Applying Low Discrepancy Sequences for Node-ID Assignment in P2PSIP

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Abstract—The IETF P2PSIP Working Group is currently designing a standard overlay protocol, named RELOAD, that employs a centralized node identifier (node-id) assignment for security reasons. Given this scenario, we propose the utilization of a Low Discrepancy Sequence (LDS) for the assignment of node-ids in the P2PSIP architecture. We perform an analytical and simulation study considering a Chord DHT that demonstrates that using a LDS-based node-id assignment guarantees a fair distribution of the node’s zone of responsibility, even in high churn scenarios. Previous studies have shown that a fairer distribution of the zones of responsibility sizes leads to a fairer distribution of the storage and routing load. Therefore we conclude that the proposed LDS node-id assignment provides these features without adding any extra overhead.

Index Terms—Node-ID assignment, P2PSIP, DHT, low discrepancy sequence.

I. LOW DISCREPANCY SEQUENCE BASED NODE-ID ASSIGNMENT

The IETF P2PSIP Working Group is designing a standard overlay protocol, named RELOAD [1], to be (potentially) used by a large number of applications (e.g. a fully distributed SIP architecture). In RELOAD nodes organize themselves forming a Distributed Hash Table (DHT). Although RELOAD can support any type of DHT, the P2PSIP WG has selected Chord [2] as the default DHT system, which is mandatory in all P2PSIP implementations. In Chord, nodes are organized in a ring overlay. The position of each node in the ring is defined by its node-id. This is computed as the hash (e.g. SHA-1) of the node’s IP address, thus having a random nature. However, this decentralized assignment has been modified by RELOAD due to its security problems [3]. In RELOAD node-ids are assigned by a central entity named Enrollment Server that allocates an username and a node-id and issues a certificate for the user including these data. Moreover, in Chord (and also in RELOAD) each node is responsible of the portion of the id-space between itself and its predecessor (the node with the next lower node-id). We name this space the node’s zone of responsibility, because all the resources associated to the keys contained in a node’s zone of responsibility must be stored by this node. Due to the random nature of Chord node-ids, the size of the different zones of responsibility is heavily unbalanced, thus leading to an unbalanced distribution in the number of stored keys [4] as well as the number of routed messages per node [5]. Hence, it seems a good idea to improve this node-id assignment in RELOAD. Previous works have addressed the fairness in Chord, although keeping the random node-id assignment process. A survey of these solutions can be found at reference [6]. Most of these solutions are complicated, incur a lot of overhead and require modifications to the Chord protocol [6], whereas some other simple solutions only address the fairness of some specific aspect such as routing [5].

In this letter, we exploit the fact that RELOAD uses a central Enrollment server, and propose to assign the node-ids following a Low Discrepancy Sequence (LDS) [7] instead of doing it purely random. A LDS is a pseudo-random sequence of equidistant numbers. Describing in detail the number generation procedure used by a LDS is out of the scope of this letter. Instead, we refer the reader to reference [7]. However, for sake of clarity, we use the following example along the letter. We consider the 16 first numbers generated by a Halton Sequence (HS) [7]. This is an specific type of LDS that generates equidistant numbers in \([0,1)\) whose 16 first members are: 0, 1/2, 1/4, 3/4, 1/8, 5/8, 3/8, 7/8, 1/16, 9/16, 5/16, 13/16, 3/16, 11/16, 7/16, 15/16. We can observe that all the 16 generated numbers are equidistant and separated 1/16. Furthermore, the first 2, 4 and 8 generated numbers are also equidistant and separated 1/2, 1/4 and 1/8 respectively.

Hence, given that the id-space to be used in RELOAD is \([0,2^{128}) \in \mathbb{Z}\), we propose the RELOAD Enrollment server to assign to the \(i^{th}\) node joining the overlay the node-id \(hs(i) \times 2^{128}\) (where \(hs(i)\) is the \(i^{th}\) value of the HS). The proposed assignment guarantees that, if the number of assigned node-ids \((n)\) is a power of two, i.e. \(n = 2^i (i \in [0, N])\), and all the nodes are online in the overlay (i.e. no-churn scenario), then the distribution of the node’s zone of responsibility size is fully balanced, since all nodes have a zone of responsibility of size 1/n. Therefore, considering a number of nodes equal to a power of two is a best case scenario. However, if the number of nodes is not a power of two, this condition does not hold. Thus, we can also consider the worst case scenario. This happens when the number of nodes \((n)\) is equal to \(2^i + 2^{i-1} (i \in N)\), which is the intermediate value between two consecutive powers of two

\(^1We have validated this through simulation: we analyzed all the possible values of \(n\) between two consecutive powers of two values concluding that the most unbalanced distribution of the zone of responsibility size happens for the intermediate value.
In the rest of the letter, we study the fairness of the node’s zone of responsibility size distribution for the proposed LDS-based node-id assignment for both the best and worst case scenarios and compare it to the widely employed Random node-id assignment. For this purpose we use the Jain’s Fairness Index (FI) [11]. This is a well-known and extensively used metric to measure the fairness that gives a value between 0 (highest unfairness) and 1 (highest fairness). Since RELOAD is being designed in order to be used in different applications that may or may not suffer from churn, we first study, both analytically and by means of large-scale simulations, the FI for a no-churn scenario. After that, we run simulations to evaluate the FI in overlays under churn.

II. MODELING THE DISTRIBUTION OF THE NODE’S ZONE OF RESPONSIBILITY SIZE IN RELOAD

Without loss of generality, our model assumes that node identifiers are distributed in the continuous id-space [0,1]. This is an accurate approximation of the real scenario in which the node-ids are distributed in the discrete id-space [0,2^{128}].

A. Random node-id assignment

Let \( X \) be the random variable that represents the size of the zone between two consecutive nodes. The probability of the node’s zone of responsibility \( (X) \) to be smaller than a given value \( x \) is equal to the probability of having at least one node inside \( X \). Since node-ids are uniformly distributed in the ring and the total id-space size is equal to 1, the probability that a node falls within a zone of size \( x \) is precisely \( x \). This yields the following cumulative distribution function for random variable \( X \), where \( n \) represents the total number of nodes:

\[
cdf_{\text{Random}}(x) = P(X \leq x) = 1 - (1 - x)^n
\]  

(1)

The probability distribution function of the node’s zone of responsibility size, \( pdf(x) \), is simply computed as the derivative of the above cdf:

\[
pdf_{\text{Random}}(x) = n(1 - x)^{n-1}
\]  

(2)

B. LDS-based node-id assignment

In this subsection, we present the model for the probability distribution function of the node’s zone of responsibility size, \( pdf(x) \), for the LDS-based node-id assignment for the best case and worst case scenarios described in Section I.

- LDS best case: This occurs when the number of nodes in the overlay is a power of two, \( n = 2^i \) \((i \in [0, N])\). In this case the node’s zone of responsibility size is \( x = 1/2^i \) for all the nodes, then the \( pdf(x) \) is:

\[
pdf_{\text{LDS best}}(x) = \begin{cases} 1, & \text{if } x = \frac{1}{2^i} \\ 0, & \text{otherwise} \end{cases}
\]  

(3)

-LDS worst case: This happens when the number of nodes in the overlay is \( n = 2^i + 2^{i-1} \) \((i \in \mathbb{N})\). By examining the simple example presented in Section I, we can intuitively infer that in this case 2/3 of the nodes have a zone of responsibility of size \( x \) equal to \( 1/2^{i+1} \) whereas 1/3 of the nodes have a zone of responsibility of size \( x \) equal to \( 1/2^i \). Thus, the \( pdf(x) \) can be formulated as follows:

\[
pdf_{\text{LDS worst}}(x) = \begin{cases} \frac{2}{3}, & \text{if } x = \frac{1}{2^{i+1}} \\ \frac{1}{3}, & \text{if } x = \frac{1}{2^i} \\ 0, & \text{otherwise} \end{cases}
\]  

(4)

III. MODELING THE FAIRNESS INDEX OF THE NODE’S ZONE OF RESPONSIBILITY SIZE

The Jain’s Fairness Index [11] of the zones of responsibility size distribution is expressed as:

\[
FI = \frac{\sum_{i=1}^{n} x_i^2}{n \int_0^1 n_x x^2 \delta x}
\]  

(5)

where \( x_i \) represents the size of the zone of responsibility of node \( i \). Since the sum of the size of the zone of responsibility of all the nodes is equal to the size of the whole id-space, the value of the numerator is 1. To compute the denominator we rearrange the sum by grouping all the nodes that have a zone of responsibility of equal size together:

\[
n \sum_{i=1}^{n} x_i^2 = n \int_0^1 n_x x^2 \delta x
\]  

(6)

Therefore, we obtain the following expression:

\[
FI = \frac{1}{n \int_0^1 pdf(x) x^2 \delta x}
\]  

(9)

If we apply to Eq. 9 the \( pdf(x) \) expression for the Random node-id assignment (Eq. 2) and the LDS-based node-id assignment for both the best (Eq. 3) and worst (Eq. 4) cases, after performing some basic calculations we obtain the closed formulas for the FI in each case:

\[
FI_{\text{Random}} = \frac{(n + 1)(n + 2)}{2n^2} \approx 0.5 \text{ (}\forall n > 100\text{)}
\]  

(10)

\[
FI_{\text{LDS best}} = 1
\]  

(11)

\[
FI_{\text{LDS worst}} = 0.889
\]  

(12)

Interestingly, the LDS-based node-id assignment offers (in both the worst and the best case) a FI that is independent of the overlay size. Hence, for any number of nodes, the LDS-based node-id assignment guarantees a \( FI \geq 0.889 \). Moreover, the Random node-id assignment shows a \( FI \approx 0.5 \) for any realistic deployment.
IV. PERFORMANCE EVALUATION

Firstly, we run large-scale simulations for the no-churn scenario. We simulate the creation of a RELOAD topology considering the best and worst cases of the LDS-based node-id assignment between $n = 2^{10}$ and $n = 2^{20}$. This is, for the best case we consider topologies with $n = 2^i$ ($i \in [10, 20]$), whereas for the worst case we simulate topologies with $n = 2^i + 2^{i-1}$ ($i \in [10, 19]$). Furthermore we simulate a Random node-id assignment for all the previous overlay sizes. We run 50 simulations for each assignment mechanism (LDS-based and Random) and overlay size, and calculate the FI of the node’s zone of responsibility size. On the one hand, the simulation results validate the accuracy of the presented model: the FI values derived from the model incur errors lower than 0.5% compared to the simulation results in all cases. On the other hand, the results confirm that the LDS-based node-id assignment guarantees a FI of the node’s zone of responsibility size $\geq 0.889$ clearly outperforming the random node-id assignment in all the studied scenarios.

Next, we run large-scale simulations in a dynamic scenario to study the effect of churn. For this purpose we use a cycle-based simulator with a simple churn model based on a churn factor ($c$); each simulation run is composed by 100 simulations cycles. In each cycle a node has a probability $c$ of being offline and $(1-c)$ of being active. We use the following set-up: $n = 2^{15}$, $c = [0, 0.1,..., 0.9]$ and 100 runs per each value of $c$ for the Random and best LDS-based node-id assignment cases. For the LDS worst case we use the same set-up but considering $n = 2^{15} + 2^{14}$. Fig. 1 presents the obtained results including the average FI (across the 100 simulation runs) and the confidence intervals3 for the different studied churn factors. On the one hand, in the case of Random node-id assignment the FI increases with the churn. The reason is that churn changes the nodes’ zone of responsibility size along the time, having some times a larger zone and in other occasions a smaller zone. Thus, when considering a large number of simulation cycles the average zone of responsibility size of nodes tends to be equalized in the long term. On the other hand, in the LDS best case our starting point with no-churn ($c = 0$) is a perfect situation with FI equal to 1 because all zones of responsibility have a size equal to $1/n$. As we increase the churn it is likely that some nodes have in some cycles zones of responsibility of size $i/n$ ($i = 2, 3, 4,...$). This leads to a higher variability in the zone of responsibility size distribution. However, the FI reduction is slight. If we consider the LDS worst case we observe a slight improvement of the FI up to $c = 0.5$ (for the same reason as in the Random node-id assignment case) and from this value the FI slightly degrades as we increase the churn. Again, the LDS-based node-id assignment outperforms the Random node-id assignment in all cases. Finally, it is worth noting that we have repeated the churn experiments for values of $n$ ranging from $2^{10}$ to $2^{20}$ obtaining similar results to those presented in Fig 1.

V. CONCLUSIONS

The proposed LDS-based node-id assignment guarantees a high fairness ($0.8 \leq \text{FI} \leq 1$) of the node’s zone of responsibility size, even for extremely high churn. Then, our proposal leads to a fair distribution of storage and routing load. Furthermore, this is achieved without adding any extra overhead or extra functionality to the RELOAD protocol since we just modify the way of assigning node-ids by the central Enrollment Server.

REFERENCES


\[\text{Fig. 1. Fairness Index of the node’s zone of responsibility size in dynamic scenarios under churn.}\]