Learning by Fund-raising*

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May 6, 2014

Abstract

From experience, fund-raisers learn to become more efficient solicitors. This paper incorporates fund-raising technology into the theory of charitable giving. A full characterization of the solicitation strategy that maximizes donations net of fund-raising costs is provided. The strategy identifies a fund-raiser incentive to invest in learning by soliciting some early donors who would give less than their solicitation costs. A notion of “excessive” fund-raising is introduced. It is shown that this may worsen with learning. Our model also accommodates a technology with overhead costs. An extension with rising solicitation costs is also considered.

Keywords: fund-raising, solicitation cost, charitable giving.
JEL Classification: H00, H30, H50

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1 Introduction

Charitable fund-raising is a highly professional activity. The Association for Professional Fundraisers (AFP) represents 30,000 members, and every year more than 115,000 nonprofit organizations consult these professionals for a total of 2 billion dollars (Kelly, 1998). As with many services and manufacturing sectors such as military aircraft, software, pizza industry, it is strongly believed that fund-raising is learned on the job.\textsuperscript{1} Hence demand for those more experienced professionals rises. For instance, a recent survey by Cygnus Applied Research reveals that most successful fund-raisers remain on the job for three to six months before being recruited for another.\textsuperscript{2} As the president of Cygnus puts it: “Only one out of three fund-raisers experiences even a day without a job”. Professional fund-raisers also place a great value on experience as suggested by a fund-raiser’s quote: “Fund Development Associates is the regional expert in fund-raising. No one has more direct, hands-on experience. By selecting our firm, you will have a team of professionals with more than one hundred years of combined successful fund-raising experience who have assisted hundreds of charitable organizations achieve their goals”.\textsuperscript{3}

Both practitioners and researchers agree that one of the most important fund-raising techniques is to directly ask people (Andreoni and Payne 2003; Yoruk 2009; Meer and Rosen 2011). It is believed that people often have good intentions to give, but unless solicited, these intentions may not materialize. In this paper, we contend that such direct solicitations are also the source of learning for the fund-raiser. She learns from experience to become a more productive solicitor. For example, consider a situation in which phone call duration of informing a potential donor shortens with each additional solicitation. The presence of learning economies may rationalize why during big fund-drives charities solicit some initial donors despite their donations being not sufficient to cover their costs, even without considering setup costs. It may also explain the emergence of soft contribution and time targets in fund-drives. Once we take into account the interactions between fund-raising technology and donor preferences, our model predicts that learning economies are worth more in sectors where there is strong warm glow, since the fund-raisers may extensively exploit it.

\textsuperscript{1}See Argote and Epple 1990, Spence 1981, Benkard (2000).
\textsuperscript{2}The survey includes 1,700 fund-raisers and 8,000 nonprofit chief executives. Results are available at http://www.cygresearch.com/files/ AFP_Intl-Conf_Vancouver_April_2_2012-PenelopeBurk.pdf
\textsuperscript{3}See http://www.funddevelopmentassociates.com/associates.html
My formal setup adds an “active” fund-raiser to the “standard” model of giving in which donors consume two goods: a private and a public good.\footnote{See, e.g., Warr (1983); Roberts (1984); Bergstrom, Blume, and Varian (1986); and Andreoni (1988).} We consider a charity that occasionally runs fund-drives. The fund-raiser’s role consists in individually informing potential donors about the charitable cause. Asking people is, however, costly. We introduce a fund-raising technology with two components: a constant marginal cost measuring minimum expenses per solicitation, and a variable part decreasing in the number of solicitations. The latter captures learning by fund-raising. I investigate how learning shapes the fund-raising strategy. This work also develops an appropriate notion of excessive fund-raising in a single charity framework examining whether learning by fund-raising is a source of it.

In a setup where donors have homogeneous preferences, the charity contacts individuals according to income, starting with the wealthiest. A sufficient condition to solicit one extra individual is that she is expected to provide a gift above the marginal cost, i.e. become a "marginal net contributor". We show that identifying these marginal net contributors in our model is equivalent to identifying the net contributors in a model with constant marginal cost when each donor’s wealth is reduced by the variable part of its marginal cost. This important equivalence allows us to partially utilize the characterization in Name-Correa and Yildirim (2013) who assume away learning. In this work, the charity ranks individual according to income and starts soliciting with the richest donor. Once some donors are in the “game”, the propensity of an additional individual to net contribute decreases due to the free-riding incentive. This effect is measured in terms of the cumulative summation of each income difference between the additional donor and the wealthier ones. Sequentially applied, this logic implies that once the charity identifies a “net free-rider”, the solicitations optimally stops.

In the presence of learning, however, the fund-raiser may solicit a marginal net free-rider, as long as this solicitation enables her to substantially move down her learning curve. Thus, negative net contributions represent the fund-raiser’s investment in learning. We provide the exact equilibrium condition of whether investing in learning is worthy or not in section 3. The charity’s optimal stopping rule must be forward-looking. It must also take into account the intensity of contributions for a subset of remaining individuals. While we assume that the solicitation set is observed by the contacted donors, our characterization is robust to unobservability under reasonable (off-equilibrium) beliefs.

We first build a benchmark in which the fund-raiser establishes for each donor a min-
imum gift size and commits to it. We show that this commitment allows the charity to obtain extra-large gifts from the wealthiest donors. With respect to this benchmark, we find that the charity conducts excessive fund-raising even with a constant return to scale solicitation technology. Besides lack of commitment, we show that learning is another source of excessive fund-raising. We find, however, that a higher learning rate may reduce both, the extent of excessive fund-raising and the accumulated experience.

It is well known that watchdog groups evaluate a charity’s efficiency according to its cost structure. They recommend managing a low fixed cost. According to Charity Navigator, administrative costs should not represent more than 20% of total costs. Our model also applies to a situation in which the presence of a fixed cost generates returns to scale in fund-raising. When a higher setup cost does not totally discourage fund-raising, it increases donations and encourages the charity to solicit more. Despite these two positive effects, the public good provision diminishes. We also show that if the solicitation technology is endogenous, it may be profitable for the charity to incur a high setup cost in exchange for a lower marginal cost when optimal fund-raising entails running big fund-drives. As an example consider relatively homogeneous donor income distribution and/or popular charitable causes. However, measures of efficiency such as cost to donation ratios may severely underestimate the efficiency of a fund-raising campaign at an interim stage. For big fund-drives taking place over several years, this may be detrimental success. As a consequence, charities may be suboptimally putting less weight on overhead costs to avoid negative advertising at the initial stages.

We, further extend the model to incorporate a warm-glow motive for giving (Andreoni 1989) and show that our results follow under such added realism. In another extension, we show that when the fund-raiser separates the population in groups and learning is group specific, the charity may favor contacting groups with lower expected income and higher potential for learning. Finally, we show that under decreasing returns to scale it is never optimal to contact a net free-rider.

In addition to the papers mentioned above, our work fits with a small body of theoretical literature on strategic fund-raising as means of: advertising and reducing donors’ search costs (Rose Ackerman 1983; and Andreoni and Payne 2003), providing prestige to donors (Glazer and Konrad 1996; Harbaugh 1998; and Romano and Yildirim 2001), signaling the project quality (Vesterlund 2003; and Andreoni 2006b), and organizing lotteries (Morgan 2000). Our work is also related to the models of strategic fund-raising to overcome
zero-contribution equilibrium under non-convex production either by securing seed money (Andreoni 1998) or by collecting donations in piece-meals (Marx and Matthews 2000). None of these papers, however, consider endogenous, costly solicitations and learning by fund-raising. Other models consider learning about the project quality by providing the charitable good within a dynamic framework. In these models learning is faster when the cumulative production of the good is larger (Bolton and Harris 1999; and Yildirim 2003).

The closest work to ours is Name-Correa and Yildirim (2013); henceforth, NY (2013). They build a model in which donors do not consider giving unless asked by the fund-raiser. They fully incorporate fund-raising costs to determine the fund-raiser’s solicitation strategy. The charity commits to that strategy and successfully launches a fund-drive. Our work is similar to theirs; instead of attaching a cost to each donor, though, we explicitly introduce a fund-raising cost structure unrelated to donors’ identities. This allows us to model the learning aspect of soliciting as decreasing marginal costs in fund-raising.

Rose-Ackerman (1982) is the first to build a model of costly fund-raising in which donors, as in mine, are unaware of a charity until they receive a solicitation letter. She, however, does not construct donors’ responses from an equilibrium play. She is also the first in positing that fund-raising is likely to be conducted in excess. Her argument is that competition among charities triggers high expenses in fund-raising without bringing further benefits to donors. This happens whenever fund-raising diverts funds from one charity that donors value to another they like the same. In contrast, in our model we build the concept of excessive fund-raising in a non-competitive framework. The term "excessive" comes from the fact that relatively more cost is incurred when charity lacks commitment to secure a minimum level of contribution from each donor.

There exists more extensive empirical and experimental literature in charitable giving. Andreoni (2006a) and List (2011) provide a good overview of this.

The paper is organized as follows. In Section 2 the model is presented. In Section 3, we determine the optimal fund-raising strategy. In Section 4 we introduce returns to scale generated by a fixed cost. In Section 5 we consider excessive fund-raising. We show how cumulative experience is affected by the rate of learning in section 6. The extensions are presented in Section 7, and conclusion in Section 8.
2 Model

Our formal setup introduces a fund-raising technology into the standard model of privately provided public goods (e.g., Warr 1983; Roberts 1984; Bergstrom et al. 1986; and Andreoni 1988). For this reason we briefly review this basic framework before introducing fund-raising costs.

**Standard Model.** There is a set of individuals, \( N = \{1, \ldots, n\} \), who each allocates his wealth, \( w_i > 0 \), between a private good consumption, \( x_i \geq 0 \), and a gift to the public good or charity, \( g_i \geq 0 \). Units are normalized so that \( x_i + g_i = w_i \). At the outset, every person is fully aware of the charitable fund-drive and is in the “contribution game”. Letting \( G = \sum_{i \in N} g_i \) be the supply of the public good, individual \( i \)'s preference is represented by the utility function \( u(x_i, G) \), which is strictly increasing, strictly quasi-concave, and twice differentiable. Given contribution by others, \( G_{-i} \), consider individual \( i \)'s maximization problem:

\[
\max_{x_i, g_i} U(x_i, G) \\
\text{s.t. } x_i + G = w_i + G_{-i} \\
G \geq G_{-i}
\]

Let \( f(w) \), individual \( i \)'s demand for the public good ignoring the inequality constraint above, where \( w = w_i + G_{-i} \). We assume that \( f(w) \) satisfies strict normality, i.e. \( 0 < f'_i(w) \leq \theta < 1 \) for some parameter \( \theta \).\(^5\) If individual \( i \) contributes, then \( g_i = f(w_i + G_{-i}) - G_{-i} > 0 \). We assume \( f_i(0) = 0 \) which, coupled with the strict normality assumption, guarantees that each individual’s stand-alone value is positive.

**Learning by Fund-raising.** As in Rose-Ackerman (1982) and Andreoni and Payne (2003), we assume that each person \( i \) becomes informed of the fund-drive only if solicited by the fund-raiser.\(^6\) We assume for simplicity that each solicitation reaches the donor with certainty. It costs \( c(i) = c + s(i) \) to solicit the \( i \)th individual in a sequence. The fixed marginal cost \( c > 0 \) reflects minimum expenses in telemarketing, face to face solicitations, envelopes procurement, and mailing costs. The variable marginal cost \( s(i) \) is non increasing.

\(^5\)The existence of parameter \( \theta \) facilitates our analysis by ensuring a finite \( G^* \) below. It is also commonly assumed in the literature (e.g., Andreoni 1988; Fries, Golding, and Romano 1991).

\(^6\)We envision a charity that occasionally runs fund-drives. In this scenario, it is reasonable to think that donors are unaware of the charitable good provision.
in $i$, due to the fund-raiser learning on the job or scale economies purchasing inputs at a discount. We assume that this cost structure is known by contacted individuals.\footnote{Absent the variable cost, our model would reduce to NY (2013) with homogeneous preferences and constant marginal cost.}

For a total fund-raising cost $C$, and given gross contributions by others, $G_{-i}$, individual $i$’s maximization problem is:

$$\max_{x_i, g_i} U(x_i, G - C)$$

s.t. $x_i + G - C = w_i + G_{-i} - C$

$$G \geq G_{-i}$$

If individual $i$ is a contributor, then $g_i = f(w_i + G_{-i} - C) - (G_{-i} - C) > 0$ and the net cost $C - G_{-i}$ must be low "enough". More precisely, $C - G_{-i}$ must be below a cutoff cost, $\widehat{C}_i \in (0, w_i]$. Otherwise, individual $i$ would prefer to devote his entire wealth to consume the private good. The cutoff cost $\widehat{C}_i$, measures the propensity of person $i$ to provide a public good by covering a fund-raising cost of $C$ in a solo economy. It can be shown that this cuttof cost is unique.\footnote{Note that person $i$ would receive utility $u_i(w_i, 0)$, if he contributed nothing. Otherwise, he would have to choose $g_i \geq C$ to maximize $u_i(w_i - g_i, g_i - C)$. Let $V_i(w_i - C)$ be $i$’s indirect utility in the latter case, which is increasing in the (net) income. For $C = 0$, clearly $V_i(w_i) > u_i(w_i, 0)$ because $f_i(w_i) > 0$, whereas for $C = w_i$, we have $V_i(0) \leq u_i(w_i, 0)$. Hence $\widehat{C}_i$ is unique.}

Let $F \subseteq N$ be the set of donors contacted by the fund-raiser. In the baseline model, we assume that the contacted donors know those in the fund-raiser set, though we relax this assumption in Section 3.2.\footnote{That donors may know the fund-raiser set prior to giving is not completely unrealistic, specially for small fund-drives. For instance, charities organize fund-raising events where donors meet each other.} Let $g^*_i(F)$ be donor $i$’s equilibrium donation in the simultaneous contribution game among donors in $F$. Then, the total fund-raising cost and the gross donations are defined by $C(F) = \sum_{i=1}^{\left|F\right|} c(i)$ and $G^*(F) = \sum_{i \in F} g^*_i(F)$ respectively. $C(\emptyset) = 0$ and $g^*_i(\emptyset) = 0$ by convention. The charity’s problem consists in choosing the set of donors $F$ that maximizes the provision of the public good understood as net donations: $\widehat{G}^*(F) = \max \left\{ G^*(F) - C(F), 0 \right\}$.

Let $F^o$ denotes the optimal solicitation set. In the case of a failed fund-raising, when gross donations received are insufficient to cover the total solicitation cost, we assume for simplicity that the donations are not refunded or they are used for other causes. We also
assume that when two distinct fund-raiser sets yield the same public good provision, the charity selects the one with the lower cost.\textsuperscript{10}

Our fund-raising game, then, proceeds as follows. First, the charity decides whether or not to launch a fund-drive. If one is launched, the charity reaches out to the (optimal) set $F_0$ of potential donors, who become aware of both, the fund-drive, and the others solicited. Finally, they contribute simultaneously to the public good, leading to equilibrium gifts $\{g_i^*(F_0)\}_{i \in F_0}$ and public good $\mathcal{G}^*(F_0)$. Our solution concept is subgame perfect Nash equilibrium in pure strategies.

3 Optimal fund-raising

In section 3.1 we fully characterize the fund-raising equilibrium in terms of the primitives of the model. In section 3.2 we show that this equilibrium is robust to incomplete information.

3.1 Characterization

In order to characterize the optimal fund-raising strategy, for any set of solicitations we observe the following: (1) The incurred cost just depends on the number of solicitations and (2) the higher the income of an individual, the more she gives in any contribution game $F$.\textsuperscript{11} Without loss of generality, we index subjects in a descending order of their wealth: $w_1 \geq w_2 \geq ... \geq w_n$. From these two facts it is intuitive that:

**Observation 1.** For any optimal fund-drive size $k$, the top $k$ individuals are the ones being solicited.

This observation says that the problem of identifying the optimal fund-raising set is linear instead of being combinatorial. Thus, it is not necessary to compare the outcomes of $2^N$ potential contribution games. Instead, the charity sequentially solicits individuals according to their incomes up to individual $k$ such that the following inequality is satisfied:

$$F_0 = \left\{ i \leq k : \sum_{i \leq k} [g_i^*({1, 2, \ldots, k}) - c(i)] > \sum_{i < j} [g_i^*({1, 2, \ldots, j}) - c(i)] \text{ for any } j \neq k \right\}.$$  

Consequently, optimal fund-raising attaches $c(i)$ to individual $i$. In other words, from charity’s standpoint, marginal fund-raising costs are identity dependent. The fundraiser’s

\textsuperscript{10}One justification for this could be that the charity is concerned about its cost/donation rating by the watchdog groups.

\textsuperscript{11}This is shown in Andreoni (1988)
problem consists then in finding a stopping rule to solicitations given homogenous preferences and individuals’ incomes and costs. Without introducing more structure to this problem the optimal solution entails the comparison of \( n \) sets. We will show below an algorithm that requires less steps to reach the optimal solution. The characterization of the fund-raiser set resulting from the iterative application of the algorithm identifies an investment in learning motive. Besides, it facilitates us to determine the effects of experience on optimal fund-raising.

According to NY (2013), when costs are purely identity dependent, the fund-raiser designs a strategy where each individual donor \( i \) is solicited if her gift exceeds her solicitation cost in the contribution game \( F = \{1, 2, \ldots, i\} \); such donor is a marginal net contributor. In that work, this marginal strategy leads to an optimal set, \( F^o \), where every solicited individual ends up being a net contributor as well. For this reason they do not distinguish between a net contributor and a marginal net contributor. Example 1 illustrates optimal fund-raising without learning, as in that work.

\textbf{Example 1. No learning.} Let \( N = \{1, 2, 3\} \) and \( u_i = x_i^{1-\alpha}(\bar{G})^\alpha \), with \( \alpha = 0.3 \).

Individuals’ wealth and solicitation costs are such that \( (w_1, w_2, w_3) = (20, 14, 14) \), \( c = 1 \).

Consider no scale economies, i.e., \( s(i) = 0 \). The following table reports donor equilibrium, and highlights the optimal fund-raiser set.

<table>
<thead>
<tr>
<th>( F )</th>
<th>( g_1 - c )</th>
<th>( g_2 - c )</th>
<th>( g_3 - c )</th>
<th>( G^* - C )</th>
</tr>
</thead>
<tbody>
<tr>
<td>{1}</td>
<td>5.7</td>
<td></td>
<td></td>
<td>5.7</td>
</tr>
<tr>
<td>{1, 2}</td>
<td>5.82</td>
<td>-0.177</td>
<td></td>
<td>5.65</td>
</tr>
<tr>
<td>{1, 2, 3}</td>
<td>5.875</td>
<td>-0.125</td>
<td>-0.125</td>
<td>5.62</td>
</tr>
</tbody>
</table>

\textbf{Table 1:} Donor equilibrium without learning.

Tables 1 reveals that it is optimal to contact only donor 1. Donor 2 and 3 are not included in the set because their contributions never exceed the marginal cost.

Once we introduce learning economies, it is possible that the fund-raiser at the margin optimally solicits an individual \( i \) who provides a gift below the cost \( c(i) \); such a donor is a marginal net free rider. By the same token, it is also possible that a solicited marginal net contributor ends up becoming a net free rider in the contribution game \( F^o \). Both situations entail an investment in learning motive. We first illustrate the former point in example 2,
which also motivates our subsequent analysis. Once we characterize optimal fund-raising we show the latter point in example 3.

**Example 2. Investing in learning.** As in the previous example, let \( N = \{1, 2, 3\} \) and \( u_i = x_i^{1-\alpha}(G)^\alpha \), with \( \alpha = 0.3 \). Individuals’ wealth and solicitation costs are such that \((w_1, w_2, w_3) = (20, 14, 14)\), \( c = 1 \).

Consider learning by fund-raising such that \( s(i) = (7, 5, 1) \). The following table reports donor equilibrium, and highlights the optimal fund-raiser set.

<table>
<thead>
<tr>
<th>( F )</th>
<th>( g_1^* - c(1) )</th>
<th>( g_2^* - c(2) )</th>
<th>( g_3^* - c(3) )</th>
<th>( G^* - C )</th>
</tr>
</thead>
<tbody>
<tr>
<td>{1}</td>
<td>3.9</td>
<td></td>
<td></td>
<td>3.9</td>
</tr>
<tr>
<td>{1, 2}</td>
<td>3.94</td>
<td>-0.06</td>
<td></td>
<td>3.88</td>
</tr>
<tr>
<td>{1, 2, 3}</td>
<td><strong>2.79</strong></td>
<td><strong>-1.21</strong></td>
<td><strong>2.79</strong></td>
<td><strong>4.37</strong></td>
</tr>
</tbody>
</table>

**Table 2:** Donor equilibrium with learning economies

Table 2 shows that it is optimal to contact donors 1, 2, and 3. Without donor 3, donor 2, whose gift remains below \( c(2) \), diminishes the charitable good provision. By additionally soliciting individual 3, however, the public good reaches its optimal level. Finally, it is clear that even with three donors, a direct approach to identify the extent of fund-raising is non trivial.

To develop a simple, intuitive characterization of the fund-raiser set in terms of cost cutoffs, we reformulate our original problem with learning economies to a constant return to scale setting with marginal cost \( c \) and nominal income distribution \( \{\hat{w}_i\} \). \( \hat{w}_i = w_i - s(i) \) can be understood as well as individual \( i \)’s "disposable income". Under this interpretation, sequential costs are equivalent to taxes on individuals and, for a given set \( F \), individual \( i \)’s gift is \( g_i(F) - s(i) \). We show the equivalence of these two frameworks in two steps. Consider person \( i \)’s maximization problem:

\[
\max_{x_i, g_i} \left( x_i, G - \sum_{j \in F} c(j) \right) \\
\text{s.t.} \ x_i + g_i = w_i
\]
As a first step, consider substituting for \( w_i \equiv w_i - s(i) - c \) and \( g_i \equiv g_i - s(i) - c \), person \( i \) can be deemed as choosing the level of the charitable good:

\[
\max_{x_i, G} U(x_i, G) \quad \text{s.t. } x_i + G = w_i + G_{-i} \quad G \geq G_{-i}
\]

The solution to this maximization yields \( i \)'s demand function for the charitable good given net contributions by others, \( G_{-i} : \)

\[
G = \max \{ f(w_i + G_{-i}), G_{-i} \}.
\]

As a second step, from this whole normalization, we define \( \tilde{w}_i \equiv w_i + c \equiv w_i - s(i) \) and \( \tilde{g}_i \equiv g_i + c \equiv g_i - s(i) \).

Let \( F_i \) be the set of the top \( i \) individuals. The next Lemma shows that individual \( i \)'s incentive to provide a donation above the marginal cost, \( c \), in \( F_i \) can be represented by a cost cutoff.

**Lemma 1** Let \( \tilde{\phi}(G) \equiv \phi(G) - G \), where \( \phi = f^{-1} \), and donor \( i \)'s cost cutoff be given by

\[
\tilde{c}_i = \tilde{w}_i - \tilde{\phi}(\sum_{j=1}^{i}(\tilde{w}_j - \tilde{w}_i)).
\]  

(1)

Individual \( i \) is a net contributor in \( F_i \) or marginal net contributor iff \( c < \tilde{c}_i \)

By strict normality \( \tilde{\phi}(.) > 0 \). Therefore, individual \( i \)'s cutoff cost decreases in others’ disposable incomes and increases in her own.

**Observation 2.** Absent the sequential component of fund-raising costs, i.e, \( c(i) = c \) we obtain: \( \tilde{c}_1 \geq \tilde{c}_2 \geq . \geq \tilde{c}_n \) and \( F^0 = \{ i \in N \mid c < \tilde{c}_1 \} \) (NY, 2013)

Note first that without sequential cost, \( \tilde{w}_i = w_i \). Hence, for any subeconomy \( F_i \), individuals ended up being ranked by the fund-raiser according to their net gifts \( g_i^*(F_i) - c \), since \( w_1 - c \geq w_2 - c . . \geq w_n - c \). It is clear that \( \tilde{c}_i \) is less than \( w_i \), except for the first individual, and it diminishes in \( i \). Intuitively, once the richest donor is solicited, the second individual is less propense to cover the marginal cost \( c \) as a consequence of the free rider problem. In general, as the charity keeps fund-raising, free riding becomes more and more severe and
it is less likely that an additional person will be solicited. Once a marginal net free rider is identified, fund-raising stops. Otherwise, given that individuals are ranked according to their net gifts’ sizes, additional solicitations would bring only negative net donations. This would hurt the public good provision. Consequently, every individual in $F^o$ ends up being a net contributor as well since $g^*_k(F_i) - c > g^*_k(F_i) - c > 0$ where individual $k$ is the highest index individual such that $g^*_k(F_i) - c > 0$ and $i < j$. Re-consider Example 1 above, when $s(i) = 0$. From eq. (1), it is easily verified that $\bar{c}_1 = 20$, $\bar{c}_2 = 0$, and $\bar{c}_3 = 0$, which implies that $F^o = \{1\}$.12

The free rider problem is still present when $s(i) > 0$. However, on the upside, by an additional solicitation the charity also accumulates experience, which partially counteracts the negative effect of free riding. Thus, it is no longer the case that the propensity of individual $i$ to be a marginal net contributor, as measured by her cost cutoff is decreasing in the number of previous solicitations. As a result, the optimal stopping rule is forward looking and it also considers the marginal intensity of giving. Let $a_{\bar{c}}(i, k)$ be the average disposable income from individuals $i$ to $k$ where $i \leq k$. By convention, $a_{\bar{c}}(i, i) = w_i - s(i)$. By applying the next proposition iteratively we obtain a full characterization of the fundraiser’s strategy.

**Proposition 1** Suppose either (1) $i = 1$ or (2) $i > 1$ and individuals $1$ to $i - 1$ are solicited by the fund-raiser. Then, $i$ is solicited iff there is an individual $k \geq i$ such that $c < \bar{c}_i(a_{\bar{c}}(i, k))$. Moreover if $k > i$ is the closest individual to $i$ satisfying the previous inequality, then donors from $i + 1$ up to $k$ must also be solicited.

Proposition 1 says that to contact an additional individual $i$, it is sufficient that she pays for her cost in the economy $F_i$ i.e., she is a marginal net contributor.

This proposition also says that despite individual $i$ being a marginal net free rider, she is solicited as long as subsequent cost decreases turn out to be substantial. The presence of net free riders in $F^o$ can be thought of a charity’s investment in acquiring experience.

**Definition.** A charity investment in learning is represented by $\sum_{\{i \in F^o: g_i(F^o) < c(i)\}} [c(i) - g_i(F^o)]$

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12 As mentioned above, without learning there is no distinction between a net free rider and a marginal net free rider, i.e., $g_i(F_i) > c$ iff $g^*_i(F^o) > c$. 

12
Re-consider Example 1 above, under learning economies. From eq. (1), it follows that \( \bar{c}_1 = 13, \bar{c}_2 = -0.33 < c < \bar{c}_3 = 22.33 \). Moreover, \( \bar{c}_3(a_{\hat{e}}(2,3)) = 6.33 > c \). Thus, according to Proposition 2, \( F^o = \{1,2,3\} \). The investment in learning is \( c(2) - g^5_{2}(\{1,2,3\}) = 1.21 \).

Even if every solicited individual is a marginal net contributor, optimal fund-raising may entail investing in learning as illustrated in the next example.

**Example 3.** (NY 2013). There are \( n \) identical individuals. Learning is drastic, i.e. the sequence of cost is strictly decreasing \( c(1) > c(2) > \ldots > c(n) \). Suppose \( c(1) \) is small enough such that each individual solo decision would be to provide a donation. Then, from 1 it easy follows that \( c < \bar{c}_1 < \bar{c}_2 < \bar{c}_n \). Indeed, every individual is a marginal net contributor. However, when the set of donors is large enough, each individual donation is close to the average cost which is clearly lower than the initial cost \( c(1) \). Thus, some initial donors are net free riders which constitutes an investment in learning.

To conclude this section, note that the fund-raiser considers the set resulting from iteratively applying proposition 2 as a candidate equilibrium strategy. This set will be optimal if, given the total fund-raising cost, \( \sum_{j=1}^{\left| F^o \right|} c(j) \), incurred, and contribution by others \( G^*_{-i} \), each solicited individual decides to contribute rather than consume only the private good.

Consequently, the solicitation set derived from Proposition 2 is the equilibrium action by the fund-raiser if each contacted individual’s net cost, \( \sum_{j=1}^{\left| F^o \right|} c(j) - G^*_{-i} \), is strictly less than her cutoff, \( \hat{C}_i \). The next condition guarantees that this happens for every donor included in the set.

**Assumption S.** Let \( k \in N \) be the largest index such that \( c < \bar{c}_i \). Then it follows that \( (i) \sum_{i=1}^{k} (w_i - c(i)) > 0 \), and \( (ii) \text{ for } i \leq k: f(w_i - \hat{C}_i) \leq \Phi^{-1}_k(\sum_{i=1}^{k} (w_i - c(i))) \), where \( \Phi_k(G) = \sum_{j=1}^{k} \phi(G) + G \).

The intuition behind assumption S is the following: Consider a solo economy. Pick any donor \( i \) in the fund-raiser set. At \( C = \hat{C}_i \), she is indifferent between providing the public good at level \( f(w_i - \hat{C}_i) \) or consuming exclusively the private good. Assumption S says that whenever aggregate wealth in \( F^o \) is high enough, each solicited donor is actually demanding more than \( f(w_i - \hat{C}_i) \). Given strict normality, it indicates that she has strict incentives to contribute.
The next section shows that the equilibrium of the fund-raising game is robust to incomplete information.

### 3.2 Unobservability of the Fund-raiser Set

Our assumption regarding the observability of the the fund-raiser set is reasonable for small fund-raising campaigns. For larger ones, it is not feasible for donors to keep track of the charity’s solicitations, but to hold beliefs about them. Let $\mathcal{F}_i$ be donor $i$’s belief about the fund-raiser set when she is contacted.

Given the optimal fund-raiser set $F^0$, one natural belief system is as follows: a solicited donor who is also in $F^0$ believes that the charity sticks to the solicitation strategy $F^0$, whereas a solicited donor outside $F^0$ believes that every richer individual is also solicited while lower income individuals are not. Each donor assumes that others act according to the stated beliefs. This belief system is grounded in a learning by fund-raising setting. To gain experience in the field, fund-raising may be carried out by few people. Thus donors may not perceive deviations from $F^0$ as uncorrelated or isolated mistakes. We show in the next proposition that when donors share these beliefs, the fund-raiser’s equilibrium strategy is the same whether or not it is observable.

**Proposition 2** Suppose the fund-raiser set is unobservable to donors. Let $\mathcal{F}_i = F^0$ if $i \in F^0$, and $\mathcal{F}_i = \{1, 2, 3, .., i\}$ if $i \notin F^0$. Then, under the beliefs $\{\mathcal{F}_i\}_{i=1}^n$, $F^0$ is sustained as a perfect Bayesian equilibrium.

It is plausible in big fund-raising campaigns that total initial donations do not cover total initial costs. Despite that, we observe that fund-drives are launched and charitable goods are provided out of net donations because initial donors expect the charity to continue fund-raising up to individual $k$ to take advantage of learning economies. Thus, they know that eventually total donations exceed total costs.

Under the belief system described above, the fund-raiser does not necessarily have a commitment problem to its target strategy. However, problems may arise if people perceive mistakes to be uncorrelated as follows: If a donor in $F^0$ is contacted, he learns about the fund-drive and believes that the rest of $F^0$ will also be contacted, whereas if a donor outside $F^0$ is contacted, he attributes this to a mistake and believes that he is the only one contacted besides $F^0$.

To illustrate the tension between charity and donors under these beliefs, notice...

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13These beliefs are similar to “passive” beliefs often used in bilateral contracting in which one party
that if \( k \) is the highest index individual being contacted under \( F^o \), any solicited individual \( i > k \) would take others’ contributions as: \( C^*(F^0) - C(F^0) - c(k + 1) \). But, in fact, as the charity keeps fund-raising more and more, subsequent cost decreases are obtained without being noticed by additional donors. Consequently, for a big enough donor database and learning potential, the free rider problem is curbed to some extent, thus undermining the charity’s credibility to \( F^o \). This is consistent with the anecdotal evidence that schools often announce a target level of funds to be raised as well as the length of the fund-drive.  

4 Effects of a Fixed cost on Optimal Fund-raising: Overhead Costs vs Marginal Costs

Fixed costs, also called overhead costs—expenses such as rent, utilities, technology, accounting costs, legal costs, and marketing costs—are an important component of a charity’s cost structure. Donors and foundations are aware of the potential detrimental impact of these costs on the charitable good provision. Indeed, watchdog groups rank charities’ efficiency based on the administrative cost to total cost ratio. For instance, Charity Navigator suggests that for an acceptable charity this ratio ranges from 15% to 20%. Moreover, a study conducted by the center of philanthropy at Indiana University shows that of the 710 foundations that responded to the survey, 69% responded that their donations were intended to support charity’s overhead expenses.

To isolate the effect of a fixed cost on optimal fund-raising, we consider the following particular cost structure: a fixed cost \( s \) and a constant marginal cost \( c \). This is captured in our model by making \( s(1) = s > 0 \) and \( s(i) = 0 \) for every \( i > 1 \). Let \( F^o(s) \) be the fund-raiser set when the fixed cost amounts to \( s \).

**Proposition 3** Consider two fixed cost levels, \( s \) and \( s' \) such that \( s < s' \) and \( F^o(s) \) as well as \( F^o(s') \) are non-empty. Then,

(a) Fund-raising increases in the setup cost, i.e., \( F^o(s) \subseteq F^o(s') \)

(b) Individual gross donations augments in the setup cost, i.e., \( g_i(F^o(s)) < g_i(F^o(s')) \) for every \( i \in F^o(s') \), but

\[ \text{privately contracts with several others (e.g., Cremer and Riordan 1987; McAfee and Schwartz 1994).} \]

\[ ^{14}\text{For example, Duke University announces in 2012 a new five-year fund-raising campaign to raise $3.25 billion for academic programs, medical education and health research, and its endowment.} \]
The public good amount falls in the setup cost, i.e., $G(F(s)) > G(F'(s'))$

The intuition behind this Proposition is simple. From (1) it is clear that for individuals $i > 1$ cutoff costs rise in the fixed cost. Thus, given a higher fixed cost, the charity solicits more because it anticipates that individuals are more willing to give in order to partially recover the cost increase. Despite the rise in total gross donations generated by current and additional solicited donors, the level of the public good falls since individuals collectively do not make up for the totality of the rise in the cost. Thus, the two positive effects of the setup cost increase, i.e., more fund-raising and more gross donations, are neutralized by the negative effect of a rising cost burden on the supplied public good.

More fund-raising, even when optimally conducted, may in some cases indicate that the charity is actually less productive. This observation contrasts with our intuitive understanding of public good provision in a costless economy where, fixing individuals’ characteristics, a larger set of contributors signals a greater supply of the public good.

In the rest of the section we approach the charity’s problem of selecting a suitable fund-raising technology to conduct fund-drives with certain characteristics. Consider a charity deciding between two technologies with different constant marginal costs and distinct marginal cost/overhead expenses ratios. These are described by a pair (fixed cost, marginal cost): $(s, c)$ and $(s', c')$ such that $s < s'$ but $c > c'$. Adopting the latter technology over the former would enable the charity to save on marginal costs at the expense of a higher overhead cost. We consider that those saving are not too high, i.e., $0 < c - c' < s' - s$, thus, selecting the most adequate cost structure is not a trivial task for the charity. It is intuitive that for small fund-drives or for charities providing public goods under strong free-riding, technology $(s, c)$ dominates $(s', c')$ in the sense that more public good is being provided under the former technology. In this case, since the fund-drive is small, a low overhead cost is critical. The opposite happens in big fund-raising campaigns, where the positive impact of a reduction on marginal costs on the public good provision is anchored. To formalize this intuition let $w = (w_1, w_2, ..., w_n)$ and $L_i(w) = \sum_{j=1}^{i} w_j$. Take two income vectors $w' \neq w''$ such that $L_n(w') = L_n(w'')$. We say that $w''$ is more Lorenz-unequal than $w'$ if $L_i(w'') > L_i(w')$ for all $i < n$. Armed with this definition of Lorenz inequality,

**Lemma 2** Consider an income profile $w$ such that $n(c, s)$ and $n(s', c') > 0$. Then, there exists a more Lorenz-unequal income profile $w'$ such that $\overline{G}(c, s, w') > \overline{G}(c', s', w')$. On the other hand, for every unequal income distribution $w$ there exists $\overline{r}(w) > 0$ such that
for any replica-economy with \( r > r(w) \), where \( r \) stands for the number of replications, it follows that \( \mathcal{G}^*(c, s, w) < \mathcal{G}^*(c', s', w) \)

Even though the previous Lemma suggests that it is optimal for a big charity to undertake a big enough initial investment in capital to be able to keep marginal costs low, coordination problems among donors may favor the employment of a less efficient technology with low overhead costs. For example consider the case in which there is an incumbent charity providing a homogeneous public good. There is also an entrant charity with more efficient technology facing higher fixed cost. Then, as in Andreoni (1988) the higher fixed cost may engender a zero equilibrium. Since the focal point is to contribute to the well established charity there may be barriers to enter rooted in donors’ inability to coordinate. Thus, securing seed money would not only allow to overcome a zero contribution equilibrium but also facilitates the emergence of more efficient technology to deal with massive fund-drives.

5 Excessive Fund-raising

Does learning through fund-raising experience incentivize charities to solicit too much in order to enable cost reduction? We approach this matter by establishing a benchmark setting in which the fund-raiser fixes a minimum gift size \( t_i \) for each donor. She publicly announces each \( t_i \) and commits to refuse donations below the respective threshold, though giving is still voluntary.\(^{15}\) Hence, the free rider problem is still present. For a fixed set \( F \), the fund-raiser maximization problem is:

\[
\max_{\{t_i\}_{i=1}^{|F|}} \sum_{i=1}^{|F|} t_i \tag{2}
\]

s.t. \( U(w_i - t_i, T - C(F)) \geq U(w_i, \max\{T - i - C(F), 0\}) \) for every \( i \in F \)

where \( T = \sum_{j=1}^{|F|} t_j \).\(^{16}\) The fund-raiser problem then consists in choosing the optimal solicitation set \( F^* \) and threshold gifts \( \{t^*_i\}_{i=1}^{|F^*|} \).

\(^{15}\)This is actually a case of multilateral "contracting" under positive externalities as in Segal (1999). It also resembles Andreoni 1998, in which the threshold for public good provision is determined by the production technology. In our setting, donors face individual thresholds endogenously determined by the fund-raiser.

\(^{16}\)The most acute form of commitment or pressure would add a target level of the charitable good such that if total donations are below that target, neither provision takes place nor refund is made. In this extreme case, the fund-raiser extracts from each individual, \( g_i^0 \) such that \( U(w_i - g_i^0, g_i^0) = U(w_i, 0) \). The critical public good level would be \( \sum_{j=1}^n g_j^0 \).
Assumption M. $U_{12} > 0$.

Assumption $M$ is satisfied for a general class of utility functions such as Cobb-Douglas. It guarantees that in the benchmark, optimal fund-raising behaves as in the case without commitment, i.e. it also dictates to solicit individuals in a sequence from the top to the bottom of the income distribution. Thus:

Observation 3. \textit{Individual $i$ does not provide a gift above $t_i^*$. Moreover $t_{i+1}^* > 0$ implies $t_i^* > t_{i+1}^*$.}

Threshold gifts leave each individual indifferent to contributing the "suggested" amount or not giving at all. As in the case with purely voluntary contributions, the wealthier the individual, the higher her threshold gift. Given the charity’s commitment power to minimum gift sizes in the benchmark, the following observation is intuitive:

Observation 4. \textit{For any fixed set of donors, the voluntary provision of the public good is below that in the benchmark.}

For a fixed fund-raising strategy, a lack of commitment directly hurts the public good provision. Noteworthy, the fund-raiser can feasibly set a minimum gift size to individual $i$ corresponding to her voluntary contribution under $F^o$, that is $t_i = g_i^*(F^o)$. In other words, the equilibrium voluntary contribution profile $\{g_i^*(F^o)\}_{i \in F^o}$, is a feasible solution to (2) when $F = F^o$. We then show that the fund-raiser can profitably deviate. To see this, suppose the charity exclusively “pressures” individual 1. By quasiconcavity of the utility function, the fund-raiser is able to extract from him a larger gift than voluntarily provided. In response, other individuals lower their contributions. Overall, the public good amount increases above the level supplied under voluntary contributions, by the strict normality assumption.

Equilibria of the fund-raising game with and without commitment can not be Pareto ranked. This is intuitive since the fund-raiser does not act as a benevolent social planner. Even though more public good is provided in the benchmark, this may come at the expense of some donors’ welfare by setting extra-high thresholds to them. Contributors are receiving their outside option utilities which may be below those levels in the purely voluntary case.

\footnote{Assumption $M$ is satisfied when individuals have homothetic homothetic preferences.}
voluntary contribution game. However, we propose a different prospective under which lack of commitment indeed introduces an "inefficiency" in the society. Consider establishing a target provision of the public good at the level reached without commitment, $\overline{G}^* (F^o)$. No solicitations are made once this target is accomplished. From observation 4, it directly follows that relatively more fund-raising is conducted under purely voluntary contributions to fulfill the target. In our model costs are interpreted ex-post as taxes from which individuals derive no utility. In this sense, we say that the charity conducts excessive fund-raising. Let $\eta(\overline{G}^* (F^o))$ the lowest number of solicitations required to reach at least $\overline{G}^* (F^o)$ under the benchmark.

**Definition 1** We say that a charity conducts excessive fund-raising whenever she solicits a larger number of donors with respect to the benchmark to reach a provision of the public good $\overline{G}^* (F^o)$. The extent of excessive fund-raising is measured as $|F^o| - \eta(\overline{G}^* (F^o))$.

This way of measuring excessive fund-raising enable us to isolate the direct negative effect of a lack of commitment on the level of the charitable good, as stated in observation 4 to focus on the relative inefficiency introduced by a costly fund-raising process. Thus, by fixing the level of the public good at the amount optimally provided under purely voluntary contributions, we focus on the the extra solicitations incurred due to the fund-raiser’s lack of commitment to gift sizes.  

**Proposition 4** Consider a set of potential donors $N$. Then, the charity conducts excessive fund-raising. Moreover, there exists at least one fund-raising technology such that the extent of excessive fund-raising is strictly positive.

Rose-Ackerman (1982) was the first to introduce the concept of excessive fund-raising in a competitive charitable market under costly fund-raising. Our work complements her by pointing out that, even without competition, excessive fund-raising emerges due to the charity’s lack of commitment to minimum gift sizes. The focus on the rest of this section is on determining whether an accumulation of experience motive constitutes an additional source of excessive fund-raising. One may intuitively consider that under learning economies, there is more extent for excessive fund-raising since the charity relatively benefits from soliciting more. By the same token one would expect that a faster learning process triggers a larger

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18Because of the integer problem, by soliciting $\eta(\overline{G}^* (F^o))$, it may be the case that the provision of the public good in the benchmark is higher than $(\overline{G}^* (F^o))$. 

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solicitation gap with respect to the benchmark. To deal with these issues in a systematic way, we consider for the rest of the section, a case in which there is strong substitutability among gifts in the benchmark when complementarities due to cost recovery motives are absent.

**Assumption:** In a costless economy \( rac{d_t}{dT_{-i}} < -1 \). \(^{19}\)

This assumption is satisfied when individuals have Cobb-Douglas preferences with demand for the public good \( \alpha w \) where \( \alpha < \frac{1}{2} \). Indeed, this case is the most relevant from an empirical perspective. Some works, as Zieschang (1985), have estimated \( \alpha \) to be 0.0342. A relative low taste for the public good is the source of strong substitution effects among feasible requested donations. Thus, when the fund-raiser increases for one individual the minimum donation size by one dollar, the potential gift size for everyone else drops by more than one dollar. As a result, optimal fund-raising entails receiving an extra-larger donation from the richest individual and nothing from the rest. Now, once cost is introduced, it may be the case that the fund-raiser solicits more individuals to partially recover the initial cost, \( c(1) \). This means that each solicited individual \( i > 1 \) provides a positive net donation and also that \( \sum_{i>1} (t_i(F^*) - c(i)) < c(1) \). Indeed, it is shown in the appendix that if at least two individuals are solicited, all of them are pivotal, in the sense that each individual contribution is critical to the public good provision. Since gifts are smaller when charity lacks commitment, it is then intuitive that relatively more fund-raising is optimally conducted to recover the initial cost, as shown in Lemma 3.

**Lemma 3** Suppose more than two individuals are solicited under \( F^0 \). Then, \( F^* \subseteq F^0 \).

Lemma 3 provides us with a stronger observation about the effect of commitment on optimal fund-raising. Commitment enables the charity to make less solicitation not just to achieve the target, \( \overline{G}^*(F^o) \), but also the optimal public good provision, \( \overline{G}^*(F^*) \). From a different angle, commitment allows the charity to extract bigger donations from initial donors which accelerates free-riding when individuals have low taste for the public good.

\(^{19}\)This is equivalent to requiring that \( U_2(w_i, T_{-i}) < U_1(w_i - t_i, T_{-i} + t_i) \). That is, the marginal utility of the public good consumption at the level contributed by others (outside option) must be lower than the marginal utility of the private good consumption given that \( t_i \) is donated. In other words, given the outside option level of consumption of both goods, there exists a strong substitution effect toward the private good consumption.
To explore the effect of learning on the extent of excessive fund-raising we build on the following sequential cost function:

\[ s(i) = \max\{s - \delta(i - 1), 0\} \]  

(3)

where \( \delta \) represents the learning rate. The next proposition shows how excessive fundraising changes when we move from constant returns to scale in fund-raising to learning by fund-raising.

**Proposition 5** Consider two scenarios: constant returns to scale, \( \delta_{nl} = 0 \), and learning by fund-raising, \( \delta_l \in (0, s) \). Excessive fund-raising is higher under learning, \( \delta_l > 0 \).

Proposition 5 says that excessive fund-raising worsens with learning. This result can be explained in terms of the effect of learning on optimal fund-raising in both cases, when charity commits to minimum gift size and when this is not feasible. On one hand, the charity fund-raises more to take advantage of cost decreases when there is no commitment. On the other hand, recall that if more than one individual is solicited in the benchmark (under commitment), it is just out of a cost recovery motive; in other words, all individuals are pivotal. Then, fund-raising shrinks with learning because for any subeconomy \( F_i, i > 1 \), total cost diminishes. This implies that the fund-raising effort required to reach a target of \( G^*(F^o) \) in the benchmark shrinks as well. Both effects push excessive fund-raising to a higher extent.

From a policy prospective, we know that a government allocation of a grant to a charity financed through taxes collected in previous periods increases the level of the public good but less than dollar by dollar. One of the reasons is that a grant displaces private giving. In addition, Andreoni and Payne (2003, 2011) identifies fund-raising crowding-out as an important component of total crowding-out. NY(2013) formalizes fund-raising crowding out for a costly fund-raising model with constant returns to scale in solicitations. When the charity learns through fund-raising experience a government grant introduces an additional benefit from a societal prospective. It may decrease the extent of excessive fund-raising through the fund-raising crowding-out channel. An additional solicitation turns out to be marginally less profitable because the free-rider problem is exacerbated, thus partially counteracting learning benefits. A similar benefit can be obtained through those income redistributions generating more inequality in a Lorenz sense that also undermine fund-raising incentive to acquire experience by soliciting more.
Following the logic underlying proposition 6, it seems intuitive that any increase in the rate of learning widens excessive fund-raising. Surprisingly, this statement is not necessarily correct.

**Proposition 6** *The extent of excessive fund-raising is (potentially) non-monotonic in \( \delta \).*

Proposition 6 shows that extent excessive fund-raising is affected by the rate of learning in a complex way. This result is explained in terms of a non-monotonic propensity to solicit an individual \( i > 2 \) in the purely voluntary contribution case, which is reflected in her cutoff cost being non-monotonic in the rate of learning, reaching an interior optimum. Now, to understand the source of this non-monotonicity, note first from (3) that there is some threshold rate for individual \( i > 2 \), \( \delta^*_1 \), such that the marginal benefit from learning, \( s - s(i) \), increases in the rate of learning, \( \delta \), for \( \delta < \delta^*_1 \) and remains constant for \( \delta \geq \delta^*_1 \). The learning benefit from fund-raising richer individuals lasts for a larger range of the rate of learning, i.e., \( \delta^*_1 > \delta^*_2 > \cdots > \delta^*_n \). From these two observations, it is intuitive that for a low learning rate the benefits from accumulating more experience becomes bigger with additional solicitations but benefits drop after a certain number of solicitations for a high learning rate. In other words, for a sufficiently high index individual \( i \), she is more likely to be a marginal net contributor for low learning rates than for high learning rates, where learning benefits from additional solicitations are exhausted quickly.

### 6 The learning rate and Cumulative Experience.

Even though a slower learning process may actually increase the extent of excessive fund-raising as noted in section 5, in the upside, it may surprisingly permit the fund-raiser to accumulate more experience as well, as formalized in the next Lemma. As in section 5, let 
\[
s(i) = \max \{s - \delta(i - 1), 0\}
\]
We measure cumulative experience through the marginal cost of soliciting the last individual in the optimal set, which we denote by \( c(F^o) \).

**Lemma 4** Consider two rates of learning: \( \delta_h \) and \( \delta_l \) such that \( \delta_h > \delta_l > 0 \). Then \( c(F^o(\delta_h)) \leq c(F^o(\delta_l)) \) is not always the case.

A slower learning process on one hand makes fund-raising a fixed number of individuals more costly, but on the other hand, may encourage the charity to solicit more people to reach scale economies. If the difference between learning rates is low enough, the latter effect
may outweigh the former one as stated in Lemma 3. Consequently, a charity learning more slowly may end up accumulating more fund-raising experience reflected in a lower marginal cost. This may be important for a charity periodically running fund-drives because learning spillovers would also be intertemporal in this case.

In summary, a slower learning process may have negative consequences in a static sense through augmenting the extent of excessive fund-raising. This same learning process may generate positive dynamic consequences because of deeper learning.

7 Extensions

In this section we provide three extensions. In the first one we introduce a more realistic framework in which donors contribute to charity based on two ulterior motives: altruism and warm-glow. In the second one we consider the case in which population is divided among professional groups and learning is group specific. The last one addresses decreasing returns to scale in fund-raising in a more general way than in section 4.

7.1 Warm-Glow Giving

In this section we consider warm-glow as an additional motive for giving and show how fund-raising incentives are affected by it. As in NY(2013), we assume that an individual obtains warm-glow from her net contribution. Thus, let \( u = u(x_i, G, g_i - c(i)) \) be person \( i \)'s utility function, which is increasing and strictly quasi-concave. Person \( i \)’s demand for the public good in a Nash equilibrium can be written as: \( G^* = \hat{f}(\overline{w}_i + G^*_{-i} - C(F_{-i}), G^*_{-i} - C(F_{-i})) \), where \( \overline{w}_i = w_i - c(i) \) and \( F_{-i} = F \setminus \{i\} \). Partial derivatives satisfy \( 0 < \hat{f}_{i1} < 1 \) and \( \hat{f}_{i2} \geq 0 \) by normality of goods. If, in addition, \( 0 < \hat{f}_1 + \hat{f}_2 \leq \theta < 1 \), then a unique Nash equilibrium obtains. Note that for \( \hat{f}_2 = 0 \), the warm-glow model reduces to the standard model.

To obtain a closed form solution that facilitates our comparative statics analysis, we consider the following utility for all \( i \):

\[
U_i(x_i, G, g_i) = (1 - \beta) \ln x_i + \beta \ln (\gamma G + (1 - \gamma)g_i)
\]

where \( \beta \in (0,1) \), \( \gamma = \frac{\alpha - \beta}{\alpha(1 - \beta)} \) and \( \alpha \in (\beta,1) \). Under this specification warm glow is a substitute for altruism. The demand for the public good in this case is \( G^* = \beta w_i + \frac{\beta}{\alpha} G_{-i} \). Ignoring the costly aspect of fund-raising, note that when \( \alpha = 1 \), \( G^* = \beta(w_i + G_{-i}) \). Thus, individuals give out of a pure public good motive. On the other hand, when \( \alpha = \beta \), then
Thus, the lower $\alpha$ is, the stronger is the warm-glow motive.

It can be shown that Proposition 2 holds under this utility specification, and individual $i$'s cutoff cost is given by

$$c_i = b \hat{w}_i - \frac{\alpha - \beta}{\beta} \sum_{j=1}^{i} (\hat{w}_j - \hat{w}_i).$$

(4)

It is intuitive that the more warm-glow people experience, the more incentive is the fund-raiser to solicit more, since the free-rider problem is less severe. Indeed, Eq.(4) implies that $c_i$ increases as $\alpha$ decreases. Thus, fund-raisers learn more on the job when the warm-glow motive is strong. We measure the extent of learning as cumulative experience, through the marginal cost of soliciting the last individual in the optimal set, which, as in section 6, we denote by $c(F^o)$. The next Lemma shows that the more intense is warm-glow, the more experience the fund-raiser accumulates.

**Lemma 5** Cumulative experiences increases in the relative intensity of warm-glow, $1 - \alpha$. That is, $c(F^o)$ is decreasing in $1 - \alpha$ in the interval $(0, 1 - \beta)$.

Warm-glow may incentivize the charity to invest more in learning in the following sense: If a net free-rider is identified at some stage of the solicitation process, the fund-raiser is more likely to solicit her as a learning investment.

### 7.2 Group specific learning

Suppose the fund-raiser divides the set of potential donors into $m \geq 1$ groups, depending on their professional activities. She believes that each member of group $i$ independently draws his income from a discrete distribution, $\bar{w}_i$, with mean $E[\bar{w}_i]$. We assume that the charity learns by fund-raising within a group, but this experience does not translate into cost decreases in soliciting members of other groups. Thus, let $s_i(j)$ be the sequential cost of fund-raising the $j$th individual in group $i$. The fund-raiser's strategy is to choose the number of donors to be contacted from each group. To focus the analysis on the fund-raiser side, we continue to assume that donors have no uncertainty about the income profile in the population. Moreover, to simplify the analysis, we consider identical homothetic preferences.

$\bar{C}_i^* = \beta w_i + G_{-i}$ and $g_i^* = \beta w_i$. Hence, individuals give motivated by pure warm-glow.\(^{20}\)

The parameter $\alpha$ represents the altruism coefficient as introduced in Andreoni (1989). It is a measure of the relative strength of the public good motive for giving.
so that $f(w) = \alpha w$ for some $\alpha \in (0, 1)$. Without loss of generality we rank groups according to their average disposable incomes: $E[\bar{w}_1] - a_{s_1} \geq \ldots \geq E[\bar{w}_m] - a_{s_m}$. The fund-raiser’s equilibrium strategy is stated in Proposition 9.

**Proposition 7** Let group $i$ s cutoff be given by

$$c_i = E[\bar{w}_i] - a_{s_i} - \frac{1 - \alpha}{\alpha} \sum_{j=1}^{i} n_j [(E[\bar{w}_j] - E[\bar{w}_i]) + (a_{s_i} - a_{s_j})]$$

Then $c_1 \geq c_2 \geq \ldots \geq c_n$. Moreover, every member of group $i$ is solicited iff $c < \overline{c}_i$.

The fund-raiser optimally treats each group member as having mean disposable income $E[\bar{w}_i] - a_{s_i}$. It is intuitive, then, that the fund-raiser either contacts no members of group $i$ iff $c \geq \overline{c}_i$ or solicits all of them iff $c < \overline{c}_i$. Thus, group $i$’s cutoff cost is interpreted as the average propensity of its members to pay for $c$. Note that an increase in the extent of learning economies within a group $i$, either due to the presence of more members or to a higher speed of learning, augments the group’s mean disposable income. Thus, group $i$ is more likely to be solicited and any other group less so.

We say that groups $i$ and $j$ merge if the fund-raiser knows $n_i$ and $n_j$ but is not able to distinguish among members of these groups. (NY, 2013)

Consider the case in which the technology for fund-raising any given group is $s(j)$ and two groups merge. We assume full learning spillovers within the merged group. That is, the fund-raising cost function for this group is still $s(j)$. One may think that since the merger brings more potential for learning, the public good provision increases. But this is not always the case. After a merger, available information becomes coarser in the sense that the fund-raiser does not distinguish individuals in the merged group. This effect potentially hurts the public good provision as shown in the next Lemma:

**Lemma 6** Suppose groups $i$ is solicited and group $j$ is not and they merge. If the merged group $ij$ is not solicited, i.e.,

$$E[\bar{w}_{ij}] - a_{s_{ij}} - \frac{1 - \alpha}{\alpha} \sum_{\{k \neq i,j: E[\bar{w}_k] > E[\bar{w}_{ij}]\}} n_k [(E[\bar{w}_k] - E[\bar{w}_{ik}]) + (a_{s_k} - a_{s_{ij}})] \leq c,$$

where $E[\bar{w}_{ij}]$ and $a_{s_{ij}}$ are respectively the mean income and average cost of the merged group $ij$, then the ex-ante public good provision after the merger diminishes

Lemma 6 makes explicit the tradeoff generated after a merger. On one hand, learning increases, i.e., $a_{s_{ij}} < a_{s_i}$, which makes $\overline{c}_{ij}$ increase with respect to $\overline{c}_i$. On the other hand,
coarser information hurts fund-raising in the sense that \(E[\tilde{w}_{ij}] < E[\tilde{w}_i]\). This effect makes \(e_{ij}\) fall below \(e_i\). If the latter effect is stronger, the merged group \(ij\) is not solicited. Thus, members of group \(i\), who were optimally solicited before the merger, are no longer identified by the fund-raiser, learning spillovers do not justify the inclusion of members of the merger group in the solicitation set. As a result, the public good provision declines.

It has been documented that a commercial firm may experience a depreciation in cumulative experience through time, for example by introducing new products or dealing with unexpected technology shocks. Benkard (2000) models organizational forgetting for the aircraft industry and tested that hypothesis empirically. In our setting, we may introduce incomplete learning spillovers in fund-raising when two groups \(i\) and \(j\) merge. For example, consider the following modification to the technology described above: whenever the \(k\)th individual being solicited pertains to one group and the \(k+1\)th individual results to belongs to the other group, the experience gained from fund-raising the \(k\)th individual vanishes. Let \(\tilde{a}_{s_{ij}}\) be the random variable representing the average cost of group \(ij\).

Note that \(E(\tilde{a}_{s_{ij}}) > a_{s_{ij}}\). Moreover, there are particular realizations of \(\tilde{a}_{s_{ij}}\) higher than \(\max\{a_{s_i}, a_{s_j}\}\). In these cases, fund-raising the merge group turns out to be more costly on average, than soliciting all members of any of the groups before they become indistinguishable. This is an illustration of how ex-post public good provision may diminish in the presence of fund-raising forgetting, even when coarser information is not detrimental, as in the case whenever both groups \(i\) and \(j\) are fund-raised before the merge. If the intensity of forgetting is strong enough, the effect on public good provision may be detrimental even from an ex-ante prospective.

**Lemma 7** Suppose that optimal fund-raising entails soliciting groups \(i\) and \(j\) before they merge. Consider a forgetting process such that \(E(\tilde{a}_{s_{ij}}) > \frac{n_iE(a_i) + n_jE(a_j)}{n_i + n_j}\). Then, the ex-ante public good provision after the merger diminishes.

Previous works, such as Andreoni (2013), find a negative effect of diversity on the public good provision. Its analysis comes from the donor’s side. The previous discussion suggests that a better understanding of this matter must include as well a complementary analysis from the fund-raiser side. Augmenting the diversity of a group, say through a merge, even controlling for the negative effect of coarser information on optimal fund-raising, may be detrimental for the public good provision due to negative learning spillovers (forgetting).
7.3 Decreasing returns to scale

In this section we consider a charity constrained by physical and human resources. We envision fund-raising as an increasingly costly process. The next proposition formalizes the fund-raiser’s solicitation strategy in this setting.

Proposition 8 Suppose $s(1) \leq s(2) \leq s(3) \ldots \leq s(n)$. Let $\bar{\phi}(G) \equiv \phi(G) - G$, and donor $i$’s cost cutoff be given by

$$c_i(\bar{w}_i) = \bar{w}_i - \Phi(\sum_{j=1}^{i-1} (\bar{w}_j - \bar{w}_i))$$

(cutoff costs)

Then, $\bar{c}_1 \geq \bar{c}_2 \geq \ldots \geq \bar{c}_n$, and $F^o = \{ i \in N | c < \bar{c}_i \}$

A technology with decreasing returns to scale reinforce the free-rider problem thus making the charity even more conservative in soliciting an additional subject, with respect to the constant marginal cost setting. Furthermore, absent a learning motive, once a net free rider is identified, the solicitation process stops. Consequently, as in NY (2013), every solicited individual is a net contributor in $F^o$. Indeed, it is intuitive that as the charity experiences more rapid diseconomies of scale, the fund-raiser set shrinks together with the public good provision. Moreover, the degree of excessive fund-raising tends to diminish in a setting where donors have a low taste for the public good.

8 Conclusion

In this paper we extend the literature on charitable fund-raising by bringing to the center of the analysis the role of solicitation technology in optimal fund-raising. It is characterized in terms of donors’ preference and incomes as well as solicitation costs. We also develop a notion of excessive fund-raising within a single charity framework with respect to a benchmark in which the fund-raiser commits to minimum gift sizes.

We specially consider a charity which becomes a more efficient fund-raiser with each additional solicitation. This fact is not innocuous in terms of optimal fund-raising and excessive fund-raising. On the contrary, on one hand, it determines an investment in learning incentive. For instance, some charities may launch a fund-drive even when initial donations are not sufficient to cover initial costs. However, it is common knowledge that the charity fund-raises more to achieve cost reductions, which ensures the charitable good provision. Consequently, measures of efficiency based on average quantities such as cost to donation
ratios may severely underestimate the prospects of learning in interim fund-raising campaigns. On the other hand, the extent of excessive fund-raising augments when we move from a constant return to scale technology to a situation of learning through fund-raising experience.

From a policy perspective, the introduction of direct government grants have relevant effects in environments where the fund-raiser learns on the job. In a static sense, it alleviates excessive fund-raising by accelerating free-riding. As a consequence, the charity accumulates less experience, which may be detrimental for the public good provision in a dynamic sense. Thus, government grants may be more beneficial for occasional fund-drives with considerable learning potential.

In big fund-drives, since donors are unable to keep track of solicitations to others, they form beliefs about those. We show that in some cases, a strong charity learning potential may undermine its commitment to its optimal fund-raising strategy. As a result, initial donors may be cautious in the size of their donations conjecturing that the charity will solicit too much. This observation is related to the common practice of charities to set soft fund-raising targets in terms of money and time. The fulfillment of those targets generates credibility for future campaigns.

We further consider an extension of the model incorporating warm-glow. Charities in sectors where donors experience strong warm-glow invest more in learning and accumulate more experience. Considering a framework where charities endogenously choose a fund-raising technology, it is more likely to observe returns to scale to solicitations in those causes related to intense warm-glow motives.

In another extension we characterize optimal fund-raising in a setting in which the population is divided among professional groups and learning takes place exclusively within each group. We find that increasing the diversity within groups by merging those may be detrimental for the public good provision due to negative learning spillovers.

For future research, it may be worthwhile to characterize optimal fund-raising when there are several charities competing with horizontally differentiated causes, as in Andreoni and Payne 2001.

Appendix

Proof of Observation 1. Suppose the optimal strategy consists in fundraising \(k\) individuals. Let \(C_j = \sum_{i \leq j} c(i)\). Consider the case in which the first \(i\) individuals are
included in the set, where $i < k$. Denote $G^*(F_{i+l}) = G^*(F \cup \{l\})$. Note that by including any individual $l \geq i$ such that $g_i^l(F_{i+l}) > 0$, by (??), it follows that every individual $j \leq i$ is also a contributor. Moreover, $G^*(F_{i+l}) - C$ solves

$$
(i + 1) \left[ \phi(G^*(F_{i+l}) - C_{i+1}) - (G^*(F_{i+l}) - C_{i+1}) \right] + G^*(F_{i+l}) - C_{i+1} = \sum_{j=1}^{i} w_i + w_l - C_{i+1}
$$

Thus, $G^*(F_{i+1}) \geq G^*(F_{i+l})$ for any $k \geq l \geq i + 1$, since $w_{i+1} \geq w_l$ and \( \phi' > 0 \). The proposition follows from applying iteratively the previous result. □

**Definition.** Let $G_i^0(c)$ be the “drop-out” level of the public good for person $i$ under net income $w_i - c$, which uniquely solves $f(w_i - c + G_i^0) = G_i^0$. By convention $G_i^0(c) = 0$ whenever $w_i - c \leq 0$.

**Lemma A1.** If $\overline{G}^*(F_i) > 0$ for some $F_i$, then $g_i^*(F_i) - c(i) > 0$ if and only if $G_i^0(c(i)) > \overline{G}^*(F_i)$

**Proof.** Following closely Lemma 1 in NY(2013), note that $\phi_i(\overline{G}^*(F_i)) - \overline{G}^*(F_i) = w_i - g_i^*(F_i)$, or equivalently $\phi_i(\overline{G}^*(F_i)) - \overline{G}^*(F_i) = (w_i - c(i)) - (g_i^*(F_i) - c(i))$ if $g_i^*(F_i) > 0$; and $\phi_i(\overline{G}^*(F_i)) - \overline{G}^*(F_i) \geq w_i$ if $g_i^*(F_i) = 0$. Since $\phi_i(G_i^0(c(i))) - G_i^0(c(i)) = w_i - c(i)$, and $\phi'_i > 1$, the Lemma follows. □

**Definition.** Let $\Phi_i(\overline{G}) \equiv \sum_{j=1}^{i} (\phi(\overline{G}) - \overline{G}) + \overline{G}$, where $\phi \equiv f^{-1}$ and $\Phi'_i(\overline{G}) > 0$. Define

$$
\Delta_i(c(i)) \equiv \Phi_i(G_i^0(c(i))) - \sum_{j=1}^{i} (w_j - c(j))
$$

**Corollary 1** If $\overline{G}^*(F_i) > 0$ for some $F_i$, then $g_i^*(F_i) - c(i) > 0$ if and only if $\Delta_i(c(i)) > 0$.

**Proof of Lemma 1.** This proof follows closely NY(2013). Define $\overline{c}_i$ the value of $c$ making $\Delta_i(c(i)) = 0$. Simplifying terms, $\overline{c}_i$ solves:

$$
i_i[\phi(G_i^0(c(i))) - G_i^0(c(i))] + G_i^0(c(i)) - \sum_{j=1}^{i} (w_j - s(j)) + ic = 0.
$$

Since $\phi(G_i^0(c(i))) - G_i^0(c(i)) = w_i - s(i) - c$, from the equation above, we have

$$
G_i^0(\overline{c}_i + s(i)) = \sum_{j=1}^{i} [(w_j - w_i) + (s(i) - s(j))].
$$

In addition, given that $\overline{\phi}(G) \equiv \phi(G) - G$, we also have $\overline{\phi}(G_i^0(\overline{c}_i + s(i))) = w_i - s(i) - \overline{c}_i = \overline{\phi} \left( \sum_{j=1}^{i} [(w_j - w_i) + (s(i) - s(j))] \right)$, which reduces to

$$
\overline{c}_i = w_i - s(i) - \overline{\phi} \left( \sum_{j=1}^{i} [(w_j - w_i) + (s(i) - s(j))] \right).
$$
Let \( \hat{w}_i = w_i - s(i) \). Then \( \varphi_i = \hat{w}_i - \overline{\varphi}(\sum_{j=1}^{i}(\hat{w}_j - \hat{w}_i)) \). Finally, notice that \( \Delta_i > 0 \) iff \( c < \varphi_i \), then, by the previous corollary, the proposition follows. ■

**Proof of Proposition 1.** By noting that if \( \overline{G}^* (F_i) > 0 \) then it satisfies \( \Phi_i(\overline{G}^* (F_i)) = \sum_{j=1}^{i}(w_j - c(j)) \) and by Lemma A1, it follows that \( \Delta_i(w_i - c(i)) > 0 \) iff \( g^*_i(F_i) - c(i) > 0 \). Consider first the case where \( i = 1 \). Take the lowest index individual \( k \geq i \) s.t. (i) \( \Delta_k(w_k - c(k)) > 0 \) and (ii) \( \sum_{j=1}^{k}(w_j - c(j)) > 0 \). Clearly \( \overline{G}^*(F_k) > G^Q_k(c(k)) > 0 = \overline{G}^*(\emptyset) \).

Now consider \( i > 1 \) and individuals \( 1, 2, ..., i-1 \) are solicited. Take the lowest index individual \( k \geq i \) s.t. \( \Delta_k(w_k - c(k)) > 0 \). Notice that \( g^*_k(F_k) > 0 \). Thus, \( \overline{G}^*(F_k) > \overline{G}^*(F_{i-1}) \) iff \( \sum_{j=1}^{k}[g^*_j(F_k) - c(j)] > 0 \). Let \( w'_j - c'(j) = w_j - c(j) \) for \( j < i \) and \( w'_j - c'(j) = a_{w-c}(ik) \) for \( i \leq j \leq k \). This implies \( \Delta_i(w'_i - c'(i)) = \Delta_{i+1}(w'_{i+1} - c'(i+1)) = ... = \Delta_k(w'_k - c'(k)) \).

Thus \( g^*_j(w'_j, F_k) - c'(j) > 0 \) for every \( j = i, i+1, ..., k \) iff \( \Delta_i(a_{w-c}(ik)) > 0 \). Note also that

\[
\sum_{j=1}^{k}[g^*_j(w'_j, F_k) - c'(j)] = \sum_{j=1}^{k}a_{w-c}(ik) + \sum_{j=1}^{i-1}(w_i - c(i)) \]

\[
- \sum_{j=1}^{i-1}[g^*_j(w'_j, F_k) - c'(j)]
\]

\[
= \sum_{j=1}^{k}(w_i - c(i)) - \sum_{j=1}^{i-1}[g^*_j(w'_j, F_k) - c'(j)]
\]

\[
= \sum_{j=1}^{k}[g^*_j(w'_j, F_k) - c'(j)]
\]

The first equality above is valid since individuals \( i, i+1, ..., k \) are gross contributors, under both income distributions. Thus, we obtain the result

\[
\overline{G}^*(F_k) > \overline{G}^*(F_{i-1}) \text{ iff } \sum_{j=1}^{k}[g^*_j(F_k) - c(j)] > 0 \text{ iff } \Delta_i(a_{w-c}(ik)) > 0.
\]

Consider the case in which \( k > i \). Suppose there is no \( i \leq l < k \) such that \( \Delta_i(a_{w-c}(lk)) > 0 \). In this case it is optimal to include \( i, i+1, ..., k \) in \( F^o \). Thus, by Lemma 1 the proposition follows. ■

**Proof of Proposition 2.** (i) follows by noticing that cost cutoff for individuals \( i > 1 \) are increasing in the setup cost and the fund-drive is launched for both fixed cost levels under consideration. To prove (ii) by Let \( \Phi_{F^o(s)}(\overline{G}) \equiv \sum_{i \in F^o(s)}(\phi(G) - \overline{G}) + \overline{G} \) and \( b \) be the number of solicitations under \( F^o(s') \). Then, by using equilibrium conditions

\[
\Phi_{F^o(s')}(\overline{G}^*) = \sum_{i \in F^o(s')}(w_i - c) - s
\]

\[
> \sum_{F^o(s')}(w_i - c) - s' = \Phi_{F^o(s')}(\overline{G}^*(F^o(s')))
\]

30
The inequality comes from $s < s'$. By strict normality, $\Phi'_{F_o(s)}(.) > 0$. Thus, $\overline{G}^* \leq \overline{G}^*(F_o(s))$, by a revealed preference argument. Thus, $\overline{G}^*(F_o(s)) > \overline{G}^*(F_o(s'))$. Finally, note that in equilibrium $g_i^*(F_o(s')) = w_i + \overline{G}^*(F_o(s')) - \phi_i(F_o(s'))$.

Since $\overline{G}^*(F_o(s)) > \overline{G}^*(F_o(s'))$ and $\phi_i' > 1$, it follows that $g_i^*(F_o(s')) > g_i^*(F_o(s))$ for every $i \in F_o(s')$. Thus, (iii) follows. ■

**Proof of Proposition 3.** Let $F_i$ be donor $i$’s belief about the fund-raiser set when he is contacted. Then, as stated in the text, $F_i = F^0$ if $i \in F^0$, and $F_i = \{1, 2, 3, \ldots, i\}$ if $i \notin F^0$.

We will show that given the beliefs $\{F_i\}_{i=1}^n$, contacting $j \notin F^0$ is not a profitable deviation for the fund-raiser. Let $k$ be the lowest index individual being solicited under $F^0$. Let $g_k^0$ be $j$’s contribution under the stated belief system. We first show that $\sum_{i=k+1}^j [g_i^0 - c(i)] \leq 0$ for any $j \geq k + 1$. By contradiction, suppose $\sum_{i=k+1}^j [g_i^0 - c(i)] > 0$ for some $j \notin F^0$.

Then, there must be some $k + 1 \leq l \leq j$ such that $g_l^0 > g_l^*(F_j)$ since by proposition 2

$$\sum_{i=k+1}^j [g_i^0 - c(i)] \leq 0. \quad (5)$$

On the other hand, if the individuals in $F_j$ knew about the presence of the other ones before contributing, then,

$$\phi(\overline{G}^*(F_j)) - \overline{G}^*(F_j) = w_l - g_l^*(F_j). \quad (6)$$

Now, $g_l^0 > g_l^*(F_j)$ implies $w_l - g_l^*(F_j) > w_l - g_l^0$. Then, since $\phi' > 1$, eq.(5) and (6) reveal that $\overline{G}^*(F_j) > \overline{G}^*(F^0) + \sum_{i=k+1}^j [g_i^0 - c(i)]$. This contradicts $\overline{G}^*(F_j) \leq \overline{G}^*(F^0)$. ■

**Lemma A2.** Suppose $G_1^0 > G_2^0 > \ldots > G_n^0 > 0$. Then

(a) there are some replicas $r_1 \leq \ldots \leq r_2 < \infty$ such that type-$i$ donors are not solicited in any $r \geq r_i$ replica economy.

(b) As $r \to \infty$, only type-1 donors are solicited, in which case each donation converges to the solicitation cost, $c_1$, but the public good level approaches to $G_1^0$.

**Proof.** We first claim that if $\overline{G}^*(F) > 0$ for some $F$, then $i \in F$ is a net contributor in equilibrium, i.e., $g_i^*(F) - c_i > 0$ if and only if $G_i^0 > \overline{G}^*(F)$. As we argued in the above proof, in a positive equilibrium, $\phi_i(\overline{G}^*(F)) - \overline{G}^*(F) = w_i - g_i^*(F)$, or equivalently $\phi_i(\overline{G}^*(F)) = (w_i - c_i) - (g_i^*(F) - c_i)$ if $g_i^*(F) > 0$; and $\phi_i(\overline{G}^*(F)) - \overline{G}^*(F) \geq w_i$ if $g_i^*(F) = 0$. Since $\phi_i(G_i^0) - G_i^0 = w_i - c_i$ by the definition of drop-out value, and $\phi_i' > 1$, the claim follows. ■
Now, note that since \( \phi_{i+1}(G^0_{i+1}) - G^0_{i+1} = w_{i+1} - c_{i+1} \), we have

\[
\Delta_i - \Delta_{i+1} = G^0_i - G^0_{i+1} + \sum_{j=1}^{i} [(\phi_j(G^0_j) - G^0_j) - (\phi_j(G^0_{i+1}) - G^0_{i+1})].
\]

Given that \( G^0_i \geq G^0_{i+1} \) and \( \phi'_{j} > 1 \), it follows that \( \Delta_i \geq \Delta_{i+1} \). Moreover, \( \Delta_1 = G^0_1 > 0 \).

Next, let \( k \in N \) be the largest number such that \( \Delta_k > 0 \). Since \( \Phi_k(0) = 0, \Phi'_{k} > 0 \), and \( \sum_{j=1}^{k} (w_j - c_j) > 0 \), there is a unique solution, \( G'^{*} > 0 \), to \( \Phi_k(G'^{*}) = \sum_{j=1}^{k} (w_j - c_j) \).

If this is an equilibrium, each \( i = 1, ..., k \) is a net contributor because \( G^0_i > G'^{*} \). \( G'^{*} \) is an equilibrium among these individuals if \( \sum_{j=1}^{k} c_j - G'^{*}_{-i} < \hat{C}_i \) for \( i = 1, ..., k \). Note that

\[
\sum_{j=1}^{k} c_j - G'^{*}_{-i} \leq \sum_{j=1}^{k} c_j - \sum_{j \neq i} c_j = c_i.
\]

But, by Assumption S, \( c_i < \hat{C}_i \). Finally, Lemma A1 implies that \( G'^{*} = \Phi_{-1}^{-1}(\sum_{j=1}^{k} (w_j - c_j)) \).

Proof of Lemma 2. Let \( w'_1 = \sum_{i=1}^{n} w_i - n \epsilon \) and \( w'_i = \epsilon \) for \( i > 1 \) and \( \epsilon \) close to zero. Then, \( F^o(w') = \{1\} \) under any of the proposed cost distributions. Since \( c(1) = s + c < c'(1) = s' + c' \), \( \Phi'(c, s, w') > G'^{*}(c', s', w') \). Now by Lemma A1, there exists \( r(c, s, w), r(c', s', w) > 0 \) such that every individual of type 1, i.e., richest individuals, are solicited in both cases. Take \( r = \max \{r(c, s, w), r(c', s', w)\} \). Then, for \( r > r \) the fund-raiser set is identical under either cost distribution. It just entails soliciting the richest income individuals (type-1-individuals). Then, for \( r \) big enough \( |F^o(r)(c - c')| > s' - s \) which implies \( G'^{*}(c, s, w) < G'^{*}(c', s', w) \) since both quantities are bounded, by Lemma A2 and the strict normality assumption.

Definition. Let \( t_i(T_{-i}) \) be the value of \( t_i \) satisfying \( U(w_i - t_i(T_{-i}), t_i(T_{-i}) + T_{-i}) - U(w_i, T_{-i}) = 0 \)

Denote \( G^0_i(0) \) simply as \( G^0_i \), i.e., \( f(w_i + G^0_i) = G^0_i \).

Let \( t^*_i \) be individual \( i \) threshold gift in a solo economy, i.e., \( t^*_i \) satisfies \( U(w_i - t^*_i, t^*_i) = U(w_i, 0) \).

Define individual \( i \)'s threshold gift as a function of others' threshold gifts as

\[
t_i(T_{-i}) = \tilde{T}(w_i + T_{-i}, T_{-i}, w_i) - T_{-i},
\]

(7)
Lemma A3. \( \hat{T} \) satisfies:

1. \( \hat{T}(w_i + T_{-i}, T_{-i}, w_i) - T_{-i} > 0 \) for every \( T_{-i} \in [0, G_i^0) \)
2. \( \hat{T}(w_i + G_i^0, G_i^0, w_i) = G_i^0 = 0 \)
3. \( \hat{T}_1 > 0, \hat{T}_2 < 0, \hat{T}_3 < 0 \)
4. \( \hat{T}(w_i + T_{-i}, T_{-i}, w_i) > f(w_i + T_{-i}) \) for every \( T_{-i} \in [0, G_i^0) \)
5. \( \hat{T}_1 + \hat{T}_3 > 0 \)

Proof. Note that by quasiconcavity and by assumption M we respectively know that (i) \( U_{ii} < 0 \) and \( U_{12} > 0 \). Moreover, by definition of \( g_i(T_{-i}) \) it follows that (ii) \( U_1(w_i - g_i(T_{-i}), g_i(T_{-i}) + g_i(T_{-i} + T_{-i}) = U_2(w_i - g_i(T_{-i}), g_i(T_{-i}) + T_{-i}). \) Quasiconcavity of the utility function also implies that (iii) \( t_i(T_{-i}) > g_i(T_{-i}). \) Therefore, (iv) \( U_1(w_i - t_i(T_{-i}), t_i(T_{-i}) + T_{-i}) > U_2(w_i - T_i(T_{-i}), T_i(T_{-i}) + T_{-i}) \) by (i), (ii) and (iii). By differentiating \( U(w_i - t_i(T_{-i}), t_i(T_{-i}) + T_{-i}) - U(w_i, T_{-i}) = 0 \) wrt \( T_{-i} \) we obtain \( \frac{\partial U}{\partial T_{-i}} = \frac{U_2(w_i, T_{-i}) - U_2(w_i - t_i(T_{-i} + t_i))}{U_1(w_i - t_i(T_{-i} + t_i))}. \) The sign of \( \frac{\partial U}{\partial T_{-i}} \) is negative by (i) and (iv). After adding \( T_{-i} \) to each side of individual i’s budget constraint we obtain \( x_i + T = w_i + T_{-i} \), then as in the standard public good model (BBV), the first term of \( \hat{T} \) captures the direct positive effect of \( w_i + T_{-i} \) on \( T \). The second term of \( \hat{T} \) captures the negative effect of \( T_{-i} \) on \( T \) through a better outside option. The third term captures the negative effect of \( w_i \) on \( T \) through a better outside option. Thus, (3) follows. Part (4) also follows by quasiconcavity. By quasiconcavity of the utility function, \( t_i^* > 0 \). Moreover \( t_i^* = \hat{T}(w_i, 0, w_i) > f(w_i) = g_i^0. \) Note that by definition of \( G_i^0, U(w_i, G_i^0) > U(w_i - g_i, G_i^0 + g_i) \) for any \( g_i > 0. \) Thus, \( \hat{T}(w_i + T_{-i}, T_{-i}, w_i) - T_{-i} > 0 \) for every \( T_{-i} \in [0, G_i^0) \) and \( \hat{T}(w_i + G_i^0, G_i^0, w_i) = G_i^0 = 0 \)

Finally to obtain (5) we differentiate \( U(w_i - t_i(T_{-i}), t_i(T_{-i}) + T_{-i}) - U(w_i, T_{-i}) = 0 \) wrt \( w_i \). Thus, \( \frac{\partial U}{\partial w_i} = \frac{U_2(w_i, T_{-i}) - U_2(w_i - t_i(T_{-i} + t_i))}{U_1(w_i - t_i(T_{-i} + t_i))} \). The sign of \( \frac{\partial U}{\partial w_i} \) is positive by (i) and (iv).

For the following lemmas and propositions we omit the third argument of \( \hat{T} \), knowing that \( \hat{T} \) is increasing in \( w_i \), by Lemma A3, part (5)

Lemma A4. In the costless case, there exists a solution to the taxation problem unique up to total taxation \( T^* \). Moreover, if \( \hat{T}_1 + \hat{T}_2 < 0 \) the optimal solution consists of at most one individual threshold gift to be positive.
Proof. Fixing a set $F$, existence follows directly from Brower’s fixed point. For the case in which $\widehat{T}_1 + \widehat{T}_2 < 0$ note that $\frac{dt_i}{dT_i} < -1$, thus it directly follows that the optimal solution entails taxing just one individual. $\blacksquare$

Lemma A5. Suppose that under costless fund-raising, $\frac{dt_i}{dT_i} < -1$, then, facing a total cost $C$, individual 1 is the only one being solicited if $C \leq t^*_1 - G^0_2$. More individuals are contacted iff $C > t^*_1 - G^0_2$ (but without being too big to totally discourages giving). In this latter case, solicited donors beyond individual 1 partially recover the cost, i.e., $\sum_{i>1} t^*_i < C$. Moreover every donor is pivotal, i.e., $\frac{dt_i}{dT_i} > 0$ for every $i$ s.t. $t^*_i > 0$

Proof. Note that $\widehat{T}_1 + \widehat{T}_2 > 0$ implies that stand-alone gifts are ranked according to incomes. Consequently, if the optimal solicitation strategy dictates to contact just a single individual, this must be individual 1. Define $t^*_i$ as individual $i$'s standalone value. Consider first the case in which $C \leq t^*_1 - G^0_2$. It means that $t_i = 0$ for any $i > 1$. Thus, just individual 1 is solicited in this case. Now, consider the case in which $C > t^*_1 - G^0_2$. It implies that $t_2(t^*_1 - C) > 0$. Therefore, at least one more individual is solicited. Suppose $\sum_{i>1} t^*_i = C$. From individual 1 prospective the economy is costless. Therefore, $t^*_1 > G^0_1$ because $\frac{dt_i}{dT_i} < -1$. Then, it also follows that $t^*_i > G^0_i$ for $i > 1$. This contradicts $t^*_i > 0$ for $i > 1$. Suppose $\sum_{i>1} t^*_i > C$, then by implementing $t^*_2 - \epsilon$ instead of $t^*_2$, the fund-raiser would be able to increase individual 1 threshold gift by more than $\epsilon > 0$ since $\frac{dt_i}{dT_i} < -1$. This contradicts the optimallity of $\{t^*_i\}_{i \in F^*}$. To consider the last part of the lemma we adapt the definition of $t^*_i$ for costly fund-raising: $t_1 - C = \widehat{T}(w_1 + T_{-1} - C, \max \{T_{-1} - C, 0\}) - \max \{T_{-1} - C, 0\}$. Hence, $\sum_{i>1} t^*_i < C$ implies $t^*_i(T_{-i}) = \widehat{T}_1 > 0$. More generally, consider the case in which the fund-raiser optimally solicits at least two individuals. Pick individuals $i$ and $j$. We show that it must be the case that $t^*_i, t^*_j > 0$. That is, all individuals are pivotal. By way of contradiction, suppose it does not follow. Let $t^*_i < 0$, i.e., $T_{-i} - C > 0$ for some $i$, then by reducing $j$’s gift size by one unit, the fund-raiser is able to raise individual $i$’s threshold gift by more than one unit, which constitutes a profitable deviation. Contradiction. $\blacksquare$

Proof Observation 3. Consider first the case in which $\frac{dt_i}{dT_i} > -1$ for $C = 0$ i.e., $0 < \widehat{T}_1 + \widehat{T}_2 < 1$. We show that $t^*_i > t^*_{i+1}$ whenever $w_i > w_{i+1}$. By way of contradiction suppose $t^*_{i+1} > t^*_i$. Note that there exists an inverse function $\widehat{\phi}(T, w_i)$, where $T = T - C$ such that $\widehat{T}_1 > 0$ and $\widehat{\phi}_2 < 0$ and

$$t^*_i = T^* - \widehat{\phi}(T^*, w_i) < t^*_{i+1} = T^* - \widehat{\phi}(T^*, w_{i+1}).$$

Since $\widehat{\phi}_2 < 0$, it implies $w_i < w_{i+1}$ which is a contradiction. Now consider $\frac{dt_i}{dT_i} < -1$ when $C = 0$, i.e., $\widehat{T}_1 + \widehat{T}_2 < 0$. We provide a local
argument to show the result. Clearly individual 1 is solicited since she is the one providing the greatest stand-alone value. If two or more solicitations are made \( t_i^* > 0 \) for every \( i \) s.t. \( t_i^* > 0 \) by Lemma A4. Therefore, \( \hat{T}_1 + \hat{T}_2 = \hat{T}_1 > 0 \). Thus, at \( T = T^* \), there exists an inverse function \( \phi(w_i) \) such that \( \phi' < 0 \) and

\[
t_i^* = T^* - \phi(w_i) < t_{i+1}^* = T^* - \phi(w_{i+1}).
\]

Since \( \phi' < 0 \), it implies \( w_i < w_{i+1} \) which is a contradiction.

Finally, to prove the last part of the observation, consider some individual providing a gift \( y_i^* > t_i^* \). Then \( U(x_i - y_i^*, y_i^* + T_i^* - C) > U(w_i, \max \{T_i^* - C, 0\}) \). This means that the threshold gift is set below individual \( i \) best response to \( T_i^* - C \). By quasiconcavity of the utility function there exists \( t_i' > y_i^* \) such that \( U(x_i - t_i', t_i' + T_i^* - C) = U(w_i, \max \{T_i^* - C, 0\}) \), which contradicts the optimality of \( \{t_i^*\}_{i \in F^*} \).

**Proof Observation 4.** The voluntary contributions \( \{g_i^s\}_{i=1}^{\vert F \vert} \) is a feasible solution to \( (2) \). Pick individual 1. Note that since \( g_1^s \) is her best response to \( G_1^s - C \) then \( U(w_1 - g_1^s, g_1^s + G_1^s - C) > U(w_1, G_1^s - C) \). Therefore, by quasiconcavity of \( U() \) it follows that \( g_1^s < t_1(G_1^s) \). Hence, by fixing \( t_1(G_1^s) \) we obtain \( G^s < t_1(G_1^s + G_{-1}(t_1)) \), by strict normality. Since \( G_{-1}(t_1) < G_1^s \) then \( U(w_1 - t_1(G_1^s), t_1(G_1^s) + G_1^s - C) = U(w_1, G_1^s - C) > U(w_1, G_{-1}(t_1(G_1^s) - C)). \) Thus, \( \{t_1(G_1^s), G_{-1}(t_1)\} \) is a feasible solution to \( (2) \). We have found a profitable deviation from \( G^s \) from the fund-raiser’s prospective. Thus, \( G^s < T^* \).

**Proof of Proposition 4.** This Lemma follows directly from observation 4.

**Proof of Lemma 3.** Suppose at least two individuals are solicited in the voluntary contribution model. We know from Lemma A4, that in this case every individual is pivotal, i.e., \( \frac{dt_i}{dx_i} > 0 \). This means that \( T_i^s - C < 0 \) for every \( i \in F^s \). For the next step, fix \( F^s \). By quasiconcavity of the utility function we know that \( t_i^s > g_i^s \). Consider \( T_i^s > G_i^s \) for individual \( i \). It follows that \( t_i(T_i^s) > t_i(G_i^s) > g_i(G_i^s) \). The first inequality comes from the pivotality condition \( T_i^s - C < 0 \), that is own gifts and gifts by other are complementary, as well as from \( T_i^s(F^s) > G_i^s(F^s) \). The second inequality comes from the quasiconcavity of the utility function. Thus \( t_i^s(F^s) > g_i^s(F^s) \) for every \( i \in F^s \). To conclude the proof we show that \( i \in F^s \) implies \( i \in F^o \). Note that \( i \in F^s \) implies \( G_i^s(F^s) - C_i < T_i^s(F^s) - C_i < c(i) \). The former inequality follows because \( G_i^s < T_i^s \). This allows us to conclude that individual \( i \) is not a free rider since no public good would be provided in \( F^s \) without her contribution.

By assumption \( S \) either \( F^s = F^o \) and by proposition 1, \( g_i^s(F^s) > c(i) \) or \( F^s \subseteq F^o \).
Lemma A6. (Voluntary contributions) For every $i > 2 \triangledown \tau_{i}(\delta = 0) < \tau_{i}(\delta = s)$. Moreover, for any: $\delta'' > \delta'$ (i) $\tau_{i}(\delta') < \tau_{i}(\delta'')$ for $0 \leq \delta', \delta'' < \frac{s}{i-1}$ and (ii) $\tau_{i}(\delta') \geq \tau_{i}(\delta'')$ for $\frac{s}{i-1} < \delta', \delta'' \leq s$

Proof. Note that

$$\tau_{i}(s) = w_{i} - \Phi(\sum_{j=1}^{i-1} [(w_{j} - w_{i}) - s]) > \tau_{i}(0)$$

$$= w_{i} - s - \Phi(\sum_{j=1}^{i-1} [(w_{j} - w_{i})])$$

establishing the first part of the Lemma. Moreover,

$$\tau_{i}(\delta) = w_{i} - s + \delta(i - 1) - \Phi(\sum_{j=1}^{i-1} [(w_{j} - w_{i}) - \delta(i - 1) \frac{s}{2}])$$

for $0 \leq \delta < \frac{s}{i-1}$. Thus, $\frac{\partial \tau_{i}(\delta)}{\partial s} = i - 1 + \Phi'(\cdot)(i - 1) \frac{s}{2} > 0$. On the other hand for $\frac{s}{i-1} \leq \delta < s$, note that $\tau(\delta) = w_{i} - \Phi(\sum_{j=1}^{i-1} [(w_{j} - w_{i}) + (\max \{i - 1)\delta, s\} - s])$. Thus,

$$\frac{\partial \tau_{i}(\delta)}{\partial s} = -\Phi'(\cdot) \left(\frac{(k-1)k}{2}\right) < 0$$

where $k$ is the highest index individual with $(k - 1)\delta < s$.

Lemma A7. For any $\delta_{1}, \delta_{2}$ such that $\delta_{1} < \delta_{2}$ it must follow that $F^*(\delta_{2}) \subseteq F^*(\delta_{1})$

Proof. Consider first the benchmark with constant marginal cost. Note that if $t_{1}(0) - c - s \geq G^0_{2}$, then, it follows that the optimal fund-raising strategy in the benchmark consists in soliciting exclusively individual 1. Moreover, this strategy is fixed for any learning rate. On the other hand, consider $|F^*| \geq 2$. By Lemma A5 we know that if an individual $i \geq 2$ is solicited, then she must be pivotal.

Note that an increase in $\delta$ lowers $C(F^*)$. Therefore, if individual $i > |F^*|$ was not necessary to cover $C(F^*)$ before the $\delta$-increase, it would not be contacted once $\delta$ increases. Thus, for any $\delta_{1}, \delta_{2}$ such that $\delta_{1} < \delta_{2}$ it must follow that $F^*(\delta_{2}) \subseteq F^*(\delta_{1})$.

Proof of Proposition 5. Consider the voluntary case. By Lemma A5, $\tau_{i}(0) < \tau_{i}(\delta)$ for any 0 \leq \delta \leq \frac{s}{i-1}$. Thus, (i) $F^0(0) \subseteq F^0(\delta)$. From Lemma A7, we know that $F^*(\delta) \subseteq F^0(0)$. Since $\eta(G^*(F^0(\delta))) \leq |F^*(\delta)| \leq |F^0(\delta)|$ and by (i) the proposition follows.

Proof of Proposition 6. We first show that fund-raising in the pure voluntary contribution case is potentially non-monotonic in $\delta$. Consider $|N| > 2$. Let $i$ be the highest index in the set. Fix $w_{1}, w_{2}, .., w_{i-1}$ such that, (i) $w_{j} \geq w_{j+1} + s$ for every $j = 1, 2, .., i - 1$, i.e., cutoff costs are monotonically decreasing, (ii) $c < \tau_{i-1}(0)$, i.e., every $j < i$ is a net
contributor for any $0 \leq \delta \leq s$, Note that there exists $\bar{w}_i > 0$ such that $c_i(\bar{w}_i, s) = c$, or, equivalently $\bar{w}_i - s - \phi(\sum_{j=1}^{i-1}(w_j - \bar{w}_i) - s) = 0$. This comes from $c_i(w_{i-1}, s) > c$, $\frac{\partial c_i(w, s)}{\partial w} = 1 + \phi'(i - 1)$. On the other hand, let $\bar{w}_i > 0$ solves $c_i(\bar{w}_i, \frac{s}{i-1}) = c$. That is, $\bar{w}_i - s - \phi(\sum_{j=1}^{i-1}[(w_j - \bar{w}_i) - \frac{i}{2}s]) > 0$. Since $c_i(.)$ is increasing in $i$, it follows that $\bar{w}_i < w_i$. Moreover, $w_{i-1} > \bar{w}_i$. Pick any $\bar{w}_i \geq w_i > w_i$. Then:

$$c_i(0) < c_i(s) \leq c < c_i\left(\frac{s}{i-1}\right) \quad (8)$$

Thus, given that every $j < i$ is in $F^0$ for any $\delta$, by (ii) and from (8) it follows that $F^0(0) = F^0(s) \subset F^0(\frac{s}{i-1})$.

By this result, Lemma A7, and the fact that $\overline{\pi}(G^*(F^0(\delta))) \leq |F^*(\delta)|$, it follows then that excessive fund-raising is potentially non-monotonic in $\delta$.

**Proof of Lemma 4.** A particular example works. Consider $N = \{1, 2, 3\}$. Suppose $w_1 > w_2 + c$ and $w_2 > w_3 + c$. Let $\bar{w}_3$ solves $\overline{\pi}(\bar{w}_3, c - c_l) = c$. That is,

$$\bar{w}_3 = \phi(\sum_{j=1}^{2}[(w_j - \bar{w}_3) - (c - c_l)]).$$

Let $w_3$ solves $\overline{\pi}(w_3, \frac{c - c_l}{2}) = c$. That is,

$$w_3 = \phi(\sum_{j=1}^{2}\left[(w_j - w_3) - \frac{3}{2}(c - c_l)\right]).$$

Check that $w_1$ and $w_2$ are big enough such that $w_3 > 0$. Pick any $\bar{w}_3 \geq w_3 > w_3$. Then, $\overline{\pi}_3(w_3, 0) < \overline{\pi}_3(w_3, c - c_l) \leq c < \overline{\pi}_3(w_3, \frac{c - c_l}{2})$. Now, let $\delta^*$ solves $\overline{\pi}_3(w_3, \delta^*) = c$. Let $\delta_h = \delta^* + c, \delta_l = \delta^*$. So, $F^0(\delta = \delta_h) = \{1, 2\}$ and $F^0(\delta = \delta_l) = \{1, 2, 3\}$. Then, $c(2, \delta_h) - c(3, \delta_l) = \delta_h - 2c > 0$ since $\delta_h > \frac{c - c_l}{2}$.

**Proof of Proposition 7.** In NY(2013) it is proven that without learning, either all members of a given group are solicited or neither of them are. Once we introduce learning, this result is reinforced in the sense that being $j$ a member of group $i$, then $E[\bar{w}_i] - s(j) < E[\bar{w}_i] - s(j + 1)$.

Thus, the cutoff cost of individual $j + 1$ is higher than the cutoff cost of individual $j$. By following the corollary of proposition 2, it, then, also follows that either all members of a given group are solicited or neither of them are. Therefore, we can redistribute income among members of group $i$ such that each of them is allocated with mean income $E[\bar{w}_i] - a_{s_i}$. As in the proof of proposition 2, such a redistribution is neutral. Thus, the result follows by applying proposition A1 in NY (2012).
Proof of Lemma 6. Notice that if group $ij$ is not solicited, then no additional learning is generated by the fund-raiser strategy. On the other hand, since group $i$ was solicited before the merger, a revealed preference argument shows that a strictly lower public good provision is expected after the merger. ■

Proof of Proposition 8. Notice that under decreasing returns to scale, the cost function is non-decreasing. Therefore, $\{\hat{w}_i\}$ is a non-increasing sequence. From (1) the result follows. ■

References


