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# TESIS DOCTORAL

# ESSAYS ON LABOUR ECONOMICS

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*A la Joana*.

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# RESUMEN

Esta tesis se centra en la aplicación de modelos de búsqueda y "mismatch" para analizar los determinantes del desempleo y de la inversión en capital humano. El primer capítulo estudia el efecto de la proporción de viviendas en propiedad sobre el desempleo. El segundo capítulo estudia como la participación de la mujer en el mercado de trabajo ha afectado ciertos agregados de la economía. El tercer capítulo analiza a nivel teórico el rol del tamaño del mercado en generar incentivos para invertir en capital humano.

El primer capítulo, titulado "Housing tenure and the labour market", estudia el efecto de la proporción de viviendas en propiedad sobre la migración y el desempleo en una economía poblada de parejas. Con este objetivo, he desarrollado un modelo de búsqueda conjunta con múltiples localizaciones donde las parejas deciden comprar o alquilar su vivienda. La calibración del modelo se ha realizado para Estados Unidos. Obtengo que los que tienen su vivienda en propiedad tienen una tasa de desempleo menor que los que alquilan a pesar de que permanecen desempleados más tiempo. Este resultado es debido a las diferencias en las tasas de transición al desempleo entre propietarios e inquilinos y la endogeneidad de la decisión de ser propietario. En un modelo de búsqueda conjunta, la mayor tasa de migración de los inquilinos implica que pueden dejar su empleo, y ser desempleados, más a menudo.

El segundo capítulo, "The effect of women participation rate on the labour market", estudia el effect de la incorporación de la mujer en el mercado de trabajo sobre el desempleo, la migración y el ahorro. Desarrollo un modelo con múltiples localizaciones donde las parejas buscan empleo, y deciden su localización y sus ahorros. Obtengo que la participación de la mujer en el mercado de trabajo incrementa la migración por motivos laborales y la tasa de desempleo masculina. Por otro lado, también encuentro una reducción sustancial del nivel de ahorro.

El tercer capítulo, "Human capital and market size", estudia como el tamaño del mercado de trabajo afecta la decisión de los trabajadores de invertir en capital humano. Considero un mercado de trabajo donde las empresas I consider a labour market where firms jerarquizan los trabajadores según su nivel de habilidades. El proceso de emparejamiento que opera en el mercado tiene la propiedad que la probabilidad de encontrar empleo de los trabajadores depende del tamaño del mercado, "market tightness" y su "ranking". Cuando la "market tightness" es alta, los mercados grandes dan más incentivos para adquirir capital humano y la distribución de habilidades converge al nivel máximo. Sin embargo, si el nivel de "market tightness" es bajo, los incentivos para invertir de los trabajadores con menor nivel disminuye con el tamaño del mercado, lo que implica una distribución de habilidades más desigual en los mercados grandes.

ABSTRACT

This thesis focuses on the application of models of search and mismatch to analyse the determinants of unemployment and human capital investment. The first chapter studies the effect of home-ownership on unemployment. The second chapter studies how women participation in the labour market has affected several aggregates of the economy. The third chapter analyses at a theoretical level the role of the size of the labour market in generating incentives to invest in human capital.

The first chapter, entitled "Housing tenure and the labour market", studies the effect of home-ownership on migration and unemployment in an economy populated by couples. To this end, a model of joint search with multiple locations and housing tenure decisions is developed and calibrated to the U.S. economy. I find that home-owners have a lower unemployment rate than renters although they suffer longer unemployment spells. This can be explained by the differences in the separation rate of jobs between owners and renters and the endogeneity of housing tenure. With joint search, the higher migration rate of renters implies that they quit their jobs, and become unemployed, more often.

The second chapter, "The effect of women participation rate on the labour market", studies the effect of women participation in the labour force on unemployment, migration and savings. I develop a model with multiple locations where couples search for jobs, and make saving and locational decisions. I find that women participation into the labour market increases work related migration and the unemployment rate of men. On the other hand, I also find a substantial decrease in the level of savings.

The third chapter, "Human capital and market size", studies how the size of the labour market affects workers' decision to invest in human capital. I consider a labour market where firms rank workers according to their level of skills. The matching process operating in the market has the property that the job finding probability of the workers depends on market size, market tightness and their ranking. When market tightness is high, bigger markets provide more incentives to acquire human capital and the distribution of skills converges to the highest level. However, if the level of market tightness is low, the incentives to invest for the workers with lower rank decrease with the size of the market, which results in a more unequal distribution of skills in bigger markets.

# 1

# HOUSING TENURE AND THE LABOUR MARKET

## 1.1 INTRODUCTION

Home-owners are less mobile than renters. In the US, 7.4% of renters migrate each year compared to only 1.7% of home-owners[1]. Being a home-owner makes migration more costly, since selling and buying a house entails some costs that renters do not need to pay. These transaction costs also reduce the incentives to migrate for job reasons. Unemployed homeowners are less willing to take jobs out of their city and remain unemployed for a longer time. At the aggregate level, this argument implies that economies with a high home-ownership rate will have a low level of migration and the labour market will be less efficient at matching workers and firms.

In this paper, we study the effects of housing tenure on mobility and unemployment. Our contribution is to consider that households consist of more than one individual. We will assume that households are composed of two members, husband and wife, that both of them belong to the labour force and make their decisions jointly. In the US, there are more couples with both spouses employed than with only one spouse employed.[2] The model economy consists of a large number of couples that can choose either to own or rent their house and that can work in different locations. In each location, there is a frictional labour market where firms and workers meet. If a firm and a worker match, wages are bargained. Job creation is determined by free entry of firms.

In a calibrated version of our model to the US economy, we find that home-owners have a lower unemployment rate than renters despite having longer unemployment spells. This is the result of two effects. First, renters have a higher separation rate from their jobs than home-owners. Second, unemployed home-owners that are liquidity constrained sell their house and become renters.

To understand the first effect, we need to take into account the interaction between the spouses. Suppose that an unemployed worker, the husband, receives a job offer from another city. If the couple decides that he should accept the job, the two spouses will migrate. However, it is likely that the wife was employed in their former city, so she will have to quit her job and move with her husband. This implies that she will be unemployed for a while until she also finds a job in the new city. Note, that the couple is better off migrating although the wife has to quit her job. Thus, the couples that are more likely to migrate, become unemployed at a higher rate. We can now include housing tenure into the argument and conclude that if owners migrate less, they will also lose their job at a lower rate than renters.

1 Inter-county migration rates from CPS 2011. The same pattern occurs for any year although the migration rate of both renters and owners have been declining over time.

2 In 2012, 58% of all married-couple families with at least one member employed were families with both spouses employed. 28% were families with only one spouse employed (Bureau of Labor Statistics).

The results of the model are consistent with the work by Coulson and Fisher (2009) that estimate a negative effect of home-ownership on the probability of being unemployed.

Given that housing tenure has implications on the labour outcomes of owners versus renters, we use our model to conduct the following policy exercise. We introduce a tax policy that favours home-ownership. Under this policy the returns from financial assets are taxed but owners imputed rents are not, which makes more profitable to keep wealth by owning a house than accumulating financial assets. The decision to buy or sell a house, thus, involves a trade-off between benefiting from the preferential tax treatment and avoiding the transaction costs. We compare this benchmark economy with an alternative economy with a positive tax on home-owners imputed rents. In this alternative economy there are less incentives to own a house and the home-ownership rate is lower. We calculate the results for an economy with a tax system such that the home-ownership rate is 10 percentage points lower than in the benchmark economy. We find that the effect on migration is important, as it rises by 8%. The change on taxes affects the unemployment rate through different channels. First, for a given level of market tightness, the matching process becomes more efficient because there are fewer workers who reject job offers. Second, the separation rate increases because there are more quits. And third, there is more entry of firms, that is, market tightness is higher. The net effect is a small decrease of unemployment of 0.1%. Therefore, we find a small positive relationship between the home-ownership rate and unemployment.

**Related literature**

The results of this paper contribute to the literature that estimates the effect of home-ownership on unemployment. Since Oswald (1997) suggested that a high home-ownership rate could be the cause behind the high unemployment rate in Europe, several authors have estimated the effect of housing tenure both at the regional level and at the individual level.

There is plenty of evidence that home-ownership has a negative effect on mobility, see for example Winkler (2010), Barceló (2006) and Caldera and Andrews (2011). However, the empirical results on unemployment duration are mixed. Taskin and Yaman (2012) for the US and Brunet and Lesueur (2003) for France estimate a positive effect of home-ownership on unemployment duration but Munch et al. (2006), van Vuuren and van Leuvensteijn (2007), Battu, Ma and Phimister (2008) and Flatau et al. (2003) find a negative effect of home-ownership on unemployment duration for Denmark, Netherlands, UK and Australia. As Flatau et al. (2003) show, liquidity constraints are an important determinant of the differences in unemployment between owners and renters. In the Australian case, they find that the shorter unemployment duration of owners is driven by the owners with a mortgage. However, the model we develop abstracts from this potential mechanism. We will only deal with the effect of transaction costs.

At the aggregate level, Blanchflower and Oswald (2013) report for the US a positive relationship between home-ownership and unemployment. Their work highlights the need of studying the effect of home-ownership on unemployment.

With respect to the theoretical models that relate home-owners' moving costs to unemployment, our model is closest to Coulson and Fisher (2009) and Head and Lloyd

(2012). As in Coulson and Fisher (2009), we model the labour market as a frictional market with endogenous job creation and wages. Coulson and Fisher (2009) find that a higher home-ownership rate may foster firm entry so that aggregate unemployment decreases. On the other hand, Head and Lloyd (2012) explicitly model the transaction frictions of the housing market and the tenure decision of households. They quantify that the effect of home-ownership on unemployment is positive but small, a decrease of 10pp of owner occupied housing decreases unemployment by 0.3pp. Our model relates to their work in the sense that we also allow households to decide whether to own or rent their house.

Finally, this paper is also related to the literature on joint search. In this literature, Guler et al. (2012) highlight that when individuals make job decisions jointly, their locational decisions are restricted, with a negative effect on unemployment. Gemici (2011) estimates the implications of joint search on wages and employment. Particularly interesting for the topic addressed here is the exercise about the effect of moving costs on labour market outcomes. Gemici (2011) finds that when moving costs are higher, the decrease in the migration rate has no effect on the husbands employment rate but has a positive effect on the wives employment rate. This result is consistent with our finding that the households who have the higher costs of moving also have a lower unemployment rate.

The rest of the paper is organized as follows. Section 2 describes the model economy. Section 3 analyses theoretically the unemployment rate by housing tenure. Section 4 covers the calibration. In section 5 we use the model to analyse the relationship between housing tenure and the labour market. Section 6 repeats the analysis for single-agent households and, finally, Section 7 concludes.

## 1.2 MODEL ECONOMY

### 1.2.1 *Setting*

Time is discrete. There are two locations, $l$ and $n$. The economy is populated by a measure 1 of infinitely lived individuals. There is also a continuum of firms. All individuals are married. A couple (or household) consists of two individuals who share their income, house and wealth. They are restricted to live and work in the same location and they must either rent or own a house. Each individual derives utility from a private composite consumption good, $c$. Individuals are risk averse.

Each firm operates a technology such that, if the firm is matched with a worker, it turns one unit of labour into $y$ units of consumption good. Output can become costlessly consumption good and housing. Thus, the price of housing is the same as the price of the consumption good, which is normalised to one.

### 1.2.2 *Households: housing and financial assets*

All houses have the same size $\bar{h}$. Households can save and borrow at a constant interest rate $i$. We will denote their level of savings by $a$. However, there is a borrowing limit that depends on whether the household rents or owns a house. Renters can only save, $a \geq 0$, whereas owners may borrow up to some proportion of the value of the house,

$a \geq -(1-\chi)\overline{h}$. Therefore, $\chi$ denotes the minimum down-payment required in order to buy a house. Let $A = \left[-(1-\chi)\overline{h}\ ,\ \overline{a}\right]$ be the set of possible assets. Finally, owners also incur a transaction cost when buying a house and when selling it, which is $\phi_b\overline{h}$ and $\phi_s\overline{h}$. The price of renting a house is $r_f$. All the houses depreciate at rate $\delta_h$ each period. Home-owners pay for the amount depreciated each period.

### 1.2.3 *Households: labour*

Each member of the couple can be either employed or unemployed. Unemployed workers receive flow income $z$ and look for a job in both locations. Let $u_k$ and $v_k$ be the number of unemployed workers in location $k$ with $k \in \{l, n\}$. In each location vacancies and job seekers meet randomly each period according to an aggregate meeting function with constant returns to scale. In order to account for the possibility that the unemployed search more efficiently locally, we will assume that $u_{-k}$ enters into the meeting function of location $k$ as $\varepsilon u_{-k}$ and that $\varepsilon < 1$. Thus, each period the number of meetings in location $k$ will be given by $M(u_k + \varepsilon u_{-k}, v_k)$. With these assumptions, an unemployed worker in $l$ will meet a vacant job in this location with probability $\alpha_{ll} = \frac{M(u_l+\varepsilon u_n, v_l)}{u_l+\varepsilon u_n}$. On the other hand, an unemployed worker in location $n$ will meet a vacant job in $l$ with probability $\alpha_{nl} = \varepsilon\alpha_{ll}$. We define $\theta_l = \frac{v_l}{u_l+\varepsilon u_n}$ as the market tightness in $l$. Similarly for location $n$. An unemployed worker may meet one, two or zero vacant jobs. After meeting them, the couple decides whether to match to one of them or to remain unemployed. If the match is realized, wages are set by Nash bargaining over fixed-wage contracts. That is, once the wage is bargained in the first period, the worker will receive the same wage until the end of the employment relationship. Denote by $P = [z, y]$ the set of possible wages.

Employed workers cannot search. The employment relationship may end exogenously with probability $s$ in each period. A match may also end if a worker quits. We only allow workers to quit if their spouse receives a job offer from the other location and they decide to accept it and migrate.

### 1.2.4 *Taxes*

In this paper we will analyse how the home-ownership rate affects the labour market. We will do that by comparing two economies whose differences in the home-ownership rate will arise from different tax codes. The tax code in the model includes a tax on financial income and labour income, $\tau$, and a tax on imputed rents, $\tau_{ir}$. Imputed rents are the rents provided by owner-occupied housing. Therefore, the higher the tax on imputed rents, the lower the return on owning a house, which implies that households will be less willing to own.

### 1.2.5 *Timing of events*

Each period is composed of the following stages:

1. Unemployed workers and vacant jobs meet in the labour market.

2. Couples where at least one spouse is unemployed choose a job for the unemployed spouse and bargaining for new matches takes place.

3. The spouses of the unemployed workers that have accepted a job in another city quit and they both migrate.

4. Production takes place, couples receive their income and decide their level of consumption, financial assets, and their housing tenure for that period.

5. Employed workers lose their job with probability $s$.

### 1.2.6 *Household's decision problem*

We describe next the Bellman equations of the couples at the point in which they have already bargained over the wage and the migration decision has already taken place.

In stage 4, $W\left(h, g, a, p_m, p_f\right)$ is the value of a couple that in the previous period had housing tenure $h$ and that in this period has migration status $g$, level of assets $a$, whose husband has labour payoff $p_m$ and whose wife has labour payoff $p_f$. Housing tenure can be either $h = \overline{h}$ if the couple owned the house in the previous period and $h = 0$ if they were renters. The migration status will be $g = 0$ if the couple has not migrated and $g = 1$ if it has migrated in stage 3. Finally, $p_m$ is the wage of the husband if he is employed and is equal to $z$ if the husband is unemployed. Similarly for the wife. To simplify notation, we omit the location from the state of the couple.

The problem of the household in stage 4 is the following:

$$W\left(h, g, a, p_m, p_f\right) = \max_{c_m, c_f, a', h' \in \left\{0, \overline{h}\right\}} \left\{\vartheta u\left(c_m\right) + \left(1 - \vartheta\right) u\left(c_f\right) + \beta \widetilde{W}\left(h', a', p_m, p_f\right)\right\} \quad (1)$$

$$\begin{aligned}
\text{st } c_m + c_f + a' + r_f\overline{h} + \left(\phi_b + 1\right) h' I_{h=0} &= inc + \left(r_f - \delta_h\right)\left(1 - \tau_{ir}\right) h' + \left(1 - \phi_s\right) h I_{h'=0} \quad \text{if g=0} \quad (1a) \\
c_m + c_f + a' + r_f\overline{h} + \left(\phi_b + 1\right) h' &= inc + \left(r_f - \delta_h\right)\left(1 - \tau_{ir}\right) h' + \left(1 - \phi_s\right) h \quad \text{if g=1} \quad (1b) \\
a' &\geq -\left(1 - \chi\right) h' \quad (1c)
\end{aligned}$$

with $inc = \left(1 - \tau I_{p_m \neq z}\right) p_m + \left(1 - \tau I_{p_f \neq z}\right) p_f + \left(1 + i\left(1 - \tau\right)\right) a.$

The couple chooses consumption, $c_m$ and $c_f$, the level of financial assets they want to keep, $a'$, and the current housing tenure, $h'$, that maximizes its lifetime utility subject to the budget constraint. Their lifetime utility is the sum of their current utility plus a continuation value, $\widetilde{W}\left(h', a', p_m, p_f\right)$. We will describe the budget constraint first.

First of all, notice that the budget constraint depends on the migration status of the couple. The first restriction corresponds to the budget constraint in the case that the couple has remained in its location, $g = 0$, and the second restriction corresponds to the budget constraint in the case that the couple has migrated at the beginning of the period, $g = 1$.

We examine first the case of no migration. The left hand side of constraint $(1a)$ includes the couple's consumption expenditure, $c_m + c_f$, the level of savings that they want to keep, $a'$, and the rent of the house, $r_f\overline{h}$. The last term, $\left(\phi_b + 1\right) h' I_{h=0}$, is what the couple has to pay if they want to buy a house[3], where $I_{h=0}$ is an indicator function equal to one when $h = 0$ and equal to zero otherwise. We now turn to the right hand side of the constraint. The first term, *inc*, includes the labour income net of taxes, $\left(1 - \tau I_{p_m \neq z}\right) p_m + \left(1 - \tau I_{p_f \neq z}\right) p_f$, their financial assets and their income from these assets net of taxes, $\left(1 + i\left(1 - \tau\right)\right) a$. The second term, $\left(r_f - \delta_h\right)\left(1 - \tau_{ir}\right) h'$, accounts for the imputed rents and the depreciation and is positive when the couple is owner. The government tax on these rents is $\tau_{ir}$. Finally, if the couple is owner and sells their house, they will receive $\left(1 - \phi_s\right) h I_{h'=0}$, where $I_{h'=0}$ is an indicator function equal to one when $h' = 0$ and equal to zero otherwise.

Consider now the budget constraint of a couple that has moved at the beginning of the period, $g = 1$, described in equation $(1b)$. The difference with the previous case is that the owners who migrate necessarily sell their house and pay for the corresponding transaction costs. Consequently, if they want to be owners again, they must also buy a house.

The last restriction, $(1c)$, tells us that only owners can be indebted up to $\left(1 - \chi\right) \overline{h}$.

After reviewing the constraints of the household we can deal with the lifetime utility of the couple. The current utility is the sum of the utility of both spouses with Pareto weights given by $\vartheta$ and $(1 - \vartheta)$. With respect to the continuation value, $\widetilde{W}\left(h', a', p_m, p_f\right)$, notice that it does not depend on the migration status, migration only affects the budget constraint. However, the continuation value differs depending on the employment status of the couple, $\left(p_m, p_f\right)$. We will explain first the case of a couple where both husband and wife are employed, then the case where only one of them is employed and last we will describe the continuation value when both are unemployed.

#### 1.2.6.1 *Husband and wife employed*

If both members of the couple are employed, we will have that $p_m = w_m$ and $p_f = w_f$. Then, their continuation value is:

$$\widetilde{W}\left(h, a, w_m, w_f\right) = \left(1 - s\right)^2 W\left(h, 0, a, w_m, w_f\right) + \left(1 - s\right) s W\left(h, 0, a, w_m, z\right) \\ + s\left(1 - s\right) W\left(h, 0, a, z, w_f\right) + s^2 W\left(h, 0, a, z, z\right)$$

That is, in the next period, they both will keep their job with probability $\left(1 - s\right)^2$, they both will lose their job with probability $s^2$and with probability $s\left(1 - s\right)$, only the husband will keep his job and with the same probability only the wife will keep her job. In neither of the cases the couple migrates, since only unemployed workers who have found a job in another location and their spouses can migrate. Finally, since wages are constant during the employment relationship, next period the workers that keep their job receive the same labour payoff as in the current period.

#### 1.2.6.2 *One spouse employed*

If only the husband is employed[4], the continuation value, $\widetilde{W}\left(h, a, w_m, z\right)$, will be the expectation over two outcomes: the value of the couple at the beginning of the period

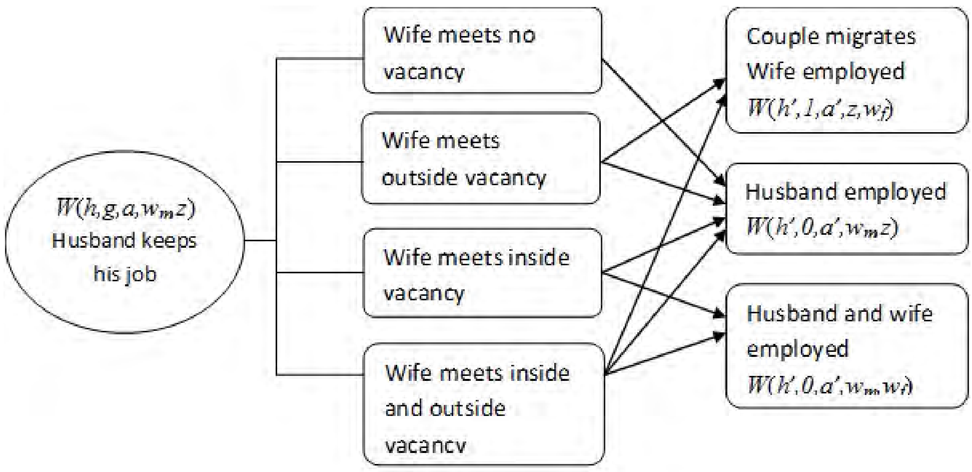
3 In the case that a couple chooses to be owner this period and they were renters the previous one.
4 The value of a couple where only the wife is employed works analogously.

if he keeps his job and the value if he loses it. We denote by $\widetilde{W}_{-s}(h, a, w_m, z)$ the first case and $\widetilde{W}_s(h, a, z, z)$ the second one. Therefore, the continuation value is:

$$\widetilde{W}(h, a, w_m, z) = (1 - s)\widetilde{W}_{-s}(h, a, w_m, z) + s\widetilde{W}_s(h, a, z, z)$$

We describe next how $\widetilde{W}_{-s}(h, a, w_m, z)$ is calculated. This value is the expectation over the outcomes represented in Figure 1.

Figure 1: Employment decision when the husband keeps his job




Since in the previous period the wife was unemployed, at the beginning of this one, she may meet a vacancy from the location where they live, "inside vacancy", from the other location, "outside vacancy", both or none. We had defined the probability of meeting an inside vacancy as $\alpha_{ll}$ when the worker lives in $l$ and $\alpha_{nn}$ when the worker lives in $n$. Since we are abstracting away from location, let the probability of meeting an inside vacancy be denoted as $\alpha_i$. Similarly, we will denote the probability of meeting an outside vacancy, which can be $\alpha_{ln}$ for a worker who lives in $l$ and $\alpha_{nl}$ for a worker who lives in $n$, as $\alpha_o$.

$\widetilde{W}_{-s}(h, a, w_m, z)$ can be calculated as the sum of four terms which correspond to the four possible outcomes. The first term corresponds to the case when she does not meet any vacancy, the second, when she only meets an outside vacancy, the third, meeting an inside vacancy and the last corresponds to the case when she meets both:

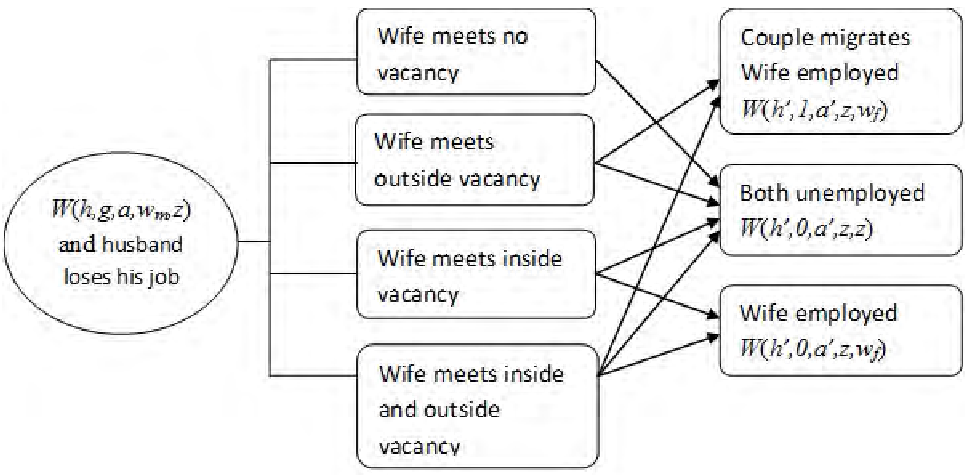
$$\begin{aligned}
\widetilde{W}_{-s}(h, a, w_m, z) = {} & (1 - \alpha_i)(1 - \alpha_o) W(h, 0, a, w_m, z) \\
& + (1 - \alpha_i)\alpha_o \max\left\{W\left(h, 1, a, z, w_f(h, 1, a, w_m)\right), W(h, 0, a, w_m, z)\right\} \\
& + \alpha_i (1 - \alpha_o) max\left\{W\left(h, 0, a, w_m, w_f(h, 0, a, w_m)\right), W(h, 0, a, w_m, z)\right\} \\
& + \alpha_i \alpha_o max\left\{W\left(h, 0, a, w_m, w_f(h, 0, a, w_m)\right), W\left(h, 1, a, z, w_f(h, 1, a, w_m)\right), W(h, 0, a, w_m, z)\right\}
\end{aligned}$$

Depending on which vacancies the wife meets, the couple must decide if she matches to some vacancy and if the husband must quit his job. As a result, they will be in one of the three cases represented in the third column of Figure 1. These cases are that the couple remains in the same employment status with the husband employed and the wife unemployed, that the wife gets employed in the same location so they both are employed or that the wife gets employed in the other location and they both migrate. In the last two cases the continuation value of the couple will depend on the wage that the wife will earn in her new job. We will derive how the wage is bargained in subsection 1.2.8, now we just take into account that the wage will depend on the previous period tenure status of the couple, $h$, their migration status, $g$, their level of assets, $a$, and both the employment status and wage of her husband, that we will summarise as $w_m$, so $w_f = w_f\left(h, g, a, w_m\right)$. We can now describe the continuation value of the couple.

If the wife does not meet any vacancy, the couple has no choice and must remain in the same employment status as before. The value of the couple will be $W\left(h, 0, a, w_m, z\right)$. If the wife only meets an outside vacancy, she can accept it or reject it. If she rejects it, they will remain in the same employment status and have value $W\left(h, 0, a, w_m, z\right)$. But if she accepts it, the husband must quit and they both migrate. Then, their value will be $W\left(h, 1, a, z, w_f\left(h, 1, a, w_m\right)\right)$. Consider now the case where the wife only meets an inside vacancy. If she accepts, in the following period they will be both employed, with value $W\left(h, 0, a, w_m, w_f\left(h, 0, a, w_m\right)\right)$, and if she rejects they will remain with the same employment status as before $W\left(h, 0, a, w_m, z\right)$. Finally, if the wife meets both vacancies, she can either match the inside vacancy, match the outside vacancy or reject both. The value of the couple in each of these cases will be, respectively, $W\left(h, 0, a, w_m, w_f\left(h, 0, a, w_m\right)\right)$, $W\left(h, 1, a, z, w_f\left(h, 1, a, w_m\right)\right)$ and $W\left(h, 0, a, w_m, z\right)$.

Let's consider now $\widetilde{W}_s\left(h, a, z, z\right)$, which is the value of the couple if the husband lost his job at the end of the previous period. Again, we must take into account that the wife may meet and outside vacancy, an inside vacancy, both or none. As the husband has lost his job, his employment status in the wage function of the wife will be denoted by, 1, so $w_f = w_f\left(h, g, a, 1\right)$. In Figure 2 are summarized the decisions of the couple for each of these cases.

Figure 2: Employment decision when the husband loses his job




Therefore, the value of the couple at the beginning of the period is:

$$\begin{aligned}\widetilde{W}_s(h,a,z,z) &= (1-\alpha_i)(1-\alpha_o)\,W(h,0,a,z,z)\\ &+(1-\alpha_i)\,\alpha_o \max\left\{W\left(h,1,a,z,w_f(h,1,a,1)\right),W(h,0,a,z,z)\right\}\\ &+\alpha_i(1-\alpha_o)\,max\left\{W\left(h,0,a,z,w_f(h,0,a,1)\right),W(h,0,a,z,z)\right\}\\ &+\alpha_i\alpha_o max\left\{W\left(h,0,a,z,w_f(h,0,a,1)\right),W\left(h,1,a,z,w_f(h,1,a,1)\right),W(h,0,a,z,z)\right\}\end{aligned}$$

If the wife does not meet any vacancy, both the husband and the wife are unemployed in the following period and their value will be $W(h,0,a,z,z)$. If the wife only meets an outside vacancy, she can accept it or reject it. If she rejects it, they will be both unemployed and have value $W(h,0,a,z,z)$. But if she accepts, they migrate and obtain the value $W\left(h,1,a,z,w_f(h,1,a,1)\right)$. The third case is that the wife only meets an inside vacancy. If she accepts, they will have value $W\left(h,0,a,z,w_f(h,0,a,1)\right)$, and if she rejects they will obtain $W(h,0,a,z,z)$. Finally, if the wife meets both vacancies, she can either match the inside vacancy, match the outside vacancy or reject both. The value of the couple in each of these cases will be, respectively, $W\left(h,0,a,z,w_f(h,0,a,1)\right)$, $W\left(h,1,a,z,w_f(h,1,a,1)\right)$ and $W(h,0,a,z,z)$.

#### 1.2.6.3 *Neither member of the couple is employed*

If both members of the couple are unemployed, their value will be given by $W(h,0,a,z,z)$. In this case, they both will search and in the following period, they can meet vacancies from both locations. We relegate the description of the possible outcomes to the Appendix.

#### 1.2.6.4 *Policy functions*

From the household's problem we obtain several policy functions. Let $\psi$ be the state of the household, that is, $\psi = \left(h,g,a,p_m,p_f\right)$. The savings rule will be given by $a^* = g_a(\psi)$ and the tenure policy will be given by $h^* = g_h(\psi)$.

### 1.2.7 *Firms problem*

We now turn to the problem of the firm. There are many firms, each with one job. Each of them produces $y$ units of consumption good each period when the job is filled. The per period profits of the firm are constant for the duration of the match, given that the wage is constant. However, the probability that the match ends depends on the state of the couple because so does the probability that a worker quits. Thus, we will describe first the value of a filled job that employs a worker whose spouse is also employed. Then, we will describe the case where the spouse is unemployed. Note also that the profits of the firm not only depend on the state of the couple but also on which member of the couple it is employing.

#### 1.2.7.1 *Both members of the couple are employed*

The value of a firm that employs the husband when the couple is in state $\psi = \left(h,g,a,w_m,w_f\right)$ is $J(m,\psi) = J\left(m,h,g,a,w_m,w_f\right)$ with:

$$J\left(m,h,g,a,w_m,w_f\right) = y - w_m + \frac{1}{1+i}\left[(1-s)^2 J\left(m,h^*,0,a^*,w_m,w_f\right)\right.$$
$$\left. + s\,(1-s)\,J\,(m,h^*,0,a^*,w_m,z) + sV\right] \qquad (2)$$

where $h^* = g_h\,(\psi)$and $a^* = g_a\,(\psi)$.

In that period the firm will obtain the productivity minus the wage. In the following period, with probability $(1-s)$ the match will continue and with probability $s$ the match will end, in this latter case the value of the firm will be $V$. However, if the match continues, either the wife keeps her job and the value of the firm is $J\left(m,h^*,0,a^*,w_m,w_f\right)$ or she loses her job and the value of the firm is $J\,(m,h^*,0,a^*,w_m,z)$.

#### 1.2.7.2 *Only one member of the couple is employed*

The value of a firm that employs the husband when the couple is in state $\psi = (h,g,a,w_m,z)$ is denoted by $J\,(m,\psi) = J\,(m,h,g,a,w_m,z)$. It consists of current profits, $y - w_m$, plus a continuation value that depends on the employment status of both spouses. If the husband loses his job, that is, if the match ends for exogenous reasons, the continuation value of the firm is $V$. If the match does not end for exogenous reasons we must take into account the value of the firm when the wife becomes employed in the same location, when she becomes employed in the other location or when she remains unemployed.

In the first case, the case in which the wife finds a job in the same location, the continuation value of the firm will be the value of employing the husband when both spouses are employed, $J\left(m,h^*,0,a^*,w_m,w_f\,(h^*,0,a^*,w_m)\right)$. The probability of this event, that we denote by $p_2$, is the sum of two terms. The first one is the probability that the wife meets an inside and an outside vacancy, $\alpha_i\alpha_o$, and matches the inside vacancy. We must take into account that acceptance will take place if the couple is not worse off than when they migrate $W\left(h^*,0,a^*,w_m,w_f\,(\cdot)\right) \geq W\left(h^*,1,a^*,z,w_f\,(\cdot)\right)$ or than when the wife remains unemployed, $W\left(h^*,0,a^*,w_m,w_f\,(\cdot)\right) \geq W\,(h^*,0,a^*,w_m,z)$. The second term is the probability that the wife only meets an inside vacancy, $\alpha_i\,(1-\alpha_o)$, and accepts. In this case acceptance will take place if the couple is not worse off than when the wife remains unemployed, $W\left(h^*,0,a^*,w_m,w_f\,(\cdot)\right) \geq W\,(h^*,0,a^*,w_m,z)$. Thus, $p_2$ can be calculated as:

$$p_2 = \left[\alpha_i\alpha_o I\left(W\left(h^*,0,a^*,w_m,w_f(\,)\right)\geq W\left(h^*,1,a^*,z,w_f(\,)\right)\right) + \alpha_i\,(1-\alpha_o)\right] I\left(W\left(h^*,0,a^*,w_m,w_f(\,)\right)\geq W(h^*,0,a^*,w_m,z)\right)$$

If the wife becomes employed in the other location, the husband will quit his job and migrate with her, so the continuation value for the firm will be $V$. The probability of this event is denoted by $p_1$. It is also the sum of two terms. The first one is the probability that the wife meets an inside and an outside vacancy, $\alpha_i\alpha_o$, and matches the outside vacancy. We must take into account that acceptance will take place if the couple is better off than when she matches the inside vacancy $W\left(h^*,1,a^*,z,w_f\,(\cdot)\right) > W\left(h^*,0,a^*,w_m,w_f\,(\cdot)\right)$ or than when the wife remains unemployed, $W\left(h^{*},1,a^{*},z,w_{f}\left(\cdot\right)\right) > W\left(h^{*},0,a^{*},w_{m},z\right)$. The second term is the probability that the wife only meets an outside vacancy, $\left(1-\alpha_{i}\right)\alpha_{o}$, and accepts. In this case acceptance will take place if the couple is better off than when the wife remains unemployed, $W\left(h^{*},1,a^{*},z,w_{f}\left(\cdot\right)\right) > W\left(h^{*},0,a^{*},w_{m},z\right)$. Thus, $p_1$ can be calculated as:

$$p_{1}=\left[\alpha_{i}\alpha_{o}I\left(W\left(h^{*},1,a^{*},z,w_{f}\left(\right)\right)>W\left(h^{*},0,a^{*},w_{m},w_{f}\left(\right)\right)\right)+\left(1-\alpha_{i}\right)\alpha_{o}\right]I\left(W\left(h^{*},1,a^{*},z,w_{f}\left(\right)\right)>W\left(h^{*},0,a^{*},w_{m},z\right)\right)$$

If the wife remains unemployed, the continuation value of the firm will be the value of employing the husband when the wife is unemployed, $J\left(m,h^{*},0,a^{*},w_{m},z\right)$. This will occur with probability $1-p_{2}-p_{1}$.

Therefore, the value of a firm that employs the husband when the wife is unemployed is given by:

$$\begin{aligned}J\left(m,h,g,a,w_{m},z\right)=y-w_{m}+\frac{1}{1+i}&\left[\left(1-s\right)p_{2}J\left(m,h^{*},0,a^{*},w_{m},w_{f}\left(h^{*},0,a^{*},w_{m}\right)\right)\right.\\&\left.+\left(1-s\right)\left(1-p_{2}-p_{1}\right)J\left(m,h^{*},0,a^{*},w_{m},z\right)+\left(s+\left(1-s\right)p_{1}\right)V\right]\end{aligned}\tag{3}$$

### 1.2.7.3 *Value of a vacancy*

To create a job, a firm first posts a vacancy. There is a flow cost of posting a vacancy, denoted by $\xi$. In the following period, the firm will hire a worker with probability $\alpha_{r}$. Let $\lambda_{hire}\left(j,h,g,a,p_{m},p_{f}\right)$ be the density of the unemployed workers that a firm can hire where $j$ identifies the spouse in the couple $j\in\{m,f\}$. To simplify notation we denote $\widetilde{\psi}=\left(j,h,g,a,p_{m},p_{f}\right)$, so the density can be written as $\lambda_{hire}\left(j,h,g,a,p_{m},p_{f}\right)=\lambda_{hire}\left(\widetilde{\psi}\right)$.[5] Notice that the value of a filled job is a function of $\widetilde{\psi}$, $J\left(\widetilde{\psi}\right)$ . The value of a vacancy is:

$$V=-\xi+\frac{\alpha_{r}}{1+i}\int_{\Psi}J\left(\widetilde{\psi}\right)\lambda_{hire}\left(\widetilde{\psi}\right)d\widetilde{\psi}\tag{4}$$

with $\Psi=\{m,f\}\text{x}\left\{0,\overline{h}\right\}\text{x}\{0,1\}\text{x}A\text{x}P\text{x}P$.

## 1.2.8 *Wage determination*

The wage is fixed for the duration of the match and determined through Nash bargaining between the firm and the worker at the beginning of their employment relationship. We will describe here the wage determination of the husband. The same logic applies for the wife. The wage of the husband will depend on the previous period tenure status of the couple, $h$, their migration status, $g$, their level of assets, $a$, and the employment status of the wife. The employment status of the wife in the wage function includes more possibilities than her employment status in the value of the couple. For the case of the wage function, we must consider the case in which the wife is employed at the moment of bargaining, the case in which she is unemployment but is also being hired that period and the case in which she is unemployed and remains unemployed that

5 $\lambda_{hire}\left(\widetilde{\psi}\right)$ and $\alpha_{r}$ are derived in the Appendix.

period. In the first case, the employment status of the wife will be given by $w_f$, so the wage of the husband will be $w_m\left(h, g, a, w_f\right)$. In the second case, the case in which the wife is also being hired, the employment status of the wife will be denoted by 2 and the wage of the husband will be $w_m\left(h, g, a, 2\right)$. Finally, if the wife remains unemployed, her employment status will be denoted by 1 and the wage of the husband will be $w_m\left(h, g, a, 1\right)$.

Consider a couple with a level of assets $a$, and that in the previous period had tenure $h$. If the wife is unemployed at the beginning of the period and does not get a job, the wage of the husband, $w_m\left(h, g, a, 1\right)$, will be the solution to:

$$\max_{w_m}\left(W\left(h, g, a, w_m, z\right) - W\left(h, 0, a, z, z\right)\right)^{\gamma}\left(J\left(m, h, g, a, w_m, z\right) - V\right)^{1-\gamma} \tag{5}$$

The first term, $W\left(h, g, a, w_m, z\right) - W\left(h, 0, a, z, z\right)$, represents the gains from the match for the couple if the husband gets employed at wage $w_m$ where $W\left(h, g, a, w_m, z\right)$ is the value of the couple if he gets matched and $W\left(h, 0, a, z, z\right)$ is the value if he remains unemployed. If the job is in the other location, they will migrate and $g = 1$, whereas if the job is in their location they will not migrate. The second term, $J\left(m, h, g, a, w_m, z\right) - V$, represents the gains from the match for the firm.

If the wife is unemployed at the beginning of the period and both the husband and the wife find a job at the same time[6], the wage of the husband, $w_m\left(h, g, a, 2\right)$, will be the solution to:

$$\begin{aligned}\max_{w_m}\left(W\left(h, g, a, w_m, w_f\left(h, g, a, 2\right)\right) - W\left(h, g, a, z, w_f\left(h, g, a, 2\right)\right)\right)^{\gamma}\cdot \\ \cdot\left(J\left(m, h, g, a, w_m, w_f\left(h, g, a, 2\right)\right) - V\right)^{1-\gamma}\end{aligned} \tag{6}$$

In this case, the value of the couple if he gets matched will be the value of being both spouses employed, $W\left(h, g, a, w_m, w_f\left(h, g, a, 2\right)\right)$, and the value of the couple if he does not get matched is the value of the couple when only the wife gets employed $W\left(h, g, a, z, w_f\left(h, g, a, 2\right)\right)$. Consistent with this, the gains from the match for the firm are $J\left(m, h, g, a, w_m, w_f\left(h, g, a, 2\right)\right) - V$. As in the previous case, $g$ depends on whether the new job is in the same location as the couple or not.

Finally, consider the case where the wife was employed at wage $w_f$ in the previous period and kept her job at the end of it. In this case, if the husband finds a job in the same location, both spouses will be employed but if the husband finds a job in the other location, they will migrate and she will quit. Therefore, the wage of the husband if he finds a job in their location, $w_m\left(h, 0, a, w_f\right)$, will be the solution to:

$$\max_{w_m}\left(W\left(h, 0, a, w_m, w_f\right) - W\left(h, 0, a, z, w_f\right)\right)^{\gamma}\left(J\left(m, h, 0, a, w_m, w_f\right) - V\right)^{1-\gamma} \tag{7}$$

On the other hand, if the husband gets employed in the other location, his wage, $w_m\left(h, 1, a, w_f\right)$, will be the solution to:

$$\max_{w_m}\left(W\left(h, 1, a, w_m, z\right) - W\left(h, 0, a, z, w_f\right)\right)^{\gamma}\left(J\left(m, h, 1, a, w_m, z\right) - V\right)^{1-\gamma} \tag{8}$$

6 This possibility can only arise if she was also unemployed the previous period. Otherwise, she could not search.

In this last case, the value if the husband does not get matched is $W\left(h, 0, a, z, w_f\right)$, that is, we assume that the wife will not quit her job until the bargaining has been realized. For this reason, the wage of the husband when he finds a job in the other location depends on the wage of his wife.

### 1.2.9 *Leasing companies*

In this economy, the houses that the couples rent are owned by leasing companies. Following Gervais (2002), leasing companies are two-period lived institutions, with a new cohort being born every period. In the first period, they buy houses, rent them and pay for the depreciation. In the second period, they sell the houses. Their problem is:

$$\underset{H_{t,f}}{max}\left\{\left(-1+r_f-\delta_h\right)\bar{h}H_{t,f}+\frac{H_{t,f}\bar{h}}{1+i}\right\}$$

where $H_{t,f}$ denotes the total amount of rental housing in period $t$. For this maximization problem to be well defined, it must be that:

$$r_f=\frac{i}{1+i}+\delta_h \tag{9}$$

which implies that the leasing companies make zero profits.

### 1.2.10 *Equilibrium*

Since both locations have the same productivity, we will consider a symmetric equilibrium in which they have the same share and distribution of the population and the same number of firms. We omit all locational subscripts.

A steady-state equilibrium for a given set of policy arrangements $\{\tau, \tau_{ir}\}$consists of a set of value functions $\{W(\psi), J(j, \psi)\}$, a set of decision rules $\{g_a(\psi), g_h(\psi)\}$, a time-invariant measure of agent types $\lambda\left(\widetilde{\psi}\right)$, a set of prices
$\left\{r_f, w_j(h, g, a, \cdot)\right\}$, and market tightness $\theta$ such that:

1. Given prices, $\theta$ and the fiscal policy, the household's decision rules solve the dynamic program given by (1).
2. Given prices and the household's decision rules, the firm solves (2)-(3).
3. $\theta$ satisfies $V=0$ with $V$ given by (4).
4. Wages satisfy (5)-(8) and the rent satisfies (9).
5. $\lambda\left(\widetilde{\psi}\right)$ is the invariant distribution generated by the meeting probability, separation rate and the household's decision rules.

## 1.3 THE UNEMPLOYMENT RATE

In this section, we will derive the unemployment rate of home-owners and renters as a function of their job finding and separation rates. For comparison purposes, we will begin by deriving the aggregate unemployment rate in this economy. In order to do so, we must take into account that the inflow and outflow from unemployment must be equal in steady state. The inflow will be generated by previous period employed workers that today are unemployed, which is the product of the separation rate, $sep$, and one minus the unemployment rate, $1-u$. The outflow will be generated by previous period unemployed workers that today are employed, which is the product of the job finding rate, $f$, and the unemployment rate, $u$. Therefore, in steady state the unemployment rate must satisfy:

$$u = \frac{sep}{sep + f} \tag{10}$$

On the other hand, we can decompose the inflows and outflows to unemployment of home-owners and renters. We define $\mu\left(h_1, e_1, h_2, e_2\right)$ as the number of workers that in this period have housing tenure $h_1$, employment status $e_1$ and in the following period will have housing tenure $h_2$ and employment status $e_2$, with employment status defined as $e = 1$ if the worker is employed and $e = 0$ if the worker is unemployed.

The inflow and the outflow of unemployed home-owners must be equal. The inflow will be generated by previous period employed home-owners that become unemployed and today do not sell their house, $\mu\left(\overline{h}, 1, \overline{h}, 0\right)$; previous period employed renters that become unemployed and today buy a house, $\mu\left(0, 1, \overline{h}, 0\right)$; and previous period unemployed renters that do not find a job and today buy a house, $\mu\left(0, 0, \overline{h}, 0\right)$. The outflow will be generated by previous period unemployed home-owners that do not find a job and today sell their house, $\mu\left(\overline{h}, 0, 0, 0\right)$ and by previous period unemployed home-owners that find a job. Notice that in this last case the outflow is generated whether they sell or not their house, $\mu\left(\overline{h}, 0, \overline{h}, 1\right) + \mu\left(\overline{h}, 0, 0, 1\right)$.

In steady state it must be that the inflow equals the outflow. Therefore:

$$\mu\left(\overline{h}, 1, \overline{h}, 0\right) + \mu\left(0, 1, \overline{h}, 0\right) + \mu\left(0, 0, \overline{h}, 0\right) = \mu\left(\overline{h}, 0, 0, 0\right) + \mu\left(\overline{h}, 0, \overline{h}, 1\right) + \mu\left(\overline{h}, 0, 0, 1\right) \tag{11}$$

We can rewrite these variables in terms of the unemployment rate of renters and home-owners.

$$\mu\left(h_1, e_1, h_2, e_2\right) = \frac{\mu\left(h_1, e_1, h_2, e_2\right)}{\mu\left(h_1, e_1\right)} \frac{\mu\left(h_1, e_1\right)}{\mu\left(h_1\right)} \mu\left(h_1\right)$$

with $\mu\left(h_1, e_1\right)$ being the amount of workers with housing tenure $h_1$ and employment status $e_1$ and $\mu\left(h_1\right)$ being the amount of workers with tenure status $h_1$. $\mu\left(h_1, e_1, h_2, e_2\right)$ is the product of three terms. When $h_1 = \overline{h}$, the third term is the home-ownership rate, we will denote it by $HR = \mu\left(\overline{h}\right)$. When $e_1 = 0$ and $h_1 = \overline{h}$, the second term is the unemployment rate of home-owners, and we will denote it by $u_h = \frac{\mu\left(\overline{h}, 0\right)}{\mu\left(\overline{h}\right)}$. Similarly, the unemployment rate of renters is $u_r = \frac{\mu(0,0)}{\mu(0)}$. Substituting these terms, equation 11 becomes:

$$\frac{\mu\left(\overline{h},1,\overline{h},0\right)}{\mu\left(\overline{h},1\right)}\left(1-u_h\right)HR+\frac{\mu\left(0,1,\overline{h},0\right)}{\mu\left(0,1\right)}\left(1-u_r\right)\left(1-HR\right)+\frac{\mu\left(0,0,\overline{h},0\right)}{\mu\left(0,0\right)}u_r\left(1-HR\right)=$$
$$=\frac{\mu\left(\overline{h},0,0,0\right)}{\mu\left(\overline{h},0\right)}u_hHR+\frac{\mu\left(\overline{h},0,\overline{h},1\right)+\mu\left(\overline{h},0,0,1\right)}{\mu\left(\overline{h},0\right)}u_hHR$$

Furthermore, we can define the separation rate of home-owners and renters as:

$$sep_h=\frac{\mu\left(\overline{h},1,\overline{h},0\right)+\mu\left(\overline{h},1,0,0\right)}{\mu\left(\overline{h},1\right)}\qquad sep_r=\frac{\mu\left(0,1,\overline{h},0\right)+\mu\left(0,1,0,0\right)}{\mu\left(0,1\right)}$$

And the job finding rate as:

$$f_h=\frac{\mu\left(\overline{h},0,\overline{h},1\right)+\mu\left(\overline{h},0,0,1\right)}{\mu\left(\overline{h},0\right)}\qquad f_r=\frac{\mu\left(0,0,\overline{h},1\right)+\mu\left(0,0,0,1\right)}{\mu\left(0,0\right)}$$

So we can rewrite equation 11 as:

$$\left(1-\phi_1^h\right)sep_h\left(1-u_h\right)HR+\phi_1^r sep_r\left(1-u_r\right)\left(1-HR\right)+$$
$$+\phi_2^r\left(1-f_r\right)u_r\left(1-HR\right)=\phi_2^h\left(1-f_h\right)u_hHR+f_hu_hHR$$

with $\phi_1^h$ representing the proportion of home-owners that sell their house out of all the previously employed home-owners that lost their job, $\phi_1^r$ representing the proportion of renters that buy their house out of all the previously employed renters that lost their job, $\phi_2^r$ representing the proportion of renters that buy their house out of all the previously unemployed renters that do not find a job and $\phi_2^h$ representing the proportion of home-owners that sell their house out of all the previously unemployed home-owners that do not find their job

In this way, the unemployment rate of home-owners becomes:

$$u_h=\frac{\left(1-\phi_1^h\right)sep_h}{\left(1-\phi_1^h\right)sep_h+f_h+\phi_2^h\left(1-f_h\right)}+\frac{\phi_1^r sep_r\left(1-u_r\right)+\phi_2^r\left(1-f_r\right)u_r}{\left(1-\phi_1^h\right)sep_h+f_h+\phi_2^h\left(1-f_h\right)}\frac{1-HR}{HR}$$

The first term of $u_h$ is similar to the equation we had found for $u$. It tells us that the unemployment rate of home-owners depends on their separation rate from jobs and their job finding rate. The only difference are $\phi_1^h$ and $\phi_2^h$, that imply that the unemployment rate will be lower if home-owners sell their house when they are unemployed. On the other hand, the second term of the equation of $u_h$ is positive only if $\phi_1^r$ and $\phi_2^r$ are positive, meaning that the unemployment rate of home-owners will be higher if renters buy a house when they are unemployed.

We can follow the same procedure to calculate renters' unemployment rate. The inflow and the outflow of unemployed renters must be equal. The inflow will be generated by previous period employed renters that become unemployed and today do not buy a house, $\mu\left(0,1,0,0\right)$; previous period employed owners that become unemployed and today sell their house, $\mu\left(\overline{h},1,0,0\right)$; and previous period unemployed home-owners that do not find a job and today sell their house, $\mu\left(\overline{h},0,0,0\right)$. The outflow will be generated by previous period unemployed renters that do not find a job and today buy a house, $\mu\left(0,0,\overline{h},0\right)$ and by previous period unemployed renters that find a job, $\mu\left(0,0,\overline{h},1\right)+\mu\left(0,0,0,1\right)$. If we equate these two flows, we will have:

$$\mu\left(0,1,0,0\right)+\mu\left(\overline{h},1,0,0\right)+\mu\left(\overline{h},0,0,0\right)=\mu\left(0,0,\overline{h},0\right)+\mu\left(0,0,\overline{h},1\right)+\mu\left(0,0,0,1\right)$$

Substituting the unemployment rates and home-ownership rate we will have:

$$\begin{aligned}&\left(1-\phi_1^r\right)sep_r\left(1-u_r\right)\left(1-HR\right)+\phi_1^h sep_h\left(1-u_h\right)HR+\\&\quad+\phi_2^h\left(1-f_h\right)u_h HR=\phi_2^r\left(1-f_r\right)u_r\left(1-HR\right)+f_r u_r\left(1-HR\right)\end{aligned}$$

Renters' unemployment rate is:

$$u_r=\frac{\left(1-\phi_1^r\right)sep_r}{\left(1-\phi_1^r\right)sep_r+f_r+\phi_2^r\left(1-f_r\right)}+\frac{\phi_1^h sep_h\left(1-u_h\right)+\phi_2^h\left(1-f_h\right)u_h}{\left(1-\phi_1^r\right)sep_r+f_r+\phi_2^r\left(1-f_r\right)}\frac{HR}{1-HR}$$

Similar to the case of home-owners, renters' unemployment rate is the sum of two terms. The first term depends on the separation and job finding rate of renters, and on the proportion of unemployed renters that become home-owners. The second term accounts for the possibility that home-owners sell their house when they are unemployed.

## 1.4 CALIBRATION

We calibrate the model to the US economy for the period 1980-2005.

### 1.4.1 *Functional forms*

We need to specify the functional form of the utility function and the meeting function. We assume that the utility function of the couple is given by:

$$u\left(c\right)=\frac{c^{1-\sigma}}{1-\sigma}$$

The meeting function is:

$$M(u_l+\varepsilon u_n,v_l)=kv_l^{1-\mu}\left(u_l+\varepsilon u_n\right)^{\mu}$$

Therefore $\alpha_{ll}=k\theta_l^{1-\mu}$ and $\alpha_{nl}=\varepsilon k\theta_l^{1-\mu}$ with $\theta_l=\frac{v_l}{u_l+\varepsilon u_n}$ the market tightness in $l$.

### 1.4.2 *Parametrization*

We now describe the parameters used in the model. Since the model period is a month, parameters are expressed at monthly frequency except for migration rates. In Table 1 we have the parameters that are set exogenously because they are measured directly from the data or because we take them directly from the literature. The down-payment for buying a house is $\chi = 0.2$, which is the usual number used in the literature on housing (Chamber et al. (2009) estimate it from the American Housing Survey of 1995). We set the interest rate to $i = 0.327\%$, which corresponds to an annual interest rate of 4%, as in Díaz and Luengo-Prado (2008). Houses depreciate at rate $\delta_h = 0.0025$. The cost of posting a vacancy is $\xi = 1.752$, consistent with Hagedorn and Manovskii (2008), who calculate that the cost of posting a vacancy is 58.4% of labour productivity. Following Shimer (2005), both the bargaining weight and the elasticity of the meeting function on unemployment are set to $\mu = \gamma = 0.72$. For the income tax, we use the estimates of Díaz and Luengo-Prado (2008), calculated for the period 1989-2004, $\tau = 0.2$. Finally, the tax on imputed rents is 0, $\tau_{ir} = 0$, since in US imputed rents are not taxed.

Table 1: Exogenous parameters

| Parameter | Source |
|---|---|
| $\sigma = 2$ | |
| $\vartheta = 0.5$ | |
| $\chi = 0.2$ | Chamber et al. (2009) |
| $i = 0.327\%$ | Díaz and Luengo-Prado (2008) |
| $\delta_h = 0.0025$ | Harding et al. (2007) |
| $\xi = 1.752$ | Hagedorn and Manovskii (2008) |
| $\gamma = \mu = 0.72$ | Shimer 2005 |
| $\tau = 0.2$ | Díaz and Luengo-Prado (2007) |
| $\tau_{ir} = 0$ | |

The remaining parameters and the relevant targets are given in Table 2. In the table we associate each parameter with a specific target for which it is particularly relevant. We explain next each of the targets in more detail.

We use the median ratio of value to current income as a target. This information is provided by the American Housing Survey (AHS), and they define it as the value of the housing unit divided by the total current (family) income. This survey is conducted every 2 years and we use as a target the average for the period 1989 to 2005, which is 2.5 in annual terms and 29 in monthly terms.

We also attempt to match an unemployment income equal to 40% of the mean wage, as in Shimer (2005). The job finding rate is calculated as the average for the period 1980-2005 of the series constructed by Robert Shimer.[7] We obtain that the job finding rate was 0.42. This series is constructed with the information from the Current Population Survey (CPS).

7 For additional details, please see Shimer (2007) and his webpage http://sites.google.com/site/robertshimer/research/flows.

Table 2: Estimated parameters

| Parameter | Target | Source |
|---|---|---|
| $\beta = 0.9942$ | home ownership rate= 66% | CPS 1980-2005 |
| $s = 0.029$ | unemployment rate= 6.2% | CPS 1980-2005 |
| $k = 0.53$ | job finding rate= 0.42 | Shimer 1980-2005 |
| $z = 0.922$ | $z = 40\%$ mean wage | Shimer 2005 |
| $\bar{h} = 147$ | house/earnings= 29 | AHS 1989-2005 |
| $\varepsilon = 0.12$ | annual migration rate= 2.4% | CPS 1999-2005 |
| $\phi_s = \phi_b = 0.0058$ | migration rate owners/renters= 33% | SIPP 2001 |

The following targets are obtained from the Current Population Survey (CPS). The home-ownership rate, which is defined as the proportion of households that are owners out of the total number of occupied households, was 66%, and the unemployment rate, 6.2%. Finally, given that the mechanism of the model relies on the effects of housing tenure on workers mobility, we need a target on how mobile workers are. This target is particularly important since the magnitude of the effect of tenure on the labor market depends on the degree at which workers migrate in order to have a job. Since 1999, the March CPS includes information on the reasons for moving. To calculate the migration rate, we only consider moves caused by work related reasons. We obtain an annual rate of 2.4% for the period from 1999 to 2005. Finally, from the Survey of Income and Program Participation (SIPP) we obtain that in 2001 the migration rate of home-owners was 33% the migration rate of renters. This last target is especially related to the transaction costs that home-owners must pay when they move. We assume that they incur the same cost for selling and for buying a house.

Table 3 summarizes the fit of the model with respect to the targeted moments.

Table 3: Calibration targets

| Moment | Data | Model |
|---|---|---|
| home-ownership rate | 66% | 65% |
| unemployment rate | 6.2% | 6.5% |
| job finding rate | 0.42 | 0.44 |
| unemp. flow/wage | 40% | 39% |
| house/earnings | 29 | 28 |
| annual migration rate | 2.4% | 2.5% |
| migration rate owners/renters | 33% | 35% |

## 1.5 HOME-OWNERSHIP, MIGRATION AND UNEMPLOYMENT

We compute numerically the equilibrium. The model predicts an unemployment rate equal to 6.3% for home-owners and equal to 6.9% for renters. This is so, although home-owners migrate less than renters. We can make use of the equations derived in section **??** to understand this result. The unemployment rate of home-owners is given by:

$$u_h = \frac{\left(1-\phi_1^h\right) sep_h}{\left(1-\phi_1^h\right) sep_h + f_h + \phi_2^h \left(1-f_h\right)} + \frac{\phi_1^r sep_r \left(1-u_r\right) + \phi_2^r \left(1-f_r\right) u_r}{\left(1-\phi_1^h\right) sep_h + f_h + \phi_2^h \left(1-f_h\right)} \frac{1-HR}{HR}$$

and for renters is

$$u_r = \frac{\left(1-\phi_1^r\right) sep_r}{\left(1-\phi_1^r\right) sep_r + f_r + \phi_2^r \left(1-f_r\right)} + \frac{\phi_1^h sep_h \left(1-u_h\right) + \phi_2^h \left(1-f_h\right) u_h}{\left(1-\phi_1^r\right) sep_r + f_r + \phi_2^r \left(1-f_r\right)} \frac{HR}{1-HR}$$

The equations show that there are three aspects that affect the unemployment rate of home-owners and renters: their job finding rates, their separation rates and their housing policy when they are unemployed. In Table 4 we find the values of these variables in the benchmark model. On one hand, renters job finding rate is higher, $f_r > f_h$ due to their greater mobility. This leads to a lower unemployment rate for renters. However, the separation rate is also higher for renters than for home-owners, $sep_r > sep_h$, which implies a higher unemployment rate for renters. This difference in the separation rates is also due to the difference in mobility rates. Since unemployed renters accept more often outside jobs, their partners, who may be employed, must quit their job in order for the couple to migrate.

Table 4: Labour statistics in the benchmark model

| | $u_h$ | $f_h$ | $sep_h$ | $\phi_1^h$ | $\phi_2^h$ |
|---|---|---|---|---|---|
| Home-owners | 0.063 | 0.43 | 0.030 | 0.02 | 0.01 |
| | $u_r$ | $f_r$ | $sep_r$ | $\phi_1^r$ | $\phi_2^r$ |
| Renters | 0.069 | 0.45 | 0.031 | 0 | 0 |

The third aspect that affects the unemployment rates by tenure is the housing policy. In the benchmark model some unemployed home-owners sell their house when they are close to their borrowing constraint, increasing the number of unemployed renters and decreasing the number of unemployed home-owners. However, no couple buys a house when at least one of its members is unemployed.

Our results are qualitatively consistent with the data from the CPS: the mean unemployment rate for the period 1980-2005 of renters was 9.8%, whereas it was 5.3% for owners. In order to further understand the role of couples in the unemployment rate and separation rate, we make use of the SIPP of 2004 to calculate the separation rate and unemployment rate of married and non married people by housing tenure for that year. The separation rate is calculated as the proportion of employed workers that become either unemployed or not in to the labour force. For the married, the home-owner separation rate was 0.013 whereas the renter separation rate was 0.026. For non married people, home-owners also had a lower separation rate than renters, but the difference is much smaller: 0.034 for home-owners and 0.035 for renters. In table 5, we report the separation rate of home-owners and renters disaggregated by marital status, educational level and sex for 35 to 55 years old. Clearly, the separation rate of married renters is higher than the separation rate of married home-owners for all educational levels. We also find that this pattern also takes place in most cases for non married workers. However, the difference is higher in the case of the married, which is consistent with the idea that being married raises the separation rate of renters. The fact that this pattern also takes place for the non-married is also reasonable. For example, the workers that expect their job to end soon, will tend to be renters, and this mechanism will affect both married and non-married workers.

Table 5: Separation rates from employment (SIPP 2004)

| | | Male | | Female | |
|---|---|---|---|---|---|
| | | Owner | Renter | Owner | Renter |
| Low skill | Married | 0,012 | 0,018 | 0,025 | 0,032 |
| | Not Married | 0,022 | 0,024 | 0,026 | 0,024 |
| Medium skill | Married | 0,009 | 0,022 | 0,014 | 0,028 |
| | Not Married | 0,017 | 0,024 | 0,015 | 0,022 |
| High skill | Married | 0,007 | 0,018 | 0,012 | 0,025 |
| | Not Married | 0,012 | 0,022 | 0,011 | 0,022 |

Mean of the monthly separation rate in 2004 for civilian noninstitutionalized U.S. population with age between 35 and 55.

When we calculate the unemployment rate of home-owners and renters disaggregated by demographic characteristics, we also find that, for all educational levels, the unemployment rate of married renters is higher than the unemployment rate of married home-owners. These rates are reported in Table 6. In most cases this pattern also takes place for non-married individuals but the differences are not so large, with the exception of non-married females with a low educational level.

Table 6: Unemployment rates (SIPP 2004)

| | | Male | | Female | |
|---|---|---|---|---|---|
| | | Owner | Renter | Owner | Renter |
| Low skill | Married | 0,041 | 0,054 | 0,076 | 0,119 |
| | Not Married | 0,078 | 0,077 | 0,058 | 0,111 |
| Medium skill | Married | 0,023 | 0,060 | 0,031 | 0,058 |
| | Not Married | 0,051 | 0,058 | 0,041 | 0,057 |
| High skill | Married | 0,016 | 0,041 | 0,023 | 0,063 |
| | Not Married | 0,034 | 0,049 | 0,029 | 0,061 |

Mean of the monthly separation rate in 2004 for civilian noninstitutionalized U.S. population with age between 35 and 55.

The results of the model are also consistent with empirical work that control for the demographic characteristics of the individuals. Coulson and Fisher (2009), using data from the 1990 Census, estimate that being home-owner decreases the probability of being unemployed by 3.6%. Our model predicts that this probability decreases by 8.7%.

Table 7: Home-ownership and the labour market

| | Benchmark | Experiment |
|---|---|---|
| income tax | 0.2 | 0.1988 |
| tax on imputed rents | 0 | 0.0021 |
| home-ownership rate | 65.4% | 55.9% |
| monthly migration rate | 0.21% | 0.23% |
| - owners | 0.130% | 0.125% |
| - renters | 0.370% | 0.366% |
| unemployment rate | 6.483% | 6.476% |
| - owners[8] | 6.261% | 6.255% |
| - renters | 6.902% | 6.758% |
| job finding rate | 43.724% | 43.909% |
| - owners | 43.115% | 43.124% |
| - renters | 44.765% | 44.832% |
| separation rate | 3.031% | 3.041% |
| - owners | 2.988% | 2.986% |
| - renters | 3.113% | 3.111% |
| market tightness | 0.439 | 0.440 |

We now conduct the experiment of decreasing the home-ownership rate by 10 percentage points. We do that by modifying the tax code in the economy. We increase the tax on imputed rents, which reduces the willingness of households to own a house. We also decrease the income tax such that the government budget is unchanged. We define the government budget as:

$$B = \int_{\widetilde{\Psi}} \lambda\left(\widetilde{\psi}\right)\left(0.5\tau ia + \tau p_j\left(1 - I\left(p_j = z\right)\right) - zI\left(p_j = z\right) + 0.5\tau_{ir}(r_f - \delta_h)g_h\left(h, g, a, p_m, p_f\right)\right) d\widetilde{\psi}$$

with $\widetilde{\Psi} = \{m, f\} \text{x} \left\{0, \overline{h}\right\} \text{x} \{0, 1\} \text{x} A \text{x} P \text{x} P$

The reduction in the home-ownership rate has a positive effect on migration, it raises by 8%. However, the effect on the unemployment rate is small, it decreases by 0.1%. The change in the unemployment rate is the result of two countervailing effects. On one hand the job finding rate increases, which leads to a lower unemployment rate. On the other hand the separation rate also increases, which implies a higher unemployment rate. Overall, the positive effect on the job finding rate offsets the negative effect on the separation rate and unemployment decreases. Our result contrasts with the model with bargaining in Coulson and Fisher (2009), who find that the effect of a lower home-ownership rate on expected profits is negative and may even lead to a higher aggregate unemployment rate. However, it is in line with Head and Lloyd (2012), although they estimate a higher effect. They find that a reduction of 10 percentage points in the home-ownership rate decreases the unemployment rate by 6%. It is interesting to note that our model finds a positive relationship between the home-ownership rate and aggregate unemployment although home-owners have a lower unemployment rate.

The increase in the job finding rate in our model is due to two reasons. First, the higher proportion of renters makes the matching process more efficient because they reject less vacancies than home-owners. But there is also a higher entry of firms which shows in the increase in the market tightness. The higher entry of firms imply that the job finding rate not only increases at the aggregate level but also if we measure it separately for home-owners and renters. On the contrary, the increase in the separation rate is only due to a compositional effect, since the separation rate of both home-owners and renters decrease.

## 1.6 SINGLE-AGENT MODEL

In this section, we consider a model identical to the benchmark case except for the fact that the household is composed of just one individual. We refer to this model as the single-agent model. We calibrate it to the same targets as in the benchmark economy. The description of this model and the estimated parameters can be found in Appendix A.1.4.

Table 8: Labour statistics in the Single-Agent model

| | $u_h$ | $f_h$ | $sep_h$ | $\phi_1^h$ | $\phi_2^h$ |
|---|---|---|---|---|---|
| Home-owners | 0.065 | 0.43 | 0.030 | 0.0002 | 0.0057 |
| | $u_r$ | $f_r$ | $sep_r$ | $\phi_1^r$ | $\phi_2^r$ |
| Renters | 0.061 | 0.47 | 0.030 | 0 | 0 |

In contrast with the benchmark model, this model predicts a lower unemployment rate for renters than for home-owners. As it can be seen in Table 8, renters unemployment rate in this case is only 6.1 whereas it is 6.5 for home-owners. This is due to the big difference in the job finding rate across these two groups. On the other hand, the two channels that contributed to the lower unemployment rate of home-owners in the benchmark model, are less important in this case. First, the separation rate is the same for renters and for home-owners. Second, the effect of housing policy is qualitatively similar to the benchmark economy, home-owners change tenure when they are unemployed and renters not, but quantitatively is less important.

## 1.7 CONCLUSIONS

We have developed a model of joint search with multiple locations and have used this framework to understand the potential effects of housing tenure on the labour market. Home-owners lack of mobility make them more likely to remain unemployed longer. However, the introduction of joint search into the analysis suggests that the workers that migrate more find jobs at a higher rate but also separate from their jobs at a higher rate. The longer unemployment duration of home-owners do not imply that they have a higher unemployment rate. The calibrated version of the model predicts that owners' unemployment rate is 8.7% lower than for renters. However, we do not find a negative relationship between the home-ownership rate and the unemployment rate. With a home-ownership rate 10 percentage points lower than in the benchmark economy, the unemployment rate would decrease only 0.1%. This analysis suggests that a policy that provides incentives for home-ownership has a small effect on aggregate unemployment.

Finally, it is important to remark that the results of this model are based on the assumption that the gains of migration only come through a higher efficiency at matching workers and firms in two locations that are identical. However, Oswald (1997) emphasizes the role of migration as a device to reduce unemployment by shifting unemployed workers from depressing areas to booming areas. The implications of introducing joint search in an environment with asymmetric locations is left for future research.

# 2

# THE EFFECT OF WOMEN PARTICIPATION ON THE LABOUR MARKET

The increasing attachment of women into the labour force has been one of the greatest transformations of the labour market of the last century. Goldin (2006) describes this process as being composed of two phases. The first one, called the "evolutionary phase", runs from the end of the XIX century to the late 1970s and was characterized by a huge increase in the participation rate of women. However, at this stage they generally did not build a professional career and were only the secondary earners in the household.

The second phase of this process takes place from the late 1970s to nowadays. In this period, the "revolutionary phase", women participation rate has increased slightly but the role of women in the labour market has changed in a fundamental way as they have placed greater importance to career success. This means that they invest more in human capital and accumulate more experience as they expect to remain attached to the labour force during most of their life.

The shift of households from a structure in which only one member worked to one in which the two spouses are part of the labour force has consequences on how economic decisions are taken. In the latter case, the worker obtains insurance from the labour income of the spouse. This allows to bargain higher wages and to reduce the level of precautionary savings. On the other hand, the employment of the spouse imposes a locational restriction to the worker, thus leading to lower wages and less mobility. This implies that the fact that both spouses work has an effect on the aggregate variables of the economy.

The objective of this paper is to study how the changing role of married women in the labour market has affected unemployment, migration and savings. We develop a model in which households are composed of couples that jointly decide their level of savings, location and labour outcomes. Taking into account the phases of women increasing attachment in the labour market, we compute the equilibrium of the model in three different situations. In the first one, only the husband participates into the labour force. In the second one, all women participate in the labour market but that they have a lower productivity than men. Finally, we also consider the case in which both spouses participate in the labour market and have the same productivity.

We find that when women participate into the labour market married men have a slightly higher unemployment rate due to their higher separation rates. This is because there can be moves generated by the wife jobs that will imply that the husband will have to quit his job. The aggregate unemployment rate is higher when there is the productivity gap because the lower productivity of women makes less profitable to create jobs, which implies that women have a higher unemployment rate in this case.

The effect on migration and the level of savings is much larger than on unemployment. When women participate into the labour market the couple migration rate doubles because moves can also be originated by the wife finding a job. On the other hand, the level of savings is reduced by half, as women jobs reduce the income fluctuations of the couple. Thus, the results of the model suggest that the increasing attachment of women into the labour market have had an impact on migration and savings and, to a lesser degree, on unemployment.

**Related literature**

There is a growing literature that takes into account the presence of two earner households into the analysis of economic decisions. Guler et al. (2012) analyse the differences in labour market outcomes between a framework where workers search for jobs jointly in couples and a framework where workers search alone. They find that income pooling within the couple allows workers to wait more time for better jobs which translates into higher lifetime incomes. On the other hand, following the argument already posed by Mincer (1978) they study how joint search imposes a cost on the relocation of workers, which implies that they accept less jobs from other locations and stay unemployed for longer. This locational tie implies that workers earn a lower lifetime income. Consistent with this, Gemici (2012) finds that the locational restriction of couples has a negative effect on wages and family stability.

The role of precautionary savings in two earner households has also been explored in the literature. Otigueira and Siassi (2013) focus on the effect of intra-household risk sharing for the level of precautionary savings set aside by the household, and specially on the implications for the crowding out effects of unemployment insurance.

Finally, Guler and Taskin (2013) study how the decrease in the gender wage gap has contributed to the evolution of migration. Contrary to our results, they find that the reduction in the wage gap should have decreased migration.

The rest of the paper is organized as follows. Section 2 describes the model economy. Section 3 covers the calibration. In section 4 we use the model to study the effect of women participation on unemployment, migration and savings and, finally, Section 5 concludes.

## 2.1 MODEL ECONOMY

### 2.1.1 *Setting*

Time is discrete. The economy is populated by a measure 1 of infinitely lived individuals. There is also a continuum of firms. There are two locations, $l$ and $n$. All individuals are married. A couple (or household) consists of a male, $m$, and a female, $f$, who share their income and wealth and work in the same location. Each individual derives utility from a private composite consumption good, $c$. They are risk averse. Employed workers produce $y_m$ units of consumption good each period if they are males and $y_f$ units if they are females.

### 2.1.2 *Households: housing and financial assets*

Households can save at a constant interest rate $i$. We will denote their level of savings by $a$ with $a \geq 0$. Let $A = [0\ ,\ \overline{a}]$be the set of possible assets.

### 2.1.3 *Households: labour*

Each member of the couple can be either employed or unemployed. Unemployed workers receive flow income $z_j$ with $j \in \{m, f\}$ and look for a job in both locations. We assume that each location has a labour market for females and a labour market for males. Let $u_l^j$ be the number of unemployed workers and $v_l^j$ the number of vacancies in location $l$ who are $j \in \{m, f\}$with $m$ meaning male and $f$ female. Similarly for location $n$. In each labour market vacancies and job seekers meet randomly each period according to an aggregate meeting function with constant returns to scale. In order to account for the possibility that the unemployed search more efficiently locally, we will assume that $u_n^j$ enters into the meeting function of location $l$ as $\varepsilon u_n^j$ and that $\varepsilon < 1$. Thus, each period the number of meetings in location $l$ for workers $j$ will be given by $M(u_l^j + \varepsilon u_n^j, v_l^j)$. With these assumptions, an unemployed worker $j$ who lives in $l$ will meet a vacant job in this location with probability $\alpha_{ll}^j = \frac{M(u_l^j + \varepsilon u_n^j, v_l^j)}{u_l + \varepsilon u_n}$. On the other hand, an unemployed worker $j$ who lives in location $n$ will meet a vacant job in $l$ with probability $\alpha_{nl}^j = \varepsilon \alpha_{ll}^j$. We define $\theta_l^j = \frac{v_l^j}{u_l^j + \varepsilon u_n^j}$ as the market tightness in $l$ for worker $j$. An unemployed worker may meet one, two or zero vacant jobs. After meeting them, the couple decides whether to match to one of them or to remain unemployed. If the match is realized, wages are set by Nash bargaining over fixed-wage contracts. That is, once the wage is bargained in the first period, the worker will receive the same wage until the end of the employment relationship. Denote by $P = [z, y]$ the set of possible wages.

Employed workers cannot search. The employment relationship may end exogenously with probability $s$ in each period. A match may also end if a worker quits. We only allow workers to quit if their spouse receives a job offer from the other location and they decide to accept it and migrate.

### 2.1.4 *Timing of events*

Each period is composed of the following stages:

1. Unemployed workers and vacant jobs meet in the labour market.

2. Couples with unemployed workers choose a job and bargaining for new matches takes place.

3. The spouses of the workers that have accepted a job in another city quit and they both migrate.

4. Production takes place, couples receive their income and decide their level of consumption and financial assets for that period.

5. Employed workers lose their job with probability $s$.

### 2.1.5 *Household's decision problem*

We describe next the Bellman equations of the couples at the point in which they have already bargained over the wage and the migration decision has already taken place.

In stage 4,$W\left(a, p_m, p_f\right)$ is the value of a couple that in this period has level of assets $a$, whose husband has labour payoff $p_m$ and whose wife has labour payoff $p_f$. The labour payoff, $p_m$ is the wage of the husband if he is employed and is equal to $z_m$ if the husband is unemployed. Similarly for the wife. Notice that the value of the couple does not depend on whether they migrated or not at the beginning of the period. To simplify notation, we omit the location from the state of the couple.

The problem of the household in stage 4 is the following:

$$\begin{aligned} W\left(a, p_m, p_f\right) &= \max_{c_m, c_f, a' \geq 0} \left\{ \vartheta u\left(c_m\right) + (1-\vartheta)\, u\left(c_f\right) + \beta \widetilde{W}\left(a', p_m, p_f\right) \right\} \\ \text{st } c_m + c_f + a' &= p_m + p_f + (1+i)\, a \end{aligned} \tag{12}$$

The couple chooses consumption, $c$, and the level of financial assets they want to keep, $a'$, that maximizes its lifetime utility subject to the budget constraint. The budget constraint states that the couple's consumption expenditure, $c_m + c_f$, plus the level of savings that they want to keep, $a'$, must be equal to the labour income of the spouses plus their financial assets and the income from them.

Their lifetime utility is the sum of their current utility plus a continuation value, $\widetilde{W}\left(a', p_m, p_f\right)$. The current utility is the sum of the utility of both spouses with Pareto weights given by $\vartheta$ and $(1-\vartheta)$. The continuation value differs depending on the employment status of the couple, $\left(p_m, p_f\right)$. We will explain first the case of a couple where both husband and wife are employed, then the case where only one of them is employed and last we will describe the continuation value when both are unemployed.

#### 2.1.5.1 *Husband and wife employed*

If both members of the couple are employed, we will have that $p_m = w_m$ and $p_f = w_f$. Then, their continuation value is:

$$\begin{aligned} \widetilde{W}\left(a, w_m, w_f\right) = (1-s)^2\, W\left(a, w_m, w_f\right) + (1-s)\, sW\left(a, w_m, z_f\right) \\ + s\,(1-s)\, W\left(a, z, w_f\right) + s^2 W\left(a, z_m, z_f\right) \end{aligned}$$

That is, in the next period, they both will keep their job with probability $(1-s)^2$, they both will lose their job with probability $s^2$and with probability $s\,(1-s)$, only the husband will keep his job and with the same probability only the wife will keep her job. In neither of the cases the couple migrates, since only unemployed workers who have found a job in another location and their spouses can migrate. Finally, since wages are constant during the employment relationship, next period the workers that keep their job receive the same labour payoff as in the current period.

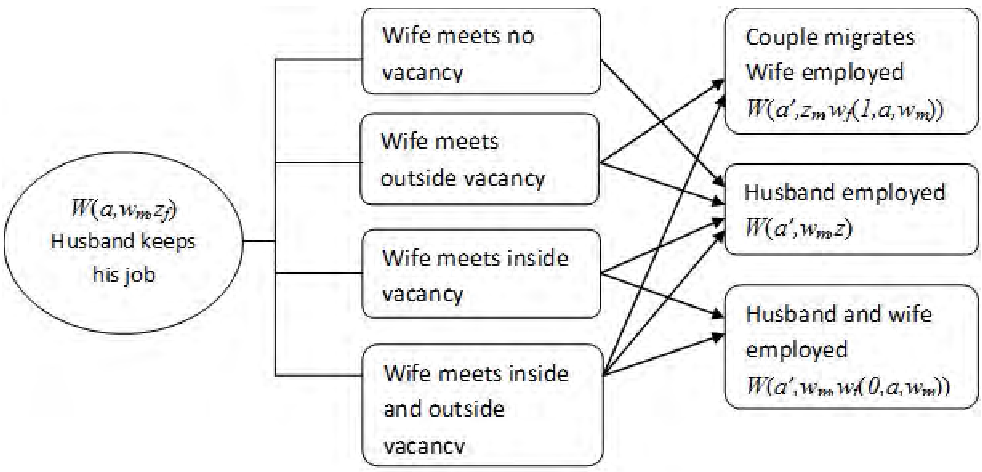
#### 2.1.5.2 *One spouse employed*

If only the husband is employed[1], the continuation value, $\widetilde{W}\left(a, w_m, z_f\right)$, will be the expectation over two outcomes: the value of the couple at the beginning of the period if he keeps his job and the value if he loses it. We denote by $\widetilde{W}_{-s}\left(a, w_m, z_f\right)$ the first case and $\widetilde{W}_s\left(a, z_m, z_f\right)$ the second one. Therefore, the continuation value is:

$$\widetilde{W}\left(a, w_m, z_f\right) = (1-s)\,\widetilde{W}_{-s}\left(a, w_m, z_f\right) + s\widetilde{W}_s\left(a, z_m, z_f\right)$$

We describe next how $\widetilde{W}_{-s}\left(a, w_m, z_f\right)$ is calculated. This value is the expectation over the outcomes represented in Figure 3.

Figure 3: Employment decision when the husband keeps his job




Since in the previous period the wife was unemployed, at the beginning of this one, she may meet a vacancy from the location where they live, "inside vacancy", from the other location, "outside vacancy", both or none. We had defined the probability of meeting an inside vacancy as $\alpha_{ll}^{j}$ when the worker lives in $l$ and $\alpha_{nn}^{j}$ when the worker lives in $n$. Since we are abstracting away from location, let the probability of meeting an inside vacancy be denoted as $\alpha_i^j$ for $j \in \{l, n\}$. Similarly, we will denote the probability of meeting an outside vacancy as $\alpha_o^j$.

The value of the couple at the beginning of the period can be calculated as the sum of four terms which correspond to the four possible outcomes. The first term corresponds to the case when she does not meet any vacancy, the second, when she only meets an outside vacancy, the third, meeting an inside vacancy and the last corresponds to the case when she meets both:

$$\begin{aligned}
\widetilde{W}_{-s}\left(a, w_m, z_f\right) = & \left(1-\alpha_i^f\right)\left(1-\alpha_o^f\right) W\left(a, w_m, z_f\right) \\
& + \left(1-\alpha_i^f\right)\alpha_o^f \max\left\{W\left(a, z_m, w_f\left(1, a, w_m\right)\right), W\left(a, w_m, z_f\right)\right\} \\
& + \alpha_i^f\left(1-\alpha_o^f\right) max\left\{W\left(a, w_m, w_f\left(0, a, w_m\right)\right), W\left(a, w_m, z_f\right)\right\}
\end{aligned}$$

1 The value of a couple where only the wife is employed works analogously.

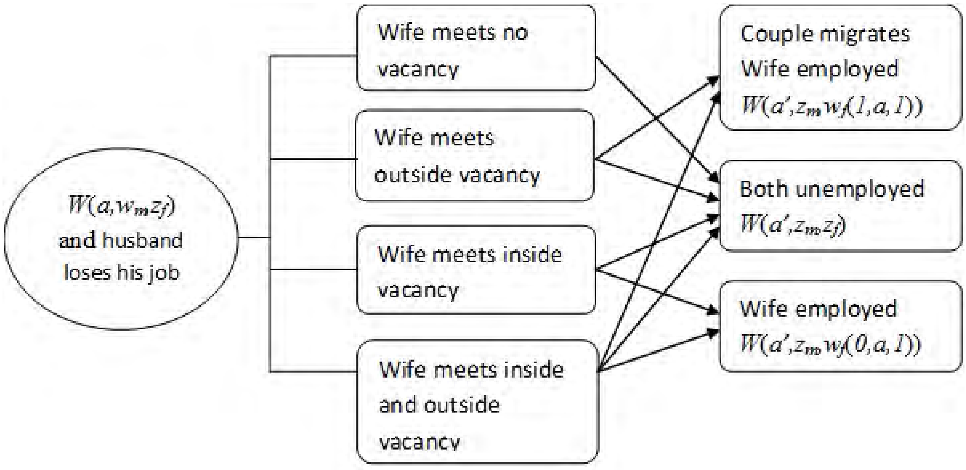
$$+\alpha_i^f \alpha_o^f max\left\{W\left(a, w_m, w_f\left(0, a, w_m\right)\right), W\left(a, z_m, w_f\left(1, a, w_m\right)\right), W\left(a, w_m, z_f\right)\right\}$$

Depending on which vacancies the wife meets, the couple must decide if she matches to some vacancy and if the husband must quit his job. As a result, they will be in one of the three cases represented in the third column of Figure 3. These cases are that the couple remains in the same employment status with the husband employed and the wife unemployed, that the wife gets employed in the same location so they both are employed or that the wife gets employed in the other location and they both migrate. In the last two cases the continuation value of the couple will depend on the wage that the wife will earn in her new job. We will derive how the wage is bargained in subsection 2.1.7, now we just take into account that the wage will depend on the migration status of the couple, which we will denote by $g$, their level of assets, $a$, and both the employment status and wage of her husband, that we will summarise as $w_m$, so $w_f = w_f(g, a, w_m)$. We can now describe the continuation value of the couple.

If the wife does not meet any vacancy, the couple has no choice and must remain in the same employment status as before. The value of the couple will be $W\left(a, w_m, z_f\right)$. If the wife only meets an outside vacancy, she can accept it or reject it. If she rejects it, they will remain in the same employment status and have value $W\left(a, w_m, z_f\right)$. But if she accepts it, the husband must quit and they both migrate. Then, their value will be $W\left(a, z_m, w_f\left(1, a, w_m\right)\right)$. Consider now the case where the wife only meets an inside vacancy. If she accepts, in the following period they will be both employed, $W\left(a, w_m, w_f\left(0, a, w_m\right)\right)$, and if she rejects they will remain with the same employment status as before $W\left(a, w_m, z_f\right)$. Finally, if the wife meets both vacancies, she can either match the inside vacancy, match the outside vacancy or reject both. The value of the couple in each of these cases will be, respectively, $W\left(a, w_m, w_f\left(0, a, w_m\right)\right)$, $W\left(a, z_m, w_f\left(1, a, w_m\right)\right)$ and $W\left(a, w_m, z_f\right)$.

Let's consider now $\widetilde{W}_s\left(a, z_m, z_f\right)$, which is the value of the couple if the husband lost his job at the end of the previous period. Again, we must take into account that the wife may meet and outside vacancy, an inside vacancy, both or none. As the husband has lost his job, his employment status in the wage function of the wife will be denoted by, 1, so $w_f = w_f(g, a, 1)$. In Figure 4 are summarized the decisions of the couple for each of these cases.

Figure 4: Employment decision when the husband loses his job




Therefore, the value of the couple at the beginning of the period is:

$$\begin{aligned}\widetilde{W}_s\left(a,z_m,z_f\right) = &\left(1-\alpha_i^f\right)\left(1-\alpha_o^f\right)W\left(a,z_m,z_f\right)\\ &+\left(1-\alpha_i^f\right)\alpha_o^f \max\left\{W\left(a,z_m,w_f\left(1,a,1\right)\right),W\left(a,z_m,z_f\right)\right\}\\ &+\alpha_i^f\left(1-\alpha_o^f\right) max\left\{W\left(a,z_m,w_f\left(0,a,1\right)\right),W\left(a,z_m,z_f\right)\right\}\\ &+\alpha_i^f\alpha_o^f max\left\{W\left(a,z_m,w_f\left(0,a,1\right)\right),W\left(a,z_m,w_f\left(1,a,1\right)\right),W\left(a,z_m,z_f\right)\right\}\end{aligned}$$

If the wife does not meet any vacancy, both the husband and the wife are unemployed in the following period and their value will be $W\left(a,z_m,z_f\right)$. If the wife only meets an outside vacancy, she can accept it or reject it. If she rejects it, they will be both unemployed and have value $W\left(a,z_m,z_f\right)$. But if she accepts, they migrate and obtain the value $W\left(a,z_m,w_f\left(1,a,1\right)\right)$. The third case is that the wife only meets an inside vacancy. If she accepts, they will have value $W\left(a,z_m,w_f\left(0,a,1\right)\right)$, and if she rejects they will obtain $W\left(a,z_m,z_f\right)$. Finally, if the wife meets both vacancies, she can either match the inside vacancy, match the outside vacancy or reject both. The value of the couple in each of these cases will be, respectively, $W\left(a,z_m,w_f\left(0,a,1\right)\right)$, $W\left(a,z_m,w_f\left(1,a,1\right)\right)$ and $W\left(a,z_m,z_f\right)$.

#### 2.1.5.3 *Neither member of the couple is employed*

If both members of the couple are unemployed, their value will be given by $W\left(a,z_m,z_f\right)$. In this case, they both will search and in the following period, they can meet vacancies from both locations. We relegate the description of the possible outcomes to the Appendix.

Finally, from the household problem we can derive the savings rule, which will be given by $a^* = g_a\left(a,p_m,p_f\right)$.

### 2.1.6 *Firms problem*

We now turn to the problem of the firm. There are many firms, each with one job. The per period profits of the firm are constant for the duration of the match, given that the wage is constant. However, the probability that the match ends depends on the state of the couple because so does the probability that a worker quits. Thus, we will describe first the value of a filled job that employs a worker whose spouse is also employed. Then, we will describe the case where the spouse is unemployed. Note also that the profits of the firm not only depend on the state of the couple but also on which member of the couple it is employing.

#### 2.1.6.1 *Both members of the couple are employed*

The value of a firm that employs the husband when the couple is in state $\left(a, w_m, w_f\right)$ is $J\left(m, a, w_m, w_f\right)$ with:

$$\begin{aligned} J\left(m, a, w_m, w_f\right) = y_m - w_m + \frac{1}{1+i}\Big[(1-s)^2 J\left(m, a^*, w_m, w_f\right) \\ + s\left(1-s\right) J\left(m, a^*, w_m, z_f\right) + sV\Big] \end{aligned} \tag{13}$$

where $a^* = g_a\left(a, p_m, p_f\right)$.

In that period the firm will obtain the productivity minus the wage. In the following period, with probability $(1-s)$ the match will continue and with probability $s$ the match will end, in the latter case the value of the job will be $V$. However, if the match continues, either the wife keeps her job and the value of the firm is $J\left(m, a^*, w_m, w_f\right)$ or she loses her job and the value of the firm is $J\left(m, a^*, w_m, z_f\right)$.

#### 2.1.6.2 *Only one member of the couple is employed*

The value of a firm that employs the husband when the couple is in state $\left(a, w_m, z_f\right)$ is denoted by $J\left(m, a, w_m, z_f\right)$. It consists of current profits, $y_m - w_m$, plus a continuation value that depends on the employment status of both spouses. If the husband loses his job, that is, if the match ends for exogenous reasons, the continuation value of the firm is $V$. If the match does not end for exogenous reasons we must take into account the value of the firm when the wife becomes employed in the same location, when she becomes employed in the other location or when she remains unemployed.

In the first case, the case in which the wife finds a job in the same location, the continuation value of the firm will be the value of employing the husband when both spouses are employed, $J\left(m, a^*, w_m, w_f\left(0, a^*, w_m\right)\right)$. The probability of this event, that we denote by $p_2$, is the sum of two terms. The first one is the probability that the wife meets an inside and an outside vacancy, $\alpha_i^f \alpha_o^f$, and matches the inside vacancy. We must take into account that acceptance will take place if the couple is not worse off than when they migrate $W\left(a^*, w_m, w_f\left(\cdot\right)\right) \geq W\left(a^*, z, w_f\left(\cdot\right)\right)$ or than when the wife remains unemployed, $W\left(a^*, w_m, w_f\left(\cdot\right)\right) \geq W\left(a^*, w_m, z\right)$. The second term is the probability that the wife only meets an inside vacancy, $\alpha_i^f\left(1-\alpha_o^f\right)$, and accepts. In this case acceptance will take place if the couple is not worse off than when the wife remains unemployed, $W\left(a^*, w_m, w_f(\cdot)\right) \geq W\left(a^*, w_m, z\right)$. Thus, $p_2$ can be calculated as:

$$p_2 = \left[\alpha_i^f \alpha_o^f I\left(W(,a^*,w_m,w_f(\,)) \geq W(a^*,z_m,w_f(\,))\right) + \alpha_i^f\left(1-\alpha_o^f\right)\right] I\left(W(a^*,w_m,w_f(\,)) \geq W(a^*,w_m,z)\right)$$

If the wife becomes employed in the other location, the husband will quit his job and migrate with her, so the continuation value for the firm will be zero. The probability of this event is denoted by $p_1$. It is also the sum of two terms. The first one is the probability that the wife meets an inside and an outside vacancy, $\alpha_i^f \alpha_o^f$, and matches the outside vacancy. We must take into account that acceptance will take place if the couple is better off than when she matches the inside vacancy $W\left(a^*, z_m, w_f(\cdot)\right) > W\left(a^*, w_m, w_f(\cdot)\right)$ or than when the wife remains unemployed, $W\left(a^*, z_m, w_f(\cdot)\right) > W\left(a^*, w_m, z_f\right)$. The second term is the probability that the wife only meets an outside vacancy, $\left(1-\alpha_i^f\right)\alpha_o^f$, and accepts. In this case acceptance will take place if the couple is better off than when the wife remains unemployed, $W\left(a^*, z_m, w_f(\cdot)\right) > W\left(a^*, w_m, z_f\right)$. Thus, $p_1$ can be calculated as:

$$p_1 = \left[\alpha_i^f \alpha_o^f I\left(W(a^*,z_m,w_f(\,)) > W(a^*,w_m,w_f(\,))\right) + \left(1-\alpha_i^f\right)\alpha_o^f\right] I\left(W(a^*,z_m,w_f(\,)) > W(,a^*,w_m,z_f)\right)$$

If the wife remains unemployed, the continuation value of the firm will be the value of employing the husband when the wife is unemployed, $J\left(m, a^*, w_m, z_f\right)$. This will occur with probability $1 - p_2 - p_1$.

Therefore, the value of a firm that employs the husband when the wife is unemployed is given by:

$$\begin{aligned} J\left(m, a, w_m, z_f\right) = y_m - w_m + \frac{1}{1+i}\Big[&(1-s)\, p_2 J\left(m, a^*, w_m, w_f\left(0, a^*, w_m\right)\right) \\ &+ (1-p_2-p_1)(1-s)\, J\left(m, a^*, w_m, z_f\right) + \left(s + p_1(1-s)\right) V\Big] \end{aligned} \tag{14}$$

### 2.1.6.3 *Value of a vacancy*

To create a job, a firm first posts a vacancy. There is a flow cost of posting a vacancy, denoted by $\xi$. In the following period, a firm in the labour market for $j$ will hire a worker with probability $\alpha_r^j$. Let $\lambda_{hire}\left(j, g, a, p_m, p_f\right)$ be the density of the unemployed workers that a firm can hire. To simplify notation we denote $\widetilde{\psi} = \left(j, g, a, p_m, p_f\right)$, so the density can be written as $\lambda_{hire}\left(j, g, a, p_m, p_f\right) = \lambda_{hire}\left(\widetilde{\psi}\right)$.[2] Notice that the value of a filled job is a function of $\widetilde{\psi}$, $J\left(\widetilde{\psi}\right)$ . The value of a vacancy in labour market $j$ is:

$$V_j = -\xi + \frac{\alpha_r^j}{1+i}\int_{\Psi} J\left(\widetilde{\psi}\right)\lambda_{hire}\left(\widetilde{\psi}\right) d\widetilde{\psi} \tag{15}$$

2 $\lambda_{hire}\left(\widetilde{\psi}\right)$ and $\alpha_r^j$ are derived in the Appendix.

with $\Psi = \{j\} \text{x} \left\{0, \overline{h}\right\} \text{x} \{0,1\} \text{x} A \text{x} P \text{x} P$.

### 2.1.7 *Wage determination*

The wage is fixed for the duration of the match and determined through Nash bargaining between the firm and the worker at the beginning of their employment relationship. We will describe here the wage determination of the husband. The same logic applies for the wife. The wage of the husband will depend on the migration status of the couple, $g$, their level of assets, $a$, and the employment status of the wife. The employment status of the wife in the wage function includes more possibilities than her employment status in the value of the couple. For the case of the wage function, we must consider the case in which the wife is employed at the moment of bargaining, the case in which she is unemployment but is also being hired that period and the case in which she is unemployed and remains unemployed that period. In the first case, the employment status of the wife will be given by $w_f$, so the wage of the husband will be $w_m\left(g,a,w_f\right)$. In the second case, the case in which the wife is also being hired, the employment status of the wife will be denoted by 2 and the wage of the husband will be $w_m\left(g,a,2\right)$. Finally, if the wife remains unemployed, her employment status will be denoted by 1 and the wage of the husband will be $w_m\left(g,a,1\right)$.

Consider a couple with a level of assets $a$. If the wife is unemployed at the beginning of the period and does not get a job, the wage of the husband, $w_m\left(g,a,1\right)$, will be the solution to:

$$\max_{w_m} \left(W\left(a,w_m,z_f\right) - W\left(a,z_m,z_f\right)\right)^{\gamma} \left(J\left(m,a,w_m,z_f\right) - V\right)^{1-\gamma} \tag{16}$$

The first term, $W\left(a,w_m,z_f\right) - W\left(a,z_m,z_f\right)$, represents the gains from the match for the couple if the husband gets employed at wage $w_m$ where $W\left(a,w_m,z_f\right)$ is the value of the couple if he gets matched and $W\left(a,z_m,z_f\right)$ is the value if he remains unemployed. The second term, $J\left(m,a,w_m,z_f\right) - V$, represents the gains from the match for the firm.

If the wife is unemployed at the beginning of the period and both the husband and the wife find a job at the same time[3], the wage of the husband, $w_m\left(g,a,2\right)$, will be the solution to:

$$\begin{aligned}\max_{w_m} \left(W\left(a,w_m,w_f\left(g,a,2\right)\right) - W\left(a,z_m,w_f\left(g,a,2\right)\right)\right)^{\gamma}\cdot \\ \cdot\left(J\left(m,a,w_m,w_f\left(g,a,2\right)\right) - V\right)^{1-\gamma}\end{aligned} \tag{17}$$

In this case, the value of the couple if he gets matched will be the value of being both spouses employed, $W\left(a,w_m,w_f\left(g,a,2\right)\right)$, and the value of the couple if he does not get matched is the value of the couple when only the wife gets employed $W\left(a,z,w_f\left(g,a,2\right)\right)$. Consistent with this, the gains from the match for the firm are $J\left(m,a,w_m,w_f\left(g,a,2\right)\right) - V$.

Finally, consider the case where the wife was employed at wage $w_f$ in the previous period and kept her job at the end of it. In this case, if the husband finds a job in the

3 This possibility can only arise if she was also unemployed the previous period. Otherwise, she could not search.

same location, both spouses will be employed and they will not migrate, $g = 0$, but if the husband finds a job in the other location, they will migrate, $g = 1$, and she will quit. Therefore, the wage of the husband if he finds a job in their location, $w_m\left(0, a, w_f\right)$, will be the solution to:

$$\max_{w_m} \left(W\left(a, w_m, w_f\right) - W\left(a, z_m, w_f\right)\right)^{\gamma} \left(J\left(m, a, w_m, w_f\right) - V\right)^{1-\gamma} \tag{18}$$

On the other hand, if the husband gets employed in the other location, his wage, $w_m\left(1, a, w_f\right)$, will be the solution to:

$$\max_{w_m} \left(W\left(a, w_m, z_f\right) - W\left(a, z_m, w_f\right)\right)^{\gamma} \left(J\left(m, a, w_m, z_f\right) - V\right)^{1-\gamma} \tag{19}$$

In this last case, the value if the husband does not get matched is $W\left(a, z_m, w_f\right)$, that is, we assume that the wife will not quit her job until the bargaining has been realized. For this reason, the wage of the husband when he finds a job in the other location depends on the wage of his wife. If the job is in the other location, they will migrate and $g = 1$, whereas if the job is in their location they will not migrate.

### 2.1.8 *Equilibrium*

Since both locations have the same productivity, we will consider a symmetric equilibrium in which they have the same share and distribution of the population and the same number of firms. We omit all locational subscripts.

A steady-state equilibrium consists of a set of value functions $\left\{W\left(a, p_m, p_f\right), J\left(j, a, p_m, p_f\right)\right\}$, a decision rule $\left\{g_a\left(a, p_m, p_f\right)\right\}$, a time-invariant measure of agent types $\lambda\left(\widetilde{\psi}\right)$, a set of prices
$\left\{w_j\left(h, g, a, \cdot\right)\right\}$, and market tightness $\left\{\theta_j\right\}$ such that:

1. Given prices and market tightness, the household's decision rules solve the dynamic program given by (12).
2. Given prices and the household's decision rules, the firm solves (13)-(14).
3. $\theta_j$ satisfies $V_j = 0$ with $V_j$ given by (15).
4. Wages satisfy (16)-(19).
5. $\lambda\left(\widetilde{\psi}\right)$ is the invariant distribution generated by the meeting probability, separation rate and the household's decision rules.

## 2.2 CALIBRATION

We calibrate the model to the US economy for 2000-2009. In this period, we assume that both males and females are in the labour force and have the same productivity: $y_m = y_f$.

### 2.2.1 *Functional forms*

We need to specify the functional form of the utility function and the meeting function. We assume that the utility function of the couple is given by:

$$u(c) = \frac{c^{1-\sigma}}{1-\sigma}$$

The meeting function is:

$$M(u_l^j + \varepsilon u_m^j, v_l^j) = k v_l^{j1-\mu} \left(u_l^j + \varepsilon u_m^j\right)^{\mu}$$

Therefore $\alpha_{ll}^j = k\theta_l^{j1-\mu}$ and $\alpha_{ml}^j = \varepsilon k \theta_l^{j1-\mu}$ with $\theta_l^j = \frac{v_l^j}{u_l^j + \varepsilon u_m^j}$ the market tightness in $l$.

### 2.2.2 *Parametrization*

We now describe the parameters used in the model. Since the model period is a month, parameters are expressed at monthly frequency except for migration rates. In Table 9 we have the parameters that are set exogenously because they are measured directly from the data or because we take them directly from the literature. We set the interest rate to $i = 0.327\%$, which corresponds to an annual interest rate of 4%, as in Díaz and Luengo-Prado (2008). The cost of posting a vacancy is $\xi = 1.752$, consistent with Hagedorn and Manovskii (2008), who calculate that the cost of posting a vacancy is 58.4% of labour productivity. Following Shimer (2005), both the bargaining weight and the elasticity of the meeting function on unemployment are set to $\mu = \gamma = 0.72$.

Table 9: Exogenous parameters

| Parameter | Source |
|---|---|
| $\sigma = 2$ | |
| $\vartheta = 0.5$ | |
| $\frac{y_m}{y_f} = 1$ | |
| $i = 0.327\%$ | Díaz and Luengo-Prado (2008) |
| $\xi = 1.752$ | Hagedorn and Manovskii (2008) |
| $\gamma = \mu = 0.72$ | Shimer 2005 |

The remaining parameters and the relevant targets are given in Table 10. In the table we associate each parameter with a specific target for which it is particularly relevant. We explain next each of the targets in more detail.

We use that median ratio of financial assets to income for married workers in 2001 was 7.95 as a target. This information calculated from the data provided by the Survey of Consumer Finances (SCF).

We also attempt to match an unemployment income equal to 40% of the mean wage, as in Shimer (2005). The job finding rate and the unemployment rate of married workers is calculated as the average for the period 2000-2004 with the information from the Current Population Survey (CPS). We obtain that the job finding rate during this period was 0.35 and the unemployment rate 3.3%.

Table 10: Estimated parameters

| Parameter | Target | Source |
|---|---|---|
| $\beta = 0.9965$ | median assets to income= 7.96 | SCF 2001 |
| $s = 0.024$ | unemployment rate= 3.3% | CPS 2000-2004 |
| $k = 0.52$ | job finding rate= 0.35 | CPS 2000-2004 |
| $z_m = z_f = 1.19$ | $z_j = 40\%$ mean wage $j$ | Shimer 2005 |
| $\varepsilon = 0.08$ | annual migration rate= 2.1% | CPS 1999-2005 |

Finally, we have as a target the migration rate for the period 1999-2005 with the information from the March Current Population Survey. In the calculations we only consider moves caused by work related reasons. We obtain an annual rate of 2.4% for the period from 1999 to 2005.

Table 11 summarizes the fit of the model with respect to the targeted moments.

Table 11: Calibration targets

| Moment | Data | Model |
|---|---|---|
| median assets to income | 7.96 | 8.00 |
| unemployment rate | 3.3% | 3.3% |
| job finding rate | 0.35 | 0.35 |
| unemp. flow/wage | 40% | 40% |
| annual migration rate | 2.1% | 2.1% |

## 2.3 RESULTS

In this section, we will study the behaviour of the model economy for different stages of the women entry into the labour force. The model will allow as to isolate the effect of women attachment into the labour force from other changes that took place during this period. We will compare the level of unemployment, migration and savings in an economy where only the husband works, which would correspond to the American economy at the beginning of the twentieth century to the current situation where both husband and wife have similar professional careers. We will also compute the equilibrium of the model for an intermediate situation, where both spouses work but women have a lower productivity than men. This situation would correspond to the end of the seventies and eighties. This is because in 1980 most of the expansion in women participation rate had already taken place. The participation rate of married white females between 35 and 44 years was 10% in 1930 and slightly above 60% in 1980[4]. On the other hand, women's earnings as a percentage of men's earnings was between 58% and 60% for all the sixties and seventies. In 1980 was still 60% but then it started increasing to above 75% in the 2000s. The lower earnings received by women were largely due to a lower level of human capital. For example, college graduation

4 Goldin (2006)

rates of males had been larger than that of women for all the first part of the twentieth century. However, this gap started to narrow until the cohorts born in 1960, thus those that went to college around 1980, and from then on women graduation rates have out-weighted men's ones.[5]

We model this process as an exogenous change in women's productivity. Goldin (2006) explains how the increase in the participation rate that took place between the thirties and fifties was mainly the result of new information technologies that created nice jobs, like clerical jobs, and the increase in high school graduation. Continued participation into the labour force was unexpected for these generation of women. This would contrast with the newer generations, that would anticipate longer periods in the labour force. Therefore, the percentage of women that graduated from college increased. We model the increase in the human capital of women as an exogenous change in their productivity.

In Table 12 we report the equilibrium of the model for the three situations we have described[6]. We find that the migration rate is lowest when women are out of the labour force and highest when both spouses have the same level of productivity. This result contrasts with the idea that having a spouse who works imposes a restriction to mobility. The reason for this finding is that when both spouses are part of the labour force, they both may find a job in another location, increasing the probability of migration, whereas in the other case only the husband may find a job opportunity outside. Comparing the two cases where women are part of the labour force, we find that migration is highest when women have the same productivity as men. This is because in this later case, the level of migration originated from wife jobs is higher.

We measure the level of savings with the median level of assets to income that couples keep. When the wife takes part of the labour force, she provides insurance to the spouse and the level of savings accumulated by the couple is lower. We find that the effect on savings is considerable, as the ratio of assets to income diminishes from 16, when only the husband works, to 8, when there is no gap in productivity between the spouses.

Finally, the model also provides with some predictions on the unemployment rate. We find that in the case in which females have a lower productivity than males, their unemployment rate is higher. This is so because it is not so profitable for firms to incur the cost of posting a vacancy for a female although the wage they pay to females is lower than to males. This can be seen, in their lower job finding rate. On the other hand, the higher unemployment rate of females is also due to their higher separation rate compared to males. As the family migrates more often for the employment prospects of the husband, the wife must also quit her job more often.

Interestingly, the unemployment rate of males differs in the three situations. When both spouses have the same productivity and are part of the labour force, males unemployment rate is higher. This result is due to the higher separation rate of males and despite their higher job finding probability. The reason for the higher separation rate of males is that if their wife finds a job in another location, they may migrate. The increase in the job finding probability can be explained from the increase in market

5 Goldin and Katz (2009), page 249.
6 For the case with the productivity gap, we assume that $z_f = 0.6 z_m$.

Table 12: Three stages of the labour market

| $y_f/y_m$ | - | 0.6 | 1 |
|---|---|---|---|
| annual migration rate | 1.0% | 1.6% | 2.1% |
| - only male finds job | 1.0% | 1.0% | 1.0% |
| - only female finds job | - | 0.6% | 1.0% |
| - both spouses find job | - | 0.0% | 0.0% |
| assets to income | 16 | 9 | 8 |
| job finding rate | 34.6% | 31.5% | 34.7% |
| - males | 34.6% | 34.4% | 34.7% |
| - females | - | 29.0% | 34.7% |
| separation rate | 1.10% | 1.16% | 1.19% |
| - males | 1.10% | 1.14% | 1.19% |
| - females | - | 1.18% | 1.19% |
| unemployment rate | 3.1% | 3.6% | 3.3% |
| - males | 3.1% | 3.2% | 3.3% |
| - females | - | 3.9% | 3.3% |
| mean wage | 2.969 | 2.374 | 2.967 |
| - males | 2.969 | 2.968 | 2.967 |
| - females | - | 1.775 | 2.967 |

tightness. The mean wage paid to males in this situation is lower, implying that it is more profitable for firms to employ males in this case.

The exercise we have realized is an attempt to understand the effect that the incorporation of married women in the labour market have produced on the economy. The results are at odds with the idea that having two workers in the family should reduce migration. This idea is posed in Molloy et al. (2011) as a possible explanation for the decrease in migration that took place from the 80s. However, as they report, the percentage of households with two earners has been stable during this period. On the contrary, the main increase in the participation rates of married women began in the 1950s, but in this period the migration rate was stable. On the other hand, Guler and Taskin (2013) propose the reduction in the wage gap that started in the 1970s as the factor generating the decrease in migration. The results of our model do not support this view.

We can also measure the relationship between women participation and migration through the variation at the cross sectional level. Consistent with our model, Gemici (2011) has documented that couples where both spouses have college degree have higher migration rates than couples when only one of the spouses belongs to this group. This evidence is in line with our results, but more direct evidence is needed on the effect of working spouses.

With respect to the evolution of personal savings rate, it has been found that the it has decreased in the US since the 1980s. This trend can be partly explained by measurement issues, wealth effects of the stock market, the increase of credit or to social security programs (Gokhale et al. (1996), Lusardi et al. (2001)). However, the survey in Guidolin and Jeneusse (2007) concludes that these explanations remain insufficient. It is also interesting to note that the savings rate has also decreed during this period in other countries like Canada, Australia and UK. The results of our model suggest that women participation into the labour force could have contributed to this trend.

Table 13: Unemployment rate

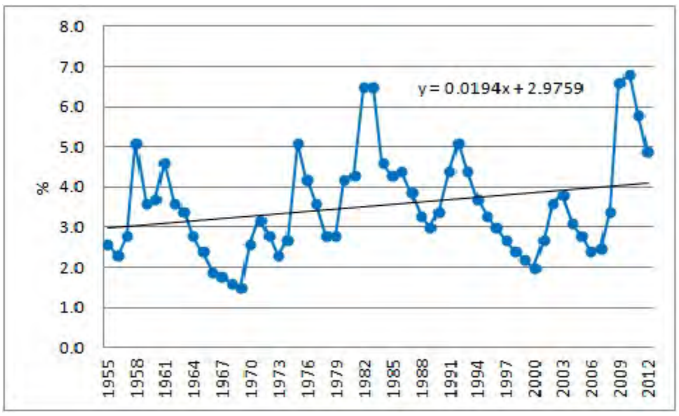
| | Married men | Married women |
|---|---|---|
| 1955-1964 | 3.5 | 5.1 |
| 1965-1974 | 2.3 | 4.7 |
| 1975-1984 | 4.5 | 6.4 |
| 1985-1994 | 4.0 | 4.5 |
| 1995-2004 | 2.9 | 3.3 |
| 2005-2012 | 4.4 | 4.4 |

Source: Bureau of Labour Statistics

Finally, we will briefly compare the results of our model with the evolution of the unemployment rate. We have data on the unemployment rate of married men and women since 1955. As it can be seen in table 13, the unemployment rate of married women had been, since 1955, higher than the unemployment rate of married men but these differences have been decreasing over the years until the current period in which they are equal.

There are few studies that deal with the unemployment rate of married women. Johnson (1983) attributes their high unemployment to a higher rate of transitions into and out of the labour force but Azmat et al. (2006) do not find strong evidence for that. Our model with differences in human capital across gender is consistent with the narrowing of the unemployment rate gap that took place in the last years.

Figure 5: Married men unemployment rate




Source: Bureau of Labour Statistics

Our model also predicts that women participation in the labour market generated a slightly higher unemployment rate for married men. In Figure 5, we can see that there was a positive trend during these years.

## 2.4 CONCLUSIONS

The labour market experienced an important transformation with the entry of women into the labour market. This process, not only increased the size of the labour force, but also changed the structure of households, with an increasing proportion of the two earner type. In this kind of households, workers must coordinate their search for jobs within the family.

In this paper, we have studied the implications of these process for the level of unemployment, migration and savings of married people. Our findings point to an increase in work related migration and a decrease in the level of savings. We also find that this process has led to a slight increase in aggregate unemployment.

These results suggest that the role of women should be taken into account in order to understand the evolution of the labour market, mobility and savings of the last decades.

# 3 HUMAN CAPITAL AND MARKET SIZE

## 3.1 INTRODUCTION

The distribution of skills differs across cities. In the US it has been found a positive, although small, relationship between average skill and city size.[1] Big cities have a higher proportion of high skilled workers. Glaeser and Resseger (2010) find a 2.8 increase in the percentage of college graduates in a city for an increase of 1% in population size for US in 2000. However, in big cities the distribution of skills is also more unequal than in small ones.[2] These differences in the skill composition is an important factor in order to understand productivity differences across locations. Combes et al. (2008) estimate that "differences in the skill composition of the labour force account from 40 to 50% of aggregate spatial wage disparities" in France.

This uneven distribution of skills across locations may be the result of workers sorting to those locations that favour more their level of skills. However, it also implies that city size must have an effect on the decision to become skilled. In this paper we consider how the size of cities affect the incentives to acquire human capital. This issue is of interest because it implies that the size of cities may be a determinant of the distribution of skills of the entire economy.

Big cities are characterized by big labour markets. Consider a big labour market along some period of time. Neither the number of workers that may be looking for a job nor the number of vancant jobs is constant. There is some variability on these numbers. However, the variability is small relative to the size of the market. Therefore, there is low uncertainty on the level of market tightness that the market will have at some moment in time. On the contrary, if the labour market is small, changes in the number of workers and jobs between different periods cause a great variability relative to the size of the market. These differences between big and small markets may have an effect on the decision to be skilled. Acquiring skills increases the chances to find a job when there is an excess of unemployed workers in the market. However, it is not so profitable when tightness is high and all workers get employed. Since in small markets there is more uncertainty on the level of tightness that the workers will encounter, there is also more uncertainty on the gains of this investment, with implications on the decision of the worker.

In this paper, we formalise this idea with a static model of mismatch with heterogeneity in human capital. Under this setting, the number of workers and vacancies in the market are random. However, the uncertainty on these numbers is decreasing with the size of the market. If there is an excess of workers, the ones with the lowest skills will be unemployed. However, workers can invest in human capital before entering the market in order to improve their ranking.

1 Bacolod et al. (2009)
2 Bacolod et al. (2009) and Eeckhout et al. (2011)

In this environment, we define market tightness as the ratio of the expected number of vacancies to the expected number of workers. Therefore, the size of the market is large if the expected number of workers is large, for a given level of market tightness.

We find that the effect of market size on the probability of finding a job for a worker with a given rank depends on market tightness. When the market is tight, the job finding probability of all workers increases with market size. However, if this ratio is low, the job finding probability of the workers with lower rank decreases with the size of the market, while the probability for the other workers increases.

The effect of market size on the job finding probability determines the effect on the distribution of human capital in equilibrium. For tight markets, the distribution converges to the top levels as market size increases. When market tightness is low, the distribution spreads to the extremes in bigger markets. The workers with lower rank invest less in human capital than if they were in smaller cities but the workers with higher rank invest more. Numerical results show that, when the level of market tightness is low, the mean of the distribution of human capital is increasing with market size, but only for small markets.

The model implies that the distribution of human capital in equilibrium is such that the job finding probability of a worker with a given level of human capital is independent of market size. That is, workers with the same rank have different probabilities depending on the size of the market, but workers with the same level of human capital do not.

Finally, the model also generates predictions with respect to the expected wage a worker can earn. We find that when market tightness is low, workers obtain higher returns to skill in bigger cities.

**Related literature**

Wheeler (2001), Bacolod et al. (2009) and Eeckhout et al. (2011) have documented the allocation of skills across locations. Wheeler (2001), using county-level data from the 1990 Census of Population and Housing, estimates a negative relationship between the percentage of population with low levels of education and population and a positive relationship for the case of population with high levels of schooling, thus implying that workers have higher levels of education in bigger cities. Bacolod et al. (2009) deal with the effect of city size on the distribution of education and other kind of skills, like intelligence or social skills, with data from the 1990 1% Census sample and the National Longitudinal Survey of Youth 1979. They find that the effect of city size on the mean education of workers is larger than on the other measures of skills. Their work also highlights that simplifying the level of skills to two levels might be too restrictive. Although they do not find a strong effect of city size on the mean level of intelligence and social skills, they do find that it affects the distribution, as larger cities accumulate more people at the top and at the bottom of the distribution. Eeckhout et al. (2011) findings also suggest that the distribution of skills in big cities have fatter tails. Using the information from the Current Population Survey of 2009, when they measure skills with years of education, they find a small positive correlation between city size and average skills but a positive relation between city size and the standard deviation of the skill measure.With respect to wages, both Wheeler (2001) and Bacolod et al. (2009) find that the urban wage premium increases with worker education, so that the wages of the workers at the top of the skill distribution increase relative to the workers at the low end of the distribution as market size increases.

There have been proposed different theories that can explain why big cities attract skilled workers. Most of them are based on different kinds of agglomeration economies. Resseger and Glaeser (2010) consider that cities foster the learning of skilled workers. In Andersson et al. (2007), bigger markets provide with better assortative matching. Hendricks (2011) relates the skill composition to the size of the business services sector. At the country level, Redding and Schott (2003), propose that skilled-intensive production has increasing returns to scale. Other theories are based on the idea that only the most skilled can afford the tougher competition of bigger markets, as in Nocke (2006) and Behrens et. al (2010). On the other hand, Eeckhout et al. (2011) explain the higher inequality in skills in big cities through complementarities between the skills of workers in the production function.

In contrast with the previous literature, in our model workers choose their level of skills[3]. Instead of an exogenous distribution of skills and workers sorting across locations, in our model workers remain in their location but can choose how much human capital to accumulate.

Our model is also related with the literature that deals with the efficiency of the matching function. One of the explanations for the existence of agglomeration economies is that the matching process is more efficient in bigger cities (Duranton and Puga 2004). This is consistent with the findings of Gan and Li (2004) for the market of PhD. economists and the evidence in Bleakley and Lin (2012) on ocuppational switching. Most of the empirical research find that the number of matches at the aggregate level is a constant-returns-to-scale function of unemployment and vacancies (Petrongolo and Pissarides 2001). However, Shimer (2007) shows that a mismatch model with a matching function that features increasing returns to scale is also consistent with the data.

The human capital investment decision in our model is closely related to Moen (1999). His formulation does not require heterogeneity of the workers or firms in order to find a non-degenerate distribution of skills. However, unlike us, he does not consider a mismatch model of the labour market and the size of the market plays no role.

The rest of the paper is organized as follows. Section 2 describes the setting. Section 3 considers the problem of the workers. In Section 4 we calculate the equilibrium. In Section 5 we use the model to analyse the relationship between the size of the labour market and the distribution of human capital. In Section 6 we consider the effect on wages, and, finally, Section 7 concludes.

## 3.2 setting

We analyse an economy that lasts for one period in which we can identify 3 stages: In the first stage a number of workers are born and vacancies are created. In the second stage, each worker decides how much human capital to accumulate. Finally, workers and vacancies go to the labour market in order to get matched and receive their payoffs.

We assume that the number of workers, $U$, and vacancies, $V$, is random: $U$ follows a uniform distribution with support $[a_u, b_u]$ and $a_u > 0$. $V$ follows a uniform distribution

3 The exception is Redding and Schott (2003), where workers can choose to be either skilled or unskilled.

with support $[a_v, b_v]$ and $a_v > 0$. We also assume that $b_u - a_u = b_v - a_v = 2k > 0$. The distribution of workers and vacancies are independent.

Let $\overline{U}$ and $\overline{V}$ be the expectation of $U$ and $V$ respectively and let $\overline{V} = \theta\overline{U}$, where we can interpret $\overline{U}$ as the size of the market and $\theta$ as market tightness. We can define the parameters of the distributions of $U$ and $V$ as $b_u = \overline{U} + k$, $b_v = \theta\overline{U} + k$, $a_u = \overline{U} - k$ and $a_v = \theta\overline{U} - k$. In this way, the distributions of $U$ and $V$ depend on $\overline{U}$ and $\theta$.

In the first stage, a realization of $U$ and $V$, which we will denote $u$, and $v$, is drawn. A continuum of workers are born and a continuum of vacancies are created. All workers and vacancies are respectively identical. We assume that each firm has one vacancy, so referring to a vacancy or a firm is equivalent. At this stage, workers do not know which realization has taken place.

In the second stage, the decision problem of each worker consists in choosing a costly level of human capital, $h$. Workers are risk neutral. The key assumption in this model is that the workers do not know which realization has been drawn before entering the market. This implies that when they decide how much human capital to accumulate they do not know $u$ and $v$ . However, we assume that they know the distribution of these two variables. If we think of education as one of the main factors to acquire human capital, this assumption is reasonable. Individuals usually decide their level of human capital at early stages in life, without knowing the exact conditions of the labour market.

Each worker can accumulate a level of human capital $h \in \mathbb{R}^+$ at a cost $C(h)$. The cost function is twice continuous differentiable and satisfies $C(0) = 0$, $C'(0) = 0$, $C'(h) > 0$ for $h > 0$, and $C''(h) > 0$. We also assume that for some $h$, $C'(h) > 1$.

After the investment decision has taken place, all workers and vacancies observe $u$ and $v$ and enter the labour market in order to get matched. We assume perfect competition in the labour market. The chances of getting matched for a worker will depend on his level of human capital and on the realizations of $U$ and $V$. If a vacancy and a worker with a level of human capital equal to $h$ match in the market, they together produce $h$ units of an homogeneous output. In this case, the payoff of the worker will be $w$ and the payoff of the firm will be $\pi = h - w$. The price of the good they produce is normalised to 1.

## 3.3 WORKERS' PROBLEM

In this section, we will deal with the workers' problem. First, we will set the problem they face at the third stage, when they enter the labour market. At that stage, the level of skills of all workers are already determined and the realization of the random variables are observed. We will derive which workers will be employed and at what wages. In the second part of this section, we will describe how workers choose their level of skills. We will assume that when workers take this decision, they have rational expectations over the equilibrium wages of the labour market.

### 3.3.1 *The labour market*

When the agents enter the labour market, the level of skills of the workers is already determined. Let $F(h)$ denote the cumulative distribution of human capital in equilibrium.

$F(h)$ can also be considered as the ranking of the worker, with higher $F(h)$ meaning a higher ranking. Initially, we will assume that this distribution has a connected support and that it is continuous with, possibly, a mass of workers at the minimum and maximum of the support. We will denote the minimum level chosen $h_{min}$, the maximum $h_{max}$ and the mass of workers at these levels, $f_0$ and $f_1$ respectively.

There is perfect competition in the labour market. Firms seek to maximize their profits and workers seek to maximize their wage. Shimer (2007) shows that, in this case, the number of matches will equal $\min\{u, v\}$ and that firms employ the most productive workers, that is, firms rank the workers according to their level of human capital. This last result and continuity of the distribution of human capital imply that a worker with a level of skills $h_0$ will find a job if there are more vacancies than workers with a higher level of skills than him, which can be expressed as $(1 - F(h_0))u \leq v$. In case there is a mass of workers at the extremes of the distribution, there are more cases that we must take into account.[4]

Let $h_m(u, v)$ denote the level of skills of the worker with the lowest level of human capital among those employed. The payoffs in equilibrium of a worker with level of skills $h \in (h_{min}, h_{max})$, when there are $u$ workers and $v$ vacancies in the market, is $w(h, u, v)$ and is given by[5]:

$$w(h,u,v) = \begin{cases} h & \text{when } v > u \\ h - h_m(u,v) & \text{when } (1 - F(h))\, u \leq v \leq u \\ 0 & \text{when } (1 - F(h))\, u > v \end{cases} \tag{20}$$

The function is derived in Appendix A.3.1. The payoff function states that if there is an excess of jobs, $v > u$, workers' wage is their productivity. However, if there is an excess of workers, $v \leq u$, then the wage of an employed worker is $h - h_m(u, v)$. If a worker does not match any firm, he does not produce and receives 0.

### 3.3.2 *Workers' expected earnings*

At the second stage of the period, workers choose their level of skills. Since they are risk neutral, their problem is to choose the level of human capital that maximizes their expected earnings, $\mathcal{E}(h)$.

The worker only knows the distributions from where $u$ and $v$ have been drawn. Notice that the lack of information at the second stage of the process is the only source

4 In case there is a mass $f_1$ of workers at the maximum of the support: If $f_1 < v$, all workers with $h_{max}$ find a job. If $v < f_1$, only $\frac{v}{f_1}$ of workers with $h_{max}$ will find a job.
In case there is a mass $f_0$ at the minimum of the support: If $v > u$, all workers with $h_{min}$ find a job. If $(1 - f_0)\, u < v < u$, only $\frac{v - u(1-f_0)}{f_0}$ of workers with $h_{min}$ will find a job. If $v < (1 - f_0)\, u$, none of workers with $h_{min}$ find a job.

5 In case there is a mass $f_1$ of workers at the maximum of the support: If $v > u$, $w(h_{max}, u, v) = h_{max}$. If $f_1 \leq v$, $w(h_{max}, u, v) = h_{max} - h_m(u, v)$. If $v < f_1$, $h_m(u, v) = h_{max}$ and $w(h_{max}, u, v) = 0$ for the employed and the unemployed.
In case there is a mass $f_0$ at the minimum of the support: If $v > u$, $w(h_{min}, u, v) = h_{min}$. If $(1 - f_0)\, u \leq v \leq u$, $h_m(u, v) = h_{min}$ and $w(h_{min}, u, v) = 0$ for the employed and the unemployed. If $v < (1 - f_0)\, u$, $w(h_{min}, u, v) = 0$.

of friction in this economy. Once the agents go to the market, the matching process is efficient, both in generating the maximum number of matches possible and in selecting the most productive workers. We assume rational expectations over the equilibrium in the labour market. The expected earnings, $\mathcal{E}(h)$, is a function of $h$ defined as:

$$\mathcal{E}(h) = \mathbb{E}\left[w(h, U, V)\right] - C(h)$$

where $\mathbb{E}[\cdot]$ is the expectation operator.

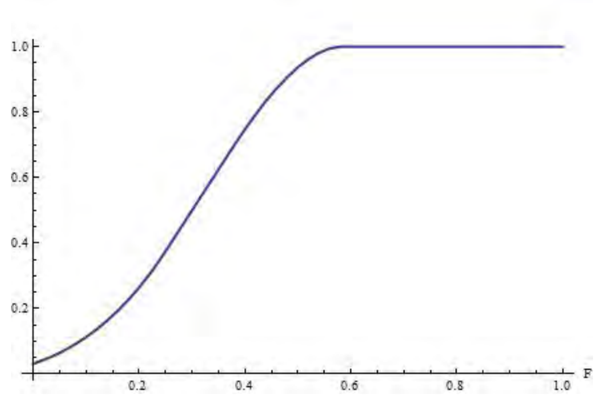
The expected payoff depends on the job finding probability of the worker. We will next derive this probability and decompose it in such a way that we can calculate the expected earnings. First of all, we will derive the probability that $(1-F)u \leq v$, for $F \in [0,1]$, since this is the job finding probability of a worker whose level of skills belongs to the continuous part of the distribution. We will denote this probability by $P(F)$. In Appendix A.3.2 we show that $P(F)$ is given by:

$$P(F) = \begin{cases} 1 & \text{for } \frac{b_u - a_v}{b_u} < F \leq 1 & [1] \\ \frac{-2a_u(b_v - a_v)(1-F) - a_v^2 + b_u(2b_v - b_u(1-F))(1-F)}{2(1-F)(b_u - a_u)(b_v - a_v)} & \text{for } \frac{a_u - a_v}{a_u} < F \leq \frac{b_u - a_v}{b_u} & [2] \\ \frac{2b_v - a_u(1-F) - b_u(1-F)}{2(b_v - a_v)} & \text{for } \frac{b_u - b_v}{b_u} < F \leq \frac{a_u - a_v}{a_u} & [3] \\ \frac{(b_v - a_r(1-F))^2}{2(1-F)(b_u - a_u)(b_v - a_v)} & \text{for } \frac{a_u - b_v}{a_u} < F \leq \frac{b_u - b_v}{b_u} & [4] \\ 0 & \text{for } 0 \leq F \leq \frac{a_u - b_v}{a_u} & [5] \end{cases} \tag{21}$$

This function is depicted in Figure 6.

Figure 6: Probability function

$P(F)$




For $b_u = 120$, $a_u = 80$, $b_v = 90$, $a_v = 50$.

**Lemma 1:**
- $P(F)$ is continuous and increasing in $F$.
- $P(1) = 1$.
- $P(0) = 0$ if $b_v \leq a_u$ and $P(0) > 0$ otherwise.

Proof: Appendix A.3.3.

Lemma 1 implies that $P(F)$ can be expressed as:

$$P(F) = P(0) + \int_0^F \frac{\partial P\left(\widetilde{F}\right)}{\partial \widetilde{F}} d\widetilde{F}$$

In the previous subsection we had defined $f_0$ as the mass of workers with human capital $h_{min}$. Therefore, we can rewrite $P(F)$ as:

$$P(F) = P(f_0) + \int_{f_0}^F \frac{\partial P\left(\widetilde{F}\right)}{\partial \widetilde{F}} d\widetilde{F} \tag{22}$$

Equation 22 represents the job finding probability of a worker with $h \in (h_{min}, h_{max})$ both in the case $f_0 > 0$ and when $f_0 = 0$.

As the worker's problem is defined in terms of his level of human capital, and not his ranking, we rewrite the job finding probability as a function of $h$, by defining $P_h(h) = P(F(h))$ since $P(F)$ and $F(h)$ are increasing functions.

Therefore, $P_h(h)$ can be represented as:

$$P_h(h) = P(F(h)) = P(f_0) + \int_{h_{min}}^h p_h\left(\widetilde{h}\right) d\widetilde{h}$$

with

$$p_h(h) = \begin{cases} \frac{\partial P(F(h))}{\partial F(h)} \frac{\partial F(h)}{\partial h} & \text{for } h_{min} < h < h_{max} \\ 0 & \text{otherwise} \end{cases}$$

Adding and subtracting $P(0)$, we obtain:

$$P_h(h) = P(0) + (P(f_0) - P(0)) + \int_{h_{min}}^h p_h\left(\widetilde{h}\right) d\widetilde{h}$$

According to this equation, $P_h(h)$, can be calculated as the sum of three terms. The first term is the probability that $(1 - F)u \leq v$ for $F = 0$, which is the probability that there is an excess of vacants. The second term is the probability that the workers with $h_{min}$ are the marginal workers. The third term is the integral of $p_h(h)$, which is the density of $h$ being the marginal worker. This is because $P_h(h)$ is the probability that $(1 - F(h))u \leq v$, so $p_h(h)$ is the density of $(1 - F(h))u = v$.

Using this definition of the job finding probability allows as to calculate the expected earnings of the worker as:

$$\mathcal{E}(h) = P(0)h + (P(f_0) - P(0))(h - h_{min}) + \int_{h_{min}}^h p_h(\widetilde{h})(h - \widetilde{h})d\widetilde{h} + (1 - P_h(h))(h - h_{max}) I(h > h_{max}) - C(h) \tag{23}$$

In this ecuation, $I(h > h_{max})$ is an indicator function equal to 1 when $h > h_{max}$. The first term of the equation accounts for the salary when there is an excess of vacancies. And the second to the forth term account for the case when there is an excess of workers for all possible values of the marginal worker. $\mathcal{E}(h)$ are the expected earnings for any worker with $h \geq 0$. We can check that is is true even for $h \leq h_{min}$ and $h \geq h_{max}$.

We can consider first the case of $h_{min}$. If $f_0 = 0$, the worker who chooses this level of human capital will only find a job if $u \leq v$ and his payoff will be $h_{min}$. Therefore, his expected earnings are $P(0)\, h_{min} - C(h_{min})$ which is equal to $\mathcal{E}(h_{min})$. On the other hand, if $f_0 > 0$, when $v$ is lower than $u$ but greater than $(1 - f_0)\, u$, some workers with $h_{min}$ will find a job and some not, but all of them will have a payoff equal to 0, since the marginal worker in this case is $h_{min}$. Therefore, also in this case, the expected earnings are $\mathcal{E}(h_{min})$, as equation 23 shows. Consider now the case of a worker that chooses $h_1 < h_{min}$. His job finding probability is $P(0)$ and, in this case, his payoff will be $h_1$. Clearly, his expected earnings can be calculated making use of $\mathcal{E}(h)$.

We can deal now with the opposite case, a worker with $h \geq h_{max}$. The argument is the same as with the minimum of the support. Consider a worker that chooses $h_{max}$ and $f_1 > 0$, then, equation 23 accounts for any case in which $v > f_1 u$. What happens if $v < f_1 u$? Then, $h_{max}$ is the marginal worker and has a payoff of 0. Thus, the expected earnings of worker $h_{max}$ can be calculated with $\mathcal{E}(h)$. However, a worker with $h > h_{max}$ would earn $h - h_{max}$ in this case. This is accounted in the fourth term of the equation.

The decision problem of the worker is:

$$\max_{\{h \geq 0\}} \{\mathcal{E}(h)\} \tag{24}$$

Note that the worker takes $F(h)$ and $w(h, u, v)$ as given.

## 3.4 EQUILIBRIUM

An equilibrium solution to the problem outlined above can be described by a tuple $(F(h), w(h, u, v))$ such that:

1. $w(h, u, v)$ satisfies (20)
2. Workers solve (24)
3. $F(h)$ satisfies $\mathcal{E}(h_1) = \mathcal{E}(h_2)$ for $\forall h_1, h_2$ on the support of $F(h)$ and $\mathcal{E}(h) < \mathcal{E}(h_1)$ for $\forall h$ that does not belong to the support.

The third point is due to the fact that, ex ante, all the individuals are equal. Thus, in equilibrium they all must obtain the same expected earnings. For any $h$ not chosen, the expected earnings must be lower. Given that the support of $F(h)$ is connected, this third condition implies that the derivative of the expected earnings with respect to human capital is zero for any $h$ in the support of $F(h)$:

$$\frac{\partial \mathcal{E}(h)}{\partial h} = 0$$

As we are dealing only with the cases in the support of $F(h)$, the indicator function is 0. We can rewrite $\mathcal{E}(h)$ in the following way

$$\mathcal{E}(h) = P(0)\, h + (P(f_0) - P(0))\,(h - h_{min}) + h\int_{h_{min}}^{h} p_h(\widetilde{h})d\widetilde{h} - \int_{h_{min}}^{h} p_h(\widetilde{h})\widetilde{h}d\widetilde{h} - C(h)$$

and take the derivative

$$\frac{\partial \mathcal{E}(h)}{\partial h} = P(0) + (P(f_0) - P(0)) + \int_{h_{min}}^{h} p_h(\widetilde{h})d\widetilde{h} + hp_h(h) - p_h(h)h - C'(h)$$

Notice that terms 4 and 5 account for the increment in the hiring probability, what it is referred to as the rat race. However, at this point the worker is the marginal worker, therefore, the gain from increasing the probability of finding a job is 0. This means that in this setup the ranking of workers does not produce rat race in contrast with Moen (1999). On the contrary, each worker chooses the optimal level of $h$ given his hiring probability. The derivative is simply:

$$\frac{\partial \mathcal{E}(h)}{\partial h} = P(0) + (P(f_0) - P(0)) + \int_{h_{min}}^{h} p_h(\widetilde{h})d\widetilde{h} - C'(h)$$

But

$$P_h(h) = P(0) + (P(f_0) - P(0)) + \int_{h_{min}}^{h} p_h\left(\widetilde{h}\right)d\widetilde{h}$$

Therefore,

$$\frac{\partial \mathcal{E}(h)}{\partial h} = P_h(h) - C'(h) = P(F(h)) - C'(h)$$

For the workers with a level of human capital $h \in (h_{min}, h_{max})$, $P_h(h)$ is their job finding probability and equal to $P(F(h))$. Therefore,

$$P(F(h)) = C'(h) \tag{25}$$

This condition means that the marginal cost of acquiring an extra unit of human capital must equal the marginal benefit, which is simply $P(F)$. The individuals with a higher ranking will enjoy a higher probability of finding a job, which increases expected earnings. However, in equilibrium, all individuals must obtain the same expected earnings. These expected earnings are equalised through the cost of obtaining human capital. Compare two workers with different levels of human capital. One worker has a higher position in the ranking than the other and a higher probability of finding a job. How much more human capital than the other he has does not affect the probability, since it only depends on the relative position in the ranking of each of them. In order for the low skilled worker to be willing to remain low skilled, the high skilled worker must accumulate a quantity of human capital such that the cost associated to it makes the expected earnings of both workers equal. The following result shows that equation 25 is also satisfied by the workers at the extrems of the distribution and that the distribution that satisfies this equality is an equilibrium.

**Proposition 1:** An equilibrium of the model exists with the associated distribution of human capital satisfying $P(F(h)) = C'(h)$.

a) If $b_v \geq a_u$

- The distribution is continuous on $[h_{min}, h_{max})$

- There is a mass of workers at $h_{max}$ equal to $f_1 = 1 - \frac{(1-\theta)\overline{U}+2k}{\overline{U}+k}$

b) If $b_v < a_u$

- The distribution is continuous on $(h_{min}, h_{max})$
- There is a mass of workers at $h_{max}$ equal to $f_1 = 1 - \frac{(1-\theta)\overline{U}+2k}{\overline{U}+k}$
- There is a mass of workers at $h_{min}$ equal to $f_0 = \frac{(1-\theta)\overline{U}-2k}{\overline{U}-k}$

Proof: Appendix A.3.4.

Proposition 1 shows that the distribution of human capital only depends on the job finding probability and the marginal cost. Furthermore, it always has a mass of workers at the maximum of the support. This is because of the properties of the job finding probability. In the following section, we will calculate and analyse the distribution that arises from this model in equilibrium.

## 3.5 THE EFFECT OF MARKET SIZE AND MARKET TIGHTNESS

Let $\overline{U}$ and $\overline{V}$ be the expectation of $U$ and $V$ respectively and let $\overline{V} = \theta\overline{U}$. We can interpret $\overline{U}$ as market size and $\theta$ as market tightness. If we redefine the parameters of the distributions of $U$ and $V$ as $b_u = \overline{U} + k$, $b_v = \theta\overline{U} + k$, $a_u = \overline{U} - k$ and $b_v = \theta\overline{U} - k$, we can derive $P(F)$ as a function of market size,$\overline{U}$ , market tightness,$\theta$ , and ranking, $F$. Given the equilibrium condition we have found, it is clear that analysing the effect of these variables on the hiring probability will help us to understand their impact on the distribution of human capital.

### 3.5.1 *The hiring probability*

We can calculate the hiring probability as a function of market size $\overline{U}$, market tightness $\theta$, and the ranking of the workers $F$ with $0 \leq F \leq 1$:

$$P(F) = \begin{cases} 1 & \text{for } \frac{(1-\theta)\overline{U}+2k}{\overline{U}+k} < F \leq 1 & [1] \\ \frac{-\left(-4+4F+F^2\right)k^2+2(2-F)k(\theta-(1-F))\overline{U}-\left((\theta-(1-F))\overline{U}\right)^2}{8(1-F)k^2} & \text{for } \frac{(1-\theta)\overline{U}}{\overline{U}-k} < F \leq \frac{(1-\theta)\overline{U}+2k}{\overline{U}+k} & [2] \\ \frac{k+(\theta-(1-F))\overline{U}}{2k} & \text{for } \frac{(1-\theta)\overline{U}}{\overline{U}+k} < F \leq \frac{(1-\theta)\overline{U}}{\overline{U}-k} & [3] \\ \frac{\left((-2+F)k+((1-F)-\theta)\overline{U}\right)^2}{8(1-F)k^2} & \text{for } \frac{(1-\theta)\overline{U}-2k}{\overline{U}-k} < F \leq \frac{(1-\theta)\overline{U}}{\overline{U}+k} & [4] \\ 0 & \text{for } 0 \leq F \leq \frac{(1-\theta)\overline{U}-2k}{\overline{U}-k} & [5] \end{cases}$$

We have seen that $P(F)$ is the probability that $(1 - F(h))\,U \leq V$. In the literature, it has been assumed that $U$ and $V$ are distributed according to independent Poisson distributions, see Shimer (2007). We have used a uniform distribution because of its simplicity, which is important when we introduce heterogeneity in human capital. The length of the support of the distribution is constant for all $\overline{U}$ and $\overline{V}$, which implies that the variance of the distributions are unaffected by the size of the market. This assumption implies that uncertainty decreases and the matching function becomes more efficient when market size increases. In Appendix A.3.5, we show that the matching function associated to this probability is increasing and concave in $\overline{U}$ and $\overline{V}$ and, for intermediate values of the market tightness, has increasing returns to scale.

The following two results consider the effect of market size and market tightness on the job finding probability. The first of them states that the probability of finding a job is increasing with market tightness, as expected.

**Proposition 2:** The job finding probability is increasing in market tightness for any $F < 1$.

Proof: Appendix A.3.7.1.

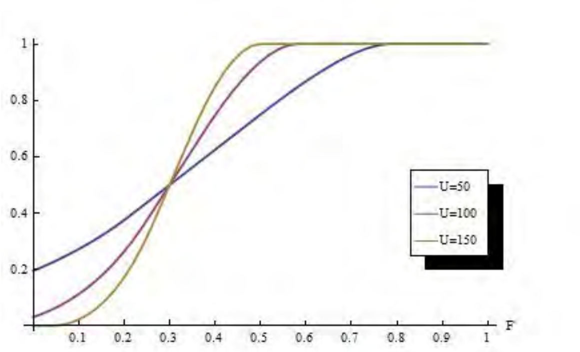
**Proposition 3:** For the workers with ranking $1 - \theta < F < 1$, the job finding probability is increasing with market size. For the workers with ranking $F < 1 - \theta$ the job finding probability is decreasing with market size. When the position in the ranking is $F = 1 - \theta$ the probability of finding a job is constant with market size.

Proof: Appendix A.3.7.2.

The last result states that the workers who have a higher position in the ranking, have a greater hiring probability in bigger markets and the workers with a lower level of skills have a lower probability. In Figure 7, $P(F)$ is represented as a function of $F$ for different market sizes. We can check that for $F > 1 - \theta = 0.3$ the job finding probability is increasing with market size. But for $F < 0.3$ the job finding probability of the workers is lower when the size of the market increases. In addition, the three functions cross at $1 - \theta = 0.3$, thus, the probability of finding a job for the worker with $F = 0.3$ is the same for all market sizes.

Figure 7: Probability and market size

$P(F)$




$\theta = 0.7$ and $k = 20$

An implication of this result is that as the point at which the effect changes sign is $F = 1 - \theta$, if $\theta > 1$, all workers have $F > 1 - \theta$. Thus, all of them will have a higher probability when the market is bigger. The lower is $\theta$ the higher the proportion of workers who will have a lower probability in a bigger market.

### 3.5.2 *The effect of market size and market tightness on the distribution of human capital*

In this section, we analyse how market size and market tightness affect the distribution of human capital. It is important to notice that, as the workers at the top of the distribution always find a job with probability 1, they choose the same level of human capital for any market size and market tightness. This is stated in Proposition 4.

**Proposition 4:** The maximum level of skills, $h_{max}$, is constant in market size and market tightness.

Proof: From Proposition 1, we know that $h_{max}$ is such that $P(1) = C'(h_{max})$ and from Lemma 1, we know that $P(1) = 1$.

Consider now an increase in market tightness, which we can interpret as an improvement in the labour market conditions. This makes the marginal benefit of investing in human capital, the job finding probability, also increase. Consequently, for any rank below the top, workers find optimal to invest more in human capital. In particular, the workers at the bottom will invest a higher level, that is, $h_{min}$ is increasing in market tightness. This implies that the proportion of workers at the lower part of the distribution decreases and the distribution of human capital concentrates in the maximum level. The effect on the cumulative distribution is Proposition 5.

**Proposition 5:** $F(h)$ is decreasing in $\theta$ for $h_{min} \leq h < h_{max}$.
Proof: A.3.7.3

An implication of this result is that $f_0$ decreases with market tightness and $f_1$ increases. In fact, for a high enough level of market tightness, $\theta > 1 + \frac{2k}{\overline{U}}$, the whole population chooses $h_{max}$. This last situation corresponds to the case when the probability that $V > U$ is one.

We can consider now the size of the market. The effect of market size on the distribution is also driven by the effect on the job finding probability. If market size increases, the probability of those workers with rank below $1-\theta$ decreases. This means that the marginal benefit of investing in human capital is lower. Thus, their level of investment will be lower. This implies that for $F < 1-\theta$, the proportion of workers at the lower part of the distribution increases and the minimum level invested, $h_{min}$, also decreases. $F(h)$ is increasing in $\overline{U}$ on this range. On the contrary, for the workers with rank above $1-\theta$, the job finding probability increases with market size and also the level invested in human capital. For this range of the distribution, workers concentrate to the top of the distribution, and $F(h)$is decreasing in $\overline{U}$.

If we consider the effect on the entire distribution, we must take into account whether $\theta$ is greater or less than 1. If $\theta > 1$, all workers have rank above $1-\theta$. As market size increases, the whole population concentrates in the maximum level, $f_1$ tends to 1. If $\theta < 1$, the distribution concentrates to the extremes. In this case, the limit of the distribution when market size tends to infinity is that $\theta$ workers choose the maximum level of human capital. The other workers do not invest at all. Proposition 6 summarizes the effects of market size on the distribution.

**Proposition 6:** Let $h_0$ be such that $F(h_0) = 1-\theta$. Then:

a. $F(h)$ is increasing in $\overline{U}$ for $h < h_0$, decreasing in $\overline{U}$ for $h_0 < h < h_{max}$ and constant for $h_0$.
b. $f_0$ tends to $1-\theta$ if $\theta < 1$ and to 0 if $\theta \geq 1$ as $\overline{U}$ tends to infinity.
c. $f_1$ tends to $\theta$ if $\theta < 1$ and to 1 if $\theta \geq 1$ as $\overline{U}$ tends to infinity.
Proof: Appendix A.3.7.4.

The findings about the distribution suggest that a higher size of the market implies a more unequal distribution, where the skilled workers have a higher level of human capital and the unskilled workers have a lower level of human capital, compared to smaller labour markets. The effect on the mean of the distribution can be calculated with numerical examples. Let $C(h) = \frac{1}{2}h^2$ and $k = 25$. In table 14, we have the mean for different values of $\overline{U}$ and $\theta$.

Table 14: Mean value of human capital

| | | $\overline{U} = 50$ | $\overline{U} = 100$ | $\overline{U} = 200$ | $\overline{U} = 500$ |
|---|---|---|---|---|---|
| $\theta = 0.6$ | $\mathbb{E}[h]$ | 0.602 | 0.612 | **0.603** | **0.601** |
| | $h_{min}$ | 0.18 | 0.02 | **0** | **0** |
| | $h_{max}$ | 1 | 1 | **1** | **1** |
| | $f_0$ | 0 | 0 | **0.17** | **0.32** |
| | $f_1$ | 0.07 | 0.28 | **0.42** | **0.52** |
| $\theta = 0.8$ | $\mathbb{E}[h]$ | 0.751 | 0.795 | 0.804 | **0.801** |
| | $h_{min}$ | 0.32 | 0.18 | 0.02 | **0** |
| | $h_{max}$ | 1 | 1 | 1 | **1** |
| | $f_0$ | 0 | 0 | 0 | **0.1** |
| | $f_1$ | 0.2 | 0.44 | 0.6 | **0.71** |

According to the table, the mean level of human capital is increasing with market size until $h_{min} = 0$. Then, it decreases as it tends to $f_1 h_{max} = \theta h_{max}$. As we had found analytically, $h_{min}$ is decreasing whereas $h_{max}$ is unaffected by market size. The distribution under these parameters is represented in Figure 8 and the density functions associated to these distributions are in Figure 9.

## 3.6 THE EFFECT OF MARKET SIZE ON WAGES

We can analyse the effect of market size on the expected wage. Let $\mathbb{E}[w(h, U, V) \mid job]$ be the expected payoffs of a worker with human capital $h$, conditional on finding a job.

$$\mathbb{E}[w(h, U, V) \mid job] = \frac{\mathbb{E}[w(h, U, V)]}{P(F(h))}$$

$\mathbb{E}[w(h, U, V)]$ can be derived from the equilibrium condition that equalises the expected earnings of all workers. We use the expected earnings of the worker at the bottom because of its simplicity.

Figure 8: Distribution of human capital

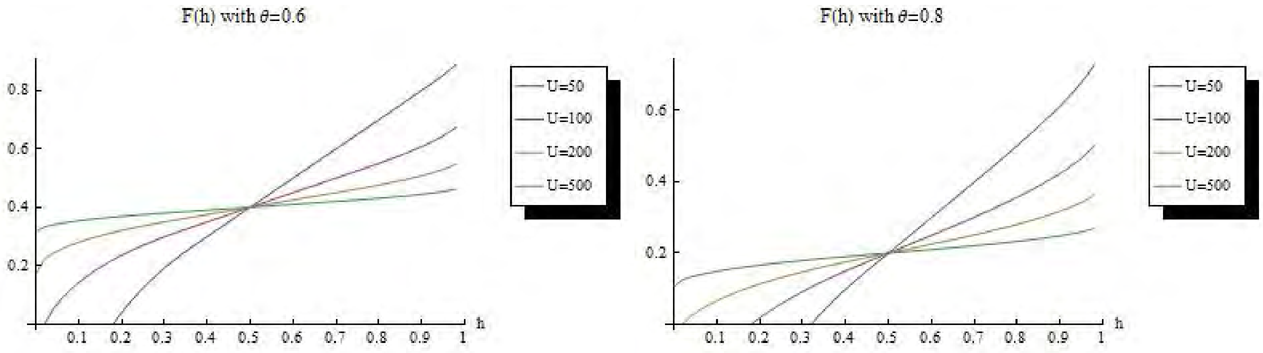
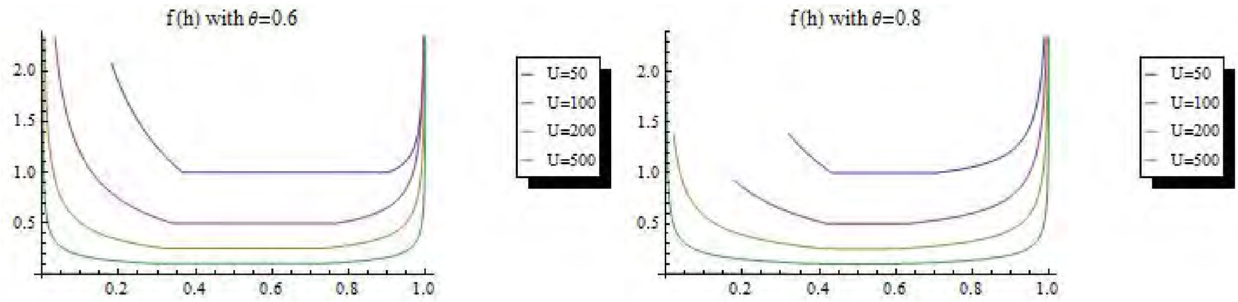



Figure 9: Density functions




$$\mathcal{E}(h) = \mathbb{E}\left[w(h, U, V)\right] - C(h) = \mathcal{E}(h_{min}) = P(0)\, h_{min} - C(h_{min})$$

The expected wage is:

$$\mathbb{E}\left[w(h, U, V) \mid job\right] = \frac{C(h) + \mathcal{E}(h_{min})}{P(F(h))}$$

To calculate the effect of market size on the expected wage, we need to know what is the effect on the expected earnings of the worker at the minimum of the distribution. It is:

$$\frac{\partial \mathcal{E}(h_{min})}{\partial \overline{U}} = \frac{\partial P(0)}{\partial \overline{U}} h_{min}$$

On the other hand, we also need to know what is the effect of market size on the job finding probability of a worker with level of human capital $h$. We can use the equilibrium condition $P(F(h)) = C'(h)$. As the job finding probability must be equal to the marginal cost in equilibrium, we see that market size does not affect the job finding probability of the worker who chooses a level of human capital $h$. The ranking of this worker will change so that the probability still equals the marginal cost. Therefore, the expected wage can be represented as:

$$\mathbb{E}\left[w\left(h,U,V\right)\mid job\right]=\frac{C(h)+\mathcal{E}\left(h_{min}\right)}{C'\left(h\right)}$$

The derivative with respect to market size is:

$$\frac{\partial\mathbb{E}\left[w\left(h,U,V\right)\mid job\right]}{\partial\overline{U}}=\left(\frac{1}{C'\left(h\right)}\right)\frac{\partial\mathbb{E}\left[w\left(h,U,V\right)\right]}{\partial\overline{U}}=\left(\frac{1}{C'\left(h\right)}\right)\frac{\partial\mathcal{E}\left(h_{min}\right)}{\partial\overline{U}}$$

Therefore, the effect on the expected wage is:

$$\frac{\partial\mathbb{E}\left[w\left(h,U,V\right)\mid job\right]}{\partial\overline{U}}=\left(\frac{1}{C'\left(h\right)}\right)\frac{\partial P\left(0\right)}{\partial\overline{U}}h_{min}$$

The derivative is negative for $1-\frac{2k}{\overline{U}}<\theta<1$, positive for $1<\theta<1+\frac{2k}{\overline{U}}$ and 0 otherwise. Therefore, in the cases where market size affects the expected wages, we see that the effect depends on market tightness again. When the labour market is tight, the expected wage is higher in bigger markets and the opposite occurs for market tightness lower than 1.

This is a consequence of the wage setting. When the market is tight, there is a higher probability that there is an excess of jobs, and workers receive higher salaries. As market size increases, the probability of this event is higher, therefore, the expected wages increase. This positive effect is decreasing with the level of skills of the worker.

On the other hand, when market tightness is low, there is a high probability that there is an excess of workers in the market. If this is the case, the wage of the workers will be lower, since the wage will be their productivity minus the productivity of the marginal worker. However, this negative effect is decreasing with the level of skills of the worker. This is an implication of the equilibrium condition. The expected earnings must be the same for all workers. As market size increases, the expected earnings of the worker at the bottom decreases. Therefore, the expected payoffs of all workers must decrease in the same quantity. As the job finding probability of a worker with low rank is small, the expected wage must decrease more in order to obtain the same reduction in the expected payoff. This implies that we can explain higher returns to skill in bigger cities through gains in matching efficiency when market tightness is lower than 1.

## 3.7 CONCLUSIONS

In this paper, we have explored the implications of city size on the incentives to invest on human capital. To this end, we have developed a model in which workers' chances of getting a job depend on the size of the market and their relative position in the distribution of skills. The results of this model are consistent with the hypothesis that investment in human capital increases with market size. However, when market tightness is low, the mass of workers with the lowest level of human capital increases with market size even when the mean level of human capital is higher.

Another implication of the results is that market size can affect the distribution of human capital even in situations when the aggregate matching function has constant returns to scale. Even in this case, market size affects the hiring probability of the workers, increasing their chances for those at the top of the distribution and decreasing the chances for those at the bottom.

# A

# APPENDIX

## A.1 CHAPTER 1

### A.1.1 *Continuation value*

The continuation value if both spouses are unemployed is:

$$\widetilde{W}(h,a,z,z) = \alpha_i^2 \widetilde{W}_i(h,a,z,z) + \alpha_i(1-\alpha_i)\widetilde{W}_{im}(h,a,z,z)$$
$$+\alpha_i(1-\alpha_i)\widetilde{W}_{if}(h,a,z,z) + (1-\alpha_i)^2\widetilde{W}_{i0}(h,a,z,z)$$

Where the first term accounts for the case that both spouses receive an inside offer, the second term is for the case when only the husband receives an inside offer, the third term is for the wife being the only one and the last term accounts for the case when neither of them receives an inside offer. Each of these values, at the same time will be an expectation over the probabilities of receiving outside offers. They are specified below.

If both receive an inside offer:

$$\widetilde{W}_i(h,a,z,z) = max\left\{W\left(h,0,a,w_m,w_f\right), W(h,0,a,w_m,z), W\left(h,0,a,z,w_f\right)\right\}$$

If only the husband receives an inside offer:

$$\widetilde{W}_{im}(h,a,z,z) = \alpha_o^2 max\left\{W\left(h,1,a,w_m,w_f\right), W(h,0,a,w_m,z), W\left(h,1,a,z,w_f\right)\right\}$$
$$+\alpha_o(1-\alpha_o)\, max\left\{W(h,0,a,w_m,z), W\left(h,1,a,z,w_f\right)\right\} + (1-\alpha_o)W(h,0,a,w_m,z)$$

If only the wife receive an inside offer:

$$\widetilde{W}_{if}(h,a,z,z) = \alpha_o^2 max\left\{W\left(h,1,a,w_m,w_f\right), W(h,1,a,w_m,z), W\left(h,0,a,z,w_f\right)\right\}$$
$$+\alpha_o(1-\alpha_o)\, max\left\{W(h,1,a,w_m,z), W\left(h,0,a,z,w_f\right)\right\} + (1-\alpha_o)W\left(h,0,a,z,w_f\right)$$

If neither receive and inside offer:

$$\widetilde{W}_{i0}(h,a,z,z) = \alpha_o^2 max\left\{W\left(h,1,a,w_m,w_f\right), W(h,1,a,w_m,z), W\left(h,1,a,z,w_f\right), W(h,0,a,z,z)\right\}$$
$$+\alpha_o(1-\alpha_o)\, max\left\{W(h,1,a,w_m,z), W(h,0,a,z,z)\right\}$$
$$+\alpha_o(1-\alpha_o)\, max\left\{W\left(h,1,a,z,w_f\right), W(h,0,a,z,z)\right\} + (1-\alpha_o)^2 W(h,0,a,z,z)$$

### A.1.2 *Distribution of unemployed workers*

The density of workers that the firm may hire can be obtained from the density of workers that are unemployed and the conditional density that these workers will be employed in the following period. Let $\hat{\psi} = \left(j,c,h,g,a,p_m,p_f\right)$ be the state of a worker that is $j \in \{m,f\}$, and belong to a couple that live in location $c \in \{l,n\}$, the previous period had tenure $h$, with migration status $g$, level of assets $a$, and employment status $p_m, p_f$. Also let $\lambda(\hat{\psi})$ be the density of workers and $Q(\hat{\psi}' \mid \hat{\psi})$ be the conditional density that a worker that is $\hat{\psi}$ becomes $\hat{\psi}'$. The quantity of workers in state $\hat{\psi}_0 = \left(j_0,c_0,h_0,g_0,a_0,p_{m0},p_{f0}\right)$ that are employed and in the previous period were unemployed is:

$$\hat{\lambda}_{hire}(\hat{\psi}_0) = \int_{\hat{\Psi}} I(p_j = z)\, I(p_{j0} \neq z)\, Q(\hat{\psi}_0 \mid \hat{\psi})\, \lambda(\hat{\psi})\, d\hat{\psi}$$

with $\hat{\Psi} = \{m, f\} \text{x} \{l, n\} \text{x} \left\{0, \overline{h}\right\} \text{x} \{0, 1\} \text{x} A \text{x} P \text{x} P$.

The number of matches in location $c = c_0$ is:

$$M_{c_0} = \int_{\Psi} I(c = c_0)\, \hat{\lambda}_{hire}(\hat{\psi})\, d\hat{\psi}$$

Therefore, the density of unemployed workers that a firm established in location $c = c_0$ may hire is:

$$\lambda_{hire}(j, c_0, h, g, a, p_m, p_f) = \frac{\hat{\lambda}_{hire}(j, c_0, h, g, a, p_m, p_f)}{M_{c_0}}$$

Moreover, the hiring probability in $c$ is the number of hirings in $c$ divided by the vacancy rate in that location.

$$\alpha_r^c = \frac{M_c}{v_c}$$

In subsection 1.2.7.3, we abstract away from locational notation and write $\lambda_{hire}(j, c, h, g, a, p_m, p_f) = \lambda_{hire}(j, h, g, a, p_m, p_f)$ and $\alpha_r^c = \alpha_r$.

### A.1.3 *Computation of the equilibrium*

We define $q = a + (1 - \chi) h$. Therefore, the household problem can be rewritten as:

$$\begin{aligned}
W\left(h, g, q, p_m, p_f\right) &= \max_{c_m, c_f, q', h' \in \{0, \overline{h}\}} \left\{\vartheta u(c_m) + (1 - \vartheta) u\left(c_f\right) + \beta \widetilde{W}\left(h', q', p_m, p_f\right)\right\} \\
\text{st } c_m + c_f + q' - (1 - \chi) h' + r_f \overline{h} + (\phi_b + 1) h' I_{h=0} &= inc + \left(r_f - \delta_h\right)(1 - \tau_{ir}) h' + (1 - \phi_s) h I_{h'=0} \quad \text{if g=0} \\
c_m + c_f + q' - (1 - \chi) h' + r_f \overline{h} + (\phi_b + 1) h' &= inc + \left(r_f - \delta_h\right)(1 - \tau_{ir}) h' + (1 - \phi_s) h \quad \text{if g=1} \\
q' &\geq 0 \quad (1c)
\end{aligned}$$

with $inc = (1 - \tau I_{p_m \neq z}) p_m + \left(1 - \tau I_{p_f \neq z}\right) p_f + (1 + i(1 - \tau))(q - (1 - \chi) h)$.

Similarly, the problem of the firm is defined on $q$.

We compute the equilibrium with the following steps:

1. We use a discrete grid on the variable $q$, and on the payoffs $p_j$.
2. We guess market tightness $\theta$ and the wage function $w_j\left(h, g, q, w_{-j}\right)$.
3. We solve the household's problem by value function iteration.
4. We calculate the invariant densities, $\lambda(\widetilde{\psi})$ and $\lambda_{hire}(\widetilde{\psi})$.
5. We compute the value of a filled job by value function iteration using the fact that in equilibrium $V = 0$.
6. We calculate the new wage function from Nash bargaining, using the value functions found in step 4 and 6. We update our guess for the wage function as a linear combination of the new wage and the old guess.
7. We compute the value of a vacant job, we raise our guess of $\theta$ if the value is positive and decrease it if it is negative.
8. We repeat this procedure from step 2 until convergence.

### A.1.4 *Single-Agent model*

The economy is populated by a measure 1 of infinitely lived workers. Each worker constitues a household. $W(h,g,a,p)$ is the value of a worker that in the previous period had housing tenure $h$ and that in this period has migration status $g$, level of assets $a$, labour payoff $p$. The problem of the household in stage 4 is the following:

$$\begin{aligned} W(h,g,a,p) &= \max_{c,a',h'\in\{0,\overline{h}\}} \left\{\vartheta u(c_m) + (1-\vartheta)\, u\left(c_f\right) + \beta\widetilde{W}\left(h',a',p\right)\right\} \qquad (26) \\ \text{st } c + a' + r_f\overline{h} + (\phi_b+1)\, h' I_{h=0} &= inc + \left(r_f - \delta_h\right)(1-\tau_{ir})\, h' + (1-\phi_s)\, h I_{h'=0} \quad \text{if g=0} \\ c + a' + r_f\overline{h} + (\phi_b+1)\, h' &= inc + \left(r_f - \delta_h\right)(1-\tau_{ir})\, h' + (1-\phi_s)\, h \quad \text{if g=1} \\ a' &\geq -(1-\chi)\, h' \end{aligned}$$

with $inc = \left(1 - \tau I_{p\neq z}\right) p + (1 + i(1-\tau))\, a$.

The continuation value of an employed worker is $\widetilde{W}(h,a,w)$ with:

$$\widetilde{W}(h,a,w) = (1-s)\, W(h,0,a,w) + sW(h,0,a,z)$$

The continuation value of an unemployed worker is $\widetilde{W}(h,a,z)$ with:

$$\begin{aligned} \widetilde{W}(h,a,z) = {} & \alpha_i\alpha_o max\left\{W(h,1,a,w(h,1,a)), W(h,0,a,z), W(h,0,a,w(h,0,a))\right\} \\ & + \alpha_i(1-\alpha_o)\, W(h,0,a,w(h,0,a)) \\ & + (1-\alpha_i)\,\alpha_o max\left\{W(h,1,a,w(h,1,a)), W(h,0,a,z)\right\} \\ & + (1-\alpha_i)(1-\alpha_o)\, W(h,0,a,z) \end{aligned}$$

In the Single-Agent model a worker never quits his job. Therefore, the value of a filled job that pays wage $w$ is:

$$J(w) = \frac{(1+i)(y-w) + sV}{i + sep} \qquad (27)$$

Let $\lambda_{hire}\left(\widetilde{\psi}\right)$be the density of the unemployed workers that a firm can hire with $\widetilde{\psi} = (h,g,a,p)$. Then, the value of a vacant job is:

$$V = -\xi + \frac{\alpha_r}{1+i}\int_{\Psi} J\left(\widetilde{\psi}\right)\lambda_{hire}\left(\widetilde{\psi}\right) d\widetilde{\psi} \qquad (28)$$

with $\Psi = \left\{0,\overline{h}\right\} \text{x} \{0,1\} \text{x} A \text{x} P$.

The wage is determined by Nash bargaining. $w(h,g,a)$ is the solution to:

$$\underset{w}{max}\,(W(h,g,a,w) - W(h,0,a,z))^{\gamma}\,(J(w) - V)^{1-\gamma} \qquad (29)$$

Leasing companies solve the same problem as in the benchmark model. Therefore, the price for renting a house satisfies:

$$r_f = \frac{i}{1+i} + \delta_h \qquad (30)$$

A steady-state equilibrium for a given set of policy arrangements $\{\tau, \tau_{ir}\}$consists of a set of value functions $\{W(\psi), J(j,\psi)\}$, a set of decision rules $\{g_a(\psi), g_h(\psi)\}$, a time-invariant measure of agent types $\lambda(\widetilde{\psi})$, a set of prices $\{r_f, w_j(h,g,a,\cdot)\}$, and market tightness $\theta$ such that:

1. Given prices, $\theta$ and the fiscal policy, the household's decision rules solve the dynamic program given by (26).
2. Given prices and the household's decision rules, the firm solves (27).
3. $\theta$ satisfies $V = 0$ with $V$ given by (28).
4. Wages satisfy (29) and the rent satisfies (30).
5. $\lambda(\widetilde{\psi})$ is the invariant distribution generated by the meeting probability, separation rate and the household's decision rules.

Table 15: Estimated parameters in the Single-Agent model

| Parameter | Target | Source |
|---|---|---|
| $\beta = 0.9960$ | home ownership rate= 66% | CPS 1980-2005 |
| $s = 0.030$ | unemployment rate= 6.2% | CPS 1980-2005 |
| $k = 0.53$ | job finding rate= 0.42 | Shimer 1980-2005 |
| $z = 0.94$ | $z = 40\%$ mean wage | Shimer 2005 |
| $\overline{h} = 73$ | house/earnings= 29 | AHS 1989-2005 |
| $\varepsilon = 0.26$ | annual migration rate= 2.4% | CPS 1999-2005 |
| $\phi_s = \phi_b = 0.035$ | migration rate owners/renters= 33% | SIPP 2001 |

Table 16: Calibration targets in the Single-Agent model

| Moment | Data | Model |
|---|---|---|
| home-ownership rate | 66% | 65% |
| unemployment rate | 6.2% | 6.5% |
| job finding rate | 0.42 | 0.44 |
| unemp. flow/wage | 40% | 39% |
| house/earnings | 29 | 28 |
| annual migration rate | 2.4% | 2.5% |
| migration rate owners/renters | 33% | 35% |

## A.2 CHAPTER 2

### A.2.1 *Continuation value*

The continuation value if both spouses are unemployed is:

$$\widetilde{W}(a,z,z) = \alpha_i^m \alpha_i^f \widetilde{W}_i(a,z,z) + \alpha_i^m \left(1-\alpha_i^f\right) \widetilde{W}_{im}(a,z,z)$$
$$+\alpha_i^f \left(1-\alpha_i^m\right) \widetilde{W}_{if}(a,z,z) + \left(1-\alpha_i^m\right)\left(1-\alpha_i^f\right) \widetilde{W}_{i0}(a,z,z)$$

Where the first term accounts for the case that both spouses meet an inside vacancy, the second term is for the case when only the husband meets an inside vacancy, the third term is for the wife being the only one and the last term accounts for the case when neither of them meets an inside vacancy. Each of these values, at the same time will be an expectation over the probabilities of meeting outside vacancies. They are specified below.

If both meet an inside vacancy:

$$\widetilde{W}_i(a,z,z) = max\left\{W\left(a,w_m,w_f\right), W\left(a,w_m,z\right), W\left(a,z,w_f\right)\right\}$$

If only the husband meets an inside vacancy:

$$\widetilde{W}_{im}(a,z,z) = \alpha_o^m \alpha_o^f max\left\{W\left(a,w_m,w_f\right), W\left(a,w_m,z\right), W\left(a,z,w_f\right)\right\}$$
$$+\alpha_o^f\left(1-\alpha_o^m\right) max\left\{W\left(a,w_m,z\right), W\left(a,z,w_f\right)\right\} + \left(1-\alpha_o^f\right) W\left(a,w_m,z\right)$$

If only the wife meets an inside vacancy:

$$\widetilde{W}_{if}(a,z,z) = \alpha_o^m \alpha_o^f max\left\{W\left(a,w_m,w_f\right), W\left(a,w_m,z\right), W\left(a,z,w_f\right)\right\}$$
$$+\alpha_o^m\left(1-\alpha_o^f\right) max\left\{W\left(a,w_m,z\right), W\left(a,z,w_f\right)\right\} + \left(1-\alpha_o^m\right) W\left(a,z,w_f\right)$$

If neither meets an inside vacancy:

$$\widetilde{W}_{i0}(a,z,z) = \alpha_o^m \alpha_o^f max\left\{W\left(a,w_m,w_f\right), W\left(a,w_m,z\right), W\left(a,z,w_f\right), W\left(a,z,z\right)\right\}$$
$$+\alpha_o^m\left(1-\alpha_o^f\right) max\left\{W\left(a,w_m,z\right), W\left(a,z,z\right)\right\}$$
$$+\alpha_o^f\left(1-\alpha_o^m\right) max\left\{W\left(a,z,w_f\right), W\left(a,z,z\right)\right\} + \left(1-\alpha_o^m\right)\left(1-\alpha_o^f\right) W\left(a,z,z\right)$$

### A.2.2 *Distribution of unemployed workers*

The density of workers that the firm may hire can be obtained from the density of workers that are unemployed and the conditional density that these workers will be employed in the following period. Let $\hat{\psi} = \left(j,c,g,a,p_m,p_f\right)$ be the state of a worker that is $j \in \{m,f\}$, live in location $c \in \{l,n\}$, in a couple with migration status $g$, financial assets $a$, and employment status $p_m$ and $p_f$. Also let $\lambda\left(\hat{\psi}\right)$ be the density of workers and $Q\left(\hat{\psi}' \mid \hat{\psi}\right)$ be the conditional density that a worker that is $\hat{\psi}$ becomes $\hat{\psi}'$. The quantity of workers in state $\hat{\psi}_0 = \left(j_0,c_0,g_0,a_0,p_{m0},p_{f0}\right)$ that are employed and in the previous period were unemployed is:

$$\hat{\lambda}_{hire}\left(\hat{\psi}_0\right) = \int_{\hat{\Psi}} I\left(p_j = z\right) I\left(p_{j0} \neq z\right) Q\left(\hat{\psi}_0 \mid \hat{\psi}\right) \lambda\left(\hat{\psi}\right) d\hat{\psi}$$

with $\hat{\Psi} = \{m, f\} \text{x} \{l, n\} \text{x} \left\{0, \overline{h}\right\} \text{x} \{0, 1\} \text{x} A \text{x} P \text{x} P$.

The number of matches in location $c = c_0$ and labour market $j = j_0$:

$$M_{c_0}^{j_0} = \int_{\Psi} I\left(c = c_0\right) I\left(j = j_0\right) \hat{\lambda}_{hire}\left(\hat{\psi}\right) d\hat{\psi}$$

Therefore, the density of unemployed workers that a firm in location $c = c_0$ and in labour market $j = j_0$ may hire is:

$$\lambda_{hire}\left(j_0, c_0, g, a, p_m, p_f\right) = \frac{\hat{\lambda}_{hire}\left(j_0, c_0, h, g, a, p_m, p_f\right)}{M_{c_0}^{j_0}}$$

Moreover, the hiring probability is the number of hirings divided by the vacancy rate.

$$\alpha_r^{j,c} = \frac{M_c^j}{v_c^j}$$

In subsection 1.2.7.3, we abstract away from locational notation and write $\lambda_{hire}\left(j, c, h, g, a, p_m, p_f\right) = \lambda_{hire}\left(j, h, g, a, p_m, p_f\right)$ and $\alpha_r^c = \alpha_r$.

## A.3 CHAPTER 3

### A.3.1 *Wages*

a) The number of matchings will equal $\min\{u, v\}$.

The reason is that if the number of matchings is less than $\min\{u, v\}$, there are workers and vacancies unmatched. Firms are willing to employ a worker as long as profits $\pi(u, v) = h - w(h, u, v) \geq 0$, and workers are willing to be employed as long as $w(h, u, v) \geq 0$. We can see that there is no salary in which neither of them wants to get matched and salaries in which only one wants to get match are not an equilibrium.

b) When $v > u$, the salary paid is $w(h, u, v) = h$ and $\pi(u, v) = 0$.

If the salary were $w'(h, u, v) < h$, profits would be $\pi'(u, v) > 0$ for a matched firm and $\pi'(u, v) = 0$ for a firm that does not get matched. Therefore, an unmatched firm would be strictly better off being matched while it is not, which is not possible in equilibrium. On the other hand, if $w'(h, u, v) > h$, profits would be strictly lower for a matched firm, which cannot be an equilibrium, either.

c) If $v \leq u$, the workers with higher human capital are employed.

Assume the contrary, then there is some $h'$ not employed such that $h' > h$ and $h$ is employed. This implies $\pi(h', u, v) \leq \pi(h, u, v)$, thus $w(h', u, v) \geq h' - h + w(h, u, v) > 0$ but in this case $h'$ strictly prefers to be employed while it is not. This cannot be an equilibrium.

d) $w(h_m, u, v) = 0$

As $h_m$ is the employed worker with lowe ranking, $\pi(h_m, u, v) \geq \pi(h, u, v)$ for $h_m > h$, thus $w(h_m, u, v) \leq h_m - h + w(h, u, v)$. But in equilibrium, unemployed workers do not prefer to be employed, thus their offered salary is 0, which means that $w(h_m, u, v) \leq h_m - h$, for any $h$ unemployed, therefore, it is enough to consider the highest $h$ in the market among the unemployed. As the supprt is connected, this means that $w(h_m, u, v) \leq 0$. On the other hand, for the marginal worker to prefer to be employed we need that $w(h_m, u, v) \geq 0$.

e) If $v \leq u$, the salary paid to an employed worker is $w(h, u, v) = h - h_m$.

All firms get matched, in equilibrium it must be that $\pi(h', u, v) = \pi(h, u, v)$, hence $w(h', u, v) = h' - h + w(h, u, v)$. In particulaer, we will have that $w(h', u, v) = h' - h_m + w(h_m, u, v) = h' - h_m$.

### A.3.2 *Calculation of $P(F)$*

$P(F)$ is the probability that $(1 - F)u \leq v$ for $F \in [0, 1]$. Let $r = (1 - F)u$, $r$ is the realization of the random variable $R$. We are interested in the probability that $r \leq v$.

We know that $U$ follows a uniform distribution with support $[a_u, b_u]$. Then, the distribution function of $U$ is:

$$F_U(u) = \frac{u - a_u}{b_u - a_u}$$

Since $R$ is a monotone function of $U$, its distribution function is:

$$F_F(r) = F_U\left(\frac{r}{(1-F)}\right) = \frac{r/(1-F) - a_u}{b_u - a_u}$$

$R$ follows a uniform distribution with support $[a_r, b_r]$, with $a_r = (1-F)\,a_u$ and $b_r = (1-F)\,b_u$. To compute the probability that $r \leq v$, we need to consider all the possible combinations in the positions of the supports of $F_F(r)$ and $F_V(v)$.

**Condition 1.** $a_r < b_r < a_v < b_v$:

If $(1-F)$ is small it could be that the support of $F_R(r)$ were on the left of the support of $F_V(v)$. In this case, $R \leq V$ for any possible realization, therefore the probability that $r \leq v$ will be 1. The condition $b_r < a_v$ implies $(1-F)\,b_u < a_v$, that we can rewrite as $F > \frac{b_u - a_v}{b_u}$.

$$P(F) = 1 \quad \text{if} \quad F > \frac{b_u - a_v}{b_u}$$

**Condition 2.** $a_r < a_v \leq b_r < b_v$

If $a_r < a_v \leq b_r < b_v$, the probability that $r \leq v$ can be calculated as the sum of the probability of three different events: the probability that $r$ lies between $a_r$ and $a_v$, the probability that $v$ lies between $b_r$ and $b_v$ and the probability that both $r$ and $v$ lie between $a_v$ and $b_r$, in this circumstance we only have to take into account the realizations where $r \leq v$. The intersection of the first two events has a positive probability.

Below we have the probability in this case. The three terms are respectively the probabilities of the three events.

$$P(F) = \int_{a_r}^{a_v}\int_{a_v}^{b_v} \frac{1}{(b_v - a_v)(b_r - a_r)} dvdr + \int_{b_r}^{b_v}\int_{a_v}^{b_r} \frac{1}{(b_v - a_v)(b_r - a_r)} drdv + \int_{a_v}^{b_r}\int_{a_v}^{v} \frac{1}{(b_v - a_v)(b_r - a_r)} drdv$$

The conditions $b_r \geq a_v$ and $a_v > a_r$ imply $(1-F)\,b_u \geq a_v$ and $a_v > (1-F)\,a_u$. Therefore, $F$ must be such that $\frac{a_u - a_v}{a_u} < F \leq \frac{b_u - a_v}{b_u}$. If we calculate the integrals:

$$P(F) = \frac{-2a_u\,(b_v - a_v)\,(1-F) - a_v^2 + b_u\,(2b_v - b_u\,(1-F))\,(1-F)}{2\,(1-F)\,(b_u - a_u)\,(b_v - a_v)} \text{ for } \frac{a_u - a_v}{a_u} < F \leq \frac{b_u - a_v}{b_u}$$

**Condition 3.** $a_v \leq a_r < b_r < b_v$

Since the support of $F_V(v)$ is larger than the support of $F_R(r)$, there is also the possibility that $a_v \leq a_r < b_r < b_v$. In this case, the probability that $r \leq v$ is the sum of the probability that $v$ lies between $b_r$ and $b_v$ plus the probability that both $r$ and $v$ lie between $a_r$ and $b_r$, in this circumstance we only have to take into account the realizations where $r \leq v$.

$$P(F) = \int_{b_r}^{b_v}\int_{a_r}^{b_r} \frac{1}{(b_v - a_v)(b_r - a_r)} drdv + \int_{a_r}^{b_r}\int_{a_r}^{v} \frac{1}{(b_v - a_v)(b_r - a_r)} drdv$$

The conditions $b_r < b_v$ and $a_v \leq a_r$ imply $b_u\,(1-F) < b_v$ and $a_v \leq (1-F)\,a_u$. Therefore, $F$ must be such that $\frac{b_u - b_v}{b_u} < F \leq \frac{a_u - a_v}{a_u}$. We obtain that:

$$P(F) = \frac{2b_v - a_u (1-F) - b_u (1-F)}{2 (b_v - a_v)} \text{ for } \frac{b_u - b_v}{b_u} < F \leq \frac{a_u - a_v}{a_u}$$

**Condition 4.** $a_v < a_r < b_v \leq b_r$

For $(1-F)$ closer to 1, it could be that $a_v < a_r < b_v \leq b_r$. In this case, when $v$ lies between $a_v$ and $a_r$, $r$ cannot be smaller than $v$. The same happens when $r$ lies between $b_v$ and $b_r$. There is only one event that we need to take into account, when $r$ and $v$ lie between $a_r$ and $b_v$ and $r \leq v$.

$$P(F) = \int_{a_r}^{b_v} \int_{a_r}^{v} \frac{1}{(b_v - a_v)(b_r - a_r)} drdv$$

The conditions $b_v \leq b_r$ and $a_r < b_v$ imply $b_v \leq (1-F)\, b_u$ and $(1-F)\, a_u < b_v$. Therefore, in this case it must be that $\frac{a_u - b_v}{a_u} < F \leq \frac{b_u - b_v}{b_u}$.

$$P(F) = \frac{(b_v - a_r (1-F))^2}{2 (1-F) (b_u - a_u) (b_v - a_v)} \text{ for } \frac{a_u - b_v}{a_u} < F \leq \frac{b_u - b_v}{b_u}$$

**Condition 5.** $a_v < b_v \leq a_r < b_r$:

Finally, if $a_v < b_v \leq a_r < b_r$, the probability that $R \leq V$ is 0. The condition $a_r \geq b_v$ implies $(1-F)\, a_u \geq b_v$. Therefore, $\frac{a_u - b_v}{a_u} \geq F$.

Taking into account the 5 conditions, the probability that $(1-F)\, U \leq V$ is the following piecewise function:

$$P(F) = \begin{cases} 1 & \text{for } \frac{b_u - a_v}{b_u} < F \leq 1 & [1] \\ \frac{-2a_u(b_v - a_v)(1-F) - a_v^2 + b_u(2b_v - b_u(1-F))(1-F)}{2(1-F)(b_u - a_u)(b_v - a_v)} & \text{for } \frac{a_u - a_v}{a_u} < F \leq \frac{b_u - a_v}{b_u} & [2] \\ \frac{2b_v - a_u(1-F) - b_u(1-F)}{2(b_v - a_v)} & \text{for } \frac{b_u - b_v}{b_u} < F \leq \frac{a_u - a_v}{a_u} & [3] \\ \frac{(b_v - a_r(1-F))^2}{2(1-F)(b_u - a_u)(b_v - a_v)} & \text{for } \frac{a_u - b_v}{a_u} < F \leq \frac{b_u - b_v}{b_u} & [4] \\ 0 & \text{for } 0 \leq F \leq \frac{a_u - b_v}{a_u} & [5] \end{cases} \quad (21)$$

### A.3.3 *Characterization of* $P(F)$

The derivative of $P(F)$ is:

$$\frac{\partial P(F)}{\partial F} = \begin{cases} 0 & \text{for } \frac{b_u - a_v}{b_u} < F \leq 1 \\ \frac{b_u^2 (1-F)^2 - a_v^2}{2(1-F)^2 (b_u - a_u)(b_v - a_v)} & \text{for } \frac{a_u - a_v}{a_u} < F \leq \frac{b_u - a_v}{b_u} \\ \frac{a_u + b_u}{2(b_v - a_v)} & \text{for } \frac{b_u - b_v}{b_u} < F \leq \frac{a_u - a_v}{a_u} \\ \frac{b_v^2 (1-F)^2 - a_u^2 (1-F)^2}{2(1-F)^2 (b_u - a_u)(b_v - a_v)} & \text{for } \frac{a_u - b_v}{a_u} < F \leq \frac{b_u - b_v}{b_u} \\ 0 & \text{for } 0 \leq F \leq \frac{a_u - b_v}{a_u} \end{cases}$$

Therefore, $P(F)$ is increasing in $F$ and strictly increasing in $F$ for $\frac{a_u-b_v}{a_u} < F < \frac{b_u-a_v}{b_u}$. Depending on the parameters, some of the conditions on $F$ that define the probability function are not part of the domain $[0,1]$. Given the assumptions in the setting, there are 4 possible cases: the support of $V$ can be either at the right or at the left of the support of $U$ and they may intersect or not. We derive next, what intervals used for the definition of $P(F)$ apply, for each of the 4 cases.

**Case 1.** $a_u < b_u \leq a_v < b_v$

In this case the support of $U$ is to the left of the support of $V$. We also know that the support of $R$ is to the left of the support of $U$ for any rank of the worker. Then, it is clear that the support of $R$ will also be to the left of the support of $V$ for any $F$. This is the same as saying that, if with probability 1 there are more vacancies than workers, then also with probability 1 there are more vacancies than some proportion of workers. Analytically, it implies that the intervals 2 to 5 used to define $P(F)$ do not belong to the domain of the function. In fact, we can check that the range of these intervals is negative, as their superior limit is $F < \frac{b_u-a_v}{b_u}$. Therefore, in this case the job finding probability can be defined as:

$$P(F) = 1 \text{ for } 0 \leq F \leq 1$$

**Case 2.** $a_u < a_v < b_u < b_v$

In this case, the support of $U$ is to the left of the support of $V$ but they intersect. Following the same argument as in the previous case, we can check that only intervals 1 to 3 belong to the domain of $P(F)$, as in these 3 cases $b_r < b_v$. Analytically, we can check that the range of conditions 4 and 5 is negative, since the maximum of these restrictions is $\frac{b_u-b_v}{b_u} < 0$. In this case, the job finding probability can be defined as:

$$P(F) = \begin{cases} 1 & \text{for } \frac{b_u-a_v}{b_u} < F \leq 1 & [1] \\ \frac{-2a_u(b_v-a_v)(1-F)-a_v^2+b_u(2b_v-b_u(1-F))(1-F)}{2(1-F)(b_u-a_u)(b_v-a_v)} & \text{for } \frac{a_u-a_v}{a_u} < F \leq \frac{b_u-a_v}{b_u} & [2] \\ \frac{2b_v-a_u(1-F)-b_u(1-F)}{2(b_v-a_v)} & \text{for } 0 \leq F \leq \frac{a_u-a_v}{a_u} & [3] \end{cases}$$

**Case 3.** $a_v \leq a_u < b_v \leq b_u$

In this case, the support of $U$ is to the right of the support of $V$ but they intersect. This means that for some $F$ the support of $R$ will also be to the right of $V$, so condition 4 applies. Clearly, conditions 1 to 3 also apply for $F$ high enough. We can check that restriction 5 does not apply. Its maximum is $\frac{b_u-b_v}{b_u} = 0$ . The job finding probability in this case is given by:

$$P(F) = \begin{cases} 1 & \text{for } \frac{b_u-a_v}{b_u} \leq F \leq 1 & [1] \\ \frac{-2a_u(b_v-a_v)(1-F)-a_v^2+b_u(2b_v-b_u(1-F))(1-F)}{2(1-F)(b_u-a_u)(b_v-a_v)} & \text{for } \frac{a_u-a_v}{a_u} < F < \frac{b_u-a_v}{b_u} & [2] \\ \frac{2b_v-a_u(1-F)-b_u(1-F)}{2(b_v-a_v)} & \text{for } \frac{b_u-b_v}{b_u} < F \leq \frac{a_u-a_v}{a_u} & [3] \\ \frac{(b_v-a_r(1-F))^2}{2(1-F)(b_u-a_u)(b_v-a_v)} & \text{for } 0 \leq F \leq \frac{b_u-b_v}{b_u} & [4] \end{cases}$$

**Case 4.** $a_v < b_v \leq a_u < b_u$

In this case the support of $U$ is to the right of the support of $V$. With probability 1 there will be more workers and vacancies. Therefore, the workers with the lowest rank will not find a job. All 5 conditions apply. And the job finding probability is given by equation 21.

Notice that condition 1 always applies for workers with $F$ high enough. As we have assumed $a_v > 0$, $P(1) = 1$, for any position of the supports. On the other hand, condition 5 only applies in case 4. This means that when $b_v > a_u$, $P(0) > 0$.

### A.3.4 *Equilibrium*

We will follow the steps below for case a and case b.

Step 1. $F(h)$ is continuous on the domain established.

Step 2. The minimum of the support is $h_{min}$ and satisfies $P(F(h_{min})) = C'(h_{min})$.

Step 3. The maximum of the support is $h_{max}$ and satisfies $P(F(h_{max})) = C'(h_{max})$.

Step 4. This equilibrium exist.

a) If $b_v \geq a_u$:

Step 1. In Appendix A.3.3, it is shown that $P(F)$ is strictly increasing for $0 \leq F < 1 - f_1$. Therefore, $F(h)$ that satisfies $P(F(h)) = C'(h)$ is a continuous distribution on $[h_{min}, h_{max})$. In section **??**, we have shown that it implies that $\mathcal{E}(h_1) = \mathcal{E}(h_2)$ for any $h_1, h_2 \in (h_{min}, h_{max})$.

Step 2. If the distribution of human capital is continuous on $[h_{min}, h_{max})$, then, the job finding probability for workers at $h_{min}$ is $P(0)$. We have to show that $\mathcal{E}(h_{min}) > \mathcal{E}(h)$ for any $h < h_{min}$, keeping the ranking fixed. The expected earnings of the worker at the minimum of the support are $\mathcal{E}(h) = P(0)h - C(h)$. Since $\mathcal{E}'(h) = P(0) - C'(h)$ and $\mathcal{E}''(h) = -C''(h) < 0$, we see that this condition is satisfied.

Step 3.The maximum of the support is $h_{max}$ and satisfies $P(F(h_{max})) = C'(h_{max})$. If the distribution of human capital is continuous on $[h_{min}, h_{max})$, then the job finding probability for workers at $h_{max}$ is $P(1)$, as firms can rank workers above and below $F = 1 - f_1$ and the probability that $v \geq f_1$ is 1.[1] We have to show that $\mathcal{E}(h_{max}) > \mathcal{E}(h)$ for any $h_1 > h_{max}$. The expected earnings of a worker with $h_1 > h_{max}$ is: [2]

$$\mathcal{E}(h_1) = P(0)h_1 + (P(f_0) - P(0))(h_1 - h_{min}) + \int_{h_{min}}^{h_1} p_h(\widetilde{h})(h - \widetilde{h})d\widetilde{h} + (1 - P_h(h_1))(h_1 - h_{max}) I(h_1 > h_{max}) - C(h_1)$$

But the fourth term is 0 since $P_h(h_1) = 1$. Then,

1 Firms can rank workers above and below $1 - f_1$

2 Remember that $P(F)$ was well calculated even if the $f_1$ workers at the top are not continuously distributed, therefore, equation_ is correctly specified. Even if they choose $h > h_{max}$ the highest level of skills of the marginal worker is the level of skills of the worker with rank just below $F_1$.

$$\mathcal{E}\left(h_1\right) = P\left(0\right) h_1 + \left(P\left(f_0\right) - P\left(0\right)\right)\left(h_1 - h_{min}\right) + \int_{h_{min}}^{h_1} p_h(\widetilde{h})(h - \widetilde{h}) d\widetilde{h} - C(h_1)$$

The difference in expected earnings, keeping the ranking fixed, are:

$$\mathcal{E}\left(h_1\right) - \mathcal{E}\left(h_{max}\right) = P\left(0\right)\left(h_1 - h_{max}\right) + \left(P\left(f_0\right) - P\left(0\right)\right)\left(h_1 - h_{min} - h_{max} + h_{min}\right) + \int_{h_{min}}^{h_{max}} p_h(\widetilde{h})(h_1 - \widetilde{h} - h_{max} + \widetilde{h}) d\widetilde{h} - C(h_1) + C(h_{max})$$

This equation simplifies to:

$$\mathcal{E}\left(h_1\right) - \mathcal{E}\left(h_{max}\right) = \left(h_1 - h_{max}\right)\left(P\left(f_0\right) + \int_{h_{min}}^{h_{max}} p_h(\widetilde{h}) d\widetilde{h}\right) - C(h_1) + C(h_{max})$$

Using that $P\left(f_0\right) + \int_{h_{min}}^{h_{max}} p_h(\widetilde{h}) d\widetilde{h} = 1$, we obtain:

$$\mathcal{E}\left(h\right) - \mathcal{E}\left(h_{max}\right) = \left(h - h_{max}\right) 1 - C(h) + C(h_{max})$$

Therefore,$\mathcal{E}\left(h\right) - \mathcal{E}\left(h_{max}\right)$ is negative since $C'\left(h_{max}\right) = 1$ and $C\left(h\right)$ is convex.

Step 4. The LHS of $P\left(F\left(h\right)\right) = C'(h)$ can range from 0 to 1. Since $C'(h)$ is assumed to be continuous and ranges from $0$ to $m > 1$, the equilibrium exists.

b) If $b_v < a_u$:

Step 1. In Appendix A.3.3, it is shown that $P\left(F\right)$ is strictly increasing for $f_0 \leq F < 1 - f_1$. Therefore, $F\left(h\right)$ that satisfies $P\left(F\left(h\right)\right) = C'(h)$ is a continuous distribution on $\left(h_{min}, h_{max}\right)$.

Step 2: If the distribution of human capital is continuous on $\left(h_{min}, h_{max}\right)$, then the job finding probability for workers at $h_{min}$ is $P\left(0\right)$. Firms can rank workers above and below $f_0$ and $P\left(F\right)$ for $F \leq f_0$ is 0. We have to show that $\mathcal{E}\left(h_{min}\right) > \mathcal{E}\left(h\right)$ for any $h < h_{min}$, keeping the ranking fixed. The expected earnings of the worker at the minimum of the distribution are $\mathcal{E}\left(h\right) = P\left(0\right) h - C\left(h\right)$. Since $\mathcal{E}'\left(h\right) = P\left(0\right) - C'\left(h\right)$ and $\mathcal{E}''\left(h\right) = -C''\left(h\right) < 0$, we see that this condition is satisfied.

Step 3: the same as step 3 in part a.

Step 4. The LHS of $P\left(F\left(h\right)\right) = C'(h)$ can range from 0 to 1. Since $C'(h)$ is assumed to be continuous and ranges from $0$ to $m > 1$, the equilibrium exists.

### A.3.5 *Matching function*

The matching process is efficient in the sense that the number of workers employed are $\min\{U, V\}$. However, when the individuals choose $h$, they still do not know $u$ and $v$, for this reason, the relevant matching function is:

$$\mathbb{E}\left[min\left\{U, V\right\}\right]$$

Let $a_u = \overline{U} - k$, $b_u = \overline{U} + k$, $a_v = \overline{V} - k$ and $b_v = \overline{V} + k$. The matching function will be denoted $M\left(\overline{U}, \overline{V}\right)$, and is a function that depends on the expected number of workers $\overline{U}$, the expected number of vacancies $\overline{V}$, and on the parameter $k$.

**Case 1.** $a_u < b_u \le a_v < b_v$
In all realizations $min\{u, v\} = U$, therefore the expected number of matchings will be:

$$M\left(\overline{U},\overline{V}\right) = \int_{a_u}^{b_u}\left[\int_{a_v}^{b_v}\frac{1}{(b_v-a_v)(b_u-a_u)}udv\right]du = \overline{U}$$

The condition $a_u < b_u < a_v < b_v$ implies $\overline{U} + 2k < \overline{V}$.

**Case 2.** $a_u < a_v < b_u < b_v$
The matching function on this range is:

$$\begin{aligned} M\left(\overline{U},\overline{V}\right) = & \int_{b_u}^{b_v}\left[\int_{a_u}^{b_u}\frac{1}{(b_v-a_v)(b_u-a_u)}udu\right]dv + \int_{a_u}^{a_v}\left[\int_{a_v}^{b_u}\frac{1}{(b_v-a_v)(b_u-a_u)}udv\right]du \\ & + 2\int_{a_v}^{b_u}\left[\int_{a_v}^{v}\frac{1}{(b_v-a_v)(b_u-a_u)}udu\right]dv \end{aligned}$$

The first term represents the case when $v$ lies between $b_u$ and $b_v$, in this case $min\{u, v\} = v$ whatever the realization of $U$. The second term represents the case when $u$ lies between $a_u$ and $a_v$, in this case $min\{u, v\} = u$ whatever the realization of $v$ (we do not include the cases already accounted in the first term). The third and fourth term represent the case when $u$ and $v$ lie between $a_v$ and $b_u$: in the third term $min\{u, v\} = v$ and in the fourth $min\{u, v\} = u$. This can be simplified to:

$$M\left(\overline{U},\overline{V}\right) = \frac{-8k^3 - 6k\left(\overline{V}-\overline{U}\right)^2 + \left(\overline{V}-\overline{U}\right)^3 + 12k^2\left(\overline{V}+\overline{U}\right)}{24k^2}$$

The condition $a_u < a_v \le b_u < b_v$ implies $\overline{U} < \overline{V} \le \overline{U} + 2k$.

**Case 3.** $a_v \le a_u < b_v \le b_u$
The matching function on this range is:

$$\begin{aligned} M\left(\overline{U},\overline{V}\right) = & \int_{b_v}^{b_u}\left[\int_{a_v}^{b_v}\frac{1}{(b_v-a_v)(b_u-a_u)}vdv\right]du + \int_{a_v}^{a_u}\left[\int_{a_u}^{b_v}\frac{1}{(b_v-a_v)(b_u-a_u)}vdu\right]dv \\ & + 2\int_{a_u}^{b_v}\left[\int_{a_u}^{u}\frac{1}{(b_v-a_v)(b_u-a_u)}vdv\right]du \end{aligned}$$

This is symmetric to case 2, with the support of $V$ in the position of the support of $U$, and vice versa.

$$M\left(\overline{U},\overline{V}\right) = \frac{-8k^3 - 6k\left(\overline{U}-\overline{V}\right)^2 + \left(\overline{U}-\overline{V}\right)^3 + 12k^2\left(\overline{U}+\overline{V}\right)}{24k^2}$$

The condition $a_v \le a_u < b_v \le b_u$ implies $\overline{U} - 2k < \overline{V} \le \overline{U}$.

**Case 4.** $a_v < b_v \le a_u < b_u$
In all realizations $min\{u, v\} = v$, the expected number of matchings will be:

$$M\left(\overline{U},\overline{V}\right) = \int_{a_v}^{b_v}\left[\int_{a_u}^{b_u}\frac{1}{(b_v-a_v)(b_u-a_u)}vdu\right]dv = \overline{V}$$

The condition $b_v \le a_u$ implies $\overline{V} + k \le \overline{U} - k$. Therefore, the support must satisfy $\overline{V} \le \overline{U} - 2k$.

The matching function that results is:

$$M\left(\overline{U},\overline{V}\right)=\begin{cases}\overline{U} & \text{for } \overline{U}+2k<\overline{V}\\ \frac{-8k^3-6k\left(\overline{V}-\overline{U}\right)^2+\left(\overline{V}-\overline{U}\right)^3+12k^2\left(\overline{V}+\overline{U}\right)}{24k^2} & \text{for } \overline{U}<\overline{V}\leq\overline{U}+2k\\ \frac{-8k^3-6k\left(\overline{U}-\overline{V}\right)^2+\left(\overline{U}-\overline{V}\right)^3+12k^2\left(\overline{U}+\overline{V}\right)}{24k^2} & \text{for } \overline{U}-2k<\overline{V}\leq\overline{U}\\ \overline{V} & \text{for } \overline{V}\leq\overline{U}-2k\end{cases}$$

The first derivatives of the matching function are:

$$\frac{\partial M\left(\overline{U},\overline{V}\right)}{\partial\overline{V}}=\begin{cases}0 & \text{for } \overline{U}+2k<\overline{V}\\ \frac{\left(2k+\overline{U}-\overline{V}\right)^2}{8k^2} & \text{for } \overline{U}<\overline{V}\leq\overline{U}+2k\\ \frac{4k^2+4k\left(\overline{U}-\overline{V}\right)-\left(\overline{U}-\overline{V}\right)^2}{8k^2} & \text{for } \overline{U}-2k<\overline{V}\leq\overline{U}\\ 1 & \text{for } \overline{V}\leq\overline{U}-2k\end{cases}$$

$$\frac{\partial M\left(\overline{U},\overline{V}\right)}{\partial\overline{U}}=\begin{cases}1 & \textbf{for } \overline{U}+2k<\overline{V}\\ \frac{4k^2+4k\left(\overline{V}-\overline{U}\right)-\left(\overline{V}-\overline{U}\right)^2}{8k^2} & \textbf{for } \overline{U}<\overline{V}\leq\overline{U}+2k\\ \frac{\left(2k-\overline{U}+\overline{V}\right)^2}{8k^2} & \textbf{for } \overline{U}-2k<\overline{V}\leq\overline{U}\\ 0 & \textbf{for } \overline{V}\leq\overline{U}-2k\end{cases}$$

Both $\overline{U}$ and $\overline{V}$ have the same derivative, so we only need to check one of them, let's say $\frac{\partial M\left(\overline{U},\overline{V}\right)}{\partial\overline{U}}$. And only, when $\overline{U}<\overline{V}\leq\overline{U}+2k$ we cannot see directly that the derivative is positive. It will be so if $4k^2+4k\left(\overline{V}-\overline{U}\right)-\left(\overline{V}-\overline{U}\right)^2>0$. On the other hand, we know that $\overline{V}\leq\overline{U}+2k$ , which implies that $\left(\overline{V}-\overline{U}\right)^2\leq(2k)^2$. Besides, we know that $\overline{U}<\overline{V}$ implies that $\left(\overline{V}-\overline{U}\right)>0$. Both conditions prove that the derivative is positive.

The second derivatives are:

$$\frac{\partial^2 M\left(\overline{U},\overline{V}\right)}{\partial\overline{V}^2}=\begin{cases}0 & \text{for } \overline{U}+2k<\overline{V}\\ -\frac{2k-\overline{V}+\overline{U}}{4k^2} & \text{for } \overline{U}<\overline{V}\leq\overline{U}+2k\\ -\frac{2k-\overline{U}+\overline{V}}{4k^2} & \text{for } \overline{U}-2k<\overline{V}\leq\overline{U}\\ 0 & \text{for } \overline{V}\leq\overline{U}-2k\end{cases}$$

$$\frac{\partial^2 M\left(\overline{U},\overline{V}\right)}{\partial\overline{U}^2}=\begin{cases}0 & \text{for } \overline{U}+2k<\overline{V}\\ -\frac{2k+\overline{U}-\overline{V}}{4k^2} & \text{for } \overline{U}<\overline{V}\leq\overline{U}+2k\\ -\frac{2k-\overline{U}+\overline{V}}{4k^2} & \text{for } \overline{U}-2k<\overline{V}\leq\overline{U}\\ 0 & \text{for } \overline{V}\leq\overline{U}-2k\end{cases}$$

Again, the second derivatives are equal. Analysing $\frac{\partial^2 M\left(\overline{U},\overline{V}\right)}{\partial\overline{U}^2}$, we can check that the derivative is negative in case 2 and 3 on the dominion defined.

Therefore, $M\left(\overline{U},\overline{V}\right)$ is increasing and concave on both arguments.

$M\left(\overline{U},\overline{V}\right)$ has increasing returns to scale if $M\left(m\overline{U},m\overline{V}\right) > m{\cdot}M\left(\overline{U},\overline{V}\right)$ for any $m > 1$.

In case 3:

$$M\left(m\overline{U},m\overline{V}\right) - m * M\left(\overline{U},\overline{V}\right) = \frac{(m-1)\left(8k^3 - 6km\left(\overline{U}-\overline{V}\right)^2 + m(1+m)\left(\overline{U}-\overline{V}\right)^3\right)}{24k^2} = \frac{(m-1)J(m)}{24k^2}$$

This is positive if $J(m) > 0$.

$J''(m) = 2\left(\overline{U}-\overline{V}\right)^3 > 0$ in case 3. It is convex.

$J(m)$ has a minimum at $m = \frac{6k-\left(\overline{U}-\overline{V}\right)}{2\left(\overline{U}-\overline{V}\right)}$ for $\overline{U} \neq \overline{V}$

$$J\left(\frac{6k-\left(\overline{U}-\overline{V}\right)}{2\left(\overline{U}-\overline{V}\right)}\right) = 0.25\left(8k - \left(\overline{U}-\overline{V}\right)\right)\left(-2k + \left(\overline{U}-\overline{V}\right)\right)^2$$

This is positive if $8k - \left(\overline{U}-\overline{V}\right) > 0$. In case 3 $\overline{U} - 2k < \overline{V}$, therefore $J\left(\frac{6k-\left(\overline{U}-\overline{V}\right)}{2\left(\overline{U}-\overline{V}\right)}\right) > 0$ and $J(m) > J\left(\frac{6k-\left(\overline{U}-\overline{V}\right)}{2\left(\overline{U}-\overline{V}\right)}\right) > 0$

If $\overline{U} - \overline{V} = 0$, $J(m) = 8k^3 > 0$ for any $m$.

Therefore, the matching function has increasing returns to scale in case 3. Similarly, it also has increasing returns to scale in case 2.

### A.3.6 *Distribution of human capital*

We can summarize the conditions on the distributions of $V$ and $U$ as conditions on $\theta$ for $\overline{U}$ and $k$ given. For low values of $\theta$, it will be likely that there will be few vacancies, compared to workers. The support of $V$ will be to the left of the support of $U$. And as $\theta$ increases, the support of $V$ will move to the right.

We can derive the continuous part of the distribution from the equilibrium condition:

$$P(F) = C'(h)$$

It is possible to invert $P(F)$, for $\frac{(1-\theta)\overline{U}-2k}{\overline{U}-k} < F < \frac{(1-\theta)\overline{U}+2k}{\overline{U}+k}$ because $P(F)$ is strictly increasing on this range. This range corresponds to conditions 2, 3 and 4, defined in Appendix A.3.2.

**Condition 2:**

$$P(F) = \frac{-\left(-4+4F+F^2\right)k^2 + 2(2-F)k(\theta-(1-F))\overline{U} - \left((\theta-(1-F))\overline{U}\right)^2}{8(1-F)k^2} \text{ for } \frac{(1-\theta)\overline{U}}{\overline{U}-k} < F < \frac{(1-\theta)\overline{U}+2k}{\overline{U}+k}$$

The equation $p(F) = C'(h)$ implies:

$$F(h) = \frac{\left(-2+4C'(h)\right)k^2 - k(\theta-3)\overline{U} + (1-\theta)\overline{U}^2}{\left(k+\overline{U}\right)^2} - \frac{2\sqrt{2}\sqrt{\left(1-C'(h)\right)k^2\left(\left(1-2C'(h)\right)k^2 + k(\theta-1)\overline{U} + \theta\overline{U}^2\right)}}{\left(k+\overline{U}\right)^2}$$

for $h_1 < h < h_{max}$

With $h_{max}$ defined by $P\left(\frac{(1-\theta)\overline{U}+2k}{\overline{U}+k}\right) = 1 = C'(h_{max})$and $h_1$ and satisfies $P\left(\frac{(1-\theta)\overline{U}}{\overline{U}-k}\right) = C'(h_1)$.

**Condition 3:**

$$P(F) = \frac{k + (\theta - (1-F))\overline{U}}{2k} \text{ for } \frac{(1-\theta)\overline{U}}{\overline{U}+k} < F \leq \frac{(1-\theta)\overline{U}}{\overline{U}-k}$$

The equation $p(F) = C'(h)$ implies:

$$F(h) = \frac{\left(2C'(h)-1\right)k + \overline{U} - \theta\overline{U}}{\overline{U}}$$

for $h_2 < h < h_1$

With $h_1$ satisfying $P\left(\frac{(1-\theta)\overline{U}}{\overline{U}-k}\right) = C'(h_1)$ and $h_2$ such that $P\left(\frac{(1-\theta)\overline{U}}{\overline{U}+k}\right) = C'(h_2)$.

**Condition 4:**

$$P(F) = \frac{\left((-2+F)k + ((1-F)-\theta)\overline{U}\right)^2}{8(1-F)k^2} \text{for } \frac{(1-\theta)\overline{U}-2k}{\overline{U}-k} \leq F \leq \frac{(1-\theta)\overline{U}}{\overline{U}+k}$$

The equation $P(F) = C'(h)$ implies:

$$F(h) = \frac{\left(2-4C'(h)\right)k^2 + k(\theta-3)\overline{U} + (1-\theta)\overline{U}^2 + 2\sqrt{2}\sqrt{C'(h)k^2\left(\left(2C'(h)-1\right)k^2 + \theta\overline{U}^2 + k(1-\theta)\overline{U}\right)}}{\left(k-\overline{U}\right)^2}$$

for $h_{min} < h < h_2$

With $h_2$ such that $P\left(\frac{(1-\theta)\overline{U}}{\overline{U}+k}\right) = C'(h_2)$ and $h_{min}$ satisfying $P(0) = C'(h_{min})$

As it happened when calculating the probability function, we should consider the 4 cases the economy can take separately.

**Case 1:** $1 + \frac{2k}{\overline{U}} \leq \theta$

All the population has a probability of 1 of finding a job and there is an excess of vacancies, so the wage of each worker equals his level of skills. The expected earnings of the worker are:

$$\mathcal{E}(h) = h - C(h)$$

This is maximum when $h = h_{max}$. All workers will choose the same level of skills and $h_{min} = h_{max}$.

**Case 2:** $1 < \theta < 1 + \frac{2k}{\overline{U}}$

In this case, only conditions 1 to 3 apply. Thus, the distribution function is continuous also at the minimum of the support:

$$F(h)=\begin{cases}1 & \text{for } h_{max}\\ \frac{(-2+4C'(h))k^2-k(\theta-3)\overline{U}+(1-\theta)\overline{U}^2-2\sqrt{2}\sqrt{(1-C'(h))k^2\left((1-2C'(h))k^2+k(\theta-1)\overline{U}+\theta\overline{U}^2\right)}}{\left(k+\overline{U}\right)^2} & \text{for } h_1<h<h_{max}\\ \frac{(2C'(h)-1)k+\overline{U}-\theta\overline{U}}{\overline{U}} & \text{for } h_{min}\le h\le h_1\end{cases}$$

The workers at the bottom of the distribution will choose $h_{min}$ such that $P(0)=C'(h_{min})$. In case 2, $P(0)=\frac{k+(\theta-1)\overline{U}}{2k}$.

**Case 3:** $1-\frac{2k}{\overline{U}}<\theta\le 1$

In this case, only conditions 1- 4 apply. Thus, the distribution function is also continuous at the minimum of the support.

$$F(h)=\begin{cases}1 & \text{for } h_{max}\\ \frac{(-2+4C'(h))k^2-k(\theta-3)\overline{U}+(1-\theta)\overline{U}^2-2\sqrt{2}\sqrt{(1-C'(h))k^2\left((1-2C'(h))k^2+k(\theta-1)\overline{U}+\theta\overline{U}^2\right)}}{\left(k+\overline{U}\right)^2} & \text{for } h_1<h<h_{max}\\ \frac{(2C'(h)-1)k+\overline{U}-\theta\overline{U}}{\overline{U}} & \text{for } h_2<h\le h_1\\ \frac{(2-4C'(h))k^2+k(\theta-3)\overline{U}+(1-\theta)\overline{U}^2+2\sqrt{2}\sqrt{C'(h)k^2\left((2C'(h)-1)k^2+\theta\overline{U}^2+k(1-\theta)\overline{U}\right)}}{\left(k-\overline{U}\right)^2} & \text{for } h_{min}\le h\le h_2\end{cases}$$

The workers at the bottom of the distribution will choose $h_{min}$ such that $P(0)=C'(h_{min})$. In case 3, $P(0)=\frac{\left(-2k+(1-\theta)\overline{U}\right)^2}{8k^2}$.

**Case 4:** $\theta\le 1-\frac{2k}{\overline{U}}$

In this case the 5 conditions apply. The workers with $F=F_0$ satisfy the optimal condition that $P(F)=C'(h)$ but $P\left(\frac{(1-\theta)\overline{U}-2k}{\overline{U}-k}\right)=0$. They choose, a level of skills equal to 0. As the level of skills cannot be negative, this is the minimum of the distribution of skills, $h_{min}=0$. There is a mass of workers $f_0=\frac{(1-\theta)\overline{U}-2k}{\overline{U}-k}$ at the minimum of the distribution.

Thus, the distribution of $h$ is:

$$F(h)=\begin{cases}1 & \text{for } h=h_{max}\\ \frac{(-2+4C'(h))k^2-k(\theta-3)\overline{U}+(1-\theta)\overline{U}^2-2\sqrt{2}\sqrt{(1-C'(h))k^2\left((1-2C'(h))k^2+k(\theta-1)\overline{U}+\theta\overline{U}^2\right)}}{\left(k+\overline{U}\right)^2} & \text{for } h_1<h<h_{max}\\ \frac{(2C'(h)-1)k+\overline{U}-\theta\overline{U}}{\overline{U}} & \text{for } h_2<h\le h_1\\ \frac{(2-4C'(h))k^2+k(\theta-3)\overline{U}+(1-\theta)\overline{U}^2+2\sqrt{2}\sqrt{C'(h)k^2\left((2C'(h)-1)k^2+\theta\overline{U}^2+k(1-\theta)\overline{U}\right)}}{\left(k-\overline{U}\right)^2} & \text{for } h_{min}<h\le h_2\\ \frac{\overline{U}-\theta\overline{U}-2k}{\overline{U}-k} & \text{for } h=h_{min}=0\end{cases}$$

### A.3.7 *Effect of market size and market tightness*

#### A.3.7.1 *Effect of market tightness on the job finding probability*

$$\frac{\partial p(F)}{\partial \theta}=\begin{cases} 0 & \text{for } \frac{(1-\theta)\overline{U}+2k}{\overline{U}+k}<F\leq 1 & [1]\\ \frac{\overline{U}\left((2-F)k-(\theta-(1-F))\overline{U}\right)}{4(1-F)k^2} & \text{for } \frac{(1-\theta)\overline{U}}{\overline{U}-k}<F\leq\frac{(1-\theta)\overline{U}+2k}{\overline{U}+k} & [2]\\ \frac{\overline{U}}{2k} & \text{for } \frac{(1-\theta)\overline{U}}{\overline{U}+k}<F\leq\frac{(1-\theta)\overline{U}}{\overline{U}-k} & [3]\\ \frac{\overline{U}\left((2-F)k+(\theta-(1-F))\overline{U}\right)}{4(1-F)k^2} & \text{for } \frac{(1-\theta)\overline{U}-2k}{\overline{U}-k}<F\leq\frac{(1-\theta)\overline{U}}{\overline{U}+k} & [4]\\ 0 & \text{for } 0\leq F\leq\frac{(1-\theta)\overline{U}-2k}{\overline{U}-k} & [5] \end{cases}$$

Condition 2 states $F\leq\frac{(1-\theta)\overline{U}+2k}{\overline{U}+k}$, which implies $(2-F)\,k-(\theta-(1-F))\,\overline{U}\geq 0$. Therefore, the derivative associated to condition 2 is positive. Condition 4 states $\frac{(1-\theta)\overline{U}-2k}{\overline{U}-k}<F$, which implies $(2-F)\,k+(\theta-(1-F))\,\overline{U}>0$. Therefore, the derivative associated to condition 4 is positive. $\frac{\partial p(F)}{\partial\theta}>0$.

#### A.3.7.2 *Effect of market size on the job finding probability*

The derivative of the hiring probability with respect to $\overline{U}$ is:

$$\frac{\partial p(F)}{\partial \overline{U}}=\begin{cases} 0 & \text{for } \frac{(1-\theta)\overline{U}+2k}{\overline{U}+k}<F\leq 1 & [1]\\ (\theta-(1-F))\,\frac{(2-F)k-(\theta-(1-F))\overline{U}}{4(1-F)k^2} & \text{for } \frac{(1-\theta)\overline{U}}{\overline{U}-k}<F\leq\frac{(1-\theta)\overline{U}+2k}{\overline{U}+k} & [2]\\ \frac{\theta-(1-F)}{2k} & \text{for } \frac{(1-\theta)\overline{U}}{\overline{U}+k}<F\leq\frac{(1-\theta)\overline{U}}{\overline{U}-k} & [3]\\ (\theta-(1-F))\,\frac{\left((2-F)k+(\theta-(1-F))\overline{U}\right)}{4(1-F)k^2} & \text{for } \frac{(1-\theta)\overline{U}-2k}{\overline{U}-k}<F\leq\frac{(1-\theta)\overline{U}}{\overline{U}+k} & [4]\\ 0 & \text{for } 0\leq F\leq\frac{(1-\theta)\overline{U}-2k}{\overline{U}-k} & [5] \end{cases}$$

Condition 2 states $\frac{(1-\theta)\overline{U}}{\overline{U}-k}<F$, which implies $\theta-(1-F)>\frac{Fk}{\overline{U}}$, which means $(\theta-(1-F))>0$. On the other hand, $F\leq\frac{(1-\theta)\overline{U}+2k}{\overline{U}+k}$ implies $(2-F)\,k-(\theta-(1-F))\,\overline{U}\geq 0$. Therefore, the derivative associated to condition 2 is positive. The derivative associated to condition 3 is positive for $F>1-\theta$, negative for $F<1-\theta$ and zero for $F=1-\theta$. Condition 4 states $F\leq\frac{(1-\theta)\overline{U}}{\overline{U}+k}$, which implies $(\theta-(1-F))\leq-\frac{FK}{\overline{U}}<0$. On the other hand, $\frac{(1-\theta)\overline{U}-2k}{\overline{U}-k}<F$ implies $(2-F)\,k+(\theta-(1-F))\,\overline{U}>0$. Therefore, the derivative associated to condition 4 is negative.

#### A.3.7.3 *Effect of market tightness on the distributions*

$F\,(h)$ satisfies $P\,(F\,(h))=C'(h)$. As $\frac{\partial P(F)}{\partial\theta}>0$ for any $h<h_{max}$ but the RHS is constant with market tightness, $F\,(h)$ must be lower. For the case of $h_{min}$, it is easy to check that also in case 4, $f_0=\frac{\overline{U}-\theta\overline{U}-2k}{\overline{U}-k}$, which is decreasing in $\theta$ until it reaches its minimum 0.

#### A.3.7.4 *Effect of market size on the distribution*

Part a:

$F(h)$ satisfies $P(F(h)) = C'(h)$. As $\frac{\partial P(F)}{\partial \overline{U}} > 0$ for any $1-\theta < F < 1$ but the RHS is constant with market tightness, $F(h)$ must be lower for this range of $F(h)$. If $F < 1-\theta$, we will have that $\frac{\partial P(F)}{\partial \overline{U}} < 0$, so $F(h)$ must increase. If $h = h_0$, $\frac{\partial P(F)}{\partial \overline{U}} = 0$, so $F(h_0)$ is constant in $\overline{U}$.

Part b:

If $\theta < 1$, the economy is either in case 3, $\theta \geq 1 - \frac{2k}{\overline{U}}$, or in case 4, $\theta < 1 - \frac{2k}{\overline{U}}$. For $\overline{U}$ large enough it will shift from case 3 to 4. In case 3, $f_0 = 0$. In case 4, $f_0 = \frac{\overline{U}-\theta\overline{U}-2k}{\overline{U}-k}$ which is increasing in $\overline{U}$, $\frac{\partial f_0}{\partial \overline{U}} = \frac{(1+\theta)k}{\left(\overline{U}-k\right)^2} > 0$. If $\theta \geq 1$, $f_0 = 0$, as we show in Proposition 1.

Part c:

$f_1 = \frac{\theta\overline{U}-k}{\overline{U}+k}$ if $f_1 < 1$, which is increasing in $\overline{U}$, $\frac{\partial f_1}{\partial \overline{U}} = \frac{(1+\theta)k}{\left(\overline{U}+k\right)^2} > 0$.

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