R&D INVESTMENTS FOSTERING HORIZONTAL MERGERS*

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Abstract
We study a homogenous good triopoly in which firms first choose their cost-reducing R&D investments and consider alternative merger proposals, and then compete à la Cournot in the ensuing industry. We identify conditions under which both horizontal mergers and non-integration are sustained by Coalition-Proof Nash equilibria (CPNE). These conditions involve the effectiveness of the R&D technology, as well as the distribution of the bargaining power between the acquirer and the acquiree, which determine the allocation of the incremental profits generated by the merger. We show that whether firms follow duplicative or complementary research paths, sustaining a merger generally requires a sufficiently effective R&D technology that creates endogenous cost asymmetries and renders the merger profitable, and a moderate distribution of bargaining power that allows to spread the benefits of the merger. We examine the welfare effects of mergers and obtain clear policy guidelines.

Keywords: Horizontal Mergers; Cost-Reducing Innovation; Endogenous Efficiency Gains; Antitrust; Coalition-Proof Nash Equilibrium.

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1 Introduction

In the context of the global economy, firms extensively use mergers as vehicles for growth and for gaining competitive advantage (UNCTAD, 2000; 2007). Regarding horizontal mergers, Röller et al. (2001) argue that their profitability as well as their opposing effects on welfare have been important topics in recent years. Gugler et al. (2003) identify nearly 16,500 horizontal mergers around the world (each worth over one million dollars) over the period from 1981 to 1998, that led to profit increases and efficiency gains.

A question that arises naturally is: Where do these efficiency gains come from? Straume (2006) argues that the most commonly indicated source of efficiency gains is the rationalization of production, from plants with higher marginal costs to those with lower marginal costs. Yet, the relevant literature typically assumes that the pre-integration cost asymmetries are exogenous. By contrast, this paper investigates the role of firms’ R&D investment decisions in endogenously creating cost asymmetries and potential efficiency gains that result in profitable horizontal mergers. In our setup, merger participants bargain over the division of the incremental profits generated by the merger. With this perspective, the paper also studies the relevant welfare effects and proposes appropriate competition policies.

We consider a two-stage Cournot triopoly model in which ex-ante symmetric firms produce a homogenous good. In the first stage, each firm decides its level of R&D investment, and may also propose a merger with one of the other two firms in the industry.\footnote{Following Inderst and Wey (2004), our model does not allow the possibility that all three firms merge to create a monopoly, which would be blocked by the antitrust authority.} If the merger proposals of any two firms are in accordance, then these firms merge; otherwise, all firms remain independent. In the second stage, the firms in the industry – either two or three, depending on whether or not there has been a merger – compete à la Cournot.

We consider that firms follow either duplicative research paths (Katsoulacos and Ulph, 1998) or complementary research paths (d’ Aspremont and Jacquemin, 1988). In the former case, the integrated entity’s cost is that of the most efficient firm in the merger, and therefore the R&D investment of the least efficient firm is wasted. In the latter case, the integrated entity’s cost is determined by the sum of the two firms’ R&D investments.

Regarding the division of the incremental profits generated by a merger, we include an exogenous parameter that captures the bargaining power of the acquirer relative to that of the acquiree. This parameter determines the distribution of the incremental profits (i.e., the difference between the integrated entity’s profits and the sum of those two firms’ profits in the ensuing triopoly if the merger does not materialize) between the acquirer and the acquiree. Thus,
under duplicative research paths, the acquiree invests in R&D in order to maximize its share of the merger’s incremental profits, which results in wasteful duplication of R&D investments. By contrast, under complementary research paths, the R&D investments of both firms contribute to reducing the unit production cost of the integrated entity.

We investigate conditions regarding the effectiveness of the R&D technology and the distribution of the bargaining power between the acquirer and the acquiree that sustain mergers. In our context, in which firms may ultimately act jointly, it is natural to assume that firms can freely discuss and reach agreements for joint action. Hence, an appropriate solution concept must take into account the possibility of deviations by coalitions of firms, as well as by individual firms. Of course, in the absence of commitment, firms’ agreements must be self-enforcing. Therefore, we employ the notion of coalition-proof Nash equilibrium (CPNE) introduced by Bernheim et al. (1987) – see also Moreno and Wooders (1996). The notion of CPNE identifies the “agreements” that are invulnerable to self-enforcing deviations by either individual firms or coalitions of firms. Note that, although this is a natural solution concept for the formation of mergers, our paper is the first to introduce it in the merger literature.

When firms follow duplicative research paths, two types of mergers must be considered: those in which the acquirer is the most efficient firm in the merger (type 1), and those in which the acquiree is the most efficient firm in the merger (type 2). We find that type 1 mergers can typically be sustained by a CPNE when the R&D technology is quite effective, i.e., when reducing the unit cost requires a small amount of R&D investment, whereas type 2 mergers can be sustained by a CPNE even when the R&D technology is relatively ineffective. Non-integration can be sustained by a CPNE except when both the R&D technology is quite effective and the acquirer’s bargaining power is rather large, in which case only mergers of either type (or both) can be sustained by a CPNE. Moreover, when the R&D technology is quite ineffective, only non-integration can be sustained by a CPNE, whereas for intermediate values of the effectiveness of the R&D technology and the acquirer’s bargaining power both non-integration and mergers may be sustained by a CPNE.

When firms follow complementary research paths, an integrated entity is significantly more efficient than the outsider firm, and thus captures a larger share of the market and realizes large profits. In this case, a merger can be sustained by a CPNE even when the R&D technology is ineffective and the bargaining power of the acquirer is large. In fact, there is a large region of parameters for which only a merger can be sustained by a CPNE. Only when the R&D technology is very ineffective, is non-integration the only outcome that can be sustained by a CPNE.

2 They can also be sustained by a CPNE for some less effective R&D technologies provided that the acquirer’s bargaining power is not too high.
CPNE.

Intuitively, sustaining a merger requires that the efficiency gains effect dominates the business stealing effect (Stigler, 1950; Salant et al., 1983). Obviously, under duplicative research paths the wasteful duplication of R&D efforts makes it more difficult for a merger to be sustained.

Assessing the welfare effects of a horizontal merger requires determining the balance of its two opposing effects: The deadweight loss resulting from the increased market concentration, and the efficiency gains that may make a merger welfare-enhancing. In our analysis, the impact of these two effects depends mainly on the effectiveness of the R&D technology. When the R&D technology is relatively effective, the efficiency gains effect dominates the deadweight loss effect, and mergers are welfare-enhancing. Then, the important task for antitrust authorities is to evaluate these two effects and to approve a merger only if it improves welfare, as measured either by total surplus or by consumer surplus.

We find that under duplicative research paths, a merger in which the technology of the acquiree is used (type 2) involves less wasteful R&D investment, and is more likely to increase total surplus than a merger in which the technology of the acquirer is used (type 1). Further, a merger is more likely to increase the total surplus under complementary research paths than under duplicative ones. Hence, when the regulator’s objective is to maximize total surplus, merger approval rules should be relatively more stringent under duplicative than under complementary research paths. Moreover, under duplicative research paths, merger approval rules should be relatively more stringent for type 1 than for type 2 mergers.

When the regulator’s objective is to maximize the consumer surplus, there are mergers generating efficiency gains that should not be approved. Our analysis reveals that a merger leads to an increase in the consumer surplus only under complementary research paths, and only when the R&D technology is too effective.

Market and societal incentives for a merger are often misaligned, i.e., there are cases in which a merger can be sustained by a CPNE, yet this merger is welfare detrimental and therefore should not be approved; or there are cases in which a welfare-enhancing merger cannot be sustained by a CPNE. This suggests that under some circumstances policy-makers should encourage firms – e.g., via R&D subsidization policies – toward a merger that otherwise would not materialize. Moreover, we identify cases in which under duplicative research paths the antitrust authority

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3Whinston (2007) states that the antitrust authority’s “enforcement practice in most countries (including the U.S. and the E.U.) is closest to a consumer surplus standard.” The *Horizontal Merger Guidelines* of the U.S. Department of Justice (2011) state that “the Agency will not challenge a merger if cognizable efficiencies . . . likely would be sufficient to reverse the merger’s potential harm to consumers in the relevant market.” The *Guidelines on the Assessment of Horizontal Mergers* of the European Union (2004) state that “the relevant benchmark in assessing efficiency claims is that consumers will not be worse off as a result of the merger.”
should approve a merger only under the remedy that the integrated entity uses the acquiree’s technology (instead of the acquirer’s). This further advocates relatively stringent approval criteria under duplicative research paths.4

1.1 Related literature and contribution

Our paper contributes to the following strands of the literature. First, to the literature on endogenous horizontal mergers with efficiency gains.5 Barros (1998) formalizes endogenous mergers through “participation constraints”, i.e., the merging firms’ profits do increase relative to their pre-merger sum of profits, and “stability constraints”, i.e., the outsider firm cannot offer a more profitable alternative to any merger participant. In Horn and Persson (2001), a merger is a cooperative game of coalition formation in which merger participants can decide on any division of the coalition’s profits. Using the core as the equilibrium concept, they compare the feasible coalitions given that transfer payments are not allowed. In this line of research, Straume (2006) argues that the most commonly indicated source of efficiency gains is the rationalization of production, from plants with higher marginal costs to those with lower ones. Moreover, the pre-merger (exogenous) marginal cost asymmetries determine the identity of the merger participants in equilibrium.6

Contrary to this line of research, the present paper (i) demonstrates that firms strategically choose their pre-merger technologies so as to induce endogenous cost asymmetries and potential efficiency gains; (ii) formalizes the bargaining between the merger participants over the division of the incremental profits generated by the merger; and (iii) considers that the antitrust authority approves the merger if and only if it improves welfare, measured by either the total surplus or the consumer surplus. In addition, the paper highlights that the merger participants’ identity results from the firms’ R&D investment and merger decisions in equilibrium. It shows that under duplicative research paths, the integrated entity always involves the most and the least

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4Derwinkel-Kalt and Wey (2016) cite evidence according to which “Remedies are increasingly applied by antitrust agencies in the United States and the European Union to clear merger proposals which are otherwise subject to serious anticompetitive concerns” as well as “The US Horizontal Merger Guidelines and the EU Merger Regulation allow for remedial offers to address competitive concerns”.

5There is also the “exogenous mergers” literature (in the taxonomy of Banal-Estañol et al., 2008). In this literature, there is only one candidate merger and each firm in the industry is assigned a role, either participant or no-participant, which is exogenously fixed and cannot be changed thereafter. Moreover, the merger is realized if and only if it increases each participant’s profits, compared with those that would be obtained in the no-merger scenario (Salant et al., 1983; Deneckere and Davidson, 1985; Perry and Porter, 1985; Farrell and Shapiro, 1990; Lommerud and Sørgard, 1997; Lommerud et al., 2005; Davidson and Ferrett, 2007).

6For large cost asymmetries, Barros (1998) finds that the two most efficient firms merge, while in Straume (2006) the merger occurs between either the least and the most efficient firm or the two least efficient firms. In Matsushima et al. (2013), symmetric firms merge, but only if R&D is not too costly; otherwise, the merger occurs between asymmetric firms. For moderate cost asymmetries, Barros (1998) and Straume (2006) find that the merger occurs between the least and the most efficient firm.
efficient firm. This is also true under complementary research paths, but only if the acquirer’s bargaining power is large; otherwise, the two most efficient firms merge.

In the context of endogenous mergers, efficiency gains can also be materialized by integrating the merger participants’ complementary resources, i.e., “synergies” in the terminology of Farrell and Shapiro (2001). Banal-Estañol et al. (2008) consider that three managers, each controlling some non-transferable resources (such as organizational or managerial capacities), first decide whether or not to merge and then choose their investments while anticipating a share of the future revenues. They find that managerial conflicts within a merger may offset possible synergies, thereby reducing the incentives to merge. Moreover, even when managers decide to merge, the integrated entity may become less efficient than the outsider firms. By contrast, in our setting, an integrated entity in a type 1 merger is always more efficient than the outsider, whereas in a type 2 merger it is more efficient than the outsider unless the R&D technology is too ineffective and the bargaining power of the acquirer is large enough.

Second, this paper contributes to the literature on optimal merger approval rules. Under exogenous pre-merger cost asymmetries and potential efficiency gains, Nocke and Whinston (2013) consider bargaining between a predetermined acquirer and possible target firms that will result in a bilateral merger that maximizes industry profit. They show that the minimum increase in consumer surplus for the proposed merger to be approved is larger, the greater the target’s pre-merger market share. In Burguet and Carminal (2015), each firm may bargain bilaterally and simultaneously with any other firm in a triopoly, and any pair of firms may merge, thus realizing efficiency gains that vary exogenously across pairs. In this context, bargaining may lead to a particular merger despite the existence of alternative mergers that would generate more profits and/or consumer surplus.

The above papers argue that the antitrust authority should optimally commit to an approval rule that is more stringent than that resulting when the welfare criterion is to maximize consumer surplus. Nocke and Whinston (2013) show that it might be optimal to reject a merger that increases consumer surplus in order to induce firms to propose alternative mergers that lead to a larger increase in consumer surplus. In Burguet and Carminal (2015), the antitrust authority announces a threshold value of marginal cost and approves a merger if and only if the merger’s marginal cost is below this threshold. Our analysis also advocates stringent approval rules for horizontal mergers. Yet, we argue that the approval rule should depend on whether

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8. The literature on optimal merger approval rules has its origin in Besanko and Spulber (1993), who consider the optimal rule for an antitrust authority. The latter cannot directly observe efficiencies but can recognize that firms know this information and decide whether to propose a merger based on their superior knowledge.
firms follow duplicative or complementary research paths, with the rule being more stringent in the former case. Moreover, under duplicative research paths, the merger guidelines should allow for an approval conditional on whether the integrated entity uses the merger participant’s technology that enhances welfare. This relates our paper to the literature on merger remedies in oligopolies (see Dertwinkel-Kalt and Wey, 2016 and the references therein). Yet, this literature typically assumes industries with fixed productive capital, in which a merger reduces the marginal cost by combining the merging firms’ complementary assets. In this context, the merger guidelines allow for approval conditional on the merging firms divesting critical assets to a competitor.

Finally, there is a strand in the literature that studies firms’ incentives to invest in cost-reducing R&D in the prospect of horizontal acquisitions. In a duopolistic industry, Stenbacka (1991) and Wong and Tse (1997) argue that the acquirer’s pre-acquisition incentives to reveal its cost reduction R&D investments to the acquiree depend on the degree of spillovers in the industry and the effectiveness of the R&D technology. In the prospect of an acquisition within a duopoly, Canoy et al. (2000) show that whether a manager over- or under-invests depends on the distribution of bargaining power between the manager and the firm’s shareholders. Socorro (2009) argues that under takeover threats, an acquiree may increase its cost-reducing R&D investment in order to signal its compatibility with the acquirer. These papers sideline the issues of bargaining between the merger participants, as well as merger control.

The rest of the paper is organized as follows. Section 2 describes the general model where we set up our inquiry. Section 3 introduces specific assumptions about the demand and the R&D technology that render a tractable setting in which the primitives are reduced to two parameters: the distribution of bargaining power between the acquirer and the acquiree and the effectiveness of the R&D technology. In Sections 4 and 5, we describe the Nash and coalition-proof Nash equilibria of the industry under duplicative and complementary research paths respectively. In Section 6, we derive the welfare implications of our results. We conclude in Section 7. The Appendix contains the proofs of our results.

2 A general model

We consider a homogenous good industry. The inverse demand function for the good is \( P(Q) \), where \( Q \in \mathbb{R}_+ \) is the firms’ total output. There are three firms that can produce the good with the same constant returns to scale technology, whose marginal (and unit) cost is initially equal to
Each firm can reduce its marginal cost by investing in R&D activities. Reducing the marginal cost by an amount \( x \in [0, c] \) requires an investment in R&D of \( C(x) \). To simplify our analysis, we assume that the inverse demand \( P(Q) \) is such that whether the industry is a duopoly or a triopoly there exists a unique Cournot equilibrium.\(^{10}\) For every \( i \in N := \{1, 2, 3\} \) we denote by \( \pi_i(3)(c_1, c_2, c_3) \) firm \( i \)'s equilibrium profit in the triopoly in which firms' marginal costs are \((c_1, c_2, c_3) \in [0, c]^3\). Similarly, for every \( i \in \{1, 2\} \) we denote by \( \pi_i(2)(c_1, c_2) \) firm \( i \)'s equilibrium profit in the duopoly in which firms' marginal costs are \((c_1, c_2) \in [0, c]^2\).

Firms maximize profits with respect to three decision variables: The level of R&D investment, whether or not to merge, and the level of output. We formalize the sequence of firms' decisions as a two-stage game with observable actions. In the first stage firms simultaneously decide their R&D investments and whether or not to merge. In the second stage, upon observing all investment and merger decisions, firms simultaneously decide their level of output. Our model responds to the rationale that R&D investments and mergers are long-term decisions.

We model merger decisions as follows: Each firm \( i \) proposes a merger with another firm; the proposal identifies not only the firm with which firm \( i \) wishes to merge but also its role in the merger as either acquirer or acquiree. Specifically, each firm \( i \) proposes a merger by selecting a pair \( m_i = (n, n') \in N \times N \) such that either \( n = i \), or \( n' = i \), or both. A proposal \( m_i = (i, j) \) indicates firm \( i \)'s desire to merge with firm \( j \) and play the role of the acquirer in the merger. A proposal \( m_i = (j, i) \) indicates firm \( i \)'s desire to merge with firm \( j \) and play the role of the acquiree. A proposal \( m_i = (i, i) \) indicates that firm \( i \) does not wish to merge. For \( i \in N \) write \( M_i := \{(n, n') \in N \times N \mid i \in \{n, n'\}\} \) for the set of possible merger proposals by firm \( i \). Also, write \( M := M_1 \times M_2 \times M_3 \). For every \( m \in M \), if \( m_i = m_j \) for some \( i, j \in N \), then firms \( i \) and \( j \) merge and the outsider firm \( k \in N \setminus \{i, j\} \) remains independent; otherwise, i.e., when \( m_1 \neq m_2 \neq m_3 \neq m_1 \), all three firms remain independent. Note that in our setting only two firms may merge, i.e., a monopoly is ruled out because it will blocked by the antitrust authorities (Inderst and Wey, 2004).

In the spirit of the generalized Nash bargaining solution, we assume that the increment in profits generated by a merger (i.e., the difference between the integrated entity’s profits and the sum of profits that merger participants would have in the ensuing triopoly if the merger does not materialize) is split between merger participants according to pre-specified shares \( \beta \in [1/2, 1] \) and \( 1 - \beta \) for the acquirer and the acquiree, respectively. The increment in profits generated by a merger of firms \( i \) and \( j \) depends on the marginal cost of the integrated entity, which in turn

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\(^9\) Assuming that the firms’ marginal cost is less than the consumers’ maximum willingness to pay for the good \( P(0) \) rules out trivial cases.

\(^{10}\) This assumption holds when the inverse demand is a truncated linear function as we assume below. General conditions for the uniqueness of a Cournot equilibrium are provided by, e.g., Einy et al. (2010).
is determined by the merger participants’ R&D investments. Let us denote by \( \varphi(x_i, x_j) \) the reduction in the marginal cost of the integrated entity formed by firms \( i \) and \( j \) when their R&D expenditures are \( C(x_i) \) and \( C(x_j) \), respectively. We consider two polar cases: In Section 4 we study the case in which R&D investments are substitutes, i.e., firms follow duplicative research paths. In Section 5 we study the case in which R&D investments are complements, i.e., firms follow complementary research paths.

Throughout the paper, we consider only pure strategies and restrict attention to subgame perfect equilibria. In a subgame perfect equilibrium, given firms’ merger and R&D investment decisions, the profile of outputs forms a Cournot equilibrium for the industry. Restricting attention to subgame perfect equilibria allows for a reduced form analysis of our model as a single-stage game \( \Gamma \), to be described shortly. An equilibrium of \( \Gamma \) unambiguously identifies a subgame perfect equilibrium of the two-stage game defined by our model. Focusing on \( \Gamma \) facilitates the study of the factors determining the firms’ R&D investments and merger decisions in a simple and manageable way.

In the game \( \Gamma \), the firms are the players and each firm \( i \)’s set of pure strategies is \([0,c] \times M_i\). Let \((x,m) \in [0,c]^3 \times M\) be a profile of strategies. If \(m_1 \neq m_2 \neq m_3 \neq m_1\), then the ensuing industry is a triopoly, and the profit of each firm \( i \in N \) net of its R&D cost is

\[
\Pi_i(x,m) = \pi_i^{(3)}(c - x_1, c - x_2, c - x_3) - C(x_i). \quad (1)
\]

If \(m_i = m_j\), for some \( i, j \in N \), then firms \( i \) and \( j \) merge and the ensuing industry is therefore a duopoly. In this case, we denote by \( \overline{\pi}(m) \in \{i,j\} \) the index of the firm playing the role of the acquirer and by \( a(m) \in \{i,j\} \setminus \{\overline{\pi}(m)\} \) the index of the acquiree. The index of the firm that remains independent, the outsider firm, is denoted by \( o(m) \in N \setminus \{i,j\}\).\(^{11}\) The net profit of the acquirer is

\[
\Pi_{\overline{\pi}(m)}(x,m) = \pi_{\overline{\pi}(m)}^{(3)}(c - x_1, c - x_2, c - x_3) + \beta \Delta(x,m) - C(x_{\overline{\pi}(m)}), \quad (2)
\]

and that of the acquiree is

\[
\Pi_{a(m)}(x,m) = \pi_{a(m)}^{(3)}(c - x_1, c - x_2, c - x_3) + (1 - \beta) \Delta(x,m) - C(x_{a(m)}), \quad (3)
\]

\(^{11}\)For example, if \(m_1 = m_3 = (3,1)\), then \(\overline{\pi}(m) = 3\), \(a(m) = 1\), and \(o(m) = 2\).
where $\Delta(x, m)$ is the increment in profits generated by the merger

$$
\Delta(x, m) = \pi_1^{(2)}(c - \phi(x_{\pi(m)}, x_{\underline{a}(m)}), c - x_{o(m)}) - \pi_1^{(3)}(c - x_1, c - x_2, c - x_3) - \pi_1^{(3)}(c - x_1, c - x_2, c - x_3).
$$

(4)

The net profit of the outsider is

$$
\Pi_{o(m)}(x, m) = \pi_2^{(2)}(c - \phi(x_{\pi(m)}, x_{\underline{a}(m)}), c - x_{o(m)}) - C(x_{o(m)}).
$$

(5)

Note that the net profit of each merger participant consists of three terms: The first term is the firm’s outside option, i.e., its profits in the Cournot equilibrium of the triopoly where firms’ costs are $c - x_i$ for $i \in N$, which would result if either firm $\pi(m)$ or firm $\underline{a}(m)$ breaks the merger unilaterally. The second term is the firm’s share of the increment in profits generated by the merger, $\Delta(x, m)$, which is the difference between the integrated entity’s profits in the duopoly and the sum of the profits of firms $\pi(m)$ and $\underline{a}(m)$ in the triopoly. The third term is the firm’s R&D cost.

It is easy to construct a Nash equilibrium (NE) of $\Gamma$ in which firms do not merge, which we refer to as non-integration. For example, the profile $(x, m)$ given for all $i \in N$ by $m_i = (i, i)$, and $x_i = x^{NI}$ where $x^{NI}$ maximizes $\pi_1^{(3)}(c - x, c - x^{NI}, c - x^{NI}) - C(x)$, is a NE. Simply note that no firm can generate a merger by unilaterally deviating, and each firm’s level of R&D investment maximizes its net profits given the investments of the rival firms. For future references, we state this simple result in Proposition 1.

**Proposition 1.** Non-integration can always be sustained by a Nash equilibrium.

In our context, it is natural to assume that firms are free to discuss their strategies and reach agreements for joint action. Obviously, in the absence of commitment firms’ agreements must be self-enforcing. We therefore employ coalition-proof Nash equilibrium (CPNE), introduced by Bernheim et al. (1987), as the solution concept. A CPNE identifies the agreements that are invulnerable to self-enforcing deviations by either individual firms or coalitions of firms. A deviation is self-enforcing if there is no further self-enforcing and improving deviation available to a proper subcoalition of firms. This notion of self-enforceability provides a useful means of distinguishing coalitional deviations that are viable from those that are not resistant to further deviations. Of course, a CPNE is also a NE. However, not all NE are CPNE, and the existence of a CPNE is not warranted under standard conditions – see Moreno and Wooders (1996) for a limited existence result. Moreover, identifying the CPNE of a game may be a difficult task, as it requires checking its robustness against all possible self-enforcing deviations. However,
the concept has proved to be useful in studying models of competition that lead to a large set of equilibria – see e.g., Delgado and Moreno (2004). Our model is simple enough that, notwithstanding these difficulties, we are able to identify the regions of parameters in which there are CPNE that sustain various types of mergers as well as non-integration.

The notion of CPNE is defined recursively on the number of players: In a single player game a strategy profile \( s^* \) is a CPNE if it maximizes the player’s payoff. Consider a game with \( n > 1 \) players, and assume that the notion of CPNE has been defined for games with \( k < n \) players. In this game, (i) a strategy profile \( s \) is self-enforcing if for every proper subcoalition \( J \subset N \) the strategy profile \( s_J \) is a CPNE of the game obtained from the original game by fixing the strategy of every player outside the coalition \( i \in N \setminus \{J\} \) as \( s_i \); (ii) A CPNE of the game is a strategy profile \( s^* \) that is self-enforcing and such that no other self-enforcing strategy profile exists that yields a greater payoff for every player.

3 A simple model

Henceforth we consider a simple version of our model in which the inverse demand function for the good is \( P(Q) = \max\{\alpha - Q, 0\} \) and the R&D cost function is \( C(x) = \gamma x^2/2 \). The parameter \( \gamma > 0 \) is a proxy for the effectiveness of the R&D technology in reducing the marginal cost: The greater \( \gamma \), the greater the expenditure required to obtain a given reduction in the marginal cost, i.e., the less effective the R&D technology. This quadratic specification of the R&D cost implies diminishing returns to R&D expenditures – see d’Aspremont and Jacquemin (1988). To guarantee that equilibrium is interior we assume in the sequel that \( \gamma \geq 3/2 \).

Consider the subgame of Cournot competition following merger proposals \( m \in M \) and firms’ R&D investments \( x = (x_1, x_2, x_3) \in [0, c]^3 \). Because in this subgame R&D investments are sunk, in equilibrium each firm \( i \) chooses its output to maximize its profit. In a subgame involving no merger, the profit of firm \( i \in N \) in the linear triopoly in which firms’ marginal costs are \( (c - x_1, c - x_2, c - x_3) \) is

\[
\pi_i^{(3)}(c - x_1, c - x_2, c - x_3) = \left(\frac{\alpha - c + 4x_i - X}{4}\right)^2, \tag{6}
\]

where \( X = x_1 + x_2 + x_3 \) is the firms’ total R&D investment. Consider a subgame involving a merger. Henceforth we write \( (\pi, \varphi, \varphi) \) for \( (\pi(m), \varphi(m), \varphi(m)) \). The marginal cost of the integrated entity is \( c - \varphi(x_\pi, x_\varphi) \) and that of the outsider is \( c - x_o \). In this linear duopoly the
profit of the integrated entity is
\[ \pi_1^{(2)}(c - \varphi(x_\pi, x_\omega), c - x_o) = \left( \frac{\alpha - c + 2\varphi(x_\pi, x_\omega) - x_o}{3} \right)^2, \] (7)
and that of the outsider is
\[ \pi_2^{(2)}(c - \varphi(x_\pi, x_\omega), c - x_o) = \left( \frac{\alpha - c + 2x_o - \varphi(x_\pi, x_o)}{3} \right)^2. \] (8)

In this setting, it is easy to calculate the firms’ cost reduction implied by their level of R&D investment in a NE that sustains non-integration: In such an equilibrium of \( \Gamma \), the R&D investment level of each firm \( i \in N \) maximizes net profits
\[ \Pi_i^{(3)}(\bar{x}_{-i}, x_i) = \left( \frac{\alpha - c + 3x_i - \bar{x}_{-i}}{4} \right)^2 - \frac{1}{2}\gamma x_i^2, \] (9)
where \( \bar{x}_{-i} = X - x_i \). Hence, firm \( i \)'s R&D reaction function is
\[ R_i^{(3)}(\bar{x}_{-i}) = \frac{3(\alpha - c - \bar{x}_{-i})}{8\gamma - 9}. \] (10)

Solving the system of reaction functions, we obtain the level of R&D investment of each firm \( x_{NI} \) in a NE of \( \Gamma \) that sustains non-integration. Then replacing the firms’ levels of R&D investments in (9), we obtain the net profit of each firm \( \Pi^{NI} \). Formulae describing these values are provided in Appendix A.1. These formulae reveal that the equilibrium level of R&D investment and output increases and net profit decrease, the more effective the R&D technology (i.e., the smaller the value of \( \gamma \)). It should be noted that for any \((\gamma, \beta)\), there is a unique Nash equilibrium that sustains non-integration.

Let \( x \in [0, c]^3 \) and assume that there is a merger. As the marginal costs of the integrated entity and the outsider are \( c - \varphi(x_\pi, x_\omega) \) and \( c - x_o \), respectively, the net profit of the outsider is
\[ \Pi_o(x, m) = \left( \frac{\alpha - c + 2x_o - \varphi(x_\pi, x_\omega)}{3} \right)^2 - \frac{\gamma}{2} x_o^2, \] (11)
the net profit of the acquirer \( \bar{a} \) is
\[ \Pi_{\bar{a}}(x, m) = \left( \frac{\alpha - c + 4x_\pi - X}{4} \right)^2 + \beta\Delta(x, m) - \frac{\gamma}{2} x_\pi^2, \] (12)
and the net profit of the acquiree \( a \) is
\[ \Pi_a(x, m) = \left( \frac{\alpha - c + 4x_a - X}{4} \right)^2 + (1 - \beta)\Delta(x, m) - \frac{\gamma}{2} x_a^2, \] (13)
where $\Delta(x, m)$ is the increment in profits generated by the merger,

$$
\Delta(x, m) = \left( \frac{\alpha - c + 2\varphi(x_\pi, x_o) - x_o}{3} \right)^2 - \left[ \left( \frac{\alpha - c + 4x_\pi - X}{4} \right)^2 + \left( \frac{\alpha - c + 4x_o - X}{4} \right)^2 \right].
$$

(14)

Calculating the firms’ reaction functions in R&D investment levels when there is a merger, and verifying whether a profile can be sustained by a CPNE of $\Gamma$, is more involved and requires specifying the function $\varphi$, which describes the technological consequences of the merger. In the next two sections we study in turn the cases of duplicative and complementary research paths.

### 4 Duplicative research paths

In this section we assume that firms’ R&D investments are perfectly substitutable, i.e., as in Katsoulacos and Ulph (1998), firms follow duplicative research paths. If two firms $i$ and $j$ merge, then the integrated entity produces its output at a marginal cost equal to the minimum of the marginal costs of both firms, i.e.,

$$
\varphi(x_i, x_j) = \max\{x_i, x_j\}.
$$

In this scenario, following a merger, the reaction function of the outsider is

$$
R_o(x_\pi, x_o) = \frac{4(\alpha - c - \max\{x_\pi, x_o\})}{9\gamma - 8}.
$$

(15)

The reaction functions of the acquirer and the acquiree are

$$
R_a(x_\pi, x_o) = \frac{(27 + 14\beta)(\alpha - c - x_o) + 27(2\beta - 1)x_o}{72\gamma + 26\beta - 81},
$$

(16)

and

$$
R_a(x_\pi, x_o) = \frac{(1 + 2\beta)(\alpha - c - x_o) - 3(2\beta - 1)x_\pi}{8\gamma - 10\beta + 1},
$$

(17)

if $x_\pi > x_o$ (i.e., the acquirer is the most efficient firm in the merger), and they are

$$
R_a(x_\pi, x_o) = \frac{(3 - 2\beta)(\alpha - c - x_o) + 3(2\beta - 1)x_\pi}{8\gamma + 10\beta - 9},
$$

(18)

and

$$
R_a(x_\pi, x_o) = \frac{(41 - 14\beta)(\alpha - c - x_o) - 27(2\beta - 1)x_\pi}{72\gamma - 26\beta - 55},
$$

(19)
if \( x_\pi < x_a \) (i.e., the acquirer is the least efficient firm in the merger).

Independent of the relative efficiency of the acquirer and the acquiree, the strategic relationship between the R&D investments of the three firms provides the following insights. First, the R&D investment of the least efficient firm in the merger does not affect the R&D investment of the outsider. Second, the R&D investment of the acquirer increases with the R&D investment of the acquiree: As the acquiree’s R&D increases, its reservation profit (i.e., its profit in the Cournot triopoly, \( \pi_a^{(3)} \)) increases, which provides incentives for the acquirer to increase its own R&D investment in order to mitigate this effect. Third, the R&D investment of the acquiree decreases with the R&D investment of the acquirer: An increase in the acquirer’s R&D investment leads to an increase in its reservation profit (i.e., its profits in the Cournot triopoly, \( \pi_a^{(3)} \)) and a decrease in the acquiree’s reservation profit \( \pi_a^{(3)} \) and might also increase the profit of the integrated entity in the duopoly \( \pi_1^{(2)} \); as the R&D investments of the acquirer and the acquiree are strategic substitutes in the triopoly, the acquiree’s incentive to invest in R&D decreases. Finally, the R&D investments of both firms in a merger are strategic substitutes for the R&D investment of the outsider.

A solution to the system of equations formed by the firms’ reaction functions (15) to (19) provides the firms’ R&D investments \( x = (x_\pi, x_a, x_o) \) in a Nash equilibrium that sustains a merger (see Appendix A.2 for the solution to this system). In contrast to non-integration, which can be sustained by a NE for all parameter values, mergers of either type 1 \( (x_\pi > x_a) \) or type 2 \( (x_\pi < x_a) \) can be sustained by a NE only on a subset of the parameter space. This is because mergers are vulnerable to unilateral deviations of either the acquirer or the acquiree. (These deviations involve breaking the merger and optimally adjusting the level of R&D investments to compete in a triopoly.) It turns out that for any given distribution of bargaining power between the acquirer and the acquiree (i.e., the value of \( \beta \)), when the R&D technology is sufficiently effective (i.e., for \( \gamma \) low enough), there are NE that sustain both types of mergers. The parameter constellations for which a type 1 and a type 2 merger are sustained as a NE are illustrated in Figure 1. The black and red lines indicate the upper bound for the parameter \( \gamma \) such that a type 1 and a type 2 merger, respectively, can be sustained by a NE. Note that, although for most parameters the two types of merger can be sustained by a NE, there are parameter constellations in which only one type of merger can be sustained by a NE. Proposition 2 summarizes these findings.

**Proposition 2.** Consider an industry in which firms follow duplicative research paths. (2.1) A type 1 (i.e., \( x_\pi > x_a \)) merger can be sustained by a NE if and only if \( \gamma \leq \hat{\gamma}_{12}(\beta) \equiv \min[\hat{\gamma}_1(\beta), \hat{\gamma}_2(\beta)] \).
A type 2 (i.e., $x'_a < x'_o$) merger can be sustained by a NE if and only if $\gamma \leq \gamma_1(\beta)$.

Figure 1 provides a graph of the functions involved in Proposition 2. Recall that according to Proposition 1, non-integration can always be sustained by a NE.

**INSERT FIGURE 1 HERE**

Given the firms' R&D investments in a NE that sustains a merger, we can calculate the firms' outputs $q = (q_m, q_o)$, where $q_m$ is the output of the integrated entity and $q_o$ is the output of the outsider. Likewise, we can calculate the firms' net profits $\Pi = (\Pi_a, \Pi_o)$. Formulae describing these values are given in Appendix A.2. These formulae reveal a number of properties of the outcomes generated by NE that sustain a type 1 and a type 2 merger, which we summarize in Proposition 3. Recall that $\Pi^{NI}$ is the profit of a firm in a NE that sustains non-integration.

**Proposition 3.** Consider an industry in which firms follow duplicative research paths and let $(x, q, \Pi)$ and $(x', q', \Pi')$ be the firms' R&D investments, outputs and net profits in a NE that sustains a type 1 (i.e., $x_a > x_o$) and a type 2 (i.e., $x'_a < x'_o$) merger, respectively.

1. $x_a > x_o > x'_o > 0$, $\min[x'_o, x'_a] > x'_\pi > 0$; moreover, $x'_a > x'_o$ except if both $\gamma$ and $\beta$ are large.
2. $q_m > q_o$; moreover, $q'_m > q'_o$ except if both $\gamma$ and $\beta$ are large.
3. $x_a > x'_a, x_o < x'_o, x_a + x_o > x'_a + x'_o$; moreover, $x_a > x'_a$ except if $\gamma$ is small.
4. $q_m > q'_m, q_o < q'_o$, and $q_m + q_o > q'_m + q'_o$.
5. $\Pi'_o > \Pi_o > \Pi^{NI}$; moreover, $\Pi_o + \Pi_\pi + \Pi'_a < \Pi'_o + \Pi'_\pi + \Pi'_a$ except if $\gamma$ is small.
6. $\Pi_\pi > \Pi^{NI} > \Pi'_a, \Pi'_a > \Pi^{NI} > \Pi'_\pi$; hence $\Pi_\pi > \Pi'_\pi$ and $\Pi'_a > \Pi'_o$.
7. $\Pi_o > \Pi_\pi$ except if $\gamma$ is small; moreover $\Pi'_o > \Pi'_a$ except if both $\gamma$ and $\beta$ are small.

In a NE that sustains a merger:

1. The outsider’s R&D investment is larger than that of the least efficient firm in the integrated entity; moreover, in a type 1 merger (i.e., $x_\pi > x'_a$), the integrated entity is more efficient than the outsider (i.e., $x_\pi > x_o$), but in a type 2 merger (i.e., $x'_\pi < x'_o$) the most efficient firm may either the integrated entity or the outsider. In fact, $x'_o > x'_o$ except if both $\beta$ and $\gamma$ are high enough.
2. The output of the integrated entity is always larger than that of the outsider in a type 1 merger. This is also true for a type 2 merger, except if the bargaining power of the acquirer is large and the technology is not too effective.

In addition, relative to a type 2 merger, in a type 1 merger...
(3.3) The integrated entity is more efficient, the outsider is less efficient, and the industry’s total effective R&D investments, i.e., the investments actually used to reduce firms’ marginal costs, are larger; further, the wasteful R&D investments are larger, except if the technology is too effective.

(3.4) The output of the integrated entity and the total output are larger, whereas the output of the outsider is smaller.

(3.5) The profit of the outsider and the total profit (except for very low \( \gamma \)) is smaller; further, in any merger equilibrium the outsider obtains higher profits than in a non-integration equilibrium.

Moreover, in a type 1 (2) merger

(3.6) The profits of the acquirer (acquiree) are larger than in the non-integration equilibrium, whereas the profits of the acquiree (acquirer) are smaller than in the non-integration equilibrium.

(3.7) The profits of the outsider are larger than those of the acquirer (acquiree), except if \( \gamma \) is low enough (if both \( \beta \) and \( \gamma \) are low enough.)

We turn now to identifying the CPNEs. Let us consider the non-integration Nash equilibrium profile where \( x_i = x^{NI} \) and \( m_i = (i,i) \) identified in Section 2. In order for this NE to be a CPNE of the game, it must be invulnerable to all self-enforcing coalitional deviations. The relevant deviations are those involving any two firms merging and choosing appropriately their R&D investments, taking as given the R&D investment of the outsider \( x_o = x^{NI} \). Of course, each deviation must be profitable and self-enforcing, i.e., robust against further deviations by each firm in the merger.

Consider next a candidate CPNE involving a merger. Proposition 1 identifies the parameter constellations in which there is a NE that sustains a merger. For this NE to be a CPNE, however, we must account for the possibility of coalitional deviations involving the acquirer \( \pi \) (or the acquiree \( \mu \)) and the outsider \( o \), in which the former breaks the merger and forms a new merger with the outsider and in which both firms optimally adjust their R&D investments (taking as given the R&D investments of the outsider firm in the new merger). In addition, we must account for a coalitional deviation of the firms in a type 1 (or 2) merger in which they adjust their R&D investments appropriately (taking as given the R&D investments of the outsider) and form a type 2 (or 1) merger. Of course, in order for such deviations to be self-enforcing, they must be invulnerable to further unilateral deviations by either firm in the deviating coalition. The non-profitability of each of the above deviations places a restriction on the parameter values. Checking these restrictions and identifying the parameter constellations for which a solution to the system of the firms’ reaction functions and inequalities is a CPNE of the game involves simple but messy algebra, which we relegate to Appendix A.2.
Proposition 4 provides a description of the parameter constellations in which non-integration and mergers of type 1 and 2 can be sustained by a CPNE. The description involves the following subsets of the relevant parameter space $\Omega = \{(\beta, \gamma) \in [1/2, 1] \times [3/2, \infty)\}$:

\[
B^d = \{ (\beta, \gamma) \mid \gamma \geq \gamma(\beta) \} \\
R^d = \{ (\beta, \gamma) \mid \bar{\gamma}_9(\beta) \leq \gamma \leq \bar{\gamma}_{12}(\beta) \} \cup \{ (\beta, \gamma) \mid \gamma_3(\beta) \leq \gamma \leq \gamma_7(\beta) \} \cup \{ (\beta, \gamma) \mid \gamma \leq \bar{\gamma}_4(\beta) \} \\
Y^d = \{ (\beta, \gamma) \mid \gamma \leq \gamma_1(\beta) \} \cup \{ (\beta, \gamma) \mid \gamma_4(\beta) \leq \gamma \leq \gamma_3(\beta) \} \\
W^d = \Omega \setminus (B^d \cup R^d \cup Y^d).
\]

**Proposition 4.** Consider an industry in which firms follow duplicative research paths.

(4.1) Non-integration can be sustained by a CPNE only if $(\beta, \gamma) \in B^d$.

(4.2) A type 1 merger can be sustained by a CPNE only if $(\beta, \gamma) \in R^d$.

(4.3) A type 2 merger can be sustained by a CPNE only if $(\beta, \gamma) \in Y^d$.

(4.4) If $(\beta, \gamma) \in W^d$, then a CPNE does not exist.

The sets $B^d$, $R^d$ and $Y^d$ correspond to the areas in blue, red and yellow in Figures 2a, 2b and 2c, respectively. The boundaries of these sets are identified by the “incentive constraints” required to sustain each type of outcome as a CPNE. Appendix A.2, which contains the proof of Proposition 4, provides the functions involved in the definitions of these sets.

**INSERT FIGURES 2a, 2b AND 2c HERE**

Figure 3 provides a partition of the parameter space that allows us to identify the outcomes that can be sustained by a CPNE for each parameter value: The blue (respectively, red, yellow) area represents the set $B^d \cap (\Omega \setminus (R^d \cup Y^d))$ (respectively, $R^d \cap (\Omega \setminus (B^d \cup Y^d))$ and $Y^d \cap (\Omega \setminus (B^d \cup R^d))$) in which only non-integration (respectively, a type 1 merger, a type 2 merger) can be sustained by a CPNE. The purple area represents the set $B^d \cap R^d \cap (\Omega \setminus Y^d)$ in which both non-integration and a type 1 merger can be sustained, but a type 2 merger cannot be sustained, by a CPNE. Likewise, the green area represents the set $B^d \cap Y^d \cap (\Omega \setminus R^d)$ in which both non-integration and a type 2 merger can be sustained, but a type 1 merger cannot be sustained, by a CPNE. The orange area represents the set $R^d \cap Y^d \cap (\Omega \setminus B^d)$ in which both types of mergers can be sustained, but non-integration cannot be sustained, by a CPNE. The grey area represents the set $B^d \cap R^d \cap Y^d$ in which all three outcomes can be sustained by a CPNE. Finally, the small white area represents the set $W^d$ in which no CPNE exists.
In contrast to Salant et al. (1983), that in a symmetric linear Cournot triopoly with homogenous products only integration to a monopoly is profitable, Proposition 4 establishes that R&D investments may endogenously generate the asymmetries needed to sustain a merger by two firms. Further, in contrast to Barros (1998) and Straume (2006) in which exogenously given cost asymmetries determine the participants in a merger, in our setup cost asymmetries arise endogenously and determine the merger participants. In particular, and in contrast to the aforementioned literature, we find that the merger is always formed between the most and the least efficient firms. Moreover, in a type 1(2) merger, the most efficient firm is the acquirer (acquiree). In this respect, not only the identity of the firms in the merger, but also whose technology is used by the integrated entity, is determined endogenously in our setup.

The profitability of a merger is limited by, first, the business stealing effect (Stigler, 1950; Salant et al., 1983), i.e., the increased sales of the outsider firm caused by the merger, and second, the wasteful duplication of R&D investments by the firms in the merger. A merger is profitable when the impact on profits of the efficiency gains and the increased market power generated by the merger are large enough to outweigh the business stealing and wasteful duplication effects.

Type 1 mergers can be sustained by a CPNE for the parameter constellations in the red area shown in Figure 2b. This is a subspace of the parameters in which type 1 mergers can be sustained by a NE (Proposition 1.1). This is because these mergers may be vulnerable to coalitional deviations whereby firms in the merger switch their roles and adjust their levels of R&D investments to those of a type 2 merger. The latter deviations are profitable when the R&D technology is ineffective (i.e., $\gamma$ is large). Yet, the upper bound on $\gamma$ depends in turn on the vulnerability of these coalitional deviations to further unilateral deviations. As a consequence, type 1 mergers can be sustained in the upper red area in Figure 2b. In addition, for given $\beta$, a deviation of the coalition of the outsider and the acquirer, which form a new integrated entity in which the latter has the role of the acquiree and choose the R&D investment levels of a type 2 merger, may also upset the merger when the R&D technology is sufficiently effective (i.e., $\gamma$ is low). The latter deviation is profitable for both firms (and also invulnerable to further unilateral deviations by each one of them) in the white area of Figure 2b. Thus, type 1 mergers cannot be sustained by a CPNE in that area. (See the proof of Proposition 4.2 in Appendix A.2.)

Likewise, type 2 mergers can be sustained by a CPNE for the parameter constellations in the yellow area shown in Figure 2c. Again, this is a subspace of the parameters in which type 2 mergers can be sustained by a NE (Proposition 1.2). This is because these mergers may be
vulnerable to a coalitional deviation of the acquiree and the outsider, which form an integrated entity in which the latter has the role of the acquiree and adjust their R&D investment levels appropriately. This deviation is profitable for both firms, and also invulnerable to further unilateral deviations when the R&D technology is sufficiently effective (i.e., $\gamma$ is low) and the acquirer’s bargaining power is high (i.e., $\beta$ is high). Thus, type 2 mergers cannot be sustained by a CPNE in the white area of Figure 2c. Moreover, as Figure 3 illustrates, type 2 mergers can be sustained by a CPNE in a wider range of parameters (i.e., for less effective R&D technologies) than type 1 mergers.

Non-integration can be sustained by a CPNE for all parameter constellations except for those in the white area of Figure 2a. In particular, if R&D technology is effective (i.e., $\gamma$ is low) and the bargaining power of the acquirer is high (i.e., $\beta$ is high), a deviation of a coalition of any two firms that form a type 2 merger and adjust their levels of R&D investments appropriately upsets non-integration. This deviation leads to a merger in which the acquiree invests more in R&D than the acquirer in order to overcome the acquirer’s large bargaining power. In fact, after the deviation the profits of the acquirer are lower than those of the acquiree, although the profits of both firms are sufficiently large to make the deviation profitable. Clearly, the latter deviation is self-enforcing because neither of the deviating firms can do better by switching back to non-integration.

Finally, for the parameter constellations in the white area of Figure 3 a CPNE does not exist. This region is the intersection of the white areas in Figures 2a, 2b and 2c. As we saw above, for $(\beta, \gamma) \in W^d$, non-integration cannot be sustained by a CPNE as it is vulnerable to a deviation by a coalition of any two firms that form a type 2 merger. In addition, neither type of merger can be sustained by a CPNE in this region: a type 1 merger is vulnerable to a deviation of the coalition of the outsider and the acquirer (using the technology of the latter) and a type 2 merger is vulnerable to a deviation of the coalition of the acquiree and the outsider (using the technology of the former).

5 Complementary research paths

In this section we assume that firms’ R&D investments are perfectly complementary; i.e., as in d’Aspremont and Jacquemin (1988), we assume that firms follow complementary research paths. If two firms $i$ and $j$ merge and invest in R&D $x_i$ and $x_j$, respectively, then the integrated entity reduces its marginal cost by an amount equal to

$$\varphi(x_i, x_j) = x_i + x_j.$$
In this scenario, following a merger, the firms’ reaction functions are

\[ \hat{R}_a(x_o, x_a) = \frac{(27 + 14\beta)(\alpha - c - x_o) + (118\beta - 27)x_a}{72\gamma + 26\beta - 81}, \]  

(20)

\[ \hat{R}_\pi(x_o, x_\pi) = \frac{(41 - 14\beta)(\alpha - c - x_o) - (118\beta - 91)x_\pi}{72\gamma - 26\beta - 55}; \]  

(21)

\[ \hat{R}_o(x_\pi, x_o) = \frac{4(\alpha - c - x_\pi - x_o)}{9\gamma - 8}. \]  

(22)

Naturally, the strategic relationship between the firms’ R&D investments differs from those of the case of duplicative research paths. With complementary research paths, the outsider’s R&D investment and the R&D investments of the acquirer and the acquiree that are jointly used by the integrated entity, are strategic substitutes. In addition, the incentive of the acquirer to increase its R&D investment as the acquiree invests more is even sharper than in the case of duplicative research paths. By contrast, the acquiree has an incentive to invest more in R&D the more the acquirer invests, but only if the bargaining power of the acquirer is not too large – specifically, if \( \beta < \frac{91}{118} \approx 0.77 \).

A solution to the system of equations formed by the firms’ reaction functions (20) to (22) provides the firms’ R&D investments \( x = (x_\pi, x_a, x_o) \) in the unique NE that sustains a merger. In this scenario, second order and stability conditions require \( \gamma \geq 5/2 \), which we assume throughout this section. When this inequality holds, the system of equations has a unique solution, i.e., the market outcome resulting from a merger sustained by a NE is uniquely determined. As above, and in contrast to non-integration which is sustained by a NE for all parameter values (Proposition 1), a merger is sustained by a NE only on a subset of the parameter space. More specifically, a merger is vulnerable to a deviation of the acquirer (breaking the merger and adjusting its R&D investment appropriately) when the R&D technology is sufficiently effective given \( \beta \), i.e., \( \gamma \leq \gamma_1^*(\beta) \) (see Figure 4). Interestingly, when the acquirer has no incentive to deviate, neither does the acquiree. Thus, the inequality above identifies the subset of parameters in which a merger can be sustained by a NE. This finding is stated in Proposition 5.

**Proposition 5:** In an industry in which firms follow complementary research paths a merger can be sustained by a NE if and only if \( \gamma \leq \gamma_1^*(\beta) \).

The equilibrium R&D investments determine firms’ equilibrium outputs \( q = (q_m, q_o) \) and net profits \( \Pi = (\Pi_\pi, \Pi_a, \Pi_o) \). Formulae describing the equilibrium outcome are given in Appendix A.3. These formulae reveal a number of properties that we summarize in Proposition 6.
Proposition 6. Consider an industry in which firms follow complementary research paths. In a NE that sustains a merger, \((x, q, \Pi)\), firms’ R&D investments, outputs and net profits are uniquely determined and satisfy:

\(\begin{align*}
(6.1) \quad x_\pi > x_\alpha, \; x_\pi > x_o; \; \text{hence,} \; x_\pi + x_\alpha > x_o > 0. \\
(6.2) \quad q_m > q_o. \\
(6.3) \quad \Pi_\pi > \Pi_\alpha; \; \text{moreover,} \; \Pi_\alpha > \Pi_o \; \text{if} \; \gamma \; \text{is small.} \\
(6.4) \quad \Pi_o > \Pi^{NI} \; \text{and} \; \Pi^{NI} > \Pi_\pi \; \text{except if} \; \gamma \; \text{is small; moreover,} \; \Pi_\alpha > \Pi^{NI} \; \text{if both} \; \gamma \; \text{and} \; \beta \; \text{are small.}
\end{align*}\)

In a NE that sustains a merger:

(6.1) The acquirer’s R&D investment level is greater than both the acquiree’s and the outsider’s. As a result, the integrated entity is significantly more efficient than the outsider.

(6.2) A consequence of the above is that the output of the integrated entity is greater than that of the outsider.

(6.3) The profits of the acquirer are higher than those of the acquiree. This is because the acquirer is more powerful than the acquiree. Moreover, the acquirer’s profits are higher than the outsider’s but only if the technology is sufficiently effective. This may also hold for the acquiree’s profits but for even lower values of \(\gamma\) (for \(\gamma\) close to \(5/2\)).

(6.4) The profits of the outsider are higher than in the non-integration equilibrium except if the technology is too effective (for values of \(\gamma\) close to \(5/2\)). By contrast, the profits of the acquirer and the acquiree are higher than in the non-integration equilibrium, but only if the technology is sufficiently effective. Therefore, firms have conflicting interests regarding their preferable equilibrium.

We next turn to identifying the parameter constellations for which either non-integration or a merger can be sustained by a CPNE. As before, we consider the non-integration NE profile where \(x_i = x^{NI}\) and \(m_i = (i, i)\) identified in Section 3. In order for this NE to be a CPNE of the game, it must be invulnerable to all self-enforcing coalitional deviations. Again, it is sufficient to consider a deviation of any two firms that adjust their R&D investments appropriately, taking as given that the outsider R&D investment level is \(x^{NI}\). Note that when two firms decide to jointly deviate and merge, the integrated entity enjoys efficiency gains simply because the two firms put together their R&D efforts. This implies that deviation incentives are stronger when research paths are complementary than when they are duplicative. Of course, these deviations should be invulnerable to further unilateral deviations of each of the firms forming the coalition.

As for CPNE involving a merger, we restrict our attention to mergers that are sustained by a NE (Proposition 4). We then consider coalitional deviations involving either the acquirer \(\pi\)
(or acquiree $a$) breaking the merger and forming a new integrated entity with the outsider firm $o$ as either the acquirer or the acquiree. (Note that a deviation of the coalition formed by firms $\overline{a}$ and $a$ switching their roles cannot be profitable.)

Proposition 7 provides a description of the parameter constellations in which non-integration and mergers can be sustained by a CPNE. The description involves identifying several subsets of the relevant parameter space $\hat{\Omega} = \{(\beta, \gamma) \in [1/2, 1] \times [5/2, \infty)\}$:

\[
B^c = \{(\beta, \gamma) \mid \gamma \geq \gamma_1^c(\beta)\} \cap \{(\beta, \gamma) \mid \gamma_2^c(\beta) \leq \gamma \leq \gamma_3^c(\beta)\}
\]

\[
G^c = \{(\beta, \gamma) \mid \gamma_2^c(\beta) \leq \gamma \leq \gamma_1^c(\beta)\}
\]

\[
W^c = \{(\beta, \gamma) \mid \gamma_4^c(\beta) < \gamma < \min[\gamma_2^c(\beta), \gamma_3^c(\beta)]\}
\]

\[
Y^c = \{(\beta, \gamma) \mid \gamma \leq \gamma_2^c(\beta)\}\setminus W^c.
\]

Figure 4 describes these sets, and Appendix A.3, which contains the proof of Proposition 7, provides the functions involved in the definitions.

**INSERT FIGURE 4 HERE**

**Proposition 7.** Consider an industry in which firms follow complementary research paths.

(7.1) Non-integration can be sustained by a CPNE only if $(\beta, \gamma) \in B^c \cup G^c$.

(7.2) A merger can be sustained by a CPNE only if $(\beta, \gamma) \in Y^c \cup G^c$.

(7.3) If $(\beta, \gamma) \in W^c$, then a CPNE does not exist.

A merger is sustained by a CPNE when the R&D technology is sufficiently effective – except in the regions $W^c$ (white area in Figure 4), in which a CPNE does not exist, and lower $B^c$ (blue area close to the horizontal axis), in which only non-integration can be sustained by a CPNE. The latter is also true when the R&D technology is very ineffective (upper $B^c$ area in blue). Unlike duplicative research paths, complementary research paths have no wasteful duplication of R&D investments. In addition, the fact that the merger participants combine their R&D investments intensifies the rationalization of production and strengthens the profitability of a merger. Yet, if the R&D technology is very ineffective, a merger cannot be sustained even by a NE because, as we saw above, the acquirer has unilateral incentives to deviate. Regarding coalitional deviations, a deviation by the acquiree and the outsider, which form a coalition in which they assume the roles of acquirer and acquiree respectively, and adjust their investments optimally, may upset a merger for intermediate values of the acquirer’s bargaining power when the R&D technology is too effective. Further, this deviation is profitable and invulnerable to further unilateral deviations in the $W^c$ and lower $B^c$ areas of Figure 4.
Non-integration can be sustained by a CPNE for the parameter values in the blue and green areas in Figure 4. If the R&D technology is sufficiently effective given the acquirer’s bargaining power, a coalitional deviation of any two firms that form an integrated entity and adjust their R&D investments appropriately is a profitable self-enforcing deviation.

Finally, for the parameter constellations in the white $W^c$ region, a CPNE does not exist: Non-integration cannot be sustained by a CPNE, as it is vulnerable to a deviation by a coalition of any two firms that form a merger, and a merger cannot be sustained by a CPNE because it is vulnerable to a deviation of the coalition formed by the acquiree and the outsider.

Note that in the green $G^c$ region in Figure 4, both a merger and non-integration can be sustained by a CPNE. As in the case of duplicative research paths, under complementary research paths too the outcomes generated by a merger and by non-integration cannot be Pareto ranked (see Proposition 6). Yet, a merger can be sustained by a CPNE in a much broader set of parameter constellations under complementary research paths than under duplicative research paths (see Figures 3 and 4). Also, unlike duplicative research paths, with complementary research paths, a merger may involve the two most efficient firms in the industry when the acquirer’s bargaining power is not too large. (When the acquirer’s bargaining power is large a merger involves the most and the least efficient firms in the industry.)

6 Welfare analysis

In this section we examine the impact of mergers on the total surplus (i.e., the sum of consumer surplus and firms’ profits). For $(\beta, \gamma)$ denote by $S^{NI}(\beta, \gamma)$ the total surplus under non-integration, by $\tilde{S}^d(\beta, \gamma)$ and $\bar{S}^d(\beta, \gamma)$ the total surplus in a type 1 and a type 2 merger, respectively, when firms follow duplicative research paths, and by $S^c(\beta, \gamma)$ the total surplus in a merger when firms follow complementary research paths. Proposition 8 summarizes our findings (see Appendix A.4 for its proof).

Proposition 8.

(8.1) If firms follow duplicative research paths, then $\tilde{S}^d(\beta, \gamma) \geq S^{NI}(\beta, \gamma)$ iff $\gamma \leq \tilde{\gamma}^d_w(\beta)$, and $\bar{S}^d(\beta, \gamma) \geq S^{NI}(\beta, \gamma)$ iff $\gamma \leq \gamma^d_w(\beta)$, where $\tilde{\gamma}^d_w(\beta) < \gamma^d_w(\beta)$. Thus, when a type 1 merger leads to a total surplus increase so does a type 2 merger, but not vice versa.

(8.2) If firms follow complementary research paths, then $S^c(\beta, \gamma) \geq S^{NI}(\beta, \gamma)$ iff $\gamma \leq \gamma^c_w(\beta)$. (8.3) If a merger leads to a total surplus increase under duplicative research paths, so does a merger under complementary research paths, but not vice versa (i.e., $\gamma^c_w(\beta) < \tilde{\gamma}^d_w(\beta)$).
The traditional approach for reviewing horizontal mergers stresses the trade-off (articulated by Williamson, 1968) between the deadweight loss resulting from the increased market concentration, which tends to reduce total surplus, and the efficiency gains arising from a larger and more effective use of the R&D investments, which tends to increase total surplus. Under both duplicative and complementary research paths, the (positive) efficiency gains effect dominates the (negative) deadweight loss effect when the R&D technology is sufficiently effective, i.e., $\gamma$ is sufficiently low given the acquirer’s bargaining power (see Figures 3 and 4). Interestingly, under duplicative research paths, a merger in which the technology of the acquiree is used (type 2 merger) is more likely to be welfare-enhancing than a merger in which the technology of the acquirer is used (type 1 merger) – see Figure 3. The main reason for this result is that a type 2 merger leads to a lower level of wasteful R&D expenditures than a type 1 merger (Proposition 3.3). Further, a merger is more likely to increase total surplus under complementary than under duplicative research paths. This is because efficiency gains are larger when both firms’ R&D efforts contribute to reducing the integrated entity’s marginal cost.

Following the seminal analyses of Williamson (1968) and Farrell and Shapiro (1990), let us assume that the objective of the antitrust authority is to maximize total surplus. Then the policy implications of Proposition 8 are clear: The criteria for approval should be more stringent under duplicative than under complementary research paths. Moreover, under duplicative research paths the criteria for approval should be more stringent for type 1 than for type 2 mergers.

Market and societal incentives for a merger are often misaligned. There are cases in which a merger can be sustained by a CPNE whereas non-integration cannot, yet the merger decreases total surplus, and therefore should not be approved. Under complementary research paths, this occurs when the R&D technology is ineffective and the acquirer’s bargaining power is small – see region $Y^c$ for $\gamma > \gamma_w^c (\beta)$ in Figure 4. Under duplicative research paths, this occurs for type 1 mergers when the R&D technology is relatively ineffective and the acquirer’s bargaining power is sufficiently high – see orange region for $\gamma > \tilde{\gamma}_w^d (\beta)$ in Figure 3. In addition, there are cases in which, besides non-integration, a merger that decreases total surplus can be sustained by a CPNE. Under duplicative research paths, this occurs for type 1 mergers in the purple region for $\gamma > \tilde{\gamma}_w^d (\beta)$ and for type 2 mergers in the green region for $\gamma > \tilde{\gamma}_w^d (\beta)$ – see Figure 3. Under complementary research paths, this occurs in the region $G^c$ for $\gamma > \gamma_w^c (\beta)$ (Figure 4).

There are also cases in which a merger increases total surplus, but it cannot be sustained by a CPNE. This occurs in regions $B^c$ and $W^c$ for $\gamma < \gamma_w^c (\beta)$ under complementary research paths (see Figure 4), and in the white region of Figure 3 under duplicative research paths. In addition, there are cases in which a merger increases total surplus even though firms may end up non-integrating in a CPNE. For instance, this occurs in region $G^c$ for $\gamma < \gamma_w^c (\beta)$ under
complementary research paths (Figure 4). These findings suggest that under some circumstances policy-makers should encourage firms, e.g., via R&D subsidization policies, toward a merger that otherwise would not materialize.

Interestingly, a more subtle issue arises in our setup: For some parameter values both type 1 and type 2 mergers can be sustained by a CPNE, with a type 2 merger increasing total surplus and a type 1 merger decreasing it. Specifically, this occurs in the orange region of Figure 3 for \(\gamma_d^w(\beta) < \gamma < \gamma^d_w(\beta)\). In such cases, the antitrust authority should evaluate horizontal mergers in a “remedy” regime (in the terminology of Dertwinkel-Kalt and Wey, 2016), which would thus allow for approval conditional on the technology used by the integrated entity. In particular, a merger should be approved by the antitrust authority only under the remedy that the integrated entity uses the acquiree’s technology (rather than the acquirer’s technology).

In a similar vein, there may be mergers that enhance total surplus that use the acquirer’s technology, whereas the antitrust authority would prefer a merger that uses the acquiree’s technology. Here, too, our analysis suggests that these mergers should be approved with the remedy that the integrated entity uses the acquiree’s technology. In fact, when the merger uses the acquiree’s technology, total surplus is larger than when it uses the acquirer’s technology – except if the R&D technology is extremely effective (\(\gamma < 1.645\)).

A note should be made regarding the antitrust authority’s welfare standard under consideration. Assessing the welfare effects of a merger by the impact on total surplus might lead to the approval of mergers in which the gains realized by producers exceed the losses experienced by consumers. If instead the antitrust authority’s objective is to maximize consumer surplus, our analysis reveals that a merger leads to an increase in consumer surplus only under complementary research paths and when the R&D technology is too effective, i.e., \(\gamma < \gamma_{cs}^c(\beta) < \gamma_{cw}^c(\beta)\) – see Figure 4. Here, too, market and consumer incentives for a merger are misaligned. For instance, for \(\gamma > \gamma_{cs}^c(\beta)\) in the yellow \(Y_c\) region of Figure 4, a merger detrimental to consumers is sustained by a CPNE, whereas non-integration cannot be sustained by a CPNE. By contrast, for \(\gamma < \gamma_{cs}^c(\beta)\) in the blue \(B_c\) region of Figure 4, a merger is beneficial for consumers but cannot be sustained by a CPNE. These findings provide clear guidance for policy intervention in connection with horizontal mergers.

7 Concluding remarks

In this paper we investigate the role of firms’ cost-reducing R&D investments in endogenously creating cost asymmetries and potential efficiency gains that render horizontal mergers profitable. Three key features determine whether the equilibrium of an industry involves a merger:
The effectiveness of the R&D technology, the extent to which firms R&D investments are complementary, and the distribution of bargaining power, which determines the division of the incremental profits generated by the merger. We identify the conditions under which mergers of different types can be sustained by coalition-proof Nash equilibria. (The use of CPNE as a solution concept, albeit natural, is novel in this literature.) Then, we study the welfare effects of alternative mergers and suggest corrective policy measures.

We demonstrate that when firms’ R&D investments follow complementary research paths, a merger is more likely to be viable than when they follow duplicative research paths. Moreover, in the latter case, mergers in which the acquiree’s technology is used by the integrated entity are more easily viable. Under duplicative research paths, the most and the least efficient firms are more likely to merge. Such mergers may also arise under complementary research paths, but only if the acquirer’s bargaining power is sufficiently large – when it is not, a merger is likely to involve the two most efficient firms.

Regarding welfare implications, our analysis suggests that an antitrust authority using the total surplus welfare standard should approve a merger only if the R&D technology is sufficiently effective. Moreover, the criteria for approving a merger should be more stringent under duplicative than under complementary research paths. When the objective of the policy maker’s welfare standard is consumer surplus, our analysis suggests that mergers should only be approved when firms follow complementary research paths and only if the R&D technology is sufficiently effective.

We find that market and societal incentives for a merger are often misaligned. Thus, under some circumstances policy-makers should encourage firms, e.g., via R&D subsidization policies, toward mergers that otherwise would not materialize. Our analysis further advocates relatively stringent approval criteria under duplicative research paths: In this case the antitrust authority should approve a merger only under the remedy that the integrated entity uses the acquiree’s technology.

We leave for future research the effect of introducing uncertainty over the efficiency gains of R&D investments, which in turn makes the gains resulting from horizontal mergers uncertain — see, e.g., Choné and Linnemer (2008), Zhou (2008), Amir et al. (2009), Hamada (2012). Incorporating uncertainty into our framework could lead to new insights on firms’ merger incentives and the design of appropriate antitrust policies.
8 Appendix

A.1. Outcome in a CPNE that sustains non-integration

In a CPNE that sustains non-integration, the firms’ levels of R&D, output and profits, and the consumer surplus are given by

\[ x^{NI} = \frac{3(\alpha - c)}{8\gamma - 3} \]
\[ q^{NI} = \frac{2(\alpha - c)\gamma}{8\gamma - 3} \]
\[ \Pi^{NI} = \frac{\gamma(8\gamma - 9)(\alpha - c)^2}{2(8\gamma - 3)^2} \]
\[ CS^{NI} = \frac{18\gamma^2(\alpha - c)^2}{(3 - 8\gamma)^2} \]  

(A) In a CPNE that sustains a merger in which \( x_\pi > x_\underline{a} \), the firms’ levels of R&D, output and profits, and the consumer surplus are given by

\[ x_\pi = D(4 - 3\gamma) \left( 4\beta^2 + 32\beta - 27\gamma - 14\beta\gamma \right) \]
\[ x_\underline{a} = D(4 - 3\gamma) \left( 4\beta^2 + 32\beta - 9\gamma - 18\beta\gamma \right) \]
\[ x_o = 4D \left( 4\beta^2 + 32\beta + 24\gamma^2 - 33\gamma - 26\beta\gamma \right) \]
\[ q_o = 3\gamma D \left( 4\beta^2 + 32\beta + 24\gamma^2 - 33\gamma - 26\beta\gamma \right) \]
\[ Q = 12\gamma D \left( 2\beta^2 - 10\beta\gamma + 12\beta + 6\gamma^2 - 7\gamma - 2 \right) \]
\[ \Pi_\pi = \gamma D_\pi D^2 (4 - 3\gamma)^2 \]
\[ \Pi_\underline{a} = \gamma D_\underline{a} D^2 (4 - 3\gamma)^2 \]
\[ \Pi_o = \gamma D^2 (9\gamma - 8) \left( 4\beta^2 + 48\beta + 48\gamma^2 - 71\gamma - 38\beta\gamma + 8 \right)^2 \]
\[ CS^d = \frac{9}{2}\gamma^2 D^2 \left( 4\beta^2 + 48\beta + 48\gamma^2 - 71\gamma - 38\beta\gamma + 8 \right)^2 \]  

where

\[ D = \frac{1}{4} \frac{(\alpha - c)}{54\gamma^3 - 6(17 + 8\beta)\gamma^2 + (6\beta^2 + 86\beta + 39)\gamma - 4\beta(8 + \beta)} \]
\[ D_\pi = 36\gamma^3 (2\beta - 9) + \frac{1}{2} \gamma^2 (27 + 14\beta)^2 + 8\beta^2 (8 + \beta)^2 - 8\beta\gamma (8\beta^2 + 73\beta + 72) \]
\[ D_\underline{a} = 36\gamma^3 (2\beta + 7) - \frac{9}{2} \gamma^2 (1 + 2\beta) (73 + 18\beta) - 8\beta^2 (8 + \beta)^2 + 4\gamma (16\beta^3 + 182\beta^2 + 180\beta + 9). \]
(B) In a CPNE that sustains a merger in which \( x_\pi < x_{\underline{a}} \), the firms’ levels of R&D, output and profits, and the consumer surplus are given by

\[
\begin{align*}
  x_\pi &= D(4 - 3\gamma) (4\beta^2 - 40\beta - 27\gamma + 18\beta\gamma + 36) \\
  x_{\underline{a}} &= D(4 - 3\gamma) (4\beta^2 - 40\beta - 41\gamma + 14\beta\gamma + 36) \\
  x_o &= 4D (4\beta^2 - 40\beta + 24\gamma^2 - 59\gamma + 26\beta\gamma + 36) \\
  q_o &= 3\gamma D (4\beta^2 - 40\beta + 24\gamma^2 - 59\gamma - 26\beta\gamma + 36) \\
  Q &= 3\gamma D (4\beta^2 - 56\beta + 48\gamma^2 - 109\gamma - 14\beta\gamma + 60) \\
  \Pi_\pi &= \gamma D x_\pi D^2 (4 - 3\gamma)^2 \\
  \Pi_{\underline{a}} &= \gamma D x_{\underline{a}} D^2 (4 - 3\gamma)^2 \\
  \Pi_o &= \gamma D (9\gamma - 8) (4\beta^2 - 40\beta + 24\gamma^2 - 59\gamma + 26\beta\gamma + 36)^2 \\
  CS^d &= \frac{9}{2}\gamma^2 D^2 (4\beta^2 - 56\beta + 48\gamma^2 - 109\gamma + 38\beta\gamma + 60),
\end{align*}
\]

where

\[
D = \frac{1}{4 \beta^2 (6\gamma - 4) + \beta (48\gamma^2 - 98\gamma + 40) + (3\gamma - 4) (18\gamma^2 - 26\gamma + 9)} \quad (\alpha - c)
\]

\[
D_\pi = 36(9 - 2\beta)\gamma^3 - \frac{9}{2}\gamma^2 (273 - 236\beta + 36\beta^2) \\
-4\gamma (-387 + 592\beta - 230\beta^2 + 16\beta^3) - 8(\beta - 9)^2 (1 - \beta)^2
\]

\[
D_{\underline{a}} = 36(7 + 2\beta)\gamma^3 - \frac{\gamma^2}{2} (41 - 14\beta)^2 - 8\gamma (-153 + 242\beta - 97\beta^2 + 8\beta^3) - 4(\beta - 9)^2 (1 - \beta)^2.
\]

**Proof of Proposition 2:**

(2.1) Consider a profile \((x, m)\) leading to a type 1 merger. In order for \((x, m)\) to be a NE, the profile of R&D investments \(x\) must satisfy \(x_\pi = \tilde{R}_\pi(x_{\underline{a}}, x_o) > x_{\underline{a}} = \tilde{R}_{\underline{a}}(x_\pi, x_o)\), and \(x_o = R_o(x_\pi, x_{\underline{a}})\) – see equations (15) to (19). As the outsider is on its reaction function, it does not have an improving unilateral deviation from \((x, m)\). Consider a unilateral deviation from \((x, m)\) by firm \(\overline{a}\) to \(\tilde{m}_{\overline{a}} = (\overline{a}, \overline{a})\) and \(\bar{x}_{\overline{a}} = R^{(3)}(x_o + x_{\underline{a}})\). Then \(\Pi_{\overline{a}}^{(3)}(x_o + x_{\underline{a}}, \bar{x}_{\overline{a}}) \leq \Pi_{\pi}(x, m)\) if and only if \(\gamma \leq \tilde{\gamma}_1(\beta)\), with \(\frac{d\tilde{\gamma}_1}{d\beta} < 0\). \(\tilde{\gamma}_1(0.5) = 3.280\) and \(\tilde{\gamma}_1(1) = 2.744\). Consider a unilateral deviation from \((x, m)\) by firm \(\underline{a}\) to \(\tilde{m}_{\underline{a}} = (\underline{a}, \underline{a})\) and \(\bar{x}_{\underline{a}} = R^{(3)}(x_o + x_\pi)\). Then \(\Pi_{\underline{a}}^{(3)}(x_o + x_\pi, \bar{x}_{\underline{a}}) \leq \Pi_{\underline{a}}(x, m)\) if and only if \(\gamma \leq \tilde{\gamma}_2(\beta)\), with \(\frac{d\tilde{\gamma}_2}{d\beta} > 0\). \(\tilde{\gamma}_2(0.5) = 3.167\) and \(\tilde{\gamma}_2(1) = 3.666\). Therefore, unilateral deviations are not profitable, and thus the profile \((x, m)\) is a NE, if and only if \(\gamma \leq \tilde{\gamma}_{12}(\beta) := \min\{\tilde{\gamma}_1(\beta), \tilde{\gamma}_2(\beta)\}\).

(2.2) Consider a profile \((x, m)\) leading to a type 2 merger. In order for \((x, m)\) to be a NE,
the profile of R&D investments $x$ must satisfy $x_\pi = R_\pi(x_a, x_o) < x_o = R_o(x_a, x_o)$, and $x_o = R_o(x_\pi, x_o)$. Again, the outsider is on its reaction function and thus has no improving unilateral deviation from $(x, m)$. Consider a unilateral deviation from $(x, m)$ of firm $\tilde{a}$ to $\tilde{m}_a = (\tilde{a}, \tilde{a})$ and $\tilde{x}_a = R^{(3)}(x_o + x_\pi)$. Then $\Pi_{\tilde{a}}^{(3)}(x_o + x_\pi, \tilde{x}_a) \leq \Pi_{\pi}(x, m)$ if and only if $\gamma \leq \gamma_{\tilde{a}}(\beta)$, with $\frac{d\gamma}{d\beta} < 0$, $\gamma_{\tilde{a}}(0.5) = 3.167$ and $\gamma_{\tilde{a}}(1) = 2.759$. Consider now a unilateral deviation from $(x, m)$ of firm $a$ to $\tilde{m}_a = (a, a)$ and $\tilde{x}_a = R^{(3)}(x_o + x_\pi)$. Then $\Pi_{\tilde{a}}^{(3)}(x_o + x_\pi, \tilde{x}_a) \leq \Pi_{\tilde{a}}(x, m)$ if and only if $\gamma \leq \gamma_{\tilde{a}}(\beta)$, with $\frac{d\gamma}{d\beta} > 0$, $\gamma_{\tilde{a}}(0.5) = 3.280$ and $\gamma_{\tilde{a}}(1) = 4.0$. Thus, $\gamma_{\tilde{a}}(\beta) < \gamma_{\tilde{a}}(\beta)$. Therefore, unilateral deviations are not profitable and thus the profile $(x, m)$ is a NE, if only if $\gamma \leq \gamma_{\tilde{a}}(\beta)$.

**Proof of Proposition 3:** Using (23), (24) and (25), and after messy algebraic manipulations, we obtain the results.\(^{12}\)

**Proof of Proposition 4.1**

Non-integration is sustained by a NE for all parameter values. This is because there is no profitable unilateral deviation, and there is no profitable self-enforcing deviation by all three firms either. We now determine the conditions under which non-integration is sustained by a CPNE. Let $x = (x^{NI}, x^{NI}, x^{NI})$ and $m = [(1, 1), (2, 2), (3, 3)]$ be the non-integration equilibrium strategy profile. Hence the deviations to consider are those involving a merger by any two firms, i.e., a deviation by a coalition $\{\tilde{a}, \tilde{a}\}$, such that $\tilde{m}_a = \tilde{m}_a = (\tilde{a}, \tilde{a})$ and some $\tilde{x}_\pi, \tilde{x}_a \in [0, c]$. Let $(\tilde{x}, \tilde{m})$ the profile of firms’ R&D investments and merger proposals after the deviation. There are two types of such deviations to consider:

$(NI.1)$ $\tilde{x}_\pi = R_\pi(\tilde{x}_a, x^{NI}) > \tilde{x}_a = R_\pi(\tilde{x}_a, x^{NI})$. Simple algebra reveals that $\Pi_{\tilde{a}}(\tilde{x}, \tilde{m}) < \Pi_{\tilde{a}}(x, m)$, and therefore that this is not a profitable deviation.

$(NI.2)$ $\tilde{x}_\pi = R_\pi(\tilde{x}_a, x^{NI}) < \tilde{x}_a = R_\pi(\tilde{x}_a, x^{NI})$. Then $\Pi_{\tilde{a}}(\tilde{x}, \tilde{m}) > \Pi_{\tilde{a}}(x, m)$, while $\Pi_{\tilde{a}}(\tilde{x}, \tilde{m}) > \Pi_{\tilde{a}}(x, m)$ if and only if $\gamma < \gamma(\beta)$, with $\frac{d\gamma}{d\beta} > 0$, $\gamma(1) = 2.278$ and $\gamma(\beta) = 0$ for $\beta = 0.719$. Assume that $\gamma < \gamma(\beta)$, and consider a unilateral deviation by firm $\tilde{a}$ from $(\tilde{x}, \tilde{m})$ to $\tilde{m}_a = (\tilde{a}, \tilde{a})$ and $\tilde{x}_a = R^{(3)}(x^{NI} + \tilde{x}_a)$. Then $\Pi_{\tilde{a}}^{(3)}(x^{NI} + \tilde{x}_a, \tilde{x}_a) < \Pi_{\pi}(\tilde{x}, \tilde{m})$, i.e., firm $\tilde{a}$ has no incentive to unilaterally deviate from $(\tilde{x}, \tilde{m})$. Consider next a unilateral deviation by firm $a$ from $(\tilde{x}, \tilde{m})$ to $\tilde{m}_a = (a, a)$ and $\tilde{x}_a = R^{(3)}(x^{NI} + \tilde{x}_\pi)$. Then $\Pi_{\tilde{a}}^{(3)}(x^{NI} + \tilde{x}_\pi, \tilde{x}_a) < \Pi_{\tilde{a}}(\tilde{x}, \tilde{m})$, i.e., firm $a$ has no incentive to unilaterally deviate from $(\tilde{x}, \tilde{m})$ either.

Thus, if $\gamma \geq \gamma(\beta)$, then a deviation of any two firms to form an integrated entity is non-profitable, and therefore non-integration is sustained by a CPNE. However, if $\gamma < \gamma(\beta)$, the deviation described in NI.2 above is profitable and self-enforcing, and therefore in this case non-integration cannot be sustained by a CPNE.

\(^{12}\) These are available from the authors upon request.
Proof of Proposition 4.2 and 4.3: Consider a profile \((x, m)\) leading to a merger that is sustained by a NE. Again, let \(\bar{a} = 1\), \(\bar{a} = 3\) and \(o = 2\), i.e., \(m = [(1, 3), (2, 2), (1, 3)]\). We study the conditions under which such a profile is a CPNE. For this we also need to consider deviations of coalitions of two firms.

\((M1)\) \(x_\pi = R_\pi(x_m, x_o) > x_m = R_u(x_\pi, x_o)\).

By symmetry, deviations of the coalition formed by the acquirer \(\bar{a}\) and the acquiree \(a\), in which they interchange their roles and their levels of investment, are not profitable. Other coalitional deviations to consider are as follows:

\((M1.1)\) Deviations of the coalition formed by the outsider \(o = 2\) and the acquiree \(a = 3\) to form a new merger in which the outsider is the acquirer, i.e., \(\hat{m}_2 = \hat{m}_3 = (2, 3)\). Simple algebra reveals that whether \(\hat{x}_2 = R_\pi(\hat{x}_3, x_\pi) > \hat{x}_3 = R_u(\hat{x}_2, x_\pi)\) or \(\hat{x}_2 = R_\pi(\hat{x}_3, x_\pi) < \hat{x}_3 = R_u(\hat{x}_2, x_\pi)\), the outsider’s profits following such a deviation are below those at \((x, m)\), i.e., \(\Pi_2(\hat{x}, \hat{m}) < \Pi_o(x, m)\), and therefore these deviations are not profitable.

\((M1.2)\) Deviations of the coalition formed by the outsider \(o = 2\) and the acquiree \(a = 3\) to form a new merger in which the outsider is the acquiree, i.e., \(\hat{m}_2 = \hat{m}_3 = (3, 2)\). Simple algebra reveals that whether \(\hat{x}_3 = R_\pi(\hat{x}_2, x_\pi) > \hat{x}_2 = R_u(\hat{x}_3, x_\pi)\) or \(\hat{x}_3 = R_\pi(\hat{x}_2, x_\pi) < \hat{x}_2 = R_u(\hat{x}_3, x_\pi)\), the outsider’s profits following such a deviation are below those at \((x, m)\), i.e., \(\Pi_2(\hat{x}, \hat{m}) < \Pi_o(x, m)\), and therefore these deviations are not profitable.

\((M1.3)\) Deviations of the coalition formed by the outsider \(o = 2\) and the acquirer \(\bar{a} = 1\) to form a new merger in which the outsider is the acquirer, i.e., \(\hat{m}_1 = \hat{m}_2 = (2, 1)\). Assume that \(\hat{x}_2 = R_\pi(\hat{x}_1, x_a) > \hat{x}_1 = R_u(\hat{x}_2, x_a)\); again simple algebra reveals that \(\Pi_1(\hat{x}, \hat{m}) < \Pi_a(x, m)\), and therefore this deviation is not profitable. Assume instead that \(\hat{x}_2 = R_\pi(\hat{x}_1, x_a) < \hat{x}_1 = R_u(\hat{x}_2, x_a)\). Then \(\Pi_2(\hat{x}, \hat{m}) > \Pi_o(x, m)\) if and only if \(\gamma < \tilde{\gamma}_3(\beta)\), with \(\frac{d\gamma}{d\beta} > 0\), \(\tilde{\gamma}_3(1) = 1.710\) and \(\tilde{\gamma}_3(\beta) = 0\) for \(\beta = 0.621\); and \(\Pi_1(\hat{x}, \hat{m}) > \Pi_u(x, m)\) if and only if \(\gamma > \tilde{\gamma}_4(\beta)\), with \(\frac{d\gamma}{d\beta} > 0\), \(\tilde{\gamma}_4(1) = 1.702\) and \(\tilde{\gamma}_4(\beta) = 0\) for \(\beta = 0.698\). Thus, \(\Pi_2(\hat{x}, \hat{m}) > \Pi_o(x, m)\) and \(\Pi_1(\hat{x}, \hat{m}) > \Pi_a(x, m)\) if and only if \(\gamma \in (\tilde{\gamma}_4(\beta), \tilde{\gamma}_3(\beta))\), with \(\tilde{\gamma}_3(\beta) < \tilde{\gamma}_12(\beta)\). Assume \(\gamma \in (\tilde{\gamma}_4(\beta), \tilde{\gamma}_3(\beta))\); a further unilateral deviation of firm 1 from \((\hat{x}, \hat{m})\) to \(\hat{m}_1 = (1, 1)\) and \(\hat{x}_1 = R^{(3)}(\hat{x}_2 + x_a)\) leads to profits \(\Pi_1^{(3)}(\hat{x}_2 + x_a, \hat{x}_1) < \Pi_1(\hat{x}, \hat{m})\); likewise, a further unilateral deviation of firm 2 from \((\hat{x}, \hat{m})\) to \(\hat{m}_2 = (2, 2)\) and \(\hat{x}_2 = R^{(3)}(\hat{x}_1 + x_a)\) leads to profits \(\Pi_2^{(3)}(\hat{x}_1 + x_a, \hat{x}_2) < \Pi_2(\hat{x}, \hat{m})\). Thus, for \(\gamma \in (\tilde{\gamma}_4(\beta), \tilde{\gamma}_3(\beta))\) this coalitional deviation is self-enforcing and improving, and therefore a merger cannot be sustained by a CPNE.

\((M1.4)\) Deviations of the coalition formed by the outsider \(o = 2\) and the acquirer \(\bar{a} = 1\) to form a new merger in which the outsider is the acquiree, i.e., \(\hat{m}_1 = \hat{m}_2 = (1, 2)\). Assume that \(\hat{x}_1 = R_\pi(\hat{x}_2, x_a) > \hat{x}_2 = R_u(\hat{x}_1, x_a)\); again simple algebra reveals that \(\Pi_2(\hat{x}, \hat{m}) < \Pi_o(x, m)\), and therefore this deviation is not profitable. Assume instead that \(\hat{x}_1 = R_\pi(\hat{x}_2, x_a) < \hat{x}_2 = \hat{x}_2 = R_u(\hat{x}_1, x_a)\); again simple algebra reveals that \(\Pi_2(\hat{x}, \hat{m}) < \Pi_o(x, m)\).
and \( \Pi \) investments to \( \tilde{R}^\beta(x, \tilde{m}) > \Pi(x, m) \) if and only if \( \gamma > \bar{\gamma}_5(\beta) \), with \( \frac{d\bar{\gamma}_5}{d\beta} < 0 \) and \( \bar{\gamma}_5(1) = 2.985 \); and \( \Pi_2(\tilde{x}, \tilde{m}) > \Pi(x, m) \) if and only if \( \gamma < \bar{\gamma}_6(\beta) \), with \( \frac{d\bar{\gamma}_6}{d\beta} > 0 \), \( \bar{\gamma}_6(0.5) = 1.947 \) and \( \bar{\gamma}_6(1) = 2.091 \). However, \( \bar{\gamma}_5(\beta) > \bar{\gamma}_6(\beta) \) and hence, the region of the parameter space on which both these inequalities hold is empty; therefore, this coalitional deviation is not profitable.

(M1.5) A deviation of the coalition formed by the acquirer \( \tilde{a} = 1 \) and the acquiree \( \tilde{a} = 3 \) maintaining their roles in the merger, i.e., \( \hat{m}_2 = \hat{m}_3 = (1, 3) \), but altering their R&D investments to \( \tilde{x}_1 = R_\tilde{a}(\tilde{x}_3, xo) < \tilde{x}_3 = R_\hat{a}(\tilde{x}_1, xo) \). Then \( \Pi_1(\tilde{x}, \hat{m}) < \Pi_\tilde{a}(x, m) \) (see Proposition 3.6), and therefore this coalitional deviation is not profitable.

(M1.6) A deviation of the coalition formed by the acquirer \( \tilde{a} = 1 \) and the acquiree \( \hat{a} = 3 \), interchanging their roles in the merger, i.e., \( \hat{m}_2 = \hat{m}_3 = (3, 1) \), and adjusting their R&D investments to \( \tilde{x}_1 = R_\tilde{a}(\tilde{x}_3, xo) > \tilde{x}_3 = R_\hat{a}(\tilde{x}_1, xo) \); then \( \Pi_3(\tilde{x}, \hat{m}) > \Pi_\hat{a}(x, m) \) always; while \( \Pi_1(\tilde{x}, \hat{m}) > \Pi_\hat{a}(x, m) \) if and only if \( \gamma > \bar{\gamma}_7(\beta) \), with \( \frac{d\bar{\gamma}_7}{d\beta} < 0 \), \( \bar{\gamma}_7(0.5) = 2.125 \) and \( \bar{\gamma}_7(1) = 2.119 \). Thus, \( \bar{\gamma}_7(\beta) < \bar{\gamma}_{12}(\beta) \). Assume that \( \gamma > \bar{\gamma}_7(\beta) \); a further unilateral deviation of firm 1 from \( (\hat{x}, \hat{m}) \) to \( \hat{m}_1 = (1, 1) \) and \( \hat{x}_1 = R(\hat{x}_3, xo + \tilde{x}_3) \) leads to profits \( \Pi_1(\hat{x}_3, xo + \tilde{x}_3, \hat{x}_1) > \Pi_1(\tilde{x}, \hat{m}) \) if and only if \( \gamma > \bar{\gamma}_{8}(\beta) \), with \( \frac{d\bar{\gamma}_8}{d\beta} > 0 \) and \( \bar{\gamma}_8(0.5) = 3.280 \); likewise, a further unilateral deviation of firm 3 from \( (\tilde{x}, \hat{m}) \) to \( \hat{m}_3 = (3, 3) \) and \( \tilde{x}_3 = R(\tilde{x}_3, xo + \hat{x}_1) \) leads to profits \( \Pi_3(\tilde{x}_3, xo + \hat{x}_1, \tilde{x}_3) > \Pi_3(\tilde{x}, \hat{m}) \) if and only if \( \gamma > \bar{\gamma}_9(\beta) \), with \( \frac{d\bar{\gamma}_9}{d\beta} < 0 \) and \( \bar{\gamma}_9(0.5) = 3.166 \). Hence, \( \bar{\gamma}_8(\beta) > \bar{\gamma}_9(\beta) \) and hence, for \( \gamma \in (\bar{\gamma}_7(\beta), \bar{\gamma}_9(\beta)) \) the deviation \( (\hat{x}, \hat{m}) \) from \( (x, m) \) is self-enforcing and profitable for the coalition, and thus, \( (x, m) \) cannot be sustained by a CPNE.

Finally, it is easy to see that if \( (x, m) \) is invulnerable to unilateral deviations and to deviations of coalitions of two firms, then it is also invulnerable to deviations of the grant coalition. Note that self-enforcing deviations must be NE since otherwise they are vulnerable to deviations of individual firms, which are self-enforcing. A deviation of the grant coalition that leads to non-integration is not improving because by Proposition 3.5, the profit of the outsider is below its profit when there is a merger. Conversely, deviations of the grant coalition that leads to a merger can be implemented by a deviation of a coalition of two firms, which we have already considered.

By Proposition 2 and the above discussion, we conclude that a type 1 merger in which the R&D investments satisfy \( x_\pi = \tilde{R}_\pi(x_d, xo) > x_\tilde{a} = \tilde{R}_\tilde{a}(x_\pi, xo) \) can be sustained by a CPNE for all \( (\beta, \gamma) \in [1/2, 1] \times [3/2, \infty) \) such that
\[
\gamma \in [3/2, \bar{\gamma}_{12}(\beta)] \setminus \{ (\bar{\gamma}_4(\beta), \bar{\gamma}_3(\beta)) \cup (\bar{\gamma}_7(\beta), \bar{\gamma}_9(\beta)) \}.
\]

(M2) \( x_\pi = \tilde{R}_\pi(x_d, xo) < x_\tilde{a} = \tilde{R}_\tilde{a}(x_\pi, xo) \).

By symmetry, a deviation of the coalition formed by the acquirer \( \tilde{a} \) and the acquiree \( a \).
in which they interchange their roles and their levels of investment, is not profitable. Other coalitional deviations to consider are as follows:

(M2.1) Deviations of the coalition formed by the outsider \( a = 2 \) and the acquiree \( a = 3 \) to form a new merger in which the outsider is the acquirer, i.e., \( \bar{m}_2 = \bar{m}_3 = (2, 3) \). Simple algebra reveals that \( \Pi_3(\bar{x}, \bar{m}) < \Pi_2(x, m) \) whenever \( \bar{x}_2 = R_\pi(\bar{x}_3, x_\pi) > \bar{x}_3 = R_\pi(\bar{x}_2, x_\pi) \), and \( \Pi_2(\bar{x}, \bar{m}) < \Pi_o(x, m) \) whenever \( \bar{x}_2 = R_\pi(\bar{x}_3, x_\pi) < \bar{x}_3 = R_\pi(\bar{x}_2, x_\pi) \), and therefore these deviations are not profitable.

(M2.2) Deviations of the coalition formed by the outsider \( o = 2 \) and the acquiree \( a = 3 \) to form a new merger in which the outsider is the acquirer, i.e., \( \bar{m}_2 = \bar{m}_3 = (3, 2) \). Simple algebra reveals that if \( \bar{x}_3 = R_\pi(\bar{x}_2, x_\pi) > \bar{x}_2 = R_\pi(\bar{x}_3, x_\pi) \), then \( \Pi_2(\bar{x}, \bar{m}) < \Pi_o(x, m) \), and therefore this deviation is not profitable. If instead \( \bar{x}_3 = R_\pi(\bar{x}_2, x_\pi) < \bar{x}_2 = R_\pi(\bar{x}_3, x_\pi) \), then \( \Pi_2(\bar{x}, \bar{m}) > \Pi_o(x, m) \) if and only if \( \gamma < \gamma_3(\beta) \), with \( \frac{d\gamma}{d\beta} < 0 \); \( \gamma_3(0.5) = 1.947 \), and \( \gamma_3(1) = 1.662 \); and \( \Pi_3(\bar{x}, \bar{m}) > \Pi_2(x, m) \) if and only if \( \gamma > \gamma_3(\beta) \), with \( \frac{d\gamma}{d\beta} = 0 \) for \( \beta = 0.914 \). Hence, \( \Pi_2(\bar{x}, \bar{m}) > \Pi_o(x, m) \) and \( \Pi_3(\bar{x}, \bar{m}) > \Pi_2(x, m) \) if and only if \( \gamma \in (\gamma_3(\beta), \gamma_3(\beta)) \), with \( \gamma_3(\beta) < \gamma_1(\beta) \). Assume that \( \gamma \in (\gamma_3(\beta), \gamma_3(\beta)) \); a further unilateral deviation of firm 2 from \( (\bar{x}, \bar{m}) \) to \( \bar{m}_2 = (2, 2) \) and \( \bar{x}_2 = R(3)_\pi(x_\pi + \bar{x}_3) \) leads to profits \( \Pi_2(3)(x_\pi + \bar{x}_3, \bar{x}_2) < \Pi_2(\bar{x}, \bar{m}) \), while a further unilateral deviation of firm 3 from \( (\bar{x}, \bar{m}) \) to \( \bar{m}_3 = (3, 3) \) and \( \bar{x}_3 = R(3)(x_\pi + \bar{x}_2, \bar{x}_3) \) leads to profits \( \Pi_3(3)(x_\pi + \bar{x}_2, \bar{x}_3) < \Pi_3(\bar{x}, \bar{m}) \). Hence for \( \gamma \in (\gamma_3(\beta), \gamma_3(\beta)) \) the deviation \( (\bar{x}, \bar{m}) \) from \( (x, m) \) is self-enforcing and profitable for the coalition, and thus, \( (x, m) \) cannot be sustained by a CPNE.

(M2.3) Deviations of the coalition formed by the outsider \( o = 2 \) and the acquirer \( a = 1 \) to form a new merger in which the outsider is the acquirer, i.e., \( \bar{m}_1 = \bar{m}_2 = (2, 1) \). Simple algebra reveals that whether \( \bar{x}_2 = R_\pi(\bar{x}_3, x_\pi) > \bar{x}_3 = R_\pi(\bar{x}_2, x_\pi) \) or \( \bar{x}_2 = R_\pi(\bar{x}_3, x_\pi) < \bar{x}_3 = R_\pi(\bar{x}_2, x_\pi) \) the outsider’s profits following a deviation are below those at \( (x, m) \), i.e., \( \Pi_2(\bar{x}, \bar{m}) < \Pi_o(x, m) \), and therefore these deviations are not profitable.

(M2.4) Deviations of the coalition formed by the outsider \( o = 2 \) and the acquirer \( a = 1 \) to form a new merger in which the outsider is the acquirer, i.e., \( \bar{m}_1 = \bar{m}_2 = (1, 2) \). Simple algebra reveals that whether \( \bar{x}_1 = R_\pi(\bar{x}_2, x_\pi) > \bar{x}_2 = R_\pi(\bar{x}_1, x_\pi) \) or \( \bar{x}_1 = R_\pi(\bar{x}_2, x_\pi) < \bar{x}_2 = R_\pi(\bar{x}_1, x_\pi) \) the outsider’s profits following a deviation are below those at \( (x, m) \), i.e., \( \Pi_2(\bar{x}, \bar{m}) < \Pi_o(x, m) \), and therefore these deviations are not profitable.

(M2.5) A deviation of the coalition formed by the acquirer \( a = 1 \) and the acquiree \( a = 3 \) maintaining their roles in the merger, i.e., \( \bar{m}_1 = \bar{m}_3 = (1, 3) \), but changing their R&D investments to \( \bar{x}_1 = R_\pi(\bar{x}_3, x_o) > \bar{x}_3 = R_\pi(\bar{x}_1, x_o) \). Then \( \Pi_3(\bar{x}, \bar{m}) < \Pi_2(x, m) \) (see Proposition 3.6), and therefore this is not a profitable deviation.

(M2.6) A deviation of the coalition formed by the acquirer \( a = 1 \) and the acquiree \( a = 3 \),
interchanging their roles in the merger, i.e., \( \hat{m}_2 = \hat{m}_3 = (3,1) \), and adjusting their R&D investments to \( \tilde{x}_3 = \tilde{R}_a(\tilde{x}_1, x_o) > \tilde{x}_1 = \tilde{R}_a(\tilde{x}_3, x_o) \). Simple algebra reveals that \( \Pi_1(\tilde{x}, \hat{m}) < \Pi_a(x, m) \), and therefore this is not a profitable deviation.

Finally, it is easy to see that if \((x, m)\) is invulnerable to unilateral deviations and to deviations of coalitions of two firms, then it is also invulnerable to deviations of the grant coalition. Note that self-enforcing deviations must be NE as otherwise they are vulnerable to deviations of individual firms, which are self-enforcing. A deviation of the grant coalition that leads to non-integration is not improving as by Proposition 3.6, the profit of the outsider is below its profit when there is a merger. Conversely, deviations of the grant coalition that lead to a merger can be implemented by a deviation of a coalition of two firms, which we have already considered.

By Lemma 1 and the above discussion, we conclude that the type 2 merger in which R&D investments satisfy \( x_a = R_a(x_a, x_o) < x_a = R_a(x_o, x_o) \) can be sustained by a CPNE for all \((\beta, \gamma) \in [1/2, 1] \times [3/2, \infty)\) such that

\[
\gamma \in [3/2, \gamma_4(\beta)] \backslash (\gamma_1(\beta), \gamma_3(\beta)).
\]

**Proof of Proposition 4.4:** By Propositions 4.1, 4.2 and 4.3 for \((\beta, \gamma) \in [1/2, 1] \times [3/2, \infty)\) such that

\[
\gamma \in [(\gamma_4(\beta), \gamma_3(\beta)) \cap (\gamma_1(\beta), \gamma_3(\beta))]
\]
a CPNE does not exist.

**A.3. Complementary research paths: Outcomes in a CPNE that sustains a merger**

In a CPNE that sustains a merger, the firms’ levels of R&D, output and profits, and the
consumer surplus are given by

\[ x_\bar{a} = E(4 - 3\gamma)(28\beta^2 - 52\beta - 27\gamma - 14\beta\gamma + 36) \]
\[ x_o = 16E(20\beta^2 - 20\beta + 6\gamma^2 - 17\gamma + 12) \]
\[ x_{\bar{a}} = E(4 - 3\gamma)(-28\beta^2 + 41\gamma + 4\beta - 14\beta\gamma - 12) \]
\[ Q = 12E[\beta(1 - \beta)(8 - 26\gamma) + \gamma(3 - 4\gamma)(4 - 3\gamma)] \]
\[ q_o = 12E\gamma[20\beta(1 - \beta) - \gamma(17 - 6\gamma) + 12] \]
\[ \Pi_{\bar{a}} = E^2E\bar{\pi}(4 - 3\gamma)^2 \]
\[ \Pi_o = E^2E_o(4 - 3\gamma)^2 \]
\[ \Pi_o = 16E^2\gamma(9\gamma - 8)[-20\beta(1 - \beta) + (2\gamma - 3)(3\gamma - 4)]^2 \]
\[ CS^c = 9E(12\gamma^3 - 25\gamma^2 + 26\beta\gamma(\beta - 1) + 12\gamma + 8\beta - 8\beta^2)^2 \] (26)

where

\[ E = \frac{1}{8}\frac{(\alpha - c)}{\gamma^2(27\gamma - 75) + \gamma(70 + 69\beta - 69\beta^2) + 52\beta(1 - \beta) - 24} \]
\[ 2E_{\bar{\pi}} = \gamma^4(144\beta + 648) + \gamma^3(196\beta^2 + 2700\beta - 2457) \\
-8\gamma^2(288\beta^3 - 730\beta^2 + 850\beta - 387) \\
-\gamma(784\beta^4 + 3232\beta^3 - 4496\beta^2 + 1632\beta + 144) \\
+2944\beta^5 - 6784\beta^4 + 7040\beta^3 - 4352\beta^2 + 1152\beta \]
\[ 2E_o = \gamma^4(144\beta + 504) - \gamma^3(196\beta^2 + 2308\beta - 47) \\
+\gamma^2(2304\beta^3 - 1072\beta^2 + 2032\beta - 168) \\
-(784\beta^4 + 3232\beta^3 - 4496\beta^2 + 1632\beta + 144)\gamma \\
+2944\beta^5 - 6784\beta^4 + 7040\beta^3 - 4352\beta^2 + 1152\beta. \]

**Proof of Proposition 5**

Consider a profile \((x, m)\) that leads to a merger, and without loss of generality let \(\bar{a} = 1, \bar{a} = 3\) and \(o = 2\), i.e., \(m = [(1, 3), (2, 2), (1, 3)]\). We study the conditions under which such a profile is a NE. In a candidate NE, the profile of R&D investments \(x\) must satisfy the system of equations (20) to (22). Let \((\bar{x}, \bar{m})\) be the profile of firms’ R&D investments after a unilateral
deviation.

As the outsider is on its reaction function, it does not have an improving unilateral deviation from \((x, m)\). Consider a unilateral deviation from \((x, m)\) by firm \(a\) to \(\hat{m}_a = (\bar{a}, \bar{a})\) and \(\hat{x}_a = R^3(x_o + x_a)\). Then \(\Pi_a^{(3)}(x_o + x_a, \hat{x}_a) \leq \Pi(x, m)\) if and only if \(\gamma \leq \gamma^*_1(\beta)\), with \(\frac{d\gamma}{dx} < 0\), \(\gamma^*_1(0.5) = 15.103\) and \(\gamma^*_1(1) = 14.019\). Consider a unilateral deviation from \((x, m)\) by firm \(a\) to \(\hat{m}_a = (a, a)\) and \(\hat{x}_a = R^3(x_o + x_m)\). Then \(\Pi_a^{(3)}(x_o + x_m, \hat{x}_a) \leq \Pi_a(x, m)\) if and only if \(\gamma \leq \gamma^*_0(\beta)\), with \(\frac{d\gamma}{dx} > 0\), \(\gamma^*_0(0.5) = 15.103\) and \(\gamma^*_0(1) = 15.659\). As \(\gamma^*_1(\beta) \leq \gamma^*_0(\beta)\), unilateral deviations are not profitable whenever \(\gamma \leq \gamma^*_1(\beta)\), and thus the merger profile \((x, m)\) is sustained by a NE.

**Proof of Proposition 6:** Using (23) and (26), and after messy algebraic manipulations, we get the results.\(^{13}\)

**Proof of Proposition 7.1**

Non-integration is sustained by a NE for all parameter values. This is because there is no profitable unilateral deviation, and there is no profitable self-enforcing deviation by all three firms either. We now determine the conditions under which non-integration is sustained by a CPNE. Let \(x = (x^{NI}, x^{NI}, x^{NI})\) and \(m = [(1, 1), (2, 2), (3, 3)]\) be the non-integration equilibrium strategy profile. Hence the deviations to consider are those involving a merger by any two firms, i.e., a deviation by a coalition \(\{\bar{a}, a\}\), such that \(\bar{m}_a = \bar{m}_a = (\bar{a}, a)\) and some \(\bar{x}_m, \hat{x}_a \in [0, c]\). Let \((\bar{x}, \bar{m})\) the profile of firms’ R&D investments and merger proposals after the deviation.

Let \(\bar{x}_m = R^3(x_a, x^{NI})\) and \(\hat{x}_a = R^3(x^{NI}_a, x^{NI})\). Then \(\Pi_a(\bar{x}, \bar{m}) > \Pi_a(x, m)\) if and only if \(\gamma < \gamma^*_1(\beta)\), with \(\frac{d\gamma}{dx} > 0\), \(\gamma^*_1(0.5) = 13.084\) and \(\gamma^*_1(1) = 14.0176\); and \(\Pi_a(\bar{x}, \bar{m}) > \Pi_a(x, m)\) if and only if \(\gamma < \gamma^*_2(\beta)\), with \(\frac{d\gamma}{dx} < 0\), \(\gamma^*_2(0.5) = 13.084\), and \(\gamma^*_2(0) = 0\) for \(\beta = 0.829\). Thus, \(\gamma^*_1(\beta) < \gamma^*_2(\beta)\). Assume that \(\gamma < \gamma^*_2(\beta)\), and consider a unilateral deviation by firm \(\bar{a}\) from \((\bar{x}, \bar{m})\) to \(\bar{m}_a = (\bar{a}, \bar{a})\) and \(\bar{x}_a = R^3(x^{NI}_a, \hat{x}_a)\). Then \(\Pi_a^{(3)}(x^{NI}_a + \hat{x}_a, \hat{x}_a) < \Pi_a(x^{NI}_a, \hat{x}_a, \hat{x}_a)\), i.e., firm \(\bar{a}\) has no incentive to unilaterally deviate from \((\bar{x}, \bar{m})\). Consider next a unilateral deviation by firm \(a\) from \((\bar{x}, \bar{m})\) to \(\hat{m}_a = (a, a)\) and \(\hat{x}_a = R^3(x^{NI} + \hat{x}_a)\). Then \(\Pi_a^{(3)}(x^{NI} + \hat{x}_a, \hat{x}_a) < \Pi_a(x^{NI} + \hat{x}_a, \hat{x}_a)\), i.e., firm \(a\) has no incentive to unilaterally deviate from \((\bar{x}, \bar{m})\) either.

Thus, if \(\gamma \geq \gamma^*_2(\beta)\), then a deviation of any two firms to form an integrated entity is non-profitable, and therefore non-integration is sustained by a CPNE.

**Proof of Proposition 7.2**

Consider a profile \((x, m)\) leading to a merger that is sustained by a NE. Again, let \(\bar{a} = 1\), \(a = 3\) and \(o = 2\), i.e., \(m = [(1, 3), (2, 2), (1, 3)]\). We study the conditions under which such a profile is a CPNE. For this we also need to consider deviations of coalitions of two firms.

\(^{13}\)These are available from the authors upon request.
(M1.1) A deviation of the coalition formed by the outsider \( o = 2 \) and the acquiree \( a = 3 \) to form a new merger in which the outsider is the acquirer, i.e., \( \hat{m}_2 = \hat{m}_3 = (2, 3) \). Simple algebra reveals that whenever \( \hat{x}_2 = \hat{R}(\hat{x}_2, x_\pi) \) and \( \hat{x}_3 = \hat{R}(\hat{x}_3, x_\pi) \), the outsider’s profits following such a deviation are below those at \((x, m)\), i.e., \( \Pi_2(\hat{x}, \hat{m}) < \Pi_o(x, m) \), and therefore this deviation is not profitable.

(M1.2) A deviation of the coalition formed by the outsider \( o = 2 \) and the acquiree \( a = 3 \) to form a new merger in which the outsider is the acquiree, i.e., \( \hat{m}_2 = \hat{m}_3 = (3, 2) \). Simple algebra reveals that whenever \( \hat{x}_3 = \hat{R}(\hat{x}_2, x_\pi) \) and \( \hat{x}_2 = \hat{R}(\hat{x}_3, x_\pi) \), \( \Pi_2(\hat{x}, \hat{m}) > \Pi_o(x, m) \) if and only if \( \gamma < \gamma^o_3(\beta) \), with \( \frac{d \gamma^o_3}{d \beta} < 0 \); and \( \gamma^o_3(0.5) = 4.245 \) and \( \gamma^o_3(\beta) = 0 \) for \( \beta = 0.844 \); and \( \Pi_3(\hat{x}, \hat{m}) > \Pi_\omega(x, m) \) if and only if \( \gamma > \gamma^o_3(\beta) \), with \( \gamma^o_3(\beta) = 0 \) for \( \beta = 0.760 \), \( \frac{d \gamma^o_3}{d \beta} < 0 \); and \( \beta > 0.56 \) and \( \gamma^o_3(\beta) = \gamma^o_3(\beta) \) for \( \beta = 0.603 \). Assume \( \gamma \in (\gamma^o_3(\beta), \gamma^o_3(\beta)) \); a further unilateral deviation of firm 3 from \((\hat{x}, \hat{m})\) to \( \hat{m}_3 = (3, 3) \) and \( \hat{x}_3 = R(3)(\hat{x}_2 + x_\pi) \) leads to profits \( \Pi_1(3)(\hat{x}_2 + x_\pi, \hat{x}_3) < \Pi_1(\hat{x}, \hat{m}) \); likewise, a further unilateral deviation of firm 2 from \((\hat{x}, \hat{m})\) to \( \hat{m}_2 = (2, 2) \) and \( \hat{x}_2 = R(3)(\hat{x}_3 + x_\pi) \) leads to profits \( \Pi_2(3)(\hat{x}_3 + x_\pi, \hat{x}_2) < \Pi_2(\hat{x}, \hat{m}) \). Thus, for \( \gamma \in (\gamma^o_3(\beta), \gamma^o_3(\beta)) \) this coalitional deviation is self-enforcing and improving, and therefore a merger cannot be sustained by a CPNE.

(M1.3) A deviation of the coalition formed by the outsider \( o = 2 \) and the acquirer \( a = 1 \) to form a new merger in which the outsider is the acquirer, i.e., \( \hat{m}_1 = \hat{m}_2 = (2, 1) \). Again simple algebra reveals that whenever \( \hat{x}_2 = \hat{R}(\hat{x}_1, x_a) \) and \( \hat{x}_1 = \hat{R}(\hat{x}_2, x_a) \), then \( \Pi_1(\hat{x}, \hat{m}) < \Pi_a(x, m) \), and therefore this deviation is not profitable.

(M1.4) A deviation of the coalition formed by the outsider \( o = 2 \) and the acquirer \( a = 1 \) to form a new merger in which the outsider is the acquiree, i.e., \( \hat{m}_1 = \hat{m}_2 = (1, 2) \). Simple algebra reveals that whenever \( \hat{x}_1 = \hat{R}(\hat{x}_2, x_a) \) and \( \hat{x}_2 = \hat{R}(\hat{x}_1, x_a) \), then \( \Pi_2(\hat{x}, \hat{m}) > \Pi_o(x, m) \) if and only if \( \gamma < \gamma^o_6(\beta) \), with \( \frac{d \gamma^o_6}{d \beta} > 0 \); and \( \gamma^o_6(0.5) = 4.245 \) and \( \gamma^o_6(1) = 5.564 \); and \( \Pi_1(\hat{x}, \hat{m}) > \Pi_\pi(x, m) \) if and only if \( \gamma > \gamma^o_6(\beta) \), with \( \frac{d \gamma^o_6}{d \beta} < 0 \); and \( \gamma^o_6(1) = 15.495 \). Since \( \gamma^o_6(\beta) < \gamma^o_3(\beta) \), this deviation is never profitable for both firms.

Finally, it is easy to see that if \((x, m)\) is invulnerable to unilateral deviations and to deviations of coalitions of two firms, then it is also invulnerable to deviations of the grant coalition. Note that self-enforcing deviations must be NE as otherwise they are vulnerable to deviations of individual firms, which are self-enforcing. A deviation of the grant coalition that leads to non-integration is not improving as by Proposition 6.4, the firms have conflicting interests regarding a merger. Conversely, deviations of the grant coalition that lead to a merger can be implemented by a deviation of a coalition of two firms, which we have already considered.

By Proposition 4 and the above discussion, we conclude that a merger can be sustained by
a CPNE for all $(\beta, \gamma) \in [1/2, 1] \times [5/2, \infty)$ such that

$$\gamma \in [5/2, \gamma^d_1(\beta)] \setminus (\gamma^4_4(\beta), \gamma^5_3(\beta)).$$

**Proof of Proposition 7.3:** By Propositions 7.1 and 7.2 for all $(\beta, \gamma) \in [1/2, 1] \times [5/2, \infty)$ such that $\gamma \in (\gamma^4_4(\beta), \gamma^5_3(\beta))$ a CPNE does not exist.

**A.4: Proof of Proposition 8**

(8.1) Using (23) and (24), we can check that

$$\bar{S}^d(\beta, \gamma) = \bar{C}S^d + \Pi_\pi + \Pi_a + \Pi_o \geq S_{NI}^d(\beta, \gamma) = CS_{NI} + 3\Pi_{NI}$$

if and only if $\gamma \leq \gamma^d_1(\beta)$, with $\frac{d\gamma^d_1}{d\beta} < 0$, $\gamma^d_1(0.5) = 2.259$ and $\gamma^d_1(1) = 1.915$. Using (23) and (25), we can check that

$$\bar{S}^d(\beta, \gamma) = \bar{C}S^d + \Pi_\pi + \Pi_a + \Pi_o \geq S_{NI}^d(\beta, \gamma) = CS_{NI} + 3\Pi_{NI}$$

if and only if $\gamma \leq \gamma^d_2(\beta)$, with $\frac{d\gamma^d_2}{d\beta} > 0$, $\gamma^d_2(0.5) = 2.259$ and $\gamma^d_2(1) = 2.726$. Moreover, $\gamma^d_2(\beta) \geq \gamma^d_1(\beta)$, and therefore the last statement immediately follows. Finally, we can check that $CS^d < \bar{C}S^d < CS_{NI}$ always.

(8.2) Using (23) and (26), we can check that

$$S^c(\beta, \gamma) = CS^c + \Pi_\pi + \Pi_a + \Pi_o \geq S_{NI}^c(\beta, \gamma) = CS_{NI} + 3\Pi_{NI}$$

if and only if $\gamma \leq \gamma^c_1(\beta)$, with $\frac{d\gamma^c_1}{d\beta} < 0$, $\gamma^c_1(0.5) = 8.157$ and $\gamma^c_1(1) = 7.425$. Moreover, $CS^c > CS_{NI}$ if and only if $\gamma \leq \gamma^c_{cs}(\beta)$, with $\frac{d\gamma^c_{cs}}{d\beta} < 0$, $\gamma^c_{cs}(0.5) = 3.359$ and $\gamma^c_{cs}(1) = 2.5$.

(8.3) As $\gamma^c_1(\beta) > \gamma^d_2(\beta)$, the statement immediately follows.
References


Figure 1: Type 1 (type 2) mergers are sustained by a NE below the black (red) line.
Figure 2a: Non-integration is sustained by a CPNE in the blue region $B^d$. 
Figure 2b: Type 1 mergers \((x_\pi > x_\alpha)\) are sustained by a CPNE in the red region \(R^d\).
Figure 2c: Type 2 mergers ($x_\pi < x_\omega$) are sustained by a CPNE in the yellow region $Y^d$. 
Figure 3: Outcomes sustained by CPNE under duplicative research paths: non-integration (areas in blue, purple, green and grey), type 1 mergers (areas in purple, grey, orange and red), type 2 mergers (areas in green, grey, yellow and orange).
Figure 4: Outcomes sustained by CPNE under complementary research paths: non-integration (blue region $B^c$), mergers (yellow region $Y^c$), non-integration and mergers (green region $G^c$), no CPNE exists (white region $W^c$).