



Working Paper # 06-04 (02)
Statistics and Econometrics Series
January 2006

Departamento de Estadística
Universidad Carlos III de Madrid
Calle Madrid, 126
28903 Getafe (Spain)
Fax (34) 91 624-98-51

USING AUXILIARY RESIDUALS TO DETECT CONDITIONAL HETEROSCEDASTICITY IN INFLATION

Carmen Broto and Esther Ruiz*

Abstract

In this paper we consider a model with stochastic trend, seasonal and transitory components with the disturbances of the trend and transitory disturbances specified as QGARCH models. We propose to use the differences between the autocorrelations of squares and the squared autocorrelations of the auxiliary residuals to identify which component is heteroscedastic. The finite sample performance of these differences is analysed by means of Monte Carlo experiments. We show that conditional heteroscedasticity truly present in the data can be rejected when looking at the correlations of observations or of standardized residuals while the autocorrelations of auxiliary residuals allow us to detect adequately whether there is heteroscedasticity and which is the heteroscedastic component. We also analyse the finite sample behaviour of a QML estimator of the parameters of the model. Finally, we use auxiliary residuals to detect conditional heteroscedasticity in monthly series of inflation of eight OECD countries. We conclude that, for most of these series, the conditional heteroscedasticity affects the transitory component while the long-run and seasonal components are homoscedastic. Furthermore, in the countries where there is a significant relationship between the volatility and the level of inflation, this relation is positive, supporting the Friedman hypothesis.

Keywords: Leverage effect, QGARCH, seasonality, structural time series models.

* Corresponding author. Tel: 34 91 6249851. Fax: 34 91 6249849. e-mail: ortega@est-econ.uc3m.es. Financial support from projects IHP-RNT-00-2 by the European program "Research Training Networks" and BEC2002.03720 from the Spanish Government are gratefully acknowledge. The usual disclaimers apply.

Abstract

In this paper we consider a model with stochastic trend, seasonal and transitory components with the disturbances of the trend and transitory disturbances specified as QGARCH models. We propose to use the differences between the autocorrelations of squares and the squared autocorrelations of the auxiliary residuals to identify which component is heteroscedastic. The finite sample performance of these differences is analysed by means of Monte Carlo experiments. We show that conditional heteroscedasticity truly present in the data can be rejected when looking at the correlations of observations or of standardized residuals while the autocorrelations of auxiliary residuals allow us to detect adequately whether there is heteroscedasticity and which is the heteroscedastic component. We also analyse the finite sample behaviour of a QML estimator of the parameters of the model. Finally, we use auxiliary residuals to detect conditional heteroscedasticity in monthly series of inflation of eight OECD countries. We conclude that, for most of these series, the conditional heteroscedasticity affects the transitory component while the long-run and seasonal components are homoscedastic. Furthermore, in the countries where there is a significant relationship between the volatility and the level of inflation, this relation is positive, supporting the Friedman hypothesis.

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1 Introduction

Friedman (1977) suggests that higher inflation levels lead to greater uncertainty about future inflation¹; see Ball (1992) for a economic theory explaining this causality relationship.

The empirical evidence on the Friedman hypothesis is diverse. The first problem faced by the empirical researcher is that the uncertainty of inflation is an unobserved variable and, consequently, there is a question about how to measure this uncertainty. The original papers analysing empirically the Friedman hypothesis, used as proxies for the uncertainty, the inflation variability or the dispersion of the forecasts; see, for example, Okun (1971), Foster (1978) or Cukierman and Wachtel (1979). Later, after the introduction of the ARCH model by Engle (1982), many authors measured the uncertainty of inflation by the conditional variance; see, for example, Engle (1983), Bollerslev (1986) and Cosimano and Jansen (1988). These authors did not find empirical support for the Friedman hypothesis. However, it has been supported by Joyce (1995), Baillie et al. (1996), Grier and Perry (1998), Kim and Nelson (1999), Krontonikas (2004), Conrad and Karanasos (2004) and Daal et al. (2005) among many others. Finally, there are studies as, for example, Hwang (2001), that find a negative relationship between level of inflation and its future uncertainty.

These contradictory results can be explained taking into account that, the original GARCH models fitted to represent the dynamic evolution of inflation have two main limitations. First, the response of the conditional variance to positive and negative inflation changes is symmetric and this property is intrinsically incompatible with the Friedman hypothesis. In this sense, Brunner and Hess (1993) propose a State-Dependent model that allows for asymmetric responses; see also Caporale and McKierman (1997) for an empirical implementation of this model. Alternatively, Daal et al. (2005) also consider modelling the uncertainty of inflation using the asymmetric power GARCH model of Ding et al. (1993). The second limitation is that the

¹The opposite type of causation, between inflation uncertainty and the level of inflation, has also been considered between others by Cukierman (1992), Fountas *et al.* (2000) and Conrad and Karanasos (2005) among many others; see Cukierman and Meltzer (1986) for a theoretical justification. However, this relationship has been found to be empirically weaker and we focus on the Friedman hypothesis.

early GARCH models did not distinguish between short and long run uncertainty. There is a number of papers that made this distinction; see, for example, Ball and Cecchetti (1990), Evans (1991), Evans and Watchel (1993), Kim (1993), García and Perron (1996), Grier and Perry (1998) and Kontonikas (2004). Most of these studies find stronger evidence of the Friedman hypothesis in the long run although there is mixed evidence.

The models previously proposed to represent the dynamic evolution of inflation do not include a seasonal component. Given that seasonality is a central characteristic of monthly inflation, its empirical analysis has been generally based on seasonally adjusted observations. To identify the presence of heteroscedasticity in the components, we propose to use the differences between the autocorrelations of squares and the squared autocorrelations of the auxiliary residuals. We analyse the finite sample behaviour of these differences and show that they can be useful to identify conditional heteroscedasticity even in series where looking at the original data or the traditional standardized residuals, we may conclude that they are homoscedastic. Furthermore, looking at auxiliary residuals may help to identify which of the components of the model is heteroscedastic. In this paper, we extend the random walk plus noise model with QGARCH disturbances proposed by Broto and Ruiz (2005) adding a seasonal component that is assumed to be homoscedastic. Furthermore, the model allows potentially different responses of long-run and short-run volatilities to positive and negative disturbances.

The paper is organized as follows. Section 2 introduces the Q-STARARCH model with seasonality and describes its properties. In section 3, we analyse the finite sample properties of the sample autocorrelations of squared auxiliary residuals proposed to detect whether a given component of the model is conditionally heteroscedastic. In section 4, we analyse the asymptotic and finite sample properties of a Quasi-Maximum Likelihood (QML) estimator based on the prediction error decomposition of the Gaussian log-likelihood. The proposed model is fitted in section 5 to monthly inflation series of the OCDE countries. Finally, section 6 concludes the paper.

2 QSTARCH model with seasonal effects

Consider that the series of interest, y_t , can be decomposed into a long run component, representing an evolving level, μ_t , a stochastic seasonal component, δ_t , and a transitory component, ε_t . If the level follows a random walk, the seasonal component is specified using a dummy variable formulation and the transitory component is a white noise, the resulting model for y_t is given by

$$\begin{aligned} y_t &= \mu_t + \delta_t + \varepsilon_t & (1) \\ \mu_t &= \mu_{t-1} + \eta_t \\ \delta_t &= - \sum_{i=1}^{s-1} \delta_{t-i} + \omega_t \end{aligned}$$

where s is the seasonal period; see Harvey (1989). The transitory and long-run disturbances are defined by $\varepsilon_t = \varepsilon_t^\dagger h_t^{1/2}$ and $\eta_t = \eta_t^\dagger q_t^{1/2}$ respectively where ε_t^\dagger and η_t^\dagger are mutually independent Gaussian white noise processes and h_t and q_t are defined as GQARCH processes² given by

$$\begin{aligned} h_t &= \alpha_0 + \alpha_1 \varepsilon_{t-1}^2 + \alpha_2 h_{t-1} + \alpha_3 \varepsilon_{t-1} & (2) \\ q_t &= \gamma_0 + \gamma_1 \eta_{t-1}^2 + \gamma_2 q_{t-1} + \gamma_3 \eta_{t-1} \end{aligned}$$

The parameters $\alpha_0, \alpha_1, \alpha_2, \alpha_3, \gamma_0, \gamma_1, \gamma_2$ and γ_3 satisfy the usual conditions to guarantee the positivity and stationarity of h_t and q_t ; see Sentana (1995). Finally, the disturbance of the seasonal component is assumed to be a Gaussian white noise with variance σ_ω^2 independent of ε_t and η_t . Model (1) is able to distinguish whether the possibly asymmetric ARCH effects appear in the permanent and/or in the transitory component. Furthermore, the conditional variances in (2) have different responses to shocks of the same magnitude but different sign.

²Alternatively, the variances of the unobserved components can be specified as Stochastic Volatility (SV) processes. However, the estimation of unobserved component models with SV disturbances is usually based on Simulated Maximum Likelihood and it is rather difficult to extend the method to allow for different components having different evolutions of the volatility; see, for example, Brandt and Kang (2004) and Koopman and Bos (2004). Another proposal of unobserved component models with heteroscedastic errors can be found in Ord *et al.* (1997), where instead of considering different disturbance processes for each unobserved component, the source of randomness is unique.

Although the series y_t is non-stationary, it can be transformed into stationarity by taking seasonal differences. The stationary form of model (1) is given by

$$\Delta_s y_t = S(L)\eta_t + \Delta\omega_t + \Delta_s \varepsilon_t \quad (3)$$

where Δ_s and Δ are the seasonal and regular difference operators given by $\Delta_s = 1 - L^s$ and $\Delta = 1 - L$ respectively and $S(L) = 1 + L + \dots + L^{s-1}$. The dynamic properties of $\Delta_s y_t$ can be analyzed by deriving its autocorrelation function (acf) that is given by

$$\rho(h) = \begin{cases} \frac{(s-1)\sigma_\eta^2 - \sigma_\omega^2}{s\sigma_\eta^2 + 2\sigma_\omega^2 + 2\sigma_\varepsilon^2} & h = 1 \\ \frac{(s-h)\sigma_\eta^2}{s\sigma_\eta^2 + 2\sigma_\omega^2 + 2\sigma_\varepsilon^2} & h = 2, \dots, s-1 \\ \frac{-\sigma_\varepsilon^2}{s\sigma_\eta^2 + 2\sigma_\omega^2 + 2\sigma_\varepsilon^2} & h = s \\ 0 & h > s \end{cases} \quad (4)$$

where $\sigma_\varepsilon^2 = \alpha_0/(1 - \alpha_1 - \alpha_2)$ and $\sigma_\eta^2 = \gamma_0/(1 - \gamma_1 - \gamma_2)$. Notice that the innovations of the reduced form of $\Delta_s y_t$ are uncorrelated although not independent neither Gaussian; see Breidt and Davis (1992). The non-Gaussianity and the lack of independence may affect the sample properties of some estimators often used in empirical applications.

The presence of asymmetric ARCH effects is reflected in the kurtosis of $\Delta_s y_t$ given by

$$\begin{aligned} \kappa_y = & (sq_\eta + 2q_\omega + 2)^{-2} \left[q_\eta^2 \left(s\kappa_\eta + 6 \sum_{i=1}^{s-1} (s-i)(1 + (\kappa_\eta - 1)\rho_2^\eta(i)) \right) + \right. \\ & \left. + 2\kappa_\varepsilon + 6(1 + (\kappa_\varepsilon - 1)\rho_2^\varepsilon(s)) + 12(sq_\eta q_\omega + sq_\eta + 2q_\omega + q_\omega^2) \right] \end{aligned} \quad (5)$$

where κ_ε and κ_η are the kurtosis coefficients of ε_t and η_t , given by $\kappa_\varepsilon = 3(1 + \alpha_1 + \alpha_2 + (\alpha_3^2/\alpha_0))(1 - \alpha_1 - \alpha_2)/(1 - 3\alpha_1^2 - \alpha_2^2 - 2\alpha_1\alpha_2)$ and $\kappa_\eta = 3(1 + \gamma_1 + \gamma_2 + (\gamma_3^2/\gamma_0))(1 - \gamma_1 - \gamma_2)/(1 - 3\gamma_1^2 - \gamma_2^2 - 2\gamma_1\gamma_2)$, respectively and $\rho_2^\varepsilon(h)$ is the autocorrelation of order

h of ε_t^2 given by

$$\rho_2^\varepsilon(h) = \begin{cases} \frac{2\alpha_1(1 - \alpha_1\alpha_2 - \alpha_2^2) + (\alpha_3/\sigma_\varepsilon^2)(3\alpha_1 + \alpha_2)}{2(1 - 2\alpha_1\alpha_2 - \alpha_2^2) + (3\alpha_3/\sigma_\varepsilon^2)}, & h = 1 \\ (\alpha_1 + \alpha_2)^{h-1}\rho_2^\varepsilon(h-1), & h > 1 \end{cases} \quad (6)$$

$\rho_2^\eta(h)$ is the autocorrelation of order h of η_t^2 which is analogous to the one of ε_t^2 . Finally, q_η and q_ω are the signal to noise ratios of the long-run and the seasonal components, respectively, given by $q_\eta = \sigma_\eta^2/\sigma_\varepsilon^2$ and $q_\omega = \sigma_\omega^2/\sigma_\varepsilon^2$. As expected, the kurtosis in (5) is 3 when all the noises are homoscedastic.

If the disturbances were homoscedastic and Gaussian, the autocorrelations of $(\Delta_s y_t)^2$ are the squared of the autocorrelations in expression (4); see Maravall (1983) and Palma and Zavallos (2004). However, the presence of conditional heteroscedasticity in at least one of the components, generates autocorrelations of squares larger than the squared autocorrelations. For the particular case of the local level model, i.e. model (1) without seasonal component, Broto and Ruiz (2005) show that the effects of the presence of asymmetries in the volatilities of the components, on the autocorrelations of squares are negligible. Therefore, for simplicity, the asymmetric parameters in equations (2) are fixed to zero. In this case, after some very tedious although straightforward algebra, we derive the following expression of the autocovariance function of $(\Delta_s y_t)^2$ in the seasonal QSTARCH model

$$\begin{aligned} Cov\{(\Delta_s y_t)^2, (\Delta_s y_{t-\tau})^2\} = \\ \sigma_\varepsilon^4 [q_\eta^2 \{(s-1)(\kappa_\eta - 1) + 2(\kappa_\eta - 1) \sum_{i=1}^{s-1} (s-i)\rho_2^\eta(i) + \rho_2^\eta(s) \\ + 4 \sum_{i=1}^{s-2} (s-i-1)(1 + (\kappa_\eta - 1)\rho_2^\eta(i))\} + 2q_\omega^2 \\ + (\kappa_\varepsilon - 1)\{2\rho_2^\varepsilon(1) + \rho_2^\varepsilon(s-1) + \rho_2^\varepsilon(s+1)\} - 4(s-1)q_\eta q_\omega], \quad \tau = 1 \\ \sigma_\varepsilon^4 [q_\eta^2 ((\kappa_\eta - 1)\{(s-h) + 2(s-h) \sum_{i=1}^{h-1} \rho_2^\eta(i) + 2 \sum_{i=h}^{s-h} (s-i)\rho_2^\eta(i) \\ + \sum_{i=s-h+1}^{s+h-1} (s+h-i)\rho_2^\eta(i)\} + 4 \sum_{i=1}^{s-h-1} (s-i-h)(1 + (\kappa_\eta - 1)\rho_2^\eta(i)) \\ + (\kappa_\varepsilon - 1)\{2\rho_2^\varepsilon(h) + \rho_2^\varepsilon(s-h) + \rho_2^\varepsilon(s+h)\}], \quad \tau = 2, \dots, s-1 \end{aligned}$$

$$\begin{aligned}
& \sigma_\varepsilon^4 [q_\eta^2 (\kappa_\eta - 1) (\sum_{i=1}^s i \rho_2^\eta(i) + \sum_{i=1}^{s-1} (s+i) \rho_2^\eta(i)) \\
& + (\kappa_\varepsilon - 1) \{1 + \rho_2^\varepsilon(s) + \rho_2^\varepsilon(2s)\}], \quad \tau = s \\
& \sigma_\varepsilon^4 [q_\eta^2 (\kappa_\eta - 1) (\sum_{i=h+1-s}^h (i-h+s) \rho_2^\eta(i) + \sum_{i=h+1}^{h+s-1} (h+s-i) \rho_2^\eta(i)) \\
& + (\kappa_\varepsilon - 1) \{2\rho_2^\varepsilon(h) + \rho_2^\varepsilon(h-s) + \rho_2^\varepsilon(s+h)\}] \quad \tau > s
\end{aligned} \tag{7}$$

Note that when the signal to noise ratio of the long-run component is small, the heteroscedasticity of this component does not affect the autocorrelations of squares. However, when this ratio is not too small, the effect of the heteroscedasticity in the long-run component is larger than the effect of the transitory component. The variance of $(\Delta_s y_t)^2$ is given by

$$\begin{aligned}
Var [(\Delta_s y_t)^2] &= \sigma_\varepsilon^4 \left[q_\eta^2 \left(2s^2 + s(\kappa_\eta - 3) + 6(\kappa_\eta - 1) \sum_{i=1}^{s-1} (s-i) \rho_2^\eta(i) \right) + \right. \\
& \left. 8q_\omega^2 + 2(\kappa_\varepsilon + 1) + (4(\kappa_\varepsilon - 1) \rho_2^\varepsilon(s)) + 8sq_\eta q_\omega + 8sq_\eta + 12q_\omega \right] \tag{8}
\end{aligned}$$

From expressions (??) and (8), it is possible to obtain the expression of the autocorrelations of $(\Delta_s y_t)^2$. Note that when the noises are homoscedastic and Gaussian these autocorrelations are equal to the squared autocorrelations of $\Delta_s y_t$ in expression (4). Figure 1 plots these autocorrelations for the following four Q-STARCh models with $s = 4$,

	α_0	α_1	α_2	α_3	γ_0	γ_1	γ_2	γ_3	σ_ω^2
<i>M0</i>	1	0	0	0	0.25	0	0	0	0.01
<i>M1</i>	0.05	0.15	0.8	0.17	0.25	0	0	0	0.01
<i>M2</i>	4	0	0	0	0.05	0.15	0.8	0.17	0.01
<i>M3</i>	0.2	0.15	0.8	0.17	0.05	0.15	0.8	0.17	0.01

The value of the parameters have been chosen to resemble the values typically estimated when analysing real time series of monthly inflation. In particular, the signal to noise ratio of the long run component, $q_\eta = 0.25$, is smaller than one, because usually the variance of the long run component of inflation is smaller than the variance of the transitory component. The variance of the seasonal component is also rather small, $\sigma_\omega^2 = 0.01$. With respect to the presence of conditional heteroscedasticity, model *M0*,

has all its components homoscedastic. However, the short run disturbance of model $M1$ is heterocedastic while in model $M2$, the long run component is heteroscedastic. Finally, both disturbances are heteroscedastic in model $M3$.

In Figure 1, we can observe that, given that q_η is rather small, the effect on the autocorrelations of $(\Delta_4 y_t)^2$ is larger when the heteroscedasticity appears in the transitory component than when it appears in the long-run component. To analyse the finite sample properties of the estimates of the autocorrelations of $(\Delta_s y_t)^2$, Figure 1 also plots the sample means through 1000 replicates of the sample autocorrelations of $\Delta_4 y_t$ and $(\Delta_4 y_t)^2$, $r(h)$ and $r_2(h)$, respectively, of series of size $T = 500$ generated by the models described before. We can observe that while the sample autocorrelations of $\Delta_4 y_t$ are unbiased, the biases of the sample autocorrelations of $(\Delta_4 y_t)^2$ are negative for small lags and positive for large lags.

Finally, note that as we have commented before, the autocorrelations of squares are larger than the squared autocorrelations if any of the components is conditionally heteroscedastic. Consequently, it is possible to identify the presence of conditional heteroscedasticity by looking at the differences between both functions. As an illustration, the third row of Figure 1 also plots, for the same four models considered before, the population differences $\rho_2(h) - (\rho(h))^2$ and the corresponding sample means through 1000 replicates of the differences $r_2(h) - (r(h))^2$. First of all, Figure 1 shows that, as expected, looking at $\rho_2(h) - (\rho(h))^2$, the presence of conditional heteroscedasticity is more evident in model $M1$ than in model $M2$. Note that, given that σ_ε^2 is larger than σ_η^2 , the characteristics of the short run component are expected to be more evident in the reduced form than those of the long run component. To analyse in more detail the finite sample properties of the differences between the autocorrelations of squares and the squared autocorrelations to identify the presence of heteroscedasticity, Table 1 reports the Monte Carlo means and standard deviations of $r_2(1) - (r(1))^2$ for sizes $T = 100, 200$ and 500 and models $M0, M1$ and $M2$. These results suggest that under the null of conditional homoscedasticity, the distribution of $r_2(h) - (r(h))^2$ has approximately zero mean and variance $\frac{1}{T}$ even for relatively small sample sizes. When the errors are heterocedastic and the sample size is sufficiently large, the mean of the differences is not zero especially in model $M1$. However, the standard deviations are

larger than $\frac{1}{T}$ and it is rather difficult to detect the presence of heteroscedasticity by looking at the differences between the autocorrelations of $(\Delta_s y_t)^2$ and the squared autocorrelations of $\Delta_s y_t$. Figure 2 plots kernel densities of $r_2(1) - (r(1))^2$ of the homoscedastic model $M0$ and show that, under the null hypothesis of homoscedasticity and for the sample sizes considered, the Normal distribution is an adequate distribution to approximate the finite sample distribution of these differences.

Alternatively, it is possible to test for conditional heteroscedasticity by looking at the differences between the autocorrelations of squares and the squared autocorrelations of the estimated innovations. Assuming that the parameters are known, the Kalman filter generates Minimum Mean Square Linear (MMSL) one-step-ahead prediction errors, given by $\nu_t = \Delta_s y_t - {}_t E_{t-1}(\Delta_s y_t)$. The t under the expectation operator means that the expectation is conditional on the information available at time t ; see, for example, Harvey (1989). The Monte Carlo results in Table 1 show that when the model is homoscedastic, the mean and variance of the differences of the autocorrelations for the innovations are approximately zero and $1/T$ respectively. Figure 2, where the corresponding kernel density is plotted, shows that the Normal approximation is adequate. On the other hand, in the heteroscedastic model $M1$, we can observe that the mean of the differences is larger than when looking at the stationary observations, $\Delta_s y_t$, while the standard deviations are similar. Therefore, the identification of heteroscedasticity will be easier although very large samples are still needed. However, the results for models $M2$ are rather disappointing in the sense that the sample mean of the differences are smaller than for $\Delta_s y_t$ making the identification of the heteroscedasticity even more difficult. This results is also clear in the first row of Figure 3 that plots the sample means through 1000 replicates of $Corr[\nu_t^2, \nu_{t-\tau}^2] - (Corr[\nu_t, \nu_{t-\tau}])^2$ for series generated by the four models described above with $T = 500$. The shape of these differences is similar to the pattern observed in Figure 1 for the differences between the sample autocorrelations corresponding to $\Delta_4 y_t$. The only noticeable difference is that the differences are slightly larger for the autocorrelations of the innovations.

When, as usual, the parameters of the model are unknown, they can be estimated by, for example, Quasi-Maximum Likelihood (QML)³, and the Kalman filter

³The QML estimator of the parameters is described in the following section.

implemented with the estimated parameters. In this case, we obtain the estimated innovations denoted by $\hat{\nu}_t$. The first row of Figure 3 also plots the Monte Carlo means of $Corr[\hat{\nu}_t^2, \hat{\nu}_{t-\tau}^2] - (Corr[\hat{\nu}_t, \hat{\nu}_{t-\tau}])^2$ for the same models as before and $T = 500$. Comparing these mean autocorrelations with the ones obtained when the parameters are known and $T = 10000$, we can observe important negative biases. In this figure, it is also clear that in model *M2*, the differences between the autocorrelations of $\hat{\nu}_t^2$ and the squared autocorrelations of $\hat{\nu}_t$ do not allow to identify the heteroscedasticity.

Looking at the differences between the autocorrelations of squares and the squares of the autocorrelations of $\Delta_4 y_t$ and $\hat{\nu}_t$, we can erroneously conclude that model *M2* is homoscedastic. Furthermore, even when these differences are different from zero, as in models *M1* and *M3*, they do not allow us to identify whether the heteroscedasticity affects the long run, the short run or both. Koopman and Bos (2004), looking at alternative statistics to detect conditional heteroscedasticity in the innovations, also conclude that these statistics have low power. Next, we analyse how to use the auxiliary residuals to solve these problems.

3 Auxiliary residuals

In unobserved component models, it can also be useful to analyze the auxiliary residuals, that estimate the disturbances of each component. Harvey and Koopman (1992) derive expressions of the MMSL smoothed estimators of ε_t , η_t and ω_t , called auxiliary residuals, and propose to use them to identify outliers; see also Durbin and Koopman (2001). In particular, the auxiliary residuals corresponding to model (1) are given by

$$\begin{aligned}\hat{\varepsilon}_t &= \frac{(1 - F^s) \sigma_\varepsilon^2}{\theta(F) \sigma^2} \xi_t \\ \hat{\eta}_t &= \frac{(1 - F^s)}{\theta(F)(1 - F)} \frac{\sigma_\eta^2}{\sigma^2} \xi_t \\ \hat{\omega}_t &= \frac{(1 - F)}{\theta(F)} \frac{\sigma_\omega^2}{\sigma^2} \xi_t\end{aligned}\tag{9}$$

where F is the lead operator such that $Fx_t = x_{t+1}$, L is the lag operator such that $Lx_t = x_{t-1}$, $\theta(F)$ is a polynomial of order $s + 1$, ξ_t are the reduced form disturbances and σ^2 its corresponding variance.

We propose to use the autocorrelations of auxiliary residuals to identify which disturbances of an unobserved components model are heteroscedastic⁴. Once more, the identification is based on whether the differences between the autocorrelations of squares and the squared autocorrelations of each auxiliary residual are different from zero.

The acf of the auxiliary residuals can be obtained from the expressions in Durbin and Koopman (2000). However, the expressions of the acf of the squared auxiliary residuals are not easy to obtain. Consequently, the analysis of the usefulness of the auxiliary residuals to identify heteroscedasticity in the components of seasonal unobserved components models is based on simulated data. We have generated 1000 replicates of size $T=10000$ by models $M0$, $M1$, $M2$ and $M3$ and plot, in Figure 3, the Monte Carlo means of the differences between the autocorrelations of $\hat{\varepsilon}_t^2$, $\hat{\eta}_t^2$ and $\hat{\omega}_t^2$ and the squared autocorrelations of $\hat{\varepsilon}_t$, $\hat{\eta}_t$ and $\hat{\omega}_t$ when the auxiliary residuals have been obtained assuming that the model parameters are known. This figure shows that in the homoscedastic model, $M0$, none of the auxiliary residuals have autocorrelations of squares larger than the squared autocorrelations. On the other hand, the results for model $M3$ show clearly that the transitory and long-run components are heteroscedastic while the seasonal component is homoscedastic. On the other hand, the results for model $M2$ also indicate that the long-run component is heteroscedastic while the transitory and seasonal components are homoscedastic. However, in model $M1$, even though the heteroscedasticity is much evident in the short run component than in the other two components, the differences $Corr[\hat{\eta}_t^2, \hat{\eta}_{t-\tau}^2] - (Corr[\hat{\eta}_t, \hat{\eta}_{t-\tau}])^2$ and $Corr[\hat{\omega}_t^2, \hat{\omega}_{t-\tau}^2] - (Corr[\hat{\omega}_t, \hat{\omega}_{t-\tau}])^2$ are different from zero. This could be due to the fact that σ_ε^2 is four times larger than σ_η^2 and, therefore, the heteroscedasticity of ε_t is somehow transmitted to $\hat{\eta}_t$. On the other hand, in this case, when η_t is heteroscedastic, there is not transmission towards $\hat{\varepsilon}_t$ ⁵.

Figure 3 also plots the differences between the squared autocorrelations and the autocorrelations of squares of the auxiliary residuals when they are estimated using the

⁴Wells (1996) have propose to use recursive residuals of the transitory component to test for heteroscedasticity; see Bhar and Hamori (2004).

⁵Harvey et al. (1992) also observe some transmission of heteroscedasticity between components when using LM tests to identify which component is heteroscedastic.

QML estimates of the parameters instead of the true parameters and the sample size is $T = 500$. Although the differences are negatively biased when the estimated parameters are used in the smoothing algorithm, the same patterns can be observed regardless of whether the parameters are known or estimated. Therefore, the differences between autocorrelations of auxiliary residuals allow to properly identify which disturbance is heteroscedastic. Furthermore, the transmission of heteroscedasticity between auxiliary residuals is smaller than when using the true parameters to run the filters. This result is also supported by the Monte Carlo results reported in Table 1 for the differences between the autocorrelations of order 1 of each of the auxiliary residuals. In Table 1, we can also see that, if the sample size is large enough under the null hypothesis, the differences between the autocorrelations of squares and the squared autocorrelations of the auxiliary residuals have zero mean and variance $1/T$. The only exception is the variance corresponding to the seasonal residual which is clearly larger than $1/T$. In model *M1*, $Corr [\hat{\varepsilon}_t^2, \hat{\varepsilon}_{t-1}^2] - (Corr [\hat{\varepsilon}_t, \hat{\varepsilon}_{t-1}])^2$ are significantly different from zero while the differences corresponding to $\hat{\eta}_t$ and $\hat{\omega}_t$ are zero. Also note that the variance of $Corr [\hat{\eta}_t^2, \hat{\eta}_{t-1}^2] - (Corr [\hat{\eta}_t, \hat{\eta}_{t-1}])^2$ can be approximated by $1/T$ while the variances corresponding to all the other correlations considered are larger.

On the other hand, the results for model *M2* indicate that the long-run disturbance, η_t , is heteroscedastic while the transitory and seasonal disturbances, ε_t and ω_t , are homoscedastic. Once more, the variance corresponding to the homoscedastic component is approximately $1/T$, while the other variances are larger. Note that the evidence is weaker in the latter model. In any case, analysing the auxiliary residuals we can observe an increase in power to detect heteroscedasticity with respect to using $\Delta_4 y_t$ or ν_t . Finally, the kernel densities plotted in Figure 2 illustrate that the Normal approximation can be used to test whether the differences between the autocorrelations of squares and the squared autocorrelations of auxiliary residuals are different from zero.

4 Estimation of the parameters

There are several alternative methods to estimate the parameters of the seasonal QSTARARCH model in (1). First, Harvey *et al.* (1992) proposed a QML estimator. The properties of the QML estimator for non-seasonal QSTARARCH models have been analysed by Broto and Ruiz (2005). To avoid the inconsistencies associated with the QML estimator, Fiorentini, Sentana and Shephard (2003) propose computationally feasible Markov Chain Monte Carlo (MCMC) algorithms that can be used to obtain exact likelihood-based estimators of the parameters of STARARCH models. Alternatively, Sentana *et al.* (2004) propose simulation based estimators which belong to the class of the Indirect Inference estimation procedures proposed by Gourieroux *et al.* (1993), Smith (1993) and Gallant and Tauchen (1996). The most important decision that we have to make is the choice of auxiliary model. Given that, in the context of heteroscedastic unobserved component models, the auxiliary model must be estimated subject to inequality constraints, which are often binding in practice, we must use the constrained indirect estimation procedures proposed by Calzolari *et al.* (2004) which can handle a mix of equality and inequality restrictions. However, this procedure can be very time consuming and Sentana and Fiorentini (2001) suggest sequential indirect estimators of the conditional variance parameters.

The MCMC procedure is rather complicated computationally and the Indirect Inference methods do not allow to obtain directly estimates of the unobserved components. Furthermore, the results in Sentana *et al.* (2004) and Broto and Ruiz (2005) show that the QML estimator may be appropriate for conditionally heteroscedastic unobserved components models and this is the method we are considering in this paper. In this section, we analyse the finite sample properties of the QML estimator for models with similar properties to the ones observed when analysing monthly series of inflation.

In any case, estimation of QSTARARCH models is easier using the following reparametrization of the variances of the unobserved noises proposed by Sentana (1995) to guarantee the positivity of the variances h_t and q_t ,

$$h_t = a_0 + a_1^2(\varepsilon_{t-1} - a_3)^2 + a_2^2 h_{t-1} \quad (10)$$

$$q_t = g_0 + g_1^2(\eta_{t-1} - g_3)^2 + g_2^2 q_{t-1}$$

where the parameters of interest are $\alpha_0 = a_0 + a_1^2 a_3^2$, $\alpha_1 = a_1^2$, $\alpha_2 = a_2^2$ and $\alpha_3 = -2a_3 a_1^2$. Similar transformations apply to the parameters of q_t . After estimating the parameter vector, $\Psi = (a_0, a_1, a_2, a_3, g_0, g_1, g_2, g_3, \sigma_\omega^2)$, these transformations can be used to obtain the original parameters of the model.

The QML estimator is based on expressing the model in an augmented state space form. The state vector is augmented by lags of μ_t in such a way that the Kalman filter gets estimates of the unobserved disturbances. For example, for quarterly data $s = 4$ and the measurement and transition equations are respectively given by

$$y_t = \mu_t + \delta_t + \varepsilon_t = \begin{bmatrix} 1 & 0 & 0 & 1 & 0 & 0 \end{bmatrix} \alpha_t + \varepsilon_t$$

$$\alpha_t = \begin{bmatrix} \mu_t \\ \mu_{t-1} \\ \eta_t \\ \delta_t \\ \delta_{t-1} \\ \delta_{t-2} \end{bmatrix} = \begin{bmatrix} 1 & 0 & 0 & 0 & 0 & 0 \\ 1 & 0 & 0 & 0 & 0 & 0 \\ 0 & 0 & 0 & 0 & 0 & 0 \\ 0 & 0 & 0 & -1 & -1 & -1 \\ 0 & 0 & 0 & 1 & 0 & 0 \\ 0 & 0 & 0 & 0 & 1 & 0 \end{bmatrix} \begin{bmatrix} \mu_{t-1} \\ \mu_{t-2} \\ \eta_{t-1} \\ \delta_{t-1} \\ \delta_{t-2} \\ \delta_{t-3} \end{bmatrix} + \begin{bmatrix} 1 & 0 \\ 0 & 0 \\ 1 & 0 \\ 0 & 1 \\ 0 & 0 \\ 0 & 0 \end{bmatrix} \begin{bmatrix} \eta_t \\ \omega_t \end{bmatrix}.$$

Even if ε_t^\dagger and η_t^\dagger are assumed to be Gaussian processes, QSTARARCH models are not conditionally Gaussian, since knowledge of past observations does not imply knowledge of past disturbances. Consequently, the QML estimator is based on treating the model as if it were conditionally Gaussian and running the Kalman filter to obtain the one-step ahead prediction errors and their variances to be used in the expression of the Gaussian likelihood given by

$$\log L = -\frac{T}{2} \log(2\pi) - \frac{1}{2} \sum_{t=1}^T \log F_t - \frac{1}{2} \sum_{t=1}^T \frac{\nu_t^2}{F_t}, \quad (11)$$

where ν_t , $t = 1, \dots, T$ are the innovations and F_t their corresponding variances. The QML estimator, $\hat{\Psi}$, is obtained by maximizing the Gaussian likelihood in (11) with respect to the unknown parameters.

The Kalman filter requires expressions of the conditional variances of the distur-

bances ε_t and η_t . The conditional variance of ε_t is given by

$$H_t = {}_t\underline{E}_{-1} \varepsilon_t^2 = a_0 + a_1^2(\tilde{\varepsilon}_{t-1} - a_3)^2 + a_1^2(P_{t-1} + P_{t-1}^\omega) + a_2^2 H_{t-1} \quad (12)$$

where $\tilde{\varepsilon}_t = y_t - m_t - d_t$, $m_t = {}_t\underline{E} \mu_t$, $d_t = {}_t\underline{E} \delta_t$, $P_t = {}_t\underline{E} (\mu_t - m_t)$ and $P_t^\omega = {}_t\underline{E} (\delta_t - d_t)^2$; see Harvey *et al.* (1992) and Broto and Ruiz (2005). Similarly, the conditional mean of the disturbance of the permanent component, η_t , is zero and its conditional variance is given by

$$Q_t = {}_t\underline{E}_{-1} \eta_t^2 = g_0 + g_1^2(\tilde{\eta}_{t-1} - g_3)^2 + g_1^2 P_{t-1}^\eta + g_2^2 Q_{t-1} \quad (13)$$

where $\tilde{\eta}_t = m_t - m_{t-1|t}$, $m_{t-1|t} = {}_t\underline{E} \mu_{t-1}$, $P_{t-1}^\eta = P_t + P_{t-1|t} - 2P_{t,t-1|t}$, $P_{t-1|t} = {}_t\underline{E}_{-1} (\mu_{t-1} - m_{t-1|t})^2$ and $P_{t,t-1|t} = {}_t\underline{E} E(\mu_t - m_t)(\mu_{t-1} - m_{t-1|t})$. The MMSL estimates of μ_t and μ_{t-1} given by m_t and $m_{t-1|t}$ respectively, are obtained in a natural way by the augmentation of the Kalman filter. The matrices P_t , $P_{t-1|t}$, $P_{t,t-1|t}$, P_t^η and P_t^ω are also provided by the Kalman filter.

In order to carry out the initialization of the filter, we set $m_1 = y_1$ and $P_1 = {}_0\underline{E} \varepsilon_1 = \sigma_\varepsilon^2 = (a_0 + a_1^2 b^2)/(1 - a_1^2)$. In the framework of a random walk plus white noise this is equivalent to use a diffuse prior. Furthermore, if the conditional variance of η_t at time $t - 1$ is also set equal to its unconditional variance, the Kalman filter can be started with ${}_{t-1}\underline{E}_{-1} (\varepsilon_2^2) = \sigma_\varepsilon^2$ and ${}_{t-1}\underline{E}_{-1} (\eta_2^2) = \sigma_\eta^2$.

Under very general conditions, the asymptotic distribution of $\hat{\Psi}$ can be approximated by a multivariate normal distribution with mean Ψ and covariance matrix $(Avar)^{-1}$. The ij 'th element of the matrix $Avar$ is given by

$$Avar_{ij}(\Psi) = \frac{1}{2} E \left[\sum_{t=1}^T \frac{1}{F_t^2} \frac{\partial F_t}{\partial \Psi} \frac{\partial F_t}{\partial \Psi'} + \sum_{t=1}^T \frac{1}{F_t} \frac{\partial \nu_t}{\partial \Psi} \frac{\partial \nu_t}{\partial \Psi'} \right]. \quad (14)$$

The derivatives in expression (14) can be numerically evaluated as explained by Harvey (1989). Once, the matrix $Avar$ has been computed, the delta method can be used to obtain the covariance matrix of the parameters of interest.

Next, we analyze the finite sample properties of the QML estimator by means of Monte Carlo experiments. The series are simulated by models $M1$, $M2$ and $M3$ with sample sizes $T = 100, 200$ and 500 . All the simulations are based on 1000 replicates and have been carried out on a Pentium desktop computer using our own FORTRAN codes. The numerical optimization of the likelihood has been performed using the IMSL subroutine DBCPOL with the parameters α_0 and γ_0 restricted to be nonnegative, and $\alpha_1 + \alpha_2$ and $\gamma_1 + \gamma_2$ restricted to be between 0 and 1. Table 2 reports the Monte Carlo means and standard deviations (brackets) together with the corresponding approximated asymptotic standard deviation computed using expression (14) (squared brackets). The results for model $M1$ show that the biases of all the parameters are rather small for moderately large sample sizes as, for example, $T = 500$. However, it is possible to observe that, it seems to be a negative correlation between the estimates of the parameters α_1 and α_2 . The parameter α_1 is overestimated while α_2 is underestimated. For example, when $T = 100$, the empirical correlation between α_1 and α_2 is -0.79 . Even when the sample size is $T = 500$, this correlation is -0.71 . Notice that these high correlations could be expected since we are estimating imposing the stationarity restriction, $\alpha_1 + \alpha_2 < 1$ and the parameters are very close to this bound. With respect to the standard deviations, Table 2 shows that for $T = 500$, the asymptotic deviations provide adequate approximations to the finite sample distributions for all the parameters of the variance of the transitory noise. However, the asymptotic standard deviations corresponding to the variances of the long-run and seasonal components clearly underestimate the standard deviations of the sample distribution.

Looking at the results for model $M2$ where the long-run component is heteroscedastic, we can observe that the biases of the QML estimator of the transitory and seasonal variances, σ_ε^2 and σ_ω^2 , are negligible when $T = 500$. However, the parameters of the long-run conditional variance are badly estimated. The parameter γ_1 is strongly overestimated while γ_2 is underestimated. In this model, the asymptotic standard deviations are smaller than the finite sample standard deviations. Notice that in model $M2$ the signal of the long-run component is very weak and, consequently, the QML estimator has problems to identify adequately the values of the parameters. Sentana et al. (2004) found similar biases in the QML estimator of the parameters of a related

unobserved components model with heteroscedastic signal. They show that the role of these biases is to ensure that the model provides a rather accurate approximation to the conditional distribution of y_t . The estimation procedure seems to force γ_1 to be bigger in an attempt to match its fourth order moments. They conclude that the performance of the QML estimator is rather good, except when the signal to noise ratio is small or the coefficient of variation of the heteroscedastic noise is large.

The same conclusions are obtained when looking at the results for the *M3* model in which both noises are heteroscedastic. In this case, the biases of the estimators of the parameters of the variance of ε_t are rather small while they are large when looking at the estimators of the parameters of the variance of η_t . On the other hand, the asymptotic standard deviations provide an adequate approximations to the finite standard deviations for the former while they strongly underestimate the finite standard deviations for the latter parameters. Finally, the QML estimator of σ_ω^2 has a negligible bias although the finite standard deviation is larger than the asymptotic approximation.

Figure 4 plots kernel estimates of the densities of the parameter estimates of model *M1*. This figure illustrates that the asymptotic Normal approximation of the QML estimator of all the parameters is adequate for relatively large sample sizes as, for example, $T = 500$. However, the above results suggest that it could be interesting to explore the use of bootstrap methods to obtain the finite sample distribution of the QML estimator in QSTARCH models.

5 Empirical analysis

In this section, monthly inflation series of eight OCDE countries are analysed to investigate whether the Friedman effect is present in these series. In particular, we have data on inflation measured as first differences of the CPI, i.e., $y_t = 100 * \Delta \log(CPI_t)$, in France, Germany, Italy, Japan, Netherlands, Spain, Sweden and United Kingdom from January 1962 until September 2004, that is, $T = 513^6$. Figure 5 plots the eighth

⁶Prior to its analysis, the series have been filtered to be rid of outliers. To detect outliers in the different components we have used the detection method of Harvey and Koopman (1992) as implemented in the program STAMP 6.20; see Koopman et al. (2000). The outliers detected affect mainly the transitory component although we found level outliers in Italy and the Netherlands

series of inflation, y_t , together with the differences between the autocorrelations of $(\Delta_{12}y_t)^2$ and the squared autocorrelations of $\Delta_{12}y_t$. Note that the autocorrelations of squares are larger than the squared autocorrelations of the levels, suggesting that these series may be conditionally heteroscedastic. Furthermore, all the series have kurtosis coefficients significantly greater than 3 which run from 4.62 for Japan up to 8.16 for France, so they seem to have non-Gaussian distributions.

In order to identify which component could be heteroscedastic, we start by fitting model (1) with homoscedastic disturbances to each of the inflation series. The estimated parameters appear in Table 3. First of all, note that for the eight inflation series, the estimates of the signal to noise ratios of the long-run component are very small running from 0.007 for Sweden to 0.203 for Italy. Furthermore, the variances of the seasonal components are also rather small when compared with the variance of the transitory component. Figure 5 plots the estimated long-run components and Figure 6 plots the seasonal components for each of the series of inflation. Note that the seasonal components of France and Italy could be well approximated by assuming that they are deterministic. However, the results for these two countries obtained assuming deterministic seasonality are similar and, therefore, we report the results obtained for stochastic seasonality. Table 3 also reports several sample moments of the estimated innovations. We can observe that they still have leptokurtic distributions although the kurtosis coefficients are smaller than in the original data. Furthermore, Table 3 reports the differences between the autocorrelations of $\hat{\nu}_t^2$ and the squared autocorrelations of $\hat{\nu}_t$. Taking into account that under conditional homoscedasticity the distribution of these differences can be approximated by a $N(0, 1/T)$, we have marked the differences which are significantly larger than zero. All countries except Germany, Netherlands and UK show symptoms of heteroscedasticity. It is interesting to know that even in these three countries, the differences between autocorrelations corresponding to seasonal orders are significantly larger than zero.

To identify which component could be causing the conditional heteroscedasticity, Figure 7 represent the autocorrelations of the squared auxiliary residuals and the corresponding squares of the autocorrelations for $\hat{\varepsilon}_t$ and $\hat{\eta}_t$, respectively. When looking at the differences for the auxiliary residuals of the transitory component, $\hat{\varepsilon}_t$, we observe

that except in France, The Netherlands and United Kingdom, all the series show signs of conditional heterocedasticity. However the differences corresponding to the long-run component are not different from zero. Therefore, these results suggest that while the long-run can be modelled with a homoscedastic noise in most of the series, the uncertainty of the transitory component of inflation seems to be heteroscedastic.

Consequently, the QSTARARCH model is fitted to each of the series of inflation with homoscedastic long-run and seasonal components, but the series corresponding to France and United Kingdom, which are modelled by a QSTARARCH model with homoscedastic short-run and seasonal components. Table 4 reports the estimated parameters. As expected given our previous results on the autocorrelations of the auxiliary residuals, the ARCH coefficients of the Netherlands and Sweden inflations are not significant. Therefore, the inflation of these series seems to be homoscedastic. The transitory components of all the other series of inflation have significant ARCH parameters. Note that, as it is usual in financial time series, the persistence estimated for the GARCH models is very close to unity running from 0.94 in Sweden to 0.99 in Italy and Japan. Finally, with respect to the estimated asymmetry parameters, we can observe that they are positive and significant in France, Germany and United Kingdom while they are not significant in Italy, Japan and Spain. Therefore, our results support the Friedman hypothesis of larger inflation increasing future uncertainty in France, Germany and United Kingdom while the uncertainty of inflation in Italy, Japan and Spain is time-varying although it does not depend on past levels of inflation. Finally, the inflation of the Netherlands and Sweden seems to be homoscedastic.

Finally, Table 5 represents the summary statistics of the standardized innovations ν_t of the eighth series of inflation. We can observe that the differences between the autocorrelations of squares and the squared autocorrelations are no longer significant except for a few lags and series. However, it is interesting to observe that the differences are significant for the seasonal lag 12 for France and Japan inflations. It is possible that the seasonal component of these two series may have some kind of heteroscedastic behavior. The extension of the model to incorporate a conditional heteroscedastic seasonal component is left for further research.

6 Conclusions

In this article, we fit a seasonal unobserved components model to monthly series of inflation. The model allows the transitory and long run components to be conditionally heteroscedastic. In particular, the variances of the unobserved noises are modelled as QGARCH processes. We first show how to use the auxiliary residuals to identify which components are heteroscedastic. We carry out Monte Carlo experiments to show that, if a component is homoscedastic, the finite sample distribution of the differences between the autocorrelations of the corresponding squared residuals and the squared autocorrelations of the residuals can be adequately approximated by a Normal distribution with zero mean and variance $\frac{1}{T}$. However, when the component is heteroscedastic, these differences have means different from zero and, consequently, the heteroscedasticity can be detected by looking at them. Our results also show that, although there are correlations through the auxiliary residuals, the transmission of volatility between them in finite sample sizes is not large enough as to identify heteroscedasticity in components which are truly homoscedastic. Furthermore, using auxiliary residuals to detect conditional heteroscedasticity increases the power with respect to detecting the heteroscedasticity using the estimated innovations.

We propose to estimate the parameters of the model by QML and illustrate with Monte Carlo experiments the finite sample properties in unobserved component models with stochastic seasonality.

Finally, the model is implemented to analyse the dynamic behaviour of inflation in eight OCDE countries. Our objective is to study whether the Friedman hypothesis, of uncertainty of inflation increasing with its level, is supported by the data. It is important to note that in some of the countries, the heteroscedasticity was undetected when looking at the usual diagnosis on the residuals while it was clear when analysing the auxiliary residuals. Furthermore, the auxiliary residuals show that when there is heteroscedasticity in most of the countries, it affects the transitory component while the uncertainty of the long-run component is constant. The estimated parameters show that, with the exception of the Netherlands, Sweden, France and United Kingdom, the uncertainty of the transitory components of inflation can be represented by GARCH(1,1) models with high persistence. The two first countries seem to have ho-

moscedastic inflation while in the last two the long run component is heteroscedastic. With the exception of Spain and Italy all the countries with time-varying uncertainty show a positive relationship between the uncertainty and past levels of inflation, supporting the Friedman hypothesis.

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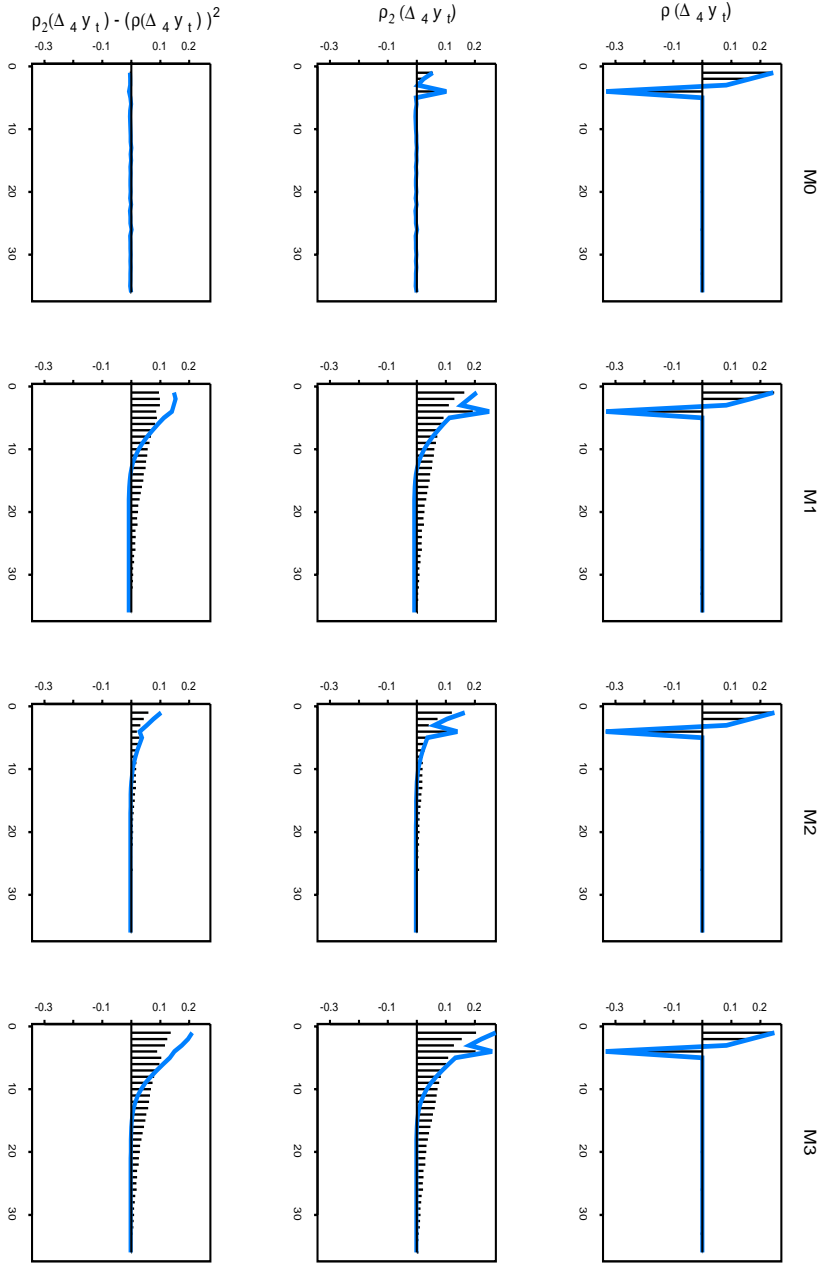
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Figure 1: Mean autocorrelation function of $\Delta^4 y_t$, $\Delta^4 y_t^2$, and differences between the mean autocorrelation function of $(\Delta^4 y_t)^2$ and mean autocorrelation function of $\Delta^4 y_t$ squared for four STARCH models (by columns). Results based on 1,000 replications of series with sample size $T=200$.



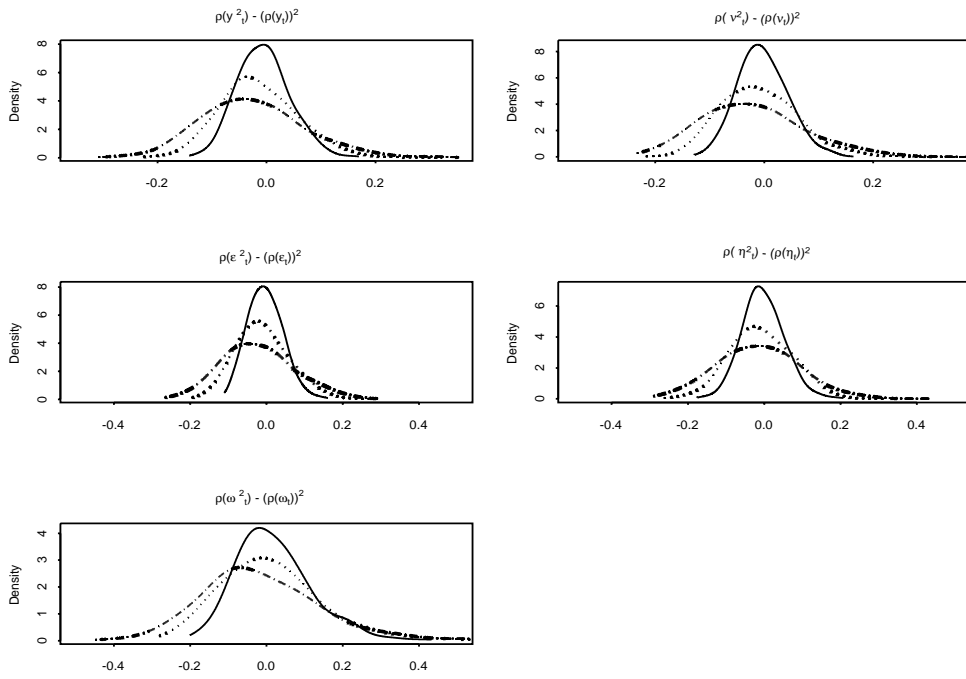


Figure 2: Kernel densities of the differences between the order 1 autocorrelation of $(\Delta_4 y_t)^2$ and the squared autocorrelation of $\Delta_4 y_t$, and analogue results for \hat{v}_t , $\hat{\varepsilon}_t$, $\hat{\eta}_t$ and $\hat{\omega}_t$ for the homoscedastic model M0.

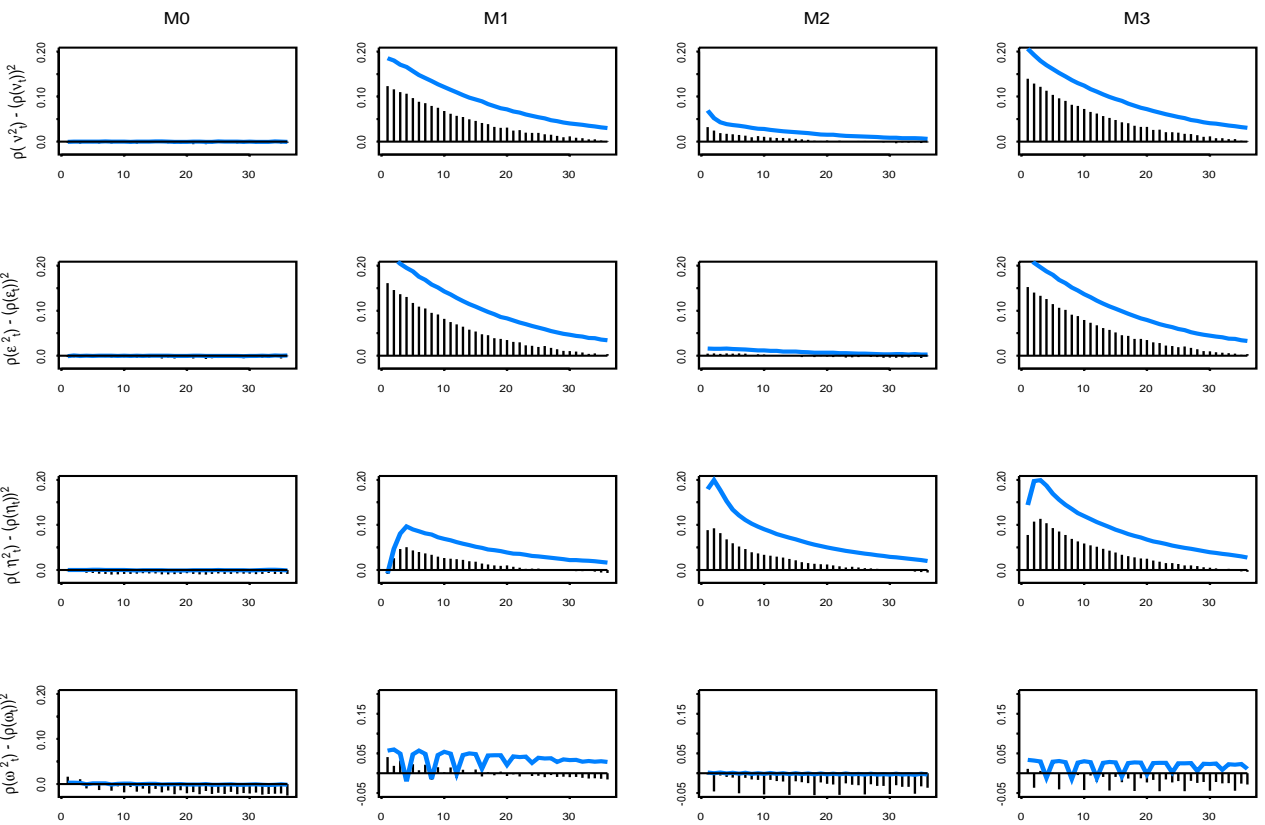


Figure 3: Mean autocorrelation function of $(\hat{\nu}_t)^2$ minus mean squared autocorrelation function of $\hat{\nu}_t$, and analogue results for $\hat{\epsilon}_t$, $\hat{\eta}_t$ and $\hat{\omega}_t$ (by rows) for four STAR models (by columns). Results are based on 1,000 replications. The continuous lines represent results for sample size $T = 10000$ and known parameters while the bands represent results for sample size $T = 500$.

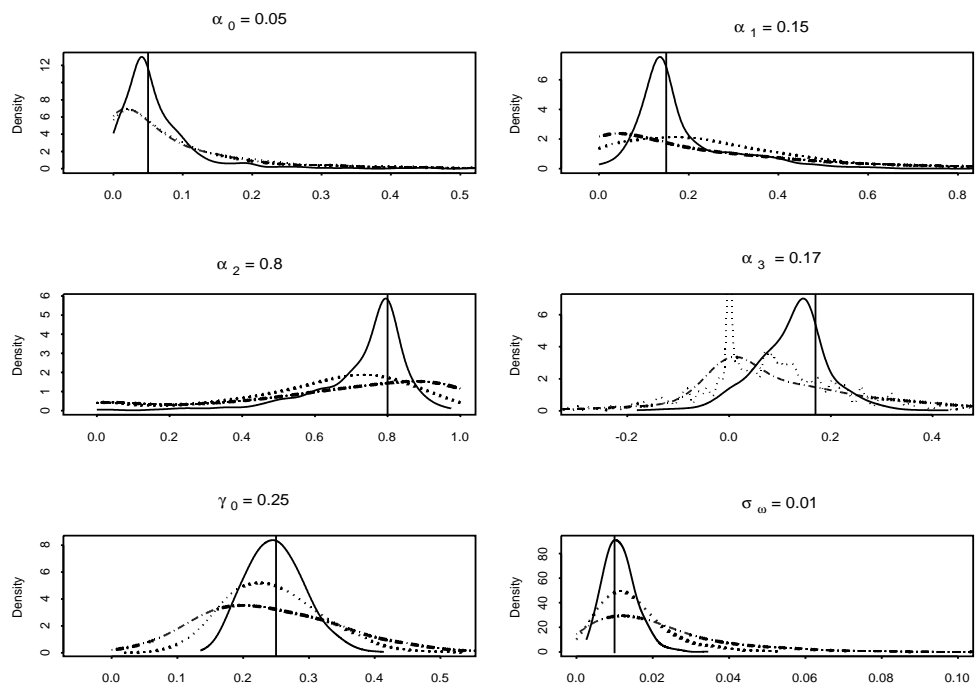


Figure 4: Kernel densities of the QML estimates of the estimated parameters of a Q-STARARCH model with $\alpha_0 = 0.05$, $\gamma_0 = 0.25$, $\alpha_1 = 0.15$, $\alpha_2 = 0.8$, $\beta = 0.17$ and $\sigma_\omega = 0.01$. The dash-dotted line corresponds to $T = 100$, the dotted line to $T = 200$ and the solid line to $T = 500$.

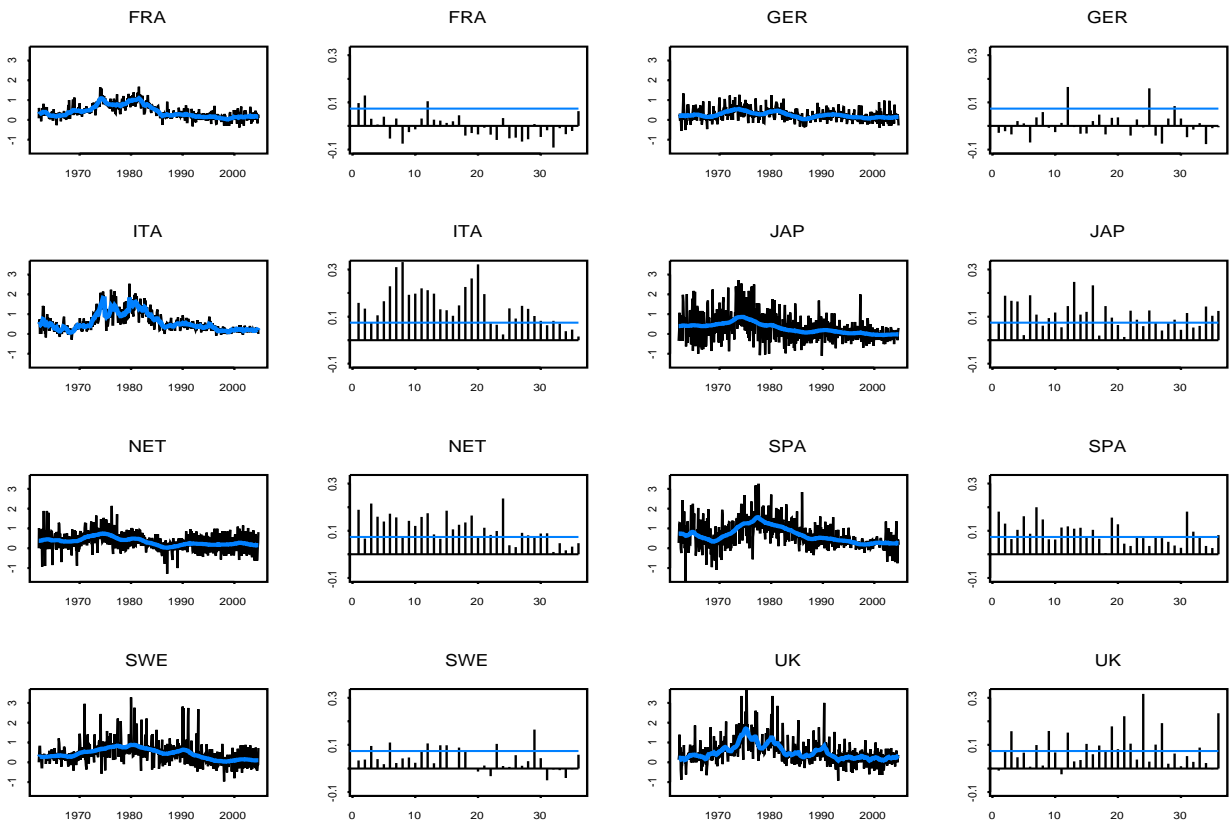


Figure 5: Inflation series for eight countries and estimated trend, together with the differences between the autocorrelations of $(\Delta_{12}y_t)^2$ and the squared autocorrelations of $\Delta_{12}y_t$.

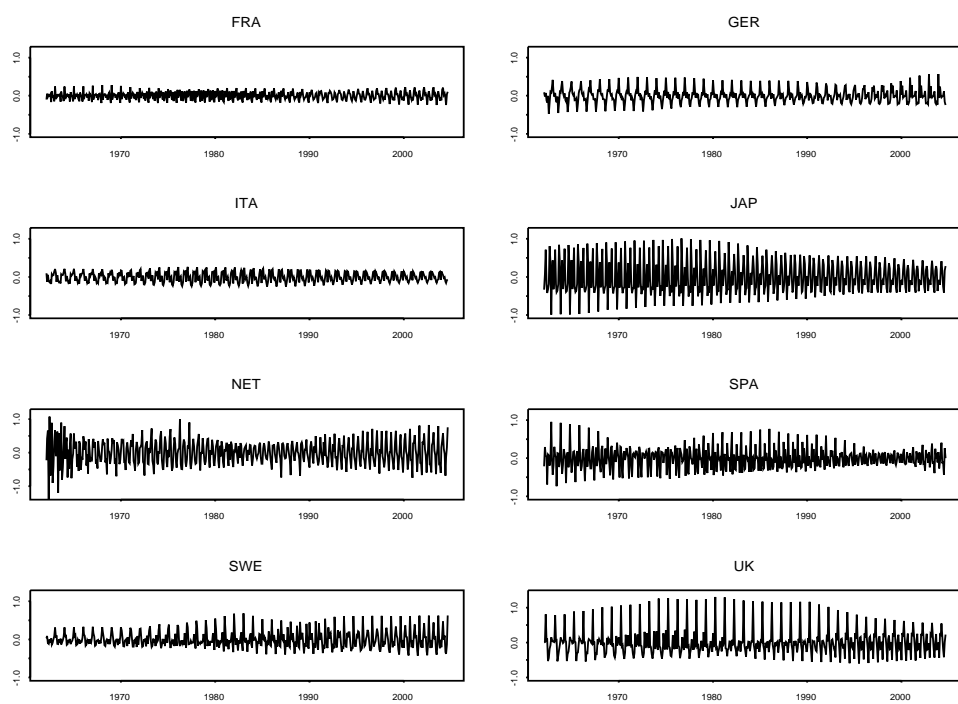


Figure 6: Seasonal component (stochastic seasonality) for the inflation series of eight countries.

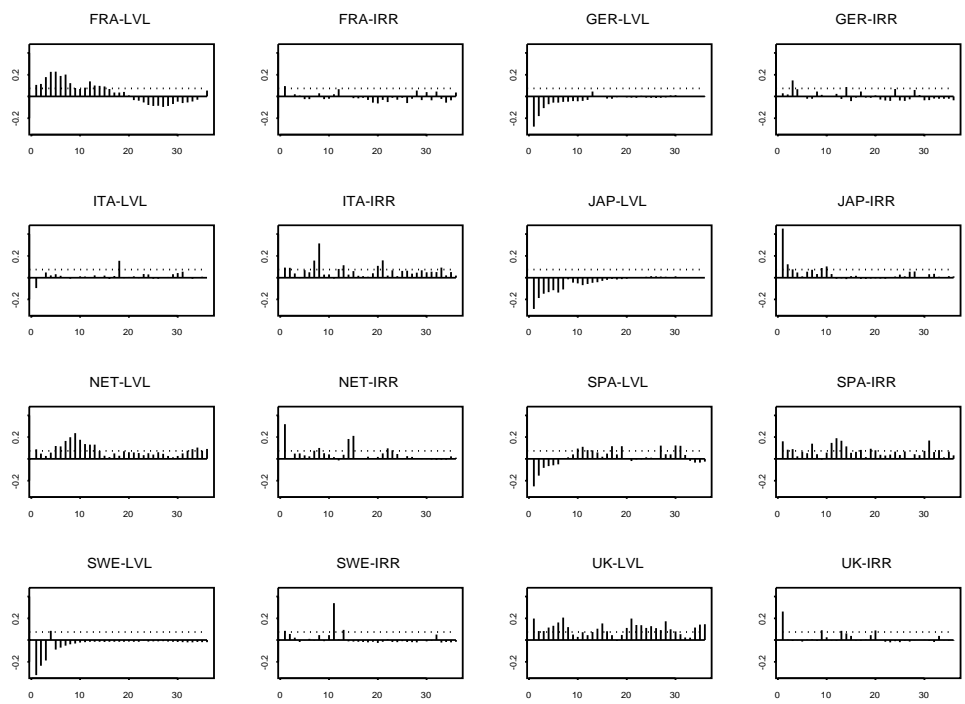


Figure 7: Differences between the autocorrelations of squares and squares of the autocorrelations of the auxiliary residuals for the inflation series of eight countries.

		$T = 100$	$T = 200$	$T = 500$
<i>M0</i>	$\Delta_4 y_t$	-0.0245 (0.0970)	-0.0137 (0.0718)	-0.0055 (0.0494)
	ν_t	-0.0203 (0.0964)	-0.0851 (0.0714)	-0.0027 (0.0453)
	$\hat{\varepsilon}_t$	-0.0212 (0.0958)	-0.0101 (0.0709)	-0.0031 (0.0457)
	$\hat{\eta}_t$	-0.0146 (0.1096)	-0.0079 (0.0816)	-0.0028 (0.0549)
	$\hat{\omega}_t$	-0.0118 (0.1606)	0.0129 (0.1366)	0.0201 (0.0993)
	<i>M1</i>	$\Delta_4 y_t$	0.0229 (0.1203)	0.0588 (0.1083)
ν_t		0.0402 (0.1257)	0.0832 (0.1127)	0.1212 (0.0945)
$\hat{\varepsilon}_t$		0.0646 (0.1258)	0.1122 (0.1106)	0.1604 (0.0988)
$\hat{\eta}_t$		-0.0151 (0.1162)	-0.0084 (0.0908)	-0.0052 (0.0669)
$\hat{\omega}_t$		-0.0036 (0.1658)	0.0248 (0.1387)	0.0446 (0.1027)
<i>M2</i>		$\Delta_4 y_t$	-0.0011 (0.1072)	0.0271 (0.0956)
	ν_t	-0.0052 (0.1026)	0.0171 (0.0875)	0.0364 (0.0714)
	$\hat{\varepsilon}_t$	-0.0190 (0.0962)	-0.0053 (0.0750)	0.0065 (0.0513)
	$\hat{\eta}_t$	0.0143 (0.1173)	0.0494 (0.0988)	0.0865 (0.0863)
	$\hat{\omega}_t$	-0.0640 (0.1660)	-0.0225 (0.1437)	-0.0016 (0.1111)

Table 1: Summary statistics of the difference between the first order autocorrelation of $\Delta_4 y_t^2$ and the squared autocorrelation of $\Delta_4 y_t$, and analogue results for $\hat{\varepsilon}_t$, $\hat{\eta}_t$ and $\hat{\omega}_t$ for M0, M1 and M2 models.

	<i>M1</i>		<i>M2</i>		<i>M3</i>	
	<i>T</i> = 100	<i>T</i> = 200	<i>T</i> = 100	<i>T</i> = 200	<i>T</i> = 100	<i>T</i> = 200
$\alpha_0 = 0.05$	0.0097 (0.1490) [0.2122]	0.1021 (0.1476) [0.1758]	3.9105 (0.8615) [0.1075]	3.9739 (0.5867) [0.0757]	0.8325 (1.0825) [0.7626]	0.5140 (0.6756) [0.5042]
$\alpha_1 = 0.15$	0.2156 (0.2446) [0.2036]	0.2483 (0.2014) [0.1940]	$\alpha_0 = 4$		0.2187 (0.2236) [0.2700]	0.2198 (0.1695) [0.1260]
$\alpha_2 = 0.8$	0.6545 (0.3037) [0.2616]	0.6087 (0.2549) [0.3044]	$\alpha_1 = 0.0$		0.5305 (0.3258) [0.3600]	0.6206 (0.2507) [0.1427]
$\alpha_3 = 0.17$	0.1119 (0.2351) [0.2166]	0.0836 (0.1640) [0.2050]	$\alpha_2 = 0.0$		0.1646 (0.4877) [0.3652]	0.1450 (0.2749) [0.2448]
$\gamma_0 = 0.25$	0.2455 (0.1112) [0.0492]	0.2469 (0.0745) [0.0291]	$\alpha_3 = 0.0$		0.2881 (0.4696) [0.1556]	0.1895 (0.2452) [0.0903]
$\gamma_1 = 0.0$			$\gamma_0 = 0.05$	0.2735 (0.4358) [0.0089]	0.3602 (0.3012) [0.0295]	0.2928 (0.2762) [0.4056]
$\gamma_2 = 0.0$			$\gamma_1 = 0.15$	0.3052 (0.2939) [0.0295]	0.3602 (0.3012) [0.0184]	0.2608 (0.2838) [0.4056]
$\gamma_3 = -0.05$			$\gamma_2 = 0.8$	0.3856 (0.3518) [0.0301]	0.3597 (0.3455) [0.0180]	0.4745 (0.3298) [0.5282]
$\sigma_\omega^2 = 0.01$	0.0211 (0.0181) [0.0047]	0.0147 (0.0090) [0.0018]	$\gamma_3 = 0.17$	0.2288 (0.4301) [0.0463]	0.3371 (0.3391) [0.0202]	0.1903 (0.3521) [0.2639]
			$\sigma_\omega^2 = 0.01$	0.0242 (0.0296) [0.0064]	0.0161 (0.0134) [0.0024]	0.2543 (0.2944) [0.1970]
				0.0113 (0.0043) [0.0007]	0.0121 (0.0063) [0.0013]	0.3251 (0.2601) [0.0345]
						0.0121 (0.0133) [0.0002]
						0.0003 (0.0002) [0.0003]

Table 2: Monte Carlo results for estimated parameters of Q-STARH models *M1*, *M2* and *M3*. Standard deviations in brackets. Asymptotic Standard deviations in squared brackets.

	FRA	GER	ITA	JAP	NET	SPA	SWE	UK
$\hat{\sigma}_\varepsilon^2$	0.0226	0.0403	0.0338	0.1855	0.0353	0.2069	0.1198	0.0645
$\hat{\sigma}_\eta^2$	0.0021	0.0006	0.0068	0.0010	0.0353	0.0029	0.0008	0.0082
$\hat{\sigma}_\omega^2$	0.0006	0.0013	0.0005	0.0021	0.0072	0.0077	0.0035	0.0032
	ν_t	ν_t	ν_t	ν_t	ν_t	ν_t	ν_t	ν_t
Mean	-0.010	-0.001	0.006	-0.029	-0.024	-0.020	-0.006	0.006
SK	0.036	0.337 *	0.506 *	0.392 *	0.195	0.421 *	0.156	0.339 *
κ	4.198 *	3.926 *	4.533 *	4.274 *	4.615 *	4.649 *	5.161 *	4.035 *
$\rho_2(\tau) - [\rho(\tau)]^2$								
1	0.1376 *	0.0822	0.1387 *	0.126 *	-0.0142	0.2053 *	0.0794 *	0.015
2	0.0883 *	0.0176	0.1206 *	0.1983 *	-0.0612	0.1394 *	0.0810 *	0.0488
3	0.0840 *	0.0588	0.0934 *	0.1676 *	0.1214 *	0.0688	0.035	0.0812 *
4	0.0609	0.0298	0.1016 *	0.0964 *	-0.012	0.1228 *	0.0282	0.0466
5	-0.0406	0.0384	0.1678 *	0.0349	-0.0144	0.2158 *	0.0108	0.0756 *
12	0.0868 *	0.0405	0.1006 *	0.0916 *	0.0510	0.2629 *	0.1366 *	0.0804 *
24	-0.027	0.1213 *	0.0760 *	0.0033	0.1813 *	0.1029 *	0.0531	0.1592 *

* Significant at 5%; SK: Skewness; κ : Kurtosis; $\rho(\tau)$: Correlation of order τ .

Table 3: Estimates of the parameters of a random walk plus noise model with stochastic seasonality and summary statistics of the corresponding innovations ν_t for inflation series.

	FRA	GER	ITA	JAP	NET	SPA	SWE	UK
α_0	0.0347 (10.0281)	0.0041 (1.2628)	0.0003 (0.9606)	0.0014 (1.0558)	0.00001 (0.9257)	0.0040 (1.4281)	0.0056 (1.3095)	0.00001 (1.1150)
α_1		0.1031 (1.9832)	0.0556 (2.2321)	0.1196 (2.0663)	0.1567 (1.2190)	0.1398 (2.6973)	0.0463 (1.6209)	
α_2		0.8770 (13.7676)	0.9418 (36.0084)	0.8778 (14.8065)	0.8431 (6.5480)	0.8280 (13.6756)	0.9014 (14.5196)	
α_3		0.0412 (2.0041)	0.0082 (1.0522)	0.0259 (1.1344)	0.0095 (0.6260)	0.0061 (0.2520)	0.0323 (2.0718)	
γ_0	0.0001 (0.9450)	0.0005 (4.0950)	0.0030 (14.7920)	0.0006 (5.7859)	0.0008 (6.3349)	0.0011 (8.7665)	0.0006 (8.0000)	0.0001 (1.4780)
γ_1	0.4413 (8.1825)							0.2990 (8.9308)
γ_2	0.5067 (9.5535)							0.7001 (20.8411)
γ_3	0.0122 (1.9572)							0.0131 (11.8506)
σ_ω^2	0.0008 (7.1271)	0.0037 (8.3849)	0.0005 (6.1968)	0.0069 (15.5946)	0.0095 (14.8151)	0.0047 (8.3973)	0.0038 (7.8182)	0.0193 (14.5179)
<i>LogL</i>	487.6183	279.9152	425.8460	142.6143	286.2358	98.9262	226.8176	222.9343

Table 4: Estimates of the Q-STARCh model with stochastic seasonality for inflation series.

	FRA	GER	ITA	JAP	NET	SPA	SWE	UK
Mean	-0.089	-0.053	-0.082	-0.104	-0.137	-0.090	-0.033	0.043
SK	-0.490*	-0.840*	-0.454*	0.875*	-0.080*	-0.055*	0.148*	0.242*
κ	5.780*	6.227*	6.238*	10.709*	5.888*	3.904*	5.068*	3.912*
$\rho_2(\tau) - [\rho(\tau)]^2$								
1	0.0638	0.0033	-0.0015	0.0127	0.0290	-0.0180	0.0585	0.0151
2	0.0943*	-0.0276	0.0313	0.0112	0.0167	-0.0003	0.0320	0.0400
3	0.0307	-0.0399	0.2312*	0.0167	0.0212	0.1339*	0.0377	0.0355
4	0.0157	-0.0658	0.0388	0.0117	0.0253	0.0210	0.0447	-0.0082
5	0.0703	-0.0236	0.0602	0.0297	0.0146	0.0222	0.0479	0.0675
12	0.1541*	-0.0202	0.0252	0.1648*	0.0655	0.0569	0.1506	0.0500
24	0.0279	0.0049	0.0194	0.0467	0.0579	0.0450	0.0739	-0.0030

* Significant at 5%; SK: Skewness; κ : Kurtosis; $\rho(\tau)$: Correlation of order τ .

Table 5: Summary statistics of the standardized innovations ν_t of a Q-STARCh model with stochastic seasonality for inflation series.