Partial Coordination in Clustered Base Station
MIMO Transmission

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Abstract—We present partial coordination strategies in a clustered cellular environment, evaluating the achievable rate in the downlink transmission. Block Diagonalization is employed for the coordinated users within the cluster to remove interference, while the interference from non-coordinated users remains. The achievable rate is evaluated resorting to an analytical expression conditioned on the position of the users in the cluster. A partial coordination approach is proposed to reduce the coordination complexity and overhead, where users close to the base station are not coordinated. Two approaches are considered, namely the non-coordinated users can be grouped and assigned separated resources from the coordinated ones, or they can be mixed.

Index Terms—Coordinated base stations, clustering, multiple-antennas, block diagonalization, network MIMO.

I. INTRODUCTION

The capacity gain of Multiple Input-Multiple Output (MIMO) techniques in cellular networks is strongly affected by the interference that characterizes these environments. The classical approach of frequency reuse to cope with interference leads to an inherent loss of spectral efficiency. To achieve spectrally-efficient communications, it is desirable that all cells operate on the same frequency channel, what is denoted as universal frequency reuse (UFR). This requires joint optimization of the resources in all cells simultaneously to improve the system performance and new techniques have emerged to manage interference, by introducing coordination among the base stations in the downlink, which are known as network MIMO, coordinated base station transmission (CBST) [1],[2] or coordinated multipoint (CoMP) [3]. Similarly to multi-user MIMO, Block Diagonalization (BD) [4], [5] may be applied for CBST as a good compromise between complexity and performance. In [6] BD is applied in a multicell scenario in combination with the interference reduction scheme of [7]. Alternatively in [8] a Singular Value Decomposition (SVD) approach is proposed that simplifies the channel estimation requirements at the expense of a performance degradation. Other linear schemes based on minimizing the mean squared error have also been proposed [9].

The main drawback of all these systems is that they require channel state information (CSI) and traffic data to be simultaneously known to all cooperating base stations. Some recent approaches have been proposed to avoid CSI and data sharing using coherent joint processing [10] at the expense of higher processing cost at the receivers with successive interference cancellation. As a practical alternative, we focus here on clustered coordination, where only a limited number of base stations can cooperate. Base stations are grouped into cooperation clusters and only the base stations of each cluster exchange information and jointly process signals. In [11] clustered coordination is analyzed, where clusters are of limited size. This has been shown to be a good trade-off between performance and overhead. Even higher performance gains can be attained if the clusters are formed dynamically [12], [13]. In [6],[14] it is shown by simulation that a small cluster size is sufficient to obtain most of the sum rate benefits from clustered coordination. In [15] a linear precoding called soft interference nulling is proposed, which is useful when clusters of limited size overlap. In [16] the joint clustering and the beamformers are studied and applied to heterogenous networks using a user-centric BS clustering.

In this work we focus on clustered BD-based CBST with non-overlapping clusters with per base station power constraints and present the evaluation of the user rates that can be achieved with different coordination strategies. In detail, starting from the observation that users close to the BS benefit only marginally from coordination, we set a coordination distance from the base station determining an area such that only users outside this area are coordinated, thus lightening the burden of coordination. The questions to consider are:

- The opportunity to coordinate users or not, in other words, the effect of the coordination distance on the achievable rate.
- The opportunity of grouping the coordinated and non-coordinated users into separate sets, assigning to each group non-interfering resources (for example different frequency bands).

Moreover, different criteria to measure the performance are considered, namely the average achievable rate among all the users, the minimum achievable rate guaranteed to at least 90% of the users and the achievable rate guaranteed to the 10% of user with higher rates. We show that the loss in terms of achievable rate with respect to the full coordination considered in the literature [1]-[3] is very limited if the coordination distance is kept within a fraction of about half the cell radius, while the effect of grouping depends on the quality criterion and on the coordination distance.
II. CLUSTER MODEL

The downlink scenario consists of cells of radius $R_{\text{cell}}$ grouped into non-overlapping clusters, where, in order to ease the analysis, we assume that all the cells have the same size. Clusters are composed of $M$ base stations (BS), that can coordinate their transmission serving a total of $N$ users in each cluster. Clusters are pre-determined in the network setup on the basis of a minimum distance criterion among the BSs or according to other criteria, which could consider the CSI. In any case we assume that the clusters do not change during the transmission, i.e. we do not address dynamic clustering. The size of the cluster $M$ is a parameter of the analysis. Each of the BSs is equipped with $t$ transmit antennas and has a maximum available power $P_{\text{max}}$ and each user terminal has $r$ receive antennas.

An example of a cluster with five cells is shown in Fig. 1.

III. PARTIAL COORDINATION

We can observe that in a coordinated base station scenario the actual effect of coordination is more useful for mobile terminals that are located far from their serving BS, close to the cell border, and experience a higher level of interference. On the other hand, if the user is close to the serving BS we can presume that the received power is high and coordination with other cells causes a loss of resources that are used unnecessarily for the coordination. Therefore, we propose a technique which considers a partial coordination scenario, where users located within the coordination distance from their nearest BS are not coordinated. Only users outside this distance are coordinated and coordination occurs only inside the cluster, in other words users belonging to other clusters are not considered, in order to limit the amount of signalling and the complexity. The coordination distance $D_c$, expressed as a fraction of the cell radius $R_{\text{cell}}$ is a design parameter.

In Fig. 1 we show an example of a cluster with five cells in which two users (marked by stars) in cells 0 and 1 are close to their BS, with a distance smaller than $D_c$, and are not coordinated and three users (marked by bullets) are at a distance greater than $D_c$ and are coordinated.

![Fig. 1. System layout with a cluster of five cells in which two users (marked by stars) are close to their BS and are not coordinated and three users (marked by bullets) are at a distance greater than $D_c$ and are coordinated.](image)

A. Coordination strategies

Within the partial coordination scenario, where only the users at a distance greater than $D_c$ are coordinated, we propose and analyze two different coordination strategies:

**Grouped:** In the first case, we assume that we can group all the users that are located within the “near area” of each cell of the cluster and assign them a separate transmission resource. The users which are located at a distance greater than the coordination distance are coordinated using another separate transmission resource. Therefore, we can split the users into two non-interfering groups, namely coordinated and non-coordinated users. Hence each cell can double the number of users served, at the expense of doubling the amount of resources. However, since we are interested in the mean average rate and not in the absolute number of users, the results of the following Section VI are not affected.

**Mixed:** In the second case no specific resource is allocated to any group of users, therefore interference occurs between coordinated and non-coordinated users.

The separate transmission resources can be different frequencies, time instants, codes,... that may be assigned to each user group by the scheduler.

IV. CHANNEL MODEL AND BLOCK DIAGONALIZATION

The channel model includes:

- Path loss with exponential power decay $d^{-\gamma}$ as a function of the distance $d$, with exponent $\gamma$.
- Rayleigh fading, so that the channel matrix entries are i.i.d. complex Gaussian variables.
- Additive Gaussian noise: a vector of i.i.d. complex Gaussian entries with zero-mean and variance $\sigma_n^2$ is added to the useful received signal vector. The value of the signal to noise ratio (SNR) is defined with reference to the power received at the cell border, as done also in e.g. [6], namely

$$\rho = \frac{P_{\text{max}} R_{\text{cell}}^\gamma}{\sigma_n^2}. \quad (1)$$

A. Coordinated users

For the $N_c$ users inside a cluster which are coordinated, transmission occurs from all the coordinated BSs, so that the channel is a $N_c r \times N_c t$ matrix $H$. If we define $H_i$ with $i = 1 \ldots N_c$ as the $r \times N_i t$ channel matrix seen by user $i$, its complex Gaussian elements have zero mean and variance $\sigma_i^2$, which is a function of the distance $d_{i,j}$ of the user $i$ to the BS $j$. Then the overall channel is $H = [H_1H_2 \ldots H_{N_c}]^T$ and the $N_c r \times 1$ received signal vector $y$ can be expressed as

$$y = H x + I + n = H W b + I + n \quad (2)$$

where $I$ is the $N_c r \times 1$ vector with the interference contribution coming from outside the cluster and $n$ is the $N_c r \times 1$ noise vector of Gaussian entries with zero-mean and variance $\sigma_n^2$. The $N_c t \times 1$ signal vector $x$ transmitted from the coordinated BSs of the cluster is obtained by applying a precoding (or beam-forming) matrix $W$, where $b = [b_1, \ldots, b_r, \ldots, b_{N_c}]^T$, $b_{ij}$ represents the $j$-th data symbol for user $i$ transmitted with
power $P_{ij}$ and $W = [\mathbf{w}_{1i}, \ldots, \mathbf{w}_{ri}, \ldots, \mathbf{w}_{Ni, ri}]^T$ is the beamforming matrix, where $\mathbf{w}_{ij} = [w_{1ij}, \ldots, w_{rij}, \ldots, w_{Ni, rij}]^T$ are the precoding vectors for the $j$-th data stream of the $i$-th user.

The beamforming matrix $W$ is obtained under a BD criteria as in [1] and [2], to guarantee that

$$P_{ii} = \sum_{m \neq i} P_{max} d_{im}^{-\gamma}$$

where $U_k$ is a unitary matrix and $S_k = \text{diag}(\lambda_{k1}^{1/2}, \lambda_{k2}^{1/2}, \ldots, \lambda_{kr}^{1/2})$. $\lambda_{ji}$ are obtained from a singular value decomposition of the interfering channels according to the procedure explained in [2]. Then, the received signal is

$$y = \begin{bmatrix} U_1 S_1 & 0 & \cdots & 0 \\ 0 & U_2 S_2 & \cdots & 0 \\ \vdots & \vdots & \ddots & \vdots \\ 0 & 0 & \cdots & U_{Ni} S_{Ni} \end{bmatrix} \mathbf{b} + \tilde{\mathbf{n}} = \mathbf{H}_i \mathbf{x}_i + \tilde{\mathbf{n}},$$

where the statistics of $\tilde{\mathbf{n}}$ remains the same because of the unitary transformation.

1) Interference: Since BD achieves cancellation of interference among the coordinated users, interference inside the cluster comes only from the BS which are not cooperating. The vector of interference $\tilde{\mathbf{I}}$ for user $i$ has components $I_{ij}$, representing the total interference power experienced in each of its antennas. We can write

$$I_{C,ij} = \sum_{m \in \mathcal{S}_i} P_{max} d_{im}^{-\gamma}$$

where $\mathcal{S}_i$ denotes the set of coordinated BSs and $d_{im}$ the distance between user $i$ and base station $m$. The sum is extended also to the BSs outside the cluster. In the following results the first tier of BS around the cluster will be considered to evaluate the interference power.

B. Non-coordinated users

In this case the received signal for user $i$ is

$$\mathbf{y}_i = \mathbf{H}_i \mathbf{x}_i + \mathbf{I} + \mathbf{n}_i,$$

where the received vector is now $r \times 1$ and $\mathbf{n}_i$ is the $r \times 1$ complex Gaussian noise vector of i.i.d zero-mean entries with variance $\sigma_n^2$. In this case $\mathbf{H}_i$ represent the channel from base station $i$ to user $i$ since no coordination is performed. Note that the matrix $\mathbf{H}_i$ can be decomposed into a form

$$\mathbf{H}_i = U_i \text{diag} \left\{ \lambda_{NC,i1}^{1/2}, \ldots, \lambda_{NC,i, r}^{1/2} \right\} \mathbf{V}_i,$$

similarly to the case (5), where in this case the singular values $\lambda_{NC,ij}$ correspond to the channel matrix without any pre-coding.

1) Interference: For a non-coordinated user $i$ the contribution of interference comes from all the BSs other than $i$.

$$I_{NC,ij} = \sum_{m \neq i} P_{max} d_{im}^{-\gamma}$$

where again $d_{im}$ is the distance between user $i$ and base station $m$ and we assume that all base stations are transmitting at maximum power. Also here the sum is extended to the BSs outside the cluster, to account for the interference from neighbour clusters.

V. ANALYSIS OF THE RATE

If user $i$ is within the “near area” at a distance $d_{ij} < D_c$ from its BS, the rate is given by

$$R_i = \sum_{j=1}^{r} \log_2 \left( 1 + \frac{P_{ij}}{\gamma} \frac{\lambda_{NC,ij}}{\sigma_n^2 + I_{NC,ij}} \right),$$

where $I_{NC,j}$ represents the total interference, expressed by (9).

If user $i$ is coordinated, its achievable rate is given by

$$R_i = \sum_{j=1}^{r} \log_2 \left( 1 + \frac{P_{ij}}{\gamma} \frac{\lambda_{NC,ij}}{\sigma_n^2 + I_{NC,ij}} \right),$$

where the total interference $I_{C,j}$ is (6). The terms $\lambda_{NC,ij}$, $\lambda_{C,ij}$ in (10), (11) account for the effects of the Rayleigh fading and of the path loss. The latter is a function of the position of the user and the effect of the distance $d_{ij}$ can be separated from the effect of fading by writing

$$\lambda_{NC,ij} = \mu_{ij} d_{ij}^{-\gamma},$$

where $\mu_{ij}$ has the same statistical characterization for both coordinated and non-coordinates users, as shown in the next section.

A. Fading effect

The normalized parameters $\mu_{ij}$ as (12) account for the channel fading (removing the effect of path loss), and the rate is averaged with respect to the probability density function (pdf) of the terms $\mu_{ij}$. These $\mu_{ij}$ represent both for coordinated and non-coordinated users the squared singular values of a Gaussian matrix, that is, the eigenvalues of the corresponding Wishart matrix. In fact the transformation giving the $\mu_{ij}$ from the normalized channel matrix (where the path loss has been removed) corresponds to the multiplication by a unitary matrices both for coordinated and non-coordinated users. Then, according to [18], the joint pdf $f(\mu_1, \ldots, \mu_r)$ is given by

$$f(\mu_1, \ldots, \mu_r) = e^{-\sum_{i=1}^{r} \mu_i \frac{1}{r-n}} \prod_{n=1}^{r} \prod_{m>n}^{r} (\mu_{mn} - \mu_n)^2.$$
Then one should average (11) with respect to the joint pdf of all the coefficients $\mu_{ij}$. However, in the evaluation of the probability distribution function the mean is

$$f(\mu) = \frac{1}{r} \sum_{k=0}^{r-1} [L_k(\mu)]^2 e^{-\mu},$$

being $L_k(\cdot)$ the Laguerre polynomials. Then we get [19]

$$\mathbb{E} = \frac{1}{r} \sum_{k=0}^{r-1} \frac{1}{r} \sum_{j=0}^{r-1} (2k+1) = r.$$  

## B. Power allocation

The power allocation should maximize some quality of service parameters, such as the sum rate (or a weighted sum rate), for the set of coordinated users. This objective is subject to a maximum transmission power $P_{\text{max}}$ at each BS, namely

$$\sum_{i=1}^{N} \sum_{j=1}^{r} P_{ij} |w_{ij}|^2 \leq P_{\text{max}},$$

for each BS $k = 1, \ldots, N_c$. The rate maximization problem has been tackled in several works, e.g. [2,7] and solutions range from the uniform power approach to optimal allocation, whose derivation requires the cumbersome numerical solution of the convex optimization problem. A power allocation scheme, resembling the well known waterfilling and performing very close to the optimum, has been presented in [2,17]. In the following we consider a uniform power allocation scheme, in which a common value $P_0$ replaces $P_{ij}$ in (11), representing the average transmitted power from the coordinated BSs to each of the $r$ parallel streams of user $i$. This value $P_0$ varies according to the number of coordinated BSs $N_i$ and decreases with $N_c$, since a fraction of the available power is spent for coordination, to null the interference. If we substitute a common value $P_0$ for each $i = 1, \ldots, N_c$ and $j = 1, \ldots, r$ the condition (16) is limited by the BS for which the following sum is maximum

$$\sum_{i=1}^{N} |w_{ij}|^2.$$  

By using a Gaussian approximation of the coefficients $w_{ij}$, $P_0$ is then related to the reciprocal value of the maximum of $N_c$ chi-squared distributed random variables $\chi_i$

$$P_0 = \frac{P_{\text{max}}}{\mathbb{E}[\chi]}$$

where

$$\chi = \max_{k=1,\ldots,N_c} \{\chi_1,\ldots,\chi_{N_c}\}.$$  

This maximum $\chi$ has probability distribution function

$$F_\chi(x) = P(t,x)^{N_c}$$

being $P(\cdot,\cdot)$ the regularized Gamma function. In terms of the probability distribution function the mean is

$$E[\chi] = \int_0^{+\infty} (1 - F_\chi(x)) \, dx$$

and can be evaluated using the bounds [19]

$$(1 - e^{-\alpha x})^\alpha \leq P(a,x) \leq (1 - e^{-\beta x})^\beta$$

with

$$\alpha = \begin{cases} 1 & 0 < a < 1 \\ d_a & a > 1 \end{cases}$$

$$\beta = \begin{cases} 1 & 0 < a < 1 \\ d_a & a > 1 \end{cases}$$

$$d_a = (\Gamma(a+1))^{-1/2},$$

where $\Gamma(\cdot)$ is the Gamma function. Then $E[\chi]$ is bounded by

$$\frac{1}{\psi(N_t+1)+\gamma_0} \leq E[\chi] \leq \frac{1}{\alpha \psi(N_t+1)+\gamma_0}$$

with $\psi$ the digamma function and $\gamma_0$ the Euler constant. In terms of $P_0$, we have

$$P_{\text{max}} \frac{\Gamma(t+1)^{-1/t}}{\psi(N_t+1)+\gamma_0} \leq P_0 \leq \frac{P_{\text{max}}}{\psi(N_t+1)+\gamma_0}$$

In the evaluation of the rate, we will consider the lower bound, giving a lower bound to the average rate of user $i$. The bounds for the power per stream derived by (26) with uniform power allocation are compared in Fig. 2 with the results obtained by simulations. We notice a very good agreement between the analytical and simulation results, for different antenna configurations.

Finally, the overall mean achievable rate for user $i$ can be expressed, using the lower bound of (26), as

$$\bar{R}_i = r \log_2 \left( 1 + \frac{\Gamma(t+1)^{-1/t}}{\psi(t+1)+\gamma_0} \frac{d_{ii}^\gamma}{\sigma_d^2 + \sum_{m \neq i} P_{\text{max}} d_{im}^\gamma} \right),$$

if the user is served by a non-coordinated BS. On the other hand, if the users belongs to a set of coordinated users, the mean achievable rate is

$$\bar{R}_i = r \log_2 \left( 1 + \frac{\Gamma(t+1)^{-1/t}}{\psi(N_t+1)+\gamma_0} \frac{d_{ii}^\gamma}{\sum_{m \neq i} P_{\text{max}} d_{im}^\gamma} \right).$$
VI. NUMERICAL RESULTS

In order to evaluate the achievable rate a semi-analytical approach is used, where, for each run, a random distribution of \( N = M \) users in the cluster is set. This determines the distances so that the achievable rate for each user can be evaluated by (27) and the statistics are collected over independent runs. The performance is considered by the mean achievable rate in the cluster (averaged over all the users) and the achievable rate at different values of the cumulative distribution function (CDF), to account for the statistical variability induced by the users location. Some parameters, if not otherwise stated, are set as \( \gamma = 3.8 \), \( R_{\text{cell}} = 1.4\,\text{km} \), \( P_{\text{max}} = 1\,\text{W} \), \( t = r = 2 \) antennas.

A. Mean achievable rate

In Fig. 3 the mean achievable rate per user is presented in clusters with different values of the cluster size \( M \), as a function of the coordination distance, with \( \text{SNR} = 15\,\text{dB} \). It can be seen that the effect of grouping the users and assigning separate resources provides in general a small advantage with respect to a mixed environment, growing with the cluster size. We observe a value of \( D_c \) up to which the performance of the mixed is slightly better, due to the different interference conditions that can occur with the two approaches: in fact in a mixed environment the variability of scenarios of coordinated/non-coordinated users can give a reduced average level of interference. This is due to the fact that if the non-coordinated users are very close to the base station (for \( D_c \simeq 0 \)) their high rate is mainly limited by interference since the SNR is high (due to the small path loss) and they can benefit from a lower interference which can happen in a mixed environment, since some other cells can have users in the coordinated area, but not in the un-coordinated area close to the BS, thus they do not interfere. On the other hand, in a grouped environment the resources double, so that for sure an interfering user will be present in any cell. The effect of the number of cells of the cluster \( M \) is not very pronounced: With full coordination (\( D_c = 0 \)) the rate slightly increases with the cluster size, due to the fact that the power loss for coordination is counterbalanced by the smaller amount of interference coming from the neighbour clusters. When the coordination area is restricted (\( D_c/R_{\text{cell}} \) approaches unity), a smaller cluster size can give better rates. A similar behaviour is observed with different values of SNR in Fig. 4, where the mean achievable rate per user in a cluster of \( M = 5 \) cells is presented, again as a function of \( D_c \). The difference between grouped and mixed

![Fig. 3. Mean achievable rate per user in a cluster of \( M \) cells with SNR=15 dB.](image)

![Fig. 4. Mean achievable rate per user for \( M = 5 \) cells/cluster, variable SNR.](image)

users becomes more noticeable at higher values of SNR, from a distance \( D_c \) up to which the mixed environment guarantees a slightly better rate, as already observed before.

B. Achievable rate for worst and best users

We now consider a different approach, so that we focus on the worst rate among the users in the cluster. In other words, we can consider a certain percentage (for example 10%) of the users who are experiencing the worst values of achievable rate, i.e. the value for which the CDF reaches the 10%. Thus, the clusters with this coordination strategy can guarantee that 90% of the users experience an achievable rate greater than the value determined with this criterion. In Fig. 5 the value of achievable rate corresponding to the 10% of the CDF is presented for a cluster of \( M = 5 \) cells, as a function of the coordination distance, with different values of SNR. We can note a dramatic change of the CDF value at 10%, i.e. the minimum guaranteed rate to the 90% of the users, in correspondence to a coordination distance between 0.3\( R_{\text{cell}} \) and 0.4\( R_{\text{cell}} \). This value of distance keeps constant with the SNR and corresponds to a point where the contribution of interference on the non-coordinated users becomes overwhelming, thus degrading the rate to a value very close to zero. When the percentage of users affected by this interference goes over 10%, then the value of minimum rate keeps constant and close to zero. We can also notice that the case of grouping leads to a slightly better performance for the minimum rate guaranteed to the majority of the users and to a bigger value of \( D_c \), where the drop in the rate occurs. If we consider all the approaches to optimize the cellular network performance, i.e. to obtain the best average
Fig. 6. Achievable rate (average, 10% and 90% of CDF) for the users in a cluster of $M = 5$ cells with variable SNR.

rate, or a minimum rate for 90% of the users, or finally to provide the best rate to a limited number of users (e.g., 10%), in Fig. 6 we present the average value together with the values at 10% and 90% of the achievable rate CDF, corresponding to the worst and best served users, for $M = 5$ cells and border SNR $= 15$ dB. We can see that if the objective is to guarantee a minimum rate for the majority of the users a strategy of grouping can achieve slight better results. On the other hand, if the maximum rate is privileged, a mixed environment leads to a better performance until a crossing point occurs, as seen in the average value, although the difference is not huge.

VII. CONCLUSIONS

We considered different strategies to perform the coordination in a clustered cell environment, easing the requirements of coordination among all the cells of the cluster by a partial coordination. We can see that, although in general a better performance is obtained by coordinating all the users in the cluster as in [1]–[2], the loss in performance derived by non-coordination is acceptable if the coordination distance is below half the radius of the cell. The advantage of a partial coordination coming from the huge reduction of the complexity and of the signalling (for control, CSI, and users data) between the coordinated base stations in the cluster can be traded-off with a small reduction of the rates.

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