



## Financial Frictions, Occupational Choice and Economic Inequality\*

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### ABSTRACT

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We develop a quantitative theory of entrepreneurship, income inequality, and financial frictions disciplined with household data from Brazil. The theory extends Lucas (1978) by modeling heterogeneity in two skills: -working and managerial skills. Consistently with the evidence, the theory implies three occupational categories: workers, employers, and self-employed entrepreneurs. We find that the removal of financial frictions decreases self-employment rates from 24% to 11% (with small effects on the number of employers), increases aggregate output by 48%, and has non-trivial effects on the distribution of income. We also find that while most households benefit from a reform that eliminates enforcement problems, the majority of employers (about two thirds) lose from the reform. By depressing the demand for labor, limited enforcement depresses the equilibrium wage rate, increasing the profits of employers. Our theory thus suggests that employers in Brazil may have a vested interest in maintaining a status quo with low enforcement.

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# 1 Introduction

A recent literature has emphasized that the misallocation of resources caused by financial frictions depress total factor productivity and, hence, output per worker (Erosa (2001), Jeong and Townsend (2007), Amaral and Quintin (2010), Buera and Shin (2011), Buera et al. (2011), Greenwood et al. (2010)). The standard approach in the financial frictions literature and, more generally in the misallocation literature (see Guner et al. (2008), and Restuccia and Rogerson (2008)) is to calibrate the model to micro data from the United States and use the calibrated model economy to simulate policy distortions in developing countries.<sup>1</sup> While this approach has the advantage that the US data is readily available, it relies on the assumption that the distribution of entrepreneurial skills or plant productivities are invariant across countries or, at the very least, do not matter for the misallocation of resources induced by policy distortions or limited enforcement in the financial markets. However, there is ample evidence suggesting that the distribution of skills do vary across rich and poor countries.<sup>2</sup> Moreover, economic theory suggests that inequality matters for the impact of micro distortions and financial frictions (see Banerjee and Newman (1993), Galor and Zeira (1993)).

We develop a quantitative theory of entrepreneurship, income inequality, and financial frictions disciplined with household level data from Brazil. The theory is used to quantitatively evaluate the impact of financial frictions on occupational decisions, resource allocation, aggregate output, and economic inequality. Conversely, we study how economic inequality shapes the impact of financial frictions in the economy. Our paper contributes to a seminal (mostly theoretical) literature that has emphasized the importance of the interaction between the distribution of wealth and financial frictions for the allocation of resources. Moreover, we use our theory of inequality to quantitatively assess the distribution of welfare gains and losses from eliminating financial frictions in the economy.

The key innovation of our theory is to extend the Lucas (1978) model in order to incorporate heterogeneity in two skills: –working and managerial skills. By modeling heterogeneity in two skills the theory can distinguish between comparative advantage in entrepreneurship (a high ratio of managerial to working skills) and absolute advantage (a high value of both skills). This distinction is necessary for the theory to be consistent with evidence on the

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<sup>1</sup>Notable exceptions are given by Midrigan and Xu (2010) and García-Santana and Pijoan-Mas (2012)

<sup>2</sup>In fact, even among developed economies, recent work on international trade theory argues that the heterogeneity in the (second moments of the ) skill distribution plays an important role for understanding trade patterns among similarly endowed economies (see Ohnsorge and Trefler (2007) and references in that paper).

income distribution across occupations in Brazil. In particular, the ratio of median earnings between entrepreneurs and workers in Brazil is equal to one, which implies that the median entrepreneur in the Brazilian data does not make higher earnings than the median worker. This implication is grossly at odds with the predominant one-skill model developed in the occupational choice literature.

By assuming that entrepreneurs can use their working and managerial skills in the operation of their businesses, our theory has the novel implication that some entrepreneurs will not hire any outside labor and be own account workers (or self-employed entrepreneurs). Building a theory that distinguishes between entrepreneurs that are employers and those that are self-employed is important because there is abundant evidence that the high rates of entrepreneurship in poor countries is mostly due to the prevalence of self-employed workers (see Figure 2). In fact, the high rates of self-employment is an important feature of the Brazilian data and, our findings, imply that self-employment is important for understanding the impact of financial frictions in Brazil.

We assume a small open economy that takes as given the international interest rate. Following Buera et al. (2011), capital market imperfections are introduced by modeling an endogenous borrowing constraint that limits the amount of capital that entrepreneurs can use. We prove that in the absence of financial frictions occupational choices are driven entirely by the ratio of managerial to working skills. Employers have a comparative advantage at managing (high  $\frac{z_m}{z_w}$ ), workers have a comparative advantage at working (low  $\frac{z_m}{z_w}$ ), and self-employed have an intermediate skill ratio. Heterogeneity in absolute advantage implies that both at the top and bottom of the income distribution there are entrepreneurs and workers. We characterize how capital market imperfections distort rates of returns on skills by making the return to managerial and working skills depend on asset holdings. We show that financial frictions have a non-trivial impact on inequality: On the one hand, they lead to higher and persistent inequality by generating variation in returns to skills and by making these returns depend on asset holdings. On the other hand, since borrowing constraints tend to be tighter for highly skill than for low skill individuals, financial frictions reduce inequality by diminishing the rents obtained by highly skilled individuals relative to a situation with perfect capital markets.

The model economy is calibrated to Brazilian household data and macro aggregates. Brazil provides a nice benchmark because it is a country that exhibits both high levels of economic inequality and of financial frictions. We simulate the effects of removing financial frictions in the calibrated model economy. We find a large drop in the rates of entrepreneurship (from 33%

to 18%), which is mostly due to a decrease in self-employment. While the self-employment rate decreases from 24% to 11%, the fraction of employers only drops by about 2 percentage points. The gain in aggregate output amounts to 48%. There are also sizable changes in the sectorial composition of output. Production by employers increase by 64% whereas production of the self-employed decreases by 53%. TFP in the economy increased by 24%, with this gain being larger than the TFP gain among self-employed (9%) and that among employers (16%). The share of aggregate capital used by the self-employed drops from 19% to 4%. Because employers tend to have a higher managerial ability than self-employed entrepreneurs, the reallocation of capital between these two groups enhances the aggregate productivity gains of removing financial frictions.

We also simulate the impact of removing financial frictions when self-employment is shut down in the baseline economy. We find that the output gain is 53%, which is higher than the 48% increase obtained in the baseline economy. Hence, self-employment decreases the negative impact of financial frictions on aggregate output. This finding can be explained as follows: Financial frictions make it hard for young and talented entrepreneurs (individuals with high managerial skills) to raise external funds. This effect is compounded by the fact that financial frictions depress the equilibrium real wage, which makes it difficult to accumulate savings by working for a paid wage when young. Self-employment allows talented entrepreneurs to circumvent the low wage and build up savings, diminishing the negative impact of financial frictions on aggregate output. Hence, our findings implies that self-employment diminishes the impact of financial frictions by being a pathway towards becoming an employer. The occupational transitions in the Brazilian household data support this prediction of the theory: Self-employed individuals are three times more likely to become employers than paid workers and about 40% of transitions into employer between two consecutive years are coming directly from self-employment. Our baseline economy matches these facts remarkably closely.

Financial frictions have important effects on the sources of income inequality and on its persistence over time. We divide household income between capital income and labor income, with labor income defined as the sum of the returns to working and managerial skill inputs. Surprisingly, we find that capital market imperfections have opposing effects on the concentration of labor income and capital income. Labor income is more evenly distributed in the economy with imperfect capital markets than in the economy with no financial frictions (with a Gini index of .52 versus a Gini index .56 in the latter economy). This is because financial frictions depress the rents earn by highly able entrepreneurs relative to an economy with perfect capital markets . On the other hand, the Gini index of capital income is about

10 percentage points higher in the economy with imperfect credit markets. Financial frictions imply that the returns to managerial ability are positively correlated with capital income. The correlation between capital and labor income is equal to 0.80 in the baseline economy, which is much larger than the 0.50 value in the economy with perfect capital markets. This effect explains why income inequality is also more persistent in the economy with financial frictions.

The skill distribution matters importantly for the impact of financial frictions. This is shown by recalibrating two new economies in which the correlation between working and managerial skills is set exogenously to a high (0.8) and a low (-0.8) value (the calibration of the baseline economy implies a skill correlation of 0.1). We find that the output and TFP gains of improving credit market institutions are large in all economies but vary substantially across the three calibrated model economies. The output gains range from 36% to 55% and the TFP gains range from 22% to 31% as the correlation between skills decreases. The skill correlation determines the extent to which talented entrepreneurs are able to self-finance their businesses. When the correlation between these two skills is high, individuals that are talented as entrepreneurs are also talented as workers. Then, if skills are also persistent over time, young and talented individuals can work when young, build savings, and use their savings to finance their businesses when old. Thus, when managerial and working skills are highly correlated and persistent over time, the effects of financial frictions on resource allocations are less important than otherwise.

Given that financial frictions are so detrimental for the efficient allocation of productive resources and aggregate output, one question that rises is why countries set up institutions causing financial frictions and why these institutions are so persistent. Acemoglu and Robinson (2012) provided many historical accounts of how political power determines economic institutions and, in turn, how political power is shaped by the political institutions and the distribution of resources in society. Given that our theory of inequality was calibrated to Brazilian household data we can gain some insights into the political economy of capital market imperfections by studying the welfare gains and losses of reforming capital markets in our calibrated model economy. We assume that the economy is in steady state and that suddenly there is a once and for all reform that makes the enforcement of credit contracts perfect. We find that the financial reform has non-trivial effects on the distribution of income and that occupational choices are crucial for understanding how the reform impacts on individuals. Keeping fixed the occupational choices of the initial steady state, workers gain with the reform since the wage rate increase. Self-employed also gain since they can borrow more. Unconstrained employers loose since wage payments go up but their managerial rents

do not increase. Constrained employers may gain or lose depending on whether the increase in managerial rents outweighs the increase in wages. Since the increase in managerial rents is likely to be more important for talented entrepreneurs, these entrepreneurs are more likely to see their income increase. The untalented entrepreneurs who were operating businesses prior to the reform are likely to see their income go down and to switch occupations.

We find that while the vast majority of households gain from the reform, about 8.7% of the population see their welfare decrease with the reform. Households that lose from the reform tend to be older, richer, and exhibit higher managerial skills and lower working skills than households that support the reform. These findings are just reflecting that occupational choices are crucial for understanding the political economy of the reform: Among the households that are worse off with the reform, about 93% of them would have been entrepreneurs on the period of the reform had the reform not taken place, and 66% would have been employers. Employers are a positive selection from the population distribution of managerial skills. Then, the fact that about two thirds of those who oppose the reform are employers explains why the managerial ability of those supporting the reform is higher than that of those opposing the reform. Nonetheless, not all employers support the reform: About 36% of employers in the initial equilibrium benefit from the elimination of enforcement problems. We find that the employers benefiting from the reform tend to be of higher managerial ability than those who oppose it. The reason is that high ability employers are more likely to be borrowing constrained than low ability entrepreneurs. As a result, they are more likely to operate at an inefficient scale and to gain more from the elimination of enforcement problems. On the other hand, the financial reform hurts many of the lower skill employers and force them to change their occupation status: About 46% of the entrepreneurs that oppose to the reform and would have been employers had the reform not taken place, do not hire any labor after the reform (most of them become self-employed after the reform). The wage hike after the reform makes it unprofitable for these entrepreneurs to hire outside labor.

Summing up, while most households benefit from a reform that eliminates enforcement problems, the majority of employers (about two thirds) lose from the reform. By depressing the demand for labor, limited enforcement depresses the equilibrium wage rate, increasing the profits of employers. Our theory thus suggests that employers may have a vested interest in maintaining a status quo with low enforcement.

The paper continues as follows. Section 2 presents some facts on entrepreneurship and economic inequality in Brazil. Section 3 presents the model economy. Section 4 presents some analytical results characterizing how financial frictions affects occupational choice decisions,

rates of returns to skills, and inequality. Section 5 calibrates the model economy, evaluates the performance of the model economy, and assesses the effects of removing financial frictions on occupational choices, aggregate output, and income inequality. This section also discusses how changes in the correlation of skills affect the impact of financial frictions on aggregate output. The paper ends with a discussion of the political economy of removing financial frictions in Brazil.

## 2 Evidence

We now document some facts on occupations and economic inequality in Brazil that guide the theory developed in this paper. The facts are based on data from the *Pesquisa Mensal de Emprego* (PME) and from the *Pesquisa de Ornamentos Familiares* (POF). The former is a monthly household employment survey, with a similar structure to the US Current Population Survey (CPS). The latter is a survey of household consumption and income. Appendix A describes how the data set used in this paper was constructed.

**Income inequality** Figure 1 presents data on the variance of log-income over the life cycle from the PME (similar findings arise from the POF). First, note that the variance of log-income at age 20 is 0.55, which is much higher than the value of 0.30 documented by Storesletten et al. (2005) for the United States. Thus, households in Brazil are quite heterogeneous at young ages. As in the United States, inequality in income grows during the life cycle suggesting the presence of persistent shocks to household earnings. By age 55, household log-income reaches a value of 1.01.

**Occupational structure** We define the occupation of a household as that of the household head. We consider two broad occupations – workers and entrepreneurs. Moreover, we further subdivide the entrepreneurial occupation in two classes – employers and own account workers (self-employed). Figure 2 uses data from the ILO to analyze occupational structure in different countries. The blue bar on Figure 2 shows the proportion of workers, the orange bar the proportion of self-employed, and the green bar the proportion of employers.<sup>3</sup> The evidence shows that developed countries have lower amount of entrepreneurs than developing countries, but this data pattern is driven by the lower proportion of self-employed in developed countries. The proportion of employers in the population of households is quite similar among countries.

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<sup>3</sup>For Canada and the United States, the ILO does not distinguish between self-employed and employers so that the orange bar is the sum of the two.

While in Brazil workers represent about 73% of households, in Germany they are about 89%. The high proportion of entrepreneurial households in Brazil is explained by self-employed households which represent about 22% of the labour force in Brazil, much lower than the 6% of self-employed households in Germany. The fraction of employers is roughly equal across these two countries (about 5%). Employers and self-employed are quite different in their average income: On average employers earn about 3 times as much as self-employed households. Moreover, self-employed earn less than the average worker.

**Distribution of earnings by occupation** We show two graphs on the distribution of earnings by occupation: The first one shows the distribution of earnings for workers and entrepreneurs, the second one shows the earnings for entrepreneurs partitioned between self-employed and employers. Figure 3 shows that the distribution of earnings of entrepreneurs is flatter than the one of workers, having a bigger mass of people with low earnings but also a bigger mass of households with high earnings. Thus, earnings are more dispersed among entrepreneurs than workers. If we further divide the Entrepreneurs in Self-Employed and Employers we can see that the first group is the one that has more mass in the lower tail of earnings. Figure 4 shows that Self-Employment is the occupation with the lowest expected returns, while Employer is the one with the highest expected returns.

Summarizing, we draw the following lessons from the above facts:

1. Income inequality in Brazil is high relative to the US, which underscores the importance of calibrating the model to Brazilian micro data. Brazilian households are highly heterogeneous early in the life cycle and inequality grows substantially with age. These observations suggest the importance of modeling heterogeneity in fixed effects (permanent skill heterogeneity) as well as persistent shocks to skills.
2. The fact that both wages and entrepreneurial income are highly dispersed, motivates us to build a model with two dimensional skill heterogeneity.
3. It is important to build a theory that distinguishes between employers and self-employed entrepreneurs since most entrepreneurial households in Brazil are self-employed (or own-account workers) households and distribution of income differ substantially across both categories of entrepreneurs. While mean income of employers is much higher than that of self-employed households, there is substantial income heterogeneity within each of these occupational categories.

4. The variation in the rates of entrepreneurship between Brazil and rich countries is entirely explained by the high rates of self-employment in Brazil, a fact that existing theories of occupational choice cannot account for.

### 3 The Model

We consider a small open economy in steady state. The model features a one sector life-cycle growth model in which households are heterogeneous in two skills – working ( $z_w$ ) and managerial abilities ( $z_m$ ). Skills evolve stochastically over the life cycle and there are no insurance markets to insure ability risk. Production is organized by entrepreneurs who combine managerial, capital, and labor inputs. As in Lucas (1978), entrepreneurs can only use their own managerial skills since there is no markets for managers. In each period households choose their occupation: whether to work for a wage or to operate a business and become entrepreneurs. Occupational choices are based on their comparative advantage as entrepreneurs and their access to capital. Following Buera et al. (2011), access to capital is limited by their wealth through an endogenous collateral constraint that arises because of enforcement problems. In order to match important aspects of the Brazilian micro data, the Lucas (1978) model is extended to distinguish between two types of entrepreneurial households – employers and self-employed households.

**Population** The economy is populated by overlapping generations, each generation consisting of a continuum of households. Households are born at age 20, retire at age 60, and die with certainty at age 75. Each household is endowed with one unit of time at every age. Before the retirement age, households decide how much of their time to allocate to working ( $t_w$ ) or to managerial ( $t_m$ ) activities. Households differ in working ( $z_w$ ) and managerial ( $z_m$ ) abilities. The logarithm of skills evolve stochastically over the life cycle according to (household  $i$  at age  $t$ )

$$\begin{aligned} \ln(z_{wit}) &= \beta_w X_t + \alpha_{wi} + u_{wit}, \\ \ln(z_{mit}) &= \beta_m X_t + \alpha_{mi} + u_{mit}, \end{aligned}$$

where  $z_{wit}$  ( $z_{mit}$ ) denote the working (managerial) skills of household  $i$  at age  $t$ ,  $X_t$  represents a quartic polynomial of age,  $\alpha_{wi}$  and  $\alpha_{mi}$  represent household fixed effects on working and managerial productivities, and  $u_{wit}$  and  $u_{mit}$  are life cycle shocks received at age  $t$  by household  $i$ . We assume that the fixed effects are drawn from a bi-variate normal distribution at the first period of life of the household (age 20):

$$\alpha = (\alpha_{wi}, \alpha_{mi}) \sim N \begin{pmatrix} 0 \\ 0 \end{pmatrix}, \begin{pmatrix} 1 & \rho\sigma_w\sigma_m \\ \rho\sigma_w\sigma_m & \sigma_m^2 \end{pmatrix}$$

where  $\rho$  is the correlation between the two fixed effects across individuals. The mean fixed effect of the distribution of working skills is normalized to 0.

The life-cycle shocks follow the stochastic process

$$u_{jit} = \rho_j u_{jit-1} + E_{jit}, \text{ for } j = w, m,$$

with  $E_t = (E_{wt}, E_{mt})$  jointly drawn from a bivariate normal distribution with correlation coefficient  $\text{corr}(E_{wt}, E_{mt}) = \rho$ . We further assume that  $\alpha_{ji}$  and  $u_{jit}$  are mutually orthogonal.

The assumptions made imply that distribution of skills at age- $t$  is log-normally distributed

$$\begin{aligned} \begin{pmatrix} \ln(z_{wt}) \\ \ln(z_{mt}) \end{pmatrix} &\sim N \begin{pmatrix} 0 \\ 0 \end{pmatrix}, \begin{pmatrix} \sigma_{wt}^2 & \rho_{wmt}\sigma_{wt}\sigma_{mt} \\ \rho_{wmt}\sigma_{wt}\sigma_{mt} & \sigma_{mt}^2 \end{pmatrix} \\ \sigma_{wt}^2 &= \sigma_{\alpha_w}^2 + \sum_{j=0}^{t-1} (\rho_w)^j \sigma_{E_w}^2 \\ \sigma_{mt}^2 &= \sigma_{\alpha_m}^2 + \sum_{j=0}^{t-1} (\rho_m)^j \sigma_{E_m}^2 \\ \rho_{wmt}\sigma_{wt}\sigma_{mt} &= \text{cov}(\alpha_w, \alpha_m) + \sum_{j=0}^{t-1} \rho_w^j \rho_m^j \text{cov}(E_w, E_m) \\ \text{cov}(\alpha_w, \alpha_m) &= \rho\sigma_{\alpha_w}\sigma_{\alpha_m} \\ \text{cov}(E_w, E_m) &= \rho\sigma_{E_w}\sigma_{E_m} \end{aligned}$$

**Production technology** Following Lucas (1978), output is produced with a constant returns to scale production technology in managerial, labor, and capital inputs. Entrepreneurs can only use their managerial input because there is no market for managers. The supply of the managerial input is equal to the product of the households' managerial ability ( $z_m$ ) and the time devoted to managing a business ( $t_m$ ). The output produced by a household supplying  $m = z_m t_m$  units of managerial input and using  $k$  units of capital and  $n$  efficiency units of labor is:

$$Y(m, k, n) = m^\gamma k^\nu n^\theta, \text{ where } \gamma + \nu + \theta = 1. \quad (1)$$

The time allocation decision of entrepreneurs ( $t_m \in [0, 1]$ ) is modeled to introduced self-employment in the Lucas (1978) framework. When  $0 < t_m < 1$  entrepreneurs supply *both* managerial and labor inputs to their own businesses. Specifically, the labor input supplied

by entrepreneurs to their business is equal to the product of their working ability ( $z_w$ ) and the time devoted to non-managerial activities ( $1 - t_m$ ). The total labor input used by an entrepreneur is the sum of the labor supplied by the entrepreneur ( $(1 - t_m)z_w$ ) and the labor hired in the market ( $n^d$ ) from workers outside the family:

$$n = n^d + (1 - t_m)z_w, \quad (2)$$

where  $z_w$  is the working ability of the household. We denote as entrepreneurs the households that choose  $t_m > 0$ . Entrepreneurs, in turn, are partitioned in two subgroups depending on whether they hire outside labor or not. The first subgroup is given by the employers, who are those entrepreneurs hiring labor outside the family ( $n^d > 0$ ). We assume that entrepreneurs that hire outside labor incur a fixed per period operating cost of  $c_f$ .<sup>4</sup> The second subgroup are those entrepreneurs that only use their own household labor input ( $n = (1 - t_m)z_w$  and  $n^d = 0$ ). Workers are those households who use all their available time as workers ( $t_m = 0$ , obtaining labor earnings  $wz_w$ ).

Summarizing, entrepreneurs produce output with a production technology that combines capital, labor, and managerial inputs. The key distinguishing feature between employers and self-employed is that the latter do not hire labor outside the household and that employers pay a fixed cost in each period of business operation. They both solve a time-allocation problem regarding the fraction of their time endowment used to supply managerial versus working skills. Below, we shall characterize how entrepreneurs optimally choose the time ( $t_m$ ) dedicated to the supply of managerial skills.

**Capital markets** We assume that the financial intermediation industry is competitive. Intermediaries take deposits from households and pay the international interest rate  $r$ . They rent capital to entrepreneurs at a rate  $r + \delta$  and loan employers the fixed cost of operation  $c_f$ . Enforcement problems limit the amount of borrowing and the capital rented to entrepreneurs. Following Buera et al. (2011), entrepreneurs may renege on the contracts after production has taken place and keep a fraction  $1 - \varphi$  of undepreciated capital and the revenue net of labor payments ( $Y(m, k, n) - wn^d + (1 - \delta)k - c_f |_{n \geq 0}$ ) but lose the financial assets  $a$  deposited with the intermediary. Entrepreneurs that default regain access to the financial markets the following period. The parameter  $\varphi \in [0, 1]$  indexes the strength of the legal institutions in the economy, with  $\varphi = 1$  indicating perfect financial markets and  $\varphi = 0$  corresponding to an economy with no credit markets. We study equilibria in which financial contracts are restricted

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<sup>4</sup>The fixed cost is introduced so that employers demand a non-trivial amount of labor (an amount bounded away from zero), thereby making the distinction between self-employed and employer meaningful.

so that there is no default in equilibrium. This occurs when the amount of capital rented is limited by the largest amount  $\bar{k}(a, z_m, z_w; \varphi)$  consistent with entrepreneurs choosing to abide by their financial contracts. To characterize rental limits, consider the profit maximization problem of entrepreneurs that take as given the capital  $k$  used in the business operation:

$$\pi(z_m, z_w, a; k) \equiv \max_{m, n, n^d, t_m} \{m^y k^v n^\theta - wn^d - r(k - a) + a - \delta k - c_f I_{n > 0}\} \quad (3)$$

subject to

$$m = t_m z_m, \quad (4)$$

$$n = (1 - t_m) * z_w + n^d, \quad (5)$$

$$\text{where } t_m \in [0, 1], n^d \geq 0, k \text{ given.} \quad (6)$$

The following proposition extends results in Buera et al. (2011) to characterize the rental limits  $\bar{k}(a, z_m, z_w; \varphi)$ .

**Proposition 1** *Capital rental  $k$  by an entrepreneur with wealth  $a$  and skills  $(z_m, z_w)$  is enforceable if and only if*

$$\pi(z_m, z_w, a; k) \geq (1 - \varphi) \max_{m, n, n^d, t_m} \{m^y k^v n^\theta - wn^d + (1 - \delta)k - c_f I_{n > 0}\}$$

subject to

$$m = t_m z_m,$$

$$n = (1 - t_m) * z_w + n^d,$$

$$\text{where } t_m \in [0, 1], n^d \geq 0.$$

*The upper bound on capital rental that is consistent with entrepreneurs choosing to abide by their contracts can be represented by a function  $\bar{k}(a, z_m, z_w; \varphi)$ , which is increasing in  $a, z_m, z_w$  and  $\varphi$ .*

Proof. See appendix.

The income of an entrepreneur in state  $(z_m, z_w, a)$  making optimal production decisions given prices and borrowing limits is given by

$$y^e(z_m, z_w, a) \equiv \max_k \{\pi(z_m, z_w, a; k)\} \quad (7)$$

subject to

$$k \leq \bar{k}(a, z_m, z_w; \varphi) \quad (8)$$

The income of a household that choose to work for a wage is  $y^w(z_m, z_w, a) = wz_w + ra$ . Household income is the maximum between the entrepreneurial and workers income:

$$y(z_m, z_w, a) = \max\{y^e(z_m, z_w, a), y^w(z_m, z_w, a)\}. \quad (9)$$

Households maximize expected discounted lifetime utility

$$\begin{aligned} & \max_{c_j, a_{j+1}} E \left\{ \sum_{j=1}^J \beta^j U(c_j) \right\} \\ & \text{subject to} \\ & c_j + a_{j+1} = y(z_{mj}, z_{wj}, a_j), \\ & c_j, a_{j+1} \geq 0, \end{aligned}$$

## 4 Time Allocation and Occupational Maps

We now study in partial equilibrium (e.g. for a fixed wage rate) how our theory can give rise to three active occupational choices: workers, self-employed, and employers. We show that when capital markets are perfect occupational choices are entirely determined by the ability ratio  $\frac{z_w}{z_m}$ . Individuals with a high  $\frac{z_w}{z_m}$  ratio have a comparative advantage at working and choose to become workers, individuals with a low  $\frac{z_w}{z_m}$  ratio have a comparative advantage at entrepreneurship and choose to become employers, and those with intermediate skill ratios prefer to be self-employed. We also characterize how tight borrowing constraints (capital market imperfections) distort occupational choices.

We start by analyzing the determinants of self-employment income. Self-employed individuals choose how much time to allocate to managerial versus working activities and how much capital to use in production. Using the linear homogeneity of the production function, the income of a self-employed individual with  $a$  units of assets who uses  $k$  units of capital can be written as

$$y_{se} = MPt_m t_m + MPt_w t_w + MPK k + ra - k(r + \delta),$$

where  $MPt_m$  and  $MPt_w$  denote the marginal products of managerial time and working time, respectively, and  $MPK$  represents the marginal product of capital. We are now ready to prove the following proposition:

**Proposition 2 (Self-employment)** *The optimal time devoted to management by self-employed entrepreneurs (entrepreneurs not hiring outside labor) is  $t_m^* = \frac{Y}{Y+\theta}$ . The marginal*

product of their time is equated across its two uses (managerial and working time) and satisfies:

$$MPT_{se} = r_{mw} (z_m^{\gamma} z_w^{\theta})^{\frac{1}{\gamma+\theta}},$$

where  $r_{mw} = \gamma v^{\frac{\gamma}{1-\gamma}} \frac{\theta}{\gamma} \frac{1}{r+\delta+\mu} v^{\frac{\theta}{1-\gamma}}$  is the rate of return to the composite skill input  $(z_m^{\gamma} z_w^{\theta})^{\frac{1}{\gamma+\theta}}$  and  $\mu$  is the Lagrange multiplier associated to the borrowing constraint. The income of a self-employed individual with assets  $a$  is given by

$$y_{se} = r_{mw} (z_m^{\gamma} z_w^{\theta})^{\frac{1}{\gamma+\theta}} + \mu k + ra,$$

where  $k = \bar{k}(z_m, z_w, a)$ .

Proof. See appendix.

Proposition 2 establishes that self-employed individuals equate the marginal product of the time allocated to managing and to working tasks. The marginal product of the self-employment time can be expressed as the product of the skill composite  $(z_m^{\gamma} z_w^{\theta})^{\frac{1}{\gamma+\theta}}$  and the rate of return  $r_{mw}$ . The skill composite  $(z_m^{\gamma} z_w^{\theta})^{\frac{1}{\gamma+\theta}}$  is a geometric average of the managerial and working abilities of the self-employed individual. The return to the skill composite ( $r_{mw}$ ) depends on parameters of the production technology, the real interest rate ( $r$ ), and the Lagrange multiplier ( $\mu$ ) associated to the borrowing constraint. Note that the return to the skill composite decreases with ( $\mu$ ). Hence, borrowing constraints generate heterogeneity in rate of returns to skills among self-employed individuals.

Since workers' income is given by

$$y_w = w z_w + ra,$$

it is immediate that  $y_{se} - y_w$  is independent of asset holdings. Hence, as shown in Proposition 3, when  $\mu = 0$  the decision of whether to work for a wage or to be self-employed only depends on the ability ratio  $\frac{z_w}{z_m}$ . On the contrary, when the borrowing constraint binds  $k = \bar{k}(z_m, z_w, a)$  occupational choice decisions depend on asset holdings because they affect the rate of return to skills (e.g. the composite input  $(z_m^{\gamma} z_w^{\theta})^{\frac{1}{\gamma+\theta}}$ ) and the rate of return to assets. These results are summarized in Proposition 3.

**Proposition 3 (Self-employed vs Worker)** Let  $R_1 \equiv \left(\frac{r_{mw}}{w}\right)^{\frac{\theta+\gamma}{\gamma}}$ , where  $r_{mw}$  is defined in Proposition 2. Then,

1. If capital markets are perfect ( $\varphi = 1$ ), working for a wage is preferred to self-employment if and only if  $\frac{z_w}{z_m} > R_1$ .

2. If capital markets are imperfect ( $\varphi < 1$ ), working for a wage is preferred to self-employment if and only if

$$\frac{z_w}{z_m} > \frac{r_{mw} + \mu k / (z_m^\theta z_w^\theta)^{\frac{1}{\gamma+\theta}}}{w} \equiv R_1,$$

where  $\mu > 0$  is the Lagrange multiplier associated to the borrowing constraint and  $k = \bar{k}(z_m, z_w, a)$ .

When capital markets are perfect ( $\varphi = 1$ ), the occupational choice decision between working for a wage or being self-employed can be represented by a ray  $R_1$  that goes through the origin in  $(z_m, z_w)$  space. Individuals with ability above this ray prefer to be a worker. In this case, occupational choice decisions are independent of asset holdings and maximize the marginal product of time. However, when capital markets are imperfect, occupational choice decisions depend on asset holdings and do not maximize the marginal product of time. The occupational choice between working and self-employment is now described by the curve  $\bar{R}_1$  in  $(z_m, z_w)$  space. Note that a proportional change in both skills decreases the income ratio  $\frac{y_w}{y_m}$  because the increase in  $z_m$  leads to a tighter borrowing constraint for a fixed asset level  $a$ , implying that the curve  $\bar{R}_1$  tilts down relatively to the ray  $R_1$  as  $z_m$  increases. An increase in assets ( $a$ ) relaxes the borrowing constraint ( $\mu$  decreases and  $r_{mw}$  increases) making it more likely that individuals will choose self-employment so that the position of the  $\bar{R}_1$  depends on asset holdings.

We now analyze the decisions of employers. Employers choose how much of their time to allocate to managerial versus working activities and how much capital ( $k$ ) and (outside) labor services ( $n_d$ ) to use in production. Using the linear homogeneity of the production function the income of an employer with  $a$  units of assets can be written as

$$y_e = MP t_m t_m + MP t_w t_w + MP n_d n_d + MP K k + ra - k(r + \delta) - w n_d - c_f,$$

where  $MP t_m$  and  $MP t_w$  denote the marginal products of managerial time and working time, respectively, and  $MP K$  and  $MP n_d$  represent the marginal product of capital and labor services. We are now ready to prove the following proposition:

**Proposition 4 (Employers)**

1. The optimal time devoted to management by employers is  $t_m^* = \min \{ \hat{t}_m, 1 \}$ , where  $\hat{t}_m \equiv \left( \frac{z_m^v k(a, z_m, z_w; \varphi)^{\theta v}}{w} \right)^{\frac{1}{\theta}} \frac{1}{\theta z_w} \theta^{-1} \frac{1}{1-\theta-\gamma}$ . Moreover, denoting by  $k^u(z_m, z_w)$  the level of capital chosen by an unconstrained entrepreneur with ability  $(z_m, z_w)$ , there exist  $a^*(z_m, z_w) < k^u(z_m, z_w)$  such that  $t_m^* = 1$  for all  $a \geq a^*(z_m, z_w)$ .
2. The marginal product of employer's time satisfies:

$$MPT_e = z_m r_m \geq z_w w \text{ (with strict inequality if } t_m = 1 \text{),}$$

where  $r_m = \gamma \frac{r}{(r+\delta+\mu)^v} \left( \frac{z_m}{\theta} \right)^{\theta} \frac{1}{1-(v+\theta)}$  is the rate of return to the managerial input  $z_m$  and  $\mu$  is the Lagrange multiplier associated to the borrowing constraint.

3. The income of an employer with ability  $(z_m, z_w)$  with assets  $a$  is given by

$$y_e = z_m r_m + \mu k + ra - c_f,$$

where  $k = \bar{k}(z_m, z_w, a)$

Proof. See appendix.

Proposition 4 states that there is a threshold level of asset holdings  $a^*(z_m, z_w)$  such that for assets below this level the marginal product of entrepreneurial time is equal to  $wz_w$  and the time allocation problem of the employer features an interior solution in which the employer performs both managing and working activities. If asset holdings are higher than the threshold  $a^*(z_m, z_w)$ , then the marginal product of entrepreneurial time is higher than that as a worker and the time allocation problem exhibits a corner solution  $t_m = 1$ .

The marginal product of employers' time ( $MPT_m$ ) can be expressed as the product of managerial skills  $z_m$  and the rate of return  $r_m$  on the employer's managerial skill. The rate of return ( $r_m$ ) depends on parameters of the production technology, the real interest rate ( $r$ ), and the Lagrange multiplier ( $\mu$ ) associated to the borrowing constraint. Note that borrowing constraints ( $\mu$ ) generate heterogeneity in rate of returns to skills among employers.

Proposition 4 shows that when capital markets are perfect ( $\mu = 0$ ) then the marginal product of employer's time is proportional to her managerial ability  $z_m$  and the income difference between being an employer and being self-employed  $y_e - y_{se}$  is independent of asset holdings. In this case, Proposition 5 shows that the decision of whether to be an employer or to be self-employed only depends on the ability ratio  $\frac{z_w}{z_m}$ , provided the fixed cost of operation faced by employers is equal to zero ( $c_f = 0$ ). There exist a constant ratio  $R_2$  such that individuals

with a skill ratio  $\frac{z_w}{z_m}$  below  $R_2$  choose to be an employer. When the fixed cost of operation is positive, then the occupational choice decision depends on comparative advantage (skill ratio  $\frac{z_w}{z_m}$ ) and on the absolute level of managerial ability  $z_m$ . Now, to be an employer rather than self-employed the ability vector  $(z_m, z_w)$  should satisfy  $\frac{z_w}{z_m} < R_2 \left(1 - \frac{c_f}{z_m r_{mw}}\right)^{\frac{v+\theta}{\theta}}$ . Intuitively, in the presence of fixed cost of being an employer, employers need a minimum level of managerial ability  $z_m$  in order to recoup the fixed cost of operation. The occupational choice decision between employer and self-employment is not only based on the skill ratio.

When capital markets are imperfect and borrowing constraints bind, occupational choice decisions depend on asset holdings because both the marginal product of time and the return to capital of both employed and self-employed individuals depend on their asset holdings (see Proposition 5). Intuitively, an increase in asset holdings increases the employer region in the occupational map in  $(z_w, z_m)$  relative to the self-employment ( $\bar{R}_2$  in Proposition 5 shifts up). The key is that borrowing constraints tend to be tighter for employers than self-employed since employers need to operate at a larger scale.

**Proposition 5 (Employer versus Self-employment)** Let  $R_2 \equiv \frac{r_m}{r_{mw}}^{\frac{\theta+v}{\theta}}$ , where  $r_{mw}$  and  $r_m$  are the rate of returns to the skill composite  $(z_m^v z_w^\theta)^{\frac{1}{v+\theta}}$  and the managerial skill defined in Propositions 2 and 4, respectively. Then,

1. If capital markets are perfect ( $\varphi = 1$ ), individuals prefer to become employers relative to self-employment when the ability ratio is such that  $\frac{z_w}{z_m} < R_2 \left(1 - \frac{c_f}{z_m r_{mw}}\right)^{\frac{\theta+v}{\theta}}$ .
2. If capital markets are imperfect ( $\varphi < 1$ ), individuals prefer to become employers relative to self-employment when the ability ratio is such that

$$\frac{z_w}{z_m} < \left( \frac{r_m}{r_{mw}} + \frac{(\mu_e k_e - c_f)}{z_m r_{mw}} - \frac{(\mu_{se} k_{se})}{z_m r_{mw}} \right)^{\frac{\theta+v}{\theta}} \equiv \bar{R}_2,$$

where  $\mu_e$  and  $\mu_{se}$  are the Lagrange multipliers associated to the borrowing constraints when the individual is an employer or is self-employed, respectively, and  $k_e$  and  $k_{se}$  are the capital used in production at these occupations.

Proposition 6 collects results characterizing occupational choice decisions when capital markets are perfect ( $\varphi = 1$ ). If the fixed cost of operation of employers is  $c_f = 0$ , occupational choices are only determined by the ability ratio  $\frac{z_w}{z_m}$ . Depending on parameter values (equilibrium returns to ability), the equilibrium may feature self-employed individuals or not. If equilibrium prices are such that  $R_1 > R_2$ , then individuals with an ability ratio  $\frac{z_w}{z_m} > R_1$

work for a wage, individuals with  $R_1 > \frac{z_w}{z_m} > R_2$  are self-employed, and those with  $\frac{z_w}{z_m} < R_2$  are employers. A positive fixed cost of operation ( $c_f > 0$ ), implies that employers require a minimum scale in order to operate a profitable business so that the decision to be an employer depends both on the skill ratio  $R_1 > \frac{z_w}{z_m} > R_2$  and on the level of managerial ability  $z_m$ .

**Proposition 6 (Occupational maps when capital markets are perfect ( $\varphi = 1$ ))**  
*Assume that  $\varphi = 1$ . Let  $R_1 \equiv \frac{r_{mw}}{z_m^{\theta+\nu}}$  and  $R_2 \equiv r \frac{m}{r_{mw}} \frac{\theta+\nu}{\theta}$ , where  $r_{mw}$  and  $r_m$  are the rate of returns to the skill composite  $\left(\frac{z_w^\nu z_m^\theta}{z_m^\nu z_m^\theta}\right)^{\frac{1}{\nu+\theta}}$  and the managerial skill defined in Propositions 2 and 4, respectively.*

1. *If there are no fixed cost of operation of being an employer ( $c_f = 0$ ), then the optimal occupational choice is the one that maximizes the marginal product of time and is only determined by the skill ratio ( $\frac{z_w}{z_m}$ ) as follows:*
  - (a) *If equilibrium prices are such that  $R_1 > R_2$ , then individuals with an ability ratio  $\frac{z_w}{z_m} > R_1$  work for a wage, individuals with  $R_1 > \frac{z_w}{z_m} > R_2$  are self-employed, and those with  $\frac{z_w}{z_m} < R_2$  are employers.*
  - (b) *If equilibrium prices are such that  $R_1 < R_2$ , there is no self-employed individuals in equilibrium. Individuals with a skill ratio such that  $z_w w < z_m r_m$  choose to become employers. Otherwise, they choose to work for a wage.*
2. *If employers incurred a positive fixed cost of operation ( $c_f > 0$ ), the decision to be an employer depends on the skill ratio ( $\frac{z_w}{z_m}$ ) and on the absolute level of managerial ability ( $z_m$ ). Individuals prefer to become employers relative to self-employment when the ability ratio is such that  $\frac{z_w}{z_m} < R_2 \left(1 - \frac{c_f}{z_m r_{mw}}\right)$ .*

Summarizing, we have developed a theory with three occupational choices and characterize occupational decisions. The theory implies that, in the absence of capital market imperfections, the skill ratio  $\frac{z_w}{z_m}$  drives occupational choices: Workers have a high  $\frac{z_w}{z_m}$  ratio, employers a low  $\frac{z_w}{z_m}$  ratio, and the self-employed have an intermediate skill ratio. Capital market imperfections distort returns to skill and, thus, occupational choices. A tight borrowing constraint depresses the rate of return to the managerial ability of employers and the return to the composite skill input supplied by self-employed individuals. It also increases the rate of return to capital faced by entrepreneurs. As a result, asset holdings matter importantly for occupational choice decisions in the presence of financial frictions. These results can be illustrated

by drawing the occupational maps in our calibrated model economy. Figure 10 draws the occupational map when perfect enforcement is introduced in the calibrated model economy. As shown in Proposition 6, when capital markets are perfect occupational choices are determined by the  $\frac{z_m}{z_w}$  skill ratio. In our baseline economy, with limited enforcement ( $\varphi < 1$ ), occupational choices are determined by the skill ratio and asset holdings since borrowing constraints affect the returns to skills and assets. Figure 11 represents graphically, for two fixed asset levels, how occupation varies across individuals that differ on  $(z_m, z_w)$ . In Panel a, the level of assets is fixed at the median income and in Panel b it is fixed at the mean income. A comparison of the occupational maps, reveal that capital market imperfections expand the region where self-employment is optimal at the expense of the regions where employer and worker are the preferred occupational choices.

## 5 Quantitative Analysis

### 5.1 Calibration

We partition the parameters in the model economy in two. The first group includes the parameters that are set using estimates from other studies in the literature. The second group consists of all the parameters that are calibrated by simulating the model economy.

**Parameters set exogenously** The model period is set to an year. The international interest rate is set at 3%. The utility function is assumed to be of the CES type:

$$u(c) = \frac{c^{1-\sigma}}{1-\sigma}$$

with  $\sigma = 1.5$ . The parameters of the production function are set to standard values in the literature:  $\gamma = .2$ ,  $\nu = .3$ ,  $\theta = .5$  (see Guner et al. (2008), Buera et al. (2011)). The annual depreciation rate is set to at  $\delta = 0.06$ .

**Calibrated parameters** For ease of exposition, below we list the parameters to be calibrated together with a corresponding target that helps identify each parameter. Nonetheless, it is important to keep in mind that the calibration is a multidimensional mapping in which all parameters and calibration targets are inter-related.

1. The discount factor  $\beta$  is chosen so that the capital to income ratio in the steady is equal to 2.4, which is consistent with the capital to income ratio in Brazil (see Júnior et al. (2004)).

2. Enforcement of credit contracts  $\varphi$  to match a credit to GDP ratio of 43% in Brazil<sup>5</sup>.
3. The coefficients on the quartic polynomial on age determining how the two working and managerial skills vary with age are set so that the age-profile of mean earnings for workers and entrepreneurs are roughly consistent with the data.
4. Following Storesletten et al. (2005), the parameters determining the stochastic process on working ability such as the variance of fixed effects  $\sigma_{\alpha_w}^2, \sigma_{\alpha}^2$ , persistence of autoregressive process  $\rho_w$ , and the variance of the innovation to working ability over the life-cycle  $\sigma_{E_w}^2$  to match the age profile of the variance of log wages.
5. There are various parameters determining the stochastic process on managerial ability (i) the variance of fixed effect on managerial skills  $\sigma_{\alpha}^2$ ; (ii) variance of innovations to managerial abilities ( $\sigma_{E_m}^2$ ); and (iii) the persistence of the auto-regressive process on managerial ability ( $\rho_m$ ). To pin down these parameters, we target: (i) the proportion of entrepreneurs and workers in the population of households (32% versus 68%); (ii) the variance of entrepreneurial log-earnings (1.06); and (iii) the persistence of being an employer between two consecutive years (68%).
6. The parameter  $\rho$  driving the correlation between managerial and working skills is pinned down by targeting the ratio of median earnings between entrepreneurs and workers.
7. The fixed cost of operation of employers  $c_f$  is set to match the fraction of employers among entrepreneurs (one fourth).

**Discretization of shocks** To solve the model numerically, we first find a finite state approximation of the following bivariate process describing the life-cycle shocks to skills

$$u_t = Au_{t-1} + E_t,$$

where  $u_t$  is a  $2 \times 1$  vector,  $A$  is a  $2 \times 2$  matrix, and  $E$  is a  $2 \times 1$  vector with mean 0 and variance-covariance matrix  $\Sigma = E(EE')$ . Using that  $\Sigma$  is a symmetric matrix, we can express it as follows:

$$\Sigma = Q\Lambda Q', \quad (10)$$

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<sup>5</sup>We use the average Private Credit/GDP from 2003 until to 2010 from the World Development Indicators from the World Bank

where  $\Lambda$  is a diagonal matrix (with the eigenvalues of  $\Sigma$  in the diagonal) and  $Q$  is the matrix of eigenvectors of  $\Sigma$  as columns. The bivariate process can be expressed

$$u_{t-1} + E_t$$

where

$E_t = QE_t$ . The key to this transformation is that  $E_t$  has a Markov chain with states given by a matrix  $Q$  with dimension  $2 \times 100$ . Then the states of the Markov chain which approximate  $u_t$  are given by the matrix  $U = QQ'$  with dimension  $2 \times 100$ . Because of our life-cycle environment, the variance of shocks grow with age. To deal with this feature, we allow the support of the shocks and the Markov chain to change with age. The Markov chain is allowed to vary with age so that the finite state approximation of the autoregressive bivariate process matches the unconditional variance of the continuous bivariate shock process at each age.

Regarding fixed effects, the bivariate normal distribution is discretized with 3 values for working skill and 5 values for managerial skills. As a result, there are 15 pairs of fixed effects. At each age, there are 1500 possible pairs of skills  $(z_w, z_m)$ .

## 5.2 Calibration results

We now discuss how the calibrated model economy matches the calibration targets. The values of the calibrated parameters are reported in Table 1.

Table 2 shows that the model economy matches reasonably well the targets for the credit to GDP ratio of 43% and the capital to income ratio of 2.4. Figure 5 compares the variance of log-earnings of workers in the model economy with the Brazilian data. The model economy is consistent with the fact that there is a large amount of inequality early in the life cycle and that inequality grows substantially with age over the life cycle. The stochastic process on working skills is characterized by a high persistence ( $\rho_w$  close to 1), which is needed to match the linear age-profile of the variance of log wages in the Brazilian data. This is consistent with the findings of Storesletten, Telmer, and Yaron (2005) for the US. Relatively to previous findings for the US economy, the calibration requires a large variance of individuals fixed effects ( $\sigma_{\alpha_w}$ ) to match the high inequality of wages at age 20 in Brazil.

The calibration implies that the variances of fixed effects and of the innovation of managerial skills are much larger than the corresponding variances of working skills (see Table 1). This is necessary for the model economy to be consistent with the large variance of entrepreneurial earnings in the Brazilian data.

Table 3 compares the fraction of households that are workers, self-employed, and employers in the calibrated model economy and in Brazil. The model economy matches quite closely the fractions of workers (68%), self-employed (24%), and employers (8%) in the data. The calibration also matches the fact that about 70% of the employers in Brazil at a given point in time are still employers one year after (see Table 3). In the calibrated model economy the persistence of entrepreneurial shocks is high ( $\rho_m = 0.78$ ), but less than the persistence of shocks on working ability ( $\rho_w = 0.98$ )

The calibration implies that the correlation between skills is positive but moderate ( $\rho = 0.1$ ). This is necessary for the economy to be consistent with the fact that ratio of median income between entrepreneurs and workers is equal to 1 in the Brazilian data. In a sensitivity analysis, we later calibrate two economies in which we exogenously set a high and low skill correlation. We find that a high skill correlation implies a counterfactually high ratio of median income between entrepreneurs and workers. Conversely, a calibration with a negative skill correlation implies that the median income of entrepreneurs are below that of workers, which is also counterfactual.

Summing up, we believe that the calibrated model economy provides a reasonable account of income inequality and occupational choices in Brazil.

### **5.3 Performance of baseline economy.**

We now discuss how the baseline economy matches some facts on occupational transitions and on the distribution of income across occupations that were not directly targeted in the calibration.

Table 4 reports predictions of the model economies on occupational transitions between the three occupations considered (worker, self-employed, employer). While there are 9 possible occupational transitions, we remind the reader that the calibration only targeted the persistence of being an employer between two consecutive years. The baseline economy matches the patterns on the persistence of occupational choices remarkably well (see Table 4). First, consistently with the data, the model economies predict that being a worker is quite persistent: 90% of workers in the model economy are workers one year later. This percentage is about 94% in the data. Both in the baseline economy and in the data, entrepreneurs are less likely to remain in their occupation than workers. Second, examining transition rates within the entrepreneurial class, the calibrated model economy matches the fact that individuals are much more likely to transit into employer from self-employment than from being a paid worker. In the data, individuals are about three times more likely to become employers if they

are self-employed rather than working for a wage. Indeed, the (annual) transition rate from self-employment into employer is 22% while the transition rate from worker to employer is only 8%. In the model these rates are 26% and 8%, respectively. Third, the (annual) transition rates out of employer in the data implies that employers are much more likely to switch to self-employment (8%) than to paid work (1%). In the model economy, these transition rates are 6% and 1%, respectively.

The model economy was calibrated to match two statistics on the distribution of income across workers and entrepreneurs (ration of median income and the variance of income of each of the two occupations). Figure 6 plots the distribution of income across these two occupation categories in the data and in the model. The model was calibrated to match the fact that the ratio of median income between entrepreneurs and workers is equal to one and that the variance of entrepreneurial income is higher than that of workers. Figure 7 plots the distribution of income across three occupations (e.g. the entrepreneurial category is subdivides in two groups: employers and self-employed). The baseline economy is consistent with the fact that there is substantial income heterogeneity in all three occupations. Moreover, as in the data, the distribution of self-employment income is shifted to the left relative to that of workers and the distribution of worker's income is shifted to the left relative to that of employers. Nonetheless, we emphasize that there is substantial heterogeneity among employers: Some low income employers make less income than the median self-employed individual.

All in all, the model economy matches reasonable well patterns on occupational transitions and distribution of income within and across occupations. While we could have built a theory with three distinct skills (one for each occupation) to better match the facts, it is remarkable that our theory with heterogeneity in two skills provides an excellent account of the patterns in the data on the transition rates into and out of the employer occupation. Below, when performing a sensitivity analysis, we evaluate other implications of the theory.

#### **5.4 Experiment: Removing financial frictions.**

In order to assess the effects of credit market institutions, we compute equilibrium in the baseline economy under the assumption of perfect credit-enforcement institutions ( $\varphi = 1$ ). We focus on how capital markets impact on occupational choices, resource allocation, aggregate output, and the distribution of income in the economy.

### 5.4.1 Occupational structure and financial frictions.

We find that removing financial frictions has important consequences on the occupational structure in the economy. Indeed, the fraction of entrepreneurs decreases from 33% to 18% (see Table 5). Moreover, most of the decrease in entrepreneurship is due to a large decrease in the rate of self-employment: While self-employment rates drop from 24% to 11%, the fraction of employers drops by about 2 percentage points. Altogether, the theory is consistent with key stylized facts on changes in the occupational structure with economic development. Consistently with the data, the theory that the changes in rates of entrepreneurship across rich and poor countries is due to changes in the self-employment rate. The theory is also consistent with the fact that the fraction of workers in the labor force tends to increase with economic development: It increases from 68% in the baseline economy to 82% with perfect capital markets.

The mechanism through which capital markets impact on the occupational structure is through its impact on equilibrium wages. When financial frictions are removed, the wage rate increases due to a better allocation of productive resources (employers are better selected and they can use more capital). The increase in the wage rate strongly discourages self-employment, as can be seen by comparing the self-employment region (green region) in the occupational maps drawn in Figure 10 and Figure 11. Note that capital market imperfections expand the region where self-employment is optimal at the expense of the regions where employer and worker are the preferred occupational choices.

### 5.4.2 Output, resource allocation, and financial frictions.

The gain in aggregate output of eliminating financial frictions is quite large: It amounts to an increase of 48% (see Table 6). There is also a substantial change in the sectorial composition of output. While production by employers increase by 64%, production by self-employed decreases by 53%. This should not be surprising given that the removal of financial frictions leads to large reduction in the rates of self-employment in the baseline economy.

We now consider the impact of financial frictions on the efficiency of production. To this end, we compute (TFP) as the (input-weighted) average productivity with which the composite capital and labor input is used in production across entrepreneurs

$$TFP = \frac{\int_{i \in E} m_i^\gamma r_i^{k_i^\nu} n_i^\theta di}{\int_{i \in E} k_i^\nu n_i^\theta di}$$

where  $m_i = (z_{mi} t_i)^\gamma$  is the managerial input used by entrepreneur  $i$  and similarly for  $n_i$  and  $k_i$ . We find that aggregate TFP increases by 24% when financial frictions are removed. TFP

increased by 9% and 16% among self-employed and employers, respectively. Note that the aggregate TFP gain (24%) is higher than the gain in each of the two sectors (9% and 16%). The reason is that the removal of financial frictions leads to a reallocation of capital from self-employed to employers: The share of aggregate capital used by the self-employed drops from 19% to 4%. Because employers tend to have a higher managerial ability than self-employed entrepreneurs, the reallocation of capital between these two groups enhances the aggregate productivity gains of removing financial frictions.

Having said that self-employed are less productive entrepreneurs than employers, it is important to recognize that self-employment reduces the impact of financial frictions on output. To assess how self-employment affects the impact of financial frictions on aggregate output, we shut down self-employment in the baseline economy and simulate the removal of financial frictions. We find that the output gain is 53%, which is higher than the 48% increase obtained when simulating the removal of financial frictions in the baseline economy. Hence, self-employment decreases the negative impact of financial frictions on aggregate output. This finding can be explained as follows: Financial frictions make it hard for young and talented entrepreneurs (individuals with high managerial skills) to raise external funds. This effect is compounded by the fact that financial frictions depress the equilibrium real wage, which makes it difficult to accumulate savings by working for a paid wage when young. Self-employment allows talented entrepreneurs to circumvent the low wage and build up savings, diminishing the negative impact of financial frictions on aggregate output.

It is also interesting to decompose the gains in TFP due to the removal of financial frictions between the ones that are due to a better allocation of capital and labor inputs across entrepreneurs (intensive margin) versus the gains coming from a better selection of entrepreneurs (extensive margin). To this end, we re-distribute the capital in the baseline economy in order to equate the marginal product of capital across all entrepreneurs.<sup>6</sup> We find that capital reallocation leads to a TFP gain of 1.3% for the self-employed and of 12% for employers (see Table 7). Hence, reallocation of capital accounts for 14% of the gains in among the self-employed and for 76% of the gains among employers. At the aggregate level, reallocation accounts for about half of the TFP gains (54%).

The result that the intensive margin accounts for a much larger fraction of the TFP gains

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<sup>6</sup>While the marginal product of the labor input is equated across employers, this equality does not hold across self-employed individuals because we assume that labor input is not movable across the self-employed. Alternatively, we could have allowed self-employed to pay a fixed cost in order to equate the marginal product of labor. However, this would effectively imply an occupational switch (from self-employer to employer) which we consider as part of the extensive margin.

of employers than of the self-employed, suggests that borrowing constraints are tight among the former but not among the latter. Figure 8 plots the histogram of the ratio of the marginal product of capital relative to the gross interest rate ( $r + \delta$ ) across employers and self-employed individuals. Note that this ratio is equal to one for about 85% of self-employed, indicating that the vast majority of the self-employed are not borrowing constraint. The reason is that self-employed individuals tend to operate their businesses at a much smaller scale than employers (their mean managerial input is about one-twentieth the one of employers) and do not need to borrow much capital. On the other hand, the marginal product of capital varies substantially across employers and the vast majority of them face a binding borrowing constraint.

### 5.4.3 Impact of financial frictions on the distribution of income.

Financial frictions affect the distribution of income in many ways. While the overall effect on income inequality is not large<sup>7</sup>, it has important effects on the sources of income inequality and on its persistence over time.

We start by analyzing how financial frictions affect the distribution of capital versus non-labor income. Capital income is computed as  $ra + \mu k$ . Non-capital income is computed as the sum of labor income, managerial rents, and self-employment rents. Abusing terminology from now on we refer to non-capital income as labor income. Table 14 compares the Gini indexes of capital and labor income across economies. We find that capital income is much more unevenly distributed than labor income both in the baseline economy and in the perfect capital market economies. Surprisingly, we find that capital market imperfections have opposing effects on the concentration of labor and capital income. Labor income is more evenly distributed in the baseline economy than in the economy with perfect enforcement, with a Gini index of .52 in the former economy and of .56 in the latter economy). On the other hand, the Gini index of capital income is about 10 percentage points higher in the baseline economy. The opposite effects of capital market imperfections on the distributions of capital income and labor income offset each other and account for the small change in the Gini index of income.

The fact that the distribution of factor income varies so much across economies is symptomatic of the resource misallocation prevalent under imperfect capital markets. The low concentration of the distribution of labor income in the baseline economy is due to the fact that borrowing constraints distort rate of returns to managerial ability (recall that  $\mu > 0$  reduces  $r_m$ ). Moreover, in the baseline economy returns to managerial ability  $r_m$  and managerial ability  $z_m$  are strongly negative correlated, with a correlation coefficient of  $-0.5$ . Thus, skill-

<sup>7</sup>The Gini index of income decreases from 0.53 to 0.52 with the removal of financial frictions

ful managers tend to receive low returns to their ability. On the other hand, the correlation coefficient between these two variables is zero in the economy with perfect capital markets, as there is no heterogeneity in rate of returns to ability.

Capital income is highly unequal in the baseline economy because there is substantial heterogeneity in the returns to capital. The interest rate on deposits (3%) is substantially smaller than the average marginal product on capital obtained by employers (13.2% net of depreciation). Moreover, the marginal product of capital across employers varies importantly and its distribution features a coefficient of variation above .60. Again, this fact is symptomatic of resource being inefficiently allocated.

The presence of borrowing constraints imply that the returns to managerial ability are positively correlated with capital income. Hence, the correlation between capital and labor income is equal to .80 in the baseline economy, which is much larger than the .50 value in the economy with perfect capital markets. In the latter, the positive correlation between capital and labor income is due to the fact that highly able people tend to hold more capital than low ability people but not to rate of return differentials.

#### 5.4.4 Capital markets and the persistence of income.

To evaluate the effect of imperfect capital markets on the persistence of income, we simulate data in the baseline economy and in the economy with perfect capital markets and run the following regression in

$$\log(y_{t,j}) = \alpha_j + \beta \log(y_{t-1,j}) + b_2 \text{age}_t + b_3 \text{age}_t^2, \quad (11)$$

where  $y_{tj}$  represents the income of individual  $j$  at age  $t$ ,  $\alpha_j$  is an individual fixed effect, and  $\beta$  measures the persistence of log-income. We find that removing financial frictions in the baseline economy reduces the estimated value of  $\beta$  from 0.81 to 0.74. Income is more persistent in the baseline economy because assets are positively correlated with rate of returns and because assets matter for occupational choices. On the other hand, when there is perfect enforcement assets do not affect rates of returns and occupational choices and the persistence of income is only driven by the persistence of shocks and asset holdings.

#### 5.4.5 Capital markets and the distribution of consumption.

Financial frictions have an heterogeneous impact across households. To assess the distributive impact of financial frictions, Table 15 compares consumption inequality in the baseline model economies with that in an economy with perfect enforcement of credit contracts  $\varphi = 1$ . We find

that financial frictions have important effects on the distribution of consumption. We find that the Gini coefficient of consumption is 2 percentage points lower in the economy with perfect enforcement of credit contracts ( $\varphi = 1$ ). It is interesting that financial frictions have opposite effects on the inequality at the top and the bottom of the consumption distribution. The ratio of consumption between the 10th percentile and the 50th percentile of the consumption distribution is equal to 0.29 in the baseline model economy. This ratio increases to 0.33 in the economy with  $\varphi = 1$ . On the other hand, the consumption ratio between households at the 90th and 50th percentile increases from 3.3 to 3.44. Hence, relative to the perfect credit economy, the baseline model economy has more inequality at the bottom of the consumption distribution but less inequality at the top. The first effect is more important than the latter effect so that overall consumption inequality, as measured by the Gini index, is higher in the baseline economy than in the  $\varphi = 1$  economy.

## **5.5 Is the skill distribution important for the aggregate effects of financial frictions?**

To show that the skill distribution matters importantly for the impact of financial frictions we consider two new economies in which we fixed exogenously the skill correlation. In one economy, the skill correlation is fixed to a high positive value ( $\rho = 0.8$  and in the other to a high negative value ( $\rho = -0.8$ ). The two economies are re-calibrated to the same targets of the baseline economy, except for the fact that we do not target the ratio of median earnings between entrepreneurs and workers. We then compare across economies the impact of removing financial frictions.

### **5.5.1 The impact of financial frictions across economies.**

The output gains of removing financial frictions vary substantially across the three calibrated model economies (see Table 10).<sup>8</sup> The output gains range from 36% to 55% as the correlation between skills decreases from 0.8 to -0.8. Similarly, the TFP gains across these economies range from 22% to 31%. Hence, financial frictions have much lower effects on output per worker and TFP when skills are (strongly) positively correlated. The correlation between skills matters for the impact of financial frictions because it shapes the correlation between savings and managerial talent in equilibrium. In particular, the financing problems faced by

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<sup>8</sup>The match of the calibrated targets is better in the baseline economy. The calibrated parameters and calibration results for the economies with very positive or very negative correlation are available upon request.

talented entrepreneurs are less severe when entrepreneurs are also talented workers. When individuals are highly skillful both at managing and at working, they can work for a wage and build savings in order to diminish the negative effects of borrowing constraints on their entrepreneurial income. On the other hand, talented entrepreneurs find it more difficult to accumulate savings when skills are negatively correlated, making borrowing constraints tighter. Moreover, talented entrepreneurs have a harder time building up savings relative to other individuals with lower entrepreneurial talent because the latter have higher working skills. By allowing individuals with low managerial talent to build savings faster, financial frictions provide low skilled managers with a comparative advantage at entrepreneurship. These effects explain why the skill correlation matters importantly for the output and TFP losses caused by financial frictions.

A key innovation of our theory is that it allows entrepreneurs to choose what fraction of their time they allocate to managing versus working. This assumption allows our theory to be consistent with the fact that self-employment is quite important in poor countries. We now show that self-employment matters for the quantitative impact of financial frictions in the three calibrated model economies. To this end, we shut down self-employment in all economies and simulate the impact of financial frictions. We find that in all the calibrated model economies the output gains due to the elimination of financial frictions increase substantially in the absence of self-employment (by between 6 to 8 percentage points). Since wages are low with financial frictions, self-employment allows individuals with high managerial skill to attain higher earnings. This effect is most important in the economy with negative correlation of skills since in this economy individuals with high managerial ability tend to have lower working ability.

To further understand how the skill correlation matters for the impact of financial frictions, we analyze how financial frictions distort the rate of returns to the various production inputs. Recall that when capital market are perfect the rate of return of all productive inputs are equalized across production units. However, rates of return do vary across production units under financial frictions (see Section 4). We now show that the skill correlation parameter  $\rho$  matters importantly for the variation in rate of returns caused by financial frictions. Table 12 compares the variation in rates of returns among employers and self-employed individuals in the calibrated model economies. The standard deviation of the marginal product of capital among employers is twice as large in the economy with  $\rho = -0.8$  than in the economy with  $\rho = 0.8$  (.14 versus .07). The variation in rates of returns to capital reflects the variation in the

tightness of the borrowing constraint across entrepreneurs.<sup>9</sup> The results in Table 12 show that when skills are positively correlated there is less heterogeneity in rates of return on capital across entrepreneurs than when skills are negatively correlated. Intuitively, the financing problems faced by talented entrepreneurs are less severe when entrepreneurs are also talented workers. This is because households with high working skills can rapidly accumulate savings and alleviate the financial constraints that limit the operation of their businesses. On the other hand, when skills are negatively correlated borrowing constraints are tighter because talented entrepreneurs find it more difficult to accumulate savings.

Borrowing constraints also generate heterogeneity on the rate of return to the managerial input among employers ( $r_m$ ) and on the rate of return on the self-employment composite ( $r_{mw}$ ). Both of these returns decrease with the tightness of financial constraints (see Section 4). Table 12 shows that the variation in rates of return to the managerial input among employers is about twice as large in the economy with  $\rho = -0.8$  than in the economy with  $\rho = 0.8$  (0.31 versus 0.17.) Moreover, while in all economies the return to the managerial skill is negatively correlated with the level of managerial ability, this correlation is the lowest in the economy with  $\rho = -0.8$  (about  $-0.63$ ). When skills are strongly negatively correlated, the tight borrowing constraints faced by entrepreneurs with high managerial skills imply that they obtain a lower return to their skills than less able entrepreneurs. In this case financial frictions generate a strong comparative advantage at entrepreneurship for households with lower managerial talent but higher working ability, reducing the average entrepreneurial ability, and total factor productivity. Table 9 shows that changes in TFP associated with the elimination of financial frictions range from 31% to 22%, with the largest (lowest) increase attained in the economy with strongly negative (positive) correlation of skills.

### 5.5.2 Comparing economies with different correlation of skills

We have shown that the impact of financial frictions vary substantially across the three calibrated model economies. Then, in order to assess the impact of financial frictions in the Brazilian economy it is important to use Brazilian data to test the predictions of the calibrated model economies.

**Skill correlation and distribution of income across occupations.** Figure 12 shows that the calibrated model economies differ importantly in the distribution of income by occupation (workers versus entrepreneurs). The economy with strongly correlated shocks  $\rho = 0.80$

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<sup>9</sup>Recall that the marginal product of capital can be expressed as  $MPK = r + \delta + \mu$ , where  $\mu$  represents the Lagrange multiplier associated to the borrowing constraint (see Section 4).

is grossly at odds with the Brazilian data: It counterfactually predicts that the distribution of earnings of workers is shifted to the left relative to that of entrepreneurs. The economy with strongly negative correlated shocks  $\rho = -0.80$  is also at odds with the Brazilian data since it implies that the distribution of earnings of workers is shifted to the right relative to that of entrepreneurs. On the other hand, our baseline economy (with  $\rho = 0.10$ ) fits the Brazilian evidence on the income distribution across occupations reasonably well. Indeed, the baseline economy was calibrated to match the ratio of median income between workers and entrepreneurs (which is 1.0 both in Brazil and in the baseline model economy). This statistic takes a value of 0.7 in the economy with  $\rho = 0.8$  and a value of 1.3 when  $\rho = -0.8$ .

Figure 13 compares the distribution of income in the calibrated model economies and Brazil when the population is divided in three occupational groups (workers, self-employed, and employers). The economy with strongly correlated shocks counterfactually predicts that the earnings distribution of self-employed individuals is shifted to the right relative to that of workers. As  $\rho$  decreases, the distribution of earnings of self-employed individuals shifts to the left. As a result, consistently with the evidence, the economies with  $\rho = 0.10$  and  $-0.80$  exhibit a distribution of earnings of self-employed households that is shifted to the left relative to that of workers. Overall, the economy with  $\rho = .10$  is the one that fits the evidence best. Relative to the data, the economy with a strong negative skill correlation implies that the self-employed individuals have too low earnings relative to workers.

We now discuss why the correlation between skills matters importantly for the income distribution across occupations. When  $\rho$  is sufficiently high, the correlation between  $\ln(z_{mt}/z_{wt})$  and  $\ln(z_{wt})$  becomes positive. In this case, a high skill ratio  $z_{mt}/z_{wt}$  is also associated with high values of  $z_{mt}$  and  $z_{wt}$  so that households that have a *comparative advantage* at managing (high  $z_{mt}/z_{wt}$ ) also have an *absolute advantage* in both skills. When entrepreneurs have an absolute advantage in both occupations, highly skilled workers tend to have a comparative advantage at managing and choose the entrepreneurial occupation. Low skill workers do not have a comparative advantage at managing and choose to work for a wage. As a result, the earnings distribution among entrepreneurs is shifted to the right relative to the earnings distribution among workers and earnings inequality between occupations is large. On the other hand, when the correlation between  $\ln(z_{mt}/z_{wt})$  and  $\ln(z_{wt})$  is negative households in one occupation tend to be better at that occupation than households choosing the other occupation. Earnings inequality across occupations is not as large as in the absolute advantage case.

It is easy to show that the skill ratio and the working skill are jointly log-normally dis-

tributed for each age  $t$ :

$$\begin{pmatrix} \ln(z_{mt}/z_{wt}) \\ \ln(z_{wt}) \end{pmatrix} \sim N \begin{pmatrix} 0 \\ 0 \end{pmatrix}, \begin{pmatrix} \sigma_{2nt} + \sigma_w^2 & \rho_{wmt}\sigma_{wt}\sigma_{mt} - \sigma_w^2 \\ \rho_{wmt}\sigma_{wt}\sigma_{mt} - \sigma_w^2 & \sigma_{mt}^2 \end{pmatrix}$$

The absolute advantage case arises when the correlation between  $\ln(z_{mt}/z_{wt})$  and  $\ln(z_{wt})$  is positive, which holds if and only if

$$\rho_{wmt} > \frac{\sigma_{wt}}{\sigma_{mt}}$$

Thus, the correlation of skills have to be sufficiently strong for the absolute advantage case to hold. Figure 9 graphs the correlation between  $\ln(z_{mt}/z_{wt})$  and  $\ln(z_{wt})$  for the three calibrated model economies. The economy with  $\rho = 0.80$  is the only one with a positive correlation. In this economy, households with high managerial ability tend to have an absolute advantage in skills (have higher managerial and working skills). The economy with  $\rho = 0.10$  exhibits a correlation between  $\ln(z_{mt}/z_{wt})$  and  $\ln(z_{wt})$  of roughly  $-0.30$ .

**Skill correlation and the persistence of earnings.** The correlation between skills also matter for the persistence of earnings over time. Intuitively, earnings are less volatile when skills are positively correlated than negatively correlated. To compare the persistence of earnings across the calibrated model economies, for each economy we simulate artificial data and run the regression in (11). We find that the persistence of log-income increases from 0.73 to 0.84 as  $\rho$  increases from  $-0.8$  to  $0.8$ . Unfortunately, we do not have panel data from Brazil to estimate the persistence of income in Brazil. Nonetheless, we can use consumption data from Brazilian households to test the predictions of the theory. The idea is that consumption theory implies that permanent income is a key determinant of consumption decisions. Hence, the higher the persistence of income the higher should be the cross-sectional correlation between consumption and income across households (e.g. the correlation between consumption and income at a given date  $t$ ). Table 13 shows that the correlation between consumption and income varies widely across the calibrated model economies: from 0.24 when  $\rho = -0.8$ , to .79 when  $\rho = 0.1$ , and up to 0.85 when  $\rho = 0.8$ . This correlation is 0.71 in the Brazilian data.

**Skill correlation and selection of entrepreneurs.** The skill correlation matters importantly for how entrepreneurs are selected from the wage distribution. This is shown by comparing wages at a given point in time between those who become entrepreneurs in the following period relative to those who did not. In particular, this is done by running the following regression on log wages:

$$\log(y_{t-1}) = b_0 + b_1 * age + b_2 age^2 + b_3 * entrepreneur_t,$$

where  $y_{t-1}$  denotes wage income in period  $t - 1$  and  $entrepreneur_t$  is a dummy that takes value of 1 if the individual becomes an entrepreneur in period  $t$ . This regression is run for the three calibrated model economies as well as for the Brazilian data.<sup>10</sup>

We find that in the Brazilian data, entrepreneurs are drawn negatively from the wage distribution (the coefficient  $b_3$  is negative and equal to -0.13). The baseline economy also implies a negative coefficient with a value of  $-0.36$  (see Table 11). The economy with  $\rho = 0.8$  a positive value of  $b_3$ . The economy with strongly negative correlated skills implies a too low value for  $b_3$  (-0.86).

To investigate whether self-employed are differently selected from the wage distribution relative to employers, we run the wage regression above but allowing for a dummy for self-employment. We then consider another specification in which we allow for a dummy to indicate those workers who switch into employer. We find that in the Brazilian data self-employed tend to be negatively selected from the wage distribution ( $b_3 = -0.24$ ) but that employers tend to be positively selected ( $b_3 = 0.43$ ). The baseline economy ( $\rho = 0.1$ ) is consistent with these patterns (the dummy for self-employment is -0.43 and the one for employers is 0.50).

Altogether, the Brazilian evidence reviewed supports the economy with a moderately positive correlation between managerial and working skills ( $\rho = 0.1$ ).

## 5.6 The political economy of financial frictions

While it is well understood that financial frictions can have a large negative impact on aggregate output and total factor productivity, it is less clear why the institutions leading to poor property rights and contract enforcement are so persistent. In this section we use our theory to gain some insights into the political economy of financial frictions. In our theory, financial frictions have an heterogeneous impact across individuals that differ on age, wealth, and skills. Since our model economy was calibrated to Brazilian household data, we use our theory to assess the distribution of welfare gains of eliminating financial frictions in the Brazilian economy.

We assume that the baseline economy ( $\rho = 0.10$ ) is in steady state and that suddenly and unexpectedly there is a once and for all institutional reform that increases  $\varphi$  to 1. On impact, the wealth distribution does not matter for occupational choice decisions. Now, the skill ratio  $\frac{z_m}{z_w}$  is the only determinant of occupational choice decisions. Workers who could not operate as entrepreneurs because of binding borrowing constraints can now start a business.

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<sup>10</sup>While the Brazilian data is not a panel, we have data on the occupational choices of individuals one year apart.

Moreover, entrepreneurs who were initially borrowing constraint see their managerial rents ( $r_m z_m$ ) and entrepreneurial income increase (see the discussion in Section 4). The importance of this effect varies across individuals: It is more important for talented entrepreneurs (high  $z_m$ ) because, *ceteris paribus*, they were more likely to be initially constrained. As talented entrepreneurs raise their demand of capital relative to less talented entrepreneurs, capital is reallocated towards more productive entrepreneurs, increasing the demand for labor, and the equilibrium wage rate. The rise in wages decrease the profits of entrepreneurs. Hence, employers' income can go up or down depending on whether the increase in managerial rents is higher or lower than the increase in labor costs.

The financial reform has non-trivial effects on the distribution of income. Occupational choices of individuals are crucial for understanding how the reform impacts on them. Keeping fixed the occupational choices of the initial steady state, workers gain with the reform since the wage rate increase. Self-employed also gain since they can borrow more. Unconstrained employers loose since wage payments go up but their managerial rents do not increase. Constrained employers may gain or loose depending on whether the increase in managerial rents outweighs the increase in wages. Since the increase in managerial rents is likely to be more important for talented entrepreneurs, these entrepreneurs are more likely to see their income increase. The untalented entrepreneurs who were operating businesses prior to the reform are likely to see their income go down and to switch occupations.

We now explain how we compute the distribution of welfare gains from the reform. The small open economy assumption simplifies the computation significantly. Once the reform takes place, the marginal product of capital will be equated across entrepreneurs and will be equal to the international interest rate plus the depreciation rate of capital. On impact, competition for workers will drive the wage rate to its new long run value, which increases on impact by about 40%. While the distribution of wealth, consumption, and income may change for some periods after the reform, all macroeconomic aggregates (capital, GDP, wage rate) will be constant after the initial period of the reform. Since there are no transitional dynamics in the macroeconomic aggregates, we can then compute the distribution of welfare gains for all individuals alive at the moment of the reform as follows:

1. Simulate the distribution of households across states  $s = (age, assets, z_m, z_w)$  from the initial steady state prior to the reform.
2. For each household in state  $s$ , compute the permanent consumption compensation in the original steady state that will let the household attain the same utility as in the perfect credit economy. Denoting by  $V^{baseline}(s)$  the discounted lifetime utility of a household

in the baseline economy, and  $V^{\varphi=1}(s)$  the value function in the perfect enforcement economy, the consumption compensation  $\lambda(s)$  is computed as follows:

$$\lambda(s) = \left( \frac{V^{\varphi=1}(s)}{V^{baseline}(s)} \right)^{\frac{1}{1-\sigma_c}} - 1,$$

where  $\sigma_c$  denotes the curvature of the period utility function in consumption ( $\sigma_c = 1.5$ ). Households with  $\lambda(s) > 0$  gain from the elimination of enforcement problems. Households with  $\lambda(s) < 0$  see their welfare decrease with the reform of financial market institutions.

We find that the average welfare gain among households alive at the period of the institutional reform is 16.5%. The standard deviation of the distribution of welfare gains is 13.5%. Figure 14 shows the distribution of welfare gains across the population and documents that there is substantial heterogeneity. While the vast majority of households gain from the reform, about 8.7% of the population see their welfare decrease with the reform. Who are the households that lose with the reform?

Figure 15, 16, 17, 18, show the age, asset, and managerial-skill distributions among those who oppose and support the reform. We find that households that lose from the reform tend to be older, richer, and exhibit higher managerial skills and lower working skills than households that support the reform. These findings are just reflecting that occupational choices are crucial for understanding the political economy of the reform: Among the households that are worse off with the reform, about 93% of them would have been entrepreneurs on the period of the reform had the reform not taken place, and 66% would have been employers.

Employers are a positive selection from the population distribution of managerial skills. Then, the fact that about two thirds of those who oppose the reform are employers explains why the managerial ability of those supporting the reform is higher than that of those opposing the reform. Nonetheless, not all employers support the reform: About 36% of employers in the initial equilibrium benefit from the elimination of enforcement problems. We find that the employers benefiting from the reform tend to be of higher managerial ability than those who oppose it. The reason is that high ability employers are more likely to be borrowing constrained than low ability entrepreneurs. As a result, they are more likely to operate at an inefficient scale and to gain more from the elimination of enforcement problems. On the other hand, the financial reform hurts many of the lower skill employers and force them to change their occupation status: About 46% of the entrepreneurs that oppose to the reform and would have been employers had the reform not taken place, do not hire any labor after the reform

(most of them become self-employed after the reform). The wage hike after the reform makes it unprofitable for these entrepreneurs to hire outside labor.

Summing up, while most households benefits from a reform that eliminates enforcement problems, the majority of employers (about two thirds) lose from the reform. By depressing the demand for labor, limited enforcement depresses the equilibrium wage rate, increasing the profits of employers. Obviously, entrepreneurs as a group will benefit even more by forming a cartel in order to restrict labor demand and depress the wage rate. This achieves the goal of depressing the wage rate but without distorting the capital markets. However, this cartel agreement is not incentive compatible as each entrepreneur will have incentives to violate the group agreement and hire labor. Importantly, limited enforcement is an incentive feasible mechanism that leads to a depress wage rate. Our theory thus suggests that employers may have a vested interested in maintaining a status quo with low enforcement.

## **6 Conclusions**

We develop a theory of entrepreneurship, financial frictions, and economic inequality in Brazil. The key innovation of our theory is to extend the Lucas (1978) model in order to incorporate heterogeneity in two skills: – working and managerial skills. This modeling feature is necessary for the theory to be consistent with evidence on the income distribution across occupations in Brazil. Moreover, the theory has the novel implication that some entrepreneurs will not hire any outside labor and be own account workers (or self-employed entrepreneurs), which is consistent with the cross-country evidence that the high rates of entrepreneurship in poor countries, such as in Brazil, is mostly due to the prevalence of self-employed workers. We show that financial frictions have a non-trivial impact on inequality: On the one hand, they lead to higher and persistent inequality by generating variation in returns to skills and by making these returns depend on asset holdings. On the other hand, since borrowing constraints tend to be tighter for highly skill than for low skill individuals, financial frictions reduce inequality by diminishing the rents obtained by highly skilled individuals relative to a situation with perfect capital markets.

The model economy is used to simulate the effects of removing financial frictions in Brazil. We find that the changes on the occupational structure are large: The fraction of entrepreneurs decreases from 33% to 18%, with the bulk of the decrease in entrepreneurship explained by the large decrease in the rate of self-employment. The gains in aggregate output of removing financial frictions amounts to 48%, with asymmetric effects in the sectorial composition of

output: While production of employers increase by 64%, production of the self-employed decreases by 53%. We also simulate the impact of removing financial frictions when self-employment is shut down in the baseline economy. We find that the output gain is 53%, which is higher than the 48% increase obtained in the baseline economy. Hence, self-employment decreases the negative impact of financial frictions on aggregate output. We find that capital market imperfections have opposing effects on the concentration of labor income and capital income. Labor income is more evenly distributed in the economy with imperfect capital markets than in the economy with no financial frictions (with a Gini index of .52 versus a Gini index .56 in the latter economy). This is because financial frictions depress the rents earned by highly able entrepreneurs relative to an economy with perfect capital markets. On the other hand, the Gini index of capital income is about 10 percentage points higher in the economy with imperfect credit markets.

We evaluate the distribution of welfare gains and losses of eliminating financial frictions. While the vast majority of households gain from a reform that eliminates capital market imperfections, about 8.7% of the population see their welfare decrease with the reform. Households that lose from the reform tend to be older, richer, and exhibit higher managerial skills and lower working skills than households that support the reform. These findings are just reflecting that occupational choices are crucial for understanding the political economy of the reform: Among the households that are worse off with the reform, about 93% of them would have been entrepreneurs on the period of the reform had the reform not taken place, and 66% would have been employers. Our theory thus suggests that employers in Brazil may have a vested interest in maintaining a status quo with low enforcement.

Table 1: Calibrated Parameters

	$\rho_{w,m} = 0.1$
$\rho_w$	0.98
$\rho_m$	0.78
$\sigma_{\epsilon,w}^2$	0.38
$\sigma_{\epsilon,m}^2$	1.59
$\sigma_w^2$	0.03
$\sigma_m^2$	0.99
$C_f$	0.10
$\varphi$	0.23
$\beta$	0.995

Table 2: Calibration Results-Model Aggregates

	Data	Model
K/Y	2.4	2.3
Credit/GDP	43%	42%
Var Log(Earn)-Entrepreneurs	1.1	1.2

Table 3: Calibration Results-Occupational Structure

Fraction	Data	Model
Workers	68%	67%
Self-Employed	24%	24%
Employers	8%	9%
Emp to Emp	70%	68%

Table 4: Performance of the Model-Transitions

Transitions	Data	Model
W to W	94%	90%
SE to W	5%	9%
E to W	1%	1%
W to SE	14%	28%
SE to SE	78%	64%
E to SE	8%	8%
W to E	8%	6%
SE to E	22%	26%
E to E	70%	68%

In the table above we use W for Workers, SE for Self-Employed and E for Employers.

Table 5: Occupational Structure and Financial Frictions

Occupation	Baseline Economy	No Frictions
Workers	67%	82%
Self-Employed	24%	11%
Employers	9%	7%

Table 6: Aggregate Effects of Removing Financial Frictions

Changes in %	Output	TFP
Aggregate	48	24
Self-Employed	-53	9

Employers 64 16

Table 7: TFP and Financial Frictions

Changes in %	TFP	$TFP_{Reallocation}$
Aggregate	24	13 (54%)
Self-Employed	9	1.3 (14%)
Employers	16	12 (76%)

Table 8: Income Inequality and Financial Frictions

	$\varphi = 0.23$	$\varphi = 1$
Gini Labor Income	0.52	0.56
Gini Capital Income	0.67	0.59
Corr (cap inc., lab inc.)	0.80	0.50
Persistence of inc.	0.81	0.74

Table 9: Changes in Output-From benchmark to  $\varphi = 1$

	$\rho = 0.1$
Output Change (%) - Baseline	48
Output Change (%) - No Labor Heterogeneity	48
Output Change (%) - No Self-Employed	54

Table 10: Changes in Output-From Benchmark to  $\varphi = 1$

	$\rho$		
	-0.8	0.1	0.8
Output Change (%) - Baseline	55	48	37
Output Change (%) - No Labor Heterogeneity	47	48	41
Output Change (%) - No Self-Employed	63	54	44

Table 11: Skill Correlation  $\rho$  and the Selection of Entrepreneurs

	$\rho$		
	-0.8	0.1	0.8
Output Change (%) - Baseline	55	48	37
Output Change (%) - No Labor Heterogeneity	47	48	41
Output Change (%) - No Self-Employed	63	54	44

Table 12: Statistics on returns for different occupations and TFP gains

	$\rho_{w,m} = 0.1$
TFP gains (%)	15
Employers	
Std. Dev. $MPK_e$	0.10
Std. Dev. $c_e$	0.19
$corr(c_e, z_m)$	-0.55
Self-Employed	
Std. Dev. $MPK_{se}$	0.06
Std. Dev. $c_{se}$	0.07
$corr(c_{se}, z_m)$	-0.30

For  $\varphi = 1$  the standard deviation is 0 for all variables

$c_e$  is the return to managerial input for employers

$c_{se}$  is the return to the composite input for self-employed

$MPK_e, MPK_{se}$  are the marginal product of capital for employers and self-employed respectively

Table 13: Implications of  $\rho$  for earnings and consumption

	Data	Model
Ratio Median Earnings Worker to Entrepreneur	1.0	1.0
$corr(earnings_t, earnings_{t-1})$	N.A.	0.81
$corr(earnings_t, consumption_t)$	0.71	0.79

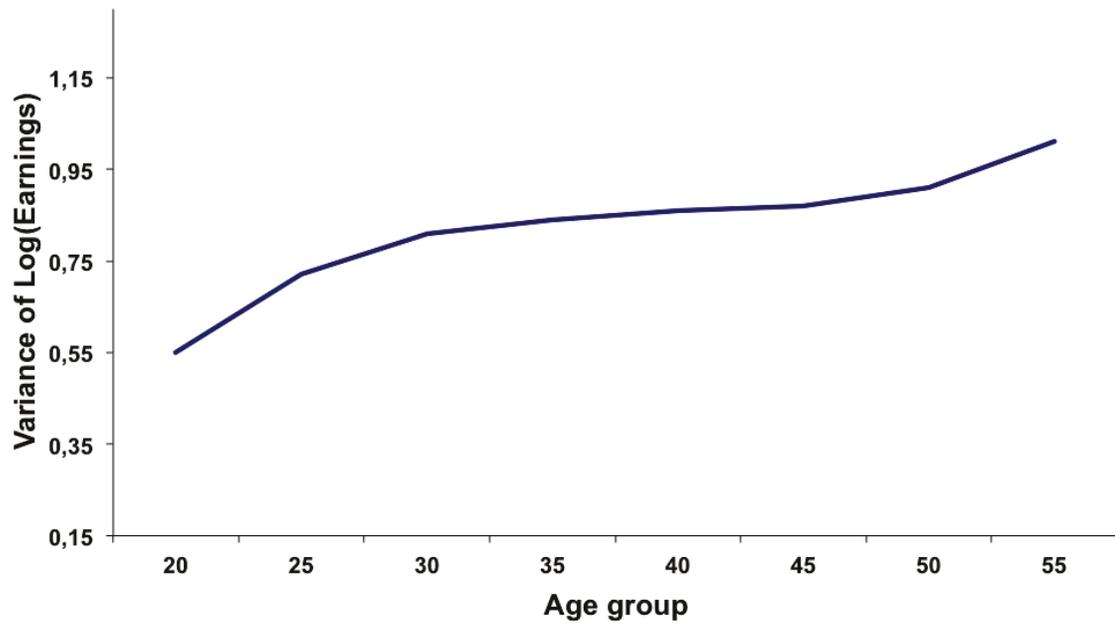
Table 14: Gini Inex-Labor and Capital Income

	Gini Index	
	$\varphi = 0.23$	$\varphi = 1$
Labor Income	0.52	0.56
Capital Income	0.67	0.59

Table 15: Consumption Inequality and Financial Frictions

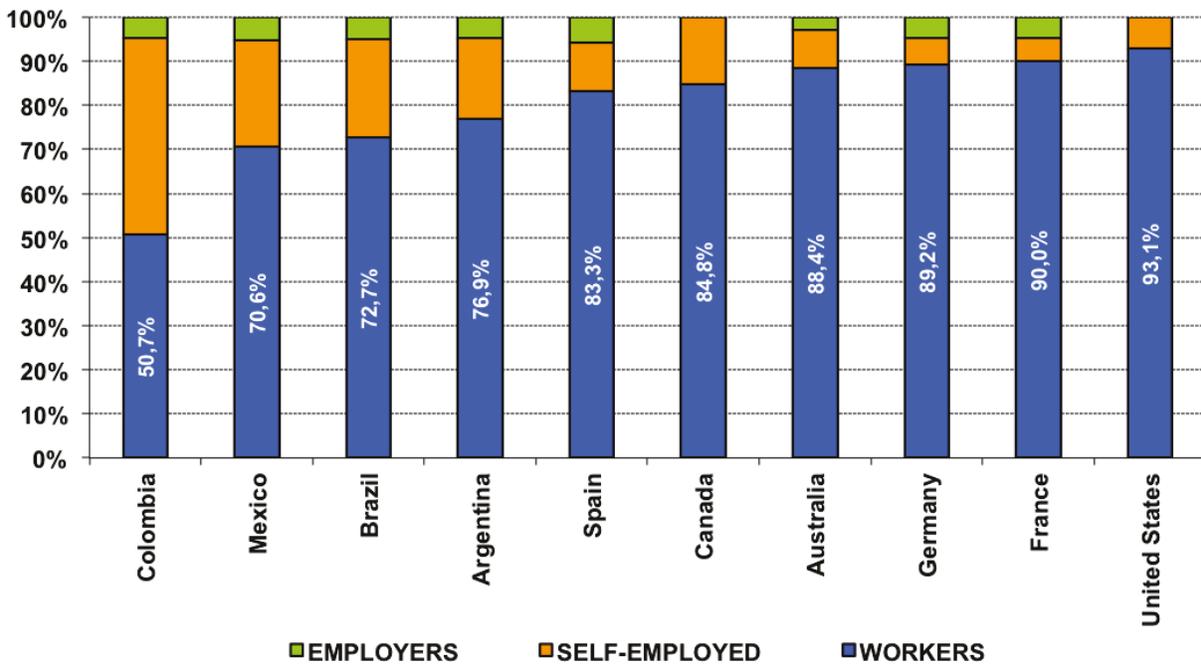
	$\varphi = 0.23$	$\varphi = 1$
Gini	0.50	0.48
p90/p10	11.5	10.33
p90/p50	3.29	3.44
p10/p50	0.29	0.33

Figure 1: Variance of Log Earnings in Brazil



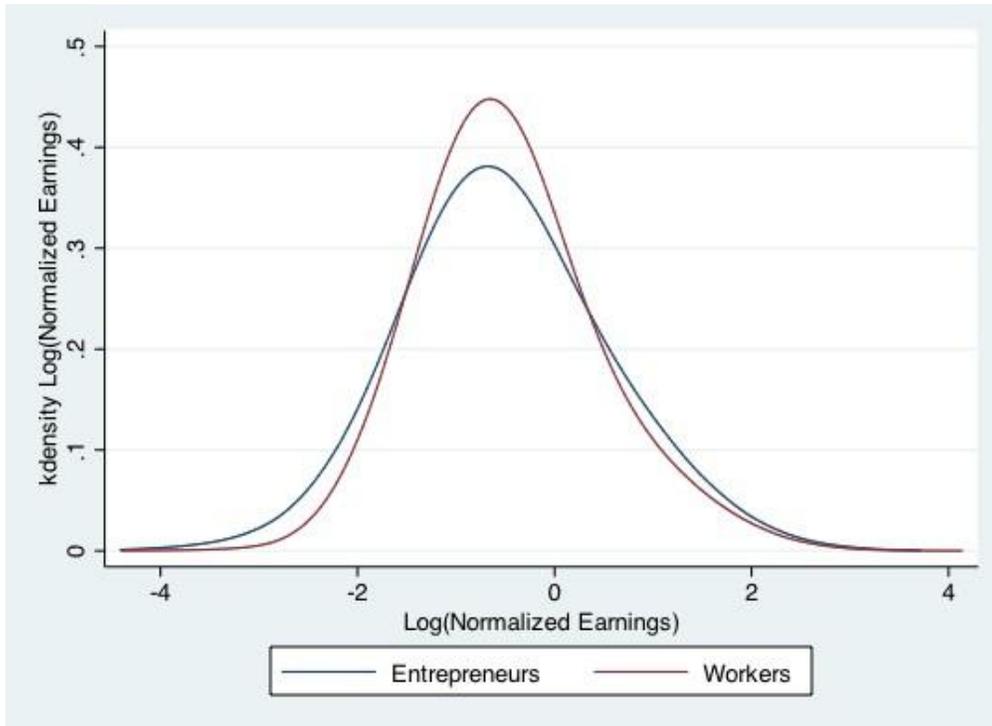
Source: Author's Elaboration based on PME 2003-2010

Figure 2: Occupational Structure across Countries



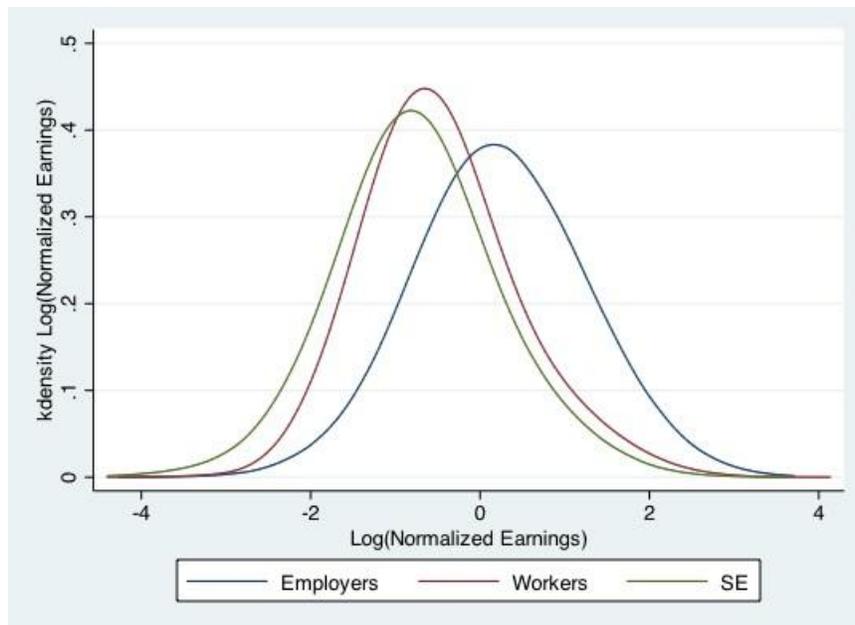
Source: Author's Elaboration based on ILO 2008

Figure 3: Distribution of Earnings in Brazil by Occupation-I



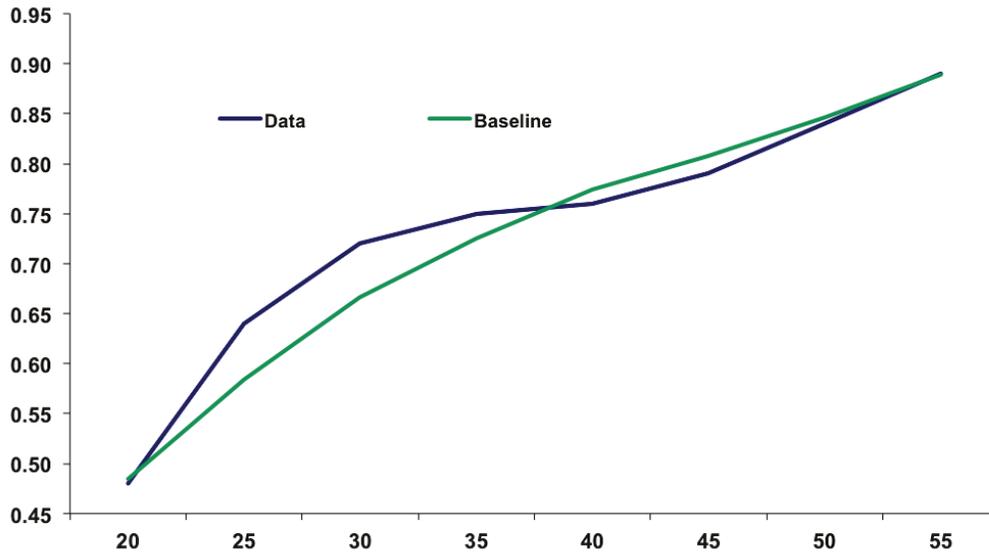
Source: Author's Elaboration based on PME 2003-2010

Figure 4: Distribution of Earnings in Brazil by Occupation-II



Source: Author's Elaboration based on PME 2003-2010

Figure 5: Variance of Log(Earnings)-Model vs Data



Source: Author's Elaboration based on PME 2003-2010

Figure 6: Distribution of Earnings-Data vs Model I

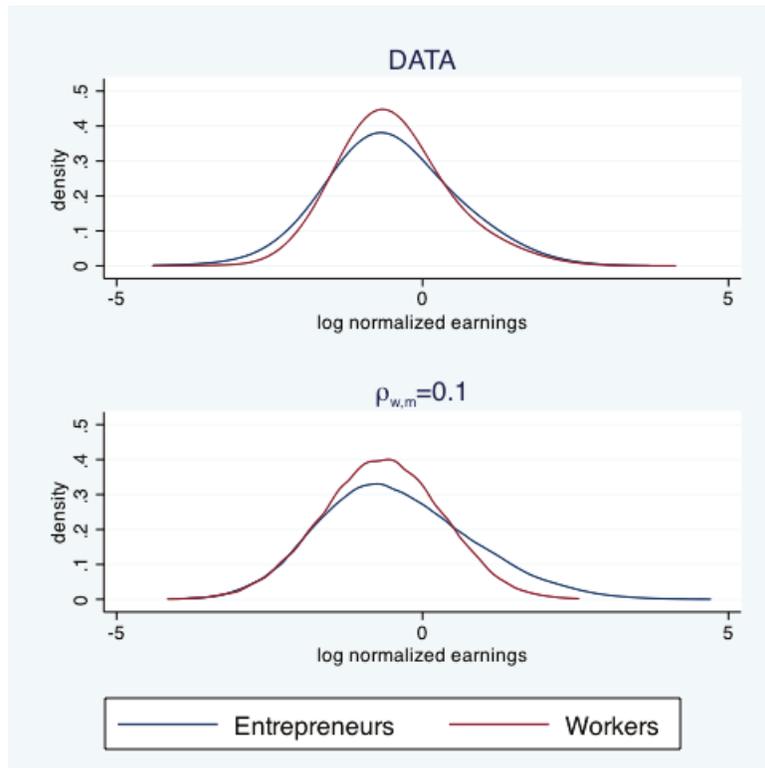


Figure 7: Distribution of Earnings-Data vs Model II

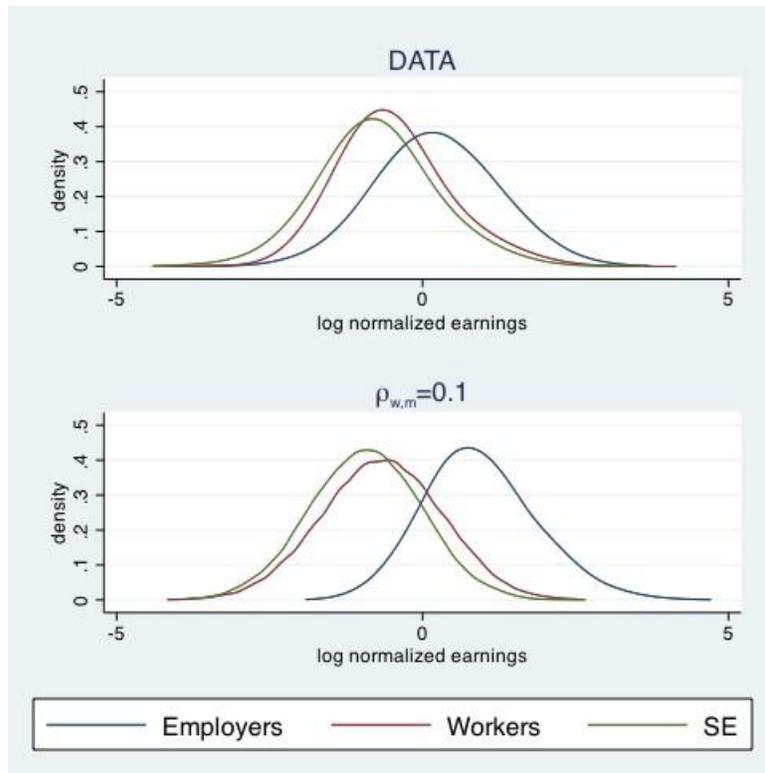


Figure 8: MPK and Market Return on Capital

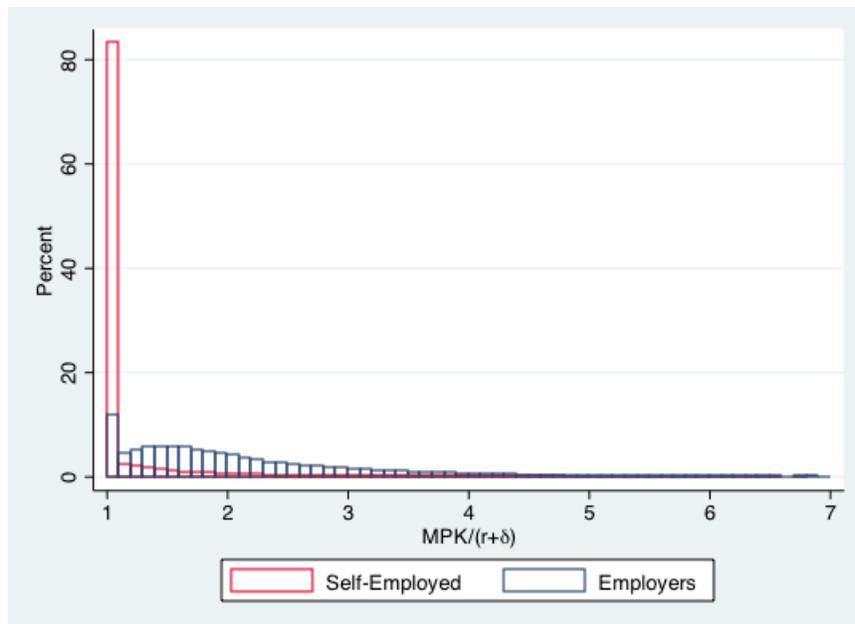


Figure 9: Correlation between  $\frac{z_m}{z_w}$  and  $z_w$  for different  $\rho_{w,m}$

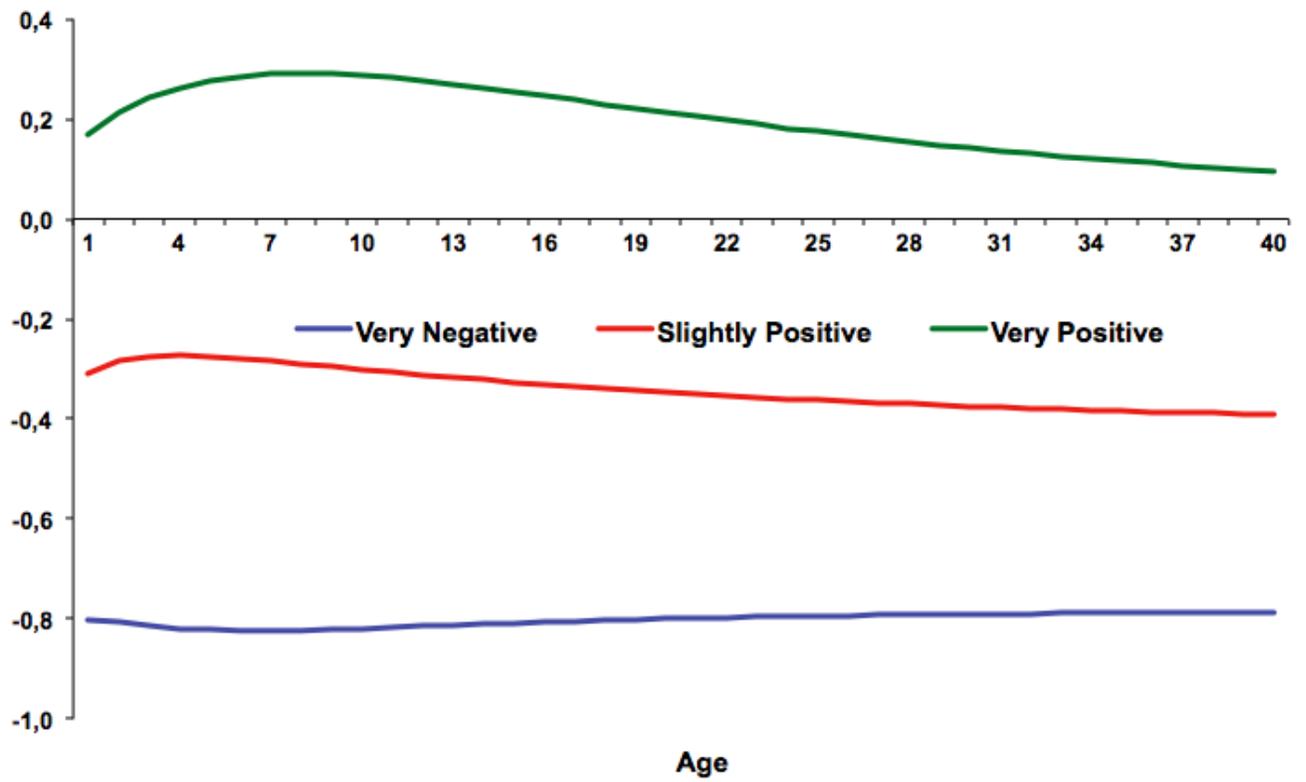


Figure 10: Occupational Map- $\varphi = 1$

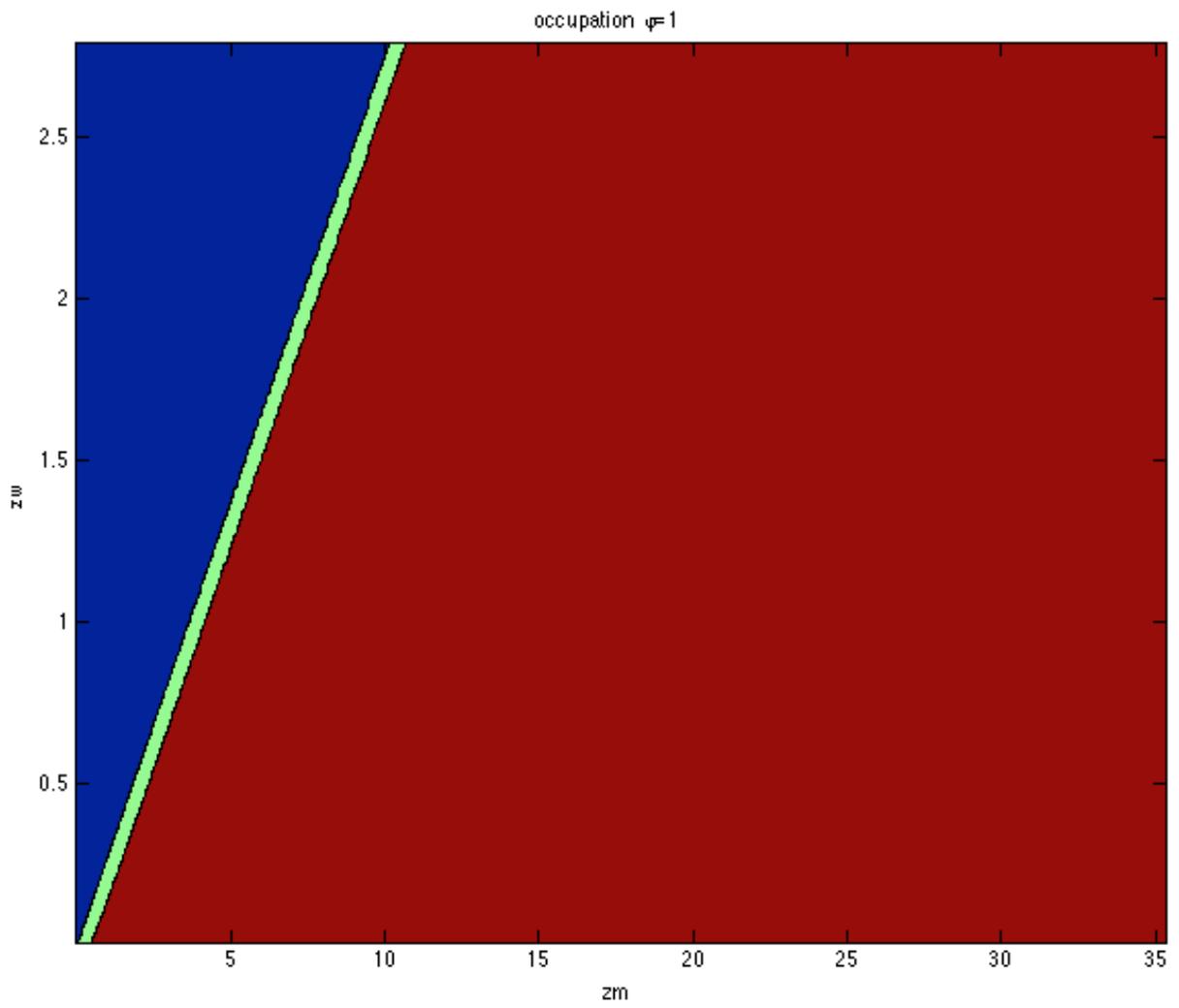


Figure 11: Occupational Maps Benchmark Economy

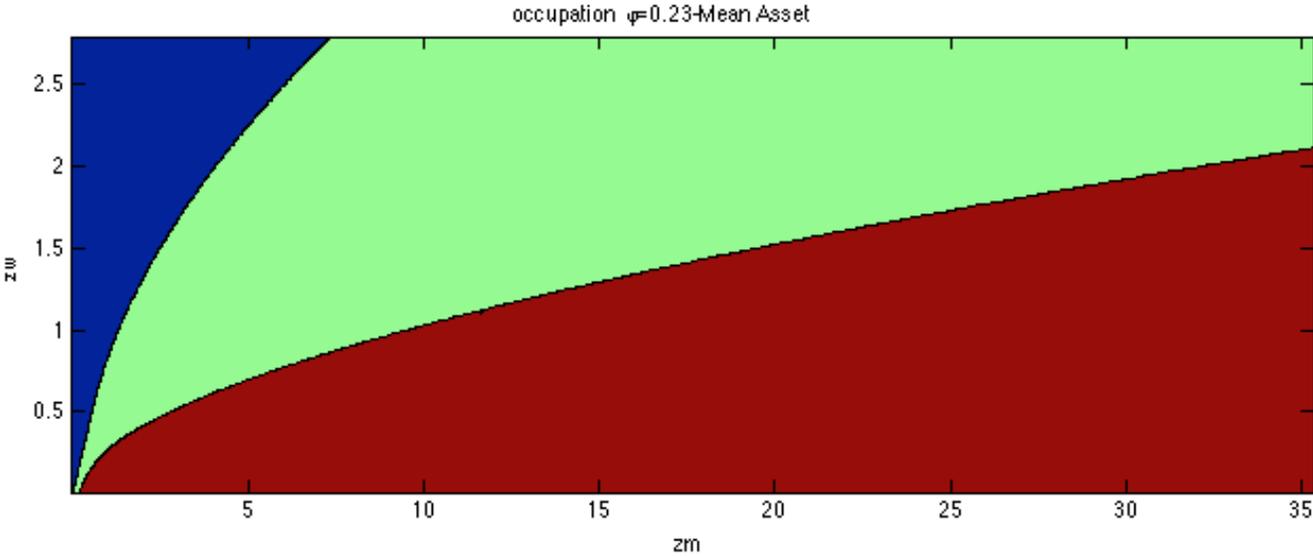
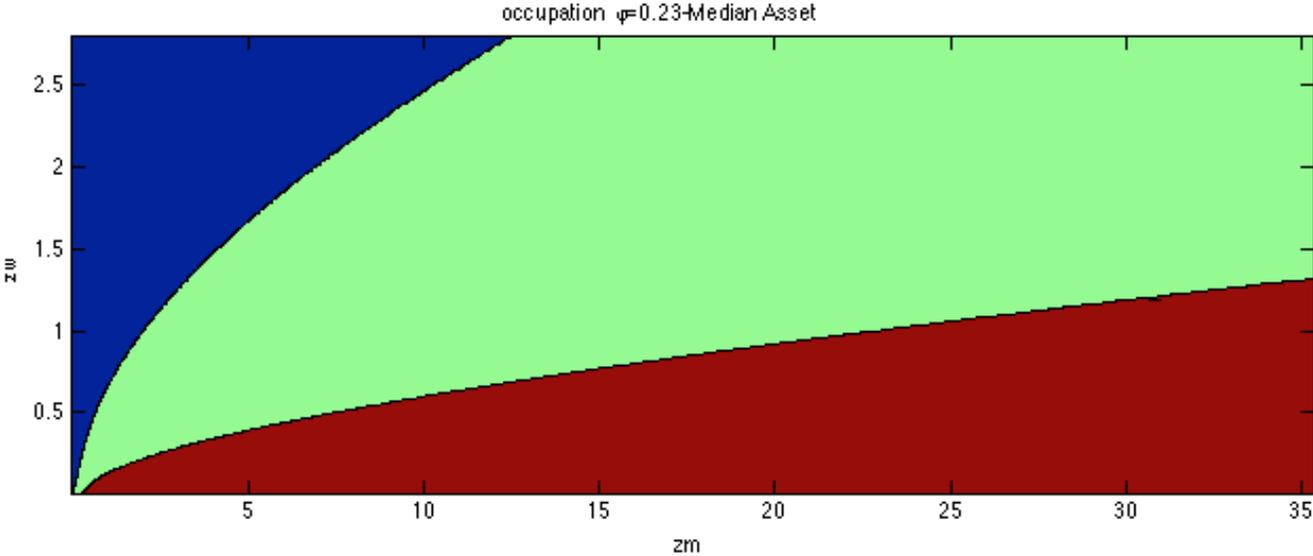


Figure 12: Distribution of Earnings-Data vs Model I

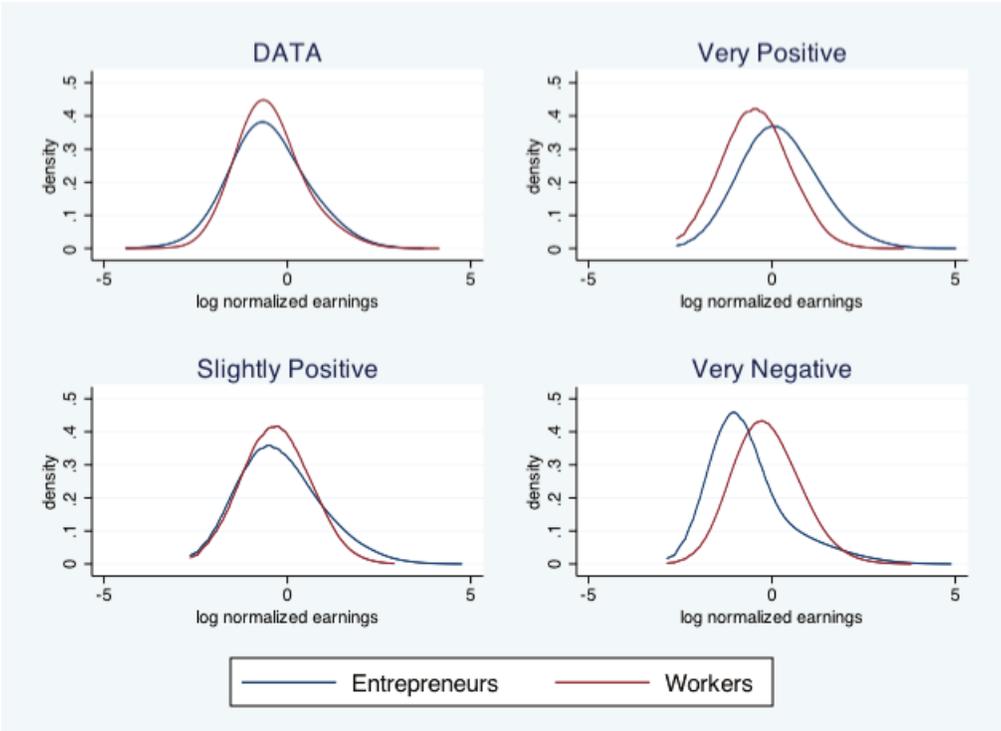


Figure 13: Distribution of Earnings-Data vs Model II

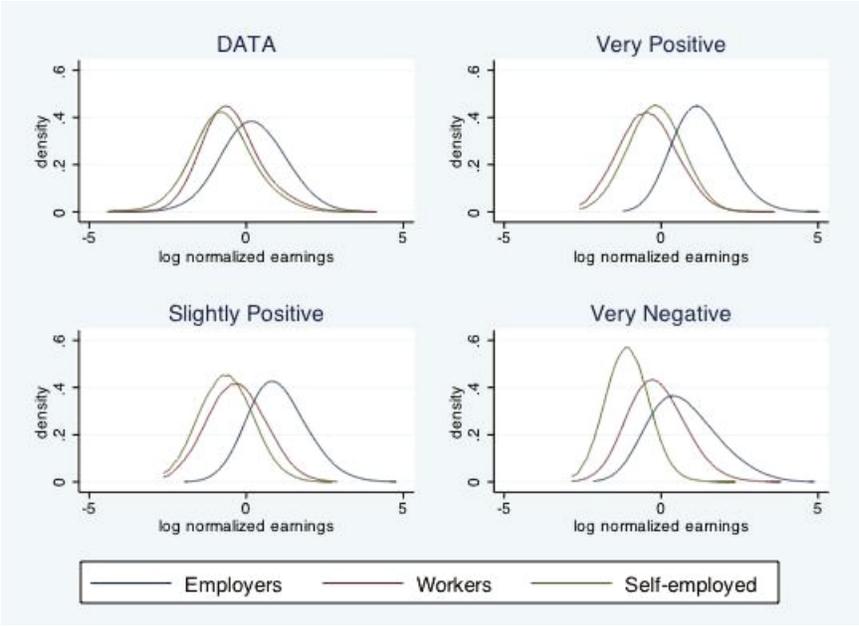


Figure 14: Welfare Gains from Financial Reform

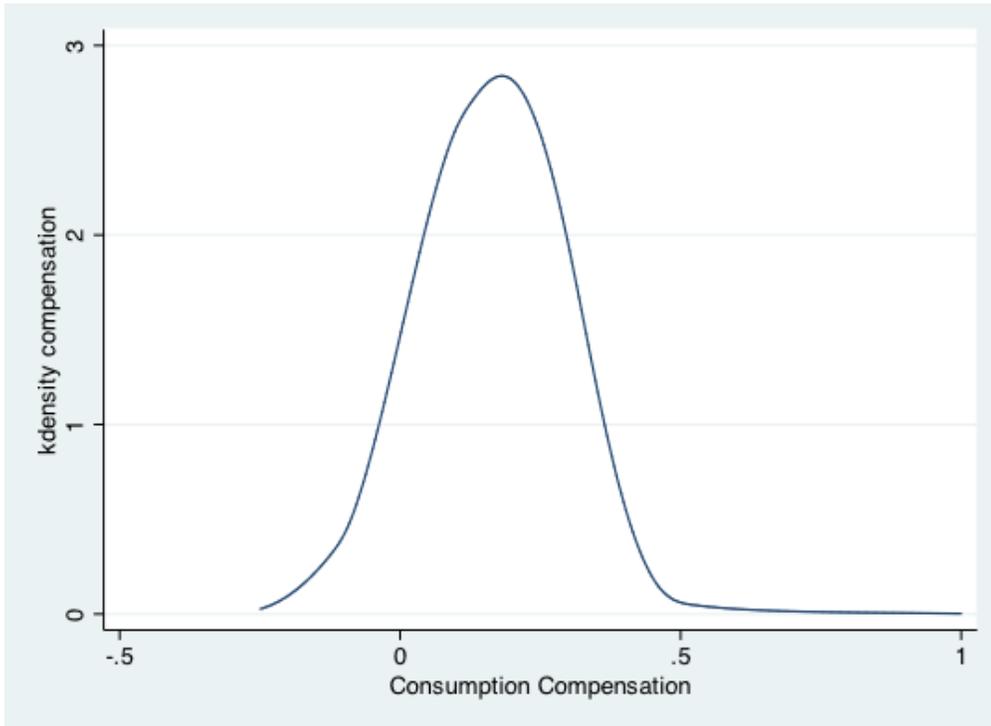


Figure 15: Age Distribution of Winners and Losers from the Reform

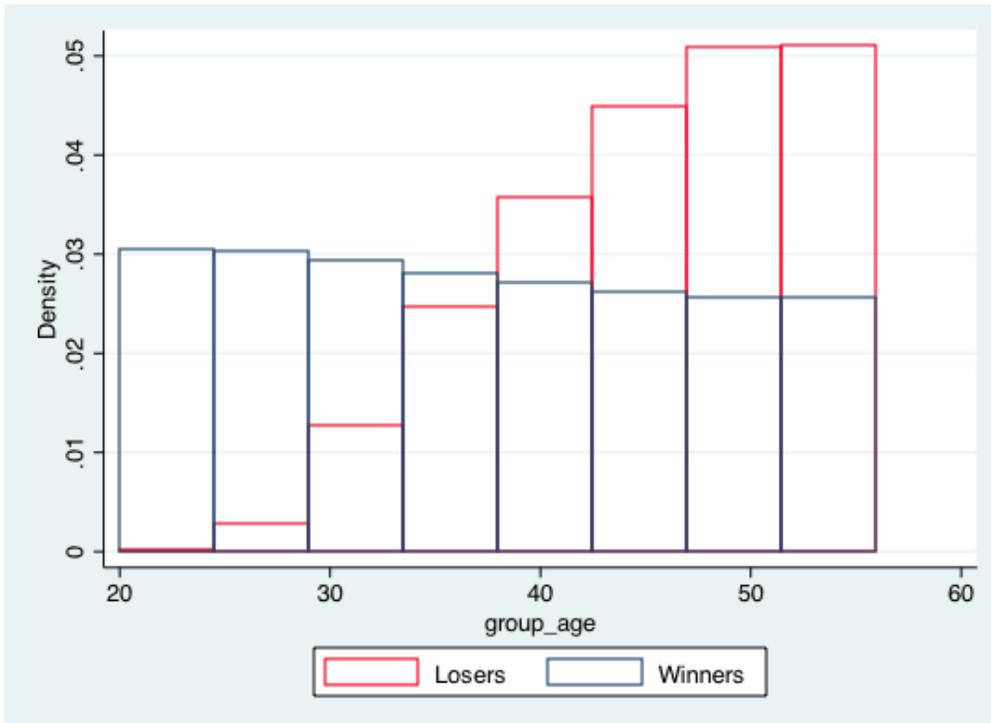


Figure 16: Wealth Distribution and the Reform

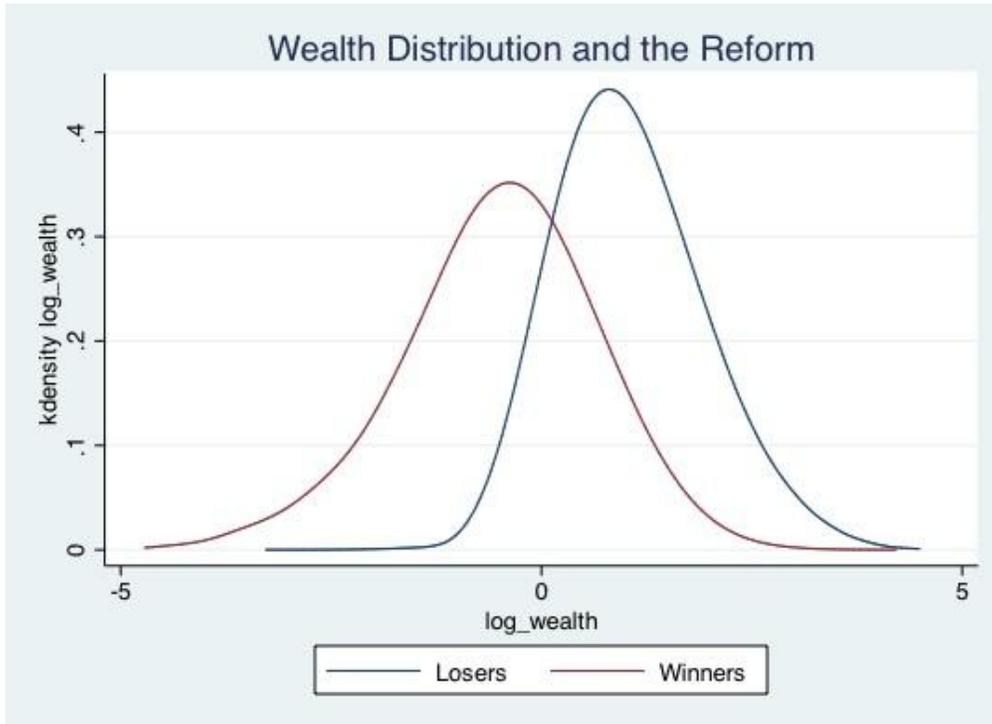


Figure 17: Distribution of Managerial Ability and the Reform

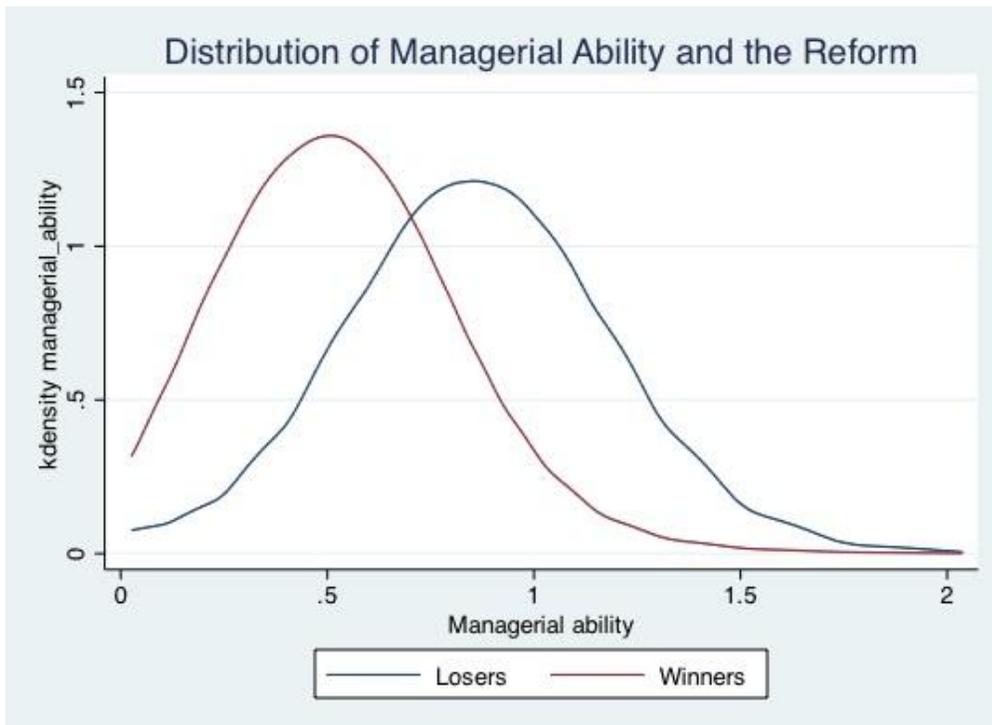
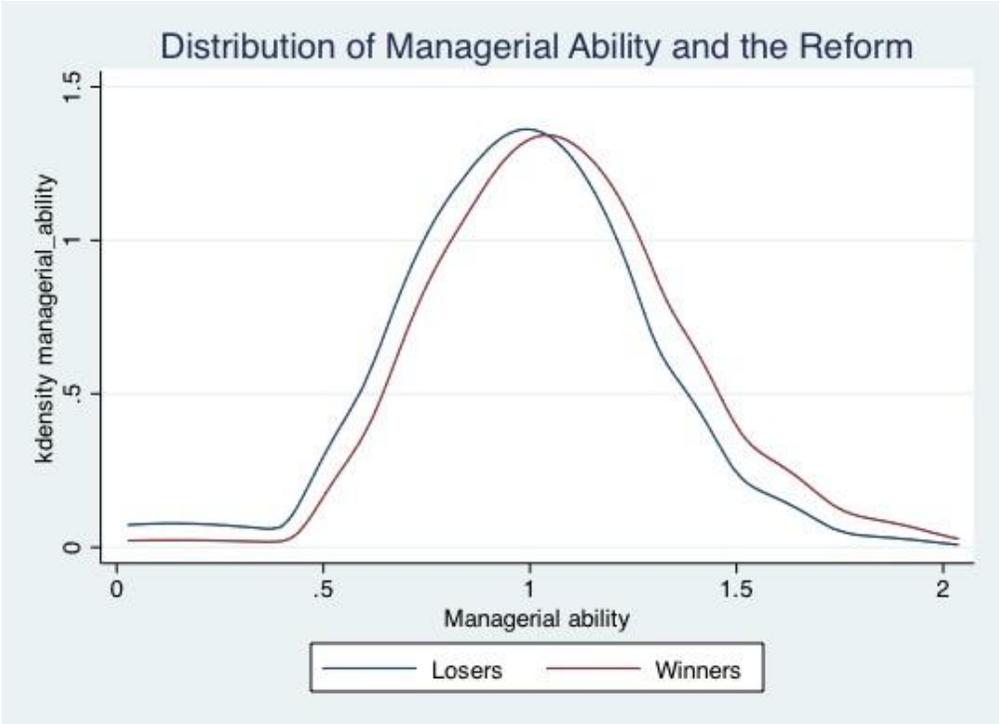


Figure 18: Distribution of Managerial Ability and the Reform among Employers



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## 7 Appendix A

**Pesquisa Mensal de Emprego** From this survey we have data for the years 2003 until 2010. The PME is a monthly household survey covering the metropolitan areas of six Brazilian regions: Rio de Janeiro, São Paulo, Porto Alegre, Belo Horizonte, Recife and Salvador. Each individual is followed for three months, left out of the sample the next eight months and interviewed again the following 4 months. We take the first and fifth interview of each individual for the years 2003 until 2010. In this way we keep two observation of each individual, which corresponds to the same month of consecutive years. We keep only household where the head is male and he is older than twenty and younger than sixty years old. The earnings of the household are the sum of the earnings of all members. In order to make the earnings comparable we deflected them with the corresponding month Consumer Price Index (CPI) and we divide them by the number of adults equivalents in the house. In addition, we only keep individuals who are employed in both periods of the survey. In the final data set we have 131,056 households with data for earnings. Individual households age is defined as the age of the household head. We use 5 years bin, centered at the age of interested, in order to compute statistics by age. To do the transition matrix of employment we consider the individual data. The variable of earnings that we consider is a constructed variable, which includes the earnings effectively perceived by the individual in the month from all the works done.

**Pesquisa de Ornamentos Familiares** The POF is a Consumption-Income survey done every five or six years. We use data from the last wave, 2008-2009. We consider households where the main earner is a male, older than twenty and younger than sixty years old. We end up with 44,930 observations. Our income variable includes: income from work, Transfers, Income from rents, other and Asset Variation. Our measure of consumption includes: food, housing, clothing, transport, health and personal care, education, recreation and culture, smoking, personal services and other current expenses. We normalize household income and consumption by dividing them by the number of adults equivalents in the house.

## 8 Appendix B

**Proof of Proposition 1.** Capital rental  $k$  by an entrepreneur with wealth  $a$  and skills  $(z_m, z_w)$  is enforceable if and only if

$$\begin{aligned} \max_{m,n,n_d,t_e} \{m^\nu k^\nu n^\theta - wn^d - r(k-a) + a - \delta k - c_f |_{n_d > 0}\} \geq \\ \max_{m,n,n_d,t_m} \{m^\nu k^\nu n^\theta - wn^d + (1-\delta)k - c_f |_{n_d > 0}\} \end{aligned}$$

which is equivalent to

$$(1+r)a \geq \varphi \left( \frac{1-\varphi+r+\delta+\delta\varphi}{\varphi} k - \varphi \max_{m,n,n_d,t_e} \{m^\nu k^\nu n^\theta - wn^d - c_f |_{n_d > 0}\} \right) \quad 1$$

Following arguments in Buera et al. (2011), the set of enforceable levels of capital rentals is characterized by a simple set of rental limits. Two cases are relevant. If the max in the RHS is attained with  $n_d = 0$ , the set of enforceable levels of capital is  $[0, \bar{k}(a, z_m, z_w; \varphi)]$  where  $\bar{k}(a, z_m, z_w; \varphi)$ , where  $\bar{k}(a, z_m, z_w; \varphi)$  is given by unique root of the equation

$$(1+r)a = \varphi \left( \frac{1-\varphi+r+\delta+\delta\varphi}{\varphi} k - \varphi \max_{m,n,n_d,t_e} \{m^\nu k^\nu n^\theta - wn^d - c_f |_{n_d = 0}\} \right) \quad 1$$

If the max in the RHS is attained with  $n_d > 0$ , then there are two positive roots of the above equation and the set of enforceable levels of capital rental is  $[\underline{k}(a, z_m, z_w; \varphi), \bar{k}(a, z_m, z_w; \varphi)]$ , where  $\underline{k}(a, z_m, z_w; \varphi)$  represents the smallest root. Nonetheless, the optimal production plan of the entrepreneur coincides with the solution to the individual problem subject to the simpler limit  $k \leq \bar{k}(a, z_m, z_w; \varphi)$ . It can also be shown that  $\bar{k}(a, z_m, z_w; \varphi)$  is strictly increasing in  $a, z_m, \varphi$  and weakly (strictly) increasing in  $z_w$  (if  $n_d = 0$ ).

**Proof of Proposition 2.** The optimal production plan of self-employed individuals solve

$$\pi^{se} = (z_m t_m)^\nu k^\nu (z_w (1-t_m))^\theta - (r+\delta)k + (1+r)a + \mu_k(k^- - k)$$

where  $\gamma + \nu + \theta = 1$ . The FOC imply:

$$\{t_m\} \quad z_m^\gamma k^\nu z_w^\theta [\gamma t_m^{\gamma-1} (1-t_m)^\theta - t_m^\nu \theta (1-t_m)^{\theta-1}] = 0 \Rightarrow t_m^* = \frac{\gamma}{\gamma + \theta}$$

$$\{k\} \quad (z_m t_m)^\gamma \nu k^{\nu-1} (z_w (1-t_m))^\theta - r - \delta - \mu_k = 0 \Rightarrow k = \left( \frac{(z_m t_m)^\gamma \nu (z_w (1-t_m))^\theta}{r + \delta + \mu_k} \right)^{\frac{1}{1-\nu}}$$

Note that the first FOC equates the marginal product of entrepreneurial time at managing and worker. Combining the FOC we obtain that the marginal product of entrepreneurial time satisfies:

$$\begin{aligned} MPT_{se} &= \gamma z_m^\gamma (t_m^*)^{\gamma-1} k^{*\nu} (z_w (1-t_m^*))^\theta \\ &= r_{mw} \left( \frac{z_m^\gamma z_w^\theta}{m w} \right)^{\frac{1}{\gamma+\theta}}, \\ &\text{where } r_{mw} = \gamma \nu^{\frac{\nu}{1-\nu}} \frac{\gamma \theta}{(\gamma + \theta)^2} \frac{1}{r + \delta + \mu} \nu^{\frac{\theta}{1-\nu}} \nu^{\frac{\nu}{1-\nu}} \end{aligned}$$

Income of self-employed individuals can then be written as

$$y_{se} = MPT_{t_m} t_m + MPT_{t_w} t_w + MPK k + ra - k(r + \delta),$$

$$y_{se} = MPT_{se} \times 1 + (r + \mu + \delta)k + ra - k(r + \delta),$$

$$y_{se} = r_{mw} \left( \frac{z_m^\gamma z_w^\theta}{m w} \right)^{\frac{1}{\gamma+\theta}} + \mu k + ra.$$

**Proof of Proposition 3.** An individual with ability  $(z_m, z_w)$  prefers to be self-employed rather than work for a wage if and only if

$$z_w w + ra < \left( \frac{z_m^\gamma z_w^\theta}{m w} \right)^{\frac{1}{\gamma+\theta}} r_{mw} + \mu k + ra,$$

which holds when the skill ratio satisfies

$$\frac{z_w}{z_m} < \frac{r_{mw} + \mu k}{w} \left( \frac{z_m^\gamma z_w^\theta}{z_m z_w} \right)^{\frac{1}{\theta+\nu}} \nu^{\frac{\theta+\nu}{\nu}}$$

If capital markets are perfect ( $\varphi = 1$ ), the Lagrange multiplier on the borrowing constraint is equal to zero ( $\mu = 0$ ) and the individual prefers to be self-employed rather than work for a wage if and only if

$$\frac{z_w}{z_m} < \frac{r_{mw}}{w} \nu^{\frac{\theta+\nu}{\nu}} \equiv R_1.$$

**Proof of Proposition 4.** The optimal production plan of employers solves

$$\pi(z_m, z_w, a) = \text{Max}_{t_m, t_w, n_d, k} (z_m t_m)^\gamma k^\nu (n_d + z_w t_w)^\theta - w n_d - (r + \delta)k + (1 + r)a$$

$$k \leq \bar{k}$$

$$t_m + t_w = 1,$$

$$t_w \geq 0.$$

The non-negativity constraint on  $t_w$  ensures that managerial time cannot be bigger than 1. Associate the multiplier  $\mu_k$  to the borrowing constraint,  $\mu_t$  to the time constraint, and  $\mu_{tw}$  to the non-negative constraint on the working time. The FOC of the problem imply

$$\begin{aligned} MPK &= (z_m t_m)^\gamma v k^{v-1} (n_d + z_w t_w)^\theta = r + \delta + \mu_k, \\ MPn_d &= (z_m t_m)^\gamma k^\gamma \theta (n_d + z_w t_w)^{\theta-1} = w, \\ MPt_m &= z_m \gamma (z_m t_m)^{\gamma-1} k^\gamma (n_d + z_w t_w)^\theta = \mu_t, \\ MPt_w &= (z_m t_m)^\gamma \theta k^\gamma (n_d + z_w t_w)^{\theta-1} z_w = \mu_t - \mu_{tw}, \end{aligned}$$

where we have assumed that parameters are such that it is optimal to hire outside labor ( $n_d > 0$ ). Combining the FOC we obtain:

$$w z_w = MPt_w \leq MPt_m, \text{ with equality only if } t_w > 0.$$

We divide the analysis in two steps.

*Step 1:* We first show that if the borrowing constraint does not bind ( $\mu_k = 0$ ), then the entrepreneur allocate all his time to managerial tasks ( $t_w = 0, t_m = 1$ ). Assume that  $\mu_k = 0$  and let  $L \equiv n_d + z_w(1 - t_m)$ . Furthermore, to find a contradiction assume that  $t_w > 0$ . Then,  $\mu_{tw} = 0$  implies  $MPt_m = MPt_w$  so that

$$z_m \gamma L = t_m z_m \theta z_w \rightarrow t_m = \frac{\gamma L}{\theta z_w}. \quad (12)$$

Combining the FOC for MPK and  $MPn_d$ , gives

$$(z_m t_m)^\gamma \frac{w v L}{(r + \delta) \theta} \theta L^{\theta-1} = w. \quad (13)$$

Combining (12)-(13) gives

$$L^{\gamma+\theta+v-1} \frac{z_m \gamma}{\theta z_m} \frac{w v}{\theta (r + \delta)} \theta = w, \quad (14)$$

which is false in general given that  $\gamma + \theta + v - 1 = 0$ . We conclude that if the borrowing constraint does not bind, then an employer optimally choose to devote all his time to managerial tasks.

*Step 2:* Assume that the borrowing constraint binds ( $k = \bar{k}$ ). We now show that there exists a threshold level of assets  $a^*(z_m, z_w)$  such that the optimal production plan features  $t_w > 0$  if  $a < a^*(z_m, z_w)$  and  $t_w = 0$  if  $a > a^*(z_m, z_w)$ . Thus, if the borrowing constraint is not too tight, employers allocate all their time to managerial activities. We now find conditions for which  $t_m < 1$  (or, equivalently,  $t_w > 0$ ). Note that  $t_m < 1$  only if  $\mu_{tw} = 0$ . In this case, the

marginal product of entrepreneurial time is equated across the two uses of time. From the FOC it can be obtained that

$$MP_{tw} = MP_{tm} \Rightarrow L = \frac{\theta z_w t_m}{\gamma}$$

Plugging  $L$  into the FOC with respect to labor demand and solving for  $t_m$  gives an expression for the optimal fraction of time dedicated to managerial tasks:

$$t_m = \frac{1}{w} \frac{\theta z_m^{\gamma} \bar{k}^{\nu}}{z_w \theta} \left( \frac{\gamma}{z_w \theta} \right)^{\frac{1}{1-\nu-\theta}}$$

Note that  $t_m < 1$  iff

$$\bar{k}(a, z_m, z_w) < k^*(z_m, z_w) \equiv \frac{1}{\theta z_m^{\gamma}} \left( \frac{z_w \theta}{\gamma} \right)^{\frac{1}{\nu}}$$

Since  $\bar{k}(a, z_m, z_w)$  is increasing in  $a$ , the inverse of this function can be used to define a threshold level of assets  $a^*(z_m, z_w)$  such that  $t_m < 1$  if and only if assets are below this threshold. Otherwise,  $t_m = 1$ .

*Step 3:* Compute the marginal product of employers time. From Step 1 and 2, when assets are below  $a^*(z_m, z_w)$  we have  $MP_{tm} = MP_{tw} = w z_w$ . On the other hand, when assets are above  $a^*(z_m, z_w)$ ,  $t_m = 1$  and  $MP_{tm} > MP_{tw}$ . To obtain an expression for  $MP_{tm}$  note that the FOC with respect to capital and outside labor imply:

$$k = \frac{w \nu}{(r + \delta + \mu_k) \theta} n_d \quad 1_{\nu} \left( \frac{\gamma}{z_w \theta} \right)^{\frac{1}{1-\nu-\theta}}$$

$$n_d = \frac{m}{w} \frac{1}{(r + \delta + \mu_k) \theta}$$

Plugging  $k$  and  $n_d$  into  $MP_{tm} = \gamma z_m^{\gamma} k^{\nu} n_d^{\theta}$  gives

$$MP_{tm} = z_m \gamma \left( \frac{\nu}{(r + \delta + \mu)} \right)^{\nu} \left( \frac{\theta}{w} \right)^{\frac{1}{1-\nu-\theta}}$$

**Proof of Proposition 5.** An individual with ability  $(z_m, z_w)$  and assets  $a$  prefers being an employer rather than self-employment if and only if

$$\left( \frac{z_w}{z_m} \right)^{\frac{1}{\theta+\nu}} r_{mw} + \mu_{se} k_{se} + ra < z_m r_m + \mu_e k_e + ra,$$

where  $\mu_e$  and  $\mu_{se}$  are the Lagrange multipliers associated to the borrowing constraints when

the individual is an employer or is self-employed, respectively, and  $k_e$  and  $k_{se}$  are the capital

used in production at these occupations. This inequality holds when the ability ratio is such that

$$\left( \frac{r_m}{r_{mw}} + \frac{(\mu_e k_e - c_f)}{z_m r_{mw}} - \frac{(\mu_{se} k_{se})}{z_m r_{mw}} \right)^{\frac{\theta+v}{\theta}} \equiv \bar{R}_2,$$

If capital markets are perfect ( $\varphi = 1$ ), the Lagrange multiplier on the borrowing constraint is equal to zero ( $\mu = 0$ ) and the individual prefers to be an employer rather than be self-employed if and only if

$$\frac{z_w}{z_m} < R_2 \left( 1 - \frac{c_f}{z_m r_{mw}} \right)^{\frac{\theta+v}{\theta}}, \text{ where } R_2 \equiv \left( \frac{f_m}{f_{mw}} \right)^{\frac{\theta+v}{\theta}}.$$

**Proof of Proposition 6.** When  $\varphi = 1$  the Lagrange multiplier on the borrowing constraint is equal to zero ( $\mu = 0$ ) and occupational choice decisions are independent of asset holdings and maximize the marginal product of time. Proposition 4 established that an individual with ability  $(z_m, z_w)$  prefers to be self-employed rather than work for a wage if and only if the skill ratio satisfies

$$\frac{z_w}{z_m} < R_1 \equiv \frac{r_{mw}}{W}^{\frac{\theta+v}{\theta}}.$$

Assuming that the fixed cost of operation  $c_f = 0$ , then Proposition 5 implies that an individual with ability  $(z_m, z_w)$  prefers to be entrepreneur instead of self-employed if and only if the skill ratio satisfies

$$\frac{z_w}{z_m} < R_2 \equiv \left( \frac{f_m}{f_{mw}} \right)^{\frac{\theta+v}{\theta}}.$$

If parameters are such that  $R_2 < R_1$ , then self-employment is dominated by either being an employer or a worker. The optimal occupational choice is to be an employer if and only if  $\frac{z_w}{z_m} > r_m$ . Otherwise, the optimal occupational choice is to work for a wage. When employers incur a fixed cost of operation, being an employer is preferred to being self-employed if and only if

$$m z_w^{1-\nu} r_{mw} < (z_m r_m - c_f),$$

$$\left( z_w^\nu \right)^{\frac{\theta+v}{\theta}}$$

which holds when  $\frac{z_w}{z_m} < \frac{r_m}{r_{mw}} - \frac{c_f}{r_{mw} z_m}^{\frac{\theta+v}{\theta}}$ .