properties can be also be diagnosed by means of natural frequency analysis.

Bone monitoring: The last main application that natural frequency brings to medicine is the possibility of monitoring wide range of different medical healing processes [5], [8]. One of the main processes benefited by these techniques are different kinds of fracture healing processes as well as the *osteointegration* of plates or screws.

II) Engineering application of natural frequency

The fact that the human femur is such a complex and incredible structure has been a great inspiration to perform this project. Therefore, it was decided to develop a 3D model simulating the actual femur, in order to understand it as an engineering component. And here is where natural frequency comes into place, as the main tool to build and analyse the femur model. As no experimental method could be used to get femur bone natural frequencies, the results had to be calculated by means of finite elements method. Therefore, using Abaqus software natural frequency values will be calculated introducing in a FEA model the structural and material characteristics of the human femur.

It has been explained above the importance that natural frequency has on the medical field, but natural frequency analysis can be also a great tool in order to understand human femur from an engineering point of view. By means of the analysis of the obtained results, the influence that the introduced properties have on femur vibrational properties can be studied. Moreover, this will also help to understand and improve femur bone structural and material properties model characterization. Therefore, the material model that originally was built to calculate natural frequency values can be improved by means of analysing its results. This idea of vibrational and material properties mutual improvement was obtained from [8] and [21] as it will be further explained.

1.2 GOALS

The initial goal of this project is to develop a reliable **FEA femur model** that leads to obtain its natural frequencies, by mean of introducing its main properties (density and Young's Modulus). In order to understand and obtain femur structural and material properties, different **investigation articles** results have to be **researched**. The model performance in this study will take into account two main considerations. On the one hand, the different **femur properties** need to be **characterized** and approximated, simulating the actual femur structure in the FEA model, so that desirable results could

be obtained. On the other hand, **natural frequency analysis** will be the way to critically measure and understand the accuracy of the model results. Moreover, this accuracy will be raised following a **model improvement process**, where different models will be performed increasing the complexity of their properties and structure characterization. Once a reliable model has been achieved, the **influence on natural frequency** results of its different input femur properties would be also deeply studied. Finally, some **medical simulations** will be performed to better understand the effect that different bone-related illnesses have on femur natural frequency values.

The organization of the document will be as follows.

Firstly, at Chapter 1 the motivations and objectives of the project are explained. Then, the introduction to the subject is presented at Chapter 2, introducing femur bone and natural frequencies, with their implications they have on models performance. The different FEA models are developed at Chapter 3, where their Abaqus development process, geometrical and material properties considered, and results are deeply analysed. In addition, at Chapter 4, some medical considerations are applied to the models performed before. Chapter 5 includes the project development diagram and its economic impact. Conclusions and future work are specified at Chapter 6. Finally, the bibliography used is listed at Chapter 7.

2 INTRODUCTION

2.1 FEMUR BONE

Bones are essential organs in the human body. Although they seem to be simple and rigid, they are built by a complex and living structure. This common bone structure is mainly composed by bone tissue, but also complemented with blood vessels and nerve fibers. Each bone carries out different important functions in the human body, such as structural support, protection of other organs, movement guide, storage of minerals or production of blood cells. And each function that a bone has to develop is strongly related to its structure. That is, their specific external and internal structure allows them to develop all of their functions in a successful way. For instance, if a bone has severe mechanical and support requirements, its structure has to combine huge strength with low weight.

2.1.1 Femur bone

The human femur is the largest, longest and strongest bone of the human body. It's the main structural element of the human body, as it has to withstand its whole weight. The average adult femur length is 48 cm and its maximum diameter at the mid-shaft is 2.84 cm. In order to have an idea about the huge femur strength, it has to be said that it can support up to 30 times human body weight.

Although femur develops different functions, like blood cells production or mineral storage, this project will be focused on its mechanical functions. Femur has to provide support, not only to the whole human body, but also to many biological elements attached to it, such as tendons or muscles. Therefore, human femur is the most important element during human mechanical activities involving the lower extremities, such as standing, walking or running.

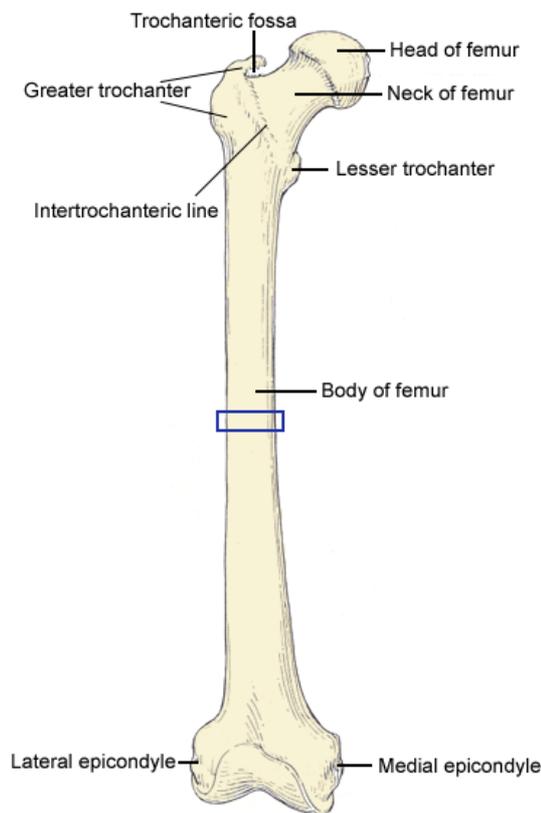
Due to of all of those reasons, femur bone is subjected to huge mechanical efforts, such as compressive, bending or torsional stresses. Femur bone can achieve all of the above due to its excellent mechanical properties, which are going to be studied in this project. Those mechanical properties are the result of a very effective combination of its structural and material characteristics. These specific characteristics of femur structure and material are now explained in detail.

2.1.2 Femur structure

Femur structure is similar to the rest of long bones that composes the human skeleton. Long bones are basically composed of a long body with two heads attached to its respective articulations. In order to deeply analyse femur structure, both external and internal structure will be discussed.

I) External structure

As explained before, femur basic structure is formed of two extremities and a central body. Its proximal extremity is connected to the hip, while its distal extremity femur forms the knee joint attached with the tibia. This section will explain the different femoral parts characteristics, which can be appreciated at Figure 1.



Adapted from Hall-Craggs, Anatomy 2nd Ed. U&S.

Figure 2.1.1: External structure of a human femur

The proximal (or upper) extremity:

Femur proximal extremity is made of the femoral head, the femoral neck and the trochanters.

Head: has a globular shape, being a little bit more than a hemisphere. It has a smooth surface, covered with cartilage with an exception on its ovoid depression (*fovea capitis femoris*), which gives the attachment to the ligamentum *teres*. The head of the femur is introduced into the cup-shaped cavity of the hip bone, called acetabulum, in order to form the “ball and socket” hip-joint. Its rounded shape allows the whole femur to move at almost any direction, including rotation into its self-axis.

Neck: Distally from the head appears the proximal neck, in which bone femur gets narrow. The main propose of this geometrical change is to provide enough space to

allow the complete leg rotation. That is the reason why this particular femur region is frequently affected by fractures. So far, the main function of both proximal head and neck is to provide the most possible movement of the femur around the hip bone. However, mechanical resistance is not really high at those regions. After the neck both trochanters appear, being the limit between neck and bone body regions.

Trochanters: At the distal end of the neck femur change its orientation in 45 degrees, to take the direction of the femur body until arriving to the distal extremity. Just after the neck finishes, two prominent bone features appear: the trochanters. Its main function is supplying support to the muscles that rotate the thigh on its axis, although they also provide additional strength at this critical stress region of the femur. There are two of them: the greater and the lesser, which are connected by the intertrochanteric line on the front side, and by the intertrochanteric crest on the back.

The Body or Shaft:

Its shape is nearly cylindrical, being wider at the distal zone than in the central one. In addition, it is slightly arched, being concave in the front view and concave behind, where the *linea aspera* is located. It has three borders, being the *linea aspera* the posterior one, and also it has one medial and one lateral.

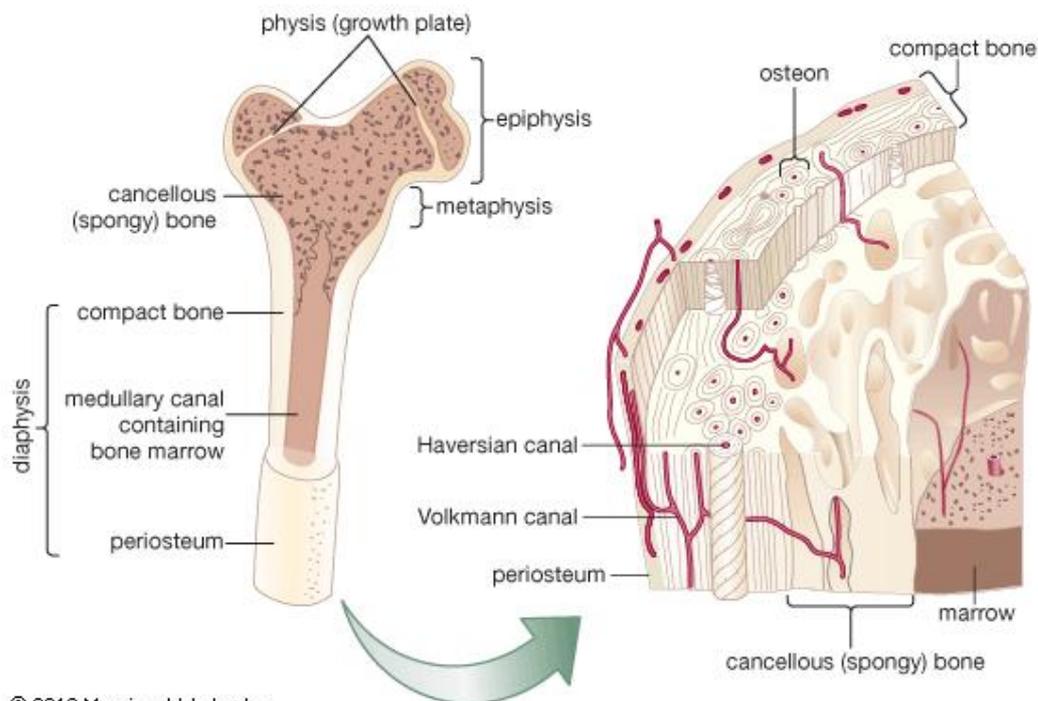
The lower extremity:

It is the distal end of the femur, and where it is attached to the tibia forming the knee-joint. It is larger than the upper extremity, and its transverse diameter is larger than the ante-posterior. It is formed by two eminences called condyles, separated by the articular depression called patellar surface at the front, and by the *intercondyloid fossa* at the back side. Both condyles are called, respectively, lateral and medial.

II) Internal structure

Once the main external regions have been specified, their internal structure is now going to be explained. Beforehand, the two main bone tissues that compose the internal structure have to be introduced: cortical (compact) and cancellous (spongy) bone. Although they will be deeply explained further, some of their basic properties will be stated: cortical bone is a rigid, compact and low porosity bone, while cancellous has a higher porosity, which makes it less stiff and dense. Both bone tissues are distributed along the whole bone geometry, although their proportion is higher at some regions (**Figure 2.1.2**). In every long bone, the external cortex is formed by cortical bone while the interior is made by cancellous tissue. There can also be observed other elements such as the medullary canal. At *femur properties* section these facts will be considered.

The thickness of these two layers is not constant along the whole bone, but a generalization between heads and the body can be made, in order to understand the main structural regions:



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Figure 2.1.2: Internal structure of a long bone

Heads: As it can be seen in the picture, heads are mainly composed by cancellous bone. The cortical bone layer that surrounds the inner cancellous bone is very thin.

Body: at the body, medullar space is surrounded with a thick cortical bone cortex. It can be noticed that the presence of cancellous bone is much lower than at the heads.

2.1.3 Femur material

Once the femur structure has been briefly explained, now the material that forms the actual bone will be exposed.

I) Bone tissue

As it has been said before, bone properties in general need to be very efficient in order to successfully perform all their functions. Focusing on their mechanical functions, all the different stresses at which bones are exposed need that their mechanical properties have to combine strength and low weight. This fact can be specially applied at femur bone, being the bone that has to withstand the biggest efforts in human body.

To fully understand bone properties, it is necessary to study its main structural material, bone tissue. Bone tissue is the main material that characterizes all human bones. It has diverse structures, depending on their function they have.

However, the bone tissue that forms any bone has in common their basic building block, the mineralized collagen fibril. The mineralized collagen fibrils are the common basic structure of all bone material family members. They are composed by three main molecular components. The major component of this structural form is the fibrous

protein collagen, an important structural element that is also present in a wide range of soft tissues, such as the skin or the tendon. The second main component will be the mineral that is formed into, or sometimes onto, this collagen structure. This mineral is named as *dahlite*, being its chemical composition carbonated apatite ($\text{Ca}_5(\text{PO}_4, \text{CO}_3)_3(\text{OH})$). The third and last main component is water.

All of these three components are associated in the complex structure that forms the fibrils, combined into a three-dimensional element matrix. However, the proportion of these components will considerably change depending on each bone family member that they will form. The way into which they associate will also depend on the bone function and requirements. Therefore, in order to form the macroscopic final bone, mineralized collagen fibrils are associated in an organized manner to form larger and more complex structures. This gradual association of the fibrils is done through different hierarchical length levels. In the next section, the different levels are going to be studied, in order to have an idea of bone tissue structure complexity.

II) Hierarchical bone structure

There are established 7 different hierarchical levels, starting from the initial sub-nanoscale level (mineralized collagen fibrils) and ending up in the final macroscopic level, which determines bone shape and its functions.

Following the chart below (**Fig. 2.1.3**), the smallest length level will be the one where the fibril components can be differentiated. Then, the next level will be referred to the combination of these components that form each of the mineralized collagen fibril. The third level will be the association between different fibrils into an array called collagen fiber, which belongs to the nanoscopic length level. Afterwards, those fibres associate between them following different patterns, also called lamellae, establishing the sub-microstructure or level four. The microstructure at the fifth level will be formed by different lamella grouped into cylindrical motifs called osteons.

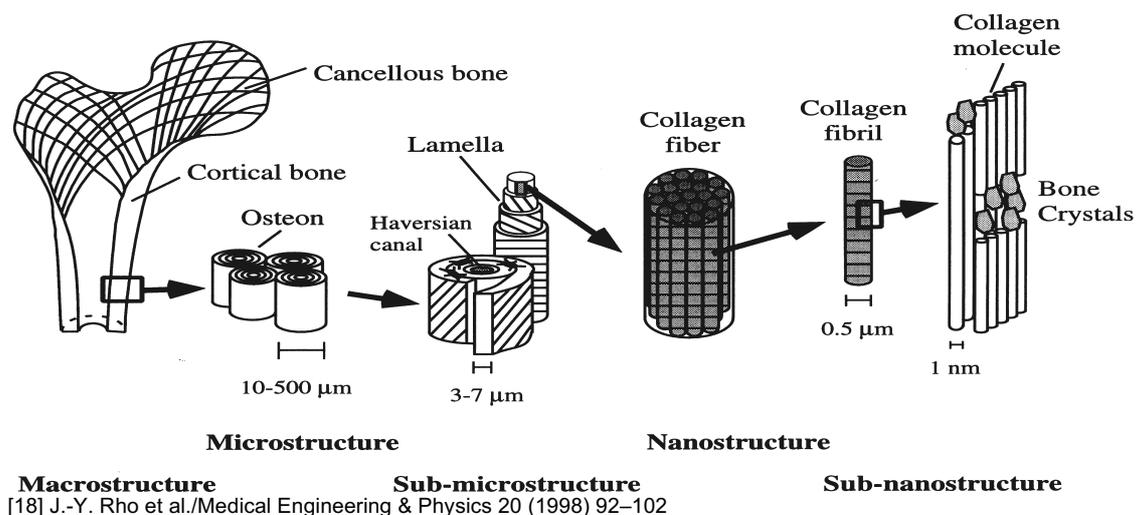


Figure 2.1.3: Hierarchical bone structure at its different size levels

Finally, at the macroscopic level, we can difference between the two main types of bone (sixth level) that are combined to end up into the whole bone (seventh level). These two main types of bone are called cortical and cancellous tissues. Their properties are a result of the combinations of the different structures explained before. Their characteristics and properties are going to be explained further.

III) Macroscopic bone tissues

At a macroscopic level, bone tissue is made by two main components: cancellous (or trabecular bone) and cortical (or compact bone). Their evident differences in porosity and density are actually a consequence on their different microscopic structures. Moreover, their distribution inside bone structure can be appreciated at **Figure 2.1.2**, at *femur internal structure* section.

Cortical bone tissue is a dense, non-porous material that surrounds bone structure. Its high stiffness is due to its microstructure, made by the shaped cylinders called osteons explained before.

Cancellous bone is less dense and more porous than cortical bone. Its irregular microstructure, made by non-organised lamellae, makes this material less stiff than cortical bone. Moreover, the structure of cancellous bone is highly vascular, as one of its main functions is to produce blood cells, by a process called *hematopoiesis*.

But this two kind of bone tissues are actually strongly related. The reason why the cortical bone cortex surrounds the cancellous inner area is because cortical bone is actually created by cancellous bone tissue. Therefore, when the disorganized lamellae that form cancellous tissue are associated into a cylindrical shape, by means of different patterns, cortical bone tissue is formed. Due to this fact, is not easy to distinguish both cortical and cancellous areas.

2.1.4 Femur properties

Femur properties are a consequence of its structural and material characteristics. In order to understand femur bone properties, the implications that both structure and material have on femur properties are now explained.

I) Structural implications

It has been seen before that the internal structure of femur bone is composed of the two main bone tissues, cortical and cancellous bone. The fact that these two materials with different properties are combined in different proportions made that the actual bone properties are not homogeneously displayed along the femur geometry. In this section it is explained how human femur properties changes longitudinally and radially.

A) Heterogeneities along femur length

Femur bone is composed of concentric layers of cortical and cancellous bone, being the external cortical layer surrounding the inner cancellous tissue. The thickness of those layers is not constant along the whole femur length, and therefore femur properties change longitudinally. It. Moreover, other features such as blood vessels or the central medulla contributes to increase this properties heterogeneity.

B) Heterogeneities at each femur section.

As explained before, in every long bone its geometrical section is composed of different concentric material layers (cortical cortex, cancellous interior). This fact makes that femur properties are also heterogeneous at each section, and therefore femur properties will change in its radially.

All of the previous femur structural characteristics make difficult the association of femur geometry with properties. However, in order to solve this problem, some assumptions and approximations have been considered to develop these project models. Those model considerations will be explained further in the geometrical approach explained at the model considerations section.

II) Material implications

Once the structural implications on femur properties have been explained, now femur material (bone tissue) influence is explained. The different information obtained regarding bone material influence on properties is now explained. There have been separated three different dimensions that bone tissue material has on femur properties will be: material orientation, material properties value and material properties relationships.

A) Material orientation

Before bone tissue properties are discussed, material orientation has to be initially analysed. It is well established that bone tissue is, by definition, an anisotropic material. That means that its mechanical properties vary depending on their orientation, which makes even more difficult to characterize femur properties.

Several studies [18], [24] ensure that, especially at bone tissues microscopic structural size levels, mechanical properties are highly oriented. However, it has been proved that those small structural elements associate in a certain way that their resultant properties orientation decreases. Therefore, at higher structural size levels, mechanical properties anisotropy is reduced.

Moreover, there are different studies [4], [21], [16], [21] which claim that human femur properties can be characterised as orthotropic. This bone material mechanical characterization has been widely used specially with the recent studies [5] that performed FEA models, with relatively good results.

Finally, there have been also studies that have considered bone material as isotropic. While some of them have negatively verified this material orientation compared with orthotropic, other investigations ensure that their accuracy of both methods is pretty close [13]. The main advantage of an isotropic material characterization is, of course, its simplification and time reduction of the calculations.

B) Material properties value

It has been proved in many studies that the properties of cortical and cancellous bone tissue cannot be generally established as constant values. That means that its own properties will change depending on its geometrical situation on the femur. Therefore, for instance, cancellous bone properties at the proximal head are not the same as their properties at the distal one. Moreover, this fact also makes that cancellous and cortical bone properties value changes within different subjects and experimental methods. The values for the two bone tissues properties found varies considerably from one article to another. However, there exist a certain value range in which bone properties happen to be limited.

Cortical bone

Its density value happens to be within the range of 1500 to 2000 Kg m^{-3} , while its Young's Modulus is roughly limited between 15 and 25 GPa. The stiffness values happen to be in the same range with either isotropic or orthotropic material orientations, as it happens with cancellous bone.

Cancellous bone

Its density value moves around 500 Kg m^{-3} , decreasing until 100 and never achieving 1000. Its Young's Modulus range goes from 300 until 2000 MPa

Those approximate values range show the big difference between both bones properties, and therefore the big impact that their combination proportion creates on femur properties.

C) Properties relationship

Many articles and studies [5], [9], [13], [16], [21] have experimentally arrived to the conclusion that, at bone tissue, there are certain relationships between density and stiffness. It has to be specified that those relationships have been observed with *apparent density* values, referred at bone tissue level, and defined as the wet mineralised mass of bone at tissue level over its volume [26]. Different equations to relate them have been performed by means of Computed Tomography-based data help.

The use of these material relationships has been especially helpful in FEA model analysis, when femur properties were characterized. By means of CT-data, geometrical and density values were obtained, and then stiffness was derived using those

relationships. Numerous articles claim that accurate density distribution models have been measured and performed [5]. Therefore, the use of these relationships has helped to introduce the mechanical properties based on density values previously measured.

Some other articles do not approve that density and stiffness present such correlation, based on microscopic bone tissue structure and stiffness variation on different subjects where its density remained constant [20]. However, they obviously accept that bone aspects like porosity affect both density and mechanical properties.

III) Complexity

Analysing both structure and material implications on femur properties, it has been seen that it is not easy to introduce them in order to perform an accurate model. In order to develop this project models, different approximations and assumptions have been considered. The different considerations used in order to perform the models, are further explained and justified at the “Model Considerations” section.

2.2 NATURAL FREQUENCY

2.2.1 Introduction

Definition

Natural frequency is defined as the rate that a body vibrates when no external force is applied on it. That is, it is the frequency value at which the body will vibrate when it is freely set in vacuum or space.

Natural frequency and resonance

Resonance is the phenomenon that occurs when there is an external load that creates on the body a vibration with the value of its natural frequency value. Therefore, knowing the natural frequency value of a body, it will be possible to create on it a physical excitation.

Natural frequency characteristics

One of the most important properties that natural frequency has is that it characterizes not only the material, but also the size and shape of the measured body. In the case of the femur, that will mean that calculating its natural frequency depends on their different material properties but also on its specific shape and size. Moreover, there is a different natural frequency values for each of its different vibration modes.

Modes of vibration

Vibration modes are related with each degree of freedom of the body. In principle, every material piece has an infinite number of vibrations modes, with successively increased frequency values.

The main vibration modes are divided into two bending modes (at each of the planes that the body longitudinal axis have) and one torsional mode (around the body longitudinal axis). Each of those modes have a successfully higher number of natural frequencies, numbered as first, second... bending or torsional modes, also including all the combinations between them.

2.2.2 Natural frequency on beams

Natural frequencies modes can be calculated knowing the body geometry and properties (density and stiffness). There are certain equations that characterize the natural vibration values for simple geometry bodies, such as beams or plates. There are more considerations that have to be taken into account, such as the load distribution along the body geometry, or its boundary conditions

For instance, for a simple beam with a distributed load, there is a certain equation to calculate its natural frequency value. It can be seen that the equation includes the beam geometrical characteristics, and also beam material and mechanical properties, such as density and stiffness. Moreover, it can be seen that the equation also includes a factor that depend on the boundary conditions of the beam.

That factor depends on the boundary conditions that limit the studied body is constrained. As it can be seen, this factor has different value for each of the vibrational

modes studied

So for example, the equation and modal factor values for a beam-like object, with uniform section and load, are the following:

BEAMS WITH UNIFORM SECTIONS AND UNIFORM LOAD

The natural frequency (ω_n) equation is defined as $\omega_n = A \sqrt{\frac{EI}{\mu L^4}}$ where

E - Young's Modulus

I - Area Moment of Inertia

L - Length of the beam

μ - Mass per unit length (linear density)

A - Modal factor (given at Table below for common configurations)

| Configuration | Mode 1 | Mode 2 | Mode 3 |
|-------------------------|--------|--------|--------|
| Fixed-Free (Cantilever) | 3.52 | 22.0 | 61.7 |
| Hinged-Hinged (Simple) | 9.87 | 39.5 | 88.9 |
| Fixed-Fixed (Builtin) | 22.4 | 61.7 | 121.0 |
| Free-Free | 22.4 | 61.7 | 121.0 |
| Fixed-Hinged | 15.4 | 50.0 | 104.0 |
| Hinged-Free | 15.4 | 50.0 | 104.0 |

Table 2.2.1: Modal factor (A) for different beam configurations [Source: Third Edition Shock and Vibration Handbook (2000)]

However, natural frequency calculation gets really complex when they are not dealing with simple geometries and material distribution. Therefore, in order to calculate femur natural frequencies, the finite elements method software Abaqus will be used.

2.2.3 Femur natural frequency

The importance of natural frequency in medicine has made that many different measurement methods to obtain their values have been performed. In this section, the most important methods to obtain frequency values are explained.

A) In-vitro experimental measurements

There have been many successful experimental studies that have obtained reasonable good results for long bones natural frequencies [8]. There have been some vibrational

analyses, (like **Campbell et al (1970)** in which femur vibrational properties were obtained following an *in vitro* experimental method. Their results seemed to give great results in order to predict the mechanical properties of the bone. Afterwards, (**Khalil et al (1981)**) obtained values for different resonant frequency modes, by means of experimental and analytical analysis.

However, in the later years, in order to obtain accurate results, it was generally established that *in vivo* experimental methods should be performed.

B) *In-vivo* experimental measurements

One of the big problems of the *in vivo* experimental methods to directly obtain its natural frequency is the presence of the *soft tissues* [8]. The *soft tissues* that surround the femur surface overcome two big problems: calculation complexity and pain. On the one hand, *soft tissues* effect on the obtained frequencies values has to be considered. This fact makes the vibration-mechanical properties correlation much complex, so approximating techniques were used to obtain the results. On the other hand, especially at the early years, the presence of soft tissues made that the pain suffered by the subject during those experiments was considerably high.

C) Finite Element Models

By the time those frequency experimental investigations were performed, there were also studies that tried to characterize bone properties by means of CT-data. Computed Tomography allows to obtain accurate values for 3D-models and density distributions [8]. Therefore, in order to get more accurate results than with experimental methods, Finite Element femur models were built with CT-data, especially to investigate bone mechanical properties, but also to obtain other bone properties.

Therefore, some of those femur models were used to obtain its natural frequencies. The process to obtain natural frequency results is as it follows. Initially, by means of CT-data bone density and geometry measurements were obtained along the whole femur length. Afterwards, mechanical properties were obtained using the known correlations between stiffness and bone density values. Finally, introducing those geometrical, material and mechanical properties on a Finite Element Model, natural frequency results were simulated. The use of FEA models in biomechanics is now widely extended, as they are able to accurately reproduce real medical situations or processes.

However, there were certain discrepancies between the revised articles which calculate femur natural frequencies by FEA models [5], [8], [21], which principally are due to the different femur properties characterization done and the CT-based data used. In the next section the criteria followed to choose a reference frequency value from all literature articles will be explained.

2.3 MODEL CONSIDERATIONS

At the previous two sections, the difficulty to obtain values for both femur properties and natural frequency has been explained. Therefore, in this section the different assumptions that have been used during the FEA models development are established. There have been used different models, where femur properties have been introduced to obtain natural frequency results. The main propose of the different considerations has been to obtain different models and improve their results. The considerations have been taken at both femur properties characterization and at its natural frequency analysis.

On the one hand, several considerations have been taken in order to approximate femur properties characterization. These femur considerations have been divided into geometrical and material approaches. While the geometrical approach deals with the structural implications explained before, the material approach is referred to the material implications. In order to get an improvement during the different models performed, the different considerations have been gradually added to each model. Therefore, the initially simple models have been increased its complexity until arriving to the final and complex models.

On the other hand, the natural frequency results obtained by the different models have been analysed. The main propose of this analysis was to ensure that each of the successive model property approaches explained before supposed a real improve on their results. The different considerations in order to analyse natural frequency values obtained are also explained in this section.

2.3.1 Femur properties considerations

As the main goal of the project is to obtain a reasonable model that predicts femur natural frequency, the properties introduced to obtain those results have to be reliable and accurate. At the initial femur bone section, there has been explained how its structural and material complexity influenced their properties. Therefore, in order to perform an accurate characterization of the femur material and mechanical properties a careful study has to be performed.

In order to apply that characterization on the FEA model, the used data precision has to reflect that complexity. Many models have been performed before, using Computed Tomography based data that accurately measured bone density and mechanical properties. In order to achieve that level of accuracy, those femur properties have been introduced in the femur FEA model by high-technological tools, which performed a successfully 3D properties distribution in each section of the whole femur bone.

However, that technological support was not available in order to perform this project. Therefore, many assumptions and approximations have been performed in order to simplify the actual femur structural and material complexity. This section will specify and justify the use of the several considerations taken into account during the FEA model development.

a) Structure approach

In order to characterize the actual shape and geometry of the human femur, a three-dimensional medical model has been used. It has been scaled in order to adapt its length to real people value. The geometrical model will be appreciated at the different pictures in the *FEA models* section, and its complexity will be manifested where its mesh accuracy is specified.

As it has been explained, femur inner structure has a complex combination of different tissues properties and geometry. The proportion and distribution of the main bone tissues varies in each section of the femur geometry. Moreover, there are also other biological elements also present in the internal bone structure, such as blood vessels and the internal medulla.

However, it also has been explained that human femur structure, like other long bones, follows a certain pattern and it can be divided into different regions. The three main regions, each of them with a pretty constant internal structure, are the two extremities and the body. In general terms, it can be said that both extremities are mainly characterized by the presence of cancellous bone, while the central region main tissue will be the cortical. This generalization has been used to divide the femur model into different geometrical regions, which try to simulate the actual structural singularities of the bone. During the models two different geometrical approaches have been used.

Three regions approach: this initial approach considered two heads at each of the distal and proximal extremities, and one central region that simulated the central body of the femur.

Five regions approach: a following improvement was made in order to simulate the transition between both extremities and the central body. Therefore, two other regions were created between the central body and both heads. The upper region simulated the proximal neck, and therefore it was applied to its actual geometry. The other region performed at the distal area tried to simulate the structural transition between the distal head and femur body.

In order to the actual bone tissue distribution, each of the femur regions was associated to different materials. Material properties belonged to the actual cortical or cancellous tissue values. The material selection process and its properties are now explained at the material approach section

b) Material approach

As said before, the different regions performed were associated to different materials, simulating actual bone tissue properties. In this section the selection and assumption in order to get those properties value is explained. As it can be noticed, the order of the different material considerations is the same as the one at the “material implications” on femur properties.

Material orientation

First of all, in the developed projects the material properties have been introduced with an isotropic orientation. This assumption is based on [13], which ensured that when developing FEA Models calculations, orthotropic material orientation compared to isotropic did not improve the results in a significant manner. Moreover, the time reduction and simplicity assuming an isotropic material orientation was also a very important reason to take into account. Finally, the constant value of 0.3 for the Poisson ratio stated in the same article [13] was also adopted. This value has been also adopted in other studies using FEA models to calculate natural frequency.

Properties value

As it has been said before, bone tissue properties cannot be considered as constant through the whole femur. This fact makes that the material properties assignation in FEA models becomes a really difficult task. Moreover, the approximate properties range established for cortical and cancellous bone happens to be very wide through the different investigations performed.

Due to this big properties value range, in this project the properties used belong to a single article [8] results. The main reason of this selection was that the properties used in the article were used to develop a FEA femur model where natural frequency values were calculated. Moreover, the properties followed an isotropic material characterization, which adapt to the material orientation chosen for this project. The article considered three different materials, in which both density and stiffness were specified: cortical bone, proximal cancellous bone and distal cancellous bone. The values were experimentally obtained from the bone of a 70 years old subject. Therefore, those values were initially assigned to the three regions model (two heads and one central body).

Moreover, when the five regions model was performed, the new neck region properties were obtained from another article [12]. According to this source, Young's Modulus of the cancellous bone at the proximal neck region was obtained. As is it going to be explained further at the FEA models section, distal neck properties were extrapolated, based on the previous head properties and the new neck stiffness value

Properties relationships

As it has been said before, it is well accepted that bone tissue density has a correlation with their mechanical properties. There have been different relationship equations obtained by experimental and numerical investigations.

However, the density-stiffness relationships considered in this project have been extracted from [13]. The main reason to choose this article results is due to its simplicity in order to apply them on this project. Instead of considering both material and apparent density, as well as bone tissue volume fractions, it simply established the relationships between the two properties used in this project: apparent density and Young's Modulus. The relationships changed between cancellous and cortical bone

material, as it was expected.

Moreover, the article established different equations regarding material orientation, distinguishing between orthotropic and isotropic properties. As it was initially explained, isotropic material orientation was used in order to simplify the calculations. Therefore, two different isotropic properties relationships have been considered for both cortical and cancellous bone tissues.

2.3.2 Femur natural frequency considerations

Natural frequency is a bone property that has been widely studied due to its different medical applications. Moreover, its importance is increasing due to recent studies that ensure that it can bring new medical benefits and solutions. Therefore, its importance has made that natural frequency has been widely measured by different methods.

a) Boundary conditions

Abaqus software was used in order to calculate femur natural frequencies from its material properties. That means that boundary conditions have to be defined in order to build the model that would simulate the results. The different revised literature articles stated that the best way to calculate human femur natural frequency with FEA models was establishing *free-free* boundary conditions [5], [8], [21]. Therefore, this type of boundary condition was established at the different models performed, so that their natural frequency results could be obtained and compared.

b) Modes of vibration considered

Any material has a resonance or natural frequency value for each of its vibrations mode. There are different kinds of vibration modes: two bending modes at two different planes and one torsional mode. Each of these three kinds can be referred to the first, the second or the “nth” bending or torsion mode. Therefore, it can be said that there are infinite modes of vibration, increasing its exponents and combining between them. However, in this study only the first five natural frequency modes are going to be considered. Moreover, in most of the literature references the general natural frequencies values obtained did not analyse more than five modes [8], [21]. The relevance of the natural frequency specially relies in the initial ones. Therefore, the accuracy of this study can be obtained by far just considering these five first natural frequency modes of vibration. Those five initial modes are the first and second bending modes in both planes (weak and strong bending planes), and the first torsion mode. In the *FEA Models* section the different frequency modes and femur planes will be explained.

c) Natural frequency values

There are a wide number of articles and studies that have come up with femur bone natural frequency values. Therefore, their values range is too big, and thus very

complex to consider all their results to be compared with the ones obtained at this project. Following the same criteria as with the mechanical properties section, an initial reference was used to start developing the model.

It was decided to be used as the reference values the results obtained at [8], which is the article used to obtain the initial bone properties. The fact that it calculated femur natural frequencies by means of finite element analysis made it a perfect reference in order to develop this thesis. The way to introduce the different properties was very well explained, due to its relatively simple assumptions, which were ideal to reduce calculation time and complexity. In addition, it also calculated natural frequency values by an experimental method, whose values will also be used as a reference for the obtained results at this project.

The idea of improving femur properties modelling by means of a reciprocal combination of mechanical and natural frequency analysis using FEA models [21] was one of the main inspirations in order to perform the present project. This concept of material and vibrational reciprocity was also remarked by the two methods (experimental and mechanical) used in [8] to fully understand human femur properties.

d) Euler-Bernoulli Beam Theory

As it has been explained, this study has no experimental data available, such as CT-based images that could be introduced in the software. The decided way to obtain a reliable material characterization was to analyse each model natural frequencies results in order to be able to improve or validate the introduced properties. One of the main analysis tools to be used in this project to validate the obtained results is based on the Euler-Bernoulli Beam Theory. According to this theory, natural frequency values follow certain relationships between them. Actually, the ratios between the second and the first bending modes at each plane (sagittal and frontal) maintain a constant value. These ratios are called Euler-Bernoulli ratios, and their values depend on the boundary conditions at which the object is subjected. In the case of “free-free” boundary conditions, this value happens to have a value equal to 2.75. The main reference article [8] used in this section ensured that femur bone can be considered as a beam-like solid, and therefore it has to obey Euler-Bernoulli Theory. As it will be further explained, Euler Bernoulli ratios have been used in this project as an important additional tool to critically analyse the natural frequency results obtained through the different models performed.

3 FEA MODELS

3.1 INTRODUCTION

To get a feasible value of a human femur natural frequencies at its different modes FEA models are going to be used. To achieve this, mechanical and geometrical properties must be implemented in the Abaqus® software 3-D model of a human femur. This block has been divided into several sections:

Abaqus modelling

This first step will be a description and explanation of the different procedures used to perform a model using Abaqus software. Moreover, it will be specified how to introduce the different **model considerations** explained at *Section 2.3*.

Initial model study

Once Abaqus procedure was clarified, an initial simple model is performed. The main propose of this initial model is to obtain the first results and **establish** the software **mesh resolution** applied on the following models. Moreover, the influence of the introduced properties on natural frequency values will be explained by means of an **initial study**. This understanding of the software inputs on the frequency results will be critical to perform the following models

Real FEA models

After this initial analysis, different models will be performed, **taking into account** the different assumptions and approximations stated at **model considerations** sections. On the one hand, the different femur properties considerations will be added to the **increasing complexity models**. On the other hand, their **results** will be **critically analysed** by means of the femur natural frequency considerations.

Model improvement

Finally, this last section will try to **analyse** in detail the **two final models** achieved in the previous section (Stiffness and Density Model). Their properties will be slightly modified to show their impact on the results. Moreover, the different possible geometrical and material model factors will be considered in order to explain and justify the results behaviour.

3.2 ABAQUS MODELLING

In this section the software procedure to obtain natural frequency results will be explained. The different models considerations have to be specified in Abaqus through its different “modules”, as is it going to be explained.

3.2.1 Abaqus “Part” module (Geometry)

The “Part” module in Abaqus is where the femur model geometry is scaled, modified and partitioned during the different models performance.

The geometrical model used in this project is a real medical model obtained from scanned-data. The 3D femur model appeared to be composed of several geometrical faces, which made possible to adapt its shape to the complex and irregular bone geometry (**Figure 3.2.1**).

Figure 1

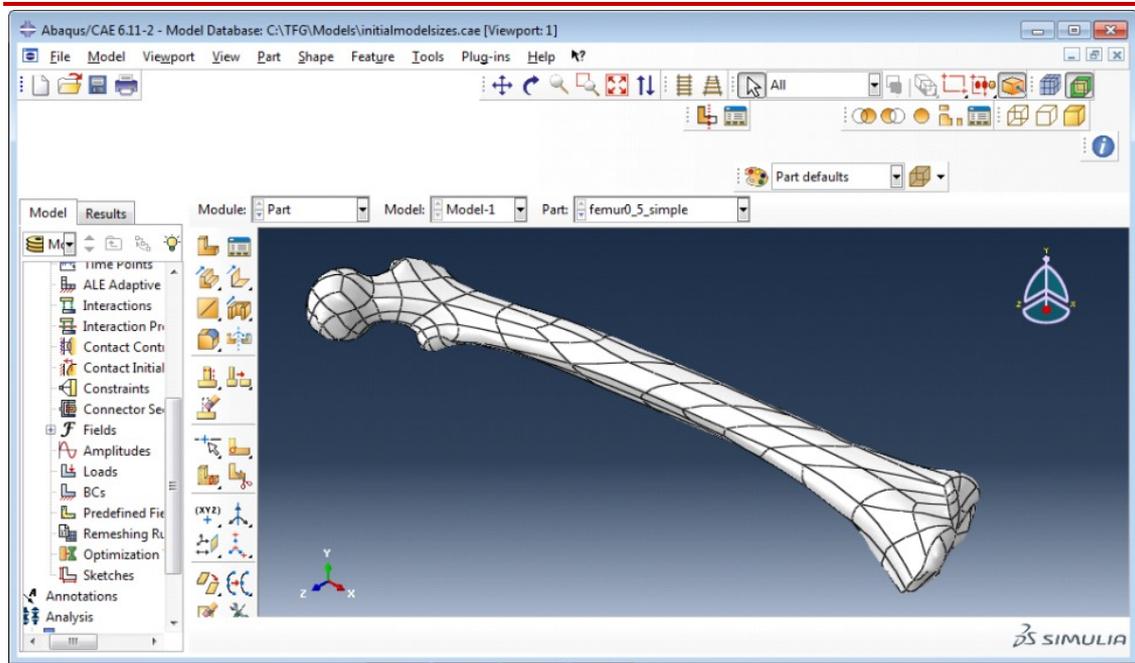


Figure 3.2.1: Abaqus part module

Partitions

As it was said before in model considerations, in order to develop the models a geometrical approach was used, dividing the femur into different regions. Partition is the software tool that allows to divide the selected geometric part into independent regions (*partitions*), where individual materials can be assigned. The type of partition creation was *cell partition*, defining the cutting plane that will divide the original bone geometry into two cell regions. Moreover, the individual cells created can be again divided to create new partitions. The Abaqus screen-shot where this procedure was done can be appreciated at **Figure 3.2.2**.

Figure 2

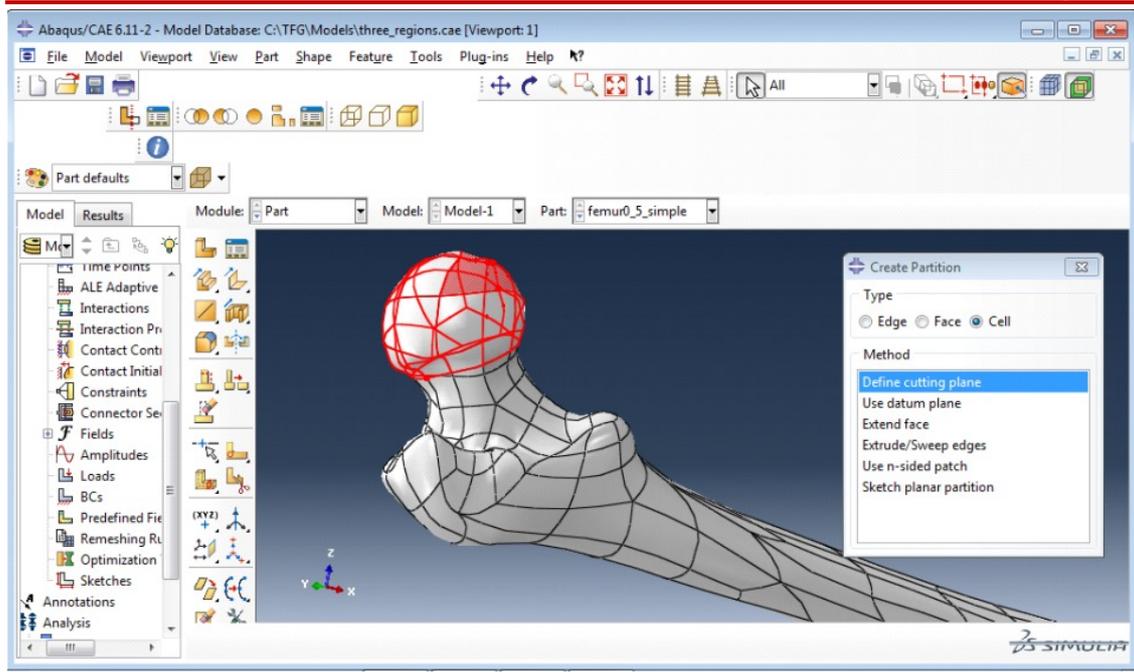


Figure 3.2.2: Partition on Abaqus part module

In our study, as it was said before two different geometrical approaches will be performed: three regions and five regions approach.

Three regions approach

This geometrical approximation was the first one to be performed. This approach considered three different regions: proximal head, central region and distal head. The procedure followed to obtain those regions on Abaqus was to perform two different cell partitions at both extremities of the original bone geometry.

Five regions approach

This second approach was performed on the previous three regions model. In this new geometrical improvement, two new regions called necks were added. The way to add those new regions was by partitioning the three-model central region at both of its ends. Therefore, those new neck regions were between both heads and the central region.

The region performance, as well as its material assignment will be fully explained at the Real FEA Model sections.

3.2.2 Abaqus “Property” module (Properties)

This will be the software module where the material properties are introduced into the model. During the different model performed, different materials were created and associated to their correspondent femur regions. The previously explained material approach considerations were introduced into Abaqus software as follows:

Initially, material characteristics and orientation were introduced, defining the material as *perfectly elastic*. Afterwards, their mechanical properties were chosen to be isotropic, and therefore a single value for its Young's Modulus and Poisson's Ratio needed to be introduced. As it was explained at *Model consideration section*, isotropic properties were decided to be used, in order to make calculations easier. A constant value of 0.3 for Poisson's Ratio has been maintained though all the models, as it was already explained. Once mechanical properties were defined, their values were introduced, as well as general properties, where density value was introduced. The units in which density and Young's Modulus had to be introduced in Abaqus were, respectively, Kg m^{-3} and Pa. **Figure 3.2.3** shows how material properties were defined as isotropic and introduced in Abaqus.

Figure 3

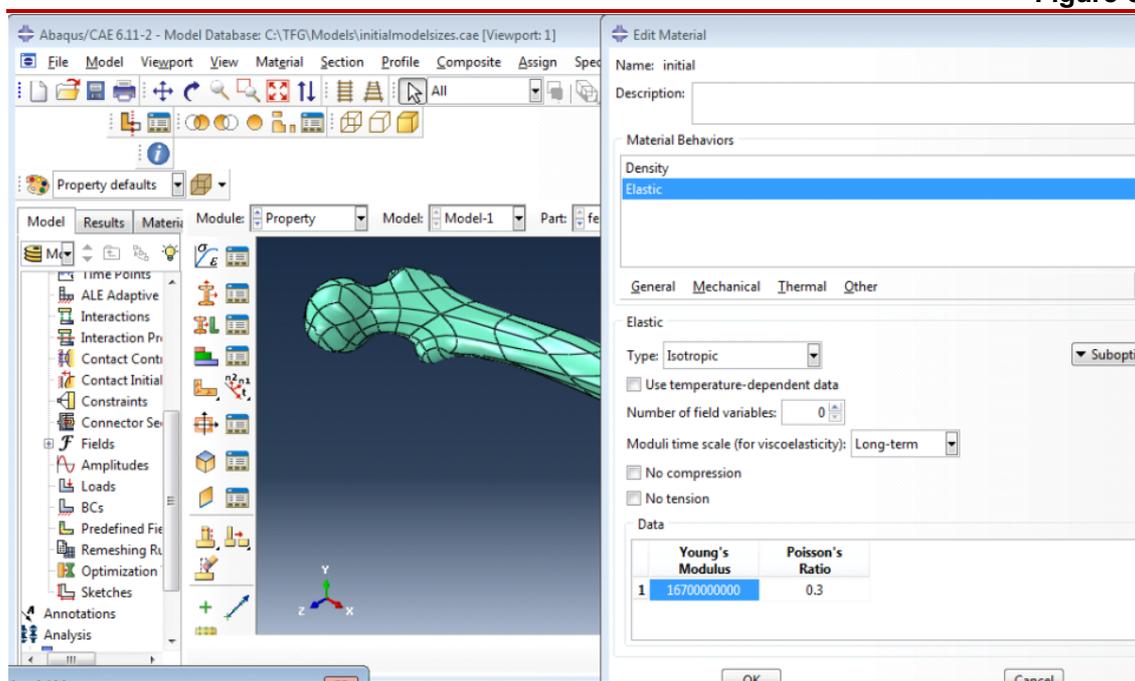


Figure 3.2.3: Abaqus property module with material edition menu

When models with several regions were performed, their regions had to be individually assigned to a different material.

For instance, in the case of the *three regions model*, three materials were introduced: Proximal cancellous bone for the proximal head, cortical bone for the central body and distal cancellous bone for the distal head, being the initial material properties considered.

In the case of the *five regions model*, two new regions were added: proximal neck and distal neck. Both necks were associated with cancellous bone properties, using the new value obtained from [12] that established proximal neck stiffness in 1000 MPa. The rest of the properties values for the neck properties were extrapolated from the new neck stiffness and the previous cancellous bone properties used before. Finally, the material relationships for density and stiffness were also applied to improve the properties associated to every region.

The specific process in order to assign material properties to each region will be fully explained at *Real FEA model section*.

3.2.3 Abaqus “Step” module

To obtain results for the natural frequency of an Abaqus model, a certain *step* needs to be defined. The kind of step that had to be selected is a *linear perturbation*, where *frequency* type could be chosen (**Figure 3.2.4**).

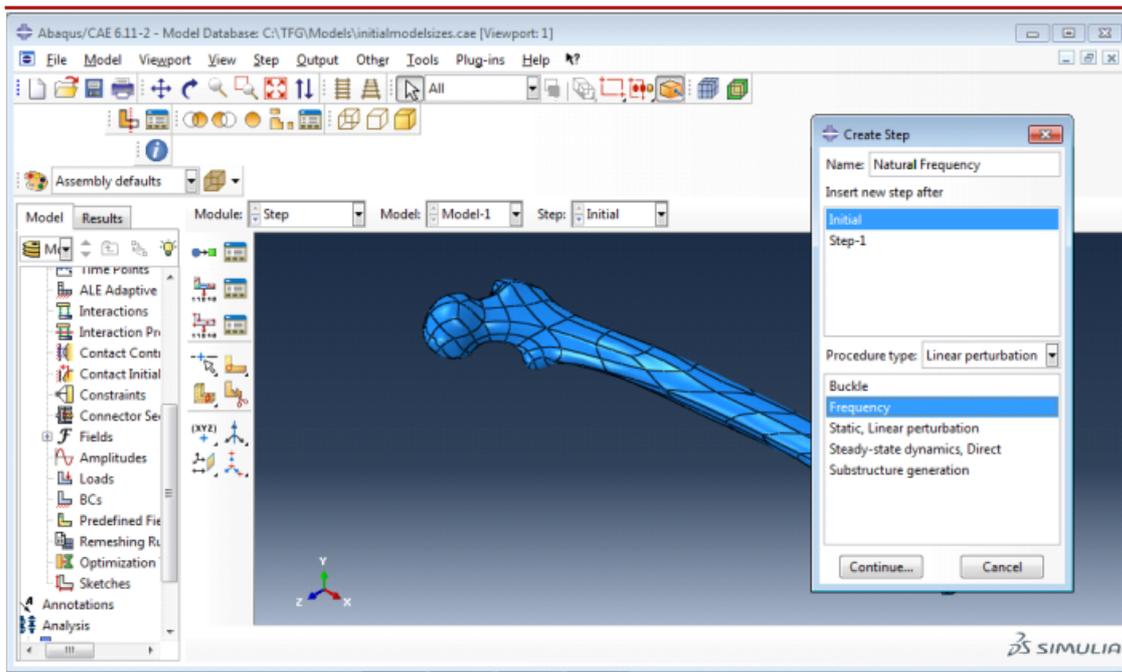


Figure 3.2.4: Abaqus step module

3.2.4 Abaqus “Load” module

This is the software module where external forces and boundary conditions are established. As it was initially said, natural frequency is defined as the value at which a body vibrates when it is set “free” in the space. Therefore, no external loads or forces can be applied to the femur. Moreover, the boundary conditions selected were “free-free” type, which allows the femur to freely move without restrictions. This decision, explained before at *section 2.3*, was based on the literature articles. All the articles which calculate femur natural frequencies with FEA Models use “free-free” type boundary conditions.

3.2.5 Abaqus “Visualization” module

In this module, the results are finally displayed on the computer screen. It has been already explained that only the first five frequency modes have been considered in this project. However, Abaqus numeration for those modes started at 7. The reason that explains this fact is that the first six Abaqus modes are referred to boundary condition restrictions. In this case, as “free-free” conditions were established, all of those modes were zero. **Table 3.2.1** shows the equivalence between the frequency modes names, Abaqus numeration and the abbreviated name used in this project.

| Frequency modes denomination | Abaqus | Abbreviated |
|-------------------------------|---------|-------------|
| 1st bending at frontal plane | Mode 7 | Mode 1F |
| 1st bending at sagittal plane | Mode 8 | Mode 1S |
| 1st torsion mode | Mode 9 | Mode T |
| 2nd bending at frontal plane | Mode 10 | Mode 2F |
| 2nd bending at sagittal plane | Mode 11 | Mode 2S |

Table 3.2.1: Frequency modes studied in this project with their names equivalence

Once they had been introduced, the different bending modes and planes have to be explained when they are applied to the femur geometry. *Frontal plane* is referred to the weakest EI femur bending plane, while *sagittal plane* refers to the strongest one. It has to be taken into account that, as the femur is longitudinally located at X-axis, frontal plane corresponds to the XZ plane, while sagittal plane is referred to the XY plane. Femur bending planes can be distinguished at **Figures 3.2.5** and **3.2.6**, while at **Figure 3.2.7** the different vibration modes applied on the femur can be appreciated.

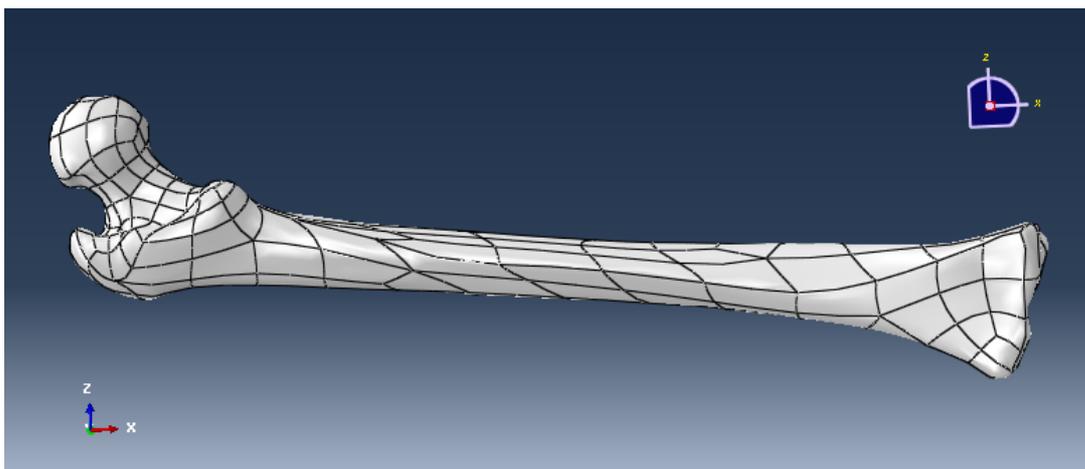


Figure 3.2.5: Femur frontal bending plane

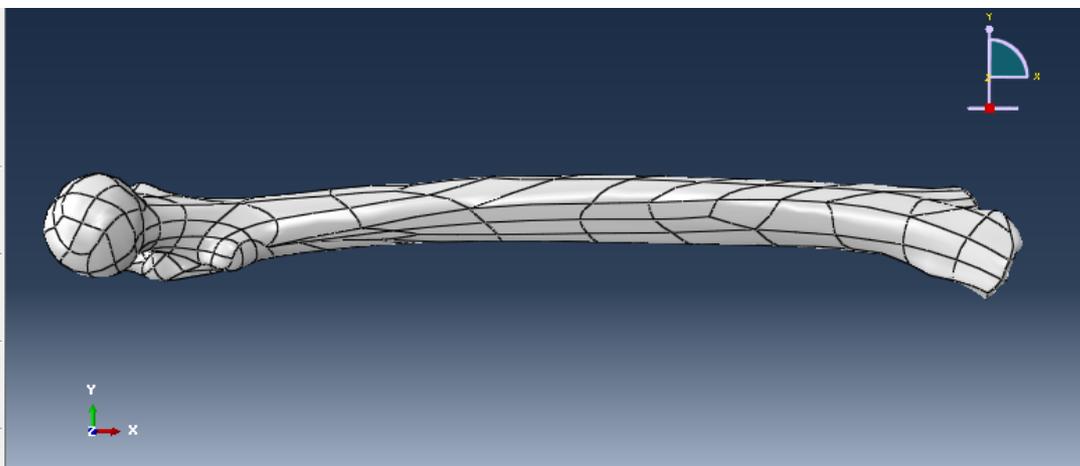


Figure 3.2.6: Femur sagittal bending plane

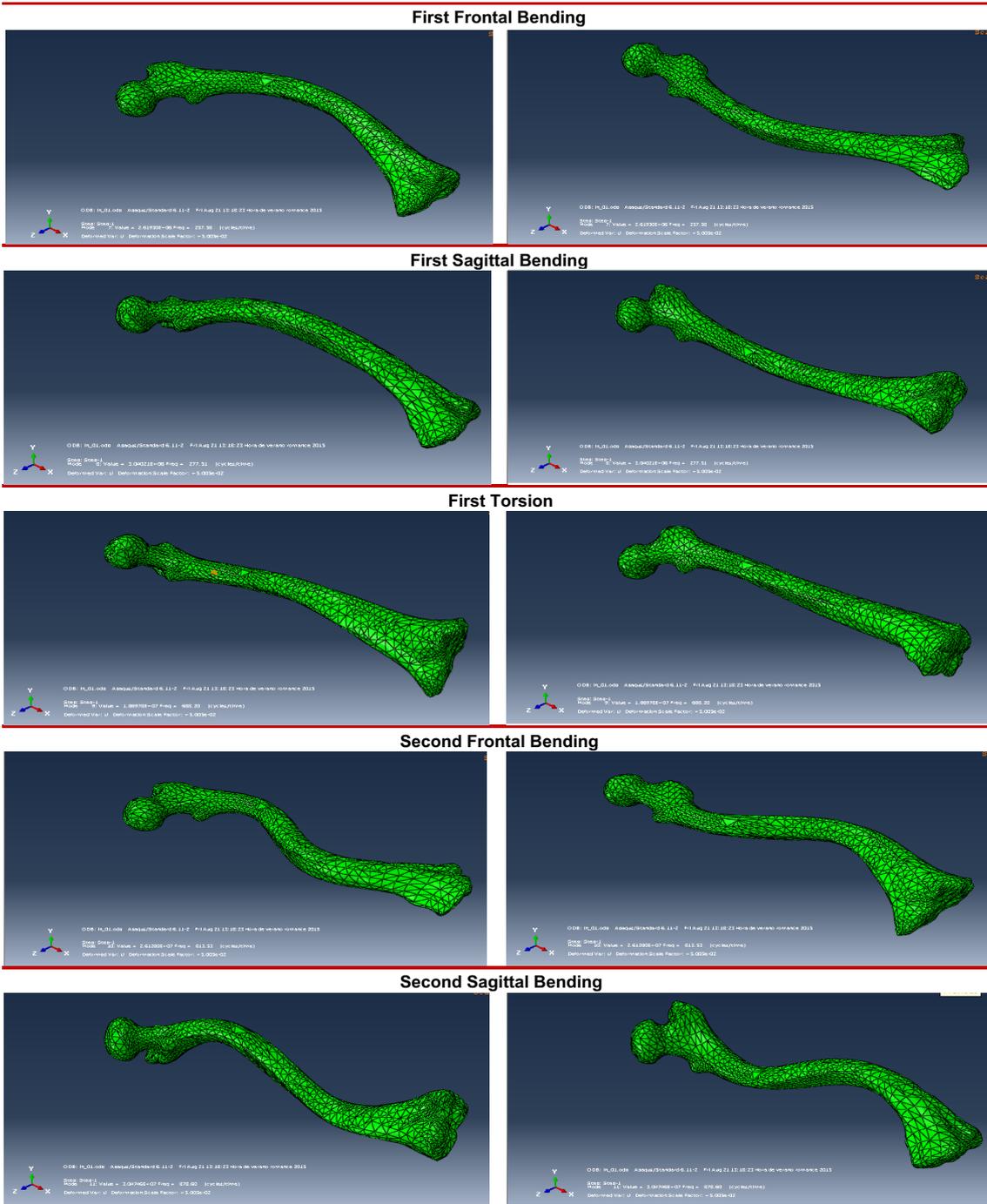


Figure 3.2.7: Considered femur vibration modes at Abaqus visualization module

3.2.6 Abaqus “Mesh” Module

This module is the one that deals with the “meshing” conditions. FEA models are based on the geometrical meshing of the used geometry. *Meshing* is actually dividing the whole body into many small elements, which will allow the software to get an approximate solution. The accuracy of the implemented mesh will determine the quality of the results. Therefore, it is critical to obtain the desirable mesh resolution needed for this model.

The mesh conditions selected were maintained at every model performed further. Taking into account this, the desirable mesh resolution to be selected considered two main factors:

- I) Mesh quality: in order to obtain accurate results, the mesh resolution has to be good enough. That means that the meshing elements used are well adapted to the femur geometry.
- II) Calculation time: it is obvious that the calculation time spent by the software increases with the meshing quality. However, it has to be considered the large amount of different models performed in this project. As the meshing conditions are going to be maintained through the whole project, the calculating time cannot be too high.

Meshing control: seeds

The previous requirements that this model meshing has to follow will be decided through meshing seeds properties. The small elements at which the femur will be divided are controlled by Abaqus “seeds”. Therefore, their proper definition is critical in order to get the appropriate mesh that will meet the desired quality and time requirements.

Element geometry

The first consideration was to decide which element geometry is the adequate to be applied in order to perform the mesh. It is important to take into account the curved and complex shapes of the femur geometry. The two main geometry for the femur elements which available on Abaqus are hexahedral and tetrahedral.

For curved shapes, the use of hexahedral is not desirable, because the only way to adapt them to a curved shape is to decrease its size too much. Even though the seed size is highly decreased, the obtained resolution is not going to be as good, and the calculation time will be too high. Moreover, Abaqus software recommends not to use this kind of *seed geometry* with the femur model, but tetrahedral or another manual method instead. Therefore, hexahedral seed geometry was rejected.

In the case of tetrahedral seed geometry, their ability to approximate to curved and complex shapes is considerably better. That means that bigger seed sizes will be highly efficient compared to the hexahedral ones. Therefore, it was finally decided to use tetrahedral *seed geometry*.

Element size

Once the geometry of our meshing elements geometry have been decided, now it is necessary to decide the most efficient size that meshing elements need to meet the accuracy requirements. In Abaqus software, *global seed size* is the parameter that measures element size at the mesh module. First of all, it has to be considered that a smaller *global seed size* is traduced into a lower element size, and therefore higher mesh quality. However, the time calculation may be excessive if the element size is too small. Therefore, the goal is to obtain the maximum possible size they need to get an accurate enough result. **The decided *global seed size* value will be chosen once the first model results were obtained, at the 3.3 Initial model study section.**

Figure 3.2.8 shows how the mesh resolution increases when *global seed size* is gradually reduced.

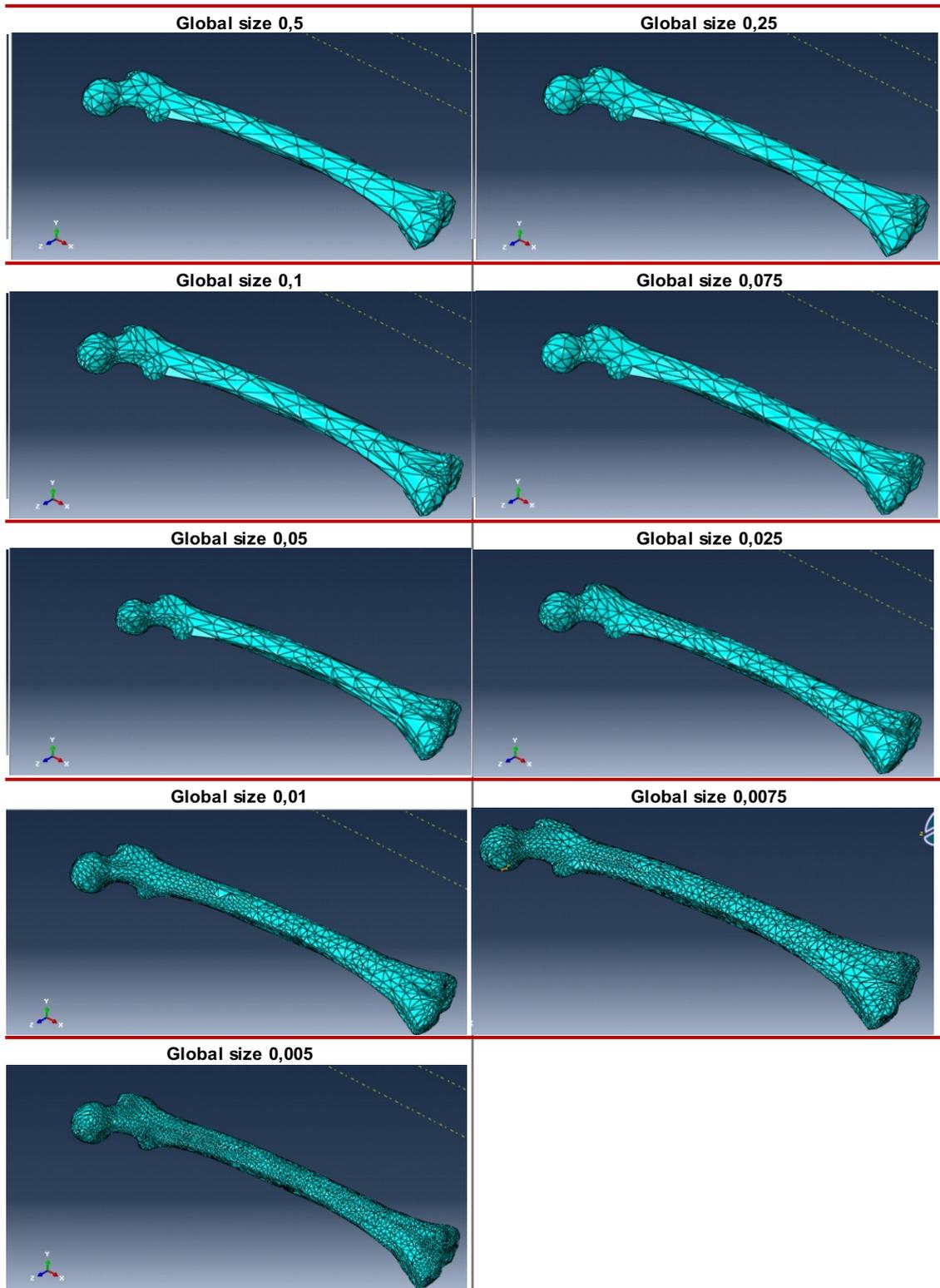


Figure 3.2.8: Femur meshes with different global seed sizes at Abaqus mesh module

3.3 INITIAL MODEL STUDY

In this initial model study the influence of the introduced variables on the natural frequency results will be analysed. Before starting the study, initial model variables and meshing considerations have to be initially specified.

3.3.1 Model variables

I) Material properties. As it was explained, the material characterization was isotropic, so the only two properties considered are Density and Stiffness (Young's Modulus). As it is going to be further explained, there exist a relationship between both of these variables at bone tissue. However, in this initial study they are taken as independent variables. The properties used in this initial model are constant for the whole femur geometry, and taken from the cortical bone values of [8].

- Stiffness (Young's Modulus) (E): 16 700 MPa

-Density (ρ): 1996 Kgm⁻³

II) Geometrical properties. The geometrical properties are based on the femur size. As the model can be scaled, the main variable to be established is going to be the total femur length, from the top of the proximal head to the to of the distal one

-Length assumed: 0.5 m

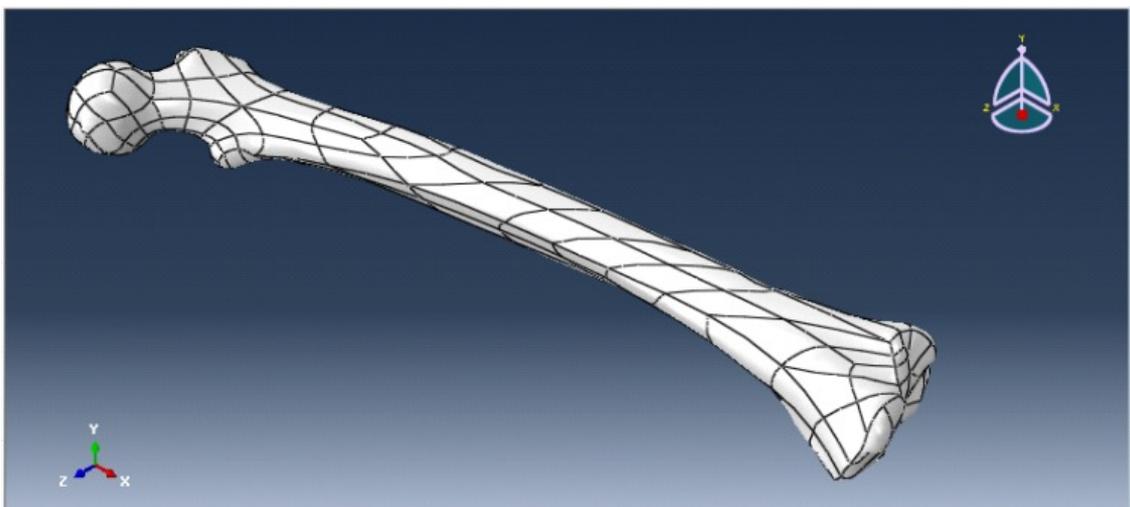


Figure 3.3.1: Femur geometry at initial model

3.3.2 Element size sensitivity analysis

By means of the initial properties stated above, the natural frequency results are going to be calculated for different meshing element sizes, in order to decide the desirable one. At **Table 3.3.1** there can be seen the different meshing parameters and results obtained with decreasing *global seed sizes* (remind that femur resolution for the different element size could be observed at **Figure 3.2.6** from the previous *section 3.2*)

| FREQUENCY MODES (Hz) | GLOBAL SEED SIZE | | | | | | | | | |
|----------------------|------------------|--------|--------|--------|--------|--------|--------|--------|---------|---------|
| | 0,5 | 0,25 | 0,1 | 0,075 | 0,05 | 0,025 | 0,01 | 0,0075 | 0,005 | |
| Mode 1F | 258,0 | 258,0 | 259,7 | 254,0 | 257,5 | 257,8 | 257,6 | 257,5 | 257,5 | |
| Mode 1S | 278,6 | 278,6 | 278,6 | 278,2 | 277,5 | 277,7 | 277,5 | 277,5 | 277,5 | |
| Mode T | 699,0 | 699,0 | 699,4 | 691,5 | 692,2 | 690,2 | 688,2 | 687,6 | 687,5 | |
| Mode 2F | 825,0 | 825,0 | 824,8 | 818,3 | 817,2 | 814,7 | 813,5 | 813,3 | 813,9 | |
| Mode 2S | 890,4 | 890,4 | 889,5 | 884,8 | 883,8 | 880,1 | 878,6 | 878,3 | 878,2 | |
| ELEMENTS | Total | 1019 | 1020 | 2424 | 1744 | 3547 | 4331 | 11766 | 19921 | 38833 |
| | Distorted | 22 | 22 | 75 | 73 | 93 | 57 | 24 | 29 | 30 |
| | % distorted | 2,2% | 2,2% | 3,1% | 4,2% | 2,6% | 1,3% | 0,2% | 0,1% | 0,1% |
| NODES | | 1884 | 1885 | 4161 | 3125 | 6023 | 7346 | 18764 | 31008 | 58838 |
| CONSUMED TIME | | 0' 43" | 0' 31" | 0' 42" | 0' 33" | 0' 55" | 0' 57" | 3' 03" | 13' 19" | 28' 55" |

Table 3.3.1: Meshing results evolution with decreasing global seed size

As it has been explained before, the two requirements in order to decide the desirable element size are mesh quality and calculation time.

Regarding mesh quality, the desired element size will be the one at which the results start to clearly converge, so that a further improvement on mesh quality will not suppose a significant increase the results accuracy. To show more clearly when does it happens, the evolution of the natural frequency result at each mode through the decreasing *global seed sizes* is represented at **Figure 3.3.2**.

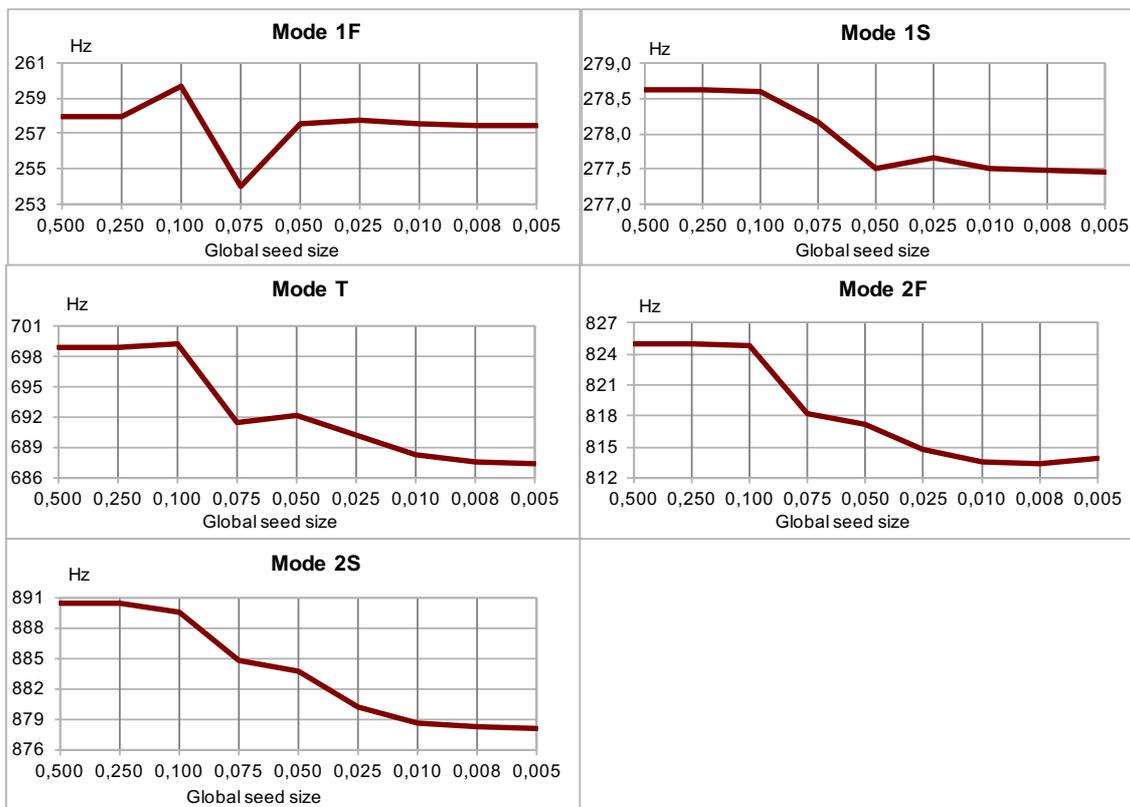


Figure 3.3.2: Frequency value evolution at each mode with global seed size

Moreover, in order to take into account time requirement, an additional graph was performed to relate distorted element percentage with calculation time (**Figure 3.3.3**).

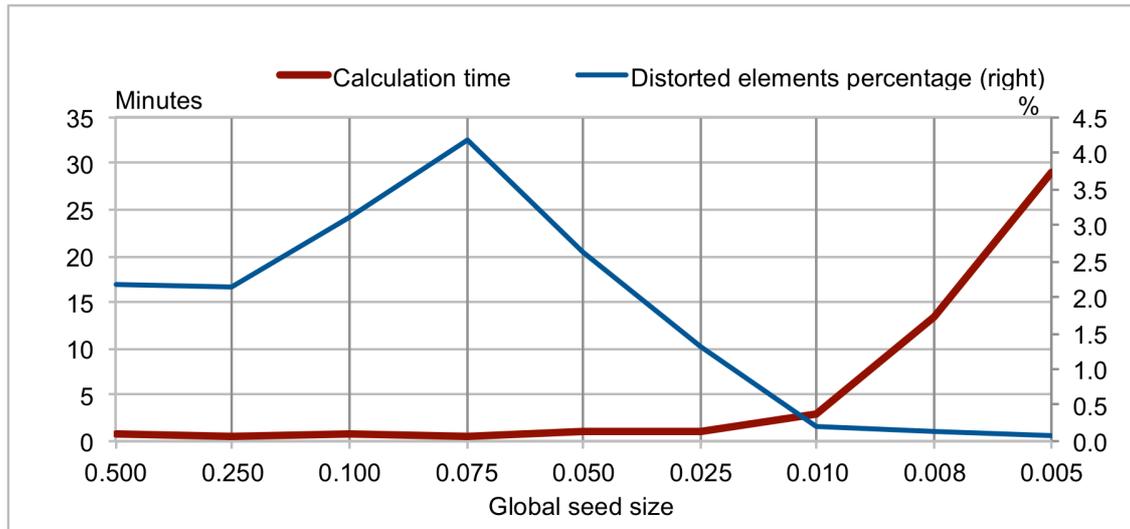


Figure 3.3.3: Calculation time and distorted elements variation with seed size

Regarding meshing quality requirements, the desired meshing *global seed size* chosen has been 0.01. As it can be seen, the frequency values at every mode clearly converge when achieving this point (**Figure 3.3.2**). Moreover, the distorted elements percentage over the total elements number at **Figure 3.3.3** is pretty low (around 0.2 %).

The last important consideration to decide the desired element size was the consumed time, because the different models and variations to be performed require a relative low calculation time. As it can be also seen in **Figure 3.3.3**, when decreasing *global seed size* to lower values than 0.01, the consumed time reaches values too high to be desired.

Therefore, 0.01 *global seed size* is the most effective element size to divide femur geometry for natural frequencies calculation, and it will be maintained during the following models.

3.3.3 Influence of each variable

As stated before, there are three main variables introduced in the Abaqus model that will affect natural frequencies values at their different modes. In order to show their influence on natural frequency, properties values are going to be modified to analyse their results variation from reference values. The initial model results calculated before (see **Table 3.3.2**) are the ones that will be used as reference values.

| | | |
|------------------------------|----------------------------|-------|
| Properties | Young's Modulus (MPa) | 16700 |
| | Density (ρ) (Kgm-3) | 1996 |
| | Length (L) (m.) | 0,5 |
| Natural Frequency Value (Hz) | Mode 1F | 257,6 |
| | Mode 1S | 277,5 |
| | Mode T | 688,2 |
| | Mode 2F | 813,5 |
| | Mode 2S | 878,6 |

Table 3.3.2: Frequency values with initial model properties

Therefore, a static comparative method is going to be used, to show the different variables influence in an effective manner. There will be three variation cases, each of them referred to a specific variable. That is, at each case two of these variables will remain constant, while one of them is going to be slightly modified. The numerical value taken into account to individually increase and decrease each variable was 10%. Finally, the different variations on the results will be studied and analysed.

3.3.3.1 Results

At the following tables (3.4, 3.5 and 3.6.) the three study cases can be appreciated for Young's Modulus, density and length variation. Each table is referred to the natural frequency results change when one variable value was modified (increased and decreased by 10%). The initial model results (reference values) are also included in each table, as well as the percentage change values of the obtained results. Therefore, the results variations from reference values at each table show the influence that each variable has on frequency.

| | Initial Model | +10% Young's modulus | | -10% Young's modulus | | |
|---------------------------|------------------|----------------------|------------------|----------------------|------------------|--------|
| | Reference values | Levels | % var. from Ref. | Levels | % var. from Ref. | |
| Young's Modulus (E)(MPa) | 16700 | 18370 | 10% | 15030 | -10% | |
| Density (ρ)(Kgm-3) | 1996 | 1996 | - | 1996 | - | |
| Length (L)(m) | 0,5 | 0,5 | - | 0,5 | - | |
| Natural frequency (Hz) | Mode 1F | 257,6 | 270,2 | 4,88% | 244,4 | -5,13% |
| | Mode 1S | 277,5 | 291,1 | 4,88% | 263,3 | -5,13% |
| | Mode T | 688,2 | 721,8 | 4,88% | 652,9 | -5,13% |
| | Mode 2F | 813,5 | 853,2 | 4,88% | 771,8 | -5,13% |
| | Mode 2S | 878,6 | 921,5 | 4,88% | 833,5 | -5,13% |

Table 3.3.3: Natural frequency results change with Young's Modulus variation

| | Initial Model | +10% Density | | -10% Density | | |
|---------------------------|------------------|--------------|------------------|--------------|------------------|-------|
| | Reference values | Levels | % var. from Ref. | Levels | % var. from Ref. | |
| Young's Modulus (E)(MPa) | 16700 | 16700 | - | 16700 | - | |
| Density (ρ)(Kgm-3) | 1996 | 2195,6 | 10% | 1796,4 | -10% | |
| Length (L)(m) | 0,5 | 0,5 | - | 0,5 | - | |
| Natural frequency (Hz) | Mode 1F | 257,6 | 245,6 | -4,65% | 271,5 | 5,41% |
| | Mode 1S | 277,5 | 264,6 | -4,65% | 292,5 | 5,41% |
| | Mode T | 688,2 | 656,2 | -4,65% | 725,4 | 5,41% |
| | Mode 2F | 813,5 | 775,7 | -4,65% | 857,5 | 5,41% |
| | Mode 2S | 878,6 | 837,7 | -4,65% | 926,1 | 5,41% |

Table 3.3.4: Natural frequency results change with density variation

| | Initial Model | +10% Length | | -10% Length | | |
|---------------------------|------------------|-------------|------------------|-------------|------------------|--------|
| | Reference values | Levels | % var. from Ref. | Levels | % var. from Ref. | |
| Young's Modulus (E)(MPa) | 16700 | 16700 | - | 16700 | - | |
| Density (ρ)(Kgm-3) | 1996 | 1996 | - | 1996 | - | |
| Length (L)(m) | 0,5 | 0,55 | 10% | 0,45 | -10% | |
| Natural frequency (Hz) | Mode 1F | 257,6 | 234,2 | -9,09% | 286,1 | 11,08% |
| | Mode 1S | 277,5 | 252,3 | -9,09% | 308,2 | 11,05% |
| | Mode T | 688,2 | 625,6 | -9,09% | 764,4 | 11,08% |
| | Mode 2F | 813,5 | 739,6 | -9,09% | 903,7 | 11,08% |
| | Mode 2S | 878,6 | 798,7 | -9,09% | 976,7 | 11,16% |

Table 3.3.5: Natural frequency results change with length variation

3.3.3.2 Analysis

From the obtained results, each variable influence on the frequency results need to be clarified. Paying attention to the percentage variation number of the results at both variable increase and decrease cases, the following relationships between variables and frequency can be taken:

Young's Modulus has a positive relationship with natural frequency values. That is, frequency variation when Young's Modulus increases is positive (**Table 3.3.4**). Density and length have a negative relationship with natural frequency. In this case, it can be appreciated at **Tables 3.3.5** and **3.3.6** that the result variation sign is negative for their variables increase case, and vice-versa.

As it can be seen, the numerical value of the influence of both stiffness and density on frequency values is very similar, but their sign is the opposite. These two

considerations will be especially taken into account at *Final model study* section. Moreover, length is the variable that has a greater impact on natural frequency results.

The last consideration is that results variation happens to be constant at every frequency mode. Therefore, each variable change is completely proportional to the frequency variation at every mode. Notice that in the next *Real Femur* section, when Euler-Bernoulli ratios are explained, this last consideration will be repeated.

One the most basic concepts have been clarified, different levels with an increasing complexity level will be performed.

3.4 REAL FEA MODELS

3.4.1 Introduction

Once the influence on the results of the different variables has been studied with the simple initial model, in this section the real heterogeneity of the femur it is going to be considered. To approximate the model to the real femur characteristics, the different approaches that have been explained at 2.3. *Model consideration* section will be taken into account.

As explained before, the combination of the two main bone types (cortical and cancellous) establishes an irregular density and stiffness distribution along the bone geometry. That is, the different bone regions have a variable proportion of these two materials, and therefore their mechanical properties will be different. The goal of this new set of models is to reach a reasonably accurate simulation of the main bone regions to obtain a gradual improvement on femur properties characterization. The process followed has been the following:

a) Three-regions model

This first approach considered the three initial materials distinguished in [8]: proximal cancellous bone, cortical bone and distal cancellous.

b) Five-regions (Initial) model

As two new regions were added for this new approach, their properties had to be established. The heads and central region maintained the previous properties, while the value from [12] for proximal neck cancellous bone was introduced. The rest of neck properties derived from this value extrapolation compared with the head properties and the new neck stiffness. The procedure obtain the results will be further explained.

c) Five-regions (revised) models

In this new improvement, the relationships between density and Young's Modulus from [13] were introduced. As it is going to be explained, two different models were obtained, *stiffness model* and *density model*, where their properties were re-calculated maintaining, respectively, Young's Modulus and density values.

3.4.2 Natural frequency considerations

As it has been explained, different partitions assigned to their correspondent materials will be created on the FEA model, trying to simulate the real bone regions. Moreover, the correlation between natural frequency and femur properties characterization is going to be used to improve the obtained models. Therefore, each model natural frequency results have to be analysed. Two main considerations were used to critically analyse and compare the successive results.

a) Reference values for femur natural frequencies

The first one will be the comparison with the literature reference [8] values, as specified at 2.3. *Model Considerations* section. It has to be taken into account that the results from the article have been obtained following two different methods: *experimental* and *numerical*. Therefore, the results obtained by the following models in this project will be compared to each of them, in order to get a wide and critical analysis of the results.

It has to be remarked that the *numerical* case values were obtained following a similar process than in this study. That is, introducing mechanical properties in a FEA geometrical model as an input to obtain the simulated frequency values. At the *experimental* one the values obtained were directly measured from a fresh human femur. It can be noticed that there exists a difference between both methods results (see **Table 3.4.1** below). As it was explained in the article, this difference relays on the possible imperfections and errors derived from both methods.

| Natural frequency values | | Experimental data [8] | Numerical data [8] |
|------------------------------|---------|-----------------------|--------------------|
| Natural Frequency Value (Hz) | Mode 1F | 301,6 | 287,8 |
| | Mode 1S | 353,3 | 364,2 |
| | Mode T | 612,0 | - |
| | Mode 2F | 886,6 | 819,0 |
| | Mode 2S | 931,6 | 931,9 |

Table 3.4.1: Literature reference natural frequency values [8]

The next chart (**Figure 3.4.1**) shows the comparison between the frequencies values of the first initial model analysed in the last section and the numerical and experimental values.

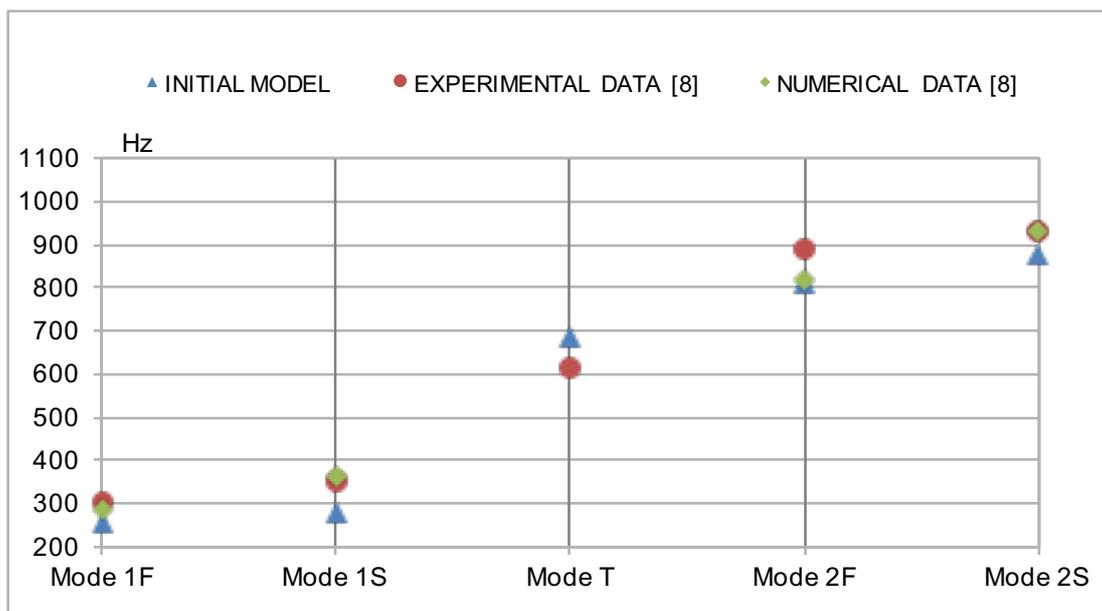


Figure 3.4.1: Initial model frequency results compared to literature values [8]

b) Euler-Bernoulli ratios

This second consideration is referred to the correlations between frequency bending modes values, already introduced at *Model considerations* section. The consideration is based on the Euler Bernoulli Theory. According to this law, there is a correlation (ratio) between 2nd bending over 1st bending modes at each bending plane which value has to be constant. For “free-free” conditions, this value that both ratio should maintain is 2,75 [8].

As Femur geometry can be considered to have a beam-like shape, its natural frequency results should follow Euler Bernoulli Theory [8]. Therefore, correlations between bending vibrating modes have been calculated for the following performed model. As their value has to approach to 2,75 in order to validate the Theory, Euler-Bernoulli ratios have been an additional way to analyse model results accuracy. Before starting the new models performance, those ratios have been calculated for both literature values (*experimental* and *numerical*) [8], as well as for the initial model. Sagittal and frontal ratios definition, values and difference from its theoretical value can be appreciated at **Table 3.4.2**.

NOTE 1: As it was said before, it is true that Euler-Bernoulli ratios only compare the values of the bending modes. The main reason why this study analysis focuses on bending modes and not in the torsion one it is due to the importance of each of them. It is obvious that the relevance of the bending efforts at which femur is subjected is much higher than the torsion ones. Therefore, the accuracy of the performed models has been focused on the four bending modes, especially on the first ones.

NOTE 2: At the initial model study, it could be observed that the frequency values variation was constant at every mode when femur length was varied. That means that Euler-Bernoulli ratios will not change with length variation. Therefore, length value will be maintained with the initial value of 0.5 m through all the following models.

| Ratio name | Definition | Theor. values | Initial model | | Exp. data [8] | | Num. data [8] | |
|------------------------------|---|---------------|---------------|-------------------|---------------|-------------------|---------------|-------------------|
| | | | Ratio | Diff. from Theor. | Ratio | Diff. from Theor. | Ratio | Diff. from Theor. |
| Frontal ratio | 2nd / 1st bending ratio at frontal plane (Mode 2F / Mode 1F) | 2,75 | 3,16 | 0,408 | 2,94 | 0,190 | 2,85 | 0,096 |
| Sagittal ratio | 2nd / 1st bending ratio at sagittal plane (Mode 2S / Mode 1S) | 2,75 | 3,17 | 0,416 | 2,64 | -0,113 | 2,56 | -0,191 |
| FREQUENCY VALUES (Hz) | | | | | | | | |
| Mode 1F | | | 257,6 | | 301,6 | | 287,8 | |
| Mode 1S | | | 277,5 | | 353,3 | | 364,2 | |
| Mode T | | | 688,2 | | 612,0 | | - | |
| Mode 2F | | | 813,5 | | 886,6 | | 819,0 | |
| Mode 2S | | | 878,6 | | 931,6 | | 931,9 | |

Table 3.4.2: Euler Bernoulli ratios definition and results for initial model and literature values [8], with their differences from its theoretical value.

3.4.3 Three regions model

This initial geometric approach will take into account each of the three main regions: proximal head, distal head and central region. As it has been explained, those regions have been performed on Abaqus software by means of cell partitioning. Therefore, from the initial bone geometry, two planes (defined by three geometrical points on the femur surface) have been used to divide both bone extremities from the central body. The partitions performed can be easily seen in Figure 8.

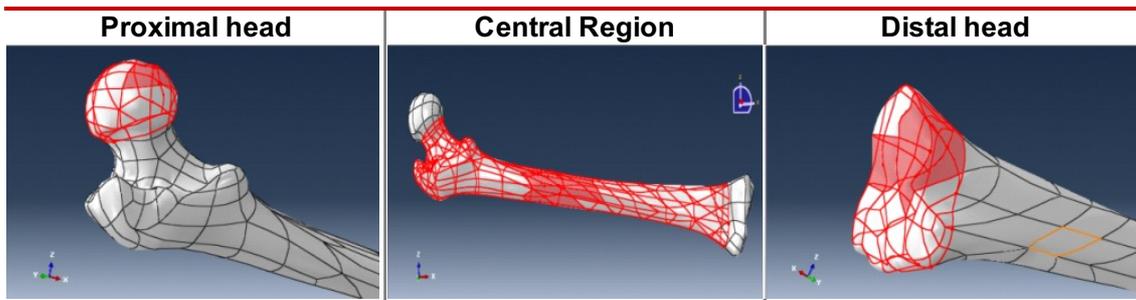


Figure 3.4.2: Different partitions of the Three Regions Model

To assign each region material, femur internal structure shown that the predominant bone tissues at each of those regions are cortical bone for the central body region and cancellous bone for both heads. The material assignation to each of the region will take into account the fact that cancellous bone properties are different at proximal and distal regions. The different materials taken into account will be, respectively: proximal cancellous bone, distal cancellous bone and cortical bone. That means that the model will have three different materials with their correspondent mechanical properties. Therefore, there will be three different regions with their different mechanical properties. The material and mechanical properties values considered at this point are the ones given by [8].

Table 3.4.3 summarize the properties and results of this model, and **Figure 3.4.3** plots their frequency results compared to literature references. Compared to initial model properties (**Figure 3.4.1**, **Table 3.4.2**), it can be shown that frequency values improve at the first two bending modes. However, torsion mode value at this Three-regions model suffers a high deviation compared to reference values. When comparing Euler Bernoulli ratios (**Tables 3.4.3** and **3.4.2**), a significant improvement can be appreciated at this new Three-regions model compared to the initial one.

| Variables | | Region name(Type of bone) | | |
|----------------------------|-------------------|---------------------------|-----------------------|------------------------|
| | | Proximal (Cancellous) | Central (Cortical) | Distal (Cancellous) |
| Young's Modulus (E) (MPa) | | 609 | 16700 | 1112 |
| Density (ρ) (Kgm-3) | | 303 | 1996 | 540 |
| Length (L) (m) | | 0,5 | | |
| Natural frequency (Hz) | Mode 1F | 312,8 | | |
| | Mode 1S | 324,0 | | |
| | Mode T | 906,6 | | |
| | Mode 2F | 961,9 | | |
| | Mode 2S | 1000,7 | | |
| Frontal ratio | Value | 3,08 | | |
| | Diff. from theor. | 0,325 | | |
| Sagittal ratio | Value | 3,09 | | |
| | Diff. from theor. | 0,339 | | |

Table 3.4.3: Three Regions Model properties and frequency results

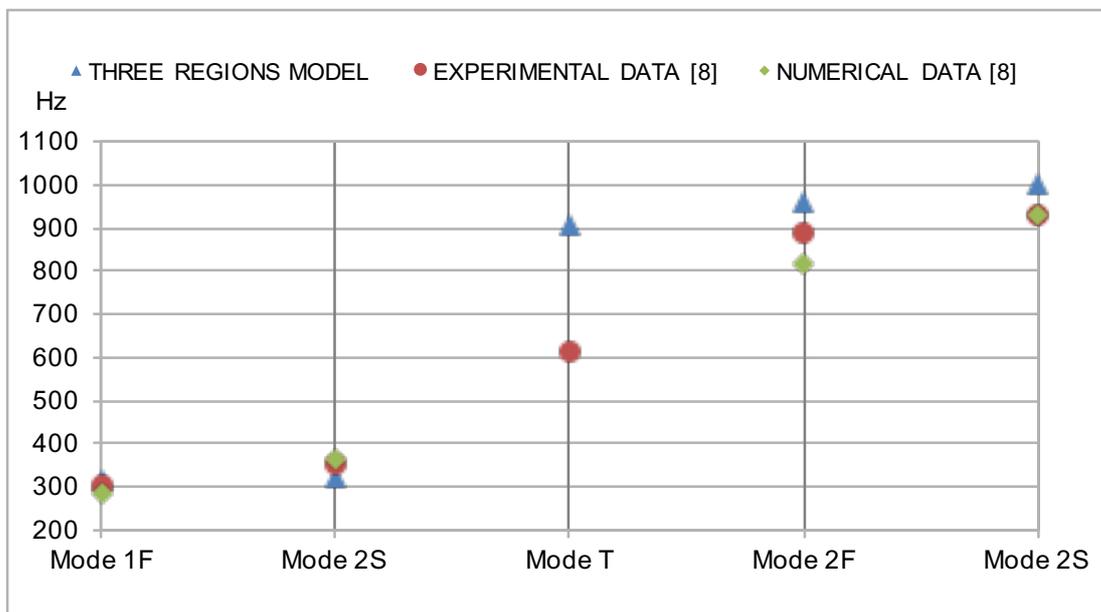


Figure 3.4.3: Tree Regions Model frequency results compared to literature values [8]

3.4.4 Second approach: Five regions model

In this new model two other regions were introduced. Those new regions will be called necks, connecting both proximal and distal heads with the central regions. This consideration came up considering the geometrical approach stated by [12]. By means of this article, the properties of the cortical tissue at the proximal neck have a very different value than the one at the central region. The addition of the distal neck considered in this model was made in order to create a transition zone between distal head and central region, which properties are highly different.

Therefore, two other regions (necks) with their own properties were added between both “heads” and the central region. In order to achieve this, both “head” region geometries were maintained, but the central region will be partitioned at both sides. The properties of both “heads” and central region will not change from the previous case, and two other materials will be added and introduced in each of the new “necks”. The different partitions performed can be appreciated at **Figure 3.4.4**

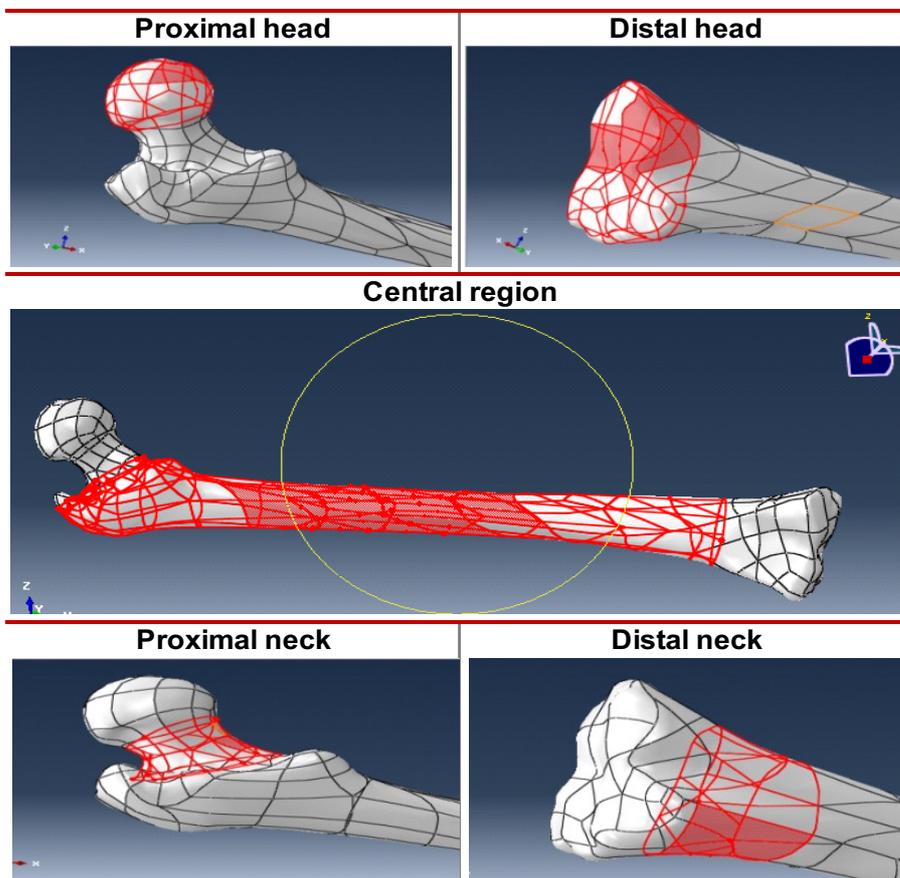


Figure 3.4.4: Different partitions of the Five Regions Model

3.4.4.1 Initial properties model

For simplicity, in this new model the properties at the proximal neck region will be considered to be cancellous bone, as this kind of tissue proportion is considerably bigger compared to cortical bone. Therefore, the Young's Modulus value for the cancellous bone at the proximal neck region from [12] was adopted to perform this model (1000 MPa). However, the density values for cancellous bone provided there do

not seem to be very realistic. Especially for cancellous bone, its density values were too high compared to other articles [1], [4], [8], [16], [21] and therefore its value was not considered as a reliable reference. Therefore, the neck density value will be an extrapolation from the known density value of the proximal head. If proximal head Young's Modulus is 609 MPa and its density is 303 Kg^m-³, at the neck region its Young's Modulus of 1000 MPa gives a density value of 497.54 Kg^m-³.

In the case of the distal region as there are no known values about it in the article. The properties for this region will be obtained increasing those from the distal head the same amount as the proximal properties have increased from head to neck. That is, if proximal neck stiffness has increased from 609 to 1000 MPa, distal neck Young's Modulus value increases from 1112 to 1824 MPa. In the case density, the density value will be obtained by an analogous way, being its value 886 Kg^m-³. The numerical value for the model properties at each regions and its frequency results can be observed at **Table 3.4.4**. Moreover, at **Figure 3.4.5** natural frequency results are plotted and compared to literature references.

| Variables | | Region name (Type of bone) | | | | |
|---|--------------------------|----------------------------|----------------------------|-----------------------|-----------------------------|-----------------------------|
| | | Prox. head (Cancellous) | Prox. neck (Cancellous) | Central (Cortical) | Distal neck (Cancellous) | Distal head (Cancellous) |
| Young's Modulus (E) (MPa) | | 609 | 1000 | 16700 | 1824 | 1112 |
| Density (ρ) (Kg ^m - ³) | | 303 | 497 | 1996 | 886 | 540 |
| Length (L) (m) | | 0,5 | | | | |
| Natural frequency (Hz) | Mode 1F | 338,1 | | | | |
| | Mode 1S | 354,7 | | | | |
| | Mode T | 862,5 | | | | |
| | Mode 2F | 959,8 | | | | |
| | Mode 2S | 1028,2 | | | | |
| Frontal ratio | Value | 2,84 | | | | |
| | <i>Diff. from theor.</i> | 0,089 | | | | |
| Sagittal ratio | Value | 2,90 | | | | |
| | <i>Diff. from theor.</i> | 0,149 | | | | |

Table 3.4.4 Five Regions Initial Model properties and frequency results

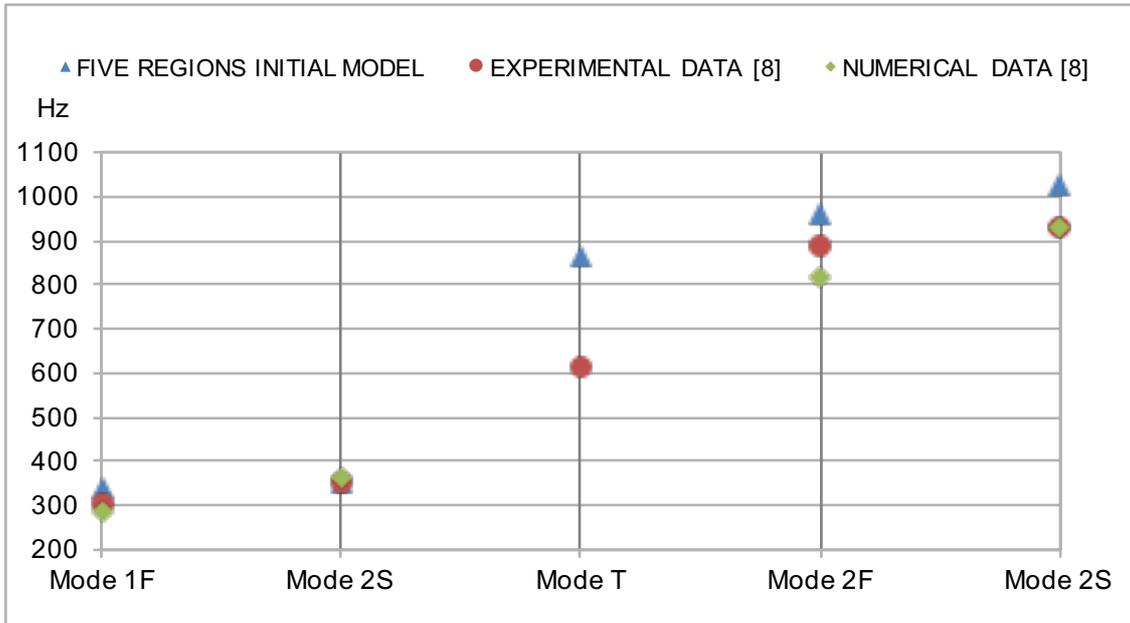


Figure 3.4.5: Five Regions Initial Model frequency results compared to literature values [8]

Notice the improvement obtained on Euler Bernoulli ratios, especially at the frontal bending planes (compare ratios *difference from theoretical value* at **Tables 3.4.3** and **3.4.4**), compared to the previous *Initial properties model* and *three regions model*. Also, almost in every mode the value has increased from the previous three single regions model, especially at first sagittal and frontal bending. In the case of the sagittal Euler Bernoulli ratio, an improvement can also be appreciated.

3.4.4.2 Revised properties models

At this point of the model improvement, the different relationships from **[13]** between density and Young's Modulus for cortical and cancellous bone were introduced. Therefore, the previous values have been re-calculated so that its density and Young's Modulus will obey their relationships. The two different equations of Young's Modulus and density can be seen in **Eq. 1 and 2** below. In order to get a complete analysis, two different models have been obtained maintaining, respectively, Young's Modulus and density values, and re-calculating the other property.

$$E = 2065\rho^{3.09} \quad (\text{cortical bone}) \quad (1)$$

$$E = 1904\rho^{1.64} \quad (\text{cancellous bone}) \quad (2)$$

Note: Young's Modulus (E) and density (ρ) units should be introduced introduced, respectively, in MPa and g/cm^3 .

I) Re-calculated properties fixing Young's Modulus: *THE STIFFNESS MODEL*

In the case of the proximal head region, both reference articles give a very similar result for its stiffness. When solving the relationship equation for cancellous bone, the previously established Young's Modulus value of 609 MPa gives a density value of 500 Kgm⁻³. As the value given by the first source [8] was 303, it does not meet the cancellous bone relationship equation. Thus, it has to be replaced by the new value obtained. In an analogous way, if Young's Modulus at the distal head region is assumed to be 1112 MPa, the density given by the equation is 720 Kgm⁻³.

At the neck regions, the same procedure has to be followed. If we consider the stiffness as the valid value, like it has been done before, densities need to be re-calculated. At the proximal neck region, the cancellous bone equation for 1000 MPa gives a density value of 675 Kgm⁻³. At the distal region, density value is 974 Kgm⁻³ when the assumed Young's Modulus is 1824 MPa.

Finally, at the central region the bone tissue considered is the cortical one. Therefore, the equation that has to be used is the first one. Following the same criterion as in the first case, the established value will be 16700 MPa. Once this value is fixed, the cortical bone relationship gives a density value of 1967 Kgm⁻³. It can be seen that this value is really similar than the initially density value provided by the first source [8] (1996 Kgm⁻³), which contributes to remark its accuracy. Anyhow, this new value obtained has been taken into account further.

The different properties values for each region as well as the obtained results can be observed at **Table 3.4.5**. At **Figure 3.4.6** natural frequency results are graphically represented and compared to literature references values.

| Variables | Region name(Type of bone) | | | | |
|---------------------------|----------------------------|----------------------------|-----------------------|-----------------------------|-----------------------------|
| | Prox. head (Cancellous) | Prox. neck (Cancellous) | Central (Cortical) | Distal neck (Cancellous) | Distal head (Cancellous) |
| Young's Modulus (E)(MPa) | 609 | 1000 | 16700 | 1824 | 1112 |
| Density (ρ) (Kgm-3) | 500 | 675 | 1967 | 974 | 720 |
| Length (L) (m) | 0,5 | | | | |
| Natural frequency (Hz) | Mode 1F | 324,6 | | | |
| | Mode 1S | 333,9 | | | |
| | Mode T | 764,4 | | | |
| | Mode 2F | 901,9 | | | |
| | Mode 2S | 970,0 | | | |
| Frontal ratio | Value | 2,78 | | | |
| | Diff. from theor. | 0,029 | | | |
| Sagittal ratio | Value | 2,91 | | | |
| | Diff. from theor. | 0,155 | | | |

Table 3.4.5: Five Regions Stiffness Model properties with natural frequency and Euler-Bernoulli ratios results.

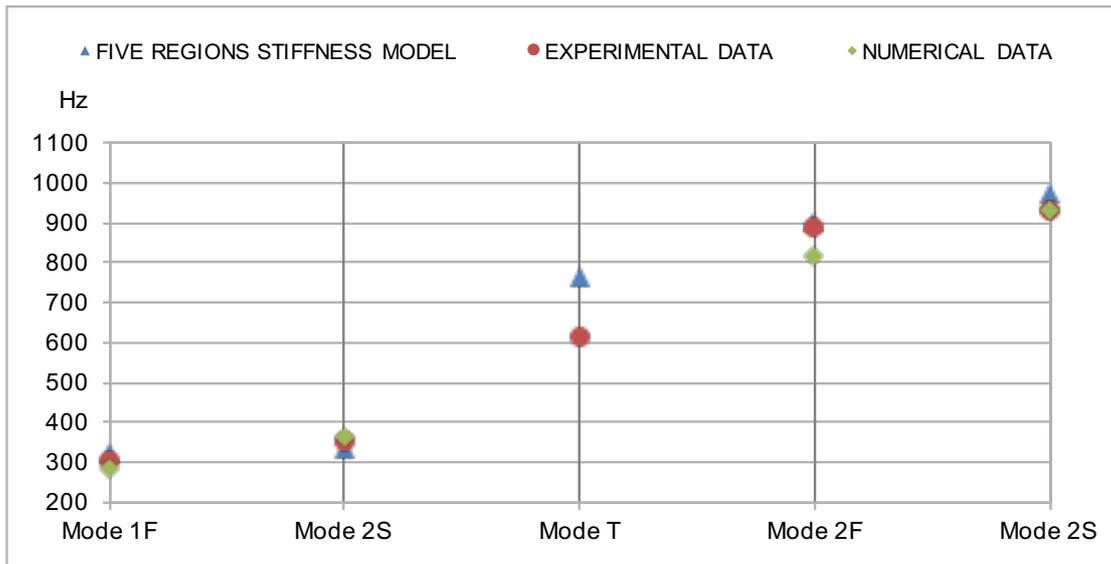


Figure 3.4.6: Five Regions Stiffness Model frequency results compared to literature values [8]

Frequency values

At the previous graph (Figure 3.4.6) it can be seen that the obtained values are really close to ones of the experimental literature measurements. In the case of the numerical model results, even though the results are slightly different, they also lay on the same value range. The only big difference between the final-stiffness model and literature results can be found in the torsional mode (Mode T). However, the torsion mode is the one that has a lower importance on femur-applied stresses.

Euler-Bernoulli ratios

It can be also appreciated the improvement achieved on these ratios from the previous models, especially at the frontal bending modes ratio (Check Table 3.4.5 results). This improvement achieved that this model was validated as a relatively accurate one, and that is why it has been further used in the Model Improvement section.

II) Re-calculated properties fixing density: *THE DENSITY MODEL*

In this case, while both neck regions properties will be remained constant, both heads and central region properties will change. The previous case established for those three regions stiffness as the fixed value, and then re-calculated density according to the equations. In this new case, density values are the ones to be fixed, and the values for stiffness (Young's Modulus) are obtained using the relationship equations used before. However, only at proximal and distal heads, as well as central one, their properties were obtained. The main reason of not changing both neck properties is because its density value was assumed not to be accurate as its stiffness was in the article. Therefore, the value of these regions properties will maintain the criteria following at the previous stiffness-model, where they also comply the cancellous bone relationships.

At the proximal head region, its fixed density value of 303 Kg m^{-3} gives a Young's Modulus of 269 MPa, with a lower value than the previous case. The density value of the central region, 1996 Kg m^{-3} , increases its Young's Modulus up to 17,475 MPa. Finally, at the distal head the obtained and reduced value for stiffness has a Young's Modulus value of 693 MPa.

In the following **Table 3.4.6**, the different properties at each region are specified, as well as the natural frequency results and their Euler-Bernoulli ratios, which have been obtained using those properties. Moreover, **Figure 3.4.7** compares the obtained natural frequency results with the literature reference [8].

| Variables | Region name (Type of bone) | | | | |
|--|----------------------------|----------------------------|-----------------------|-----------------------------|-----------------------------|
| | Prox. head (Cancellous) | Prox. neck (Cancellous) | Central (Cortical) | Distal neck (Cancellous) | Distal head (Cancellous) |
| Young's Modulus (E) (MPa) | 269 | 999 | 17475 | 1824 | 693 |
| Density (ρ) (Kg m ⁻³) | 303 | 675 | 1996 | 974 | 540 |
| Length (L) (m) | 0,5 | | | | |
| Natural frequency (Hz) | Mode 1F | 342,3 | | | |
| | Mode 1S | 357,8 | | | |
| | Mode T | 858,1 | | | |
| | Mode 2F | 968,4 | | | |
| | Mode 2S | 1031,8 | | | |
| Frontal ratio | Value | 2,83 | | | |
| | Diff. from theor. | 0,079 | | | |
| Sagittal ratio | Value | 2,88 | | | |
| | Diff. from theor. | 0,134 | | | |

Table 3.4.6: Five regions Density Model properties with natural frequency and Euler-Bernoulli ratios results.

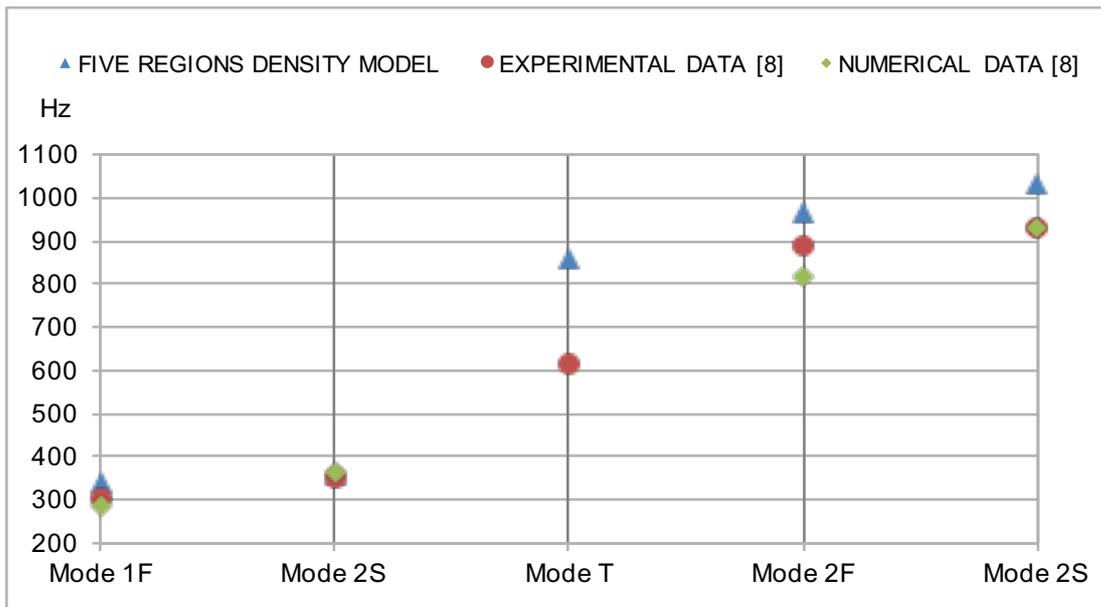


Figure 3.4.7: Five Regions Density Model frequency results compared to literature values [8]

Frequency values

It can be seen that the values obtained by this model are quite higher than the ones taken from literature. Compared to the previously obtained stiffness-model, in this case the frequency values are quite higher at the last three modes. As well as it happened before, torsion mode values are the ones which have less similarities between density-model and literature references. However, in order to validate this model accuracy, also Euler-Bernoulli ratios have to be analysed.

Euler-Bernoulli ratios

As it was achieved before, in this model there has been an improvement on Euler-Bernoulli ratios. In this case both ratios happened to be lower, improving the results obtained at the initial five regions model. That is the main reason that has made this density-model considered as a valid model, and therefore it will also be used in the next Model Improvement section.

3.4.5 Real FEA Models: Summary and results evolution

In this last part of the real models section, the result evolution obtained through the different models is generally compared. On the one hand, natural frequency values are going to be plotted at **Figure 3.4.8**, in order to obtain discuss the result and obtain some conclusions. On the other hand, the Euler-Bernoulli ratios evolution is plotted at **Figure 3.4.9**, being the main improvement tool that has been used.

Frequency values

As it can be seen at **Figure 3.4.8**, frequency values happen to be very similar at every model performed. While the initial model has the lowest values, the *Three-regions* one is the one with highest frequencies. The different Five-regions models are in between those values, being *Five-initial model* results very close to the ones obtained with the *density-model* ones. In the case of the *stiffness-model*, their results are considerably lower. As it has been said before, the absolute value of natural frequency is not enough to show the reliability of the obtained modes. Moreover, the main improvement tool used was Euler-Bernoulli ratios, which does not compare the absolute frequency value but the relationships between its different modes.

Euler-Bernoulli ratios

The evolution of **Figure 3.4.9** results shows that during the successive models performed in this section, the ratios between their second and first bending modes have been gradually improved. As it was initially said, this project used the assumption that femur bone has to follow Euler-Bernoulli beam theory, due to its geometry which can be considered beam-like. Remind that, according to this theory, the ratio between its second and first bending modes happens to be constant and equal to 2.75. Therefore, one of the parameters to measure results accuracy is the proximity of those ratios to value to 2,75, and hence their proximity to an Euler-Bernoulli beam shape. As it can be seen, this difference between obtained and theoretical values has been reduced in each model.

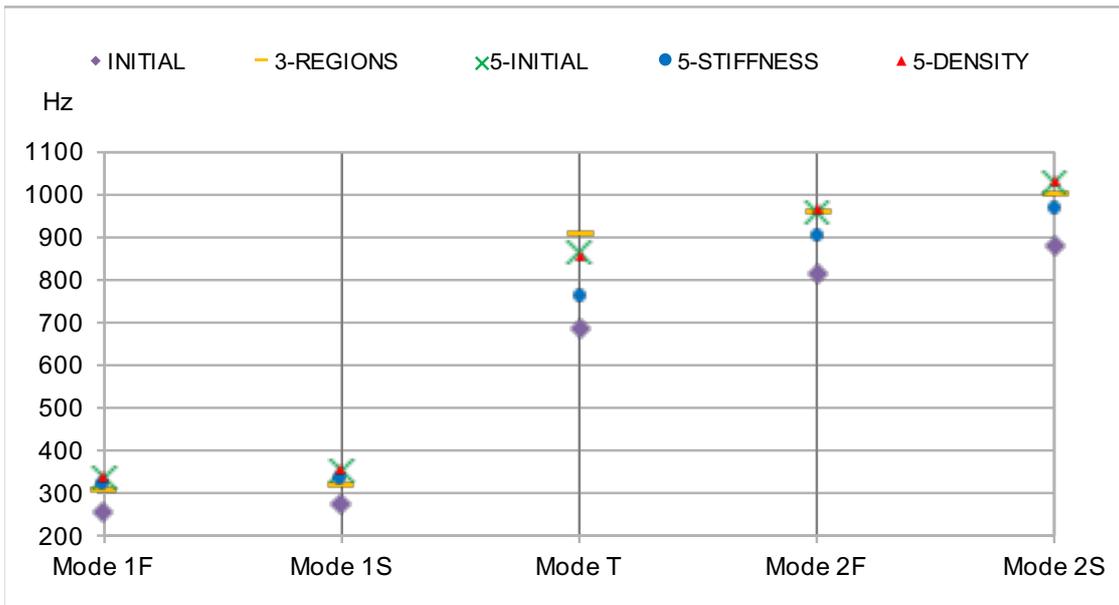


Figure 3.4.8: Natural Frequency values for the different performed models

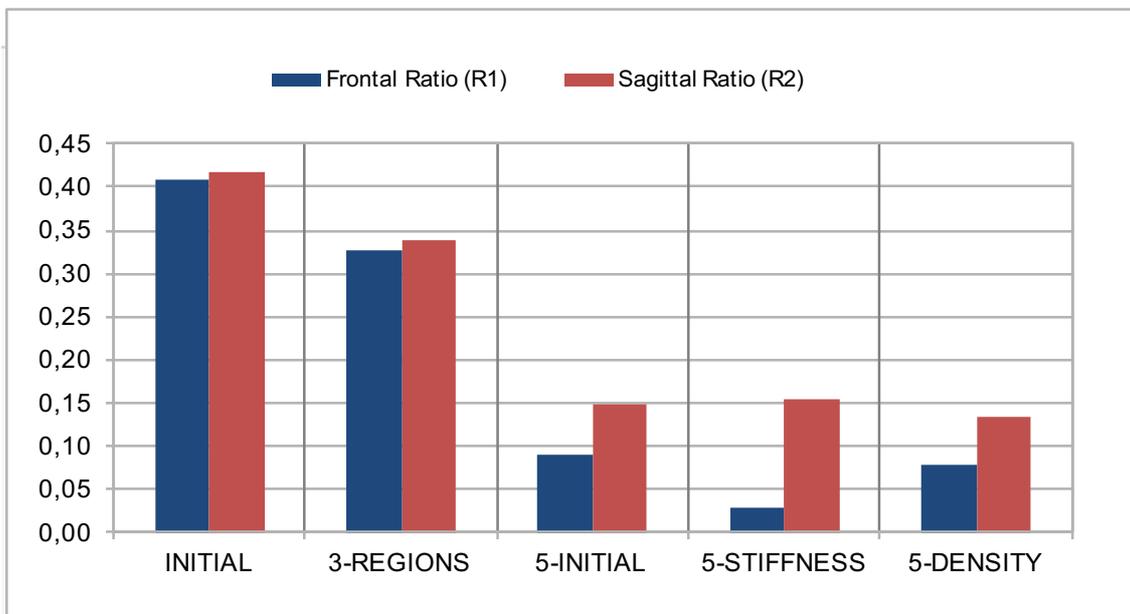


Figure 3.4.9: Euler Bernoulli ratios difference from its theoretical value (2.75) at performed models

3.5 FINAL MODEL STUDY

3.5.1 Goal

The main propose of this section is to understand the influence of each femur region on the natural frequency values. The two last models obtained in the previous section will be analysed, as they are the most complex ones. Therefore, all the different considered assumptions used to build those models will be critically analysed. This information can be very helpful in order improve the present model results, and also can allow to develop future models with higher accuracy.

3.5.2 Procedure

The followed procedure to show the regions influence on the frequency values has been the variation of each region properties from the initial reference model (*stiffness-model* and *density-model*). There were performed five different cases that showed the influence of each geometrical region: proximal head, proximal neck, central region, distal neck and distal head. Therefore, in each of the performed models, only one region modified its properties, being the rest of the regions maintaining the same properties as the initial reference model.

Once the different region cases properties were defined, their natural frequency results were calculated. As it is explained further, the results obtained through the five new models were compared to those of the reference model. By means of this analysis, the influence of each region on frequency variation could be explained, considering different *region influence factors* introduced further. In order to perform a complete analysis, by means of different graphs some *analysis parameters* will be also explained to measure results variation.

3.5.2.1 Comparison with initial study

The method to obtain the results was similar than the one initially done to get the influence of the three variables: density, stiffness and length (at *Model initial study* section). Therefore, the idea of a static comparison method was also repeated, being in this case individually compared the influence of each region properties variation.

At the *Model initial study* performed before, the influence of density, stiffness and length on natural frequency was obtained. The three variables were independently increased and decreased to obtain their relationships with the calculated frequency values. However, in this new case frequency results are going to vary in a different way than the initial one. There are several facts that will make the frequency values variation more complex.

1) Density-Stiffness relationships

At the initial study performed, density and stiffness were varied independently. It could be shown that there was a positive relationship between stiffness and natural

frequency, and a negative one in the case of density (see results at *Initial Model study* section).

However, in this new study, density and stiffness are related to each other, following the relationship equations from [13] (Eq.1 and 2) explained before. Therefore, an increase or decrease in one of those properties will suppose the correspondent increase or decrease in the other property value. That means that before calculating the region variation models, the behaviour of density and Young's Modulus variation for each type of bone has to be clear. The propose then will be trying to determine how this simultaneous change of both related variables influences the natural frequency.

II) Different geometrical regions

Initially, the variation of density and stiffness was applied along the whole bone, taken as a single material region. Therefore, no geometrical consideration was taken in order to obtain the results.

However, this new analysis considers five different geometrical regions that will individually change their properties. That means that each region will affect frequency results in a different manner, which has to be also determined. That is why there have been defined different *region influence factors* that will be considered to analyse the results. Those factors will be further explained, and some of them are region size, material properties or geometrical position.

3.5.2.2 Reference models

Two different models (stiffness-model and density-model) have been obtained in the previous section. As it has been explained, in each of those models their properties (Young's Modulus, density) follow the relationships given by its material bone type (cortical or cancellous). Both models were used as the reference models for the different region variation cases in this new model improvement section.

That means that there were performed two comparative studies based, respectively, on each of those reference models. For each comparative study, five region variation cases were used to show the influence of each of the femur regions on frequency results. The results obtained from each region variation case were compared to the reference model values, by means of different charts where its reference model is specified. It was decided to use two different references so that results analysis conclusions would be more complete.

3.5.2.3 Properties variation at each region

As it has been explained, each study case was performed varying the properties of a single region, while the rest of them maintained its reference value. In this section the criteria to change each region properties is explained. As it was done in the initial study, properties at each region should increase and decrease their value, so that their

relationship with frequency change will be clear. Properties variation will be obtained increasing and decreasing its value by a certain percentage amount over its initial value (reference model value). Moreover, to check if there exist a certain linearity with properties and frequency variation, properties values were increased and decreased by a different percentage amount.

As it was explained before, Young's Modulus and density are related by certain equations, so their variation will be simultaneous. In order to simplify the calculations using the density - Young's Modulus equations (**Eq. 1** and **2**), density value is the one that was initially modified. Once density value was changed, its correspondent Young's Modulus value was derived from the equations as explained. As the relationship between them is positive for both bone tissues, an increase on density value will also create an increase on Young's Modulus, and vice-versa. However, for each bone tissue the numerical value of the relationships is not the same. That means that Young's Modulus change is different for the same density variation. Relationship equations when density suffers a variation are stated below, and both bone materials density-stiffness relationships are plotted at **Figure 3.5.1**.

It was decided, in order to obtain reasonable variations on frequency results, to vary density in 20 and 10 percent over its initial value. The reason of increasing and decreasing properties by different amounts is to analyse if the frequency results change proportionally with properties variation. It can be observed at **Table 3.5.1** the variation that stiffness experiments for each variation cases, following the material relationships stated before.

| DENSITY (ρ) (Kgm-3) | YOUNG'S MODULUS (E) (MPa) | |
|----------------------------|---|---|
| | Type of bone | |
| | Cancellous | Cortical |
| | $E = 1904 \rho^{1.64}$ $\Delta E (\%) = [(1 + \Delta\rho)^{1.64} - 1] 100$ | $E = 2065 \rho^{3.09}$ $\Delta E (\%) = [(1 + \Delta\rho)^{3.09} - 1] 100$ |
| +10% INCREASE | 16.9% | 34.2% |
| +20% INCREASE | 34.9% | 75.7% |
| -10% DECREASE | -15.9% | -27.8% |
| -20% DECREASE | -30.6% | -49.8% |

Table 3.5.1: Relationships between density and Young's Modulus variations for each type of bone [13]

As it was explained before, the same analysis is going to be performed regarding both stiffness and density reference model properties. Therefore, their results were individually analysed for each reference model. Once the different values were calculated, both model results were compared.

Summarizing, two comparative studies were done, based on each of the two reference models obtained in the previous section (stiffness-fixed and density-fixed models). In each of those comparative studies, five region models where performed, varying the properties of its correspondent region. Finally, region properties variation was obtained

increasing and decreasing their density value by a 20% and 10% from its original value. Young's Modulus value was then derived by the new density value following the bone material relationship equations.

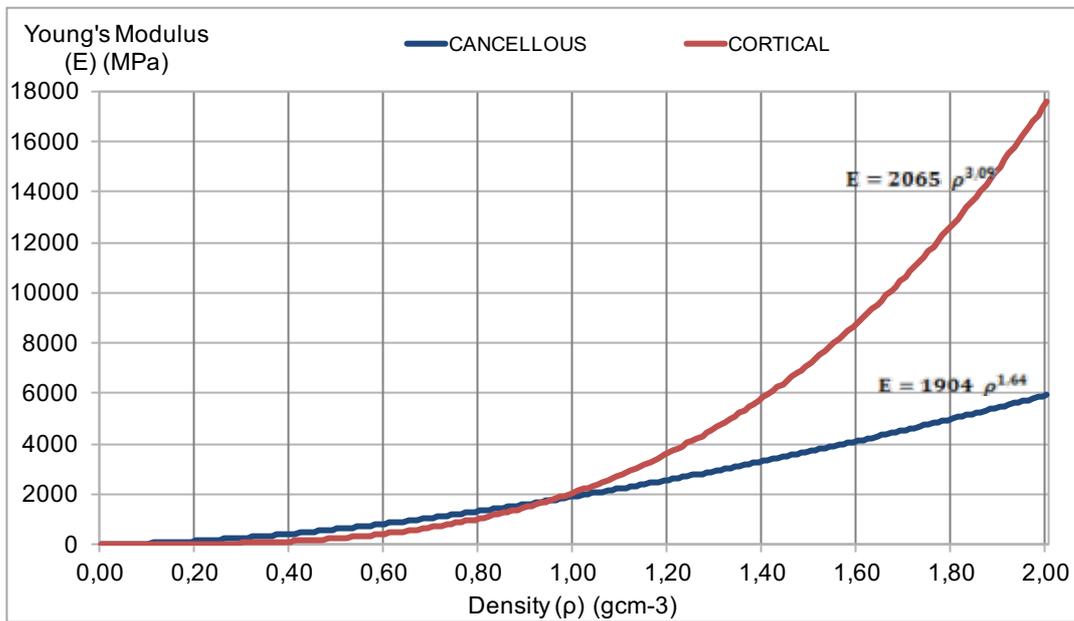


Figure 3.5.1: Relationships equations (Eq. 1 and 2) between density and Young's Modulus for each type of bone [13]

3.5.3 Results

As explained before, each table is referred to a certain properties variation case (density increase by 10% or 20%, and density decrease by 10% or 20%). At each table its reference model (*stiffness* or *density*) properties and results are specified. Moreover, each table contains properties and results for each modified region model (*affected region*). It can be seen below natural frequency values that a percentage variation is also included. In addition, below Euler Bernoulli ratios its variation from their original value are also included. The charts performed at the next section specify how those values were calculated.

Note: Before analysing the following tables, some abbreviations used there have to be explained. PH, PN, CT, DN and DH are referred to the different femur regions, which respectively are proximal head, proximal neck, central, distal neck and distal head. Moreover, Euler Bernoulli ratios are defined as SAG (Sagittal) and FRT (Frontal).

3.5.3.1 Stiffness reference models

| Affected region | Properties | Regions | | | | | Natural frequency (Hz) with % change over reference | | | | | Euler Bernoulli ratios | |
|-----------------|----------------------------|---------|------|-------|------|------|--|-------|-------|-------|-------|------------------------|--------|
| | | PH | PN | CT | DN | DH | M1F | M1S | MT | M2F | M2S | FRT | SAG |
| REFERENCE | Young M. (E) (MPa) | 609 | 1000 | 16700 | 1824 | 1112 | 325 | 334 | 764 | 902 | 970 | 2,78 | 2,91 |
| | Density (ρ) (Kgm-3) | 500 | 675 | 1967 | 974 | 720 | | | | | | | |
| Proximal head | Young M. (E) (MPa) | 714 | 1000 | 16700 | 1824 | 1112 | 323 | 332 | 752 | 893 | 967 | 2,76 | 2,91 |
| | Density (ρ) (Kgm-3) | 550 | 675 | 1967 | 974 | 720 | -0,4% | -0,5% | -1,7% | -1,0% | -0,3% | -0,015 | 0,007 |
| Proximal neck | Young M. (E) (MPa) | 609 | 1168 | 16700 | 1824 | 1112 | 324 | 334 | 782 | 909 | 973 | 2,80 | 2,92 |
| | Density (ρ) (Kgm-3) | 500 | 743 | 1967 | 974 | 720 | -0,1% | -0,1% | 2,3% | 0,8% | 0,3% | 0,026 | 0,012 |
| Central | Young M. (E) (MPa) | 609 | 1000 | 22422 | 1824 | 1112 | 364 | 375 | 815 | 982 | 1065 | 2,70 | 2,84 |
| | Density (ρ) (Kgm-3) | 500 | 675 | 2164 | 974 | 720 | 12,0% | 12,2% | 6,6% | 8,9% | 9,8% | -0,078 | -0,083 |
| Distal neck | Young M. (E) (MPa) | 609 | 1000 | 16700 | 2131 | 1112 | 323 | 333 | 765 | 904 | 972 | 2,79 | 2,92 |
| | Density (ρ) (Kgm-3) | 500 | 675 | 1967 | 1071 | 720 | -0,3% | -0,3% | 0,1% | 0,2% | 0,2% | 0,015 | 0,015 |
| Distal head | Young M. (E) (MPa) | 609 | 1000 | 16700 | 1824 | 1299 | 322 | 331 | 758 | 895 | 961 | 2,78 | 2,90 |
| | Density (ρ) (Kgm-3) | 500 | 675 | 1967 | 974 | 792 | -0,7% | -0,9% | -0,8% | -0,7% | -1,0% | 0,000 | -0,002 |

Table 3.5.2: 10% Density increase at each region of the bone. Properties, natural frequency and Euler Bernoulli ratios changes from stiffness reference model.

| Affected region | Properties | Regions | | | | | Natural frequency (Hz) with % change over reference | | | | | Euler Bernoulli ratios | |
|-----------------|----------------------------|---------|------|-------|------|------|--|-------|-------|-------|-------|------------------------|--------|
| | | PH | PN | CT | DN | DH | M1F | M1S | MT | M2F | M2S | FRT | SAG |
| REFERENCE | Young M. (E) (MPa) | 609 | 1000 | 16700 | 1824 | 1112 | 325 | 334 | 764 | 902 | 970 | 2,78 | 2,91 |
| | Density (ρ) (Kgm-3) | 500 | 675 | 1967 | 974 | 720 | | | | | | | |
| Proximal head | Young M. (E) (MPa) | 824 | 1000 | 16700 | 1824 | 1112 | 322 | 330 | 740 | 885 | 965 | 2,75 | 2,92 |
| | Density (ρ) (Kgm-3) | 600 | 675 | 1967 | 974 | 720 | -0,9% | -1,0% | -3,2% | -1,9% | -0,5% | -0,030 | 0,014 |
| Proximal neck | Young M. (E) (MPa) | 609 | 1348 | 16700 | 1824 | 1112 | 324 | 333 | 795 | 914 | 976 | 2,82 | 2,93 |
| | Density (ρ) (Kgm-3) | 500 | 810 | 1967 | 974 | 720 | -0,3% | -0,2% | 4,0% | 1,3% | 0,6% | 0,044 | 0,024 |
| Central | Young M. (E) (MPa) | 609 | 1000 | 29339 | 1824 | 1112 | 403 | 416 | 853 | 1052 | 1152 | 2,61 | 2,77 |
| | Density (ρ) (Kgm-3) | 500 | 675 | 2360 | 974 | 720 | 24,2% | 24,5% | 11,6% | 16,6% | 18,8% | -0,170 | -0,134 |
| Distal neck | Young M. (E) (MPa) | 609 | 1000 | 16700 | 2458 | 1112 | 322 | 332 | 765 | 905 | 974 | 2,81 | 2,93 |
| | Density (ρ) (Kgm-3) | 500 | 675 | 1967 | 1169 | 720 | -0,7% | -0,6% | 0,1% | 0,3% | 0,4% | 0,027 | 0,028 |
| Distal head | Young M. (E) (MPa) | 609 | 1000 | 16700 | 1824 | 1498 | 320 | 328 | 752 | 889 | 952 | 2,78 | 2,90 |
| | Density (ρ) (Kgm-3) | 500 | 675 | 1967 | 974 | 864 | -1,5% | -1,7% | -1,6% | -1,4% | -1,9% | 0,001 | -0,004 |

Table 3.5.3: 20% Density increase at each region of the bone. Properties, natural frequency and Euler Bernoulli ratios changes from stiffness reference model.

| Affected region | Properties | Regions | | | | | Natural frequency (Hz) with % change over reference | | | | | Euler Bernoulli ratios | |
|-----------------|----------------------------|---------|------|-------|------|------|--|--------|-------|-------|--------|------------------------|--------|
| | | PH | PN | CT | DN | DH | M1F | M1S | MT | M2F | M2S | FRT | SAG |
| REFERENCE | Young M. (E) (MPa) | 609 | 1000 | 16700 | 1824 | 1112 | 325 | 334 | 764 | 902 | 970 | 2,78 | 2,91 |
| | Density (ρ) (Kgm-3) | 500 | 675 | 1967 | 974 | 720 | | | | | | | |
| Proximal head | Young M. (E) (MPa) | 514 | 1000 | 16700 | 1824 | 1112 | 326 | 336 | 778 | 910 | 973 | 2,79 | 2,90 |
| | Density (ρ) (Kgm-3) | 450 | 675 | 1967 | 974 | 720 | 0,4% | 0,6% | 1,8% | 1,0% | 0,4% | 0,015 | -0,007 |
| Proximal neck | Young M. (E) (MPa) | 609 | 841 | 16700 | 1824 | 1112 | 325 | 334 | 737 | 886 | 966 | 2,73 | 2,89 |
| | Density (ρ) (Kgm-3) | 500 | 608 | 1967 | 974 | 720 | 0,1% | 0,1% | -3,6% | -1,7% | -0,4% | -0,051 | -0,014 |
| Central | Young M. (E) (MPa) | 609 | 1000 | 12061 | 1824 | 1112 | 286 | 294 | 702 | 814 | 870 | 2,85 | 2,96 |
| | Density (ρ) (Kgm-3) | 500 | 675 | 1770 | 974 | 720 | -11,9% | -12,0% | -8,1% | -9,8% | -10,3% | 0,067 | 0,055 |
| Distal neck | Young M. (E) (MPa) | 609 | 1000 | 16700 | 1534 | 1112 | 326 | 335 | 763 | 899 | 966 | 2,76 | 2,89 |
| | Density (ρ) (Kgm-3) | 500 | 675 | 1967 | 876 | 720 | 0,3% | 0,3% | -0,2% | -0,3% | -0,4% | -0,019 | -0,019 |
| Distal head | Young M. (E) (MPa) | 609 | 1000 | 16700 | 1824 | 935 | 327 | 337 | 771 | 909 | 980 | 2,78 | 2,91 |
| | Density (ρ) (Kgm-3) | 500 | 675 | 1967 | 974 | 648 | 0,8% | 0,9% | 0,9% | 0,8% | 1,0% | 0,000 | 0,002 |

Table 3.5.4: 10% Density decrease at each region of the bone. Properties, natural frequency and Euler Bernoulli ratios changes from stiffness reference model.

| Affected region | Properties | Regions | | | | | Natural frequency (Hz) with % change over reference | | | | | Euler Bernoulli ratios | |
|-----------------|----------------------------|---------|------|-------|------|------|--|--------|--------|--------|--------|------------------------|--------|
| | | PH | PN | CT | DN | DH | M1F | M1S | MT | M2F | M2S | FRT | SAG |
| REFERENCE | Young M. (E) (MPa) | 609 | 1000 | 16700 | 1824 | 1112 | 325 | 334 | 764 | 902 | 970 | 2,78 | 2,91 |
| | Density (ρ) (Kgm-3) | 500 | 675 | 1967 | 974 | 720 | | | | | | | |
| Proximal head | Young M. (E) (MPa) | 424 | 1000 | 16700 | 1824 | 1112 | 327 | 338 | 794 | 919 | 978 | 2,81 | 2,89 |
| | Density (ρ) (Kgm-3) | 400 | 675 | 1967 | 974 | 720 | 0,8% | 1,3% | 3,8% | 1,9% | 0,8% | 0,030 | -0,014 |
| Proximal neck | Young M. (E) (MPa) | 609 | 693 | 16700 | 1824 | 1112 | 325 | 334 | 706 | 867 | 962 | 2,66 | 2,88 |
| | Density (ρ) (Kgm-3) | 500 | 540 | 1967 | 974 | 720 | 0,2% | 0,1% | -7,6% | -3,9% | -0,8% | -0,114 | -0,025 |
| Central | Young M. (E) (MPa) | 609 | 1000 | 8382 | 1824 | 1112 | 248 | 254 | 629 | 720 | 765 | 2,91 | 3,01 |
| | Density (ρ) (Kgm-3) | 500 | 675 | 1574 | 974 | 720 | -23,6% | -23,8% | -17,7% | -20,2% | -21,1% | 0,127 | 0,102 |
| Distal neck | Young M. (E) (MPa) | 609 | 1000 | 16700 | 1264 | 1112 | 327 | 336 | 761 | 894 | 961 | 2,74 | 2,86 |
| | Density (ρ) (Kgm-3) | 500 | 675 | 1967 | 779 | 720 | 0,6% | 0,5% | -0,5% | -0,9% | -1,0% | -0,043 | -0,043 |
| Distal head | Young M. (E) (MPa) | 609 | 1000 | 16700 | 1824 | 770 | 330 | 340 | 778 | 917 | 991 | 2,78 | 2,91 |
| | Density (ρ) (Kgm-3) | 500 | 675 | 1967 | 974 | 576 | 1,6% | 2,0% | 1,8% | 1,7% | 2,1% | 0,001 | 0,005 |

Table 3.5.5: 20% Density decrease at each region of the bone. Properties, natural frequency and Euler Bernoulli ratios changes from stiffness reference model.

3.5.3.2 Density reference models

| Affected region | Properties | Regions | | | | | Natural frequency (Hz) with % change over reference | | | | | Euler Bernoulli ratios | |
|-----------------|----------------------------|---------|------|-------|------|-----|--|-------|-------|-------|-------|------------------------|--------|
| | | PH | PN | CT | DN | DH | M1F | M1S | MT | M2F | M2S | FRT | SAG |
| REFERENCE | Young M. (E) (MPa) | 269 | 999 | 17475 | 1824 | 693 | 342 | 358 | 858 | 968 | 1032 | 2,83 | 2,88 |
| | Density (ρ) (Kgm-3) | 303 | 675 | 1996 | 974 | 540 | | | | | | | |
| Proximal head | Young M. (E) (MPa) | 314 | 999 | 17475 | 1824 | 693 | 342 | 356 | 846 | 964 | 1027 | 2,82 | 2,88 |
| | Density (ρ) (Kgm-3) | 333 | 675 | 1996 | 974 | 540 | -0,2% | -0,5% | -1,4% | -0,5% | -0,4% | -0,007 | 0,001 |
| Proximal neck | Young M. (E) (MPa) | 269 | 1168 | 17475 | 1824 | 693 | 342 | 357 | 872 | 971 | 1037 | 2,84 | 2,91 |
| | Density (ρ) (Kgm-3) | 303 | 743 | 1996 | 974 | 540 | -0,2% | -0,2% | 1,6% | 0,2% | 0,5% | 0,012 | 0,022 |
| Central | Young M. (E) (MPa) | 269 | 999 | 23460 | 1824 | 693 | 383 | 402 | 922 | 1063 | 1131 | 2,77 | 2,82 |
| | Density (ρ) (Kgm-3) | 303 | 675 | 2196 | 974 | 540 | 11,9% | 12,2% | 7,4% | 9,7% | 9,6% | -0,055 | -0,067 |
| Distal neck | Young M. (E) (MPa) | 269 | 999 | 5915 | 2132 | 693 | 341 | 356 | 859 | 971 | 1031 | 2,85 | 2,89 |
| | Density (ρ) (Kgm-3) | 303 | 675 | 1996 | 1071 | 540 | -0,4% | -0,4% | 0,1% | 0,2% | 0,0% | 0,019 | 0,010 |
| Distal head | Young M. (E) (MPa) | 269 | 999 | 5915 | 1824 | 810 | 340 | 355 | 852 | 961 | 1023 | 2,83 | 2,88 |
| | Density (ρ) (Kgm-3) | 303 | 675 | 1996 | 974 | 594 | -0,7% | -0,7% | -0,7% | -0,7% | -0,8% | -0,002 | -0,003 |

Table 3.5.6: 10% Density increase at each region of the bone. Properties, natural frequency and Euler Bernoulli ratios changes from density reference model.

| Affected region | Properties | Regions | | | | | Natural frequency (Hz) with % change over reference | | | | | Euler Bernoulli ratios | |
|-----------------|----------------------------|---------|------|-------|------|-----|--|-------|-------|-------|-------|------------------------|--------|
| | | PH | PN | CT | DN | DH | M1F | M1S | MT | M2F | M2S | FRT | SAG |
| REFERENCE | Young M. (E) (MPa) | 269 | 999 | 17475 | 1824 | 693 | 342 | 358 | 858 | 968 | 1032 | 2,83 | 2,88 |
| | Density (ρ) (Kgm-3) | 303 | 675 | 1996 | 974 | 540 | | | | | | | |
| Proximal head | Young M. (E) (MPa) | 362 | 999 | 17475 | 1824 | 693 | 341 | 355 | 834 | 959 | 1023 | 2,81 | 2,89 |
| | Density (ρ) (Kgm-3) | 364 | 675 | 1996 | 974 | 540 | -0,4% | -0,9% | -2,8% | -1,0% | -0,8% | -0,016 | 0,003 |
| Proximal neck | Young M. (E) (MPa) | 269 | 1348 | 17475 | 1824 | 693 | 341 | 356 | 881 | 972 | 1041 | 2,85 | 2,92 |
| | Density (ρ) (Kgm-3) | 303 | 810 | 1996 | 974 | 540 | -0,3% | -0,5% | 2,7% | 0,4% | 0,9% | 0,020 | 0,040 |
| Central | Young M. (E) (MPa) | 269 | 999 | 30696 | 1824 | 693 | 424 | 446 | 971 | 1149 | 1222 | 2,71 | 2,74 |
| | Density (ρ) (Kgm-3) | 303 | 675 | 2395 | 974 | 540 | 23,9% | 24,6% | 13,2% | 18,6% | 18,5% | -0,120 | -0,142 |
| Distal neck | Young M. (E) (MPa) | 269 | 999 | 5915 | 2459 | 693 | 340 | 355 | 859 | 972 | 1030 | 2,86 | 2,90 |
| | Density (ρ) (Kgm-3) | 303 | 675 | 1996 | 1169 | 540 | -0,8% | -0,8% | 0,1% | 0,4% | -0,2% | 0,034 | 0,017 |
| Distal head | Young M. (E) (MPa) | 269 | 999 | 5915 | 1824 | 935 | 338 | 353 | 846 | 955 | 1015 | 2,83 | 2,88 |
| | Density (ρ) (Kgm-3) | 303 | 675 | 1996 | 974 | 648 | -1,3% | -1,5% | -1,5% | -1,4% | -1,6% | -0,003 | -0,005 |

Table 3.5.7: 20% Density increase at each region of the bone. Properties, natural frequency and Euler Bernoulli ratios changes from density reference model.

| Affected region | Properties | Regions | | | | | Natural frequency (Hz) with % change over reference | | | | | Euler Bernoulli ratios | |
|-----------------|----------------------------|---------|-----|-------|------|-----|--|--------|-------|--------|-------|------------------------|--------|
| | | PH | PN | CT | DN | DH | M1F | M1S | MT | M2F | M2S | FRT | SAG |
| REFERENCE | Young M. (E) (MPa) | 269 | 999 | 17475 | 1824 | 693 | 342 | 358 | 858 | 968 | 1032 | 2,83 | 2,88 |
| | Density (ρ) (Kgm-3) | 303 | 675 | 1996 | 974 | 540 | | | | | | | |
| Proximal head | Young M. (E) (MPa) | 226 | 999 | 17475 | 1824 | 693 | 343 | 360 | 871 | 973 | 1037 | 2,84 | 2,88 |
| | Density (ρ) (Kgm-3) | 273 | 675 | 1996 | 974 | 540 | 0,2% | 0,5% | 1,5% | 0,4% | 0,5% | 0,006 | 0,001 |
| Proximal neck | Young M. (E) (MPa) | 269 | 841 | 17475 | 1824 | 693 | 343 | 359 | 839 | 964 | 1025 | 2,81 | 2,86 |
| | Density (ρ) (Kgm-3) | 303 | 608 | 1996 | 974 | 540 | 0,2% | 0,2% | -2,2% | -0,4% | -0,6% | -0,017 | -0,024 |
| Central | Young M. (E) (MPa) | 269 | 999 | 12619 | 1824 | 693 | 302 | 315 | 782 | 869 | 952 | 2,88 | 3,03 |
| | Density (ρ) (Kgm-3) | 303 | 675 | 1796 | 974 | 540 | -11,8% | -12,1% | -8,9% | -10,2% | -7,7% | 0,049 | 0,143 |
| Distal neck | Young M. (E) (MPa) | 269 | 999 | 5915 | 1534 | 693 | 344 | 359 | 856 | 965 | 1031 | 2,81 | 2,87 |
| | Density (ρ) (Kgm-3) | 303 | 675 | 1996 | 877 | 540 | 0,4% | 0,4% | -0,2% | -0,4% | 0,0% | -0,023 | -0,012 |
| Distal head | Young M. (E) (MPa) | 269 | 999 | 5915 | 1824 | 583 | 345 | 361 | 865 | 976 | 1041 | 2,83 | 2,89 |
| | Density (ρ) (Kgm-3) | 303 | 675 | 1996 | 974 | 486 | 0,7% | 0,8% | 0,8% | 0,8% | 0,9% | 0,002 | 0,003 |

Table 3.5.8: 10% Density decrease at each region of the bone. Properties, natural frequency and Euler Bernoulli ratios changes from density reference model.

| Affected region | Properties | Regions | | | | | Natural frequency (Hz) with % change over reference | | | | | Euler Bernoulli ratios | |
|-----------------|----------------------------|---------|-----|-------|------|-----|--|--------|--------|--------|--------|------------------------|--------|
| | | PH | PN | CT | DN | DH | M1F | M1S | MT | M2F | M2S | FRT | SAG |
| REFERENCE | Young M. (E) (MPa) | 269 | 999 | 17475 | 1824 | 693 | 342 | 358 | 858 | 968 | 1032 | 2,83 | 2,88 |
| | Density (ρ) (Kgm-3) | 303 | 675 | 1996 | 974 | 540 | | | | | | | |
| Proximal head | Young M. (E) (MPa) | 186 | 999 | 17475 | 1824 | 693 | 344 | 361 | 884 | 977 | 1044 | 2,84 | 2,89 |
| | Density (ρ) (Kgm-3) | 242 | 675 | 1996 | 974 | 540 | 0,4% | 1,0% | 3,0% | 0,8% | 1,2% | 0,012 | 0,004 |
| Proximal neck | Young M. (E) (MPa) | 269 | 693 | 17475 | 1824 | 693 | 343 | 359 | 812 | 956 | 1028 | 2,79 | 2,86 |
| | Density (ρ) (Kgm-3) | 303 | 540 | 1996 | 974 | 540 | 0,3% | 0,4% | -5,4% | -1,2% | -0,3% | -0,044 | -0,022 |
| Central | Young M. (E) (MPa) | 269 | 999 | 8769 | 1824 | 693 | 262 | 272 | 696 | 767 | 813 | 2,92 | 2,99 |
| | Density (ρ) (Kgm-3) | 303 | 675 | 1597 | 974 | 540 | -23,3% | -23,9% | -18,9% | -20,8% | -21,2% | 0,094 | 0,103 |
| Distal neck | Young M. (E) (MPa) | 269 | 999 | 5915 | 1265 | 693 | 345 | 361 | 853 | 958 | 1029 | 2,78 | 2,85 |
| | Density (ρ) (Kgm-3) | 303 | 675 | 1996 | 779 | 540 | 0,8% | 0,8% | -0,5% | -1,0% | -0,2% | -0,052 | -0,029 |
| Distal head | Young M. (E) (MPa) | 269 | 999 | 5915 | 1824 | 481 | 347 | 364 | 872 | 984 | 1051 | 2,83 | 2,89 |
| | Density (ρ) (Kgm-3) | 303 | 675 | 1996 | 974 | 432 | 1,4% | 1,6% | 1,6% | 1,6% | 1,8% | 0,005 | 0,006 |

Table 3.5.9: 20% Density decrease at each region of the bone. Properties, natural frequency and Euler Bernoulli ratios changes from density reference model.

3.5.4 Discussion

3.5.4.1 Analysis charts

To make an organized analysis of the obtained results, different charts have been performed to clarify the impact on femur natural frequencies produced by modifying each region material properties.

A) FREQUENCY CHANGE CHARTS

Note: Although natural frequency results have discrete and independent values at every mode, their results have been lined together. The main reason of this fact is that continuous data will make the result visualization and analysis more intuitive, having the idea of graph patterns instead of just some discrete points.

This set of graphs represents the natural **frequency change of the femur when material properties are modified** at a certain region. The next formula **Eq. 3** explains how frequency change values were calculated for each frequency mode:

$$\text{Frequency change (\%)} = \frac{\omega_x^i - \omega_r^i}{\omega_r^i} \quad (3)$$

Where:

ω_r^i is the natural frequency value (Hz) at mode “i” of the original model (density or stiffness).

ω_x^i is the natural frequency value (Hz) at mode “i” when properties are changed (varying density by 10% or 20%) at a certain region “x”.

Frequency values at every mode are obtained from **Tables 3.5.2 to 3.5.9**. The first row of natural frequency results belongs to the reference values of each model. The following rows belong to the new natural frequency values obtained when the affected region properties are modified. Frequency change value is expressed as a percentage variation below.

There have been performed two different set of charts to show frequency change values:

Individual charts (Figure 3.5.2 to 3.5.6)

As it has been said before, results have been obtained by changing the properties of a single region from its original (reference) value. That is, each chart represents the frequency change due to properties variations of the five different regions. Frequency change values can be positive (its value has increased from reference) or negative (its value has decreased). Remind that region properties were either increased or decreased. Note that the *green colour* values correspond to increasing properties variations (by increasing its density in 10% and 20%), and *red colour* to decreasing properties (by decreasing density value in 10% and 20%). There are two different sets of charts for each reference model (*stiffness* and *density*).

General charts (Figure 3.5.7)

In this case, the frequency variation values due to properties change of each region are plotted in the same graph. **The main reason to perform these new charts is that the scale at the previous ones (Figure 3.5.2 to 3.5.6) was different for each region case, which made frequency change results difficult to compare.** However, at **Figure 3.5.7** frequency variation values are plotted together for different region properties variation (increase and decrease by 20%), so their obtained results can be easily compared.

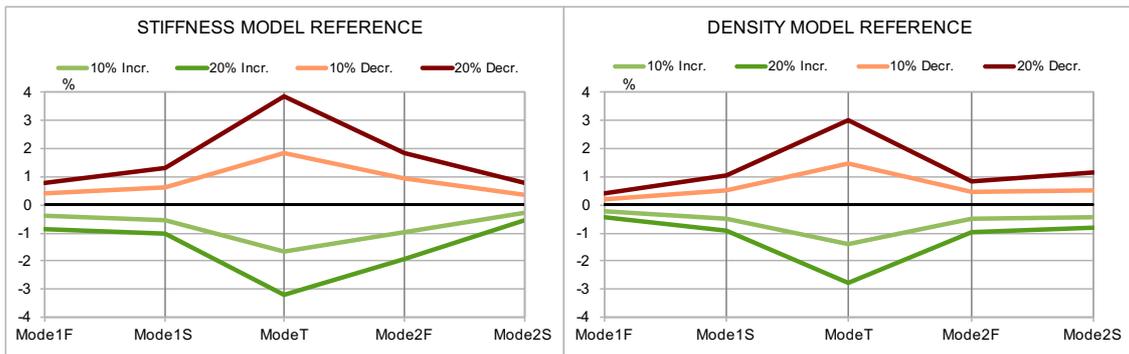


Figure 3.5.2: Frequency change (%) at every mode due to proximal head mechanical properties variation

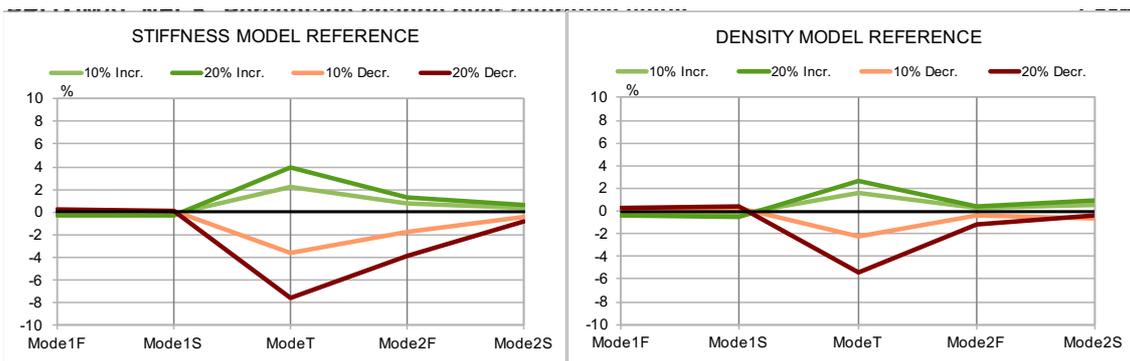


Figure 3.5.3: Frequency change (%) at every mode due to proximal neck mechanical properties variation

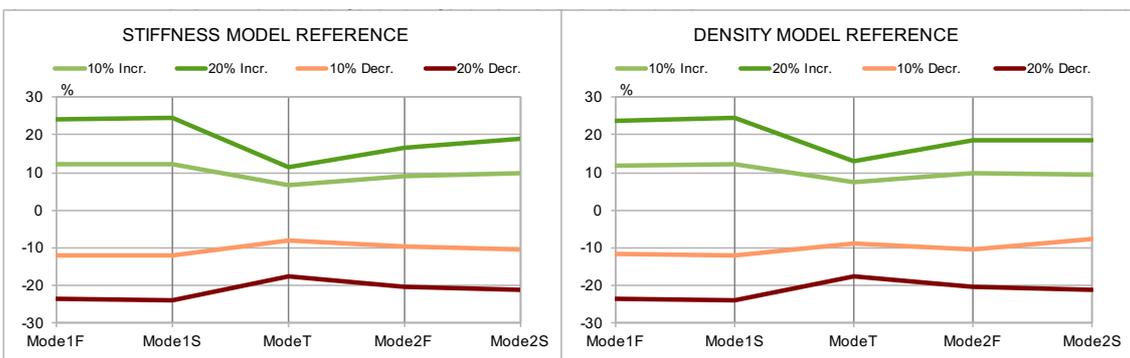


Figure 3.5.4: Frequency change (%) at every mode due to central region mechanical properties variation

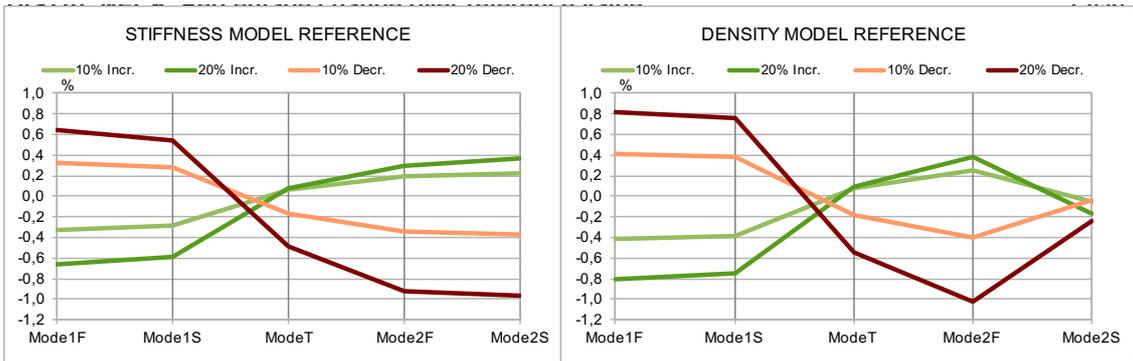


Figure 3.5.5: Frequency change (%) at every mode due to distal neck mechanical properties variation

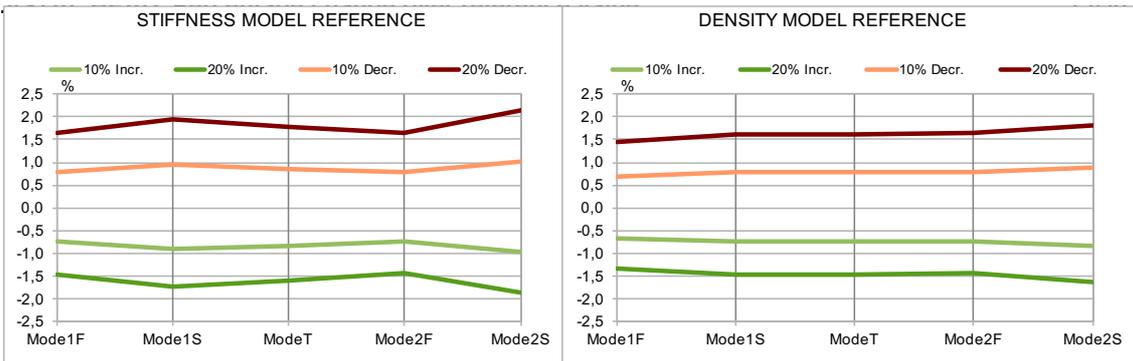


Figure 3.5.6: Frequency change (%) at every mode due to distal head mechanical properties variation

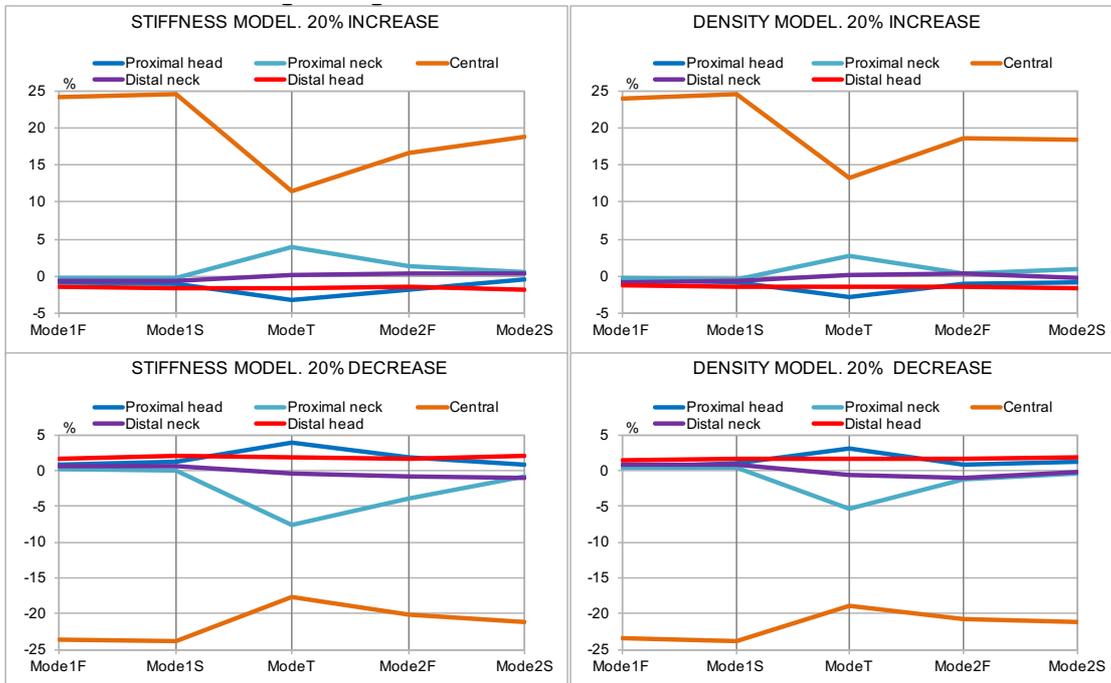


Figure 3.5.7: Frequency changes (%) at every mode due to different regions properties variation (20% density increase and decrease)

By means of the individual charts (Figure 3.5.2 to 3.5.6), different facts can be observed:

Natural frequency variation due to modified region properties is not constant at every mode.

Comparing the different values of frequency change, it can be seen that each region material modification affects femur natural frequency modes in a different way. When proximal head and neck properties are modified, frequency change is mainly produced at torsion mode, while at the two first bending modes frequency change is especially low. By contrary, when central region properties are modified, the first bending modes are the ones more affected, while torsion is the least affected. Meanwhile, frequency variation is almost constant at every mode when distal head properties change.

Comparing the two model results, graph patterns are very similar for each modified region case, except for distal neck regions, where stiffness and density models differences are remarkable.

Frequency change and region properties variation relationship varies for each region:

Remind that frequency change values could be positive or negative. Moreover, those values were obtained for increasing (*green colour*) or decreasing (*red colour*) properties at a certain region. Therefore, it can be said that there exists a relationship between frequency change and region properties variation, which sign can be positive or negative. For instance, at central region (**Figure 3.5.4**), the positive frequency change values correspond to the *green* graphs (increasing properties), and vice-versa. It can be said that natural frequency has a positive relationship to material properties of this region.

However, at both proximal and distal heads (**Figures 3.5.2** and **3.5.6**) the positive values for frequency change correspond to the red colour graphs (decreasing properties), and vice-versa. Those region properties have a negative relationship to femur natural frequency. In the case of proximal and distal necks (**Figures 3.5.3** and **3.5.5**), at the first bending modes the relationship sign is negative, but it becomes positive at torsion and second bending modes. It can be noticed that all previous facts are observed for both material model results (density and stiffness models).

Frequency change is higher when region properties are decreased:

It can be observed that, at every graph, when region properties decrease the numerical value for frequency change is higher (without taking into account if its value is positive or negative). Therefore, it can be said that a decrease in density and Young's Modulus values of a certain region has a higher impact on femur natural frequency than when they increase.

By means of general charts (**Figure 3.5.7**), some other ideas can be obtained.

Some regions have a higher impact on femur frequencies when their properties are modified

What most affects femur natural frequencies values is a variation at its central region material properties. At the graph it can be observed that femur frequency change are really high modifying central region properties than any other region. Regarding the other four regions, femur frequency happens to be more affected by properties changes at proximal zone regions (proximal neck and proximal head).

B) FREQUENCY RATIOS CHARTS

Note: Although chart results have discrete and independent values at every mode, they have been plotted together. The main reason of this fact is that attaching the different modes values the result visualization will be more intuitive, having the idea of graph patterns instead of just some independent points.

This set of graphs (**Figure 3.5.8**) represents the ratios between the *frequency change* values calculated before (**Eq. 3**). Two type of ratios values have been calculated, one for increasing region properties (*blue colour*) and another for decreasing region properties (*orange colour*). Therefore, **for each frequency mode two different ratios were obtained**. The next **equations 4** and **5** shows how the different ratios were calculated:

$$\text{Increase ratio value} = \frac{\Delta\omega_{20inc}}{\Delta\omega_{10inc}} \quad (4)$$

Where:

$\Delta\omega_{20inc}$ is the *frequency change* (%) when region density increases by 20%.

$\Delta\omega_{10inc}$ is the *frequency change* (%) when region density increases by 10%.

$$\text{Decrease ratio value} = \frac{\Delta\omega_{20dec}}{\Delta\omega_{10dec}} \quad (5)$$

Where:

$\Delta\omega_{20dec}$ is the *frequency change* (%) when region density decreases by 20%.

$\Delta\omega_{10dec}$ is the *frequency change* (%) when region density decreases by 10%.

At the previous *frequency change* charts, a certain relationship was observed between region properties and femur natural frequencies. The main objective of these charts is to show if that relationship is linearly proportional.

Density variation goes from 10% to 20%, which means that its variation is doubled at the second case. Therefore, in order to maintain a linear relationship with density:

$$\text{if } \omega \propto \rho \rightarrow \omega = k \rho \rightarrow \frac{\Delta\omega_{20}}{\Delta\omega_{10}} = \frac{k \Delta\rho_{20}}{k \Delta\rho_{10}} = \frac{20\%}{10\%} = 2 \quad (6)$$

Equation 6 shows that, if natural frequency (ω) is proportional to region density (ρ), *increase and decrease ratios* ($\frac{\Delta\omega_{20}}{\Delta\omega_{10}}$) values should approach to 2. **Figure 3.5.8** represents the increase and decrease ratios due to each modified region material (*Note that the scale at distal neck is different from the general one*).

Analysing the results, it can be seen that for almost every case ratios values approach to be equal to 2. This fact will show that the relationship between region properties and femur natural frequency values can be considered as linear. However, there are some exceptions where frequency change ratios values do not approach to 2: values obtained due to neck region properties variation and values at high frequency modes.

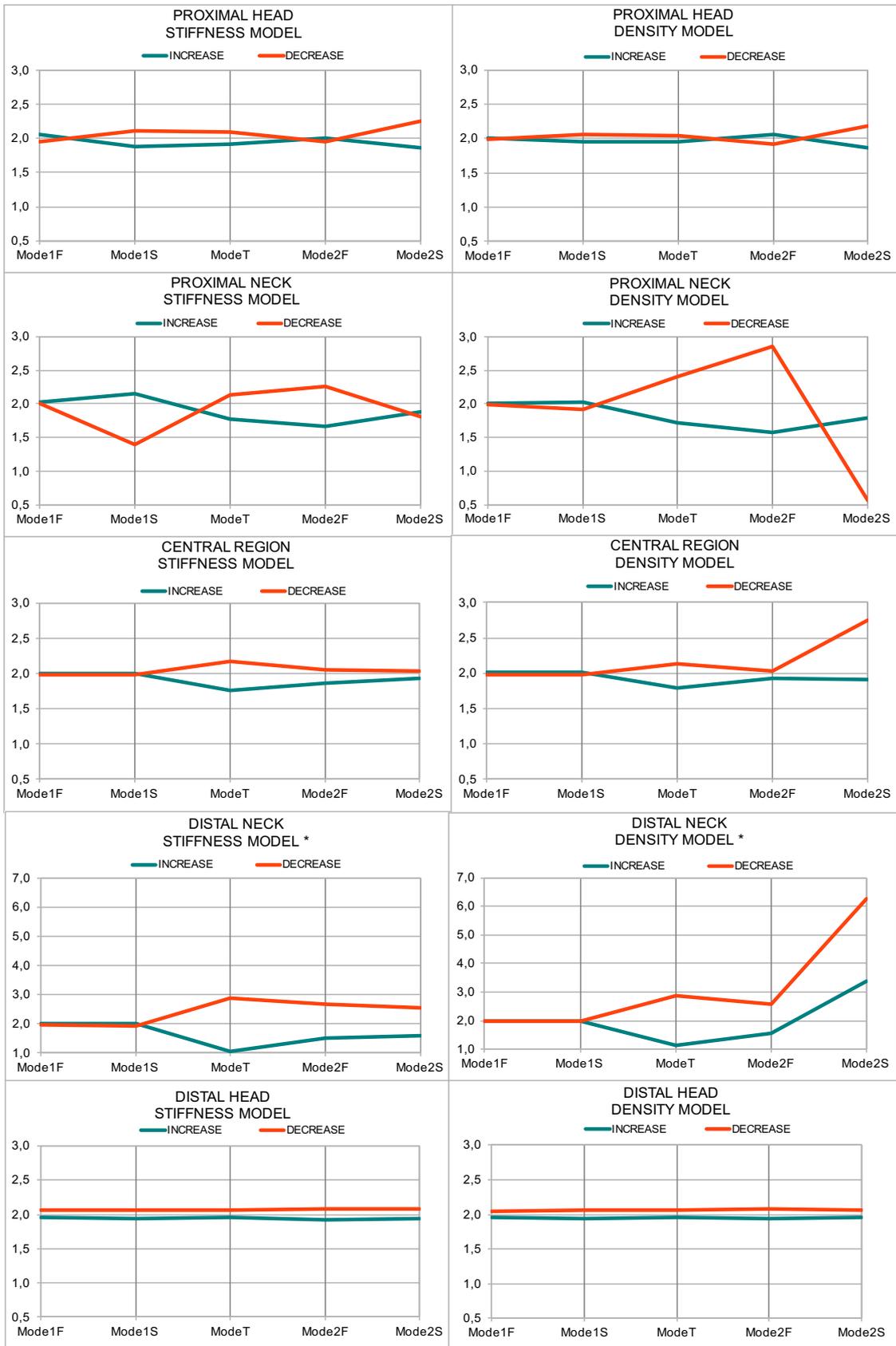


Figure 3.5.8: Frequency changes ratios ($\frac{\Delta\omega_{20}}{\Delta\omega_{10}}$) due to 20% over 10% density increase and decrease at the different regions.

C) EULER-BERNOULLI RATIOS CHARTS

The main propose of these charts is to show how the different variations on region properties affects the frequency Euler-Bernoulli Ratios. Remember that those ratios relate the different bending modes results. The next **equations 7** and **8** explains how Euler-Bernoulli ratios variation values were calculated:

$$\Delta EBS = EBS_x - EBS_r = \frac{\omega_x^{2S}}{\omega_x^{1S}} - \frac{\omega_r^{2S}}{\omega_r^{1S}} \quad (7)$$

Where:

ΔEBS is the **variation of Sagittal Euler-Bernoulli ratio**

EBS_x is Sagittal Euler-Bernoulli ratio at a region "x"

EBS_r is Sagittal Euler Bernoulli ratio for the original model (density or stiffness)

ω_x^{2S} and ω_x^{1S} are, respectively, natural frequency values at second and first sagittal modes when properties are changed at region "x"

ω_r^{2S} and ω_r^{1S} are, respectively, natural frequency values at second and first sagittal modes for the original model (reference)

$$\Delta EBF = EBF_x - EBF_r = \frac{\omega_x^{2F}}{\omega_x^{1F}} - \frac{\omega_r^{2F}}{\omega_r^{1F}} \quad (8)$$

Where:

ΔEBF is the **variation of Frontal Euler-Bernoulli ratio**

EBF_x is Frontal Euler-Bernoulli ratio at a region "x"

EBF_r is Frontal Euler Bernoulli ratio for the original model (density or stiffness)

ω_x^{2F} and ω_x^{1F} are, respectively, natural frequency values at second and first frontal modes when properties are changed at region "x"

ω_r^{2F} and ω_r^{1F} are, respectively, reference natural frequency values at second and first frontal modes for the original model

All the previous values are specified at **Tables 3.5.2 to 3.5.9**.

Figure 3.5.9 and **3.5.10** represent, respectively, the variation of Sagittal and Frontal Euler-Bernoulli ratios due to properties change at each femur region. Femur regions have been named as: PH (Proximal head), PN (proximal neck), CTRL (central region), DN (Distal neck) and PN (Proximal neck). Moreover, at each of the graphs results have been calculated for all properties variation cases (density increase and decrease by 20% and 10%).

As it was reminded at **eq. 7** and **8**, Euler-Bernoulli ratios are actually relating natural frequency results at different bending modes. Therefore, is obvious to think that properties variation at a certain region will have an impact on these ratios. Euler-Bernoulli ratios variation has been used as an another indicator to analyse the relationship between region properties variation with femur natural frequencies

At this time, Euler-Bernoulli ratios will not be used to check if they approach to its theoretical value of 2,75, and hence verifying femur bone as a Euler-Bernoulli beam. That is why the **variation** of their values from its original value has been plotted at the graphs, instead of its actual numerical value.

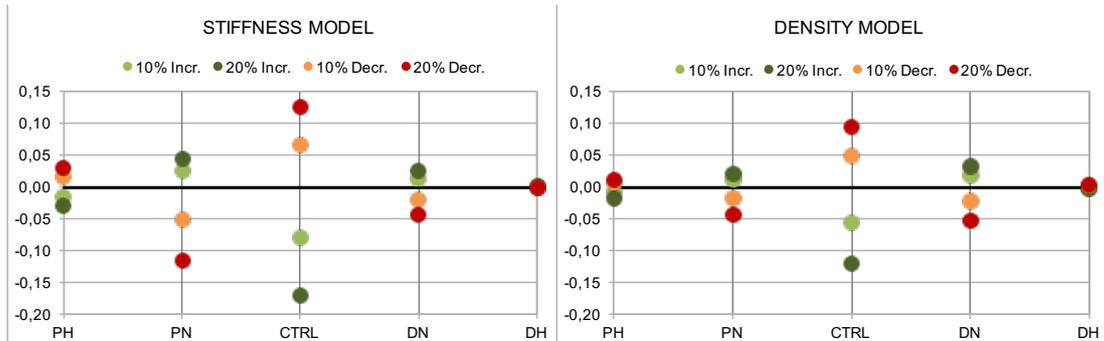


Figure 3.5.9: Variation of Frontal Euler Bernoulli ratios due to properties change at each region

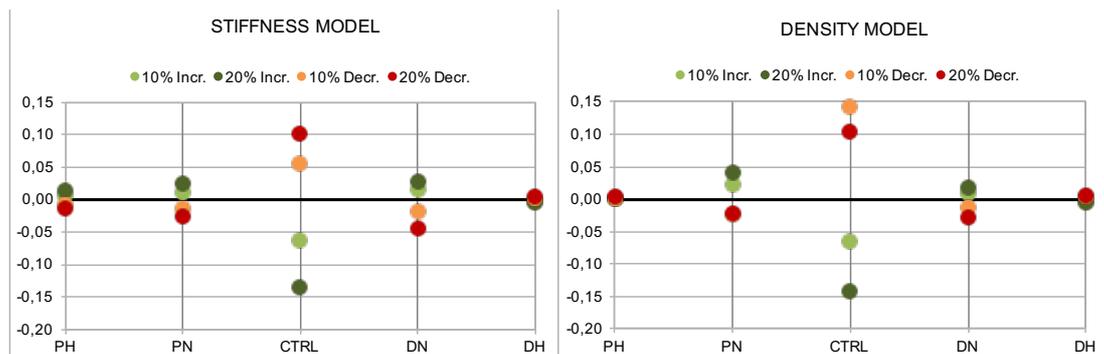


Figure 3.5.10: Variation of Sagittal Euler Bernoulli ratios due to properties change at each region

Graphs at Figures 3.5.9 and 3.5.10 shows that the biggest variation occurs de to changes at central region, followed by both necks.

Both model results have certain similarities, although the numerical values for ratios difference varies from one model to another. The most remarkable difference is that at *stiffness* model Frontal ratio is the most affected one by region material modification, while at *density* model Sagittal ratio is the most affected one.

Finally, it was also surprising to find that the relationship between ratios variation from its original value and properties change was different for each region (green values correspond to region properties increase, and vice-versa). Heads and central region properties were positively related with ratios variation (positive values were obtained with properties increase), while necks properties were negatively related with those ratios change (positive values were obtained with properties decrease).

3.5.4.2 Region characteristics to explain the obtained results

Different region characteristics are now explained, in order to get a possible explanation for the obtained results. The main goal will be to get some general conclusions about which region characteristics have a bigger impact on the results, which will be done at the next section.

I) Geometrical location and shape of the region

On the one hand, regarding geometrical location, regions can be divided into three different groups: region located at extremities (heads), region located at a transition zone (necks) and region located at the middle of the femur (central region).

On the other hand, it has to be also considered that femur geometry is not constant through its length, and therefore their regions will have a different shape. Femur proximal end has an irregular shape, while its distal end is more regular. Moreover, the central region can be considered as the most regular one,

II) Region size.

As it has been explained in previous sections, the different partitions made on the femur represent a different fraction of the whole femur volume. Therefore, region size has to be also considered as an important parameter that can influence the obtained results. It can be obvious to think that regions with higher size will have a bigger impact on natural frequency results, when their properties are modified. It can be remarked that the size of the central region is quite bigger compared to the size of the rest of them. Moreover, the size of the heads and necks is pretty similar.

III) Type of bone of the region

The relationships that density and stiffness obey are different for cortical and cancellous bone tissues. As it has been explained before, the variations performed in this study affected both density and stiffness values at each region. Density-stiffness relationships determine the variation amount between those properties, and is associated to a certain bone material (cortical or cancellous tissue) As it has been previously said, the same variation on density creates a different variation on stiffness, which is much bigger at cortical bone compared to cancellous (**Table 3.5.1**). The only region associated to cortical bone tissue is central region, while the rest of them (heads and necks) are considered cancellous bone tissue.

IV) Type of material model: stiffness or density

It has to be considered that the results have been obtained using two different models. Except for proximal and distal necks, material properties values are different at both models regions. Compared to density model, at the stiffness model density and Young's modulus values are higher at both heads, but lower at the central region. Those properties values differences made that the obtained results were not the same for both models. The similarities and differences between both models will be further analysed.

3.5.4.3 Result analysis and conclusions

Once the main region characteristics have been said, they will be now used to explain the obtained results. The main propose of this procedure is to decide which are the regions characteristics that seem more reasonable to explain the different ideas extracted from the results obtained.

Natural frequency variation due to modified region properties is not constant at every mode

As was observed at **Figures 3.5.2 to 3.5.6**, frequency change graphs due to a certain region properties variation are very different at each frequency mode. That means that material properties at each region affects frequency modes in a different way. The propose now is to determine which region characteristics seem more reasonable to explain this fact.

First of all, comparing the graph shape for both density and stiffness models, it can be seen that they have small differences (as the ones observed at last modes of distal neck at **Figure 3.5.5**). This fact shows that the type of model used cannot be considered as a relevant factor to determine which frequency modes are more affected by each region properties.

In addition, no similarities between cancellous bone regions (heads and necks) could be found in order to consider type of bone as a factor that determines which modes are affected by each region. Actually, the graphs obtained with two different bone regions (central and distal head) happens to be very similar. Therefore, type of bone cannot be considered as a characteristic involved.

Moreover, region size is not found to be determinant, due to the reason explained above (central and distal head regions have very high size differences, but their graph patterns are very similar).

Therefore, the only characteristic that may determine which modes are more affected by properties variation at a certain region is the geometrical location of that region. For instance, it can be appreciated that modifying properties at one of the proximal regions (head or neck), femur frequency modes affected are practically the same.

Summarizing, the geometrical location and shape of the region can be considered as the main factor that determines which frequency modes are more affected by each region. In addition, the type of material model used may have a certain impact too, due to some differences, especially at the numerical values for frequency change comparing both model results.

Some regions have a higher impact on femur frequencies when their properties are modified

At **Figure 3.5.7**) it was observed that, for both material model results, what most affected femur natural frequencies value was a variation on central region properties. The results due to properties variation at the other regions happen to be similar in magnitude, but very small compared to central region properties change. Taking into

account that the central region is the only one associated to cortical bone and that its size is really big compared to the other ones, two different considerations can be taken:

The first one is considering type of bone to determine region material influence on natural frequency, being cortical bone influence higher than cancellous bone. Due to the analysis performed on cortical and cancellous bone relationships, it could be shown the fact that density variation affected Young's Modulus value in a very different way for each bone tissue (**Table 3.5.1**). As it was explained, the same variation in material density created a considerably higher variation on Young's Modulus for cortical bone tissue. Therefore, its reasonable to assume that its higher variation at Young's Modulus makes that cortical bone tissue properties have a higher effect on femur natural frequencies.

On the other hand, it also has been said that region size is much bigger at the central region. It seems obvious that femur natural frequencies will be more affected by region properties modification when its size is higher.

Comparing now frequency change values due to material modification at the other regions, which have the same type of bone and similar sizes, some differences could be appreciated. Actually, frequency change happened to be more affected by properties variation at proximal head and neck regions that by the distal ones.

Moreover, when comparing both material models results, it could be seen that region properties variation do not affect femur natural frequency in the same amount. The impact on natural frequency results of both heads and necks properties variation was higher for *stiffness model*. However, for *density model* central region properties variation creates a higher femur natural frequency change.

Summarizing, when the properties of a certain region are modified, a higher region size and cortical bone type increase its effect on femur natural frequencies. But also the type of material model used and the geometrical location of the modified region influenced femur natural frequency change, having the proximal regions a bigger impact than the distal ones.

Relationship between femur natural frequencies and region properties

From **Figures 3.5.2 to 3.5.6** it could be noticed that there exists a certain relationship between femur natural frequency and properties at each region. The sign for this relationship was positively related for central region and negatively related at both heads. The relationship sign at both necks was different for each mode, being negative for the first ones and positive for the third last modes.

Type of bone seems to be a region characteristic that should influence on this relationship sign. However, the fact that for the different cancellous regions (heads and necks) the relationship does not maintain its sign makes that type of bone cannot be considered to be determining.

The only way to explain the observed results is to consider geometrical location of the region as the main region characteristic that determines if their properties are positively or negatively related with femur natural frequency. At the bone extremities (proximal

and distal head), material properties are negatively related with femur frequencies. However, at the middle of the bone (central region), material properties are positively related with femur natural frequencies. Necks will be considered as transition zones, where material properties combine positively and negatively relationships with each mode of femur natural frequency.

Effect on femur natural frequency that region properties have when they increase or decrease

It was observed at **Figures 3.5.2 to 3.5.6** that that the frequency change values happen to be higher when region properties decreased. That is, brown graphs lines (*properties decrease case*) have higher values than the green ones (*properties increase case*).

When analysing the density-stiffness equations for both cortical and cancellous tissues at **Table 3.5.1**, it could be seen that the same density variation supposes a different stiffness change, when comparing density increase or decrease cases. Actually, as it can be seen at the table, Young's Modulus variation is slightly lower when density value decreases. It has to be also considered that the effect of density and Young's Modulus on natural frequency is opposite: Young's Modulus is negatively related to natural frequency, and density positively related. Due to this fact, it could be reasonable to think that femur natural frequency variation is higher when a certain region density is reduced.

Concluding, the only explanation to explain that frequency change was higher when region properties decrease is the combined effect that density and Young's Modulus have at each type of bone tissue.

Proportionality of the relationship between femur frequencies and region properties

In order to get an idea of region properties and femur frequency proportionality, the increase and decrease ratios charts for frequency change (**Figure 3.4.8**) have been used. It was surprising to find that, for almost every case, ratios between frequency changes happened to be very close to 2. As **Eq. 6** said, this fact shows that the relationship that exist between femur frequencies and region properties approaches to be linear.

However, relationship linearity could not be considered to be linear for all frequency modes, especially at *density model* results (ratios graph values were not close to 2). Those exceptions were especially found at the highest frequency modes, where natural frequency has higher numerical values. Therefore, as a general fact it can be said that the relationship between femur frequencies and region properties is less linear at higher frequency values.

Another important observation is that the biggest results discontinuities were found at both proximal and distal neck. This may indicate that the relationship proportion is especially weak at those neck regions. Obviously, as it happened with frequencies-

properties relationship sign, necks can be considered as geometric transition zones and hence, problematic regions.

Concluding, frequency ratios chart (**Figure 3.4.8**) showed that the relationship between region properties and femur natural frequencies happens to be, in general terms, constant and linearly proportional. However, at high frequency values and at both neck regions this linearity is weaker. The main region characteristics that seem to affect this relationship linearity were the geometrical location of the region (results difference for every region, especially for the necks) and the material model used (difference from stiffness and density models results).

Euler-Bernoulli ratios variation

The analysis of Euler-Bernoulli ratios differences at **Figure 3.5.9** and **3.5.10** helped to confirm the conclusions taken before.

The most important variation at the graphs occurred when central region properties were changed (remind that this region is the one that has a higher impact on natural frequencies values). Therefore, the size of the region and its type of bone tissue also are determining factors for these ratios variation.

Euler-Bernoulli ratios variation results contributed to confirm the idea of femur frequencies and region properties relationship, due to the symmetry of its results. Euler-Bernoulli variation with properties was found to be positively related for some regions but negative for others. This fact remarks the importance that geometrical location of the region has on the relationship between femur frequencies and region properties.

When comparing both density and stiffness models results, some difference on their numerical value could be found. The most remarkable one is that at each model a certain ratio is more affected by region properties change. The fact that reference values have different Euler-Bernoulli ratios (sagittal and frontal), makes that it can be expected that their variations are going to be partially conditioned by them.

Concluding, Euler-Bernoulli results helped to remark the considerations obtained through the previous results.

4 MEDICAL APPLICATIONS

4.1 AGE CONSIDERATION

4.1.1 *Medical context*

It is a fact that mechanical and geometrical properties of bone change with the age of the subject. There have been many experiments and investigations that tried to correlate age with human bone properties changes, but none of them has come up with a consistent theory.

However, it is well known that mechanical properties of bone start to deteriorate when the subject passes its fourth decade of life. Of course, gender also influence in a great manner the declining value in each of the following decades. This is a very complex subject, due to the values based on population averages, and many articles claim that it cannot be done a reliable correlation of mechanical properties decrease with age [17], [19], [23]. One of the main reasons is the huge range of different subject values that all this mentioned investigators deal with.

Therefore, this project section propose will not be to find a real and complex correlation, nor dealing with big samples data. The only goal in this section is to get an approximate idea of how age can change the mechanical properties used at the previously calculated models, and its impact on natural frequency results.

Following the general idea of the declining mechanical properties since the fourth decade of life, [25] has been chosen to work as the main reference at this point. The article main propose is trying to explain the effect of bone collagen ageing (and its associated degradation) on bone fragility. Although the article claims that toughness is the main mechanical property determined by this collagen ageing, more mechanical properties changes were also measured.

4.1.2 *Properties variation due to age*

Stiffness has been the main mechanical property studied in this article, and therefore its decrease due to age is going to be used. [25] claims that, starting at 40 years old age, cortical bone Young's Modulus value declines at an approximate rate of 2.3% per decade. As said before, this article value is of course approximated and it is conditioned more by collagen degradation than other causes, like mineral content decrease. Anyhow, taking into account theses limitations, its value will be used due to its simplicity and homogeneity between both male and female subjects. As no reliable value has been found for cancellous bone, the same criteria for cortical bone Young's Modulus decrease has been used.

First of all, it has to be taken into account that, at the main literature reference [8] used to obtain femur properties in this project, measurements have been taken from a 70 years old male subject. Therefore, what it is going to be done at this section is simulating the results at different ages, taking into account Young's Modulus reduction with age explained before. Those results are going to be predicted from the 70 years old subject reference properties, the one used in the previous models. The mechanical

properties variation prediction will be performed at each decade, starting at 40 years old until 90. The different age properties will be calculated from the two final model references used at *Model final study* section (*stiffness* and *density models*).

Finally, bone material relationships (**Eq. 1 and 2**) between Young's Modulus and density have to be maintained. Therefore, starting from both reference values of the 70 years old models (*stiffness* and *density models*), Young's Modulus has been increased or decreased for, respectively, lower or higher ages. Density values have been then derived by means of the bone tissue equations used. At **Tables 4.1.1** and **4.1.2** the different ages properties and results variations can be appreciated, for both *stiffness* and *density model* references.

4.1.3 Results and analysis

| Age (years) | Properties | Regions | | | | | Natural frequency (Hz) with % change over reference | | | | | Euler Bernoulli ratios | |
|-------------|----------------------------|---------|------|-------|------|------|--|-------|-------|-------|-------|------------------------|-------|
| | | PH | PN | CT | DN | DH | M1F | M1S | MT | M2F | M2S | FRT | SAG |
| 90 | Young M. (E) (MPa) | 581 | 955 | 15941 | 1741 | 1061 | 320 | 330 | 756 | 891 | 957 | 2,78 | 2,90 |
| | Density (ρ) (Kgm-3) | 485 | 656 | 1938 | 947 | 700 | -1,3% | -1,3% | -1,1% | -1,2% | -1,3% | | |
| 80 | Young M. (E) (MPa) | 595 | 977 | 16316 | 1782 | 1086 | 322 | 332 | 760 | 896 | 964 | 2,78 | 2,90 |
| | Density (ρ) (Kgm-3) | 492 | 666 | 1952 | 960 | 710 | -0,7% | -0,6% | -0,5% | -0,6% | -0,7% | -0,001 | 0,000 |
| 70 (REF.) | Young M. (E) (MPa) | 609 | 1000 | 16700 | 1824 | 1112 | 325 | 334 | 764 | 902 | 970 | 2,78 | 2,91 |
| | Density (ρ) (Kgm-3) | 499 | 675 | 1967 | 974 | 720 | | | | | | | |
| 60 | Young M. (E) (MPa) | 623 | 1023 | 17084 | 1866 | 1138 | 327 | 336 | 769 | 908 | 976 | 2,78 | 2,91 |
| | Density (ρ) (Kgm-3) | 506 | 685 | 1981 | 988 | 730 | 0,7% | 0,6% | 0,6% | 0,6% | 0,6% | -0,003 | 0,001 |
| 50 | Young M. (E) (MPa) | 637 | 1047 | 17477 | 1909 | 1164 | 329 | 338 | 773 | 913 | 983 | 2,78 | 2,91 |
| | Density (ρ) (Kgm-3) | 513 | 694 | 1996 | 1001 | 741 | 1,3% | 1,3% | 1,2% | 1,3% | 1,3% | -0,004 | 0,002 |
| 40 | Young M. (E) (MPa) | 652 | 1071 | 17879 | 1952 | 1191 | 331 | 340 | 778 | 919 | 989 | 2,78 | 2,91 |
| | Density (ρ) (Kgm-3) | 520 | 704 | 2011 | 1015 | 751 | 2,0% | 1,9% | 1,7% | 1,9% | 2,0% | -0,005 | 0,002 |

Table 4.1.1: Results for different ages. Properties, natural frequency and Euler Bernoulli ratios change from stiffness model reference (70 years)

| Age (years) | Properties | Regions | | | | | Natural frequency (Hz) with % change over reference | | | | | Euler Bernoulli ratios | |
|-------------|----------------------------|---------|------|-------|------|-----|--|------|-------|------|------|------------------------|--------|
| | | PH | PN | CT | DN | DH | M1F | M1S | MT | M2F | M2S | FRT | SAG |
| 90 | Young M. (E) (MPa) | 256 | 955 | 16680 | 1741 | 662 | 338 | 353 | 848 | 955 | 1019 | 2,83 | 2,88 |
| | Density (ρ) (Kgm-3) | 295 | 656 | 1966 | 947 | 525 | 4,0% | 5,8% | 10,9% | 5,9% | 5,0% | | |
| 80 | Young M. (E) (MPa) | 262 | 977 | 17073 | 1782 | 677 | 340 | 355 | 853 | 962 | 1025 | 2,83 | 2,88 |
| | Density (ρ) (Kgm-3) | 299 | 666 | 1981 | 960 | 532 | 4,8% | 6,5% | 11,6% | 6,7% | 5,7% | 0,048 | -0,020 |
| 70 (REF.) | Young M. (E) (MPa) | 269 | 1000 | 17475 | 1824 | 693 | 342 | 358 | 858 | 968 | 1032 | 2,83 | 2,88 |
| | Density (ρ) (Kgm-3) | 303 | 675 | 1996 | 974 | 540 | | | | | | | |
| 60 | Young M. (E) (MPa) | 275 | 1023 | 17877 | 1866 | 709 | 345 | 360 | 863 | 975 | 1038 | 2,83 | 2,88 |
| | Density (ρ) (Kgm-3) | 307 | 685 | 2011 | 988 | 548 | 6,2% | 7,8% | 12,9% | 8,1% | 7,0% | 0,048 | -0,021 |
| 50 | Young M. (E) (MPa) | 281 | 1047 | 18288 | 1909 | 725 | 347 | 362 | 868 | 981 | 1045 | 2,83 | 2,88 |
| | Density (ρ) (Kgm-3) | 312 | 694 | 2026 | 1001 | 555 | 6,9% | 8,5% | 13,5% | 8,8% | 7,7% | 0,047 | -0,021 |
| 40 | Young M. (E) (MPa) | 288 | 1071 | 18709 | 1952 | 742 | 349 | 365 | 873 | 988 | 1051 | 2,83 | 2,88 |
| | Density (ρ) (Kgm-3) | 316 | 704 | 2041 | 1015 | 563 | 7,6% | 9,2% | 14,2% | 9,5% | 8,4% | 0,047 | -0,021 |

Table 4.1.2: Results for different ages. Properties, natural frequency and Euler Bernoulli ratios change from density model reference (70 years)

The result analysis will be similar to the one performed at the last Final model study section. In this case, only frequency change and Euler-Bernoulli variation will be analysed. Therefore, the same analysis variables are going to be considered.

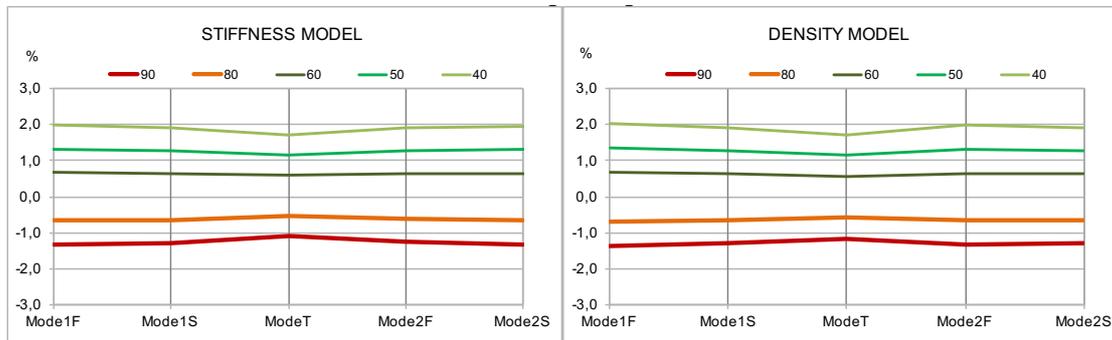


Figure 4.1.1: Frequency percentage changes over reference value (70 years old) at different ages (40 to 90 years)

It can be seen at **Figure 4.1.1** that frequency changes from 70 years old reference are almost constant at every mode, which did not happen at *Final Model study* section cases where properties were only modified at a certain region. This fact can be obviously explained because now all regions suffer the same simultaneous properties variations (Young's Modulus 2.3% variation for each decade of life). In addition, the graphs show that the relationship between material properties and frequency variation is positive. That means that for ages lower than 70 years old, properties increase at the whole femur, natural frequency values also grow, and vice-versa for higher ages. As it could be expected, a constant variation of the properties of all regions results into a homogeneous and proportional change on their natural frequency value.

The previous behaviour happens to be very similar regarding both reference models. Moreover, in both of them also can be noticed that the torsion mode (Mode T) is the only frequency mode that does not vary with the same proportion as the others.

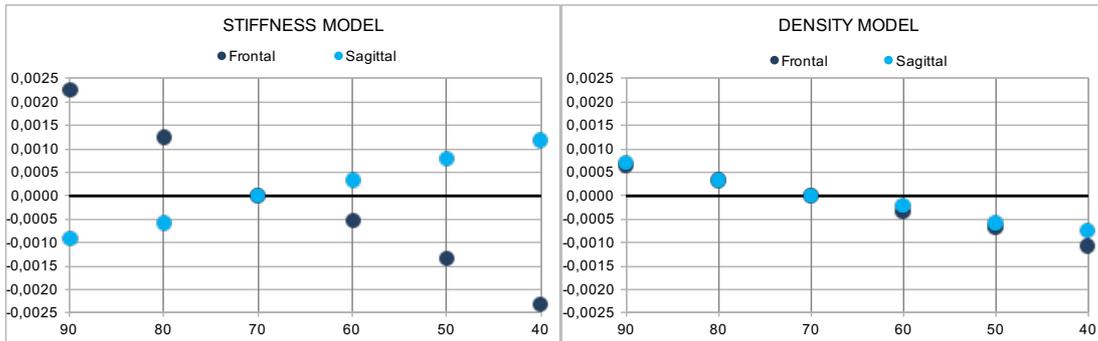


Figure 4.1.2: Euler-Bernoulli ratios difference from reference value (70 years old) at different ages for both models

In the case of the Euler-Bernoulli ratios, their values are also really close to be proportionally related to the properties variation (**Figure 4.1.2**). However, there are some differences between both reference models in values and slope sign, which may indicate that Euler-Bernoulli ratios are also determined by the differences between both material models,

But when analysing the numerical value of Euler Bernoulli ratios difference, the results obtained are really lower (around 100 times) than those ones obtained at the *Model final study* section. That means that Euler Bernoulli ratios difference with age from the reference value is almost negligible. This fact is obvious if we take into account that Euler-Bernoulli ratios relate the value of the second and first bending modes. Therefore, as every frequency mode value changed similarly, especially at the bending modes, the ratios between them have to remain constant with age.

4.2 MAIN BONE ILLNESS: OSTEOPOROSIS AND OSTEOARTHRITIS

4.2.1 *Medical context*

Most of the finite element models have been widely used either to predict, diagnose or characterize bone properties under these two main bone degradation illnesses. Their properties have been massively compared to normal bone, in order to understand the exact causes of bone degradation. There have been made numerous articles following this investigation line [1], [2], [3], [17], [20].

As it happened in the previous case with age, data correlations and predictions are not clear, and there are huge differences between the different investigations that have studied this subject. There have been generally accepted, of course, that bone fragility increases in each of these two illnesses (Osteoporosis and Osteoarthritis), but neither the reasons nor the main properties involved are clear [3]. Therefore, as it happened in the previous section, this project goal is not to open a new investigation line, but just trying to understand the possible effects that mechanical properties change will create on their associated vibrational properties. In addition, several articles have studied the possible benefits of ill bone natural frequencies in order to repair it by mechanical loading at those values. Therefore, predicting problematic bone natural frequencies from their properties is an important issue. Moreover, knowing the natural frequencies range of this problematic bone illnesses can also reciprocally help to develop reliable models to characterize their properties.

4.2.2 *Properties variation due to Osteoporosis and Osteoarthritis*

In order to simulate Osteoporosis and Osteoarthritis effects on bone properties, and hence on natural frequency results, [1] has been used to obtain properties variation due to these illnesses. The investigation article remarked the fact that the most affected area by those diseases is the proximal region. Therefore, properties variation should affect only proximal head and neck regions in this project model. Moreover, in the bone illnesses article used to obtain bone illness properties values, age average of patients happens to be around 70 years old.

Taking into account that the properties used in this thesis came from a 70 years old healthy subject [8], the accuracy of using the values given at bone illnesses article is remarked. Properties variation at the most affected bone regions by Osteoporosis and Osteoarthritis, proximal head and neck, depend on each disease. For each of those bone illnesses, their properties have been calculated using their variation values regarding healthy bone. At Osteoarthritis, Young's Modulus value increases in a 14.8% over normal bone value, and density also increases in a 51.1%; at the Osteoporosis case, Young's Modulus decreases in a 20.3 % over healthy bone value, and density also decreases in a 19.1%. **Note that both illnesses effect on femur properties is the opposite.** Using those properties variation percentages, osteoporotic and osteoarthritic bone properties have been obtained applying them to healthy bone reference properties. This fact means that Young's Modulus-density relationships are not going to be maintained for bone material of both illness, which will be taken into account at the result analysis.

As well as with age consideration, *stiffness* and *density models* have been used as the two references for the normal or healthy bone, divided into two different tables (**Tables 4.2.1 and 4.2.2**). It has to be clarified that at proximal neck region properties are the same for *stiffness* and *density* models.

4.2.3 Results and analysis

| Illness | Properties | Regions | | Natural frequency (Hz) with % change over reference | | | | | Euler Bernoulli ratios | |
|----------------|----------------------------|------------|------------|--|-------|-------|-------|-------|------------------------|--------|
| | | Prox. Head | Prox. Neck | M1S | M1F | MT | M2S | M2F | SAG | FRT |
| REFERENCE | Young M. (E) (MPa) | 609 | 1000 | 325 | 334 | 764 | 902 | 970 | 2,78 | 2,91 |
| | Density (ρ) (Kgm-3) | 500 | 675 | | | | | | | |
| OSTEOPOROSIS | Young M. (E) (MPa) | 485 | 797 | 328 | 339 | 767 | 910 | 973 | 2,77 | 2,87 |
| | Density (ρ) (Kgm-3) | 405 | 546 | 1,1% | 1,6% | 0,3% | 0,9% | 0,3% | -0,004 | -0,038 |
| OSTEOARTHRITIS | Young M. (E) (MPa) | 699 | 1148 | 314 | 325 | 719 | 862 | 956 | 2,74 | 2,95 |
| | Density (ρ) (Kgm-3) | 756 | 1020 | -3,2% | -2,8% | -5,9% | -4,4% | -1,4% | -0,036 | 0,040 |

Table 4.2.1: Femur properties with frequency change for Osteoporosis and Osteoarthritis compared to healthy bone reference for *stiffness model*

| Illness | Properties | Regions | | Natural frequency (Hz) with % change over reference | | | | | Euler Bernoulli ratios | |
|----------------|----------------------------|------------|------------|--|-------|-------|-------|-------|------------------------|--------|
| | | Prox. Head | Prox. Neck | M1F | M1S | MT | M2F | M2S | FRT | SAG |
| REFERENCE | Young M. (E) (MPa) | 269 | 1000 | 342 | 358 | 858 | 968 | 1032 | 2,83 | 2,88 |
| | Density (ρ) (Kgm-3) | 303 | 675 | | | | | | | |
| OSTEOPOROSIS | Young M. (E) (MPa) | 214 | 797 | 345 | 363 | 865 | 975 | 1036 | 2,83 | 2,85 |
| | Density (ρ) (Kgm-3) | 245 | 546 | 0,8% | 1,6% | 0,8% | 0,7% | 0,4% | -0,002 | -0,032 |
| OSTEOARTHRITIS | Young M. (E) (MPa) | 308 | 1148 | 335 | 346 | 807 | 938 | 1011 | 2,80 | 2,92 |
| | Density (ρ) (Kgm-3) | 457 | 1020 | -2,1% | -3,2% | -6,0% | -3,2% | -2,0% | -0,031 | 0,035 |

Table 4.2.2: Femur properties with frequency change for Osteoporosis and Osteoarthritis compared to healthy bone reference for *density model*

Figure 4.2.1 shows, in this case, that the frequency change happens to be different at each mode. The difference with age consideration results (Figure 4.1.1) explanation is obvious, because in this case proximal head and neck are the only regions where properties change.

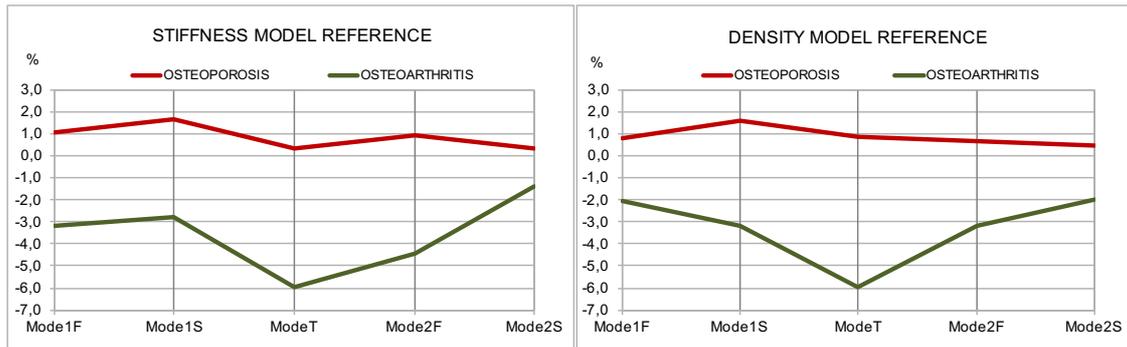


Figure 4.2.1: Frequency change (%) due to osteoarthritis and osteoporosis effect on properties for both models

If we compare both illnesses results, some big differences between them can be found. One of the most important happens to be the absolute value of frequency change. The graphs indicate that the influence of Osteoarthritis on femur frequency changes is around three times bigger than the one of Osteoporosis. The reason of this fact is due to both illnesses properties changes, where density and stiffness varies from healthy bone in a different way. The fact that in Osteoarthritis density variation is much higher than Young's Modulus variation makes that frequency change is higher. By contrary, as at Osteoporosis both properties variations are similar, which results in a lower frequency variation. Remind that density and Young's Modulus effect on femur frequency is opposite.

In addition to the absolute value, frequency change at each mode is also pretty different for both illnesses, even though material was modified at the same regions. At *Final model study*, the geometrical location of the region was concluded to be the most relevant characteristic to determine which mode was more affected by properties variation. However, now geometrical location of the region can not obviously be considered. Frequency change at the different modes has to be only determined by the different material variations due to each bone illness.

It can also be observed that frequency change values have opposite sign for both illnesses. However, the relationship between properties and frequency variation happens to be negatively related at both illnesses. This can be explained because, at the Osteoporosis case, properties were decreased from healthy bone reference, but their frequency results increase, and vice-versa for Osteoarthritis (check **Tables 4.2.1** and **4.2.2** for properties and frequency variation at each illness). This negatively related relationship between properties and frequency correspond to results obtained for proximal head region at *Final Model Study*. Therefore, this fact remarks the importance that the geometrical location of a region has on femur natural frequency variation.

The results obtained on Euler-Bernoulli ratios seem to clarify some patterns at each illness (**Figure 4.2.2**). In the case of Osteoarthritis, both ratios change by a similar variation value, although the frontal one happens to decrease while the frontal one increase its value. In the case of Osteoporosis, the variation of the first Euler-Bernoulli ratio can be neglected compared to the reduction on the second ratio. Moreover, their absolute variation on these ratios variation is considerably higher than at age consideration case, so the effect of Osteoarthritis and Osteoporosis on femur natural frequencies seems to be relevant.

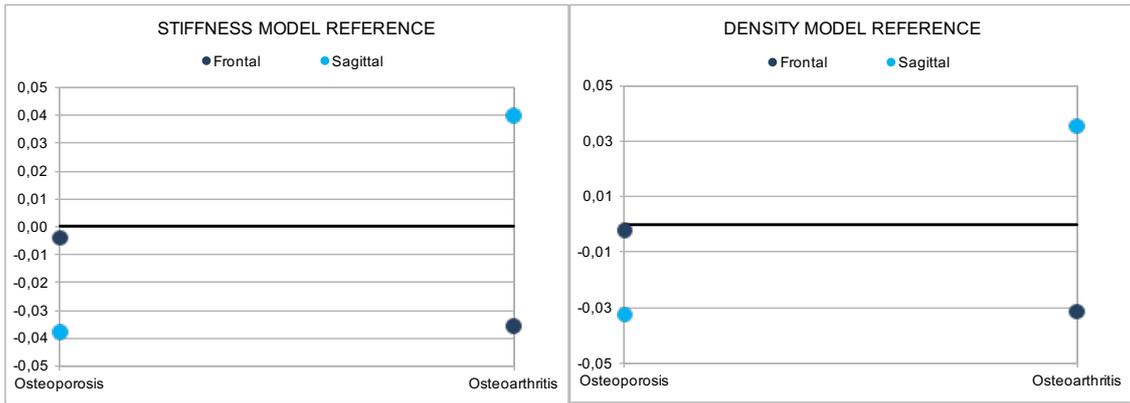


Figure 4.2.2: Euler-Bernoulli ratios variation for Osteoporosis and Osteoarthritis compared to healthy bone for both models

5 PROJECT DEVELOPMENT DIAGRAM AND BUDGET

5.1 PROJECT DEVELOPMENT DIAGRAM

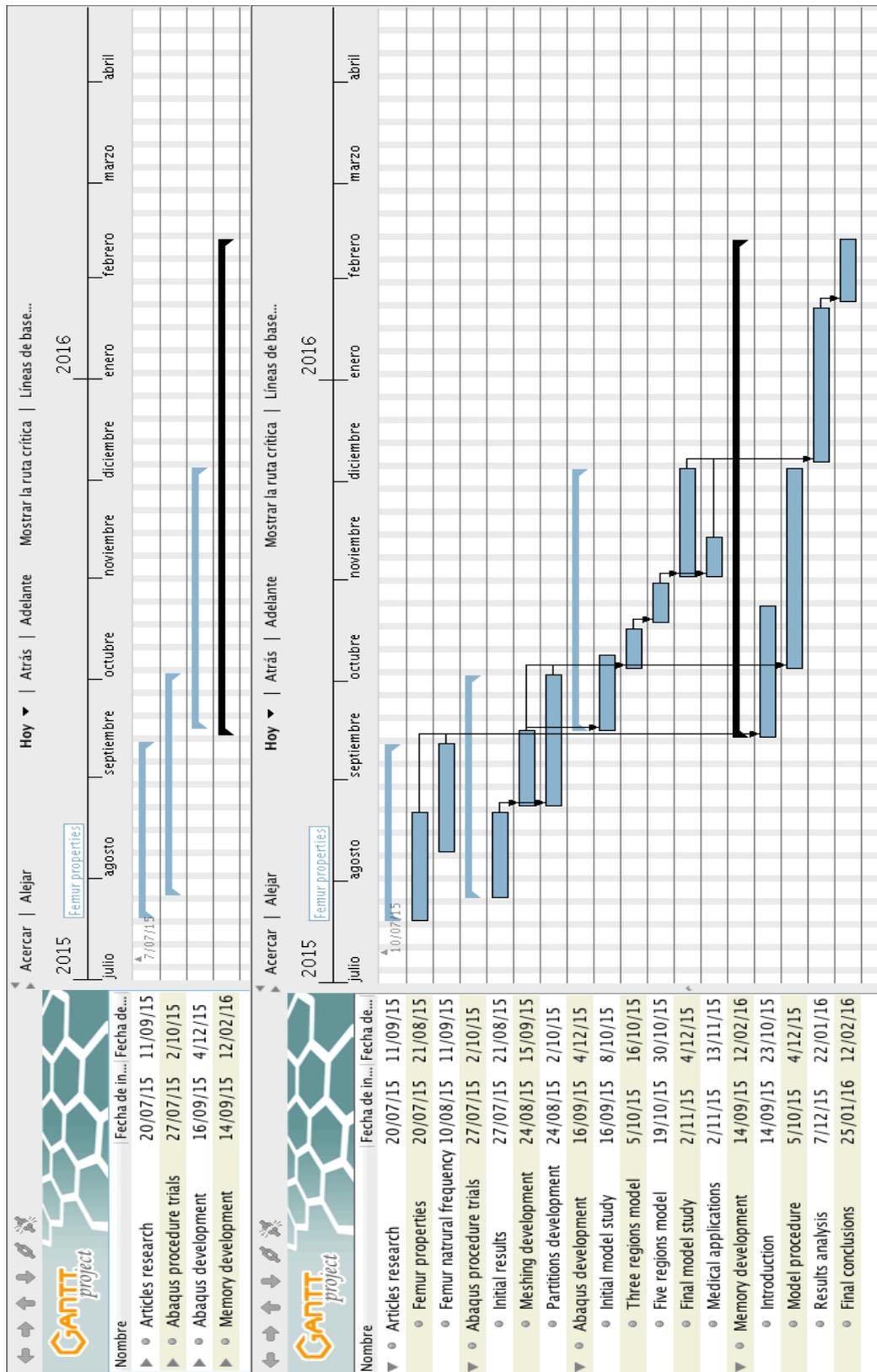


Figure 5.1.1: Project development GANTT diagram using GanttProject software

5.2 BUDGET

This project section will estimate the total cost of the project development, as if an external company was asking to obtain a detailed budget for the study performance. The different costs have been separated into labour costs (human resources) and product costs (software and hardware).

5.2.1 Labour cost

This part of the budget considers the different project sections development time, according to the project development diagram obtained before. It also takes into account the salary of the two categories of engineers involved in this project (junior and senior). Moreover, the average daily working time for each project section is also defined so that real cost can be calculated. The different variables and final results are explained at **Table 5.2.1**.

| Project section | Total time (days) | Average daily work (hours / day) | Total working time (hours) | Worker category | Salary (€ / hour) | Labour cost (€) |
|------------------------------|-------------------|----------------------------------|----------------------------|-----------------|-------------------|--------------------|
| Information research | 40 | 3 | 120 | Junior engineer | 15 | 1,800.00 € |
| Abaqus procedure trials | 50 | 4 | 200 | Junior engineer | 15 | 3,000.00 € |
| Abaqus development | 58 | 4 | 232 | Junior engineer | 15 | 3,480.00 € |
| Memory development | 110 | 3 | 330 | Junior engineer | 15 | 4,950.00 € |
| Project supervision | 15 | 2 | 30 | Senior engineer | 25 | 750.00 € |
| Total Labour cost (€) | | | | | | 13,980.00 € |

Table 5.2.1: Labour cost

5.2.2 Hardware and software costs

This second part of the budget refers to the different products used to develop the present project. Therefore, the different hardware and software devices needed to perform the project are specified, as well as its amortization time and use. All the products specifications and costs are detailed at **Table 5.2.2**.

| Product | Product cost (€) | Amortization period (months) | Product use (months) | Amortization use (%) | Final Cost (€) |
|--------------------------------|------------------|------------------------------|----------------------|----------------------|-------------------|
| Toshiba Satellite Z30-A-1DJ i5 | 893 | 48 | 7 | 15% | 130.23 € |
| Abaqus CAE License | 21950 | 12 | 5 | 42% | 9,145.83 € |
| Microsoft Office License | 149 | 48 | 6 | 13% | 18.63 € |
| GanttProject License | Free | - | 1 | - | - |
| Total products cost (€) | | | | | 9,294.69 € |

Table 5.2.2: Hardware and software cost

5.2.3 Total project cost

Combining the total cost for labour and hardware/software products, the total cost of the project is explained at Table 5.2.3.

| | |
|-------------------------------|--------------------|
| Labour total cost | 13.980,00 € |
| Hardware/software cost | 9.294,69 € |
| Total project cost (€) | 23.274,69 € |

Table 5.2.3: Total project cost

6 CONCLUSIONS AND FUTURE WORK

Before to start this conclusions section, a brief summary of the different goals at the beginning of this thesis is going to be performed, so that it can be seen that they have been successfully achieved.

Initially, at *model considerations* section the different approximations and assumptions taken from literature used in the model performance were discussed. In addition, at *Initial model study* it has been explained both geometrical and material approaches used to characterize femur properties, and how they were introduced in Abaqus software.

During *Real femur models* development, the evolution of the different properties criteria has been explained, as well as the analysis tools based on frequency results that helped to improve those models. Moreover, at the *Final model study* section a deep study of the final models results was performed, obtaining the influence that the different model regions had on natural frequency value.

Finally, two medical considerations were used in order to modify the obtained models properties and analyse the changes observed on natural frequencies.

Once this brief summary revised that the initially explained goals have been achieved, the actual conclusions obtained through the different sections are going to be explained.

6.1 Femur properties approaches success

During the different models performed at *Real FEA models* section, an improvement process has been obtained on the different models accuracy, by means of natural frequency analysis and the gradual approaches used. That means that **each model** performed included **more complex** considerations, and their **results** happened to **improve**. The **last two models** performed (*stiffness* and *density*) were the **most complete** ones, as they included five different regions and bone tissue relationships between Young's Modulus and density. The main improvement achieved in the last two models performed has been referred to the **accuracy** of the **Euler-Bernoulli ratios** obtained (ratios between second over first bending modes). Moreover, natural frequency values, specially the ones obtained at **stiffness model** were **really accurate** compared to literature reference values.

6.2 Final models influence on natural frequency

The relatively good accuracy obtained in the **last two models** (*stiffness* and *density*) has lead to deeply analyse them in order to obtain different conclusions. The **analysis** performed helped to understand and **explain the influence** of those model **considerations** (geometrical and material) **on** the obtained **vibrational results**.

Material properties were modified at each region in order to show their impact on femur natural frequencies. It has been observed that the effect on femur natural frequencies was difference depending which region properties were modified. The main region characteristics found to determine femur frequency variation observed have been type of bone tissue, region size and region geometry. **Cortical bone tissue,**

a **higher region size** and **irregular shape** happened to **increase** the **region influence on natural frequency values**. Femur frequency modes variate in a different amount when each region modified its properties. It was found that the **most affected modes were determined by the geometrical location of the region** which properties were modified.

Moreover, a clear **linearly proportional relationship between each region properties variation and femur natural frequencies** results was found. **Region geometrical situation** was the only factor found to **affect the relationship sign** (being the heads positively related and the central body negatively related). However, some **problematic areas** at which the **linearity of the relationship was relatively low** were **both necks**, where also relationship sign was not clear. This fact is understandable considering them as geometrical transition areas of two regions with very different properties (heads and central body). Moreover, the general statement of a linear and proportional relationship between region properties and femur frequency values happened to be **less accurate at higher frequency modes**.

The final study of Euler-Bernoulli ratios variation with material region modification helped to remark the previous conclusions obtained. Finally, the small differences obtained from density and stiffness models results also increase the validity of those conclusions.

6.3 Medical applications

The stiffness decrease due to bone **collagen degradation concluded in a gradual decrease of femur natural frequency value with age**, once the age of 40 years is reached. However, this variation was not considered to be really determining, being **femur frequency value decreased only around 0.65% of its value per decade**. This variation value was found to be proportional and constant at every frequency mode. This frequency values homogeneity supposed a small variation of the Euler Bernoulli ratios compared to the previous section study.

As well as it was done with age, the effect on femur natural frequencies of the two main bone illness, **Osteoporosis and Osteoarthritis**, was studied. Both illnesses **affect femur properties at proximal head and neck regions**, and hence femur natural frequencies results suffered a variation from its original value. There were certain differences on femur natural frequencies variation due to each illness. First of all, results showed that femur **frequencies values increased for Osteoporosis and decreased for Osteoarthritis**. However, it has to be considered that the relationship between bone properties and femur frequencies was negatively related for both cases (healthy bone properties are reduced for Osteoporosis and increased for Osteoarthritis). In addition, it was observed that **Osteoarthritis influence on femur frequencies was three times higher than Osteoporosis**. Moreover, frequency changed differently at each mode when comparing both illnesses results. The reason to those differences was the fact that Young's Modulus and density relationships for each illness bone material was very different. Summarizing, assuming the validity of this simulation, the **big differences** of both illnesses influence on healthy bone frequency results makes that their **diagnosis could be relatively easily distinguished by natural frequency analysis**.

6.4 Future work

First of all, femur models accuracy can be highly improved taking into account the actual femur structural and material properties introduced in this project. The different aspects that define femur bone are explained at *Femur bone properties* section. Regarding structural considerations, the main improvement that can be performed is considering the **radial material distribution** of the femur, or any other long bone. That is, reproducing the femur internal structure: bone medullae at the interior with cancellous tissue surrounding it and the external cortical tissue cortex. Regarding material considerations, another idea that must be taken into account in order to create an accurate bone model is to **deeply study density and stiffness relationships**.

As it could be proved in this project, the relationship between bone properties and natural frequency results is very intense. Therefore, the idea of **studying femur regions influence on natural frequency** results could be applied in order to improve the previously explained model characterization, combining experimental and numerical data. Once natural frequency femur values for a certain patient are experimentally measured, **numerical model accuracy can be increased** modifying its regions properties, which influence on frequency modes has been deeply studied and analysed. The most direct application of those high accurate frequency simulation models is the use of **vibrational medical techniques that need its exact value** to enhance and repair bone tissue.

The study of bone properties and natural frequency variations with age performed in this project opens also new possibilities that will help to determine personal factors strongly related with bone characteristics. By means of careful studies made with big data samples, **complex bone models to predict frequency results** can be performed **including different input criteria** such as age, gender, weight, height or physical activity.

The simple simulation performed at this project with Osteoporosis and Osteoarthritis bone illnesses corroborates that natural frequency analysis can be very helpful to **predict any kind of unhealthy bone case**. Moreover, linking this fact with bone region influence on natural frequency, it is obvious that also vibrational results can be used to **diagnose any kind of fracture or anomalous part into the whole femur bone**, and also as a great control tool to follow bone rehabilitation and healing processes.

Finally, it also has to be said that bone study applications not only lay on the medical field. As it was said at the beginning of this project, engineering field can be also inspired by bone structure and material. The high efficient stress and properties distribution of **femur** are originated by means of the association of **different structural elements at many different sizes levels**. Therefore, studying how those elements are associated can bring many **inspiring ideas in order to develop, for instance, new composite materials** combining high strengths with light weights. Moreover, **understanding bones as a type of beam**, the present study of femur natural frequencies can also contribute to improve and bring new ideas to find new vibrational techniques to diagnose beams failure.

As it was initially said, it cannot be forgotten that nature has been always the main inspiration for engineering.

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