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ESSAYS IN TRADE, INNOVATION AND PRODUCTIVITY

UNIVERSIDAD CARLOS III DE MADRID

TESIS DOCTORAL

ESSAYS IN TRADE, INNOVATION AND PRODUCTIVITY

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Firma del Tribunal Calificador:

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Getafe, de de .

A mi querida familia.

La vida es extraña.
Cuando eres niño no pasa el tiempo, y de pronto un día, tienes 50 *años.*
Y lo que te queda de la niñez, cabe en una caja oxidada.

— *Le fabuleux destin d'Amélie Poulain* (2001)

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RESUMEN

Mi tesis se centra en el Comercio Internacional y Macroeconomía Aplicada. Mi investigación actual se ocupa de los efectos de las políticas comerciales y de innovación sobre las decisiones de las empresas y su impacto en el bienestar de la economía.

El primer capítulo, titulado *"Trade and Process Innovation: Extensive and Intensive Margin"*, considera un modelo de comercio con empresas heterogéneas que no sólo deciden cuándo y cuánto exportar sino también cuándo y cuánto innovar. Aunque la literatura siempre ha reconocido la interdependencia entre la innovación y el comercio, hasta ahora no ha analizado el impacto de la liberalización del comercio sobre la productividad y el bienestar en un modelo que incorpore los márgenes extensivos e intensivos tanto del comercio como de la innovación. El objetivo principal de este capítulo es mostrar que la introducción de estos márgenes diferentes es clave para entender el impacto de la liberalización del comercio.

Distintos equilibrios pueden darse en función de los costes relativos del comercio y la innovación. En una economía con costes comerciales bajos en relación con los costes de la innovación, empresas con una productividad media exportan pero no innovan, mientras que en una economía con costes comerciales altos en relación con los costes de innovación, las empresas con una productividad media innovan pero no exportan. En un tercer equilibrio, en medio de los otros dos, algunas empresas con una productividad media exportan e innovan, mientras que otras ni exportan ni innovan.

En el segundo capítulo, titulado *"Trade, Process Innovation and Productivity: A Quantitative Analysis of Europe"*, se muestra que los distintos equilibrios teóricos descritos en el primer capítulo son cuantitativamente posibles a través de una calibración del modelo para cinco países europeos (Francia , Alemania, Italia, España y Reino Unido), utilizando los datos a nivel de empresas del proyecto European Firms in a Global Economy (EFIGE).

A continuación, muestro que el impacto de la liberalización del comercio sobre la productividad agregada y el bienestar depende crucialmente del equilibrio en el que se encuentra la economía. El primer ejercicio cuantitativo consiste en estimar el efecto de una reducción de los costes comerciales variables sobre la productividad agregada y

el bienestar. Atkeson y Burstein (2010) encuentran que permitir a las empresas innovar no cambia los beneficios del comercio, ya que la mayor intensidad en la innovación de las empresas exportadoras se compensa con la disminución de la intensidad en la innovación de las empresas no exportadoras. A pesar de encontrar resultados similares generalmente, en las economías donde el comercio es caro en relación con la innovación, la disminución de los costes del comercio variables hace que sea más difícil para las empresas nacionales innovar. En comparación con Atkeson y Burstein (2010), este efecto negativo proveniente del margen extensivo de la innovación puede dar lugar a pérdidas de bienestar. El segundo ejercicio cuantitativo se centra en los efectos sobre el bienestar de la reducción de los costes fijos del comercio o la innovación. Encuentro que una reducción de los costes fijos de comercio no siempre tiene un efecto positivo. En particular, en una economía en la que muchas empresas exportan, pero pocas empresas innovan, al reducir los costes fijos de comercio, aumenta el número de exportadores, y puede hacer que la innovación sea más cara, lo que disminuye la productividad agregada.

Estos resultados subrayan la importancia de contar con un modelo que analiza conjuntamente los márgenes extensivos e intensivos de comercio y de la innovación. No hacerlo no sólo daría lugar a una estructura teórica menos rica, sino que también nos impediría evaluar correctamente el impacto de diferentes políticas destinadas a fomentar el comercio y la innovación.

En el tercer capítulo, titulado *"Understanding Competitiveness"* y conjunto con Rubén Segura Cayuela, se analizan los factores que impulsan la evolución de los costes laborales totales unitarios (CLU), el principal indicador de la competitividad europea, en Francia, Alemania, Italia y España. Utilizando datos a nivel de empresas, calculamos un cambio ponderado de los CLU agregados entre 2002 y 2007, y lo descomponemos en tres elementos principales: el primero captura los cambios a nivel de empresa de los costes laborales unitarios, manteniendo las cuotas iniciales del mercado interno de las empresas constante; el segundo cuantifica la reasignación de cuotas de mercado en la economía nacional, manteniendo los costes laborales unitarios iniciales constantes; y el tercero mide la interacción entre las dos primeros elementos. Los resultados indican que la evolución de los CLU agregados no se debe a la evolución de los CLU a nivel de empresa, y por lo tanto falla en capturar adecuadamente la heterogeneidad existente a nivel de empresa. En cambio, la evolución de los CLU agregados es impulsada por la reasignación de recursos entre las empresas de la economía.

Motivados por el importante papel de la reasignación de recursos para explicar la evolución de los CLU agregados, aplicamos la metodología de Hsieh y Klenow (2009) para cuantificar la medida en que las diferencias de productividad en Europa se deben a una asignación ineficiente de los recursos. Como resultado de las distorsiones que afectan la producción, las empresas producen diferentes cantidades que lo dictaría su productividad. Una asignación eficiente de los recursos aumentaría la TFP manufacturera agregada en 2008 un 22,7% en Francia, 27,9% en Alemania, 43,5% en Italia y 28,2% en España.

El análisis empírico de los costes laborales unitarios como medida competitividad, pone de manifiesto la necesidad de utilizar datos microeconómicos para entender los factores que impulsan la evolución de los agregados macroeconómicos.

ABSTRACT

My Dissertation focuses on International Trade and Applied Macroeconomics. My current research deals with the effects of trade and innovation policies on firms' decisions and their impact on the welfare of the economy.

The first chapter, entitled *"Trade and Process Innovation: Extensive and Intensive Margin"*, considers a trade model with heterogeneous firms that decide not just whether and how much to export but also whether and how much to innovate. While the literature has long recognized the interdependence between innovation and trade, it has so far not analyzed the impact of trade liberalization on productivity and welfare in a model that incorporates both the extensive and intensive margins of both trade and innovation. The main point of the chapter is to show that introducing these different margins is key for understanding the impact of trade liberalization.

Different equilibria may arise, depending on the relative costs of trade and innovation. In an economy with trade costs that are low relative to innovation costs, medium productivity firms export without innovating, whereas in an economy with trade costs that are high relative to innovation costs, medium productivity firms innovate without exporting. In a third equilibrium, in between the other two, some medium productivity firms export and innovate, whereas others neither export nor innovate.

In the second chapter, entitled *"Trade, Process Innovation and Productivity: A Quantitative Analysis of Europe"*, I show that the theoretical equilibria discussed in the first chapter are quantitatively plausible by calibrating the model to five European countries (France, Germany, Italy, Spain and United Kingdom) using the firm-level data set European Firms in a Global Economy (EFIGE).

I then show that the impact of trade liberalization on aggregate productivity and welfare depends crucially on the equilibrium the economy is in. A first quantitative exercise consists of estimating the effect of a reduction in variable trade costs on aggregate productivity and welfare. Atkeson and Burstein (2010) find that allowing for firms to innovate does not change the gains from trade, since the increased innovation intensity of the exporting firms is offset by the decreased innovation intensity of non-exporting firms. Although I find overall similar results, in economies where trade is expensive relative to

innovation, a drop in variable trade costs makes it harder for domestic firms to innovate. Compared to Atkeson and Burstein (2010), this negative effect coming from the extensive margin of innovation may lead to welfare losses. A second quantitative exercise focuses on the welfare effects of lowering the fixed costs of trade or innovation. I find that a drop in fixed trade costs need not always have a positive effect. Indeed, in an economy in which many firms export, but few firms innovate, lowering the fixed costs of trade, by increasing the number of exporters, may make innovating more expensive, thus lowering aggregate productivity.

These findings stress the importance of having a model that jointly analyzes the extensive and intensive margins of both trade and innovation. Not doing so would not just result in a less rich theoretical structure, it would also keep us from correctly assessing the impact of different policies aimed at fomenting trade and innovation.

In the third chapter, entitled *"Understanding Competitiveness"* and joint with Rubén Segura Cayuela, we analyze the factors that drive the evolution of the aggregate Unit Labor Costs(ULC), the main European competitiveness indicator, in France, Germany, Italy and Spain. Using firm level data we calculate a weighted change of the aggregate ULC between 2002 and 2007, and decompose it into three main elements: the first captures changes in firm-level unit labor costs, keeping the initial domestic market shares of firms constant; the second quantifies the reallocation of market shares within the domestic economy, keeping the initial unit labor costs constant; and the third measures the interaction between the first two. The results suggest that the evolution of the aggregate ULC is not driven by the evolution of the firm level ULC, and therefore fails at capturing adequately the heterogeneity existent at the firm level. Instead, the evolution of the ULC is driven by the reallocation of resources among the firms of the economy.

Motivated by the significant role of the reallocation of resources to explain the evolution of the aggregate ULC, we apply the methodology of Hsieh and Klenow (2009) to quantify how much of the differences in productivity in Europe is due to an inefficient allocation of resources. As a result of distortions that affect production, firms produce different amounts that what would be dictated by their productivity. An efficient allocation of resources would boost aggregate manufacturing TFP in 2008 by 22.7% in France, 27.9% in Germany, 43.5% in Italy and 28.2% in Spain.

The empirical analysis of the unit labor costs as a competitiveness measure, reveals the need to use microeconomic data to understand the driving factors behind the evolution of macroeconomic aggregates.

1

TRADE AND PROCESS INNOVATION: EXTENSIVE AND INTENSIVE MARGIN

Abstract. This paper proposes a trade model with heterogeneous firms that decide not just whether and how much to export but also whether and how much to innovate, where innovation reflects the ability of firms to increase their productivity. Incorporating both the extensive and intensive margins of trade and innovation leads to different equilibria. Depending on how costly trade is relative to innovation, medium-productivity firms may either export without innovating, innovate without exporting, do both or do neither. The impact of trade on aggregate productivity and welfare depends crucially on the firms' typology distribution in equilibrium, and which distribution arises depends on the cost-benefit ratio of innovation and the exporting cost.

1.1 INTRODUCTION

There is substantial heterogeneity across firms in both process innovation and export activities. Some firms neither innovate nor export, others both innovate and export, and still others may do only one of the activities. In addition, within these different groups of firms, the intensity of both activities also differs across firms. While the literature has long recognized the interdependence between process innovation and trade, it has so far not analyzed the impact of trade liberalization on productivity and welfare in a model that incorporates both the extensive and the intensive margins of both trade and innovation.

The aim of the paper is to show that introducing these different margins is key for understanding the impact of trade liberalization. Different equilibria may arise, depending on the relative costs of trade and innovation. After discussing the properties of each of those equilibria, I show theoretically that the impact of opening up to trade depends crucially on the equilibrium the economy is in the open economy. For example, I show that if trade is relatively more expensive than process innovation, opening up to trade implies a decrease in the number of innovators in the economy, while if trade is relatively less expensive than innovation, opening up to trade leads to an increase in the number of innovators in the economy.

The paper proposes a trade model with heterogeneous firms in the spirit of Melitz (2003) with a basic difference: once a firm learns about its productivity, it can decide to spend resources on process innovation to lower its marginal costs. Process innovation[1] is a costly activity that involves both fixed and variable costs, hence firms decide whether and how much to innovate. This is key to explore how trade liberalization affects the extensive and intensive margins of trade and innovation. The model is rich enough to explore the interdependence between the innovation and export decisions, and yet tractable enough to aggregate up from firm level decisions and analyze how aggregate productivity and welfare respond to changes in trade and innovation policies.

Three different equilibria may arise, depending on how costly trade is relative to innovation. In all three equilibria, high-productivity firms always export and innovate, while low-productivity firms never export or innovate. What differs across equilibria is the behavior of medium-productivity firms. In the *low cost innovation equilibrium*,

1 Process innovation in the literature is defined as the adoption of a production technology which is significantly improved;

trade is relatively costly compared to innovation, so that medium-productivity firms innovate but do not export. In the *low cost trade equilibrium*, trade costs are relatively low compared to innovation, so that medium-productivity firms export but do not innovate. In between these two extremes, there is the *intermediate equilibrium*, characterized by medium-productivity firms engaging in either both activities or none of them. Depending on which equilibrium the economy is in, the theory illustrates that the effect of trade liberalization on aggregate productivity and welfare may be very different.

A key contribution of my work is that it joins the two branches of the literature on trade and process innovation. On the one hand, there is the literature that focuses on how firms make joint decisions on exporting and innovating. Yeaple (2005) and Bustos (2011) consider models in which there is a binary technology choice, and highlight how firms decide to both enter the export market and adopt the new technology. The cost of innovation is therefore modeled as a fixed cost. Costantini and Melitz (2008) extend this type of joint decision to a dynamic framework where firms face both idiosyncratic uncertainty and sunk costs for both exporting and technology adoption. On the other hand, there is the literature that focuses on examining the impact of trade on the intensity of innovation. Vannoorenberghe (2008) and Rubini (2011) consider models in which firm productivity is endogenously determined through innovation, and highlight that innovation is affected by the existence of foreign markets. Closely related to these is the work of Atkeson and Burstein (2010). They propose a dynamic trade model to include a process innovation decision by incumbent firms following Griliches (1979) model of knowledge capital.

The chapter is organized as follows. In Section 1.2, I present the model of the closed economy, where firms only take decisions on innovation. In Section 1.3, I present the model of the open economy and explore the equilibria determined by the interaction between the exporting and innovation choices creates. In Section 1.4, I discuss the implication on aggregate productivity. And Section 1.5 concludes.

1.2 CLOSED ECONOMY

The model is based on the monopolistic competition framework proposed by Melitz (2003) which I expand to allow firms to engage in process innovation.

1.2.1 *Demand*

There is a continuum of consumers of measure L. Given the set Ω of varieties supplied, the consumer's preferences are represented by the standard C.E.S. utility function

$$\left[\int_{\omega\in\Omega} q^{\rho}(\omega)d\omega\right]^{\frac{1}{\rho}},$$

where $q(\omega)$ denotes the quantity consumed of variety ω, and $\sigma = \frac{1}{1-\rho} > 1$ is the elasticity of substitution across varieties. The market is subject to the expenditure-income constraint:

$$\int_{\omega\in\Omega} p(\omega)q(\omega)d\omega = R,$$

where R is the total revenues obtained.

Standard utility maximization implies that the demand for each individual variety is:

$$q(\omega) = [p(\omega)]^{-\sigma}\frac{R}{P^{1-\sigma}}, \tag{1.1}$$

where $p(\omega)$ is the price of each variety ω and $P = \left[\int_{\omega\in\Omega} p(\omega)^{1-\sigma}d\omega\right]^{\frac{1}{1-\sigma}}$ denotes the price index of the economy.

1.2.2 *Supply*

There is a continuum of firms, each producing a different variety ω. Each firm draws its productivity φ from a distribution $G(\varphi)$ with support $(0, \infty)$ after paying a labor sunk cost of entry f_E. Since a firm is characterized by its productivity φ, it is equivalent to talk about variety ω or productivity φ.

Production requires only labor, which is inelastically supplied at the aggregate level L, and therefore can be taken as an index of country's size. In contrast to the Melitz model where firms use a constant returns to scale production technology, firms can affect their marginal cost through process innovation. To enter the economy, a firm needs $f_D > 0$

labor units. Therefore, to produce output $q(\varphi)$, a firm requires $l(\varphi)$ labor units:

$$l(\varphi) = f_D + c(z(\varphi)) + \frac{q(\varphi)}{\varphi} \frac{1}{(1 + z(\varphi))^{\frac{1}{\sigma - 1}}},$$

where $z(\varphi)$ is a measure of the productivity increase from innovation that has an associated cost function $c(z(\varphi))$.

The cost function of the innovation follows Klette and Kortum (2004), Lentz and Mortensen (2008), and Stähler et al. (2007), therefore $c(z)$ is a strictly convex cost function, twice differentiable with $c(0) = 0$. Firms pay a fixed cost that can be attributed to the acquisition and implementation of the technology, plus a variable cost that depends directly on the process innovation performed by each firm. Hence, the cost function $c(z)$ is defined as

$$c(z(\varphi)) = \begin{cases} z(\varphi)^{\alpha+1} + f_I & \text{if } z(\varphi) > 0, \\ 0 & \text{if } z(\varphi) = 0, \end{cases}$$

where f_I is the fixed cost required to implement the process innovation, and $\alpha > 0$ measures the rate at which the marginal cost of the innovation increases.

The fixed cost of innovation provides a partition in the firms, there will be innovators and non-innovators in the economy, allowing to study changes along the extensive margin of innovation. The variable costs explain differences among the innovation performed by firms — the higher the level of innovation, the higher the cost— allowing to study changes along the intensive margin of innovation.

Even though it can be argued that the cost of innovation can be simplified by imposing a linear variable cost, the existence of convex innovation costs is a standard feature in the literature because it ensures that innovation is finite. Another simplification would be to have either a fixed cost or a variable cost but not both. Nevertheless, maintaining a flexible cost function is important. For example, Vannoorenberghe (2008) assumes away a fixed innovation cost, which implies that all firms engage in process innovation. This eliminates the possibility of studying the interaction between the export and innovation decisions along the extensive margin, which is one of the purposes of this paper.

1.2.3 *Firm's Problem*

Figure 1.1 represents the timing of the firm's problem. [2] In a first stage, as in Melitz (2003), entering the market means paying a labor sunk cost f_E in order to get a draw of the productivity parameter φ. In the second stage, with the knowledge of their own productivity, firms decide how much to innovate. Since innovation requires paying a labor fixed cost, f_I, there will be two types of firms in the closed economy: *Type D* firms are active in the market and do not perform innovation and; and *Type DI* firms are those active in the market that innovate; Finally, in the third stage, firms choose prices. I solve the firms problem through backward induction.

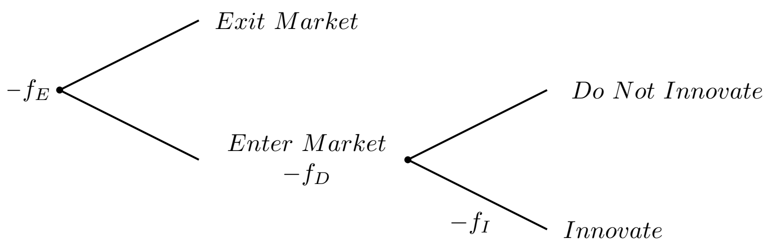


Figure 1.1: Timing in the Closed Economy

OPTIMAL PRICING RULE. In the last stage of the problem the firm sets its optimal price, given its innovation decision and the market conditions which are summarized by the price index P and R.

$$\max_{p(\varphi)} p(\varphi)\, q(\varphi) - f_D - \frac{q(\varphi)}{\varphi\left[(1+z(\varphi))^{\frac{1}{\sigma-1}}\right]} - c\,(z(\varphi))$$

The corresponding first order condition is

$$p(\varphi) = \left(\frac{\sigma}{\sigma-1}\right)\frac{1}{\varphi}\cdot\frac{1}{(1+z(\varphi))^{\frac{1}{\sigma-1}}} \quad \forall\, z(\varphi). \tag{1.2}$$

OPTIMAL INNOVATION DECISION. The optimal innovation rule is obtained from the first order condition of the maximization of $\pi(\varphi) = [p(\varphi)q(\varphi) - l(\varphi)]$ with respect to

2 As in Melitz (2003), the dynamics are trivial, since firms make all decisions at birth.

$z(\varphi)$, provided that the firm makes higher profits by innovating than by choosing not to innovate. This gives

$$z(\varphi)=\begin{cases}\left[\left(\frac{1}{\alpha+1}\right)\frac{p(\varphi)q(\varphi)}{\sigma}\right]^{\frac{1}{\alpha}} & \text{if } \pi_{DI}(\varphi)\geqslant\pi_D(\varphi),\\ 0 & \text{if } \pi_{DI}(\varphi)<\pi_D(\varphi),\end{cases}\tag{1.3}$$

where $\frac{1}{\alpha}$ is the parameter that shapes the optimal innovation function and tells us how innovation rises with size, where I take the productivity parameter $\varphi^{\sigma-1}$ to be the indicator of size. If the function is linear $(\alpha=1)$, then innovation rises proportionately with size, however, if the function is concave $(\alpha>1)$, then the amount of innovation performed will rise less than proportionally with size, and if the function is convex $(0<\alpha<1)$ the amount of innovation performed will increase more than proportionally with the productivity.

Firms will choose the option that yields the highest profits.

– Profits of a non-innovator firm (Type D):

$$\pi_D=\frac{R\,(P\rho)^{\sigma-1}}{\sigma}\varphi^{\sigma-1}-f_D.$$

– Profits of an innovator firm (Type DI):

$$\pi_{DI}=\frac{R\,(P\rho)^{\sigma-1}}{\sigma}\varphi^{\sigma-1}\,(1+z\,(\varphi))-f_D-c\,(z\,(\varphi))\,.$$

1.2.4 *Equilibrium in a Closed Economy*

In Autarky only the most productive firms will innovate. The conditions of entry in the domestic market plus the innovation condition allow to solve for the different productivity cutoffs in the closed economy equilibrium.

The Zero Profit Condition (ZCP) $\pi_D\left(\varphi_D^*\right)=0$, so that:

$$(\varphi_D^*)^{\sigma-1}=\frac{f_D}{\left(\frac{R(P\rho)^{\sigma-1}}{\sigma}\right)}.\tag{1.4}$$

The Innovation Profit Condition (IPC) determines the productivity cutoff φ_{DI}^* which is the productivity of the firm indifferent between innovating or not, i.e. $\pi_{DI}(\varphi_{DI}) = \pi_D(\varphi_{DI})$

$$(\varphi_{DI}^*)^{\sigma-1} = \frac{\left(\frac{f_I}{\alpha}\right)^{\frac{\alpha}{\alpha+1}} \cdot (\alpha+1)}{\left(\frac{R(P\rho)^{\sigma-1}}{\sigma}\right)}. \tag{1.5}$$

By combining equations Equation 1.4 and Equation 1.5 we have the relation between the innovation cutoff and the entry cutoff in terms of the fixed cost to produce, the fixed cost to innovate and the parameters of the innovation:

$$(\varphi_{DI}^*)^{\sigma-1} = \frac{\left(\frac{f_I}{\alpha}\right)^{\frac{\alpha}{\alpha+1}} \cdot (\alpha+1)}{f_D} (\varphi_D^*)^{\sigma-1}, \tag{1.6}$$

where $\frac{\left(\frac{f_I}{\alpha}\right)^{\frac{\alpha}{\alpha+1}} \cdot (\alpha+1)}{f_D}$ can be interpreted as the ratio between the innovation costs and the operating costs. The numerator is composed by the sunk cost of innovation, whose effect is determined by the shape of the innovation and the elasticity [3] of the variable costs to innovate, while the denominator is simply the operating costs that every firm must incur into in order to actively participate in the market.

Proposition 1. *The economy is in equilibrium and $\varphi_{DI}^* > \varphi_D^*$, if the following parameter restriction holds*

$$\left(\frac{f_I}{\alpha}\right)^{\frac{\alpha}{\alpha+1}} \cdot (\alpha+1) > f_D$$

Proof. Selection into innovation ($\varphi_{DI}^* > \varphi_D^*$) requires innovation costs to be high enough relative to production costs. Equations Equation 1.4 and Equation 1.5 along with the Free Entry (FE) condition, which requires that the sunk entry cost equals the present value of expected profits:

$$\left[\int_{\varphi_D^*}^{\varphi_{DI}^*} \pi_D(\varphi)\, dG(\varphi) + \int_{\varphi_{DI}^*}^{\infty} \pi_{DI}(\varphi)\, dG(\varphi)\right] = \delta f_E \tag{1.7}$$

3 $\varepsilon_C = \frac{zcdotc'_V(z)}{c_V(z)} = (\alpha+1)$

uniquely determine the equilibrium price (P), the number of firms (M), and the distribution of active firms productivity in the economy along with the productivity cutoffs φ^*_{DI}, and φ^*_D. See Appendix A.1 for a formal proof. □

Finally, notice that the cutoff productivity level in this economy is higher compared to the one found in Melitz (2003) where firms have no choice to invest in a productivity enhancing technology (See Appendix for proof). The reasoning behind this result is that the ability of some firms to invest in a cost reducing technology enables them to have more market shares than they would without the presence of innovation. Logically, those market shares are 'stolen' from the less productive firms of the economy.

1.3 OPEN ECONOMY

1.3.1 *Set-up*

I now examine the impact of trade in a world that is composed of countries whose economies are of the type that was previously described. I assume that the economy under study can trade with $n \geqslant 1$ other identical countries, that is, the world is then comprised of $n+1$ symmetric countries. The symmetry of both countries ensures that factor price equalization holds, countries have a common wage which can be still taken as a numerary and they share the same aggregate variables.

I denote the source country by i and the destination country by j, where $i, j = 1, 2, ...,$ $n+1$. To enter country j, firm i needs $f_{ij} > 0$ labor units and there are iceberg trade cost, so that $\tau_{ij} > 1$ units of the good have to be produced by a firm of country i to deliver one unit to country j. Without loss of generality, I assume that $\tau_{ii} = 1$ and thus denote $\tau_{ij} = \tau$ for all $i \neq j$.[4] Thus, to produce output $\sum_j q_{ij}(\varphi)$, a firm requires $\sum_j l_{ij}(\varphi)$ labor units:

$$\sum_j l_{ij}(\varphi) = \frac{\sum_j \tau_{ij} q_{ij}(\varphi)}{\varphi \cdot (1 + z_i(\varphi))^{\frac{1}{\sigma-1}}} + \sum_j f_{ij} + c\left(z_i(\varphi)\right).$$

Figure 1.2 represents the timing of the firm's problem in the open economy.[5] In a first stage, as in Melitz (2003), entering the market means paying a labor sunk cost f_E in order

4 Note that $\tau_{ij} = \tau_{ji}$ by symmetry and there is no possibility of transportation arbitrage
5 As in Melitz (2003), the dynamics are trivial, since firms make all decisions at birth.

to get a draw of the productivity parameter φ. In the second stage, with the knowledge of their own productivity, firms decide which activities to undertake. Since both exporting and innovation require paying a labor fixed cost, f_X and f_I, there will be four types of firms in the open economy: *Type D* firms are only active in the domestic market and do not perform innovation; *Type DI* firms are those active only in the domestic market that innovate; *Type X* firms are those active in both the domestic and the foreign market that do not perform any innovation; and *Type XI* firms are active in the domestic and foreign markets and engage in innovation activities. Finally, in the third stage, firms choose prices. I solve the firms problem through backward induction.

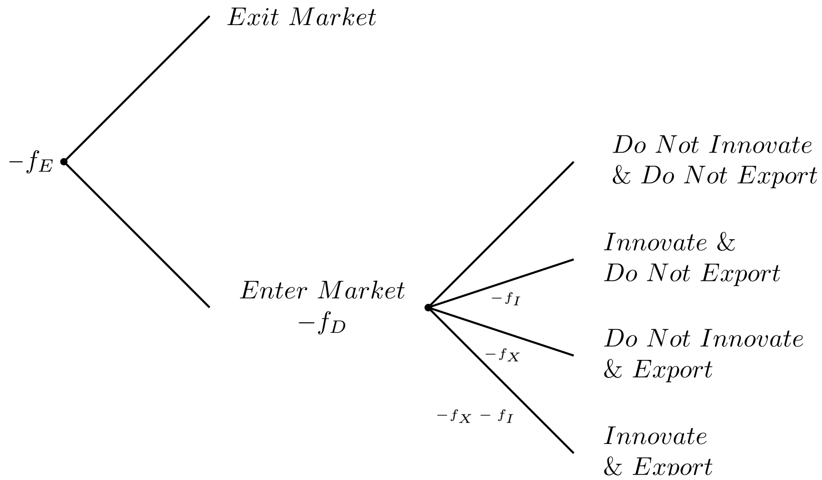


Figure 1.2: Timing in the Open Economy

OPTIMAL PRICING RULE. In the last stage of the problem the firm sets its optimal price, given its innovation decision and the market conditions which are summarized by the price index P_j and R_j.

$$\max_{p_{ij}(\varphi)} p_{ij}(\varphi)\, q_{ij}(\varphi) - f_{ij} - \frac{\tau_{ij} q_{ij}(\varphi)}{\varphi\left[(1+z_i(\varphi))^{\frac{1}{\sigma-1}}\right]} - c\,(z_i(\varphi))$$

The corresponding first order condition is

$$p_{ij}(\varphi) = \left(\frac{\sigma}{\sigma-1}\right)\frac{\tau_{ij}}{\varphi} \cdot \frac{1}{(1+z_i(\varphi))^{\frac{1}{\sigma-1}}} \quad \forall\, z_i(\varphi). \tag{1.8}$$

OPTIMAL INNOVATION DECISION. The returns of process innovation increase with the participation in more countries. Thus, the optimal innovation rule for firm i is obtained from the first order condition of the maximization of $\sum_j \pi_{ij}(\varphi) = \sum_j \left[p_{ij}(\varphi)q_{ij}(\varphi) - l_{ij}(\varphi)\right]$ with respect to $z_i(\varphi)$, provided that the firm makes higher profits by innovating than by choosing not to innovate. This gives

$$z_i(\varphi) = \begin{cases} \left[\left(1 + \sum_{j \neq i} \tau_{ij}^{1-\sigma}\right)\left(\frac{1}{\alpha+1}\right)\frac{p_{ii}(\varphi)q_{ii}(\varphi)}{\sigma}\right]^{\frac{1}{\alpha}} & \text{if } \sum_j \pi_{ij}^{I}(\varphi) \geqslant \sum_j \pi_{ij}^{NI}(\varphi), \\ 0 & \text{if } \sum_j \pi_{ij}^{I}(\varphi) < \sum_j \pi_{ij}^{NI}(\varphi). \end{cases} \tag{1.9}$$

Notice that the intensity of process innovation increases with the participation in foreign markets as can be seen by comparing Equation Equation 1.3 and Equation 1.9.

To make the joint decision of whether to enter the foreign markets and whether to innovate or not, firms will choose the option that yields the highest profits. Since countries are symmetric I can drop the subscripts and classify firms in four types.

- Profits of a domestic non-innovator firm (Type D):

$$\pi_D = \frac{R\,(P\rho)^{\sigma-1}}{\sigma}\varphi^{\sigma-1} - f_D.$$

- Profits of a domestic innovator firm (Type DI):

$$\pi_{DI} = \frac{R\,(P\rho)^{\sigma-1}}{\sigma}\varphi^{\sigma-1}\left(1 + z_D\left(\varphi\right)\right) - f_D - c\left(z_D\left(\varphi\right)\right).$$

- Profits of an exporter non-innovator firm (Type X):

$$\pi_X = \left(1 + n\tau^{1-\sigma}\right)\frac{R\,(P\rho)^{\sigma-1}}{\sigma}\varphi^{\sigma-1} - nf_X - f_D.$$

- Profits of an exporter innovator firm (Type XI):

$$\pi_{XI} = \left(1 + n\tau^{1-\sigma}\right)\frac{R\,(P\rho)^{\sigma-1}}{\sigma}\varphi^{\sigma-1}\left(1 + z_X\left(\varphi\right)\right) - nf_X - f_D - c\left(z_X\left(\varphi\right)\right).$$

where $f_D = f_{ii}$, $f_X = f_{ij} = f_{ji}$ for all $j \neq i$, $z_D(\varphi) = \left[\frac{1}{\alpha+1}\left(\frac{R(P\rho)^{\sigma-1}}{\sigma}\right)\varphi^{\sigma-1}\right]^{\frac{1}{\alpha}}$, and

$$z_X(\varphi) = \left[1 + n\tau^{1-\sigma}\right]^{\frac{1}{\alpha}} \left[\frac{1}{\alpha+1}\left(\frac{R(P\rho)^{\sigma-1}}{\sigma}\right)\varphi^{\sigma-1}\right]^{\frac{1}{\alpha}}.$$

1.3.2 *Equilibrium*

There will be three different equilibria that will cover the whole parameter space. First, the *low-cost innovation equilibrium*, where the activity of exporting is relatively costly in comparison to innovation, and therefore only the most productive firms will carry out both activities, middle productivity firms will innovate but not export, and the lower productivity firms will neither innovate nor export. Second, the *low-cost trade equilibrium*, where the activity of innovation is relatively costly in comparison to exporting, and therefore only the most productive firms will carry out both activities, middle productivity firms will export but not engage in innovation and the lower productivity firms will neither innovate nor export. Thirdly, between these two equilibria there will be the *intermediate equilibrium* where firms are either very productive and can undertake both activities or do not perform any of them.

The existence of these three equilibria is consistent with the empirical evidence found both in the trade and the innovation literature. Costantini and Melitz (2008) suggest that exporting and innovation are performed by the most productive firms while domestic producers are typically less innovative and less productive, a feature common to all the equilibria. Vives (2008) provides intuition for the decisions taken by middle productivity firms in each equilibrium. If trade costs are relatively high, middle productivity firms are domestic innovators because being an exporter without innovating is not profitable. A decrease in trade costs attracts the most productive firms from the foreign country, discouraging middle productivity domestic firms to undertake innovation. The disappearance of domestic innovators as trade costs fall can be explained by this Schumpeterian effect and is also predicted by the dynamic model of Costantini and Melitz (2008). However, a fall in trade costs enables more firms to participate actively in both markets which explains the existence of exporter non-innovators when trade costs are low enough.

Different theoretical papers have identified these equilibria separately, but never all in a single model. Bustos (2011) identifies the equilibrium where there are no domestic innovators firms since it is an unprofitable choice. In Vannoorenberghe (2008) all firms innovate, therefore it is not possible to study the interaction between both decisions. Finally, Navas-Ruiz and Sala (2007) identify the two extreme equilibria, but fail to identify the intermediate equilibrium. The main contribution of the theoretical model is the identification of all the equilibria with the ability to study the transitions between them and the possible productivity gains that might occur through the intensive and extensive margins of innovation. In the numerical section I will analyze whether these different equilibria are relevant when calibrating the model to different European countries. In what follows I describe each of the equilibria, the parameter restrictions that give rise to the different equilibria, and conclude by focusing on the effect that trade has on innovation.

1.3.2.1 *Low Cost Innovation Equilibrium*

The *low cost innovation equilibrium* is characterized by exporting being less attractive than innovation.

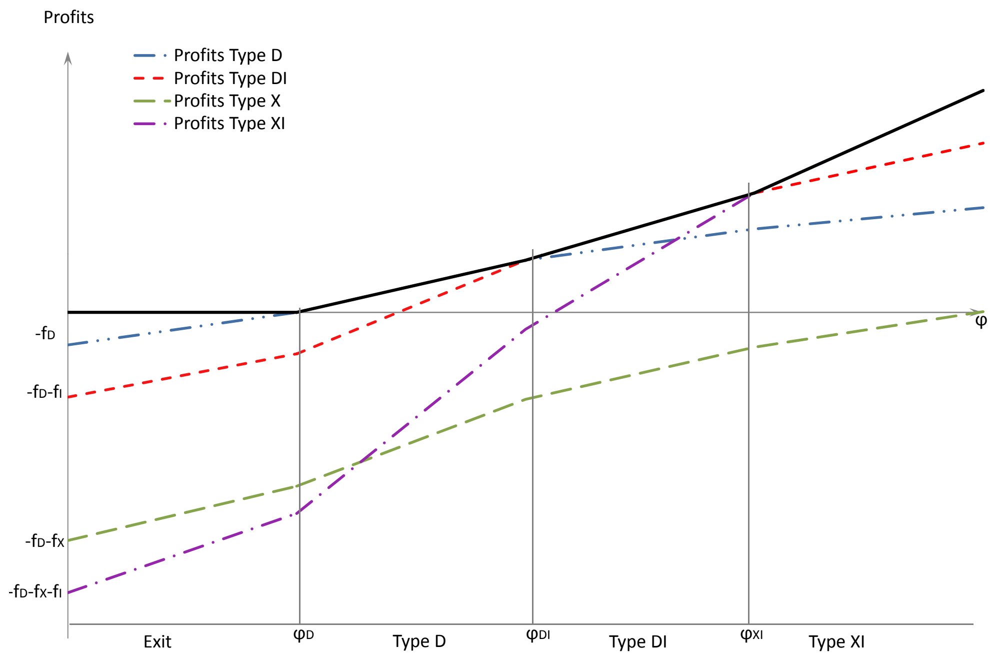


Figure 1.3: Low Cost Innovation Selection Path

In Figure 1.3, I depict the profits of all types of firms as a function of productivity when trade costs are relatively high in comparison to innovation costs. The envelope line shows the type of firm that will be chosen by a firm with productivity φ as it maximizes profits. In this equilibrium, the least productive firms $(\varphi < \varphi_D)$ exit, the low productivity firms $(\varphi_D < \varphi < \varphi_{DI})$ are active in the domestic market but do not innovate or export, middle productivity firms $(\varphi_{DI} < \varphi < \varphi_{XI})$ are active only on the domestic market but innovate, and the most productive firms $(\varphi > \varphi_{XI})$ are active both in the domestic and export market, and innovate. Note that there is no range of productivity level where exporting without innovating is profitable, that is, the marginal exporter is an innovator as well.

The conditions of entry in the domestic and export markets plus the innovation condition allow to solve for the different productivity cutoffs in the *low cost innovation equilibrium.*

The Zero Profit Condition (ZPC) in the domestic market is $\pi_D\left(\varphi_D^*\right) = 0$, so that:

$$(\varphi_D^*)^{\sigma-1} = \frac{f_D}{\left(\frac{R(P\rho)^{\sigma-1}}{\sigma}\right)}. \tag{1.10}$$

The Innovation Profit Condition (IPC) determines the productivity cutoff φ_{DI}^* which is the productivity of the firm indifferent between innovating or not while operating only on the domestic market, i.e. $\pi_{DI}\left(\varphi_{DI}^*\right) = \pi_D\left(\varphi_{DI}^*\right)$, so that:

$$(\varphi_{DI}^*)^{\sigma-1} = \frac{\left(\frac{f_I}{\alpha}\right)^{\frac{\alpha}{\alpha+1}}(\alpha+1)}{\left(\frac{R(P\rho)^{\sigma-1}}{\sigma}\right)}. \tag{1.11}$$

The Innovation Export Profit Condition (IXPC) determines the exporting innovation cutoff φ_{XI}^* which is the productivity of an innovating firm indifferent between participating also on the exporting market or not:

$$\pi_{XI}(\varphi_{XI}) - \pi_{DI}(\varphi_{XI}) = 0. \tag{1.12}$$

The following proposition shows for which part of the parameter space the *low cost innovation equilibrium* exists.

Proposition 2. *The economy is in the low cost innovation equilibrium,* $\varphi^*_{XI} > \varphi^*_{DI} > \varphi^*_D$*, if the following parameter restrictions hold*

1. $\tau^{\sigma-1} f_X \geqslant \frac{\left[\left(1+n\tau^{1-\sigma}\right)^{\frac{\alpha+1}{\alpha}}-1\right]}{n\tau^{1-\sigma}} f_I + \left(\frac{f_I}{\alpha}\right)^{\frac{\alpha}{\alpha+1}} (\alpha+1)$

2. $\left(\frac{f_I}{\alpha}\right)^{\frac{\alpha}{\alpha+1}} (\alpha+1) \geqslant f_D$

Proof. The formal proof can be found in the Appendix A.2.1. The proof is divided in two parts. First I show that there exist a single solution to Equation 1.12. The non linearity present in the optimal innovation decision is the source of the complexity of finding a closed form for the cutoff φ^*_{XI}. Nevertheless, I show that selection into exporting and innovation ($\varphi^*_{XI} > \varphi^*_{DI}$) requires that condition 1 of Proposition 2 holds, that is exporting costs should be high enough relative to innovation costs. Notice that condition 2 of Proposition 2 ensures that there is selection into innovation ($\varphi^*_{DI} > \varphi^*_D$). Secondly, I show that equations Equation 1.10 to Equation 1.12 along with the Free Entry (FE) condition, which requires that the sunk entry cost equals the present value of expected profits:

$$\frac{1}{\delta}\left[\int_{\varphi^*_D}^{\varphi^*_{DI}} \pi_D(\varphi)\, dG(\varphi) + \int_{\varphi^*_{DI}}^{\varphi^*_{XI}} \pi_{DI}(\varphi)\, dG(\varphi) + \int_{\varphi^*_{XI}}^{\infty} \pi_{XI}(\varphi)\, dG(\varphi)\right] = f_E \qquad (1.13)$$

uniquely determine the equilibrium price (P), the number of firms (M), and the distribution of active firms productivity in the economy along with the productivity cutoffs φ^*_D, φ^*_{DI} and φ^*_{XI}. □

1.3.2.2 *Low Cost Trade Equilibrium*

The *low cost trade equilibrium* is characterized by exporting being more attractive than innovation. In Figure 1.4 , I depict the profits of all types of firms as a function of productivity when trade costs are relatively low in comparison to innovation costs. The envelope line shows the type of firm that will be chosen by a firm with productivity φ as it maximizes profits. In this equilibrium, the least productive firms $(\varphi < \varphi_D)$ exit, the low productivity firms $(\varphi_D < \varphi < \varphi_{DI})$ are active in the domestic market but do not innovate or export, middle productivity firms $(\varphi_{DI} < \varphi < \varphi_{XI})$ are active only on the domestic market but innovate, and the most productive firms $(\varphi > \varphi_{XI})$ are active both in the domestic and export market, and innovate. Note that there is no range of productivity level where innovation without exporting is profitable, that is, the marginal innovator is an exporter.

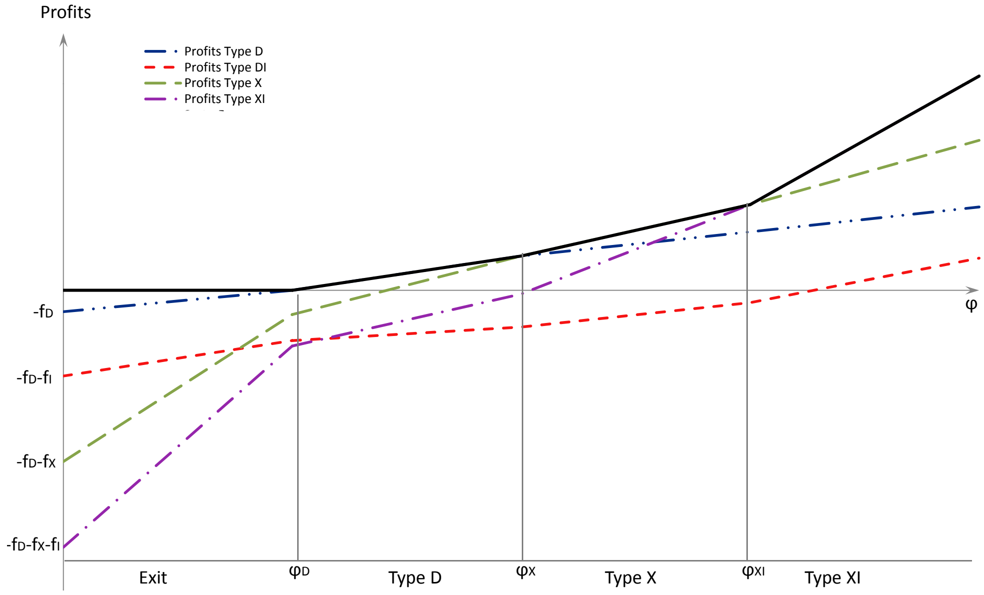


Figure 1.4: Low Cost Trade Selection Path

The conditions of entry in the domestic and export markets, plus the innovation conditions, allow to solve the different productivity cutoffs in the *low cost trade equilibrium.*

The Zero Profit Condition (ZPC) in the domestic market[6] is $\pi_D\left(\varphi_D^*\right) = 0$ so that:

$$(\varphi_D^*)^{\sigma-1} = \frac{f_D}{\left(\frac{R(P\rho)^{\sigma-1}}{\sigma}\right)}. \tag{1.14}$$

The Exporting Profit Condition (XPC) determines the exporting-entry productivity cutoff φ_X^* which is the productivity of the firm indifferent between staying in the domestic market and participating in the export market i.e. $\pi_X\left(\varphi_X^*\right) = \pi_D\left(\varphi_X^*\right)$:

$$(\varphi_X^*)^{\sigma-1} = \frac{f_X}{\left(\frac{R(P\rho)^{\sigma-1}}{\sigma}\right)\tau^{1-\sigma}}. \tag{1.15}$$

6 The ZPC condition is defined theoretically in the same way in every equilibrium. However, since the aggregates in each situation are different, the entry cutoff will also be different.

The Exporting Innovation Profit Condition (XIPC) determines the innovation exporting productivity cutoff φ^*_{XI}, which is the productivity of an exporting firm indifferent between innovating or not, i.e. $\pi_{XI}\left(\varphi^*_{XI}\right) = \pi_X\left(\varphi^*_{XI}\right)$:

$$(\varphi^*_{XI})^{\sigma-1} = \frac{\left(\frac{f_I}{\alpha}\right)^{\frac{\alpha}{\alpha+1}}(\alpha+1)}{\left(\frac{R(P\rho)^{\sigma-1}}{\sigma}\right)(1+n\tau^{1-\sigma})}. \tag{1.16}$$

The following proposition shows for which part of the parameter space the *low cost trade equilibrium* exists.

Proposition 3. *The economy is in the low cost trade equilibrium,* $\varphi^*_{XI} > \varphi^*_X > \varphi^*_D$*, if the following parameter restrictions hold*

$$\frac{\left(\frac{f_I}{\alpha}\right)^{\frac{\alpha}{\alpha+1}}(\alpha+1)}{(1+n\tau^{1-\sigma})} \geqslant \tau^{\sigma-1} f_X \geqslant f_D$$

Proof. Selection into exporting and innovation ($\varphi^*_{XI} > \varphi^*_X$) requires innovation costs to be high enough relative to trade costs and selection into exporting ($\varphi^*_X > \varphi^*_D$) requires trade costs to be high enough relative to production costs. Equation 1.14 to Equation 1.16 along with the Free Entry (FE) condition, which requires that the sunk entry cost equals the present value of expected profits:

$$\frac{1}{\delta}\left[\int_{\varphi^*_D}^{\varphi^*_X} \pi_D(\varphi)\, dG(\varphi) + \int_{\varphi^*_X}^{\varphi^*_{XI}} \pi_X(\varphi)\, dG(\varphi) + \int_{\varphi^*_{XI}}^{\infty} \pi_{XI}(\varphi)\, dG(\varphi)\right] = f_E \tag{1.17}$$

uniquely determine the equilibrium price (P), the number of firms (M), and the distribution of active firms productivity in the economy along with the productivity cutoffs φ^*_{XI}, φ^*_X, and φ^*_D. See Appendix A.2.2 for a formal proof. □

1.3.2.3 *Intermediate Equilibrium*

The *intermediate equilibrium* is characterized by exporting and innovation being relatively equally attractive. In Figure 1.5, I depict the profits of all types of firms as a function of productivity when trade costs are neither very high nor very low in comparison to innovation costs. The envelope line shows the type of firm that will be chosen by a firm with productivity φ as it maximizes profits. In this equilibrium, the least productive firms ($\varphi < \varphi_D$) exit, the low productivity firms ($\varphi_D < \varphi < \varphi_{XI}$) are active in the domestic market but do not innovate or export, and the most productive firms ($\varphi > \varphi_{XI}$) are active both in the domestic and export market, and innovate. Note that there is no range of productivity level where exporting without innovating or innovating without exporting is profitable, that is, the marginal exporter is an innovator as well.

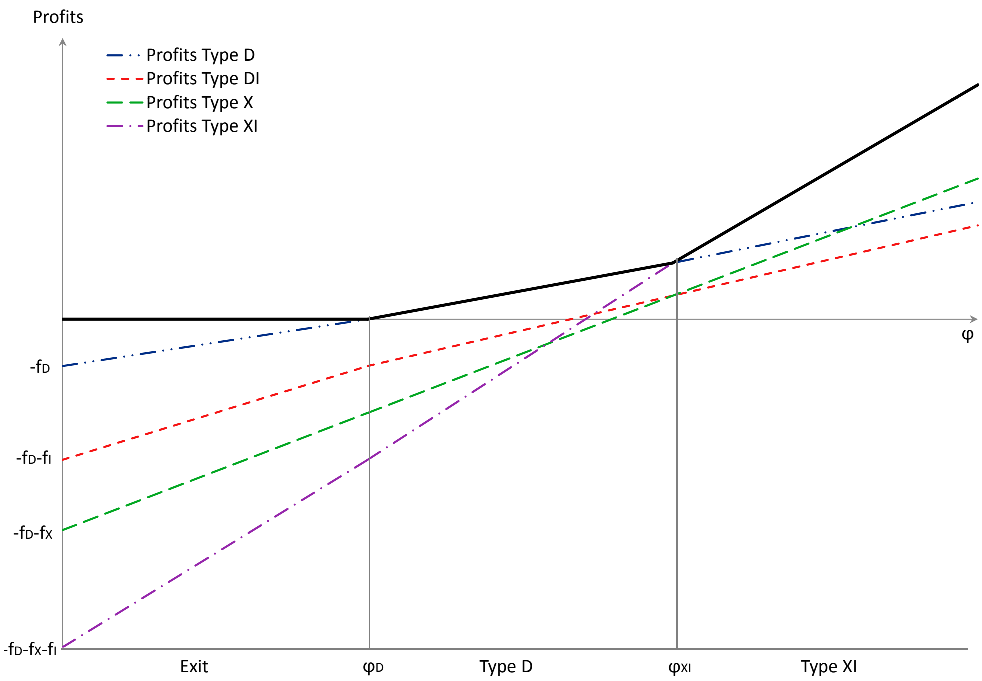


Figure 1.5: Intermediate Selection Path

The conditions of entry in the domestic markets, plus the innovation and export condition, allow to solve the different productivity cutoffs in the *intermediate equilibrium.*

The Zero Profit Condition (ZPC) in the domestic market[7] is $\pi_D\left(\varphi_D^*\right) = 0$ so that:

$$(\varphi_D^*)^{\sigma-1} = \frac{f_D}{\left(\frac{R(P\rho)^{\sigma-1}}{\sigma}\right)}. \tag{1.18}$$

7 The ZPC condition is defined theoretically in the same way in every equilibrium. However, since the aggregates in each situation are different, the entry cutoff will also be different.

The Exporting Innovation Profit Condition (XIPC) determines the innovation exporting productivity cutoff φ^*_{XI}, which is the productivity of a firm indifferent between exporting and innovating or not.

$$\pi_{XI}\left(\varphi^*_{XI}\right) - \pi_D\left(\varphi^*_{XI}\right) = 0 \tag{1.19}$$

The following proposition shows for which part of the parameter space the *intermediate equilibrium* exists.

Proposition 4. *The economy is in the intermediate equilibrium, $\varphi^*_{XI} > \varphi^*_D$, if the following parameter restrictions hold*

1. $\dfrac{\left[\left(1+n\tau^{1-\sigma}\right)^{\frac{\alpha+1}{\alpha}}-1\right]}{n\tau^{1-\sigma}} f_I + \left(\dfrac{f_I}{\alpha}\right)^{\frac{\alpha}{\alpha+1}}(\alpha+1) \geqslant \tau^{\sigma-1} f_X$
2. $\tau^{\sigma-1} f_X \geqslant \dfrac{\left(\frac{f_I}{\alpha}\right)^{\frac{\alpha}{\alpha+1}}(\alpha+1)}{\left(1+n\tau^{1-\sigma}\right)}$
3. $\dfrac{\left(\frac{f_I}{\alpha}\right)^{\frac{\alpha}{\alpha+1}}(\alpha+1)}{\left(1+n\tau^{1-\sigma}\right)} \geqslant f_D$

Proof. If the first parameter restriction does not hold, then for some firms is profitable to innovate without exporting. If the second parameter restriction does not hold, then for some firms is profitable to export without innovating. Therefore, the trade costs must be in between the limits of innovation, so that firms either export and innovate or simply remain in the domestic market. The non linearity present in the optimal innovation decision is the source of the complexity of finding a closed form for the cutoff φ^*_{XI}, nevertheless I show that conditions 1 and 2 hold. Furthermore, I show that Equation 1.18 and Equation 1.19 along with the Free Entry (FE) condition, which requires that the sunk entry cost equals the present value of expected profits:

$$\frac{1}{\delta}\left[\int_{\varphi^*_D}^{\varphi^*_{XI}} \pi_D(\varphi)\, dG(\varphi) + \int_{\varphi^*_{XI}}^{\infty} \pi_{XI}(\varphi)\, dG(\varphi)\right] = f_E \tag{1.20}$$

uniquely determine the equilibrium price (P), the number of firms (M), and the distribution of active firms productivity in the economy along with the productivity cutoffs φ^*_{XI}, and φ^*_D. See Appendix A.2.3 for a formal proof. □

1.3.3 *Trade, Innovation and Aggregate Productivity*

Trade has indirect effects on the average productivity through innovation. Moving from the *low cost innovation equilibrium* to the *low cost trade equilibrium*, the cost of exporting relative to the cost of innovation decreases, therefore the effect trade has on innovation will be differentiated according to the level of transportation costs.

low cost innovation equilibrium.

Combine Equation 1.10 and Equation 1.11, the relation between the cutoffs can be written explicitly as:

$$(\varphi_{DI}^{*})^{\sigma-1} = \frac{\left(\frac{f_I}{\alpha}\right)^{\frac{\alpha}{\alpha+1}}(\alpha+1)}{f_D}(\varphi_{D}^{*})^{\sigma-1} = \Lambda\,(\varphi_{D}^{*})^{\sigma-1}\,. \tag{1.21}$$

The relationship between the productivity cutoffs of domestic innovators and domestic non innovators is the same as in Autarky (see Equation 1.6), however in the open economy there are indirect effects via the input factor market that will impact the entry cutoff and therefore in the domestic innovation cutoff. In the *the low cost innovation equilibrium* survival becomes tougher, the presence of foreign firms pushes up real wages and it is harder for lower productivity firms to earn positive profits, hence the least productive firms exit the economy $\left(\varphi_D^{LCI} > \varphi_D^{AUT}\right)$ and firms need to be more productive in order to undertake innovation $\left(\varphi_{DI}^{LCI} > \varphi_{DI}^{AUT}\right)$.

Therefore, in the *low cost innovation equilibrium* the number of firms that perform innovation in the economy is reduced with respect to the Autarky case. Nevertheless, we cannot say anything on the amount of process innovation, since it could be possible that the increase in innovation by the firms who export make up for the loss of the firms who do not innovate anymore and the domestic firms who innovate less intensively than before. In other words, gains along the intensive margin of innovation might offset the loss along the extensive margin of innovation.

low cost trade equilibrium.

Combine Equation 1.14 to Equation 1.16, the relation between the cutoffs can be written explicitly as:

$$(\varphi_{XI})^{\sigma-1} = \frac{\left(\frac{f_I}{\alpha}\right)^{\frac{\alpha}{\alpha+1}}(\alpha+1)}{(1+\tau^{1-\sigma})}\frac{(\varphi_D)^{\sigma-1}}{f_D} = \Lambda^t(\varphi_D)^{\sigma-1} \quad (1.22)$$

$$(\varphi_{XI})^{\sigma-1} = \left[\left(\frac{f_I}{\alpha}\right)^{\frac{\alpha}{\alpha+1}}(\alpha+1)\right]\frac{\tau^{1-\sigma}}{(1+\tau^{1-\sigma})}\frac{(\varphi_X)^{\sigma-1}}{f_X} = \Lambda^t_X(\varphi_X)^{\sigma-1} \quad (1.23)$$

$$(\varphi_X)^{\sigma-1} = \frac{f_X\tau^{\sigma-1}}{f_D}(\varphi_D)^{\sigma-1} = \frac{\Lambda^t}{\Lambda^t_X}(\varphi_D)^{\sigma-1} \quad (1.24)$$

where $\Lambda^t_X = \frac{f_X\tau^{\sigma-1}}{f_D}\Lambda^t$, $\Lambda^t = \frac{\Lambda}{(1+\tau^{1-\sigma})}$ and Λ is the relationship between the innovation and the entry cutoff in Autarky (Equation 1.6) and in the *low cost innovation equilibrium* (Equation 1.21) .

In the open economy there are indirect effects via the general equilibrium that will impact the entry cutoff and therefore the innovation cutoff. Survival becomes tougher, the presence of foreign firms pushes up real wages and it is harder for lower productivity firms to earn positive profits. Hence the least productive firms exit the economy $(\varphi^{LCT}_D > \varphi^{AUT}_D)$ and firms need to be more productive in order to undertake innovation (φ^{LCT}_{XI} is pushed upwards via general equilibrium). However, there is an effect via artial equilibrium on the innovation cutoff int he opposite direction. It follows that $\Lambda \geqslant \Lambda^t$, which means that the cost-to benefit ratio is smaller in the *low cost trade equilibrium* than in Autarky or in the *low cost innovation equilibrium*. This difference is numerically given by $\frac{1}{(1+\tau^{1-\sigma})}$ whose denominator indicates the further revenue differential associated to innovation on the foreign market available through trade. Economically, since exporters expand their scales of operation, the variable cost and benefits of the productivity enhancing innovation performed are spread on more units while the up-front cost of innovation is unchanged, creating the difference we are talking about. The comparison of Equation 1.6 and Equation 1.22 shows that trade decreases (*ceteris paribus*) the innovation productivity cutoff boosting within-plant innovation (φ^{LCT}_{XI} is pushed downwards via partial equilibrium).

In the *low cost trade equilibrium*, trade has a positive impact in the intensive margin of innovation. Moreover, if the partial equilibrium effect offsets the general equilibrium effect, then trade also has a positive impact in the extensive margin of innovation.

Proposition 5. *In the low cost trade equilibrium, if productivity draws are distributed according to a Pareto distribution, then the proportion of firms doing innovation activities rises with respect to Autarky* $(\varphi_{DI}^{AUT} > \varphi_{XI}^{LCT})$.

Proof. See Appendix A.3 for the formal proof. □

1.4 DISCUSSION

The firm productivity distribution varies along the parameter space according to the relation between trade costs and the relative innovation costs. This is especially relevant for firms with an intermediate level of productivity, as their decisions will be most sensitive to these costs. In particular, in the *low cost innovation equilibrium*, when trade costs are high enough, they are domestic innovators. In the *low cost trade equilibrium*, when trade costs are low enough in relation to innovation costs, middle productivity firms will be exporters, and the most productive of them will export, and innovate. In between these two equilibria, there is the *intermediate equilibrium*, where trade costs are not relatively high enough for firms to be domestic innovators nor low enough for firms to be exporters non-innovators. That is, middle productivity firms are either exporter innovators or domestic firms. These choices are the ones that determine the parameter restrictions associated to each equilibrium. Furthermore, notice that the three equilibria cover the whole parameter space, and therefore the firm productivity distribution and the effects of opening up to trade of an economy can be always determined. Table 1.1 summarizes all the possible equilibria in the open economy and the parameter restrictions associated to each one.

The model has implications for the aggregate productivity level. Firstly, trade induces the exit of the less productive firms and the reallocation of market shares towards the more productive firms, raising the industry average productivity in the long run. This is the selection effect described in Melitz (2003). And secondly, trade has indirect effects on the average productivity through innovation. Moving from the *low cost innovation equilibrium* to the *low cost trade equilibrium*, the cost of exporting relative to the cost of innovation decreases, therefore the effect trade has on innovation will be differentiated according to the level of transportation costs. On the one hand, there is an effect through the intensive margin of innovation. The innovation intensity increases with the participation in foreign markets, and thus the effect will be larger in the *low cost trade equilibrium* where the economy is more open, followed by the *intermediate equilibrium*. In the *low cost innovation equilibrium* such effect is undetermined, since there is a positive effect from the exporters being more innovative but a negative effect from the domestic firms that innovate being less innovative. On the other hand, there is an effect through the extensive margin of innovation. In the section before was shown that the impact on average productivity through the extensive margin will be negative in the *low cost innovation equilibrium*, undetermined in the *intermediate equilibrium*, and can be positive in the *low cost trade equilibrium*.

Equilibrium	Conditions
Low Cost Innovation Equilibrium	$\tau^{\sigma-1} f_X \geqslant \frac{\left[\left(1+n\tau^{1-\sigma}\right)^{\frac{\alpha+1}{\alpha}}-1\right]}{n\tau^{1-\sigma}} f_I + \left(\frac{f_I}{\alpha}\right)^{\frac{\alpha}{\alpha+1}} (\alpha+1)$ & $\left(\frac{f_I}{\alpha}\right)^{\frac{\alpha}{\alpha+1}} (\alpha+1) \geqslant f_D$
Intermediate Equilibrium	$\frac{\left[\left(1+n\tau^{1-\sigma}\right)^{\frac{\alpha+1}{\alpha}}-1\right]}{n\tau^{1-\sigma}} f_I + \left(\frac{f_I}{\alpha}\right)^{\frac{\alpha}{\alpha+1}} (\alpha+1) \geqslant \tau^{\sigma-1} f_X$ & $\tau^{\sigma-1} f_X \geqslant \frac{\left(\frac{f_I}{\alpha}\right)^{\frac{\alpha}{\alpha+1}} (\alpha+1)}{(1+n\tau^{1-\sigma})} \geqslant f_D$
Low Cost Trade Equilibrium	$\frac{\left(\frac{f_I}{\alpha}\right)^{\frac{\alpha}{\alpha+1}} (\alpha+1)}{(1+n\tau^{1-\sigma})} \geqslant \tau^{\sigma-1} f_X \geqslant f_D$

Table 1.1: Equilibria in the Open Economy

1.5 CONCLUSIONS

This paper has proposed a trade model with heterogeneous firms that decide not just whether or how much to export but also whether or how much to innovate. By incorporating the extensive and intensive margins of trade and innovation, three equilibria may arise. In all equilibria high-productivity firms export and innovate, whereas low-productivity neither export nor innovate. What differs across equilibria is the behavior of medium-productivity firms. In an economy with trade costs that are low relative to innovation costs, medium-productivity firms export without innovating, whereas in an economy with trade costs that are high relative to innovation costs, medium-productivity firms innovate without exporting. In a third equilibrium, in between the other two, some medium-productivity firms export and innovate, whereas others do neither.

Trade has indirect effects on the aggregate productivity through innovation. Moving from the *low cost innovation equilibrium* to the *low cost trade equilibrium*, the cost of exporting relative to the cost of innovation decreases, therefore the effect a trade policy has on innovation will be differentiated according to the level of transportation costs. On the one hand, there is an effect through the intensive margin of innovation. As long as the marginal innovator is an exporter (*low cost trade equilibrium* and *intermediate equilibrium*), the effect of a trade policy will always be positive. If the marginal innovator innovator is not an exporter (*low cost innovation equilibrium*), while a trade policy will induce exporters to be more innovative, the non-exporters will be less innovative. On the other hand, there is an effect through the extensive margin of innovation. The impact of a trade policy on aggregate productivity through the extensive margin of innovation will be negative in the *low cost innovation equilibrium*, undetermined in the *intermediate equilibrium* and can be positive in the *low cost trade equilibrium*.

These findings stress the importance of having a model that jointly analyzes the extensive and intensive margins of both trade and innovation. Not doing so would not just result in a less rich theoretical structure, it would also keep us from correctly assessing the impact of different policies aimed at fomenting trade and innovation.

Of course, this model has abstracted from a number of potentially relevant features that go beyond the scope of this paper. First, I have exclusively focused on a steady state environment, thus ignoring the transition dynamics. As shown by Alessandria and Choi

(2011) and Burstein and Melitz (2011), not taking into account transition dynamics may significantly impact the welfare effects of trade liberalization. Second, the model does not consider uncertainty in innovation. While most of the literature on trade and innovation assumes there is no risk involved,[8] the empirical evidence suggests otherwise: there is risk that an innovator will not identify important needs, that innovation teams disrupt the regular operations of a business, or that even a promising idea is not accepted by the customers whose need it was meant to address. Third, I have assumed that there is no strategic interaction between firms and therefore the innovation activities of one firm do not have any influence in the innovation activities of the other firms. The existence of externalities in process innovation could have a significant effect on the results. Fourth, the model could be used to analyze the effect of joint trade and innovation policies. The right mix of policies could lead to greater gains in aggregate productivity.

8 An exception is Atkeson and Burstein (2010) who introduce uncertainty in the outcome of the investment in process innovation, although firms always get some returns (no innovation fails).

2

TRADE, INNOVATION AND PRODUCTIVITY: A QUANTITATIVE ANALYSIS OF EUROPE

Abstract. This paper evaluates quantitatively the model of trade and innovation proposed in Chapter 1. After calibrating the model to five European countries, I show that the different equilibria are plausible, and provide quantitative evidence that supports the predictions of my theory. The impact of trade on aggregate productivity and welfare depends crucially on the equilibrium the economy is in. When lowering the variable costs of trade, the welfare effects arising from reallocating market shares across firms may be non-negligible, and when lowering the fixed cost of trade, aggregate productivity need not always increase.

2.1 INTRODUCTION

Chapter 1 provides a framework in which to analyze the impact of trade liberalization on productivity and welfare in a model that incorporates both the extensive and the intensive margins of both trade and innovation.

The aim of the paper is to show that introducing these different margins is key for understanding the impact of trade liberalization. Different equilibria may arise, depending on the relative costs of trade and innovation. I show that they are quantitatively plausible by calibrating the model to five European countries. I then show that the impact of trade liberalization depends crucially on the equilibrium the economy is in and the nature of the liberalization. For example, in the case of a drop in variable trade costs, this paper shows that the effects on welfare from changes in firms' decisions to export and innovate may be non-negligible, in contrast to the literature.[1] As another example, a drop in the fixed cost of trade need not always have a positive effect on aggregate productivity. Indeed, in an economy in which many firms export, but few firms innovate, lowering the fixed cost of trade, by increasing the number of exporters, may make innovating more expensive, thus lowering aggregate productivity.

To assess the plausibility of the theory in Chapter 1, I calibrate the model to five European countries. In particular, the model is calibrated to match a number of salient features of innovation, firm size distribution, and international trade in France, Germany, Italy, Spain, and United Kingdom, using the firm-level data set *European Firms in a Global Economy* (EFIGE). The survey, conducted during the year 2009, is representative of the manufacturing sector in each country. Especially relevant for my analysis is the information on employment, internationalization, and innovation. A first result is that the different equilibria are not only theoretically relevant, but also empirically plausible: different countries are in different equilibria. This is important, since the theory predicts that the effect of trade liberalization on aggregate productivity and welfare depends crucially on the equilibrium a country is in.

A first quantitative exercise consists of quantifying the effect of a reduction in variable trade costs on aggregate productivity. The analysis is based on the *ideal measure* of aggregate productivity defined by Atkeson and Burstein (2010). I focus on this measure, because it captures the productivity that is relevant for welfare. Apart from the direct cost

1 See Arkolakis et al. (2012) and Atkeson and Burstein (2010) on this topic.

savings effects of a drop in variable trade costs, the theory predicts that there are a number of indirect effects. First, it induces the exit of less productive firms and the reallocation of market shares towards the more productive firms. This is the selection effect described in Melitz (2003). Second, the innovation intensity increases with the participation in foreign markets, so the effect through the intensive margin of innovation should be positive.[2] Third, the theory predicts that the effect through the extensive margin of innovation can be positive or negative. In the *low cost trade equilibrium* and the *intermediate equilibrium*, all innovators are exporting. In this case a decrease in variable trade costs increases the incentives to be an exporter (and to be an exporter innovator), so that the effect through the extensive margin of innovation is positive. In contrast, in the *low cost innovation equilibrium*, some of the innovators do not export. In this case, a drop in trade costs makes it harder for domestic firms to innovate, so that the effect through the extensive margin of innovation is negative.

My findings corroborate the theoretical predictions. In particular, in most countries the effect of a drop in variable trade costs on aggregate productivity through the extensive margin is positive, except in those that are in the *low cost innovation equilibrium*, where the effect is negative. My findings also shed new light on which channels matter when analyzing the impact of trade liberalization on aggregate productivity Atkeson and Burstein (2010) have suggested that the indirect effects of trade liberalization on productivity are negligible. That is, liberalizing trade improves productivity through the standard direct effect of saving resources on trade, whereas indirect effects coming from changes in firms' decisions related to exit, trade, and innovation are essentially zero. In contrast, my findings show that this depends crucially on the equilibrium an economy is in. While in most countries the indirect effects are indeed negligible, this is not the case of countries in the *low cost innovation equilibrium*. This underscores the importance of having a model that encompasses both the extensive and intensive margins of trade and innovation.

A second quantitative exercise focuses on the effectiveness of lower the fixed costs of trade or innovation to increase productivity . While my first exercise focused on a reduction in variable trade costs, I now show that a reduction in fixed trade or innovation

2 Despite the intensity of innovation from domestic firms decreasing (if there are in the economy), the increase on the intensity of innovation of exporter firms ensures that the final effect through the intensive margin is positive.

costs may also have very different effects depending on the equilibrium the economy is in. While in general the effect of lowering the fixed cost of trade is positive, I find that in the *low cost trade equilibrium* it is negative. The intuition is as follows. In such equilibrium, there are many exporters, but only the most productive innovate. Since all innovators are also exporters, by increasing the incentives to enter the export market, a drop in the fixed costs of trade pushes up real wages, reducing the incentives to innovate. As a result, both the number of innovators and the intensity of the remaining innovators decline, which translates in the final effect on welfare being negative.

The simulations reveal that a discrete drop in fixed trade costs, can induce productivity gains from 1% to 20% in total, and only if the economy is already very open (in the *low cost trade equilibrium*) might a further drop in fixed trade costs be damaging to the economy, which suggest that a fixed trade cost liberalization does not have the same nature than a variable trade cost liberalization. In contrast to a fixed trade cost reduction, a fixed innovation cost drop has little effect on productivity, the maximum increase being around 2%, and has far more damaging effects if it induces economies to be less export-oriented, since then the productivity might decrease by up to 7%.

A key contribution of my work is that it explores quantitatively the responses of firms along both the extensive and intensive margins of innovation to changes in the environment. My results echoe those of Atkeson and Burstein (2010) in that welfare gains from trade do not depend on how a change in variable trade costs affects firms' exit, export, and innovation decisions, if the extensive margin of innovation is not affected by the policy. At the same time, my result complements theirs by explaining carefully how a negative incentive to innovate, driven by a drop in variable trade costs, actually implies that firms' exit, export, and innovation decisions can have an impact on welfare gains.

My work here is also related to a large literature on the aggregate implications of trade liberalization. Baldwin and Robert-Nicoud (2008) study a variant of Melitz's model that features endogenous growth through spillovers. They show that depending on the nature of the spillovers, a reduction in international trade costs can increase or decrease growth through changes in product innovation. My model centers on process innovation and abstracts from such spillovers. Arkolakis et al. (2012) calculate the welfare gains from trade in a wide class of trade models, including Krugman (1980) and Melitz (2003) models with Pareto productivities. The main differences between this paper and mine is that they abstract from innovation and focus only on changes in marginal trade costs.

The chapter is organized as follows. In Section 2.2, I describe the dataset. In Section 2.3, I calibrate the model to match five main European economies. In Section 2.4, I analyze the effects in aggregate productivity and welfare of a drop in variable trade costs, a drop in fixed trade cost and a drop in fixed innovation costs. Section 2.5 concludes.

2.2 DATA

The data I analyze comes from the European Project EFIGE, where EFIGE stands for "European Firms in a Global Economy". The objective of this project is to examine the pattern of internationalization of European firms.

The data was collected in 2010 covers the years 2007 to 2009. The data consist of a representative sample at the country level for the manufacturing industry of firms owning establishments with more than ten employees in seven European economies: Austry, France, Germany, Hungary, Italy, Spain and United Kingdom. The distribution by firm size for the sample and the reference population are shown for each country in Table 2.1.[3]

	Between 10 and 49		Between 50 and 249		More than 250		Total	
Country	S	P	S	P	S	P	S	P
Austria	339	5,568	107	1,524	46	459	492	7,551
France	2,151	32,019	608	7,365	214	1,986	2,973	41,370
Germany	1,836	52,489	793	16,988	306	3,970	2,935	79,144
Hungary	325	6,505	118	1,874	45	460	488	8,839
Italy	2,447	77,092	429	10,062	145	1,408	3,021	88,562
Spain	2,280	38,116	406	6,241	146	1,010	2,832	45,367
U.K.	1,515	27,187	529	7,794	112	1,758	2,156	36,739

Table 2.1: Distribution by size, sample (S)/reference population(P)

3 The sample design over-represents large firms, therefore sampling weights have been constructed in terms of size-sector cells to make the sample representative of the underlying population.

The database, for the first time in Europe, combines measures of firms' international activities (e.g. exports, outsourcing, FDI, import) as well as quantitative and qualitative information on around 150 items ranging from R&D and innovation, labor organization, financing and organizational activities.[4]

The survey contains information on several dimensions of innovation and exporting. On the one hand, regarding export activities there are both qualitative and quantitative questions. Particularly I center in firms that are regular exporters, and how much did those export activities represent in their 2008 turnover. On the other hand, regarding innovation activities there is quite an extensive classification on the kind of innovation performed as well as several quantitative measures for it. Particularly, firms are asked if they did carry out during the years 2007 to 2009 process innovation, where process innovation is defined as the adoption of a production technology which is either new or significantly improved (the innovation should be new to the firm but the firm must not necessarily have to be the first to introduce this process). And finally, as a quantitative measure of innovation we use the number of employees involved in R&D activities, where R&D consist of creative activities aimed at increasing the knowledge and using this knowledge in new applications, such as in the development of technologically new or improved products and processes. I use this information to calibrate the model described in Chapter 1 and analyze quantitatively the relevance of the theoretical predictions.

2.3 Calibration

In this section I calibrate the model to match a number of salient features of innovation, the firm size distribution, and international trade in five European countries (France, Germany, Italy, Spain and United Kingdom), using firm-level data survey by the EFIGE project.[5]

Parameters common to all countries are taken directly from the empirical literature, while parameters specific to each country are calibrated such that particular firm-level moments in the model match those moments in the data. Parameters common to all

4 Altomonte and Aquilante (2012) provides more information on the construction of the dataset and a comprehensive set of validation measures that have been used to assess the comparability of the survey data with official statistics.

5 The model is not calibrated to the other two countries in the sample, Austria and Hungary, due to missing observations in the main variables used in the calibration.

countries are the elasticity of substitution, the elasticity of innovation, the probability of firm exit, and the sunk cost of entry. The elasticity of substitution is set to be consistent with empirical estimates provided by Broda and Weinstein (2006). The medians reported vary from 2.2 to 4.8 depending on the level of aggregation, thus I set $\sigma = 3.2$ which lies within the estimated range of values. A clear limitation of the data is the lack of a panel dimension. This affects the calibration of the probability of exit and the sunk cost entry, since the dataset does not have information on the entry and exit of firms. But more importantly, affects as well the calibration of the innovation parameter α, crucial in the determination of the equilibrium of an economy, since the elasticity of innovation across the European countries cannot be computed. Hence, these three parameters are taken as well as common to all the countries. The innovation parameter α is taken to be 0.9. This is consistent with the estimate of Rubini (2011), who sets the elasticity of productivity to resources devoted to innovation to match a 5% gain in labor productivity in Canada following the tariff reduction in the U.S.-Canada Free Trade Agreement between 1980 and 1996. The probability of exit and the sunk cost entry, which determine the entry and exit of firms, are set to $\delta = 0.05$ and $f_E = 1$ following Bernard et al. (2007).

Parameters specific to each country are innovation fixed costs (f_I), export fixed costs (f_X), variable trade costs (τ), domestic fixed costs (f_D), the productivity distribution, and the number of trading partners. The first four are calibrated jointly to match the number of workers in innovation, the percentage of exporters innovators in the economy, the ratio of exports to revenue, and the percentage of executives (including entrepreneurs and middle management) in the labor force. To match the productivity distribution, I target the slope of the firm size distribution in terms of employees, and similarly to Helpman et al. (2004) and Chaney (2008), I assume the productivity is distributed according to a Pareto with a probability density function

$$g(\varphi) = \frac{\theta}{\varphi^{\theta+1}},$$

where $\varphi \in [1, \infty)$ and θ is the curvature parameter. In accordance to the model considered, I estimate by maximum likelihood the curvature parameter associated to the distribution of firms, $\tilde{\theta} = \theta/(\sigma-1)\left(\frac{\alpha+1}{\alpha}\right)$. Given that the model assumes symmetric country sizes, in order to account for the differences in size of the domestic market[6], instead of considering

6 For example, Italy has roughly 20 million persons more than Spain, hence the domestic demand is larger.

that each country is trading with one single symmetric country the number of a country's trading partners n is determined by the country's size relative to the size of the other countries. For example, in a three country world, if country A has two employees and country B and C have only one employee, the number of trading partners for country A is one ($n = 1$), since it is as if they were trading with a partner of their size, and the number of trading partners for country B and for country C is three ($n = 3$). The targets are reported in Table 2.2.

Country	Slope	Employees	Executives .	Export Volume	Exporters Innovators	R&D Workers
France	1.06	2,903,820	17.4%	27.30%	22.82%	6.81%
Germany	1.10	5,565,414	9.3%	19.48%	27.59%	6.16%
Italy	1.43	3,555,052	7.6%	32.81%	27.73%	5.81%
Spain	1.27	2,010,424	9.5%	21.50%	19.89%	4.85%
U.K.	1.01	3,729,340	14.5%	25.84%	24.31%	7.38%

Table 2.2: Calibration Targets

Several facts stand out in Table 2.2 that will help us interpret the differences in the calibrated parameters. There are important differences in export shares across countries. While exports make up 33% of revenues for Italian firms, that figure drops to 21.5% in Spain, and 19.5% in Germany. Similarly, while 28% of Italian and German firms export and innovate, that share drops to 20% in Spain. The differences in R&D workers are not as substantial across countries: U.K. is the country that employs most workers in R&D (7.4%) while Spain is the country that employs least (4.85%). As for the slope of the distribution of exporting firms, a higher number indicates a steeper slope, and therefore a smaller proportion of larger firms exporting. Consistent with this, in Italy and Spain the typical exporter is relatively smaller, whereas in France and the U.K. there are many large exporting firms. The percentage of executives and middle management also differs quite a bit across countries. France and United Kingdom appear to have a more horizontal structure given that the percentage of executives (included entrepreneurs and middle management) is 17.4% and 14.5% respectively, whereas for Italian firms it drops to 7.6%, indicating a more vertical structure. The calibrated parameters for each country are in Table 2.3.

Country	θ	n	f_D	τ	f_X	f_I	$\Omega = f_X \tau^{\sigma-1}$	$f_X^E = n f_X$
France	4.9	6	1.0	1.88	0.4	5.8	1.8	2.6
Germany	5.1	2	2.0	1.14	8.4	10.6	11.2	16.8
Italy	6.6	4	1.5	1.19	5.5	6.0	8.1	22.0
Spain	5.9	8	2.0	1.93	4.3	2.6	18.3	34.4
U.K.	4.7	4	1.3	1.68	0.6	8.5	1.9	2.4

Table 2.3: Calibrated Parameters

Several of these results require some further explanation. First, Germany's fixed trade costs are relatively high with respect to other countries such as Spain, in spite of being a more open economy. This is easily explained by the fact that f_X represents the fixed trade cost paid by export destination. Because Germany's domestic market is much larger than Spain's, my assumption on symmetric countries implies that Germany has 2 trading partners, compared to 8 in the case of Spain. Therefore, as shown in Table 2.3, the effective fixed trade costs of a German exporter is 16.8, while the effective fixed trade costs of a Spanish exporter is 34.4 labor units. Second, France has a relatively high variable trade cost, similar to Spain, but this is partly offset by the relatively low fixed export cost. Finally, in spite of Spanish innovation fixed costs being the lowest, this does not imply higher innovation. In Spain, exporting is a very expensive activity in comparison to innovation, which explains why some domestic firms innovate without exporting. However, those firms innovate less intensively than the exporter innovators, so that the overall intensity of innovation in Spain is lower than in other countries.

The calibration predicts in which of the three equilibria described in the theory is each of the countries considered. The prediction is in Table 2.4, each equilibrium is determined by the openness of the economy and the level of innovation. The openness depends on both the fixed and the variable trade cost. The parameter Ω in Table 2.3 captures their joint effect, so that a country with a lower Ω is more open. In agreement with the theory, France, and United Kingdom, the most open countries with relatively high innovation, are in the *low cost trade equilibrium*. Germany and Italy, which are less open and have average innovation are in the *intermediate equilibrium*. Spain, the most closed and least innovative country of the five, is in the *low cost innovation equilibrium*.

Country	Predicted Equilibrium
France	Low Cost Trade Equilibrium
Germany	Intermediate Equilibrium
Italy	Intermediate Equilibrium
Spain	Low Cost Innovation Equilibrium
U.K.	Low Cost Trade Equilibrium

Table 2.4: Predicted Equilibrium

2.4 QUANTITATIVE ANALYSIS

In the numerical analysis I consider the effect on aggregate productivity and welfare of the following experiments: a decrease in variable trade costs, a decrease in fixed trade costs, and a decrease in fixed innovation costs.

The theory described in Chapter 1 predicts that a decrease in variable trade cost can have a substantial impact on individual firms' decisions, and thus on aggregate productivity. In addition to a direct effects on productivity, coming from trade being less wasteful and independent from changes in firms' decisions, it identifies three channels through which indirect productivity gains can happen: the selection effect, the extensive margin of innovation, and the intensive margin of innovation. The first quantitative exercise focuses on the decomposition of the change in aggregate productivity into these components and quantifying their relevance. The second quantitative exercise focuses on the effect of lowering the fixed costs of trade and innovation on productivity. Much of the literature has limited its attention to the decrease in variable trade costs. However, in a model with both trade and innovation, liberalizing trade by lowering fixed costs or by reducing variable costs may have very different results.

The section is structured as follows. First, I define the aggregate productivity measure used in the quantitative exercises, as well as its relation to welfare, following the definition of Atkeson and Burstein (2010). Second, I decompose changes in aggregate productivity following a drop in variable trade costs into its different components. Finally, I analyze the effectiveness of a trade liberalization policy versus the effectiveness of an innovation policy on aggregate productivity.

2.4.1 *Aggregate Productivity*

Assume the economy is in steady-state. To solve for aggregate quantities I define indices of aggregate productivity across firms implied by firms decisions. The first of these, Ψ_D, is an index of productivity aggregated across all operating, non-exporting domestic firms, excluding their innovation activities:

$$\Psi_D = \int_{\varphi_D}^{\varphi_X} \varphi^{\sigma-1} dG(\varphi).$$

The second, Ψ_X, is an index of productivity aggregated across all exporting domestic firms, excluding their innovation activities:

$$\Psi_X = \int_{\varphi_X}^{\infty} \varphi^{\sigma-1} dG(\varphi).$$

The third, Ψ_I, is an index of the productivity coming exclusively from the innovation activities. Since in some equilibria there are only exporter innovators, while in others there are exporter and domestic innovators, Ψ_I is defined slightly differently for each of the equilibria:

$$\Psi_I^{LCIE} = \int_{\varphi_{DI}}^{\varphi_{XI}} \varphi^{(\sigma-1)\left(\frac{\alpha}{\alpha+1}\right)} dG(\varphi) + \left[1+\tau^{(1-\sigma)}\right]^{\frac{\alpha+1}{\alpha}} \int_{\varphi_{XI}}^{\infty} \varphi^{(\sigma-1)\left(\frac{\alpha}{\alpha+1}\right)} dG(\varphi);$$

$$\Psi_I^{IE} = \int_{\varphi_{XI}}^{\infty} \varphi^{(\sigma-1)\left(\frac{\alpha}{\alpha+1}\right)} dG(\varphi);$$

$$\Psi_I^{LCTE} = \int_{\varphi_{XI}}^{\infty} \varphi^{(\sigma-1)\left(\frac{\alpha}{\alpha+1}\right)} dG(\varphi);$$

where the superscripts LCIE, IE, and LCTE refer to, respectively, the *low cost innovation equilibrium*, *the intermediate equilibrium*, and *the low cost trade equilibrium*.

The output per production worker measures aggregate productivity, Ψ, whereas the output per worker measures welfare, W. Both measures can be expressed as a function of the productivity indices previously described:

$$\Psi = \frac{Q}{L_p} = \left[M\left(\Psi_D + \left(1+\tau^{1-\sigma}\right)\Psi_X + F(\tau)I\Psi_I\right)\right]^{\frac{1}{\sigma-1}}; \tag{2.1}$$

$$W = \frac{Q}{L} = \left(\frac{\sigma-1}{\sigma}\right)\left[M\left(\Psi_D + \left(1+\tau^{1-\sigma}\right)\Psi_X + F(\tau)I\Psi_I\right)\right]^{\frac{1}{\sigma-1}}; \tag{2.2}$$

where I is the minimum level of innovation of an innovating firm in each equilibrium, and $F(\tau)$ is a function of the variable trade costs different in each equilibrium. Appendix B.1 provides a formal derivation of the aggregate productivity in the different equilibria. I focus on this measure of productivity because it is the measure of productivity that is relevant for welfare in my model and is similar to the ideal measure of productivity defined by Atkeson and Burstein (2010), hence making the results comparable.[7]

2.4.2 *Decomposing the Productivity Effect of a Reduction in Variable Trade Costs*

In this section, I analytically and quantitatively study the impact of a change in marginal trade costs on the measure of aggregate productivity. Following Atkeson and Burstein (2010), I do a first order approximation of the effect of a reduction in marginal international trade costs τ, decomposing its impact on productivity into a direct effect and an indirect effect. The direct effect takes all firms' decisions as given, and simply measures the productivity gains from trade being less wasteful because of the change

7 This measure of aggregate productivity does not necessarily correspond to aggregate productivity as measured in the data. If all differentiated products are intermediate goods used in production of final goods, changes in the price level for final expenditures can be directly measure using final goods, and $\Delta \log \Psi$ is the variation of measured productivity. If all different products are consumed directly as final goods, then the problem of measuring changes in the price level for final expenditures is more complicated. See Atkeson and Burstein (2010) and Bajona et al. (2008) for a discussion of related issues.

in trade costs. Notice that the magnitude of this direct effect is determined simply by the share of exports in production and is independent of changes decisions, whereas the indirect effect arises from changes in firms' entry, export, and innovation decisions, which are themselves responding to the change in trade costs. The following proposition shows the decomposition.

Proposition 6. *The total change in productivity from a change in trade costs and be decomposed into a direct effect and an indirect effect. Moreover, the indirect effect can be decomposed into an entry effect, a reallocation effect, and an innovation effect.*

$$\begin{aligned}\Delta\log\Psi \quad = \quad & \left.\underbrace{-s_X\Delta\log(\tau)}_{\text{Exports}} \underbrace{- \left(\frac{\Delta F(\tau)}{\tau}\right) s_{Inn} I \Delta\log(\tau)}_{\text{Exporters' Innovation}}\right\} \textit{Direct Effect} \\ & \left.\begin{aligned} & + \tfrac{1}{\sigma-1}\Bigg[\underbrace{\Delta\log(M)}_{\text{Entry Effect}} \\ & + \underbrace{s_D\Delta\log(\Psi_D)}_{\text{Domestic Market}} + \underbrace{\left(\frac{1+n\tau^{1-\sigma}}{n\tau^{1-\sigma}}\right) s_X\Delta\log(\Psi_X)}_{\text{Export Market}} \\ & + \underbrace{\underbrace{s_{Inn} I\Delta\log(I)}_{\text{Extensive Margin}} + \underbrace{s_{Inn} I\Delta\log(\Psi_I)}_{\text{Intensive Margin}}\Bigg]}_{\textit{Innovation}} \end{aligned}\right\} \textit{Indirect Effect}\end{aligned}$$

Proof. Since in each equilibria the decisions on innovation are different, I use a general syntax to point out the different components of the decomposition. The exact equations along with the full proof are in Appendix B.2. In what follows, I sketch briefly the algebra behind the decomposition. [8]

Recall that for every $x \in \mathcal{R}$

$$\frac{\Delta x}{x} = \Delta\log(x).$$

8 This derivation works well only for infinitesimal changes

Take logs of Ψ

$$\Psi = \frac{1}{\sigma - 1}\left[\log(M) + \log\left(\Psi_D + \left(1 + \tau^{1-\sigma}\right)\Psi_X + F(\tau)I\Psi_I\right)\right].$$

And derivatives

$$\Delta \log \Psi = \frac{1}{\sigma - 1}\left[\Delta \log(M) + \Delta \log \hat{\Psi}\right].$$

$$\Delta \log \hat{\Psi} = \frac{1}{\hat{\Psi}}\left[\Delta \Psi_D + \Delta\left(1 + \tau^{1-\sigma}\right)\Psi_X + \left(1 + \tau^{1-\sigma}\right)\Delta\Psi_X \right.$$
$$\left. + \Delta F(\tau) I \Psi_I + F(\tau)\Delta I \Psi_I + F(\tau) I \Delta \Psi_I\right].$$

Define the share of domestic production excluding innovation in the value of production $s_D = \frac{\Psi_D}{\hat{\Psi}}$, the share of export production excluding innovation in the value of production $s_X = \frac{n\tau^{1-\sigma}\Psi_X}{\hat{\Psi}}$, and the share of exporters innovation activities in the value of production $s_{XI}^{LCI} = \frac{\left(1+n\tau^{1-\sigma}\right)^{\frac{\alpha+1}{\alpha}}\Psi_{XI}}{\hat{\Psi}}$ and $s_{XI}^{IE,LCT} = \frac{\left(1+n\tau^{1-\sigma}\right)\Psi_{XI}}{\hat{\Psi}}$.

□

The purpose of the decomposition is to test the prediction of the theoretical model in Chapter 1 and to quantify the importance of the different effects. I now discuss each effect, and its expected theoretical sign. The direct effect takes all firms' decisions as given and has two positive components: the first captures the productivity gain of exporters which lose less output from exporting, and the second captures the additional return from innovation by exporters that now face lower trade costs. The indirect effect has five components: the first three correspond to the selection effect described in Melitz (2003), whereas the last two correspond to the change in innovation. As for the selection effect, the first component corresponds to a drop in trade costs inducing the exit of less productive firms, implying the entry effect should be negative. The second and third components have to do with the reallocation of market shares between the remaining domestic and exporting firms. Less productive firms lose market share to more productive exporting firms, hence the domestic indirect effect should be negative, and the exporters indirect effect positive. As for the innovation effect, it can be decomposed into the intensive and extensive margin of innovation. The innovation intensity increases with the participation in foreign markets, and thus the effect through the intensive margin of innovation of the exporters innovators should be positive. For the extensive margin, the theory predicts that the effect can be positive or negative. In the *low cost trade equilibrium* and the *intermediate equilibrium*, all innovators are exporting. In that case a decrease in iceberg trade costs increases the incentives to be an exporter (and to be an exporter innovator), so that the effect through the extensive margin of innovation should be positive.

In the *low cost innovation equilibrium*, innovation happens by both exporting and domestic firms. Hence, while a decrease in iceberg trade costs increases the incentives of exporters to innovate, for the domestic firms innovation becomes harder, as real wages are pushed up. This implies that the productivity cutoff of domestic innovators moves to the right, so that the effect through the extensive margin of innovation will be negative. Table 2.5 shows the elasticity of each component with respect to a decrease in variable trade costs in the five countries. All the elasticities have the predicted signs. A decrease in iceberg trade costs induces in all countries an increase in total productivity. The direct effect on exporting through innovation is stronger the more closed the economy is, since they react more strongly to variations in trade costs. There is a negative effect through the entry of firms, and through the loss of market share by domestic firms, while there is a positive effect coming from the gain in market share by exporting firms, and the intensive margin of innovation. Finally, as predicted, the extensive margin of innovation has a positive effect in the economies that are in the *low cost trade equilibrium* or *intermediate equilibrium*, while it is negative in the *low cost innovation equilibrium* economies.

Atkeson and Burstein (2010) predict that although a drop in iceberg trade costs changes individual firms' decisions, the total indirect effect is essentially zero. In contrast, my simulations show that this is not always the case. If the effect through the extensive margin is small, as in the case of U.K., then the indirect effect on total productivity is close to 0, since the response through the intensive margin of innovation is offset by the changes in firms' exit. However, if the effect through the extensive margin is large, as happens in Spain, this is no longer the case, and the indirect effect substantially differs from zero.

The difference between Atkeson and Burstein (2010) and my paper is that I have an extensive margin of innovation. Taking into account the extensive margin is particularly important in the *low cost innovation equilibrium*, where the number of total innovators in the economy decreases after a reduction of trade costs, and therefore the impact on

	France	Germany	Italy	Spain	U.K.
Total Effect	0.643	0.642	0.806	0.650	0.597
Direct Effect	0.590	0.593	0.714	1.294	0.560
Exporter	0.021	0.017	0.055	0.031	0.005
Exporters' Innovation	0.569	0.576	0.659	1.263	0.555
Indirect Effect	0.053	0.049	0.092	−0.644	0.038
Entry	−1.182	−1.265	−2.033	−1.555	−1.115
Domestic Market	−0.010	−0.003	−0.014	−0.006	−0.003
Export Market	0.062	0.038	0.167	0.087	0.015
Innovation	1.183	1.279	1.973	0.830	1.141
Extensive Margin	0.108	0.099	0.089	−0.568	0.065
Intensive Margin	1.075	1.181	1.884	1.399	1.076
Equilibrium	LCT	IE	IE	LCI	LCT

Table 2.5: Elasticities Lowering Iceberg Trade Costs 1%

aggregate productivity is negative. However, in all the equilibria where the impact is positive, since the number of innovators in the economy increases, the effect through the extensive margin of innovation is quite small. Consistent with this, I observe that a 1% drop in trade costs leads to a reduction of 1.84% in innovating firms in Spain (the only country in the low cost innovation equilibrium), whereas in Germany the number of innovating firms increases only by 0.41%, hence I expect a larger effect through the extensive margin of innovation in Spain than in Germany.

2.4.3 *Lowering Fixed Costs of Trade and Innovation*

The model described in Chapter 1 is particularly suitable to study the effectiveness of trade and innovation policies. In this section I compare the response of aggregate productivity to a decrease in fixed trade costs versus the response to a decrease in fixed innovation costs. While much of the trade literature focuses on decreases in variable trade costs, evaluating the effect of lowering fixed costs is also important. This is especially true in model where firms take both export and innovation decisions.[9]

First, I will describe the effects of a drop in fixed trade costs and a drop in fixed innovation costs on the decisions of the firms in the economy. Second, I will quantitatively assess the elasticity of total productivity, and therefore welfare, to fixed costs. Third, I will analyze the impact on aggregate productivity of a change in the economies' equilibria as a consequence of a large drop in fixed costs.

2.4.3.1 *Effects on Firms' Decisions of a Drop in Fixed Costs*

A reduction in fixed trade costs increases the incentives to enter the export market. In the *low cost innovation equilibrium* and the *intermediate equilibrium* this implies that there is an increase in the firms that export and innovate. In the *low cost trade equilibrium* it implies that more firms export but that less firms export and innovate. In this equilibrium, the firms choosing whether to innovate or not are already exporting (and therefore are paying the fixed export costs), so they only care about innovation costs and variable trade costs. For them a drop in fixed trade costs lowers the incentives to innovate, since it induces

9 In a pure trade model, without innovation, lowering variable or fixed costs tend to have qualitatively similar results on welfare. See (Melitz, 2003) for a more comprehensive explanation.

more entry into the industry, reducing the price index, and lowering the profits coming from innovation. In the next proposition I prove this latter result.

Proposition 7. *In the low cost trade equilibrium, if fixed trade costs fall*

1. *The domestic cutoff increases* $\partial\varphi_D/\partial f_X < 0$
2. *The productivity cutoff for exporting decreases* $\partial\varphi_X/\partial f_X > 0$
3. *The productivity cutoff for exporting and innovation increases* $\partial\varphi_{XI}/\partial f_X < 0$

Proof. Assume that $G(\varphi) = 1 - \left(\frac{1}{\varphi}\right)^{\theta}$. Differentiating Equation B.2 with respect to f_X and using $\partial\varphi_X/\partial f_X = (\varphi_X/\varphi_D)\,\partial\varphi_D/\partial f_X + [1/(\sigma-1)]\,\varphi_X/f_X$ and $\partial\varphi_{XI}/\partial f_X = (\varphi_{XI}/\varphi_D)\,\partial\varphi_D/\partial f_X$ from Equation 1.14, Equation 1.15 and Equation 1.16 yields:

$$\frac{\partial\varphi_D^{LCT}}{\partial f_X} = \frac{n\frac{1}{\varphi_X^{\theta}}}{-nf_X\left(\frac{\sigma-1}{\theta-(\sigma-1)}\right)\frac{\theta}{\varphi_X^{\theta}}\frac{1}{\varphi_D} - f_I\left(\frac{(\sigma-1)\left(\frac{\alpha+1}{\alpha}\right)}{\theta-(\sigma-1)\left(\frac{\alpha+1}{\alpha}\right)}\right)\frac{\theta}{\varphi_{XI}^{\theta}}\frac{1}{\varphi_D}} < 0,$$

$$\frac{\partial\varphi_X^{LCT}}{\partial f_X} = \frac{1}{\theta f_X} - \frac{f_I\left(\frac{(\sigma-1)\left(\frac{\alpha+1}{\alpha}\right)}{\theta-(\sigma-1)\left(\frac{\alpha+1}{\alpha}\right)}\right)\frac{\theta}{\varphi_{XI}^{\theta+1}}\frac{\varphi_{XI}}{\varphi_X}}{nf_X\left(\frac{\sigma-1}{\theta-(\sigma-1)}\right)\frac{\theta}{\varphi_X^{\theta+1}}}\left(\frac{\varphi_X}{\varphi_D}\right)\frac{\partial\varphi_D}{\partial f_X} > 0,$$

$$\frac{\partial\varphi_{XI}^{LCT}}{\partial f_X} = \left(\frac{\varphi_{XI}}{\varphi_D}\right)\frac{\partial\varphi_D}{\partial f_X} < 0.$$

□

Similarly, a reduction in fixed innovation costs increases the incentives to start innovating. In the *low cost trade* equilibrium and the *intermediate* equilibrium this implies that there is an increase in the firms that export and innovate (because all innovators are exporting). In the *low cost innovation* equilibrium, it implies that more firms innovate but that less firms export and innovate. A drop in fixed innovation costs lowers the incentives to export, since it induces more entry into the industry, reducing the price index, and the profits coming from exporting.

2.4.3.2 *Elasticity of Total Productivity to Fixed Costs*

Table 2.6 reports the elasticity of aggregate productivity with respect to a reduction in the fixed costs of trade and innovation, and compares them to the elasticity of aggregate productivity with respect to a reduction in the marginal trade cost. The aggregate productivity of the economy responds much more strongly to a change in marginal trade costs than to a change in fixed trade costs or fixed innovation costs. While the elasticities with respect to the fixed costs are both small, there are significant differences between them.

	France	Germany	Italy	Spain	U.K.
$\epsilon_{\Psi,\tau}$	0.643	0.642	0.806	0.65	0.597
ϵ_{Ψ,f_X}	−0.0156	0.0124	0.0578	0.0374	−0.0197
ϵ_{Ψ,f_I}	0.0129	0.0078	0.0155	0.0174	0.0030

Table 2.6: Effects of a Small Reduction in τ, f_X, and f_I.

On the one hand, the elasticity of aggregate productivity with respect to the fixed innovation costs is very similar across countries and always positive. For countries in the *low cost trade* or the *intermediate* equilibrium, lower fixed innovation costs imply more firms exporting and innovating. However, in the *low cost innovation* equilibrium, which characterizes Spain, there are two opposing effects. While the cost of innovating has dropped, there is the negative effect coming from a reduction in the incentives to export, so that the number of exporters innovators falls. As can be seen from Table 2.6, the direct positive effect more than offsets the negative effect, so that the overall productivity (and welfare) increases in Spain.

On the other hand, the elasticity of aggregate productivity with respect to fixed trade costs is in absolute terms greater than the elasticity with respect to fixed innovation costs, therefore a decrease in fixed trade costs appears to be more effective than a decrease in fixed innovation costs. However, the response of aggregate productivity to a drop in fixed export costs is negative in two countries, France, and United Kingdom. Both economies are in the *low cost trade* equilibrium, and Proposition 7 shows that a reduction in fixed trade costs increases the incentives to enter the export market, but lowers the incentives to innovate. The intuition is that the increased presence of foreign firms pushes up real wages, which reduces the number of innovators, and the intensity of the remaining innovators. Since the investment in innovation decreases, so do the total revenues (and profits) of these firms. Therefore, there is a reallocation of market shares from the most productive firms in the economy towards slightly less productive firms (the new exporters), which lowers the total productivity of the economy, and therefore welfare.[10]

2.4.3.3 *Effect on productivity from large changes in fixed costs*

Figure 2.1 and Figure 2.2 show the response of total productivity to larger changes in fixed trade costs and fixed innovation costs.

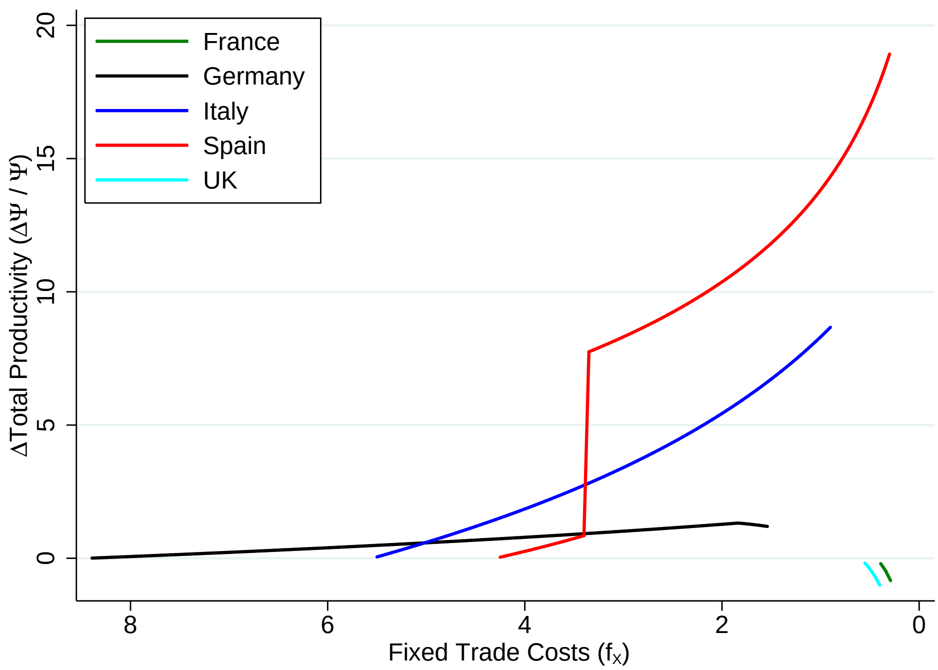


Figure 2.1: Change in Total Productivity and Fixed Trade Costs

10 Monopolistic competition between firms implies that the equilibria are not efficient in terms of Pareto, and therefore, it is possible that a reduction in some costs decreases welfare.

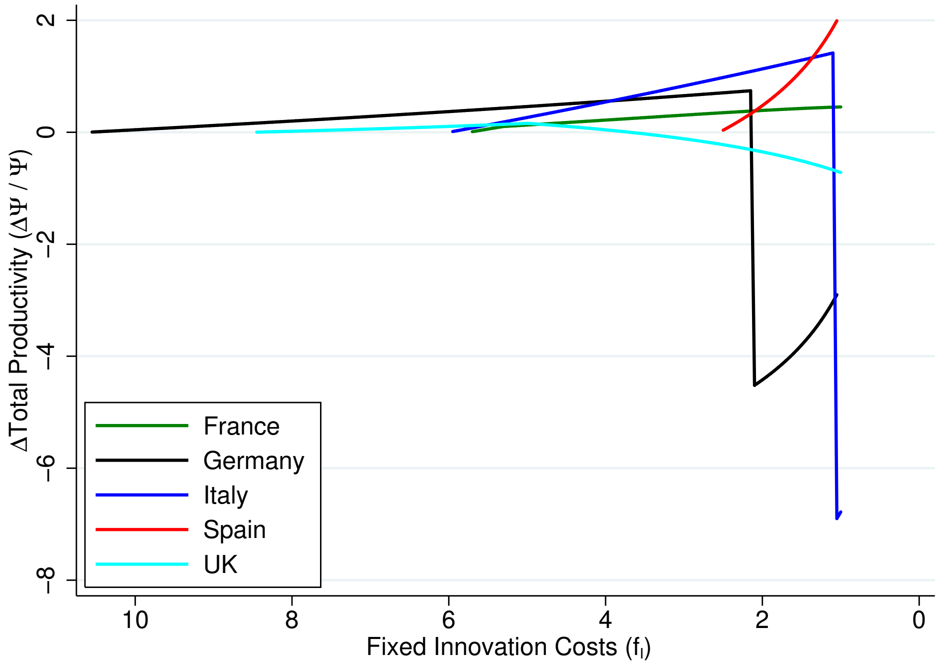


Figure 2.2: Change in Total Productivity and Fixed Innovation Costs

On the horizontal axes are the fixed costs (in reverse order, from high to low) and on the vertical axes is the variation in total productivity with respect to the initial total productivity. An upward-sloping schedule for a given country implies that total productivity (and therefore also welfare) increases when fixed costs drop. For each country the starting point is their initial fixed costs, and I only consider decreases.

Several facts stand out in these two figures. First, the response of productivity to changes in fixed trade costs is stronger than the response to changes in fixed innovation costs. Second, if the economy is in the *low cost trade equilibrium*, the total productivity decreases as fixed trade costs decrease. This is the case of France and United Kingdom. Third, if fixed innovation costs decrease, total productivity increases the most if the economy is the *low cost innovation equilibrium*. This the case of Spain. These three facts are similar to the ones found when computing the elasticities in Table 2.6.

However, the figures also reveal that the largest changes in productivity happen when countries move from one equilibrium to another as a consequence of the drop in fixed costs. This is especially relevant if the movement from one equilibrium to another has a big impact on the number of firms in the economy. These changes in productivity can be positive or negative, large or small, therefore studying what drives them is important to be able to asses the effectiveness of innovation policies and trade policies.

If the fixed trade cost drops sufficiently, Spain goes from the *low cost innovation equilibrium* to the *intermediate equilibrium*. In Figure 2.1 this change in equilibrium shows up as a large upward spike. In this transition 8% of the firms in the economy exit. This negative effect is more than compensated by an increase of 29% in the productivity of the economy when ignoring changes on the entry of firms. The large productivity increase is due to domestic innovators becoming exporting innovators thanks to the increased ease of entering the export market.

Similarly, if the fixed cost of innovation drops sufficiently, Italy and Germany also change equilibrium, this time in the other direction, from the *intermediate equilibrium* to the *low cost innovation equilibrium*. Once again, this shows up as a large spike in Figure 2.2. Since trade becomes relatively more expensive, after the transition there are less exporter innovators and more firms enter in the domestic market. The loss through the exporter innovators dominates the entry of more firms in the economy, hence the spike down in both economies during the change. Finally, notice that once in the the *low cost innovation equilibrium*, the total productivity starts increasing again.

But there are other shifts in equilibria. For example, if the fixed trade cost drops sufficiently, Germany goes from the *intermediate equilibrium* to the *low cost trade equilibrium*. And if the fixed cost of innovation drops sufficiently, France and United Kingdom go from the *low cost trade equilibrium* to the *intermediate equilibrium*. In all these cases, the change between equilibria is smooth and only the slopes change. In Figure 2.1, when Germany transitions to the *low cost trade equilibrium*, the trend becomes negative, although there are still gains in productivity with respect to the initial productivity since it is now in a more open economy. The negative effect is consistent with Proposition 7, where a decrease in fixed trade costs induces losses both through the extensive and the intensive margins of innovation. Note that France and the United Kingdom, which are already in the *low cost trade equilibrium*, display a similar behavior, whereby a drop in fixed trade costs lowers total productivity. However, since both of them are already in a very export-oriented economy, there are no gains with respect to the initial productivity, and the decrease translates in a drop in productivity.

If I turn to the opposite case, going from the *low cost trade equilibrium* to the *intermediate equilibrium*, as France and United Kingdom do in Figure 2.2, I see that both countries react differently. While there is an increase of total productivity in France with respect to the initial situation, in the United Kingdom the trend is negative and if fixed innovation costs are low enough, the total productivity decreases with respect to the initial situation. The decrease in fixed innovation costs induces firms to become exporters innovators, increasing the market shares of these firms while the most inefficient exit the economy. While in France the positive effect through the reallocation of market shares towards the more efficient firms dominates the negative effect through the exit of firms, in the United Kingdom it is the negative effect through the exit of firms which dominates.

Summarizing, Figure 2.1 and Figure 2.2 reveal that a drop in fixed trade costs is more effective in raising productivity (and welfare) than a drop in fixed innovation costs. Depending on the country, it can induce productivity gains from 1% to 20% in total, and only if the economy is already very open might a further drop in fixed trade costs be damaging to the economy. In contrast, a fixed innovation cost drop has little effect on the productivity, the maximum increase being around 2%, and if it induces economies to be less export-oriented, then the productivity might decrease by up to 7%.

2.5 EXTENSIONS

In this section I examine two particular cases of my model: the case where all the firms in the economy innovate and the case where all the firms in the economy can adopt a predetermined innovation. By closing down the extensive margin of innovation or the intensive margin of innovation as channels through which trade indirectly affects aggregate productivity, not only can I analyze better the role each channel has in the previous findings, but also the importance of studying them jointly.

First, I present the results under the assumption that there is no extensive margin of innovation. Then, I present the results under the assumption that there is no intensive margin of innovation. Finally, I discuss the importance of jointly analyzing the extensive and intensive margins of innovation.

2.5.1 *No Extensive Margin of Innovation*

I examine the quantitative results under the assumption that all the firms in the economy innovate. Thus, I eliminate the extensive margin of innovation as channel through which trade can indirectly affect the aggregate productivity of an economy. Closing this channel but allowing firms to differ in the intensity of innovation enables us to study the importance of the extensive margin of innovation for the quantitative results exposed above.

First, I describe the characteristics of the model under the assumption that all firms in the economy innovate. Second, I reevaluate the effects of a decrease in variable trade costs and a decrease in fixed trade costs in the aggregate productivity of the economy. Finally, I compare these results with those from the general model to analyze what the extensive margin of innovation adds to the policy analysis.

2.5.1.1 *Theoretical Model*

The model is based on the framework proposed in Chapter 1, which I simplify to allow all the firms of the economy to engage in process innovation. This model is similar conceptually to Vannoorenberghe (2008), Atkeson and Burstein (2010) and Rubini (2011).

The set up of the economy is the same as the one described in Chapter 1, but I assume that there are no fixed cost of innovation ($f_I = 0$). Then to set an innovation level $z(\varphi)$, firms must incur $c(z(\varphi))$ units of labor, where:

$$c(z(\varphi)) = z(\varphi)^{\alpha+1} \quad \alpha > 0.$$

The timing in this economy is as follows. In a first stage, as in Melitz (2003), entering the market means paying a labor sunk cost f_E in order to get a draw of the productivity parameter φ. In the second stage, with the knowledge of their own productivity, firms decide how much to innovate and whether to export or not. Since exporting requires paying a labor fixed cost, f_X, but innovating does not require any labor fixed costs, there will be two types of firms in the open economy: *Type DI* firms innovate but are active only in the domestic market; and *Type XI* firms innovate and are active in the domestic and foreign markets. Finally, in the third stage, firms choose prices. I solve the firms problem through backward induction.

To make the joint decision of whether to enter the foreign markets or not, and taking into account that the presence of foreign markets affects the firms' innovation, firms will choose the option that yields the highest profits.[11]

– Profits of a domestic innovator firm (Type DI):

$$\pi_{DI} = \frac{R\left(P\rho\right)^{\sigma-1}}{\sigma}\varphi^{\sigma-1}z_D\left(\varphi\right) - f_D - c\left(z_D\left(\varphi\right)\right).$$

– Profits of an exporter innovator firm (Type XI):

$$\pi_{XI} = \left(1+n\tau^{1-\sigma}\right)\frac{R\left(P\rho\right)^{\sigma-1}}{\sigma}\varphi^{\sigma-1}z_X\left(\varphi\right) - nf_X - f_D - c\left(z_X\left(\varphi\right)\right).$$

Where $z_D(\varphi) = \left[\frac{1}{\alpha+1}\left(\frac{R(P\rho)^{\sigma-1}}{\sigma}\right)\varphi^{\sigma-1}\right]^{\frac{1}{\alpha}}$, and $z_X(\varphi) = \left[1+n\tau^{1-\sigma}\right]^{\frac{1}{\alpha}} z_D(\varphi)$.

Proposition 8. *The economy is in equilibrium,* $\varphi^*_{XI} > \varphi^*_{DI}$*, if the following conditions hold:*

1. *Zero profit condition :* $\pi_{DI}(\varphi^*_{DI}) = 0$
2. *Exporting profit condition:* $\pi_{XI}(\varphi^*_{XI}) - \pi_{DI}(\varphi^*_{XI}) = 0$
3. *Free entry condition:* $\frac{1}{\delta}\left[\int_{\varphi^*_{DI}}^{\varphi^*_{XI}}\pi_{DI}(\varphi)dG(\varphi) + \int_{\varphi^*_{XI}}^{\infty}\pi_{XI}(\varphi)dG(\varphi)\right] = f_E$
4. *There is selection into exporting:* $\frac{nf_X}{(1+n\tau^{1-\sigma})^{\frac{\alpha+1}{\alpha}}-1} \geqslant f_D$

2.5.1.2 *Counterfactuals*

The key variable of the quantitative analysis is the aggregate productivity. In this economy, the aggregate productivity and the aggregate welfare of the economy, Ψ and W, are defined as:

$$\Psi = \frac{Q}{L_p} = \left[M\left(\Psi_{DI} + (1+n\tau^{1-\sigma})^{\frac{\alpha+1}{\alpha}}\Psi_{XI}\right)\right]^{\frac{1}{\sigma-1}}; \tag{2.3}$$

11 To ease the mathematics and have closed form solutions, I modify the gains from innovation of the baseline model in Chapter 1 from $(1+z(\varphi))^{\frac{1}{\sigma-1}}$ to $z(\varphi)^{\frac{1}{\sigma-1}}$.

$$W = \frac{Q}{L} = \left(\frac{\sigma-1}{\sigma}\right)\left[M\left(\Psi_{DI} + (1+n\tau^{1-\sigma})^{\frac{\alpha+1}{\alpha}}\Psi_{XI}\right)\right]^{\frac{1}{\sigma-1}}; \tag{2.4}$$

where $\Psi_{DI} = \int_{\varphi_{DI}}^{\varphi_{XI}} \varphi^{(\sigma-1)\left(\frac{\alpha}{\alpha+1}\right)} dG(\varphi)$ and $\Psi_{XI} = \int_{\varphi_{XI}}^{\infty} \varphi^{(\sigma-1)\left(\frac{\alpha}{\alpha+1}\right)} dG(\varphi)$.

The purpose of the counterfactual is to understand how the extensive margin of innovation determined the results from the quantitative exercises. Thus, I do not re-calibrate the model, and use instead the calibrated parameters in Table 2.3. Given this parameters, I decompose the effects of a decrease of variable trade costs and analyze a decrease of fixed trade costs in aggregate productivity for Germany, Italy and Spain. For France and United Kingdom, if there are no fixed innovation costs and all firms innovate, the calibrated parameters suggest that all the firms in the economy would export and innovate. Since I am interested in the joint decision of innovation and exporting, we exclude these two countries in the analysis.

DECREASE OF VARIABLE TRADE COSTS

The effect of a decrease in variable trade costs on aggregate productivity, can be decomposed using a first order approximation into a direct effect and an indirect effect. The direct effect takes all firms' decisions as given, and simply measures the productivity gains from trade being less wasteful because of the change in trade costs. Notice that the magnitude of this direct effect is determined simply by the share of exports in production and is independent of changes in decisions, whereas the indirect effect arises from changes in firms' entry, export, and innovation decisions, which are themselves responding to the change in trade costs. More formally, from equation Equation 2.3, the change in aggregate productivity from a change in variable trade costs is

$$\begin{aligned} \Delta\log\Psi &= \underbrace{-\frac{\alpha+1}{\alpha}\frac{n\tau^{1-\sigma}}{1+n\tau^{1-\sigma}} s_{XI}\Delta\log\tau}_{\text{Direct Effect}} \\ &+ \underbrace{\frac{1}{\sigma-1}\left[\underbrace{(1-s_{XI})\Delta\log\Psi_{DI} + s_{XI}\Delta\log\Psi_{XI}}_{\text{Productivity}} + \underbrace{\Delta\log M}_{\text{Entry}}\right]}_{\text{Indirect Effect}} \end{aligned}$$

where s_{XI} is the share of export production in the value of production.

The indirect effect of a change in trade costs on aggregate productivity itself has two components. The first component is the indirect effect of a change in trade costs on the productivity of the average firm, and the second component is the indirect effect of a change in the number of firms active in the economy.

Table 2.7 shows the elasticity of each component with respect to a decrease in variable trade costs. The change in the productivity of the average firm includes any gain/loss that may happen through the intensive margin of innovation. Notice that the indirect effect is always negligible, because the gains from the changes in productivity are offset by the loss through the exit of firms, just like Atkeson and Burstein (2010) predict. The extensive margin of innovation is key to explain why the indirect effect of a change in trade costs may not be always negligible.

Furthermore, the size of the total effect is considerably smaller in this set up, where if the indirect effect through the extensive margin of innovation is not considered, than in the general model (see Table 2.5). However, they are quite close to the elasticities reported by Atkeson and Burstein (2010) to whom we compare, which indicates that the intensity of innovation may not be as relevant as the extensive margin of innovation to have a large impact in productivity through a decrease in trade costs.

	Germany	Italy	Spain
Total Effect	0.077	0.067	0.041
Direct Effect	0.070	0.066	0.041
Indirect Effect	0.007	0.001	0.000
Entry	−1.407	−2.054	−1.026
Productivity	1.414	2.055	1.026

Table 2.7: Elasticities Lowering Iceberg Trade Costs 1%

DECREASE OF FIXED TRADE COSTS

Figure 2.3 shows the response of aggregate productivity to large changes in fixed trade costs. On the horizontal axis are the fixed trade costs (in reverse order, from high to low) and on the vertical axis are the variation in productivity with respect to the initial total productivity. An upward-sloping schedule for a given country implies that total productivity increases when fixed costs drop. For each country the starting point is their initial fixed costs, I only consider decreases and only consider the economy as long as the parameter conditions specified in Proposition 8 hold.

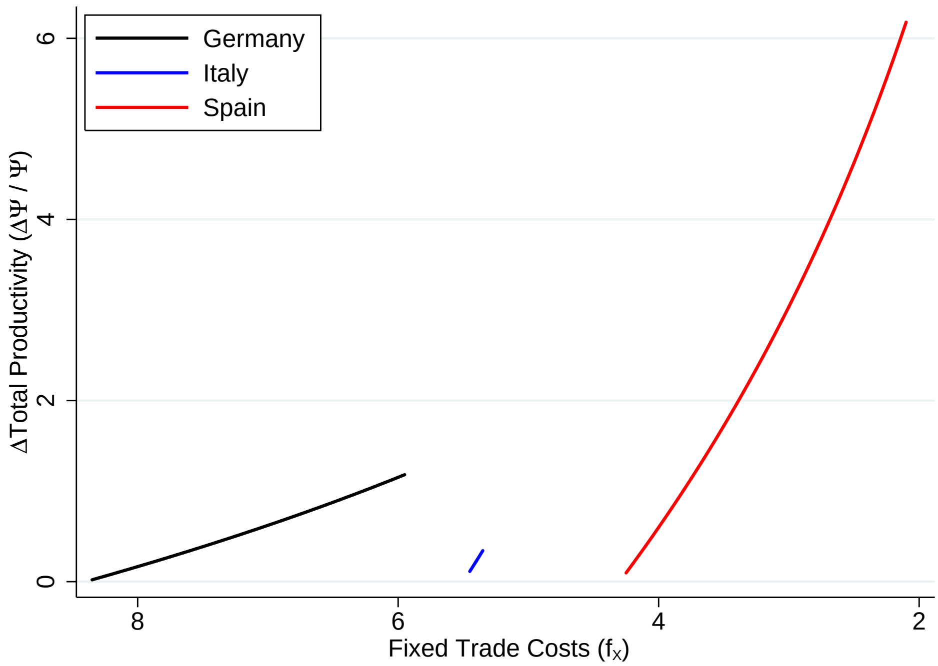


Figure 2.3: Change in Total Productivity and Fixed Trade Costs (No Extensive Margin)

In comparison to the response to a drop in fixed trade costs in the general model (see Figure 2.1), two things stand out if we only consider the intensive margin of innovation. First, the effect of a drop in fixed trade costs on total productivity is always positive. Second, the effect is more subdued in Spain and more pronounced in Germany, making the whole effect more homogeneous among the countries. That is, the differences in the aggregate productivity gains after a drop in fixed trade costs is smaller than when we consider both the extensive and intensive margin of innovation. This suggests that the presence of an extensive margin of innovation may play a key role in the differences we observed earlier, and thus for some economies it might be more important than for others to affect such margin through a trade policy.

2.5.2 *No Intensive Margin of Innovation*

I now examine the quantitative results under the assumption that firms can choose to adopt a better technology, and this innovative technology is predetermined. Thus, I eliminate the intensive margin of innovation as channel through which trade can indirectly affect the aggregate productivity of an economy. Closing this channel but allowing firms to freely choose between adopting or not the "better" technology enables us to study the importance of the intensive margin of innovation for the quantitative results exposed above.

First, I describe the characteristics of the model under the assumption that firms in the economy can choose to adopt a predetermined innovative technology. Second, I reevaluate the effects of a decrease in variable trade costs and a decrease in fixed trade costs in the aggregate productivity of the economy. Finally, I compare these results with those from the general model to analyze what the intensive margin of innovation adds to the policy analysis.

2.5.2.1 *Theoretical Model*

The model is based on the framework proposed in Chapter 1, which I simplify to let firms choose between two technologies — the innovative or the baseline technology. This model is similar conceptually to Navas-Ruiz and Sala (2007), Costantini and Melitz (2008) and Bustos (2011).

Innovating or adopting the innovative technology allows firms to increase their marginal productivity with respect to the baseline technology, but comes at the expense of incurring an implementation cost. The increase in productivity, which I denote $\bar{z}$, is independent of the firm's size and of the presence of foreign markets, that is, the productivity increase of a firm that innovates is the same regardless of their export activities. The adoption of the innovative technology requires paying a fixed labor cost, which I denote f_I. The rest of economy's set up is the same as the one described in Chapter 1.

The timing in this economy is as follows. In a first stage, as in Melitz (2003), entering the market means paying a labor sunk cost f_E in order to get a draw of the productivity parameter φ. In the second stage, with the knowledge of their own productivity, firms decide whether to export or not and whether to innovate or not. Since exporting and innovating require paying a labor fixed cost, f_X and f_I, there will be four types of firms in the open economy: *Type D* firms are active only in the domestic market and do not innovate; *Type DI* firms are active only in the domestic market and innovate; and *Type X* firms are active in the domestic and foreign markets but do not innovate. and *Type XI* firms are active in the domestic and foreign markets and innovate. Finally, in the third stage, firms choose prices. I solve the firms' problem through backward induction.

To make the joint decision of whether to enter the foreign markets or not and whether to innovate or not, and taking into account that the level of innovation does not change with the participation in foreign markets, firms will choose the option that yields the highest profits.

– Profits of a domestic non-innovator firm (Type D):

$$\pi_D = \frac{R\left(P\rho\right)^{\sigma-1}}{\sigma}\varphi^{\sigma-1} - f_D.$$

– Profits of a domestic innovator firm (Type DI):

$$\pi_{DI} = \frac{R\left(P\rho\right)^{\sigma-1}}{\sigma}\varphi^{\sigma-1}\left(1+\bar{z}\right) - f_D - f_I.$$

– Profits of an exporter non-innovator firm (Type X):

$$\pi_X = \left(1+n\tau^{1-\sigma}\right)\frac{R\left(P\rho\right)^{\sigma-1}}{\sigma}\varphi^{\sigma-1} - nf_X - f_D.$$

– Profits of an exporter innovator firm (Type XI):

$$\pi_{XI} = \left(1+n\tau^{1-\sigma}\right)\frac{R\left(P\rho\right)^{\sigma-1}}{\sigma}\varphi^{\sigma-1}\left(1+\bar{z}\right) - nf_X - f_D - f_I.$$

In equilibrium, there are three possible firm type distributions depending on the relation between the cost-benefit ratio of innovation and the exporting costs. The three equilibria are similar to the ones described in Section 1.3.2 (which are summarized in Table 1.1). Table 2.8 summarizes all the possible equilibria in the open economy, the firm type distributions, and the parameter restrictions associated to each one when firms can choose to adopt a predetermined innovative technology.

Equilibrium	Firms' Type Distribution	Conditions
Low Cost Innovation Equilibrium	Type D - Type DI - Type XI $\left(\varphi^*_{XI} > \varphi^*_{DI} > \varphi^*_D\right)$	$\tau^{\sigma-1} f_X \geqslant f_I \frac{(1+\bar{z})}{\bar{z}}$ & $\frac{f_I}{\bar{z}} \geqslant f_D$
Intermediate Equilibrium	Type D - Type XI $\left(\varphi^*_{XI} > \varphi^*_D\right)$	$f_I \frac{(1+\bar{z})}{\bar{z}} \geqslant \tau^{\sigma-1} f_X$ & $\tau^{\sigma-1} f_X \geqslant \frac{f_I}{\bar{z}(1+n\tau^{1-\sigma})} \geqslant f_D$
Low Cost Trade Equilibrium	Type D - Type X - Type XI $\left(\varphi^*_{XI} > \varphi^*_X > \varphi^*_D\right)$	$\frac{f_I}{\bar{z}(1+n\tau^{1-\sigma})} \geqslant \tau^{\sigma-1} f_X \geqslant f_D$

Table 2.8: Equilibria in the Open Economy with a "fixed" innovative techonology

2.5.2.2 *Counterfactuals*

The key variable of the quantitative analysis is the aggregate productivity. In this economy, the aggregate productivity and welfare of the economy, Ψ and W, are defined as:

$$\Psi^{LCIE} = \frac{Q^{LCIE}}{L_p} = \left[M\left(\Psi_D + \bar{z}\Psi_{DI} + (1+n\tau^{1-\sigma})(1+\bar{z})\Psi_{XI}\right)\right]^{\frac{1}{\sigma-1}}; \qquad (2.5)$$

$$\Psi^{IE} = \frac{Q^{IE}}{L_p} = \left[M\left(\Psi_D + (1+n\tau^{1-\sigma})(1+\bar{z})\Psi_{XI}\right)\right]^{\frac{1}{\sigma-1}}; \qquad (2.6)$$

$$\Psi^{LCTE} = \frac{Q^{LCTE}}{L_p} = \left[M\left(\Psi_D + (1+n\tau^{1-\sigma})\Psi_X + (1+n\tau^{1-\sigma})\bar{z}\Psi_{XI}\right)\right]^{\frac{1}{\sigma-1}}; \quad (2.7)$$

$$W = \frac{Q}{L} = \left(\frac{\sigma-1}{\sigma}\right)\Psi; \quad (2.8)$$

where $\Psi_D = \int_{\varphi_D}^{\varphi_X} \varphi^{\sigma-1} dG(\varphi)$, $\Psi_{DI} = \int_{\varphi_{DI}}^{\varphi_{XI}} \varphi^{\sigma-1} dG(\varphi)$, $\Psi_X = \int_{\varphi_X}^{\infty} \varphi^{\sigma-1} dG(\varphi)$, $\Psi_{XI} = \int_{\varphi_{XI}}^{\infty} \varphi^{\sigma-1} dG(\varphi)$, and the superscripts LCIE, IE and LCTE refer to, respectively, the low cost innovation equilibrium, the intermediate equilibrium, and the low cost trade equilibrium.

In the counterfactuals below, I decompose the effects of a decrease of variable trade costs and analyze a decrease of fixed trade costs in aggregate productivity. The purpose of the counterfactuals is to understand how the extensive margin of innovation determined the results from the quantitative exercises. Thus, I do not re-calibrate the model, and use instead the calibrated parameters in Table 2.3. Furthermore, I set the innovation step $\bar{z}$ to 0.5, matching the productivity increase of 20% suggested by Costantini and Melitz (2008). In Table 2.9 can be seen the predicted equilibria each economy is in given the calibrated parameters in Table 2.3 and the innovation step.

Country	Predicted Equilibrium
France	Low Cost Trade Equilibrium
Germany	Intermediate Equilibrium
Italy	Intermediate Equilibrium
Spain	Low Cost Innovation Equilibrium
United Kingdom	Low Cost Trade Equilibrium

Table 2.9: Predicted Equilibrium

DECREASE OF VARIABLE TRADE COSTS

The effect of a decrease in variable trade costs on aggregate productivity, can be decomposed using a first order approximation into a direct effect and an indirect effect.

The direct effect takes all firms' decisions as given, and simply measures the productivity gains from trade being less wasteful because of the change in trade costs. Notice that the magnitude of this direct effect is determined simply by the share of exports in production and is independent of changes in decisions, whereas the indirect effect arises from changes in firms' entry, export, and innovation decisions, which are themselves responding to the change in trade costs. More formally, from equations Equation 2.5 to Equation 2.7, the change in aggregate productivity from a change in variable trade costs is generally

$$\Delta \log \Psi = \underbrace{-\frac{n\tau^{1-\sigma}}{1+n\tau^{1-\sigma}}(s_X + \bar{z}s_{XI})\Delta \log \tau}_{\text{Direct Effect}} + \underbrace{\frac{1}{\sigma-1}\left[\underbrace{s_D \Delta \log \Psi_D + \bar{z}s_{DI}\Delta \log \Psi_{DI} + s_X \Delta \log \Psi_X + \bar{z}s_{XI}\Delta \log \Psi_{XI}}_{\text{Productivity}} + \underbrace{\Delta \log M}_{\text{Entry}}\right]}_{\text{Indirect Effect}}$$

where s_D is the share of domestic firms' production in the value of production, s_{DI} is the share of domestic innovators firms' production in the value of production (which will be zero in the intermediate equilibrium and the low cost trade equilibrium), s_X is the share of export firms' production in the value of production (which will be zero in the low cost innovation equlibrium and the intermediate equilibrium) and s_{XI} is the share of export production in the value of production.

The indirect effect of a change in trade costs on aggregate productivity itself has two components. The first component is the indirect effect of a change in trade costs on the productivity of the average firm, and the second component is the indirect effect of a change in the number of firms active in the economy.

Table 2.10 shows the elasticity of each component with respect to a decrease in variable trade costs. The change in the productivity of the average firm includes any gain/loss that may happen through the extensive margin of innovation. Notice that the indirect effect is not negligible for the cases of Germany, Italy and Spain which are in the intermediate equilibrium and the low cost innovation equilibrium. Particularly, in these countries the gains from the changes in productivity are more than offset by the loss through the exit of firms. In comparison to the results in the general model (see Table 2.5), the non-negligible indirect effects are now present in countries in the intermediate equilibrium as well. A decrease in variable trade costs induces some firms to both export and adopt the innovative technology, but at the same time, the increased competition reduces the gains from the domestic firms in the economy and induces the less productive of them to exit. In the general model, the existence of the intensive margin of innovation channel implied that a decrease in variable trade costs had an impact in the innovation performed by the firms. The positive effect through the intensive margin of innovation is missing here, which explains the differences in the indirect effect.

Furthermore, if only the extensive margin of innovation is considered, instead of both the extensive and intensive margins of innovation, then the strong negative effect from the indirect effect may offset the positiveness from the direct effect. And therefore, we may wrongly conclude that a decrease in variable trade costs does not have an effect on aggregate productivity.

	France	Germany	Italy	Spain	U.K.
Total Effect	0.439	−0.009	−0.003	0.037	0.456
Direct Effect	0.430	0.246	0.402	0.181	0.445
Indirect Effect	0.009	−0.255	−0.405	−0.144	0.011
Entry	−0.961	−0.512	−0.861	−0.144	−0.995
Re-allocation	0.970	0.256	0.456	0.001	1.006
Equilibrium	LCT	IE	IE	LCI	LCT

Table 2.10: Elasticities Lowering Iceberg Trade Costs 1%

DECREASE OF FIXED TRADE COSTS

Figure 2.4 shows the response of aggregate productivity to large changes in fixed trade costs. On the horizontal axes is the fixed trade costs (in reverse order, from high to low) and on the vertical axes is the variation in productivity with respect to the initial total productivity. An upward-sloping schedule for a given country implies that total productivity increases when fixed costs drop. For each country the starting point is their initial fixed costs, and I only consider decreases.

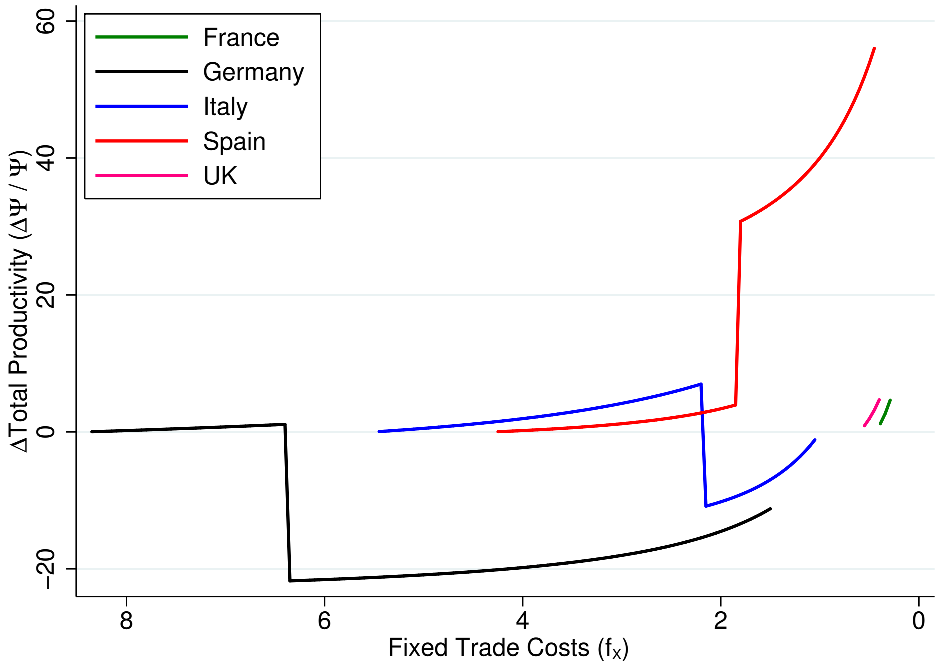


Figure 2.4: Change in Total Productivity and Fixed Trade Costs (No Intensive Margin)

In comparison to the response to a drop in fixed trade costs in the general model (see Figure 2.1), two things stand out if we only consider the intensive margin of innovation. First, the effect of a drop in fixed trade costs on total productivity is always positive when firms remain within an equilibrium, even in the low cost trade equilibrium. Second, large changes in productivity happen when countries move from one equilibrium to another as a consequence of the drop in fixed trade costs, and this changes in productivity are always large.

If the fixed trade costs drops sufficiently, Spain goes from the low cost innovation equilibrium to the intermediate equilibrium. In Figure 2.4 this change in equilibrium shows up as a large upward spike. In this transition 11% of the firms in the economy exit, but the negative effect is more than compensated by an increase of 48% in the productivity of the economy when ignoring changes on the entry of firms. The large productivity increase is due to domestic innovators becoming exporting innovators thanks to the increased ease of entering the export market. This effect is exactly the same we observed in the general specification.

Similarly, if the fixed cost of trade drops sufficiently, Italy and Germany also change equilibrium from the intermediate equilibrium to the low cost trade equilibrium. Differently than before, in Figure 2.4 this change in equilibrium shows up as a large downward spike. In this transition 43% and 34% of the firms in Germany and Italy respectively exit, but the negative effect is not compensated by an increase in the productivity of the economy when ignoring changes on the entry of firms. This effect is completely opposite to the one observed in the general specification which highlights the importance of the intensive margin of innovation to curve the negative effects from transitioning from the intermediate equilibrium to the low cost trade equilibrium.

2.5.3 *The benefits of analyzing jointly the extensive and intensive margins of innovation.*

There are substantial differences in the effects of trade policies in aggregate productivity when considering a trade model of heterogenous firms with innovation where there is both an extensive and intensive margin of innovation, only an intensive margin of innovation or just an extensive margin of innovation.

First, changes in firms' decisions regarding entry, exit export and innovation after a drop in variable trade costs are non-negligible only if we consider that not all the firms in the economy innovate. That is, if there are changes in the extensive margin of innovation driven by a drop in variable trade costs. However, the presence of the intensive margin of innovation in the analysis is key to not underestimate the total effect of a drop in variable trade cost on aggregate productivity.

Second, the response of aggregate productivity to a drop in fixed trade costs leads to large differences in the transition from one equilibrium to another when we consider only the extensive margin of innovation. These large differences are smoothed by the effects through the intensive margin of innovation. In the case of a change from the intermediate equilibrium to the low cost trade equilibrium, the positive effect from the intensive margin of innovation completely smooths the transition in the general setting. In the case of a change from the low cost innovation equilibrium to the intermediate equilibrium, the effect through the intensive margin of innovation dampens the large difference present when we only consider the extensive margin of innovation, but not enough to have a smooth transition.

Finally, if effects through the extensive and intensive margin of innovation are not consider jointly, then a drop in fixed trade costs appears to lead to an increase in the total productivity. However, when considered jointly and if an economy is in the low cost trade equilibrium, this is no longer the case. In the joint analysis, the lose of market shares from the more productive firms of the economy is amplified by the intensive margin of innovation, thus the negative effect through the reallocation of market shares towards less productive firms of the economy is greater than in an economy without an intensive margin of innovation.

2.6 CONCLUSIONS

Chapter 1 proposed a trade model with heterogeneous firms that decide not just whether or how much to export but also whether or how much to innovate. By incorporating the extensive and intensive margins of trade and innovation, three equilibria may arise. In all equilibria high-productivity firms export and innovate, whereas low-productivity neither export nor innovate. What differs across equilibria is the behavior of medium-productivity firms. In an economy with trade costs that are low relative to innovation costs, medium-productivity firms export without innovating, whereas in an economy with trade costs that are high relative to innovation costs, medium-productivity firms innovate without exporting. In a third equilibrium, in between the other two, some medium-productivity firms export and innovate, whereas others do neither.

In this paper I have shown that these equlibria are empirically plausible by calibrating the model to five European countries. The numerical exercises reveal the importance of considering both the intensive and extensive margin of innovation to understand the interdependence between trade and innovation. More generally, the effect of trade liberalization on productivity and welfare depends crucially on the equilibrium the economy is in. A standard result in the literature is that the aggregate productivity effect of a drop in variable trade costs on firms' decisions to exit, export and innovate is minimal. In my setup this is also true in most equilibria, but not in the *low cost innovation equilibrium*. In that case a drop in variable trade costs has a negative impact on the extensive margin of innovation, thus lowering the overall positive effect of trade liberalization.

In addition to analyzing a drop in variable trade costs, I also assessed the impact of a drop in fixed trade costs and fixed innovation costs. Once again, although in most equilibria these policies lead to an improvement in aggregate productivity and welfare, this is not always the case. For example, in the *low cost trade equilibrium*, a drop in fixed trade costs increases the number of exporters, making innovating more expensive. This lowers both the number of innovators and the intensity of innovation, leading to a reduction in aggregate productivity and welfare.

These findings stress the importance of having a model that jointly analyzes the extensive and intensive margins of both trade and innovation. Not doing so would not just result in a less rich theoretical structure, it would also keep us from correctly assessing the impact of different policies aimed at fomenting trade and innovation.

3

UNDERSTANDING COMPETITIVENESS

joint with Rubén Segura-Cayuela

Abstract. Using firm level data, we analyze the factors that drive the evolution of the aggregate Unit Labor Costs — the main European competitiveness indicator — in France, Germany, Italy and Spain. The evolution of the aggregate Unit Labor Cost is not driven by the evolution of the firm level Unit Labor Costs, but rather by an important factor for the competitiveness of a country: the reallocation of resources among the firms of the economy. Using the methodology of Hsieh and Klenow (2009), we show the importance of an efficient allocation of resources for productivity gains.

3.1 introduction

The latest world crisis and the increase of debt in Europe have reopened in the last few years a debate forgotten in the good times, the competitiveness of an economy. Currently the relevant measure of competitiveness in the European Union is the evolution of unit labor costs. The unit labor cost is a macroeconomic aggregate that measures the labor cost per unit of product and is calculated as the ratio of total labor costs to real output. A rise in labor costs higher than the rise in labor productivity may be a threat to an economy's cost competitiveness if other costs are not adjusted in compensation.

The use of aggregate price-cost based indicators, like the unit labor costs, may not be informative enough to determine the competitiveness of a country. For example, Spain's aggregate unit labor cost has grown faster than in the other European countries in the last decade. Then, we should see a decrease in the world's export shares reflecting the decrease in the ability to sell their products. However, the exports shares have decreased less than those of the other European countries. This "Spanish paradox" is explained by the different relative weight of firms in the unit labor costs and the economy's total exports. Firms that export are usually the largest and most productive of the economy (Clerides et al. (1998) and Bernard and Bradford Jensen (1999)), and they account for the main share of firms that export. However, for the aggregate unit labor cost all the firms in the economy are taken into account, not just the exporters. Recent literature in industrial organization and international trade (di Giovanni and Levchenko (2009) and Bernard et al. (2011)) has provided abundant empirical evidence supporting the idea that the evolution of macroeconomic aggregates is determined closely by the decisions and characteristics of the firms in the economy, and in particular by the behavior and productivity of a subgroup of them: the most productive ones. Then, an adequate competitiveness measure should be able to take into account the role of firms and their heterogeneity.

In this paper, we analyze the ability of the aggregate unit labor costs evolution to capture adequately the firm heterogeneity of a country. We calculate, using firm level data, a weighted change of the aggregate unit labor costs between 2002 and 2007 for four European countries: France, Germany, Italy and Spain. The components of the weighted average are then decomposed according to a Laspeyres decomposition into three main elements: the first captures changes in firm-level unit labor costs, keeping the initial domestic market shares of firms constant; the second quantifies the reallocation of market shares within the domestic economy, keeping the initial unit labor costs constant; and the third measures the interaction between the first two. If the aggregate ULC was a measure that captured adequately the heterogeneity existent at the firm level ULC, its evolution should be driven by the evolution of the firm level ULC. Then we should observe the within component to be the most relevant in the explanation of the aggregate ULC evolution.

The results reveal that the evolution of the firm-level unit labor cost does not explain the evolution of the aggregate unit labor costs, rather it is the resource reallocation and the interaction effect that explain around 90% of the changes in ULCs for all the countries in the sample. Furthermore, Germany is the country that presents a greater reallocation of resources in the period 2002 to 2007. In comparison with Germany, the lower resource reallocation led to competitiveness losses of around 4.3% in the case of France, 6.4% in Italy and 8% in Spain.

Motivated by the significant role of the reallocation of resources to explain the evolution of the aggregate ULC, we apply the methodology of Hsieh and Klenow (2009) to explain how much of the differences in productivity in Europe is due to an inefficient allocation of resources. As a result of distortions that affect production, firms produce different amounts than what would be dictated by their productivity. In order to determine the gains from an efficient allocation of resources, we calculate the hypothetical "efficient" output in each country — the output if these distortions did not exist — and compare it with actual output levels.

An efficient allocation of resources would boost aggregate manufacturing TFP in 2008 by 22.7% in France, 27.9% in Germany, 43.5% in Italy and 28.2% in Spain. More interestingly, we observe that over the period of 2002 to 2008, the "misallocation" of resources decreases in Germany, remains fairly constant in France and increases in Italy and Spain. This is actually consistent with the higher reallocation of resources present in the evolution of Germany's aggregate unit labor costs, which is followed by France, Italy and Spain.

Our empirical analysis of the unit labor costs as a competitiveness measure reveals the need to open the "black boxes" that the macroeconomic indicators often are, by using firm level data to understand clearly what are the driving factors behind their evolution. While the evolution of the aggregate unit labor cost does not reflect adequately the evolution of the firm level unit labor costs, and therefore does not capture the firm heterogeneity present in an economy, it highlights the importance of the reallocation of resources between firms in an economy. Our results suggest that an efficient reallocation of resources leads to productivity gains of at least 20% in all countries. Attending to the definition of Porter (1990), the competitiveness of a nation is the productivity with which a nation utilizes its human, capital and natural resources. Therefore, our results indicate that the evolution of the ULC is driven by an important factor for the competitiveness of a country.

This paper contributes to the competitiveness literature by showing that the evolution of the aggregate unit labor costs is driven by the reallocation of resources in the economy, and by quantifying potential gains through an efficient reallocation of resources. Our paper relates to two strands in the literature. First, the literature that studies the effectiveness of aggregate macroeconomic indicators and their effectiveness to be used as policy indicators (Boone et al. (2007) and Felipe and Kumar (2011)). Boone et al. (2007) claim that the use of the price cost margin as a competitiveness measure may be potentially misleading since it tends to misrepresent the development of competition over time in markets with few firms and high concentration. And Felipe and Kumar (2011) analyze if the reduction of unit labor costs through a significant reduction in nominal wages is the best policy to exit the current crisis for some countries of the eurozone. Their analysis reveals that the aggregate unit labor costs reflects actually the distribution of income between wages and profits, and that the unit capital costs have also increased in the last decade. Therefore, a large reduction in nominal wages simply will not solve the problem. Second, our paper is related to the literature that studies the efficient allocation of resources. In particular, we follow the methodology of Hsieh and Klenow (2009) who use micro data on manufacturing establishments to quantify the potential extent of resource misallocation in China and India versus the United States.

The rest of the paper is organized as follows. In Section 3.2, we describe the firm level data used throughout the exercise. In Section 3.3, we discuss the traditional indicators of competitiveness and their limitations, particularly regarding their inability to account for the role of firms and their heterogeneity. In Section 3.4, we analyze if the aggregate evolution of the unit labor costs captures adequately the evolution of the same variable for the individual firms. In Section 3.5, we explain how much of the differences in productivity and output is due to an inefficient allocation of resources. Section 3.6 concludes.

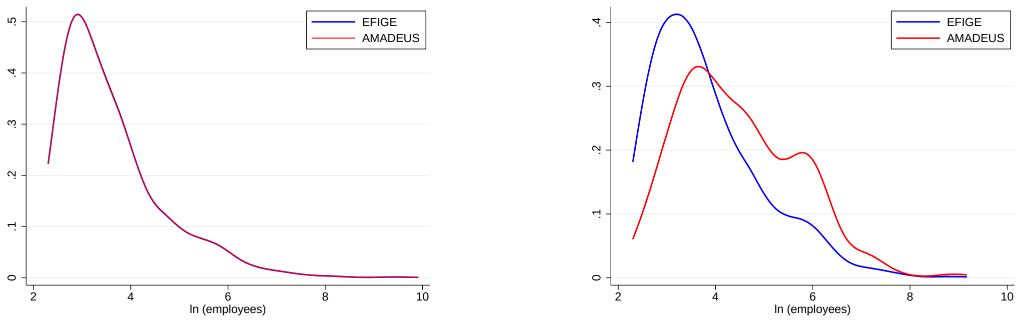
3.2 DATA

We analyze balance sheet data from the AMADEUS dataset, managed by Bureau van Dijk, which has been integrated with the EFIGE survey, a representative sample[1] at the country level for the manufacturing industry of several European economies.

The analysis is centered on France, Germany, Italy and Spain.[2] While for the analysis of the ULC only the cost of employees and the turnover of the firm are needed, the study of the impact of an efficient reallocation of resources requires data both from the balance sheet and the survey which we specify in detail later.

For each surveyed firm, nine years of usable balance sheet information has been retrieved, from 2001 to 2009. France, Italy and Spain are the countries with best quality in the balance sheet data, with a coverage[3] of 88.6%, 86.86% and 90.56% respectively. For Germany, the coverage is irregular. For the period of 2004-2008, there is a fairly good coverage of 70% to 80% of the firms, however for the years 2001-2003 and 2009 it drops to levels between 30-45% on average.

In Figure 3.1, we present the distribution of firms by employment size for all the surveyed firms in EFIGE and the sample covered by the AMADEUS database.



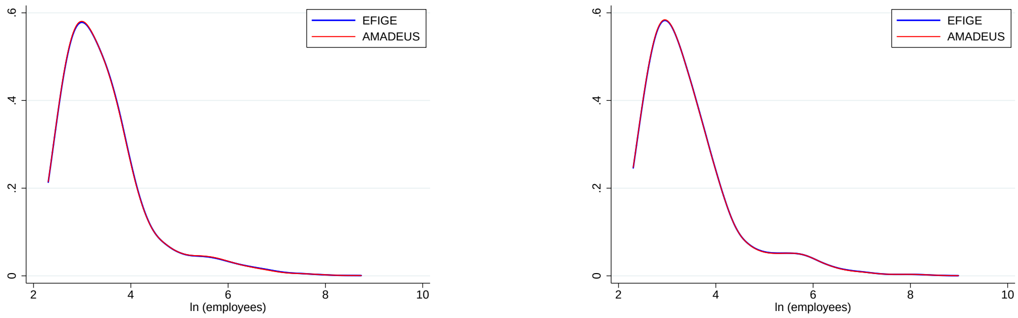
(a) France (b) Germany

Figure 3.1: Distribution of Plant Size

1 Altomonte and Aquilante (2012) provide more information on the construction of the dataset and a comprehensive set of validation measures .

2 In the EFIGE dataset there is also information about three more European countries: Austria, Hungary and United Kingdom. Due to the poor quality of the balanced data for these countries, they have not been included in the analysis.

3 The reference variable for the coverage is the turnover of the firm.



(c) Italy

(d) Spain

Figure 3.1: Distribution of Firm Size (Cont.)

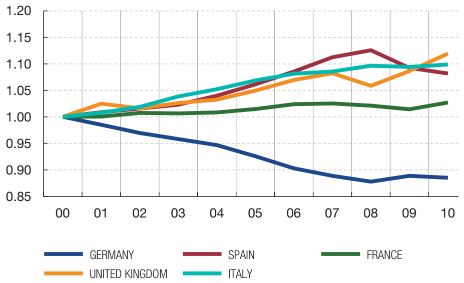
For all the countries with the exception of Germany, the firm size distribution of the subsection of firms present in AMADEUS matches almost perfectly the firm size distribution of the surveyed firms in EFIGE. Within the subsection of firms present in the AMADEUS dataset for Germany, the number of small firms is slightly under-represented while the number of medium firms is slightly over-represented with respect to the distribution of all the surveyed firms in EFIGE. Hence, we should be cautious in the interpretation of results for Germany and make sure is that they are not biased by this fact.

3.3 LIMITATIONS OF THE TRADITIONAL COMPETITIVENESS INDICATORS

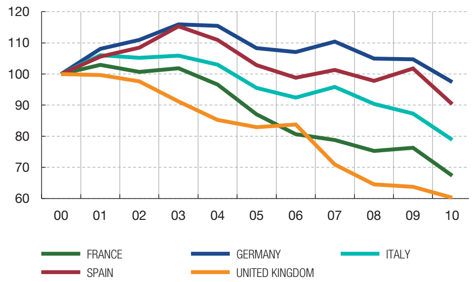
Porter (1990) defines the competitiveness of a nation as the productivity with which a nation utilizes its human, capital and natural resources. The OECD considers the ability of a country to sell its products in the international markets while Krugman (1994) refers to competitiveness as a poetic way of speaking about productivity, and warns about the danger of obsessing about the competitiveness of a country. Most of these definitions of competitiveness allude to the relative position of a country in international trade. This position, in principle, depends on price and cost factors because if they have a negative evolution in relation with those from others economies, the ability to sell products at home and abroad is damaged. This argument, combined with the easy availability of data, makes price-cost competitiveness indicators especially attractive for the analysis of a country's economic situation. This is why the classical macroeconomic textbooks relate the competitiveness of nations to the comparison of their relative prices.

Currently the price-cost indicator of reference to measure competitiveness in the European Union is the unit labor cost (ULC), which measures the labor cost by unit of product and is calculated as the ratio of total labor costs to real output.[4] A rise in an economy's ULC represents an increased reward for labor's contribution to output. However, a rise in labor costs higher than the rise in labor productivity may be a threat to an economy's cost competitiveness, if other costs are not adjusted in compensation.

A simple comparison of the evolution of prices and costs between two countries may not be informative enough to determine the competitiveness of a country, and therefore, the ULC may be a measure of competitiveness with a very limited prediction power. If an increase in the ULC index indicates a loss in competitiveness of the country, then we should see a decrease in a country's export shares whenever aggregate ULC goes up. Figure 3.2 shows the so called *Spanish competitiveness paradox*, an example that a loss in competitiveness does not imply necessarily a loss in the world's export shares. Figure 3.2a shows the evolution of the ULC for Spain and the main developed economies, while in Figure 3.2b shows the evolution of these countries worlds' export share during the 2000's. The Spanish ULC has grown faster than in the main developed countries, but on the other hand, its export shares have decreased less than those of other countries, the only exception being Germany.



(a) Unit Labor Cost (ECB)



(b) Market Share Index (WTO)

Figure 3.2: Competitiveness Indicators Vis-á-Vis the Euro Area

4 An assumption implicit in the use of cost based indicators is that in the short run the capital is fixed, and therefore the cost of capital should not differ between similar countries. This assumption can be a limitation of the cost-competitiveness measures, see Felipe and Kumar (2011) for further details.

Antràs et al. (2010) show that large Spanish firms experienced both lower ULC growth and higher export growth than other countries, yet this differential is not reflected in aggregate price indicators due to aggregation and dispersion bias (Altomonte et al. (2012)). In the calculation of the ULC all the firms are taken into account while to calculate the economy's total exports, only the exporters are taken into account. Firms that export are usually the largest and most productive of the economy (Clerides et al. (1998) and Bernard and Bradford Jensen (1999)). The different relative weight in the aggregate ULC and in the economy's total export, helps therefore to explain the *Spanish paradox*.

An adequate competitiveness measure should be able to capture the role of firms and their heterogeneity. Several questions arise then. First, why is heterogeneity so important? Second, why should a competitiveness measure take into account the heterogeneity within the firms of an economy? And third, how adequately do traditional competitiveness measures capture the heterogeneity?

To understand the importance of the heterogeneity between firms, the concept of productivity is essential since it allows high wages and high capital returns in an economy (See Porter (2005)). Recent literature in industrial organization and international trade has provided abundant empirical evidence supporting the idea that the evolution of macroeconomic aggregates is determined closely by the decisions and characteristics of the firms in the economy, and in particular by the behavior and productivity of a subgroup of them: the most productive ones. This is evident in the case of exporting firms. Exporter firms from a sector or a country are a minority and, in general, they are those that behave better in terms of productivity, size and innovation. The higher performance is present before these firms become exporters (see Clerides et al. (1998) and Bernard et al. (2011)).

Table 3.1 and Table 3.2 illustrate why a competitiveness measure should take into account this heterogeneity. Table 3.1 shows the export probability (extensive margin) of a firm in relation to its size for each of the countries in the database of EFIGE, while Table 3.2 reports the percentage of production that each firm exports (intensive margin). It is observed that for two similar sized firms from different countries, the probability of exporting and the export proportion are roughly similar. For example, among firms with 50 to 249 employees in France and Spain, the probability of exporting is 75.4% and 76.2% respectively, less than a 1 percentage point difference. Furthermore, the difference in the export intensity of these firms is only 0.3 percentage points. In the aggregate, the differences between France and Spain in the export probability and the export intensity are higher. These differences in the exports aggregated by size, sector or country do not come from differences in two similar firms from different countries, they are due to differences in the allocation of resources between the sectors of the economy and differences in the firm size distribution within sectors.

Employees	Austria	France	Germany	Hungary	Italy	Spain	UK
10 – 19	69.8	44.7	45.7	58.0	65.4	51.2	54.9
20 – 49	63.8	59.1	65.4	64.7	73.3	63.5	62.8
50 – 249	88.6	75.4	78.2	79.3	86.6	76.2	76.8
Over 249	90.8	87.6	84.0	97.4	92.6	88.0	90.7
Aggregate	72.6	57.9	63.4	67.3	72.2	61.1	61.0

Table 3.1: Extensive margin of exports (%), by country and company size.

Employees	Austria	France	Germany	Hungary	Italy	Spain	UK
10 – 19	26.2	23.0	25.9	30.2	30.4	21.4	26.2
20 – 49	33.3	27.0	28.1	43.6	34.2	24.5	27.8
50 – 249	55.9	33.0	33.9	53.2	42.2	33.3	33.2
Over 249	64.7	41.2	37.8	66.6	52.6	40.6	34.2
Aggregate	40.4	28.5	30.0	44.8	34.6	25.9	29.1

Table 3.2: Intensive margin of exports (%), by country and company size.

Barba-Navaretti et al. (2011) estimate that if Spain had the industrial structure and firm size distribution of Germany, the exports of Spain would increase 25%. The differences in the aggregates were due to differences in the allocation of resources between the sectors of the economy and differences in the firm size distribution of the firms within sectors. That is, within a sector there can be as much firm heterogeneity as there can be between firms in different sectors.

To address how adequately traditional competitiveness measures capture firm heterogeneity, in the next section we study whether the firm level ULC evolution drives the aggregate ULC evolution or whether it is driven by other factors.

3.4 ULC DECOMPOSITION

In this section we analyze how adequately the evolution of the Unit Labor Cost captures the firm heterogeneity present in a country. We decompose the evolution of the ULCs of four European countries given the firm-level information in EFIGE. The exercise analyzes if the aggregate evolution of the ULC between years 2002 and 2007 captures adequately the evolution of the same variable for the individual firms.[5]

For that purpose, we calculate at firm level a weighted change of the ULC as:

$$ULC_{t+1} - ULC_t = \sum_{i \in I_{t+1}} ms_{i,t+1} ulc_{i,t+1} - \sum_{i \in I_t} ms_{i,t} ulc_{i,t}$$

where $ulc_{i,t}$ is the ULC of a given firm i at time t and $ms_{i,t}$ is its market share at that time. The components of the weighted average are decomposed as follows, according to a Laspeyres decomposition.[6]

$$\begin{aligned}
ULC_{t+1} - ULC_t &= \sum_{i \in I_{t+1}} ms_{i,t+1} ulc_{i,t+1} - \sum_{i \in I_t} ms_{i,t} ulc_{i,t} \\
&= \underbrace{\sum_{i \in I} ms_{i,t} \left(ulc_{i,t+1} - ulc_{i,t} \right)}_{\text{Within}} + \underbrace{\sum_{i \in I} ulc_{i,t} \left(ms_{i,t+1} - ms_{i,t} \right)}_{\text{Reallocation}} \\
&+ \underbrace{\sum_{i \in I} \left(ms_{i,t+1} - ms_{i,t} \right) \left(ulc_{i,t+1} - ulc_{i,t} \right)}_{\text{Interaction}} \\
&+ \underbrace{\sum_{i \in I_{t+1} \setminus I} ms_{t+1} ulc_{t+1} - \sum_{i \in I_t \setminus I} ms_{i,t} ulc_{i,t}}_{\text{Entry}-\text{Exit}} .
\end{aligned}$$

5 Unfortunately, the bad coverage of Amadeus for Germany does not let us use the whole sample from 2001 to 2009.

6 Note that the latter decomposition is also discussed by Boone et al. (2007), as the starting point of the indicator of competition, and by (Altomonte et al., 2010).

The first element, the *within* component, is the change attributable to the evolution of the firms' ULC given their market share: a positive sign would imply a relevant loss in competitiveness at the firm level. The second element, the *reallocation* component, accounts for the redistribution of market shares among the firms, holding the ULC constant: a negative sign implies a reallocation of market shares towards firms with initial lower ULC. The third element, the *interaction* component, gives information about the underlying dynamics: a negative sign would show that ULCs and market shares are moving in different directions, either because their activity is expanding thanks to a reduction in ULC or because the importance of their sector is decreasing after an increase in the ULC. The fourth element, the *entry and exit* component is indicative of the market dynamics that follow the removal of barriers fostering entry, and the exogenous shocks that can oblige some firms to exit. As we already discussed in Section 3.2, the EFIGE survey is not designed to keep track of entry and exit of firms, therefore this element is simply a residual of the calculation, and will be ignored in the discussion.

If the aggregate ULC was a measure that captured adequately the heterogeneity existent at the firm level ULC, its evolution should be driven by the evolution of the firm level ULC. Then we should observe the *within* component to be the most relevant in the explanation of the aggregate ULC evolution.

Table 3.3 shows the result of the of the decomposition of the change in aggregate ULC in manufacturing between years 2002 and 2007 annualized. First, on average, for the period considered, the real ULCs have decreased in all countries indicating an improvement in the cost competitiveness of the countries — which is supported as well by results using the EU-KLEMS database. Second, the weight of the change in competitiveness within firms is small, particularly in Italy and Spain, where it is 0.17% and −0.21% respectively. Third, the interaction effect has the desired sign, negative. Unfortunately we can not infer if is due to to the activity of firms expanding thanks to a reduction in ULC or because the importance of their sector is decreasing after an increase in the ULC. Fourth, the reallocation of resources is the component that explains most of the evolution of the ULC for all the countries in the sample. The relative intensity differs between countries: the largest reallocation of resources occurs in Germany, followed by France, then Italy and Spain. Not only is the the reallocation of resources in France and Germany larger, but it is also the most important factor in the explanation of the evolution of the aggregate

ULC. In Italy and Spain, the interaction effect has a similar weight as the reallocation of resources effect in the explanation of the evolution of the aggregate ULC.

	Total	Within	Reallocation	Interaction	Entry-Exit
France	−2.62	−1.19	−1.87	−0.61	1.06
Germany	−3.25	−1.55	−2.69	−0.43	1.42
Italy	−1.38	0.17	−1.35	−1.42	1.22
Spain	−2.06	−0.21	−1.19	−1.27	0.61

Table 3.3: Changes in the ULCs of each country (annualized rate), 2002-2007

Table 3.4 shows the relative accumulated evolution of the ULC of each country with respect to the evolution of Germany for the period 2002 to 2007. A positive number indicates the possible gain associated with each effect if these countries had had the evolution of Germany. The change in competitiveness within firms was particularly small in Italy and Spain, which implies losses of competitiveness with respect to Germany of 8.75% in Italy and 7% in Spain. More importantly, the smaller reallocation of resources with respect to Germany between 2002 and 2007 implies losses of competitiveness around 4.3% in France, 6.4% in Italy and 8% in Spain.

	Total	Within	Reallocation	Interaction
France	5.22	1.86	4.27	−0.91
Italy	10.37	8.75	6.39	−4.77
Spain	10.82	7.00	7.95	−4.14

Table 3.4: Changes in the ULCs of each country relative to Germany, 2002-2007

Even though the exercise has limitations since we are only looking at manufacturing firms, recent empirical research with sectoral data shows that the reallocation of resources within the sector is key to understand the evolution of aggregate ULC. Given the importance of the reallocation of resources to explain the evolution of the ULC, in the next section we will focus in understanding what would be the productivity gains in each of these countries if there were no misallocation, that is, if all the resources were allocated efficiently.

3.5 RESOURCES' MISALLOCATION: SOURCE OF COUNTRY DIFFERENCES IN PRODUCTIVITY

The ability to reallocate resources within the firms of the economy has a very significant role in the explanation of the evolution of the aggregate ULC. In this section we apply the methodology of Hsieh and Klenow (2009) to explain the impact of an efficient allocation of resources in the productivity and output of France, Germany, Italy and Spain.

3.5.1 *Hsieh and Klenow (2009) Methodology*

Hsieh and Klenow (2009) propose an empirical framework to investigate if large differences in output per worker across countries (or sectors) are due to the fact that there is "misallocation" across plants, firms and sectors. The empirical framework proposed, while based on specific parametric assumptions on preferences and production technology, enables a clean representation of the potential impact of "misallocation" on sectoral or aggregate productivity.

Consider an economy consisting of S sectors and aggregate output is defined as:

$$Y = \prod_{s=1}^{S} Y_s^{\theta_s} \text{ where } \sum_{s=1}^{S} \theta_s = 1. \tag{3.1}$$

Let $P = \prod_{s=1}^{S} \left(\frac{P_s}{\theta_s}\right)^{\theta_s}$ represent the price of the final food, where P_s refers to the price of industry output Y_s. Then, cost minimization implies

$$P_s Y_s = \theta_s PY. \tag{3.2}$$

Industry output Y_s is itself a C.E.S. aggregate of M_s differentiated products:

$$Y_s = \left(\sum_{i=1}^{M_s} Y_{si}^{\frac{\sigma-1}{\sigma}} \right)^{\frac{\sigma}{\sigma-1}},$$

and each firm in sector s has a Cobb-Douglas production function that depends on firm TFP, capital and labor[7]:

$$Y_{si} = A_{si} K_{si}^{\alpha_s} L_{si}^{1-\alpha_s}.$$

Hsieh and Klenow (2009) assume that there are firm specific distortions affecting total production and capital which are modelled as taxes. They denote distortions that increase the marginal products of capital and labor by the same proportion as an output distortion τ_Y, and denote distortions that raise the marginal product of capital relative to labor as the capital distortion τ_K. As a result of these distortions, firms produce different amounts than what would be dictated by their productivity and also may have different capital-labor ratios.[8]

Combining the aggregate demand for capital and labor in a sector, the expression for the price of aggregate industry output and Equation 3.2, aggregate output can then be expressed as a function of K_s, L_s, and industry TFP:

$$Y = \prod_{s=1}^{S} \left(TFP_s \cdot K_s^{\alpha_s} \cdot L_s^{1-\alpha_s} \right)^{\theta_s}. \tag{3.3}$$

To determine the formula for industry productivity TFP_s it has to be noted that when industry deflators are used, differences in plant specific prices show up in the customary measure of plant TFP. Foster et al. (2008) stress the distinction between "physical productivity" (TFPQ) and "revenue productivity" (TFPR).

$$\begin{aligned} TFPQ_{si} &\triangleq A_{si} = \frac{Y_{si}}{K_{si}^{\alpha_s} L_{si}^{1-\alpha_s}}, \\ TFPR_{si} &\triangleq P_{si} A_{si} = \frac{P_{si} Y_{si}}{K_{si}^{\alpha_s} L_{si}^{1-\alpha_s}}. \end{aligned}$$

7 Note that capital and labor shares may differ across industries but not across firms within an industry.
8 See the Appendix for a full derivation of the firm's maximization problem.

If there are no firm specific distortions and all firms within a sector have the same markup (assumed by this framework but obviously not true in general), TFPR will be equalized across firms. In the absence of distortions, more labor and capital should be allocated to plants with higher TFPQ to the point where their higher output results in a lower price and the exact same TFPR as smaller plants. TFPR is proportional to a geometric average of the plant's marginal revenue products of labor and capital:

$$\mathrm{TFPR}_{si} \propto \frac{(1+\tau_{K_{si}})^{\alpha_s}}{1-\tau_{Y_{si}}}. \tag{3.4}$$

High plant TFPR is a sign that the plant faces barriers that raise the plant's marginal products of labor and capital, rendering the plant smaller than optimal. In general, variation of TFPR within a sector will be a measure of misallocation.

Then, the relevant measure of sectoral TFP can be written as:[9]

$$\mathrm{TFP}_s = \left(\sum_{i=1}^{M_s} \left(\mathrm{TFPQ}_{si} \cdot \frac{\overline{\mathrm{TFPR}_s}}{\mathrm{TFPR}_{si}} \right)^{\sigma-1} \right)^{\frac{1}{\sigma-1}}, \tag{3.5}$$

where $\overline{\mathrm{TFPR}_s}$ is the geometric average of the average marginal revenue product of capital and labor in sector s. Intuitively, the extent of misallocation is worse when there is greater dispersion of marginal products.

To see this more clearly, consider a special case where TFPQ_{si} and TFPR_{si} are jointly lognormally distributed, then the expression in Equation 3.5 implies:

$$\log \mathrm{TFP}_s = \frac{1}{\sigma-1} \log \left(\sum_{i=1}^{M_s} A_{si}^{\sigma-1} \right) - \frac{\sigma}{2}\, \mathrm{var}(\log \mathrm{TFPR}_{si}),$$

so that the negative effect of distortions can be summarized by the variance of log TFPR.

3.5.2 *Gains of an Efficient Allocation of Resources in Europe*

In order to determine the gains from an efficient allocation of resources, we calculate "efficient" output in each country so we can compare it with actual output levels. If there

9 See the Appendix C for the full derivation of Equation 3.5.

are no firm specific distortions, TFPR will be equalized across firms within a sector. Then, industry TFP would be $\overline{A_s} = \left(\sum_{i=1}^{M_s} A_{si}^{\sigma-1}\right)^{\frac{1}{\sigma-1}}$. For each industry, we calculate the ratio of actual TFP (Equation 3.5) to this efficient level of TFP, and then aggregate this ratio across sectors using the Cobb-Douglas aggregator (Equation 3.1):

$$\frac{Y}{Y_{efficient}} = \prod_{s=1}^{S} \left[\sum_{i=1}^{M_s} \left(\frac{A_{si}}{\overline{A_s}} \frac{\overline{TFPR_s}}{TFPR_{si}} \right)^{\sigma-1} \right]^{\frac{\theta_s}{\sigma-1}} \tag{3.6}$$

To calculate the effects of resource misallocation, we need to estimate key parameters (industry output shares, industry capital shares, and the firm-specific distortions) from the data.

The data for France, Germany, Italy and Spain are drawn from the joint EFIGE-Amadeus dataset. The information we use are the plant's industry (four-digit level), age (based on reported birth year), wage payments, value-added, export revenues, and capital stock. For labor input we use the plant's wage bill[10] rather than its employment to measure L_{si}. As a later robustness check, we measure L_{si} as employment. We define capital stock as the book value of fixed capital net of depreciation.

We set the rental price of capital (excluding distortions) to $R = 0.10$, we have in mind a 5% real interest rate and a 5% depreciation rate.[11] We set the elasticity of substitution between plant value added to $\sigma = 3$, which ranges within the estimates of the substitutability of competing manufactures in the trade and industrial organization literature (Broda and Weinstein (2006)). Later, we entertain the higher value of 5 and a lower value of 2 for σ as a robustness check. We set the elasticity of output with respect to capital in each industry (α_s) to be 1 minus the labor share in the corresponding industry in Germany in 2008. We adopt the German shares as the benchmark.

On the basis of the other parameters and the plant data, we infer the distortions and productivity for each plant in each country-year as follows:

$$1 + \tau_{K_{si}} = \frac{\alpha_s}{1-\alpha_s} \frac{wL_{si}}{RK_{si}} \tag{3.7}$$

10 The Amadeus data only report wage payments; the information on non-wage compensation is not reported.
11 The actual cost of capital faced by plant i in industry s is denoted $(1+\tau_{K_{si}})R$, so it differs from 10% if $\tau_{K_{si}} \geqslant 0$. Because our hypothetical reforms collapse $\tau_{K_{si}}$ to its average in each industry, if R is set incorrectly, it will affect the average capital distortion but not the experiment itself.

$$1 - \tau_{Y_{si}} = \frac{\sigma}{\sigma - 1} \frac{wL_{si}}{(1-\alpha_s)P_{si}Y_{si}} \quad (3.8)$$

$$A_{si} = \frac{(P_{si}Y_{si})^{\frac{\sigma}{\sigma-1}}}{K_{si}^{\alpha_s} L_{si}^{1-\alpha_s}} \quad (3.9)$$

Before calculating the gains from our hypothetical liberalization, we trim the 1% tails of $\log(TFPR_{si}/\overline{TFPR_s})$ and $\log(A_{si}/\overline{A_s})$ across industries to make the results robust to outliers. We then recalculate wL_s, K_s, P_sY_s, $\overline{TFPR_s}$ and $\overline{A_s}$.

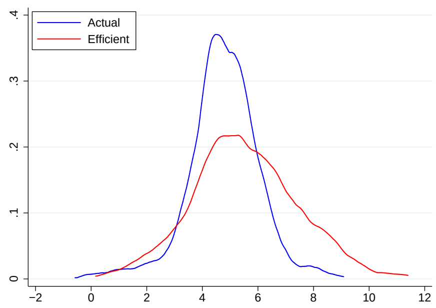
Table 3.5 provides percent TFP gains in each country from fully equalizing TFPR across plants in each industry for the years 2002 to 2008, where the entries are $100(Y_{efficient}/Y - 1)$. As we discussed in Section 3.2, a major shortcoming of the unification of the EFIGE and AMADEUS dataset is that the coverage of Amadeus for the firms surveyed is not 100%. In this exercise, for the years 2002 to 2008, for France, Italy and Spain there is a coverage of 80% to 90% of the firms, whereas for Germany it is considerably lower. Particularly, for the years 2002 and 2003 there is information for less that 50% of the firms, and for the years 2004 to 2008 it ranges between 50% and 70%. Hence, in Table 3.5 we do not report hypothetical gains from an efficient allocation of resources for Germany for the years 2002 and 2003, and the variation in these gains is calculated for the years 2008-2004 instead of 2008-2002.

Removing all barriers, by this calculation, would boost aggregate manufacturing TFP in 2008 by 22.7% in France, 27.9% in Germany, 43.5% in Italy and 28.2% in Spain. More interestingly, we observe that between the years 2002 to 2008, the gains from efficient allocation decrease in Germany (−8.50%), increase in Italy and Spain (6.93% and 6.97%), and are constant in France (−0.82%). This reveals that within this period, in Italy and Spain the "misallocation" of resources within the sector has increased while in France it remains constant and in Germany it decreases. An increase in the "misallocation" of resources in Italy and Spain, reveals an increase in the distortions or barriers to production present in these countries which is consistent with their smaller ability to reallocate market shares towards firms with initially smaller ULC as reported in Table 3.3. At the same time, the decrease in the "misallocation" of resources in Germany is also reflected by the greater ability of reallocating market shares to firms initially lower ULC. The results of the decomposition in the evolution of ULC and an hypothetical efficient allocation of resources are complementary to each other.

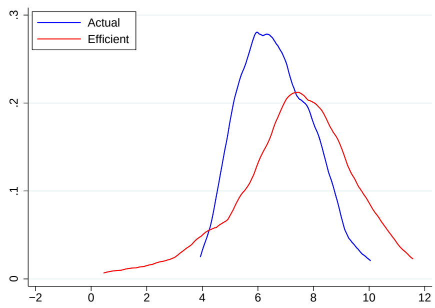
Year	France	Germany	Italy	Spain
2002	23.55		36.41	21.23
2003	19.29		30.46	21.68
2004	22.07	36.41	32.75	23.30
2005	22.43	31.90	30.46	24.66
2006	23.88	32.30	32.97	24.70
2007	20.95	33.25	34.54	28.71
2008	22.74	27.92	43.34	28.20
$\Delta_{2008-2002}$	−0.82	−8.50	6.93	6.97

Table 3.5: TFP Gains from Equalizing TFPR within Industries

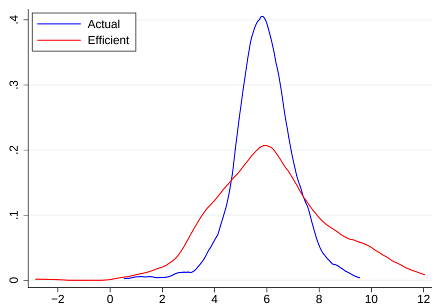
Figure 3.3 plots the "efficient" versus actual size distribution of plants in year 2008, where size is measured as plant value added. In all the countries except Germany, the hypothetical efficient distribution is more dispersed than the actual one. In particular, in all countries, there should be fewer mid-sized plants and more small and large plants. The popular belief is that there are less large firms then there should be due to distortions in the economy, but not that there are less small firms than there should be like the flattening of these distributions is predicting. Hsieh and Klenow (2009) find similar predictions for the analysis they do of China, India and the United States, which suggest that the shape of the efficient plant size distribution is robust across countries. In Germany, the efficient distribution is more dispersed as well, but we observe a shift to the right in the distribution rather than a flattening like it happens in the other countries. The reason behind the different behavior in Germany lies probably in the bias in the size distribution of the German firms present in the AMADEUS dataset that we have explained in Section 3.2. The small firms in terms of employment are very under-represented in the subsection of German firms present in the AMADEUS side of the data (see Figure 3.1), hence the explanation to why there is no flattening in the efficient distribution and the exercise predicts that a large group of the medium sized firms in terms of output should decrease their size.



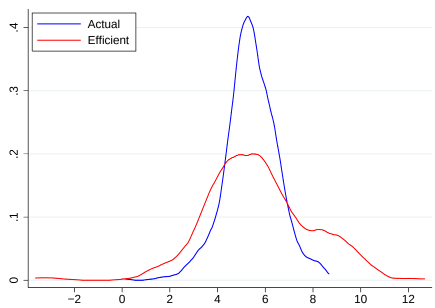
(a) France



(b) Germany



(c) Italy



(d) Spain

Figure 3.3: Distribution of Plant Size

Table 3.6 shows how the size of initially big vs. small plants would change if TFPR were equalized in each country. The entries are unweighted shares of plants. The rows are actual plant size quartiles, and the columns are bins of efficient plant size relative to actual size: 0% – 50% — the plant should shrink by a half or more, 50% – 100% — the plant should shrink by less than half, 100% – 200% — the plant should increase but not double in size, > 200% — the plant should at least double in size.

In all countries, firms with initial smaller size should increase. Particularly for Italy and Spain, not only there is a large number of firms that should increase their size but also that should at least double in size. In all countries, firms with initial size in the 2nd quartile should either shrink by half or at least double in size. This indicates that there is a large number of small medium sized firms that should not be there. In all countries, firms with initial size in the 3rd quartile should shrink. This is particularly relevant for Germany. Finally, firms with initial size in the top quartile should not shrink as much and actually should increase their size, but not double it. That is, large firms should be larger in all countries, whereas medium productivity firms should shrink and there are some small firms that should increase their size given their real productivities.

		$[0\% - 50\%]$	$[50\% - 100\%]$	$[100\% - 200\%]$	$> 200\%$
France	1st quartile	3.84	2.25	8.70	10.29
	2nd quartile	11.97	0.47	0.47	12.07
	3rd quartile	8.04	14.87	1.50	0.56
	Top quartile	1.22	7.39	14.31	2.06
Germany	1st quartile	1.75	2.62	10.92	10.04
	2nd quartile	10.48	2.62	0.0	12.23
	3rd quartile	10.48	14.41	0.0	0.0
	Top quartile	2.18	5.68	14.41	2.62
Italy	1st quartile	2.44	0.57	5.61	16.41
	2nd quartile	14.13	3.49	0.16	7.23
	3rd quartile	7.31	13.81	3.57	0.32
	Top quartile	1.14	7.15	15.68	0.97
Spain	1st quartile	2.91	0.97	9.06	12.08
	2nd quartile	12.84	0.65	0.76	10.79
	3rd quartile	8.20	16.07	0.54	0.22
	Top quartile	1.08	7.34	14.67	1.83

Table 3.6: Actual size vs. Efficient size (Percent of Plants)

We now provide a number of robustness checks to our baseline Table 3.5 calculations of hypothetical efficiency gains. We have measured plant labor input using its wage bill. The logic is that wages per worker adjust for plant differences in hours worked per worker and worker skills. However, wages could also reflect rent sharing between the plant and its workers. If so, we might be interpreting differences in TFPR across plants because the most profitable plants have to pay higher wages. We therefore recalculate the gains from equalizing TFPR in France, Germany, Italy and Spain using simply employment as our measure of plant labor input. The gains from an efficient allocation remain almost unchanged for all countries with the exception of Germany — 21.18% for France, 35.44% for Germany, 42.56% for Italy and 27.58% for Spain in 2008. The intuition behind the smaller gains for Germany when we use the wage bill rather than the employees is that wage differences may be limiting the TFPR differences.

We have assumed an elasticity of substitution within industries (σ) of 3. However the literature on business cycles puts it at 2 while the literature more close to international trade puts it at 5. Our estimates are sensitive to this parameter, with an increase between 10% and 20% in the gains from efficient allocation if $\sigma = 5$, and a decrease of 5% to 10% if $\sigma = 2$. The intuition behind these results, is that when the elasticity of substitution within industries is larger, then TFPR gaps are closed more slowly in response to reallocation of inputs from low to high TFPR plants, enabling bigger gains from equalizing TFPR gains.

Given the dispersion in the size of the firms within the sectors and between countries,[12] a last valid concern might be that the trimming of the productivity measures is large. Firms with extreme productivity values have a high relative weight (following a trend more similar to a Pareto distribution than a Normal distribution), which means that the behaviour of the sector aggregates are strongly influenced by the behaviour of the largest firms (di Giovanni and Levchenko (2009), Altomonte et al. (2010) and Altomonte et al. (2011)). Hence, less trimming (or no trimming at all) in the right tail of the distribution, implies a higher dispersion in the data observed, and we expect larger gains from an hypothetical efficient allocation of resources. To analyze the robustness of the calculations to the dispersion in firm size, we trim only 0.5% of the right tail of $\log(\mathrm{TFPR}_{si}/\overline{\mathrm{TFPR}_s})$ before calculating the hypothetical gains. While the results prove to be sensitive to this trimming, and as expected there is an increase in the gains from an efficient allocation, this

12 In Italy and Spain there are less large firms than in Germany and France. See Crespo (2012) and Rubini et al. (2012).

increase is similar across countries (around 5%) — 26.86% in France, 33.97% in Germany, 49.33% in Italy and 35.46% in Spain. Between 2002 and 2008, the predicted gains from an efficient allocation decrease in 3.64% in France, decrease in 9.20% in Germany, increase in 9.07% in Italy and increase in 10.56%. While the variations are slightly larger, the ranking is unchanged and therefore the conclusions of our exercise are consistent.

3.6 CONCLUSIONS

In this paper, we have analyzed the ability of the change in the aggregate unit labor cost to capture the change in the competitiveness of a country.

Using firm level data, we calculate a weighted change of the aggregate unit labor costs between 2002 and 2007 for four European countries: France, Germany, Italy and Spain. The components of the weighted average are then decomposed according to a Laspeyres decomposition into three main elements: the first captures changes in firm-level unit labor costs, keeping the initial domestic market shares of firms constant; the second quantifies the reallocation of market shares within the domestic economy, keeping the initial unit labor costs constant; and the third measures the interaction between the first two. The results reveal that the evolution of the firm-level unit labor cost does not explain the evolution of the aggregate unit labor costs, rather it is the resource reallocation that drives the evolution of the aggregate unit labor costs.

Motivated by the significant role of the reallocation of resources to explain the evolution of the aggregate ULC, we apply the methodology of Hsieh and Klenow (2009) to analyze the extent to which aggregate productivity differences between these four European countries relate to inefficient resource reallocation. As a result of distortions that affect production, firms produce different amounts than what would be dictated by their productivity. An efficient allocation of resources would boost aggregate manufacturing TFP in 2008 by 22.7% in France, 27.9% in Germany, 43.5% in Italy and 28.2% in Spain.

The empirical analysis of the unit labor costs as a competitiveness measure reveals the need to use microeconomic data to understand the driving factors behind the evolution of macroeconomic aggregates. And the decomposition of the aggregate indicator shows that there are relevant differences among countries which in the aggregate cannot be observed due to the noisiness of the measure.

A

APPENDIX A: PROOFS OF CHAPTER 1

A.1 CLOSED ECONOMY

PRODUCTIVITY DISTRIBUTION AND WEIGHTED AVERAGES

Let us denote by $\eta_D(\varphi)$ and $\eta_{DI}(\varphi)$ respectively, the productivity distribution of domestic producers and active innovators.

$$\eta_D(\varphi) = \begin{cases} \frac{g(\varphi)}{G(\varphi_{DI}) - G(\varphi_D)} & , \varphi_I > \varphi \geqslant \varphi_D; \\ 0 & , otherwise; \end{cases}$$

$$\eta_{DI}(\varphi) = \begin{cases} \frac{g(\varphi)}{1 - G(\varphi_{DI})} & , \varphi \geqslant \varphi_{DI}; \\ 0 & , otherwise; \end{cases}$$

The distributions $\eta_D(\varphi)$ and $\eta_{DI}(\varphi)$ are not affected by the simultaneous entry and exit since the successful entrants and failing incumbents draw their productivity level from the common distribution $\mu(\varphi)$.

Let $\widetilde{\varphi} = \left[\int_{\varphi_D}^{\infty} \varphi^{\sigma-1} \mu(\varphi)\, d\varphi\right]^{\frac{1}{(\sigma-1)}}$ which represents the average productivity of all the firms in the economy prior to innovation and $\widetilde{\varphi_I} = \left[\int_{\varphi_{DI}}^{\infty} \left(\varphi^{\sigma-1}\right)^{\frac{(\alpha+1)}{\alpha}} \eta_{DI}(\varphi)\, d\varphi\right]^{\frac{\alpha}{(\alpha+1)}\frac{1}{(\sigma-1)}}$ which represents the average productivity of the innovators after innovation.

AGGREGATE VARIABLES

Denote by m_I and m respectively the mass of active innovators and non-innovator producers, where

$$m_I = \frac{1 - G(\varphi_{DI})}{1 - G(\varphi_D)} M;$$

$$m = \frac{G(\varphi_{DI}) - G(\varphi_D)}{1 - G(\varphi_D)} M;$$

with M being the mass of incumbent firms in the economy.

Then, it can be shown that the aggregates will take the following expressions

– Aggregate Price Index

$$P^{1-\sigma} = M\left[p_D(\widetilde{\varphi})\right]^{1-\sigma} + m_I z(\varphi_{DI}) \left(\frac{1}{\varphi_{DI}^{\sigma-1}}\right)^{\frac{1}{\alpha}} \left[p_D\left(\widetilde{\varphi_I}^{\left(\frac{\alpha+1}{\alpha}\right)}\right)\right]^{1-\sigma}.$$

Notice that the first term coincides exactly with the aggregate price of the Melitz 2003 economy, therefore we can distinguish exactly the effect that having an innovation choice has on the aggregates of the economy, since this term will be distinguished in every one of the aggregates.

– Aggregate Production

$$Q^{\rho} = M\left[q_D(\widetilde{\varphi})\right]^{\rho} + m_I z(\varphi_{DI}) \left(\frac{1}{\varphi_{DI}^{\sigma-1}}\right)^{\frac{1}{\alpha}} \left[q_D\left(\widetilde{\varphi_I}^{\left(\frac{\alpha+1}{\alpha}\right)}\right)\right]^{\rho}.$$

– Aggregate Revenue

$$R = M \cdot r_D(\widetilde{\varphi}) + m_I z(\varphi_{DI}) \left(\frac{1}{\varphi_{DI}^{\sigma-1}}\right)^{\frac{1}{\alpha}} r_D\left(\widetilde{\varphi_I}^{\left(\frac{\alpha+1}{\alpha}\right)}\right).$$

– Aggregate Profits

$$\Pi = M\frac{r_D(\widetilde{\varphi})}{\sigma} - Mf_D - m_I f_I + m_I \alpha \left(\frac{1}{\alpha+1}\right)^{\frac{\alpha+1}{\alpha}} \left(\frac{r_D(\widetilde{\varphi_I})}{\sigma}\right)^{\left(\frac{\alpha+1}{\alpha}\right)}. \tag{A.1}$$

A.1.1 *Closed Economy Equilibrium*

PROOF OF PROPOSITION 1

Equation 1.4 and Equation 1.5 along with the Free Entry condition (Equation 1.7) uniquely determine the equilibrium and the productivity cutoffs. Rearrange the FE condition conveniently for the characterizing of the equilibrium as a function of φ_D^*.

$$\begin{aligned}\delta f_E &= [1 - G(\varphi_D^*)] \\ \delta f_E &= f_D k_1(\varphi_D^*) + \left(\frac{m_I}{M}\right)\alpha\left(\frac{f_D}{\alpha+1}\right)^{\frac{\alpha+1}{\alpha}} k_2(\varphi_D^*) - \left(\frac{m_I}{M}\right) f_I. \end{aligned} \tag{A.2}$$

where $k_1(\varphi_D^*) = \left[\left(\frac{\tilde{\varphi}(\varphi_D^*)}{\varphi_D^*}\right)^{\sigma-1} - 1\right]$ and $k_2(\varphi_D^*) = \left[\left(\frac{\tilde{\varphi}_I(\varphi_D^*)}{\varphi_D^*}\right)^{(\sigma-1)}\right]^{\frac{\alpha+1}{\alpha}}$.

Proof. We are going to prove that the RHS of Equation A.2 is decreasing in φ_D^* on the domain (φ_D^*, ∞), so that φ_D^* is uniquely determine by the intersection of the latter curve with the flat line δf_E in the (φ_D^*, ∞) space. The last term on Equation A.2 is constant, therefore we only need to show that the other two terms are decreasing.
Remember that $k_1(\varphi_D^*) = \left[\left(\frac{\tilde{\varphi}(\varphi_D^*)}{\varphi_D^*}\right)^{\sigma-1} - 1\right]$, then its derivative with respect to φ_D^* is

$$k_1'(\varphi_D^*) = \frac{g(\varphi_D^*)}{1 - G(\varphi_D^*)} k_1(\varphi_D^*) - \frac{(\sigma-1)\left[k_1(\varphi_D^*) + 1\right]}{\varphi_D^*}.$$

Similarly, $k_2(\varphi_D^*) = \left[\frac{\tilde{\varphi}_I(\varphi_D^*)}{\varphi_D^*}\right]^{(\sigma-1)b}$, and its derivative with respect to φ_D^* is

$$k_2'(\varphi_D^*) = \Lambda^{\frac{1}{\sigma-1}} \frac{g(\varphi_I^*)}{1 - G(\varphi_I^*)}\left[k_2(\varphi_D^*) - \Lambda^{\frac{\alpha+1}{\alpha}}\right] - \left(\frac{\alpha+1}{\alpha}\right)(\sigma-1)\frac{k_2(\varphi_D^*)}{\varphi_D^*},$$

where $\frac{\partial \varphi_{DI}^*}{\partial \varphi_D^*} = \left[\frac{\left(\frac{f_I}{\alpha}\right)^{\frac{\alpha}{\alpha+1}}(\alpha+1)}{f_D}\right]^{\frac{1}{\sigma-1}} = \Lambda^{\frac{1}{\sigma-1}}$.
Define $j_1(\varphi_D^*) = \left[1 - G(\varphi_D^*)\right] k_1(\varphi_D^*)$, and $j_2(\varphi_D^*) = \left[1 - G(\varphi_{DI}^*)\right] k_2(\varphi_D^*)$ which are non-negative.
Then the derivative and elasticity of each of the expressions are respectively

$$j_1'(\varphi_D^*) = -\frac{(\sigma-1)\left[k_1(\varphi_D^*) + 1\right]}{\varphi_D^*}[1 - G(\varphi_D^*)] < 0,$$

$$\frac{j_1'\left(\varphi_D^*\right)\cdot\varphi_D^*}{j_1\left(\varphi_D^*\right)} = \underbrace{-\left(\sigma-1\right)\left[1+\frac{1}{k_1\left(\varphi_D^*\right)}\right]}_{<0 \text{ and bounded away of it}} < -\left(\sigma-1\right),$$

and

$$\begin{aligned} j_2'\left(\varphi_D^*\right) &= -g\left(\varphi_{DI}^*\right)\Lambda^{\frac{1}{\sigma-1}}\Lambda^{\frac{\alpha+1}{\alpha}} - \theta\left(\alpha+1\right)\left(\sigma-1\right)\frac{k_2\left(\varphi_D^*\right)}{\varphi_D^*}\left[1-G\left(\varphi_{DI}^*\right)\right] < 0, \\ \frac{j_2'\left(\varphi_D^*\right)\cdot\varphi_D^*}{j_2\left(\varphi_D^*\right)} &= \underbrace{-\frac{g\left(\varphi_{DI}^*\right)}{\left[1-G\left(\varphi_{DI}^*\right)\right]}\frac{\Lambda^{\frac{1}{\sigma-1}}\Lambda^{\frac{\alpha+1}{\alpha}}}{k_2\left(\varphi_D^*\right)}\varphi_D^* - \left(\frac{\alpha+1}{\alpha}\right)\left(\sigma-1\right)}_{<0 \text{ and bounded away of it}} \\ &< -\left(\frac{\alpha+1}{\alpha}\right)\left(\sigma-1\right). \end{aligned}$$

Therefore, $j_1\left(\varphi_D^*\right)$ and $j_2\left(\varphi_D^*\right)$ must be decreasing to zero as φ goes to infinite. Furthermore, it must be that $\lim_{\varphi_D^*\to 0} j_1\left(\varphi_D^*\right) = \infty$ since $\lim_{\varphi_D^*\to 0} k_1\left(\varphi_D^*\right) = \infty$ and $\lim_{\varphi_D^*\to 0} j_2\left(\varphi_D^*\right) = \infty$ since $\lim_{\varphi_D^*\to 0} k_2\left(\varphi_D^*\right) = \infty$. Hence, $j_1\left(\varphi_D^*\right)$ and $j_2\left(\varphi_D^*\right)$ decrease from ∞ to zero on the parameter space $\left(0,\infty\right)$,and thus the RHS of Equation A.2 is drecreasing on the parameter space. □

COMPARISON OF THE ENTRY CUTOFF WITH MELITZ'S (2003)

Let's denote the cutoff productivity level in a closed economy found in Melitz (2003) by φ_M^*, then we have that $\varphi_D^* > \varphi_M^*$.

Proof. Using the ZPC and the labor market clearing condition, which are common to both models:

$$\left(\frac{\varphi_D^*}{\varphi_M^*}\right)^{\sigma-1} = \frac{P^{1-\sigma}}{P_M^{1-\sigma}},$$

where $P^{1-\sigma} = M\left[p_D\left(\widetilde{\varphi}\right)\right]^{1-\sigma} + m_I\left(\frac{f_I}{\alpha}\right)^{\frac{1}{\alpha+1}}\left(\frac{1}{\varphi_I^{\sigma-1}}\right)^{\frac{1}{\alpha}}\left[p_D\left(\widetilde{\varphi_I}^{\frac{\alpha+1}{\alpha}}\right)\right]^{1-\sigma}$ and $P_M^{1-\sigma} = M\left[p_D\left(\widetilde{\varphi}\right)\right]^{1-\sigma}$.

Let $\varphi_M^* \geqslant \varphi_D^*$, then $P_M^{1-\sigma} \geqslant P^{1-\sigma}$, which implies that

$$\int_{\varphi_M^*}^{\infty} [p_D(\varphi)]^{1-\sigma} \mu(\varphi) \, d\varphi - \int_{\varphi_D^*}^{\infty} [p_D(\varphi)]^{1-\sigma} \mu(\varphi) \, d\varphi \geqslant$$

$$\geqslant \frac{m_I}{M} \left(\frac{f_I}{\alpha}\right)^{\frac{1}{\alpha+1}} \left(\frac{1}{\varphi_I^{\sigma-1}}\right)^{\frac{1}{\alpha}} \left[p_D\left(\widetilde{\varphi_I}^{\frac{\alpha+1}{\alpha}}\right)\right]^{1-\sigma},$$

which is impossible since the RHS is positive and the LHS is negative.
Therefore, it must be that $\varphi_D^* > \varphi_M^*$. □

The ability of some firms to invest in a cost reducing technology enables them to have more market shares than they would without the presence of innovation, logically, those market shares are "stolen" from the less productive firms of the economy, i.e. to enter in the market in this economy a firm must be more productive than in an economy without technology. Hence we have firms that are more efficient but less varieties on the economy.

A.2 OPEN ECONOMY

A.2.1 *Low Cost Innovation Economy*

PRODUCTIVITY DISTRIBUTION AND WEIGHTED AVERAGES

Let us denote by $\mu_D(\varphi)$, $\mu_{DI}(\varphi)$ and $\mu_{XI}(\varphi)$ respectively, the productivity distribution of domestic producers, active innovators and active innovators and exporters prior to innovation.

$$\mu_D(\varphi) = \begin{cases} \frac{g(\varphi)}{G(\varphi_{DI}) - G(\varphi_D)} & , \varphi_{DI} > \varphi \geqslant \varphi_D; \\ 0 & , \text{otherwise}; \end{cases}$$

$$\mu_{DI}(\varphi) = \begin{cases} \frac{g(\varphi)}{G(\varphi_{XI}) - G(\varphi_{DI})} & , \varphi_{XI} \geqslant \varphi \geqslant \varphi_{DI}; \\ 0 & , \text{otherwise}; \end{cases}$$

$$\mu_{XI}(\varphi) = \begin{cases} \frac{g(\varphi)}{1 - G(\varphi_{XI})} & , \varphi \geqslant \varphi_{XI}; \\ 0 & , \text{otherwise}; \end{cases}$$

The distributions $\mu_D(\varphi)$, $\mu_{DI}(\varphi)$ and $\mu_{XI}(\varphi)$ are not affected by the simultaneous entry and exit since the successful entrants and failing incumbents draw their productivity level from the common distribution $\mu(\varphi)$.

Let $\widetilde{\varphi} = \left[\int_{\varphi_D}^{\infty} \varphi^{\sigma-1}\mu(\varphi)\, d\varphi\right]^{\frac{1}{(\sigma-1)}}$ and $\widetilde{\varphi}_X = \left[\int_{\varphi_{XI}}^{\infty} \varphi^{\sigma-1}\mu_{XI}(\varphi)\, d\varphi\right]^{\frac{1}{(\sigma-1)}}$ denote the average productivity levels of, respectively, all firms and exporting firms only prior to innovation. Then the weighted productivity average that reflects the combined market share of all firms can be defined as

$$\widetilde{\varphi}_t = \left\{\frac{1}{M_t}\left[M\widetilde{\varphi}^{\sigma-1} + nM_X\left(\tau^{-1}\widetilde{\varphi}_X\right)^{\sigma-1}\right]\right\}^{\frac{1}{\sigma-1}}.$$

And let $\widetilde{\varphi}_{DI} = \left[\int_{\varphi_{DI}}^{\infty} \left(\varphi^{\sigma-1}\right)^{\frac{(\alpha+1)}{\alpha}} \mu_{DI}(\varphi)\, d\varphi\right]^{\frac{\alpha}{(\alpha+1)}\frac{1}{(\sigma-1)}}$ and $\widetilde{\varphi}_{XI}$ represent the average productivity the domestic innovators and exporter innovators get from innovation. Then the weighted productivity average that reflects the combined market share of innovation can be defined as

$$\widetilde{\varphi}_t^I = \left\{\frac{1}{M_t^I}\left[M_I\widetilde{\varphi}_{DI}^{(\sigma-1)\left(\frac{\alpha+1}{\alpha}\right)} + m_{XI}\left(\left(1+n\tau^{1-\sigma}\right)^{\frac{\alpha+1}{\alpha}} - 1\right)\widetilde{\varphi}_{XI}^{(\sigma-1)\left(\frac{\alpha+1}{\alpha}\right)}\right]\right\}^{\left(\frac{\alpha}{\alpha+1}\right)\left(\frac{1}{\sigma-1}\right)}.$$

AGGREGATE VARIABLES

Denote by m_{XI}, m_{DI} and m_D respectively the mass of active innovators and exporters, active innovators but non-exporters and non-innovators and non-exporters present in the economy,

$$\begin{aligned} m_{XI} &= \frac{1-G(\varphi_{XI})}{1-G(\varphi_D)}M; \\ m_{DI} &= \frac{G(\varphi_{XI})-G(\varphi_{DI})}{1-G(\varphi_D)}M; \\ m_D &= \frac{G(\varphi_{DI})-G(\varphi_D)}{1-G(\varphi_D)}M; \end{aligned}$$

with M being the mass of incumbent firms in the economy, $M_I = m_{DI} + m_{XI}$ the number of firms that perform innovation activities and $M_X = m_{XI}$ the number of firms performing exporting activities. The total number of varieties sold in the economy (by

symmetry) will be $M_t = M + nM_X$, and the total number of varieties coming from innovators will be $M_t^I = M_I + nM_X$.
It can be shown that the aggregates will take the following expressions

- Aggregate Price Index

$$P^{1-\sigma} = M_t \left[p_D\left(\widetilde{\varphi}_t\right)\right]^{1-\sigma} + M_t^I z_D\left(\varphi_{DI}\right)\left(\frac{1}{\varphi_{DI}^{\sigma-1}}\right)^{\frac{1}{\alpha}}\left[p_D\left(\left(\widetilde{\varphi}_t^I\right)^{\frac{\alpha+1}{\alpha}}\right)\right]^{1-\sigma}.$$

- Aggregate Production

$$Q^{\rho} = M_t \left[q_D\left(\widetilde{\varphi}_t\right)\right]^{\rho} + M_t^I z_D\left(\varphi_{DI}\right)\left(\frac{1}{\varphi_{DI}^{\sigma-1}}\right)^{\frac{1}{\alpha}}\left[q_D\left(\left(\widetilde{\varphi}_t^I\right)^{\frac{\alpha+1}{\alpha}}\right)\right]^{\rho}.$$

- Aggregate Revenue

$$R = M_t r_D\left(\widetilde{\varphi}_t\right) + M_t^I z_D\left(\varphi_{DI}\right)\left(\frac{1}{\varphi_{DI}^{\sigma-1}}\right)^{\frac{1}{\alpha}} r_D\left(\left(\widetilde{\varphi}_t^I\right)^{\frac{\alpha+1}{\alpha}}\right).$$

- Aggregate Profits

$$\begin{aligned}\Pi \;=\; & M_t \frac{r_D\left(\widetilde{\varphi}_t\right)}{\sigma} - M f_D - n M_X f_X - M_I f_I + M_I \alpha\left(\frac{1}{\alpha+1}\cdot\frac{r_D\left(\widetilde{\varphi}_I\right)}{\sigma}\right)^{\frac{\alpha+1}{\alpha}} \\ & + m_{XI}\left[\left(1+n\tau^{1-\sigma}\right)^{\frac{\alpha+1}{\alpha}} - 1\right]\alpha\left(\frac{1}{\alpha+1}\right)^{\frac{\alpha+1}{\alpha}}\left(\frac{r_D\left(\widetilde{\varphi}_{XI}\right)}{\sigma}\right)^{\frac{\alpha+1}{\alpha}}. \qquad \text{(A.3)}\end{aligned}$$

A.2.1.1 *Low Cost Innovation Equilibrium*

PROOF OF PROPOSITION 2, PART II

If there are sufficiently high fixed export cost, there exist a single cutoff φ_{XI}^* *that solves Equation 1.12*

Proof. The proof is divided in three sections

First, I show that the LHS of Equation 1.12 is positive with respect to the productivity parameter. $\pi_{XI}\left(\varphi_{XI}\right) - \pi_{DI}\left(\varphi_{XI}\right) \geqslant 0$

$$\left[\left(1+n\tau^{1-\sigma}\right)^{\frac{\alpha+1}{\alpha}}-1\right]\alpha\left(\tfrac{1}{\alpha+1}\right)^{\frac{\alpha+1}{\alpha}}\left[\left(\tfrac{R(P\rho)^{\sigma-1}}{\sigma}\right)\varphi^{\sigma-1}\right]^{\frac{\alpha+1}{\alpha}}+n\tau^{1-\sigma}\left(\tfrac{R(P\rho)^{\sigma-1}}{\sigma}\right)\varphi^{\sigma-1}-nf_x\geqslant 0;$$

$$\begin{aligned} C_1\left(\varphi^{\sigma-1}\right)^{\frac{\alpha+1}{\alpha}}+C_2\varphi^{\sigma-1}-nf_x &\geqslant 0; \\ \frac{\partial LHS}{\partial\varphi}=C_1\left(\frac{\alpha+1}{\alpha}\right)(\sigma-1)\,\varphi^{\left(\frac{\alpha+1}{\alpha}\right)(\sigma-1)-1}+C_2(\sigma-1)\,\varphi^{\sigma-2} &> 0. \end{aligned}$$

Secondly, I show that $\pi_{XI}(\varphi_{DI})-\pi_{DI}(\varphi_{DI})<0$, otherwise the firm would choose to export and innovate instead of being indifferent between innovating or not while staying in the domestic market.

$$\begin{aligned} &\pi_{XI}(\varphi_{DI})-\pi_{DI}(\varphi_{DI})<0, \\ &\left[\left(1+n\tau^{1-\sigma}\right)^{\frac{\alpha+1}{\alpha}}-1\right]f_I+n\tau^{1-\sigma}\left(\frac{f_I}{\alpha}\right)^{\frac{\alpha}{\alpha+1}}(\alpha+1)-nf_X<0. \end{aligned}$$

Thus, for f_X large enough, that is for

$$f_X>\left[\left(1+n\tau^{1-\sigma}\right)^{\frac{\alpha+1}{\alpha}}-1\right]\frac{f_I}{n}+\tau^{1-\sigma}\left(\frac{f_I}{\alpha}\right)^{\frac{\alpha}{\alpha+1}}(\alpha+1)$$

it holds that $\pi_{XI}(\varphi_{DI})-\pi_{DI}(\varphi_{DI})<0$

Finally, I show that the difference between the profits of the exporting and non-exporting strategies while innovation goes to infinite as the productivity of the firm is larger.

If $\varphi\to\infty$,then $\pi_{XI}(z_D(\varphi))-\pi_{DI}(z_D(\varphi))\to\infty$, since by definition $\pi_{XI}(z_X(\varphi))>\pi_{XI}(z_D(\varphi))$ then it must be that $\pi_{XI}(z_X(\varphi))-\pi_{DI}(z_D(\varphi))\to\infty$ as $\varphi\to\infty$

$$\begin{aligned} \pi_{XI}(z_D(\varphi))-\pi_{DI}(z_D(\varphi)) &= n\tau^{1-\sigma}[1+z]\left(\frac{R(P\rho)^{\sigma-1}}{\sigma}\right)\varphi^{\sigma-1}-nf_X \\ &= n\tau^{1-\sigma}\left(\frac{1}{\alpha+1}\right)^{\frac{1}{\alpha}}\left[\left(\frac{R(P\rho)^{\sigma-1}}{\sigma}\right)\varphi^{\sigma-1}\right]^{\frac{\alpha+1}{\alpha}} \\ &\quad +n\tau^{1-\sigma}\left(\frac{R(P\rho)^{\sigma-1}}{\sigma}\right)\varphi^{\sigma-1}-nf_X. \end{aligned}$$

$$\lim_{\varphi\to\infty}\left[\pi_{XI}\left(z_D\left(\varphi\right)\right)-\pi_{DI}\left(z_D\left(\varphi\right)\right)\right]=\lim_{\varphi\to\infty}\left[C_4\left[\varphi^{\sigma-1}\right]^{\frac{\alpha+1}{\alpha}}+C_5\varphi^{\sigma-1}-C_6\right]$$
$$=\lim_{\varphi\to\infty}\left[C_4\left[\varphi^{\sigma-1}\right]^{\frac{\alpha+1}{\alpha}}\right]+\lim_{\varphi\to\infty}\left[C_5\varphi^{\sigma-1}\right]-\lim_{\varphi\to\infty}\left(C_6\right)\to\infty.$$

□

PROOF OF PROPOSITION 2, PART I

Equation 1.10 to Equation 1.12 along with the Free Entry condition (Equation 1.13) completely determine the equilibrium and the productivity cutoffs. Rearrange the FE conveniently for the characterizing of the equilibrium as a function of φ_D^*

$$\begin{aligned}\delta f_E &= \left[1-G\left(\varphi_D^*\right)\right]\overline{\pi}\\ \delta f_E &= f_D j_1\left(\varphi_D^*\right)+n\tau^{1-\sigma}f_D j_2\left(\varphi_X^*\left(\varphi_D^*\right)\right)-\left[1-G\left(\varphi_{XI}^*\right)\right]nf_X\\ &\quad-\left[1-G\left(\varphi_{DI}^*\right)\right]f_I+\alpha\left(\frac{1}{\alpha+1}\right)^{\frac{\alpha+1}{\alpha}}f_D^{\frac{\alpha+1}{\alpha}}j_3\left(\varphi_D^*\right)\\ &\quad+\alpha\left(\frac{1}{\alpha+1}\right)^{\frac{\alpha+1}{\alpha}}f_D^{\frac{\alpha+1}{\alpha}}\left[\left(1+n\tau^{1-\sigma}\right)^{\frac{\alpha+1}{\alpha}}-1\right]j_4\left(\varphi_D^*\right).\end{aligned}\tag{A.4}$$

where $j_1\left(\varphi_D^*\right)=\left[\left(\tilde{\varphi}\left(\varphi_D^*\right)/\varphi_D^*\right)^{\sigma-1}-1\right]$,
$j_2\left(\varphi_D^*\right)=\left(\tilde{\varphi}_x\left(\varphi_D^*\right)/\varphi_D^*\right)^{\sigma-1}\left[1-G\left(\varphi_{XI}^*\right)\right]$,
$j_3\left(\varphi_D^*\right)=\left[\left(\tilde{\varphi}_{DI}\left(\varphi_D^*\right)/\varphi_D^*\right)^{\sigma-1}\right]^{\frac{\alpha+1}{\alpha}}\left[1-G\left(\varphi_{DI}^*\right)\right]$,
and $j_4\left(\varphi_D^*\right)=\left[\left(\tilde{\varphi}_{XI}\left(\varphi_D^*\right)/\varphi_D^*\right)^{\sigma-1}\right]^{\frac{\alpha+1}{\alpha}}\left[1-G\left(\varphi_{XI}^*\right)\right]$.

Proof.

Assume the parameter restrictions $\tau^{\sigma-1}f_X\geqslant\frac{\left[\left(1+n\tau^{1-\sigma}\right)^{\frac{\alpha+1}{\alpha}}-1\right]}{n\tau^{1-\sigma}}f_I+\left(\frac{f_I}{\alpha}\right)^{\frac{\alpha}{\alpha+1}}\left(\alpha+1\right)$ and $\left(\frac{f_I}{\alpha}\right)^{\frac{\alpha}{\alpha+1}}\left(\alpha+1\right)\geqslant f_D$ hold, then the Low Cost Innovation Equilibrium exists and is unique. I shall proof that the RHS of Equation A.4 is decreasing in φ_D^* on the domain $\left(\varphi_D^*,\infty\right)$, so that φ_D^* is uniquely determined by the intersection of the latter curve with the flat line δf_E in the $\left(\varphi_D^*,\infty\right)$ space.
Let $k_1\left(\varphi_D^*\right)=\left[\left(\tilde{\varphi}\left(\varphi_D^*\right)/\varphi_D^*\right)^{\sigma-1}-1\right]$, then

$$k_1'\left(\varphi_D^*\right) = \frac{g\left(\varphi_D^*\right)}{1-G\left(\varphi_D^*\right)} k_1\left(\varphi_D^*\right) - \frac{(\sigma-1)\left[k_1\left(\varphi_D^*\right)+1\right]}{\varphi_D^*}.$$

Similarly, $k_3\left(\varphi_D^*\right) = \left[\left(\widetilde{\varphi}_{DI}\left(\varphi_D^*\right)/\varphi_D^*\right)^{\sigma-1}\right]^{\frac{\alpha+1}{\alpha}}$, thus

$$k_3'\left(\varphi_D^*\right) = \Lambda^{\frac{1}{\sigma-1}} \frac{g\left(\varphi_I^*\right)}{1-G\left(\varphi_I^*\right)}\left[k_2\left(\varphi_D^*\right) - \Lambda^{\frac{\alpha+1}{\alpha}}\right] - \left(\frac{\alpha+1}{\alpha}\right)(\sigma-1)\frac{k_2\left(\varphi_D^*\right)}{\varphi_D^*},$$

where $\frac{\partial \varphi_{DI}^*}{\partial \varphi_D^*} = \left[\frac{\left(\frac{f_I}{\alpha}\right)^{\frac{\alpha}{\alpha+1}}(\alpha+1)}{f_D}\right]^{\frac{1}{\sigma-1}} = \Lambda^{\frac{1}{\sigma-1}}$.

Define $j_1\left(\varphi_D^*\right) = \left[1-G\left(\varphi_D^*\right)\right] k_1\left(\varphi_D^*\right)$, and $j_3\left(\varphi_D^*\right) = \left[1-G\left(\varphi_{DI}^*\right)\right] k_3\left(\varphi_D^*\right)$ which are non-negative.

Then the derivative and elasticity of $j_1\left(\varphi_D^*\right)$ and $j_3\left(\varphi_D^*\right)$ are

$$j_1'\left(\varphi_D^*\right) = -\frac{(\sigma-1)\left[k_1\left(\varphi_D^*\right)+1\right]}{\varphi_D^*}\left[1-G\left(\varphi_D^*\right)\right] < 0,$$

$$\frac{j_1'\left(\varphi_D^*\right)\cdot\varphi_D^*}{j_1\left(\varphi_D^*\right)} = \underbrace{-(\sigma-1)\left[1+\frac{1}{k_1\left(\varphi_D^*\right)}\right]}_{<0 \text{ and bounded away of it}} < -(\sigma-1),$$

and

$$j_3'\left(\varphi_D^*\right) = -g\left(\varphi_{DI}^*\right)\Lambda^{\frac{1}{\sigma-1}}\Lambda^{\frac{\alpha+1}{\alpha}} - \theta(\alpha+1)(\sigma-1)\frac{k_3\left(\varphi_D^*\right)}{\varphi_D^*}\left[1-G\left(\varphi_{DI}^*\right)\right] < 0,$$

$$\frac{j_3'\left(\varphi_D^*\right)\cdot\varphi_D^*}{j_3\left(\varphi_D^*\right)} = \underbrace{-\frac{g\left(\varphi_{DI}^*\right)}{\left[1-G\left(\varphi_{DI}^*\right)\right]}\frac{\Lambda^{\frac{1}{\sigma-1}}\Lambda^{\frac{\alpha+1}{\alpha}}}{k_2\left(\varphi_D^*\right)}\varphi_D^* - \left(\frac{\alpha+1}{\alpha}\right)(\sigma-1)}_{<0 \text{ and bounded away of it}} < -\left(\frac{\alpha+1}{\alpha}\right)(\sigma-1)$$

$$< -\left(\frac{\alpha+1}{\alpha}\right)(\sigma-1).$$

Thus, $j_1\left(\varphi_D^*\right)$ and $j_3\left(\varphi_D^*\right)$ must be decreasing to zero as φ goes to infinite. Furthermore, it must be that $\lim_{\varphi_D^*\to 0} j_1\left(\varphi_D^*\right) = \infty$ since $\lim_{\varphi_D^*\to 0} k_1\left(\varphi_D^*\right) = \infty$.and $\lim_{\varphi_D^*\to 0} j_3\left(\varphi_D^*\right) = \infty$ since

$\lim_{\varphi_D^* \to 0} k_3 \left(\varphi_D^*\right) = \infty$. Since $j_1 \left(\varphi_D^*\right)$ and $j_3 \left(\varphi_D^*\right)$, it follows that $j_2 \left(\varphi_D^*\right)$ and $j_4 \left(\varphi_D^*\right)$ do also monotonically decrease from infinite to zero on the $(0, \infty)$ parameter space. Therefore, the RHS of Equation A.4 is a monotonic decreasing function from infinity to zero on the space $(0, \infty)$ that cuts the FE flat line from above identifying a unique cutoff level φ_D^*. □

A.2.2 *Low Cost Trade Economy*

PRODUCTIVITY DISTRIBUTION AND WEIGHTED AVERAGES

Let us denote by $\mu_D(\varphi)$, $\mu_X(\varphi)$ and $\mu_{XI}(\varphi)$ respectively, the productivity distribution of domestic producers, exporters and innovators exporters.

$$\mu_D(\varphi) = \begin{cases} \frac{g(\varphi)}{G(\varphi_X) - G(\varphi_D)} & , \varphi_X > \varphi \geqslant \varphi_D; \\ 0 & , \text{otherwise}; \end{cases}$$
$$\mu_X(\varphi) = \begin{cases} \frac{g(\varphi)}{G(\varphi_{XI}) - G(\varphi_X)} & , \varphi_{XI} \geqslant \varphi \geqslant \varphi_X; \\ 0 & , \text{otherwise}; \end{cases}$$
$$\mu_{XI}(\varphi) = \begin{cases} \frac{g(\varphi)}{1 - G(\varphi_{XI})} & , \varphi \geqslant \varphi_{XI}; \\ 0 & , \text{otherwise}; \end{cases}$$

The distributions $\mu_D(\varphi)$, $\mu_X(\varphi)$ and $\mu_{XI}(\varphi)$ are not affected by the simultaneous entry and exit since the successful entrants and failing incumbents draw their productivity level from the common distribution $\mu(\varphi)$.

Let $\tilde{\varphi} = \left[\int_{\varphi_D}^{\infty} \varphi^{\sigma-1} \mu(\varphi)\, d\varphi\right]^{\frac{1}{(\sigma-1)}}$ and $\tilde{\varphi}_X = \left[\int_{\varphi_{XI}}^{\infty} \varphi^{\sigma-1} \mu_{XI}(\varphi)\, d\varphi\right]^{\frac{1}{(\sigma-1)}}$ denote the average productivity levels of, respectively, all firms and exporting firms only prior to innovation. Then the weighted productivity average that reflects the combined market share of all firms can be defined as

$$\tilde{\varphi}_t = \left\{\frac{1}{M_t}\left[M\tilde{\varphi}^{\sigma-1} + nM_X\left(\tau^{-1}\tilde{\varphi}_X\right)^{\sigma-1}\right]\right\}^{\frac{1}{\sigma-1}}.$$

And let $\widetilde{\varphi}_{XI} = \left[\int_{\varphi_{XI}}^{\infty} \left(\varphi^{\sigma-1}\right)^{\frac{(\alpha+1)}{\alpha}} \mu_{XI}(\varphi)\, d\varphi\right]^{\frac{\alpha}{(\alpha+1)}\frac{1}{(\sigma-1)}}$ represent the average productivity the innovators get from innovation.

AGGREGATE VARIABLES

Denote by m_{XI}, m_X and m_D respectively the mass of active innovators and exporters, only exporters and non-innovators non-exporters present in the economy,

$$\begin{aligned} m_{XI} &= \frac{1-G(\varphi_{XI})}{1-G(\varphi_D)} M; \\ m_X &= \frac{G(\varphi_{XI})-G(\varphi_X)}{1-G(\varphi_D)} M; \\ m_D &= \frac{G(\varphi_X)-G(\varphi_D)}{1-G(\varphi_D)} M; \end{aligned}$$

with M being the mass of incumbent firms in the economy, $M_I = m_{XI}$ the number of firms that perform innovation activities and $M_X = m_X + m_{XI}$ the number of firms performing exporting activities. The total number of varieties sold in the economy (by symmetry) will be $M_t = M + nM_X$.

It can be shown that the aggregates will take the following expressions

- Aggregate Price Index

$$\begin{aligned} P^{1-\sigma} &= M_t \left[p_D\left(\widetilde{\varphi}_t\right)\right]^{1-\sigma} \\ &+ m_{XI}\left(1+n\tau^{1-\sigma}\right) z_X(\varphi_{XI}) \left(\frac{1}{\varphi_{XI}^{\sigma-1}}\right)^{\frac{1}{\alpha}} \left[p_D\left(\widetilde{\varphi}_{XI}^{\left(\frac{\alpha+1}{\alpha}\right)}\right)\right]^{1-\sigma}. \end{aligned}$$

- Aggregate Production

$$\begin{aligned} Q^{\rho} &= M_t \left[q_D\left(\widetilde{\varphi}_t\right)\right]^{\rho} \\ &+ m_{XI}\left(1+n\tau^{1-\sigma}\right) z_X(\varphi_{XI}) \left(\frac{1}{\varphi_{XI}^{\sigma-1}}\right)^{\frac{1}{\alpha}} \left[q_D\left(\widetilde{\varphi}_{XI}^{\left(\frac{\alpha+1}{\alpha}\right)}\right)\right]^{\rho}. \end{aligned}$$

– Aggregate Revenue

$$R = M_t r_D(\widetilde{\varphi}_t) + m_{XI}\left(1+n\tau^{1-\sigma}\right) z_X(\varphi_{XI})\left(\frac{1}{\varphi_{XI}^{\sigma-1}}\right)^{\frac{1}{\alpha}} r_D\left(\widetilde{\varphi}_{XI}^{\left(\frac{\alpha+1}{\alpha}\right)}\right).$$

– Aggregate Profits

$$\begin{aligned}\Pi &= M_t\frac{r_D(\widetilde{\varphi}_t)}{\sigma} - Mf_D - nM_Xf_X - m_{XI}f_I \\ &\quad + m_{XI}\left(1+n\tau^{1-\sigma}\right)\alpha\left(\frac{1}{\alpha+1}\right)^{\frac{\alpha+1}{\alpha}}\left(\frac{r_D(\widetilde{\varphi}_I)}{\sigma}\right)^{\frac{\alpha+1}{\alpha}}.\end{aligned} \tag{A.5}$$

A.2.2.1 *Low Cost Trade Economy Equilibrium*

PROOF OF PROPOSITION 3

Equation 1.14 to Equation 1.16 along with the Free Entry condition (Equation 1.17) completely determine the equilibrium and the productivity cutoffs. Rearrange the FE conveniently for the characterizing of the equilibrium as a function of φ_D^*

$$\begin{aligned}\delta f_E &= [1-G(\varphi_D^*)]\,\overline{\pi} \\ \delta f_E &= f_D l_1(\varphi_D^*) + n f_X l_2(\varphi_X^*(\varphi_D^*)) \\ &\quad + \alpha\left(\frac{1}{\alpha+1}\right)^{\frac{\alpha+1}{\alpha}}\left[f_D\left(1+\tau^{1-\sigma}\right)\right]^{\left(\frac{\alpha+1}{\alpha}\right)} l_3(\varphi_D^*) - [1-G(\varphi_{XI}^*)]\,f_I.\end{aligned} \tag{A.6}$$

where $j_1(\varphi_D^*) = \left[\left(\widetilde{\varphi}(\varphi_D^*)/\varphi_D^*\right)^{\sigma-1} - 1\right]$,

$j_2(\varphi_X^*(\varphi_D^*)) = \left[\left(\widetilde{\varphi}(\varphi_X^*)/\varphi_X^*\right)^{\sigma-1} - 1\right][1-G(\varphi_X^*)]$,

and $j_3(\varphi_D^*) = \left[\left(\widetilde{\varphi}_{XI}(\varphi_D^*)/\varphi_D^*\right)^{\sigma-1}\right]^{\left(\frac{\alpha+1}{\alpha}\right)}[1-G(\varphi_{XI}^*)]$.

Proof.

Assume the parameter restriction $\frac{\left(\frac{f_I}{\alpha}\right)^{\frac{\alpha}{\alpha+1}}(\alpha+1)}{(1+n\tau^{1-\sigma})} \geqslant \tau^{\sigma-1}f_X \geqslant f_D$ holds, then the Low Cost Trade Equilibrium exists and is unique. I shall proof that the RHS of Equation A.6 is

decreasing in φ_D^* on the domain $\left(\varphi_D^*, \infty\right)$, so that φ_D^* is uniquely determined by the intersection of the latter curve with the flat line δf_E in the $\left(\varphi_D^*, \infty\right)$ space.
Let $k_1\left(\varphi_D^*\right) = \left[\left(\widetilde{\varphi}\left(\varphi_D^*\right)/\varphi_D^*\right)^{\sigma-1} - 1\right]$, then

$$k_1'\left(\varphi_D^*\right) = \frac{g\left(\varphi_D^*\right)}{1-G\left(\varphi_D^*\right)} k_1\left(\varphi_D^*\right) - \frac{(\sigma-1)\left[k_1\left(\varphi_D^*\right)+1\right]}{\varphi_D^*}.$$

Similarly, $k_3\left(\varphi_D^*\right) = \left[\left(\widetilde{\varphi}_{DI}\left(\varphi_D^*\right)/\varphi_D^*\right)^{\sigma-1}\right]^{\frac{\alpha+1}{\alpha}}$, thus

$$k_3'\left(\varphi_D^*\right) = \Lambda^{\frac{1}{\sigma-1}} \frac{g\left(\varphi_I^*\right)}{1-G\left(\varphi_I^*\right)}\left[k_2\left(\varphi_D^*\right) - \Lambda^{\frac{\alpha+1}{\alpha}}\right] - \left(\frac{\alpha+1}{\alpha}\right)(\sigma-1)\frac{k_2\left(\varphi_D^*\right)}{\varphi_D^*},$$

where $\frac{\partial \varphi_{DI}^*}{\partial \varphi_D^*} = \left[\frac{\left(\frac{f_I}{\alpha}\right)^{\frac{\alpha}{\alpha+1}}(\alpha+1)}{f_D}\right]^{\frac{1}{\sigma-1}} = \Lambda^{\frac{1}{\sigma-1}}$.
Define $j_1\left(\varphi_D^*\right) = \left[1-G\left(\varphi_D^*\right)\right] k_1\left(\varphi_D^*\right)$, and $j_2\left(\varphi_D^*\right) = \left[1-G\left(\varphi_{DI}^*\right)\right] k_2\left(\varphi_D^*\right)$ which are non-negative.
Then the derivative and elasticity of $j_1\left(\varphi_D^*\right)$ and $j_3\left(\varphi_D^*\right)$ are

$$j_1'\left(\varphi_D^*\right) = -\frac{(\sigma-1)\left[k_1\left(\varphi_D^*\right)+1\right]}{\varphi_D^*}\left[1-G\left(\varphi_D^*\right)\right] < 0,$$
$$\frac{j_1'\left(\varphi_D^*\right)\cdot\varphi_D^*}{j_1\left(\varphi_D^*\right)} = \underbrace{-(\sigma-1)\left[1+\frac{1}{k_1\left(\varphi_D^*\right)}\right]}_{<0 \text{ and bounded away of it}} < -(\sigma-1),$$

and

$$\begin{aligned}
j_3'\left(\varphi_D^*\right) &= -g\left(\varphi_{DI}^*\right)\Lambda^{\frac{1}{\sigma-1}}\Lambda^{\frac{\alpha+1}{\alpha}} - \theta(\alpha+1)(\sigma-1)\frac{k_3\left(\varphi_D^*\right)}{\varphi_D^*}\left[1-G\left(\varphi_{DI}^*\right)\right] < 0, \\
\frac{j_3'\left(\varphi_D^*\right)\cdot\varphi_D^*}{j_3\left(\varphi_D^*\right)} &= \underbrace{-\frac{g\left(\varphi_{DI}^*\right)}{\left[1-G\left(\varphi_{DI}^*\right)\right]}\frac{\Lambda^{\frac{1}{\sigma-1}}\Lambda^{\frac{\alpha+1}{\alpha}}}{k_2\left(\varphi_D^*\right)}\varphi_D^* - \left(\frac{\alpha+1}{\alpha}\right)(\sigma-1)}_{<0 \text{ and bounded away of it}} \\
&< -\left(\frac{\alpha+1}{\alpha}\right)(\sigma-1).
\end{aligned}$$

Thus, $j_1\left(\varphi_D^*\right)$ and $j_3\left(\varphi_D^*\right)$ must be decreasing to zero as φ goes to infinite. Furthermore, it must be that $\lim_{\varphi_D^*\to 0} j_1\left(\varphi_D^*\right)=\infty$ since $\lim_{\varphi_D^*\to 0} k_1\left(\varphi_D^*\right)=\infty$ and $\lim_{\varphi_D^*\to 0} j_3\left(\varphi_D^*\right)=\infty$ since $\lim_{\varphi_D^*\to 0} k_3\left(\varphi_D^*\right)=\infty$. Since $j_1\left(\varphi_D^*\right)$ and $j_3\left(\varphi_D^*\right)$ are decreasing from infinity to zero on $(0,\infty)$,from the closed economy case, it follows that $j_2\left(\varphi_X^*\left(\varphi_D^*\right)\right)$ does also monotonically decrease from infinite to zero on the $(0,\infty)$ parameter space.

Therefore, the RHS of Equation A.6 is a monotonic decreasing function from infinity to zero on the space $(0,\infty)$ that cuts the FE flat line from above identifying a unique cutoff level φ_D^*. □

A.2.3 *Intermediate Economy*

PRODUCTIVITY DISTRIBUTION AND WEIGHTED AVERAGES

Let us denote by $\mu_D(\varphi)$, and $\mu_{XI}(\varphi)$ respectively, the productivity distribution of domestic producers, and active innovators and exporters prior to innovation.

$$\mu_D(\varphi)=\begin{cases}\frac{g(\varphi)}{G(\varphi_{XI})-G(\varphi_D)} & ,\ \varphi_{XI}>\varphi\geqslant\varphi_D;\\ 0 & ,\ otherwise;\end{cases}$$

$$\mu_{XI}(\varphi)=\begin{cases}\frac{g(\varphi)}{1-G(\varphi_{XI})} & ,\ \varphi\geqslant\varphi_{XI};\\ 0 & ,\ otherwise;\end{cases}$$

The distributions $\mu_D(\varphi)$, and $\mu_{XI}(\varphi)$ are not affected by the simultaneous entry and exit since the successful entrants and failing incumbents draw their productivity level from the common distribution $\mu(\varphi)$.

Let $\widetilde{\varphi}=\left[\int_{\varphi_D}^{\infty}\varphi^{\sigma-1}\mu(\varphi)\,d\varphi\right]^{\frac{1}{(\sigma-1)}}$ and $\widetilde{\varphi}_X=\left[\int_{\varphi_{XI}}^{\infty}\varphi^{\sigma-1}\mu_{XI}(\varphi)\,d\varphi\right]^{\frac{1}{(\sigma-1)}}$ denote the average productivity levels of, respectively, all firms and exporting firms only prior to innovation. Then the weighted productivity average that reflects the combined market share of all firms can be defined as

$$\widetilde{\varphi}_t=\left\{\frac{1}{M_t}\left[M\widetilde{\varphi}^{\sigma-1}+nM_X\left(\tau^{-1}\widetilde{\varphi}_X\right)^{\sigma-1}\right]\right\}^{\frac{1}{\sigma-1}}.$$

And let $\widetilde{\varphi}_{XI} = \left[\int_{\varphi_{XI}}^{\infty} \left(\varphi^{\sigma-1}\right)^{\frac{(\alpha+1)}{\alpha}} \mu_{DI}(\varphi)\, d\varphi\right]^{\frac{\alpha}{(\alpha+1)}\frac{1}{(\sigma-1)}}$ represent the average productivity exporter innovators get from innovation.

AGGREGATE VARIABLES

Denote by m_{XI} and m_D respectively the mass of active innovators and exporters, and non-innovators and non-exporters present in the economy,

$$\begin{aligned} m_{XI} &= \frac{1-G(\varphi_{XI})}{1-G(\varphi_D)} M; \\ m_D &= \frac{G(\varphi_{XI})-G(\varphi_D)}{1-G(\varphi_D)} M; \end{aligned}$$

with M being the mass of incumbent firms in the economy, $M_I = m_{XI}$ the number of firms that perform innovation activities and $M_X = m_{XI}$ the number of firms performing exporting activities. The total number of varieties sold in the economy (by symmetry) will be $M_t = M + nM_X$.

It can be shown that the aggregates will take the following expressions

- Aggregate Price Index

$$\begin{aligned} P^{1-\sigma} &= M_t \left[p_D(\widetilde{\varphi}_t)\right]^{1-\sigma} \\ &+ M_I z_D(\varphi_{XI}) \left(\frac{1}{\varphi_{XI}^{\sigma-1}}\right)^{\frac{1}{\alpha}} \left(1+n\tau^{1-\sigma}\right)^{\frac{\alpha+1}{\alpha}} \left[p_D\left(\left(\widetilde{\varphi}_t^I\right)^{\frac{\alpha+1}{\alpha}}\right)\right]^{1-\sigma}. \end{aligned}$$

- Aggregate Production

$$\begin{aligned} Q^{\rho} &= M_t \left[q_D(\widetilde{\varphi}_t)\right]^{\rho} \\ &+ M_I z_D(\varphi_{XI}) \left(\frac{1}{\varphi_{XI}^{\sigma-1}}\right)^{\frac{1}{\alpha}} \left(1+n\tau^{1-\sigma}\right)^{\frac{\alpha+1}{\alpha}} \left[q_D\left(\left(\widetilde{\varphi}_t^I\right)^{\frac{\alpha+1}{\alpha}}\right)\right]^{\rho} \end{aligned}$$

.

- Aggregate Revenue

$$R = M_t r_D(\widetilde{\varphi}_t) + M_I z_D(\varphi_{XI}) \left(\frac{1}{\varphi_{XI}^{\sigma-1}}\right)^{\frac{1}{\alpha}} \left(1+n\tau^{1-\sigma}\right)^{\frac{\alpha+1}{\alpha}} r_D\left(\left(\widetilde{\varphi}_t^I\right)^{\frac{\alpha+1}{\alpha}}\right).$$

– Aggregate Profits

$$\begin{aligned}\Pi &= M_t \frac{r_D(\widetilde{\varphi}_t)}{\sigma} - Mf_D - nM_X f_X - M_I f_I \\ &+ M_I \left(1+n\tau^{1-\sigma}\right)^{\frac{\alpha+1}{\alpha}} \alpha \left(\frac{1}{\alpha+1}\right)^{\frac{\alpha+1}{\alpha}} \left(\frac{r_D(\widetilde{\varphi}_{XI})}{\sigma}\right)^{\frac{\alpha+1}{\alpha}}. \qquad \text{(A.7)}\end{aligned}$$

A.2.3.1 *Intermediate Equilibrium*

PROOF OF PROPOSITION 4, PART II

There exist a single cutoff φ^*_{XI} *that solves Equation 1.19*

Proof. The proof is divided in three sections

First, I show that the LHS of Equation 1.19 is positive with respect to the productivity parameter. $\pi_{XI}(\varphi_{XI}) - \pi_D(\varphi_{XI}) \geqslant 0$

$$\left(1+n\tau^{1-\sigma}\right)^{\frac{\alpha+1}{\alpha}} \alpha \left(\frac{1}{\alpha+1}\right)^{\frac{\alpha+1}{\alpha}} \left[\left(\frac{R(P\rho)^{\sigma-1}}{\sigma}\right)\varphi^{\sigma-1}\right]^{\frac{\alpha+1}{\alpha}} + n\tau^{1-\sigma}\left(\frac{R(P\rho)^{\sigma-1}}{\sigma}\right)\varphi^{\sigma-1} - nf_X - nf_I \geqslant 0;$$

$$\begin{aligned} C_1\left(\varphi^{\sigma-1}\right)^{\frac{\alpha+1}{\alpha}} + C_2\varphi^{\sigma-1} - nf_X - f_I &\geqslant 0; \\ \frac{\partial LHS}{\partial \varphi} = C_1\left(\frac{\alpha+1}{\alpha}\right)(\sigma-1)\,\varphi^{\left(\frac{\alpha+1}{\alpha}\right)(\sigma-1)-1} + C_2(\sigma-1)\,\varphi^{\sigma-2} &> 0. \end{aligned}$$

Secondly, I show that $\pi_{XI}(\varphi_D) - \pi_D(\varphi_D) < 0$, otherwise the firm would choose to export and innovate instead of being indifferent between innovating or not while staying in the domestic market.

$$\begin{aligned} &\pi_{XI}(\varphi_D) - \pi_D(\varphi_D) < 0; \\ &\left(1+n\tau^{1-\sigma}\right)^{\frac{\alpha+1}{\alpha}} \alpha \left(\frac{f_D}{\alpha+1}\right)^{\frac{\alpha}{\alpha+1}} + n\tau^{1-\sigma} f_D - nf_X - f_I < 0. \end{aligned}$$

It holds that $\pi_{XI}(\varphi_D) - \pi_D(\varphi_D) < 0$ if:

$$\tau^{\sigma-1} f_X > \frac{\left(\frac{f_I}{\alpha}\right)^{\frac{\alpha}{\alpha+1}}(\alpha+1)}{\left(1+n\tau^{1-\sigma}\right)}$$

$$\left(\frac{f_I}{\alpha}\right)^{\frac{\alpha}{\alpha+1}}(\alpha+1)+\frac{\left[\left(1+n\tau^{1-\sigma}\right)^{\frac{\alpha+1}{\alpha}}-1\right]}{n\tau^{1-\sigma}}f_I>\tau^{\sigma-1}f_X.$$

□

PROOF OF PROPOSITION 4, PART I

Equation 1.18 and Equation 1.19 along with the Free Entry condition (Equation 1.20) completely determine the equilibrium and the productivity cutoffs. Rearrange the FE conveniently for the characterizing of the equilibrium as a function of φ_D^*

$$\begin{aligned}\delta f_E &= \left[1-G\left(\varphi_D^*\right)\right]\overline{\pi}\\ \delta f_E &= f_D j_1\left(\varphi_D^*\right)+n\tau^{1-\sigma}f_D j_2\left(\varphi_X^*\left(\varphi_D^*\right)\right)-\left[1-G\left(\varphi_{XI}^*\right)\right]nf_X\\ &+\alpha\left(\frac{f_D}{\alpha+1}\right)^{\frac{\alpha+1}{\alpha}}\left(1+n\tau^{1-\sigma}\right)^{\frac{\alpha+1}{\alpha}}j_3\left(\varphi_D^*\right)-\left[1-G\left(\varphi_{XI}^*\right)\right]f_I.\end{aligned} \tag{A.8}$$

where $j_1\left(\varphi_D^*\right)=\left[\left(\widetilde{\varphi}\left(\varphi_D^*\right)/\varphi_D^*\right)^{\sigma-1}-1\right]$,

$j_2\left(\varphi_D^*\right)=\left(\widetilde{\varphi}_x\left(\varphi_D^*\right)/\varphi_D^*\right)^{\sigma-1}\left[1-G\left(\varphi_{XI}^*\right)\right]$,

and $j_3\left(\varphi_D^*\right)=\left[\left(\widetilde{\varphi}_{XI}\left(\varphi_D^*\right)/\varphi_D^*\right)^{\sigma-1}\right]^{\frac{\alpha+1}{\alpha}}\left[1-G\left(\varphi_{XI}^*\right)\right]$.

Proof.

Assume the parameter restrictions $\frac{\left[\left(1+n\tau^{1-\sigma}\right)^{\frac{\alpha+1}{\alpha}}-1\right]}{n\tau^{1-\sigma}}f_I+\left(\frac{f_I}{\alpha}\right)^{\frac{\alpha}{\alpha+1}}(\alpha+1)>\tau^{\sigma-1}f_X$ and $\frac{\left(\frac{f_I}{\alpha}\right)^{\frac{\alpha}{\alpha+1}}(\alpha+1)}{\left(1+n\tau^{1-\sigma}\right)}>\tau^{\sigma-1}f_X$ hold, then the Intermediate Equilibrium exists and is unique. I shall proof that the RHS of equation (A.8) is decreasing in φ_D^* on the domain $\left(\varphi_D^*,\infty\right)$, so that φ_D^* is uniquely determined by the intersection of the latter curve with the flat line δf_E in the $\left(\varphi_D^*,\infty\right)$ space.

Let $k_1\left(\varphi_D^*\right)=\left[\left(\widetilde{\varphi}\left(\varphi_D^*\right)/\varphi_D^*\right)^{\sigma-1}-1\right]$,and $k_2\left(\varphi_D^*\right)=\left(\widetilde{\varphi_X}\left(\varphi_X^*\right)/\varphi_D^*\right)^{\sigma-1}$, then

$$k_1'\left(\varphi_D^*\right)=\frac{g\left(\varphi_D^*\right)}{1-G\left(\varphi_D^*\right)}k_1\left(\varphi_D^*\right)-\frac{(\sigma-1)\left[k_1\left(\varphi_D^*\right)+1\right]}{\varphi_D^*},$$

$$k_2'\left(\varphi_D^*\right) = \frac{g\left(\varphi_{XI}^*\right)}{1-G\left(\varphi_{XI}^*\right)}\left[k_2\left(\varphi_D^*\right) - \left(\frac{\varphi_{XI}}{\varphi_D}\right)^{\sigma-1}\right] - \frac{(\sigma-1)\,k_2\left(\varphi_D^*\right)}{\varphi_D^*}.$$

Similarly, $k_3\left(\varphi_D^*\right) = \left[\left(\widetilde{\varphi}_{XI}\left(\varphi_D^*\right)/\varphi_D^*\right)^{\sigma-1}\right]^{\frac{\alpha+1}{\alpha}}$, thus

$$\begin{aligned} k_3'\left(\varphi_D^*\right) &= \frac{g\left(\varphi_{XI}^*\right)}{1-G\left(\varphi_{XI}^*\right)}\left[k_3\left(\varphi_D^*\right) - \left(\frac{\varphi_{XI}^{\sigma-1}}{\varphi_D^{\sigma-1}}\right)^{\frac{\alpha+1}{\alpha}}\right]\frac{\partial\varphi_{XI}^*}{\partial\varphi_D^*} \\ &- \left(\frac{\alpha+1}{\alpha}\right)(\sigma-1)\,\frac{k_3\left(\varphi_D^*\right)}{\varphi_D^*}. \end{aligned}$$

Define $j_1\left(\varphi_D^*\right) = \left[1-G\left(\varphi_D^*\right)\right]k_1\left(\varphi_D^*\right)$, and $j_2\left(\varphi_D^*\right) = \left[1-G\left(\varphi_{XI}^*\right)\right]k_2\left(\varphi_D^*\right)$ and $j_3\left(\varphi_D^*\right) = \left[1-G\left(\varphi_{XI}^*\right)\right]k_3\left(\varphi_D^*\right)$ which are non-negative.
Then the derivative and elasticity of $j_1\left(\varphi_D^*\right)$, $j_2\left(\varphi_D^*\right)$ and $j_3\left(\varphi_D^*\right)$ are

$$\frac{j_1'\left(\varphi_D^*\right)\cdot\varphi_D^*}{j_1\left(\varphi_D^*\right)} = \underbrace{-(\sigma-1)\left[1+\frac{1}{k_1\left(\varphi_D^*\right)}\right]}_{<0 \text{ and bounded away of it}} < -(\sigma-1),$$

$$\frac{j_2'\left(\varphi_D^*\right)\cdot\varphi_D^*}{j_2\left(\varphi_D^*\right)} = \underbrace{\frac{g(\varphi_{XI})}{1-G(\varphi_{XI})}\frac{\left(\frac{\varphi_{XI}}{\varphi_D}\right)^{\sigma-1}\frac{\partial\varphi_{XI}}{\partial\varphi_D}}{k_2\left(\varphi_D^*\right)} - (\sigma-1)}_{<0 \text{ and bounded away of it}} < -(\sigma-1),$$

$$\begin{aligned} \frac{j_3'\left(\varphi_D^*\right)\cdot\varphi_D^*}{j_3\left(\varphi_D^*\right)} &= \underbrace{-\frac{g\left(\varphi_{DI}^*\right)}{\left[1-G\left(\varphi_{DI}^*\right)\right]}\frac{\Lambda^{\frac{1}{\sigma-1}}\Lambda^{\frac{\alpha+1}{\alpha}}}{k_2\left(\varphi_D^*\right)}\varphi_D^* - \left(\frac{\alpha+1}{\alpha}\right)(\sigma-1)}_{<0 \text{ and bounded away of it}} \\ &< -\left(\frac{\alpha+1}{\alpha}\right)(\sigma-1). \end{aligned}$$

Thus, $j_1(\varphi_D^*)$, $j_2(\varphi_D^*)$ and $j_3(\varphi_D^*)$ must be decreasing to zero as φ goes to infinite. It must be that $\lim_{\varphi_D^* \to 0} j_1(\varphi_D^*) = \infty$ since $\lim_{\varphi_D^* \to 0} k_1(\varphi_D^*) = \infty$, $\lim_{\varphi_D^* \to 0} j_2(\varphi_D^*) = \infty$ since $\lim_{\varphi_D^* \to 0} k_2(\varphi_D^*) = \infty$ and $\lim_{\varphi_D^* \to 0} j_3(\varphi_D^*) = \infty$ since $\lim_{\varphi_D^* \to 0} k_3(\varphi_D^*) = \infty$. Then $j_1(\varphi_D^*)$, $j_2(\varphi_D^*)$ and $j_3(\varphi_D^*)$, monotonically decrease from infinite to zero on the $(0, \infty)$ parameter space.

Therefore, the RHS of Equation A.8 is a monotonic decreasing function from infinity to zero on the space $(0, \infty)$ that cuts the FE flat line from above identifying a unique cutoff level φ_D^*. □

A.3 PROOF OF TRADE EFFECTS ON INNOVATION

PROOF OF PROPOSITION 5 *In the low cost trade equilibrium, if productivity draws are distributed according to a Pareto distribution, then the proportion of firms doing innovation activities rises with respect to Autarky* $(\varphi_{DI}^{AUT} > \varphi_{XI}^{LCT})$.

Proof.
Use Equation 1.6 and Equation 1.22 to get $\frac{\varphi_{DI}^A}{\varphi_{XI}^T} = \left(1+\tau^{1-\sigma}\right)^{\frac{1}{\sigma-1}} \frac{\varphi_D^A}{\varphi_D^T}$.

The FE conditions in autarky and free trade give us the following relationship between profits and cutoffs $\frac{\bar{\pi}^A}{\bar{\pi}^T} = \left(\frac{\varphi_D^A}{\varphi_D^T}\right)^\theta$

Hence, we need to show that $\left(1+\tau^{1-\sigma}\right)^{\frac{\theta}{\sigma-1}} \bar{\pi}^A > \bar{\pi}^T$, from where it follows that $\varphi_{DI}^A > \varphi_{XI}^T$.

Using Equation A.1 and Equation A.5, we can express $\bar{\pi}^A$, and $\bar{\pi}^T$ as

$$\begin{aligned}
\bar{\pi}^A &= Af_D + B\left(\frac{1}{\Lambda}\right)^{\frac{\theta}{\sigma-1}}, \\
\bar{\pi}^T &= Af_D + B\left(\frac{1+\tau^{1-\sigma}}{\Lambda}\right)^{\frac{\theta}{\sigma-1}} + Af_X\left(\frac{f_D}{f_X \tau^{\sigma-1}}\right)^{\frac{\theta}{\sigma-1}},
\end{aligned}$$

where $A = \frac{\sigma-1}{\theta-(\sigma-1)}$ and $B = \left(\frac{\left(\frac{\alpha+1}{\alpha}\right)(\sigma-1)}{\theta-\left(\frac{\alpha+1}{\alpha}\right)(\sigma-1)}\right) f_I$.

Thus

$$
\begin{aligned}
& \left(1+\tau^{1-\sigma}\right)^{\frac{\theta}{\sigma-1}} \bar{\pi}^{A} > \bar{\pi}^{T} \\
\hookrightarrow \quad & f_D \left[\left(1+\tau^{1-\sigma}\right)^{\frac{\theta}{\sigma-1}} - 1\right] > f_X \left(\frac{f_D}{f_X \tau^{\sigma-1}}\right)^{\frac{\theta}{\sigma-1}} \\
\hookrightarrow \quad & \left[\left(1+\tau^{1-\sigma}\right)^{\frac{\theta}{\sigma-1}} - 1\right] > \tau^{\theta} \left(\frac{f_D}{f_X}\right)^{\frac{\theta}{-}(\sigma-1)\sigma-1}
\end{aligned}
$$

From the parameter restriction we know that $1 > \frac{f_D}{f_X \tau^{\sigma-1}}$, then it follows that $1 > \left(\frac{f_D}{f_X \tau^{\sigma-1}}\right)^{\frac{\theta-(\sigma-1)}{\sigma-1}} \Rightarrow \tau^{1-\sigma} > \left(\frac{f_D}{f_X}\right)^{\frac{\theta}{\sigma-1}} \tau^{-\theta}$ then:

$$
\left(1+\tau^{1-\sigma}\right)^{\frac{\theta}{\sigma-1}} > 1+\tau^{1-\sigma} > 1+\left(\frac{f_D}{f_X}\right)^{\frac{\theta}{\sigma-1}} \tau^{-\theta}.
$$

□

B

APPENDIX B: PROOFS OF CHAPTER 2

B.1 AGGREGATE PRODUCTIVITY

In what follows I show that the output of the economy can be expressed as a function of the number of workers in the economy, their productivity and the elasticity of substitution and that Equation 2.1 is the general form of such expression in the open economy. For the proof I use the facts that in equilibrium $L = R$, that the budget constraint is $PQ = R$ and the price rule given by Equation 1.8.

LOW COST INNOVATION EQUILIBRIUM

$$\begin{aligned}
R &= M_t r_D\left(\tilde{\varphi}_t\right) + M_t^I z_D\left(\varphi_{DI}\right)\left(\frac{1}{\varphi_{DI}^{\sigma-1}}\right)^{\frac{1}{\alpha}} r_D\left(\left(\tilde{\varphi}_t^I\right)^{\frac{\alpha+1}{\alpha}}\right) \\
&= M\left(\frac{\sigma}{\sigma-1}\right)^{1-\sigma} QP^{\sigma}\left\{\int_{\varphi_D}^{\infty}\varphi^{\sigma-1}\mu(\varphi)d\varphi + n\tau^{1-\sigma}\int_{\varphi_X}^{\infty}\varphi^{\sigma-1}\mu(\varphi)d\varphi\right. \\
&+ z_D\left(\varphi_{DI}\right)\left(\frac{1}{\varphi_{DI}^{\sigma-1}}\right)^{\frac{1}{\alpha}}\int_{\varphi_{DI}}^{\varphi_{XI}}\varphi^{(\sigma-1)\left(\frac{\alpha+1}{\alpha}\right)}\mu(\varphi)d\varphi \\
&+ \left. z_D\left(\varphi_{DI}\right)\left(\frac{1}{\varphi_{DI}^{\sigma-1}}\right)^{\frac{1}{\alpha}}\left(1+n\tau^{1-\sigma}\right)^{\frac{\alpha+1}{\alpha}}\int_{\varphi_{XI}}^{\infty}\varphi^{(\sigma-1)\left(\frac{\alpha+1}{\alpha}\right)}\mu(\varphi)d\varphi\right\}.
\end{aligned}$$

Then,

$$\begin{aligned}
L &= \left(\frac{\sigma}{\sigma-1}\right) QM^{\frac{1}{1-\sigma}}\left\{\int_{\varphi_D}^{\infty}\varphi^{\sigma-1}\mu(\varphi)d\varphi + n\tau^{1-\sigma}\int_{\varphi_X}^{\infty}\varphi^{\sigma-1}\mu(\varphi)d\varphi\right. \\
&+ \left. z_D\left(\varphi_{DI}\right)\left(\frac{1}{\varphi_{DI}^{\sigma-1}}\right)^{\frac{1}{\alpha}}\left(1+n\tau^{1-\sigma}\right)^{\frac{\alpha+1}{\alpha}}\int_{\varphi_{XI}}^{\infty}\varphi^{(\sigma-1)\left(\frac{\alpha+1}{\alpha}\right)}\mu(\varphi)d\varphi\right\}^{\frac{1}{1-\sigma}}.
\end{aligned}$$

And

$$Q = \rho\left[M\left(\Psi_D + \left(1+n\tau^{1-\sigma}\right)\Psi_X + I^{LCI}\Psi_{INN}^{LCI}\right)\right]^{\frac{1}{\sigma-1}} L, \tag{B.1}$$

where $\Psi_{INN}^{LCI} = \left(\Psi_{DI} + \left(1+n\tau^{1-\sigma}\right)^{\frac{\alpha+1}{\alpha}}\Psi_{XI}\right)$, $I^{LCI} = z_D\left(\varphi_{DI}\right)\left(\frac{1}{\varphi_{DI}^{\sigma-1}}\right)^{\frac{1}{\alpha}}$,
$\Psi_D = \int_{\varphi_D}^{\varphi_{XI}} \varphi^{(\sigma-1)}\mu(\varphi)d\varphi$, $\Psi_X = \int_{\varphi_{XI}}^{\infty} \varphi^{(\sigma-1)}\mu(\varphi)d\varphi$,
$\Psi_{DI} = \int_{\varphi_{DI}}^{\varphi_{XI}} \varphi^{(\sigma-1)\left(\frac{\alpha+1}{\alpha}\right)}\mu(\varphi)d\varphi$ and $\Psi_{XI} = \int_{\varphi_{XI}}^{\infty} \varphi^{(\sigma-1)\left(\frac{\alpha+1}{\alpha}\right)}\mu(\varphi)d\varphi$.

INTERMEDIATE EQUILIBRIUM

$$\begin{aligned} R &= M_t r_D\left(\widetilde{\varphi}_t\right) + m_{XI}\left(1+n\tau^{1-\sigma}\right) z_X\left(\varphi_{XI}\right)\left(\frac{1}{\varphi_{XI}^{\sigma-1}}\right)^{\frac{1}{\alpha}} r_D\left(\left(\widetilde{\varphi}_{XI}\right)^{\frac{\alpha+1}{\alpha}}\right) \\ &= M\left(\frac{\sigma}{\sigma-1}\right)^{1-\sigma} QP^{\sigma}\left\{\int_{\varphi_D}^{\infty} \varphi^{\sigma-1}\mu(\varphi)d\varphi + n\tau^{1-\sigma}\int_{\varphi_X}^{\infty} \varphi^{\sigma-1}\mu(\varphi)d\varphi\right. \\ &+ \left. z_X\left(\varphi_{XI}\right)\left(\frac{1}{\varphi_{XI}^{\sigma-1}}\right)^{\frac{1}{\alpha}}\left(1+n\tau^{1-\sigma}\right)\int_{\varphi_{XI}}^{\infty} \varphi^{(\sigma-1)\left(\frac{\alpha+1}{\alpha}\right)}\mu(\varphi)d\varphi\right\}. \end{aligned}$$

Then,

$$\begin{aligned} L &= \left(\frac{\sigma}{\sigma-1}\right) QM^{\frac{1}{1-\sigma}}\left\{\int_{\varphi_D}^{\infty} \varphi^{\sigma-1}\mu(\varphi)d\varphi + n\tau^{1-\sigma}\int_{\varphi_X}^{\infty} \varphi^{\sigma-1}\mu(\varphi)d\varphi\right. \\ &+ \left. z_X\left(\varphi_{XI}\right)\left(\frac{1}{\varphi_{XI}^{\sigma-1}}\right)^{\frac{1}{\alpha}}\left(1+\tau^{1-\sigma}\right)\int_{\varphi_{XI}}^{\infty} \varphi^{(\sigma-1)\left(\frac{\alpha+1}{\alpha}\right)}\mu(\varphi)d\varphi\right\}^{\frac{1}{1-\sigma}}. \end{aligned}$$

And

$$Q = \rho\left[M\left(\Psi_D + \left(1+n\tau^{(1-\sigma)}\right)\Psi_X + I^{IE}\left(1+\tau^{1-\sigma}\right)\Psi_{XI}\right)\right]^{\frac{1}{\sigma-1}} L, \tag{B.2}$$

where $I^{IE} = z_X\left(\varphi_{XI}\right)\left(\frac{1}{\varphi_{XI}^{\sigma-1}}\right)^{\frac{1}{\alpha}}$, $\Psi_D = \int_{\varphi_D}^{\varphi_{XI}} \varphi^{(\sigma-1)}\mu(\varphi)d\varphi$,
$\Psi_X = \int_{\varphi_{XI}}^{\infty} \varphi^{(\sigma-1)}\mu(\varphi)d\varphi$ and $\Psi_{XI} = \int_{\varphi_{XI}}^{\infty} \varphi^{(\sigma-1)\left(\frac{\alpha+1}{\alpha}\right)}\mu(\varphi)d\varphi$.

LOW COST TRADE EQUILIBRIUM

$$\begin{aligned} R &= M_t r_D\left(\tilde{\varphi}_t\right) + m_{XI}\left(1+n\tau^{1-\sigma}\right) z_X(\varphi_{XI}) \left(\frac{1}{\varphi_{XI}^{\sigma-1}}\right)^{\frac{1}{\alpha}} r_D\left(\tilde{\varphi}_{XI}^{\left(\frac{\alpha+1}{\alpha}\right)}\right) \\ &= M\left(\frac{\sigma}{\sigma-1}\right)^{1-\sigma} QP^{\sigma}\left\{\int_{\varphi_D}^{\infty}\varphi^{\sigma-1}\mu(\varphi)d\varphi + n\tau^{1-\sigma}\int_{\varphi_X}^{\infty}\varphi^{\sigma-1}\mu(\varphi)d\varphi \right. \\ &+ \left. \left(1+n\tau^{1-\sigma}\right) z_X(\varphi_{XI})\left(\frac{1}{\varphi_{XI}^{\sigma-1}}\right)^{\frac{1}{\alpha}}\int_{\varphi_{XI}}^{\infty}\varphi^{(\sigma-1)\left(\frac{\alpha+1}{\alpha}\right)}\mu(\varphi)d\varphi\right\}. \end{aligned}$$

Then,

$$\begin{aligned} L &= \left(\frac{\sigma}{\sigma-1}\right) QM^{\frac{1}{1-\sigma}}\left\{\int_{\varphi_D}^{\infty}\varphi^{\sigma-1}\mu(\varphi)d\varphi + n\tau^{1-\sigma}\int_{\varphi_X}^{\infty}\varphi^{\sigma-1}\mu(\varphi)d\varphi \right. \\ &+ \left. \left(1+n\tau^{1-\sigma}\right) z_X(\varphi_{XI})\left(\frac{1}{\varphi_{XI}^{\sigma-1}}\right)^{\frac{1}{\alpha}}\int_{\varphi_{XI}}^{\infty}\varphi^{(\sigma-1)\left(\frac{\alpha+1}{\alpha}\right)}\mu(\varphi)d\varphi\right\}^{\frac{1}{1-\sigma}}. \end{aligned}$$

And

$$Q = \rho\left[M\left(\Psi_D + (1+n\tau^{1-\sigma})\Psi_X + I^{LCT}\left(1+n\tau^{1-\sigma}\right)\Psi_{XI}\right)\right]^{\frac{1}{\sigma-1}} L, \tag{B.3}$$

where $I^{LCT} = z_X(\varphi_{XI})\left(\frac{1}{\varphi_{XI}^{\sigma-1}}\right)^{\frac{1}{\alpha}}$, $\Psi_D = \int_{\varphi_D}^{\varphi_X}\varphi^{(\sigma-1)}\mu(\varphi)d\varphi$,
$\Psi_X = \int_{\varphi_X}^{\infty}\varphi^{(\sigma-1)}\mu(\varphi)d\varphi$ and $\Psi_{XI} = \int_{\varphi_{XI}}^{\infty}\varphi^{(\sigma-1)\left(\frac{\alpha+1}{\alpha}\right)}\mu(\varphi)d\varphi$.

B.2 PROOF OF PROPOSITION 6

LOW COST INNOVATION EQUILIBRIUM

$$\begin{aligned} \Psi^{LCI} &= \left[M\left(\Psi_D + \left(1+n\tau^{1-\sigma}\right)\Psi_X + I^{LCI}\left(\Psi_{DI} + \left(1+n\tau^{1-\sigma}\right)^{\frac{\alpha+1}{\alpha}}\Psi_{XI}\right)\right)\right]^{\frac{1}{\sigma-1}}. \\ \Delta\log\Psi^{LCI} &= \underbrace{-s_X\Delta\log(\tau)}_{\text{Exports}} \underbrace{-\left(\frac{\alpha+1}{\alpha}\right)\left(\frac{n\tau^{1-\sigma}}{1+n\tau^{1-\sigma}}\right)s_{XI}I^{LCI}\Delta\log(\tau)}_{\text{Exporters' Innovation}} \end{aligned}$$

$$
\begin{aligned}
&+ \quad \frac{1}{\sigma-1}\Bigg[\underbrace{\Delta\log(M)}_{\text{Entry Effect}} \quad + \quad \underbrace{s_D\Delta\log\left(\Psi_D\right)}_{\text{Domestic Market}} \\
&+ \quad \underbrace{\left(\frac{1+n\tau^{1-\sigma}}{n\tau^{1-\sigma}}\right)s_X\Delta\log\left(\Psi_X\right)}_{\text{Export Market}} + \underbrace{s_I I^{LCI}\Delta\log(I^{LCI})}_{\text{Extensive Margin}} \\
&+ \quad \underbrace{I^{LCI}\left[(s_I - s_{XI})\,\Delta\log(\Psi_{DI})\Delta + s_{XI}\log(\Psi_{XI})\right]}_{\text{Intensive Margin}}\Bigg].
\end{aligned}
$$

Proof. Recall that for every $x \in \mathcal{R}$

$$\frac{\Delta x}{x} = \Delta\log(x).$$

Take logs of Ψ^{LCI}

$$\log\left(\Psi^{LCI}\right) = \tfrac{1}{\sigma-1}\left[\log(M) + \log\left(\Psi_D + \left(1+n\tau^{1-\sigma}\right)\Psi_X + I^{LCI}\left(\Psi_{DI} + \left(1+n\tau^{1-\sigma}\right)^{\frac{\alpha+1}{\alpha}}\Psi_{XI}\right)\right)\right].$$

And derivatives

$$
\begin{aligned}
\Delta\log\Psi &= \frac{1}{\sigma-1}\left[\Delta\log(M) + \Delta\log\hat{\Psi}\right] \\
\Delta\log\hat{\Psi} &= \frac{1}{\hat{\Psi}}\left[\Delta\Psi_D + \Delta\left(1+\tau^{1-\sigma}\right)\Psi_X + \left(1+\tau^{1-\sigma}\right)\Delta\Psi_X\right. \\
&+ \Delta\left[\left(1+n\tau^{1-\sigma}\right)^{\frac{\alpha+1}{\alpha}}\right] I\Psi_I + \left(1+n\tau^{1-\sigma}\right)^{\frac{\alpha+1}{\alpha}}\Delta I\Psi_I + \left.\left(1+n\tau^{1-\sigma}\right)^{\frac{\alpha+1}{\alpha}} I\Delta\Psi_I\right].
\end{aligned}
$$

Define the share of domestic firms excluding innovation $s_D = \frac{\Psi_D}{\hat{\Psi}}$, the share of export firms excluding innovation $s_X = \frac{n\tau^{1-\sigma}\Psi_X}{\hat{\Psi}}$, the share of innovation activities $s_I = \frac{\Psi_{INN}}{\hat{\Psi}}$ and the share of exporters innovation activities $s_{XI} = \frac{\left(1+n\tau^{1-\sigma}\right)^{\frac{\alpha+1}{\alpha}}\Psi_{XI}}{\hat{\Psi}}$. Then, the variation in productivity can be decomposed in the following terms:

- Direct Effect on Exports $= -s_X\Delta\log(\tau)$
- Direct Effect on Exporters' Innovation $= -\left(\frac{\alpha+1}{\alpha}\right)\left(\frac{n\tau^{1-\sigma}}{1+n\tau^{1-\sigma}}\right)s_{XI}I^{LCI}\Delta\log(\tau)$

– Indirect Entry Effect $= \left(\frac{1}{\sigma-1}\right) \Delta \log(M)$

– Indirect Domestic Market Effect $= \left(\frac{1}{\sigma-1}\right) s_D \Delta \log\left(\Psi_D\right)$

– Indirect Export Market Effect $= \left(\frac{1}{\sigma-1}\right) \left(\frac{1+n\tau^{1-\sigma}}{n\tau^{1-\sigma}}\right) s_X \Delta \log\left(\Psi_X\right)$

– Indirect Extensive Margin Innovation Effect $= \left(\frac{1}{\sigma-1}\right) s_I I^{LCI} \Delta \log(I^{LCI})$

– Indirect Intensive Margin Innovation Effect
$= \left(\frac{1}{\sigma-1}\right) I^{LCI}\left(s_I - s_{XI}\right) \Delta \log(\Psi_{DI}) + \left(\frac{1}{\sigma-1}\right) I^{LCI} s_{XI} \Delta \log(\Psi_{XI})$

□

INTERMEDIATE EQUILIBRIUM

$$\Psi^{IE} = \left[M\left(\Psi_D + \left(1 + n\tau^{(1-\sigma)}\right)\Psi_X + I^{IE}\left(1 + n\tau^{1-\sigma}\right)\Psi_{XI}\right)\right]^{\frac{1}{\sigma-1}}.$$

$$\begin{aligned}
\Delta \log \Psi^{IE} \quad = \quad & \underbrace{-s_X \Delta \log(\tau)}_{\text{Exports}} \underbrace{- \left(\frac{n\tau^{1-\sigma}}{1+n\tau^{1-\sigma}}\right) s_{XI} I^{IE} \Delta \log(\tau)}_{\text{Exporters' Innovation}} \\
+ \quad & \frac{1}{\sigma-1}\Bigg[\underbrace{\Delta \log(M)}_{\text{Entry Effect}} + \underbrace{s_D \Delta \log\left(\Psi_D\right)}_{\text{Domestic Market}} \\
+ \quad & \underbrace{\left(\frac{1+n\tau^{1-\sigma}}{n\tau^{1-\sigma}}\right) s_X \Delta \log\left(\Psi_X\right)}_{\text{Export Market}} \\
+ \quad & \underbrace{s_{XI} I^{IE} \Delta \log(I^{IE})}_{\text{Extensive Margin}} + \underbrace{I^{IE} s_{XI} \log(\Psi_I)}_{\text{Intensive Margin}}\Bigg].
\end{aligned}$$

Proof. Recall that for every $x \in \mathcal{R}$

$$\frac{\Delta x}{x} = \Delta \log(x).$$

Take logs of Ψ^{IE}

$$\log\left(\Psi^{IE}\right) = \frac{1}{\sigma-1}\left[\log(M) + \log\left(\Psi_D + \left(1+n\tau^{1-\sigma}\right)\Psi_X + I^{IE}\left(1+n\tau^{1-\sigma}\right)\Psi_I\right)\right].$$

And derivatives

$$\begin{aligned}
\Delta\log\Psi &= \frac{1}{\sigma-1}\left[\Delta\log(M) + \Delta\log\hat{\Psi}\right].\\
\Delta\log\hat{\Psi} &= \frac{1}{\hat{\Psi}}\left[\Delta\Psi_D + \Delta\left(1+n\tau^{1-\sigma}\right)\Psi_X + \left(1+n\tau^{1-\sigma}\right)\Delta\Psi_X\right.\\
&+ \left.\Delta\left(1+n\tau^{1-\sigma}\right)I\Psi_I + \left(1+n\tau^{1-\sigma}\right)\Delta I\Psi_I + \left(1+n\tau^{1-\sigma}\right)I\Delta\Psi_I\right].
\end{aligned}$$

Define the share of domestic firms excluding innovation $s_D = \frac{\Psi_D}{\hat{\Psi}}$, the share of export firms excluding innovation $s_X = \frac{n\tau^{1-\sigma}\Psi_X}{\hat{\Psi}}$ and the share of exporters innovation activities $s_{XI} = \frac{(1+n\tau^{1-\sigma})\Psi_I}{\hat{\Psi}}$. Then, the variation in productivity can be decomposed in the following terms:

- Direct Effect on Exports $= -s_X\Delta\log(\tau)$
- Direct Effect on Exporters' Innovation $= -\left(\frac{n\tau^{1-\sigma}}{1+n\tau^{1-\sigma}}\right)s_{XI}I^{IE}\Delta\log(\tau)$
- Indirect Entry Effect $= \left(\frac{1}{\sigma-1}\right)\Delta\log(M)$
- Indirect Domestic Market Effect $= \left(\frac{1}{\sigma-1}\right)s_D\Delta\log\left(\Psi_D\right)$
- Indirect Export Market Effect $= \left(\frac{1}{\sigma-1}\right)\left(\frac{1+n\tau^{1-\sigma}}{n\tau^{1-\sigma}}\right)s_X\Delta\log\left(\Psi_X\right)$
- Indirect Extensive Margin Innovation Effect $= \left(\frac{1}{\sigma-1}\right)s_{XI}I^{IE}\Delta\log(I^{IE})$
- Indirect Intensive Margin Innovation Effect $= \left(\frac{1}{\sigma-1}\right)s_{XI}I^{IE}\Delta\log(\Psi_{XI})$

□

LOW COST TRADE EQUILIBRIUM

$$\begin{aligned}
\Psi^{LCT} &= \left[M\left(\Psi_D + (1+n\tau^{1-\sigma})\Psi_X + I^{LCT}\left(1+n\tau^{1-\sigma}\right)\Psi_{XI}\right)\right]^{\frac{1}{\sigma-1}}.\\
\Delta\log\Psi^{LCT} &= \underbrace{-s_X\Delta\log(\tau)}_{\text{Exports}}\ \underbrace{-s_{XI}I^{LCT}\Delta\log(\tau)}_{\text{Exporters' Innovation}}
\end{aligned}$$

$$+ \quad \frac{1}{\sigma-1}\left[\underbrace{\Delta\log(M)}_{\text{Entry Effect}} + \underbrace{s_D\Delta\log\left(\Psi_D\right)}_{\text{Domestic Market}}\right.$$

$$+ \quad \underbrace{\left(\frac{1+n\tau^{1-\sigma}}{n\tau^{1-\sigma}}\right)s_X\Delta\log\left(\Psi_X\right)}_{\text{Export Market}} + \underbrace{\left(\frac{1+n\tau^{1-\sigma}}{n\tau^{1-\sigma}}\right)s_{XI}I^{LCT}\Delta\log(I^{LCT})}_{\text{Extensive Margin}}$$

$$\left.+ \quad \underbrace{\left(\frac{1+n\tau^{1-\sigma}}{n\tau^{1-\sigma}}\right)s_{XI}I^{LCT}\log(\Psi_I)}_{\text{Intensive Margin}}\right]$$

Proof. Recall that for every $x \in \mathcal{R}$

$$\frac{\Delta x}{x} = \Delta\log(x).$$

Take logs of Ψ^{LCT}

$$\log\Psi^{LCT} = \frac{1}{\sigma-1}\left[\log(M) + \log\left(\Psi_D + \left(1+n\tau^{1-\sigma}\right)\Psi_X + I^{LCT}\left(1+n\tau^{1-\sigma}\right)\Psi_I\right)\right].$$

And derivatives

$$\begin{aligned}
\Delta\log\Psi &= \frac{1}{\sigma-1}\left[\Delta\log(M) + \Delta\log\hat{\Psi}\right]. \\
\Delta\log\hat{\Psi} &= \frac{1}{\hat{\Psi}}\left[\Delta\Psi_D + \Delta\left(1+n\tau^{1-\sigma}\right)\Psi_X + \left(1+n\tau^{1-\sigma}\right)\Delta\Psi_X\right. \\
&+ \left.\Delta\left(1+n\tau^{1-\sigma}\right)I\Psi_I + \left(1+n\tau^{1-\sigma}\right)\Delta I\Psi_I + \left(1+n\tau^{1-\sigma}\right)I\Delta\Psi_I\right].
\end{aligned}$$

Define the share of domestic firms excluding innovation $s_D = \frac{\Psi_D}{\hat{\Psi}}$, the share of export firms excluding innovation $s_X = \frac{n\tau^{1-\sigma}\Psi_X}{\hat{\Psi}}$ and the share of exporters innovation activities $s_{XI} = \frac{(1+n\tau^{1-\sigma})\Psi_{XI}}{\hat{\Psi}}$. Then, the variation in productivity can be decomposed in the following terms:

– Direct Effect on Exports $= -s_X\Delta\log(\tau)$

- Direct Effect on Exporters' Innovation $= -\left(\frac{n\tau^{1-\sigma}}{1+n\tau^{1-\sigma}}\right) s_{XI} I^{LCT} \Delta\log(\tau)$
- Indirect Entry Effect $= \left(\frac{1}{\sigma-1}\right) \Delta\log(M)$
- Indirect Domestic Market Effect $= \left(\frac{1}{\sigma-1}\right) s_D \Delta\log\left(\Psi_D\right)$
- Indirect Export Market Effect $= \left(\frac{1}{\sigma-1}\right) \left(\frac{1+n\tau^{1-\sigma}}{n\tau^{1-\sigma}}\right) s_X \Delta\log\left(\Psi_X\right)$
- Indirect Extensive Margin Innovation Effect $= \left(\frac{1}{\sigma-1}\right) s_{XI} I^{LCT} \Delta\log(I^{LCT})$
- Indirect Intensive Margin Innovation Effect $= \left(\frac{1}{\sigma-1}\right) s_{XI} I^{LCT} \Delta\log(\Psi_{XI})$

□

C

APPENDIX C: FIRM MAXIMIZATION PROBLEM OF CHAPTER 3

The production function for each differentiated product is given by a Cobb Douglas function of firm TFP, capital and labor:

$$Y_{si} = A_{si} K_{si}^{\alpha_s} L_{si}^{1-\alpha_s}.$$

Since there are distortions affecting the production of firms, the profits of a firm are given by:

$$\pi_{si} = (1 - \tau_{Y_{si}}) P_{si} Y_{si} - w L_{si} - (1 + \tau_{K_{si}}) R K_{si}.$$

Profit maximization yields the standard optimal price and capital-labor ratio:

$$\begin{aligned} P_{si} &= \frac{\sigma}{\sigma - 1} \left(\frac{R}{\alpha_s} \right)^{\alpha_s} \left(\frac{w}{1 - \alpha_s} \right)^{1-\alpha_s} \frac{(1 + \tau_{K_{si}})^{\alpha_s}}{A_{si} (1 - \tau_{Y_{si}})}, \\ \frac{K_{si}}{L_{si}} &= \frac{\alpha_s}{1 - \alpha_s} \cdot \frac{w}{R (1 + \tau_{K_{si}})}. \end{aligned}$$

The marginal revenue product of labor is proportional to revenue per worker:

$$MRPL_{si} = P_{si} \frac{\partial Y_{si}}{\partial L_{si}} = (1 - \alpha_s) \frac{P_{si} Y_{si}}{L_{si}} = \left(\frac{\sigma}{\sigma - 1} \right) \frac{w}{1 - \tau_{Y_{si}}}.$$

The marginal revenue product of capital is proportional to the revenue-capital ratio:

$$MRPK_{si} = P_{si} \frac{\partial Y_{si}}{\partial K_{si}} = \alpha_s \frac{P_{si} Y_{si}}{K_{si}} = \left(\frac{\sigma}{\sigma - 1} \right) \frac{R(1 + \tau_{K_{si}})}{1 - \tau_{Y_{si}}}.$$

To derive K_s and L_s, first we derive the aggregate demand for capital and labor in a sector by aggregating the firm-level demands for the two factor inputs. Then, we combine the aggregate demand for the factor inputs in each sector with the allocation of total expenditure across sectors.

$$L_s \equiv \sum_{i=1}^{M_s} L_{si} = L\frac{(1-\alpha_s)\theta_s/\overline{MRPL_s}}{\sum_{s'=1}^{S}(1-\alpha_{s'})\theta_{s'}/\overline{MRPL_{s'}}},$$

$$K_s \equiv \sum_{i=1}^{M_s} K_{si} = K\frac{\alpha_s\theta_s/\overline{MRPK_s}}{\sum_{s'=1}^{S}\alpha_{s'}\theta_{s'}/\overline{MRPK_{s'}}},$$

where

$$\overline{MRPL_s} \triangleq \frac{w}{\sum_{i=1}^{M_s}(1-\tau_{Y_{si}})\frac{P_{si}Y_{si}}{P_sY_s}}$$

$$\overline{MRPK_s} \triangleq \frac{R}{\sum_{i=1}^{M_s}\frac{1-\tau_{Y_{si}}}{1+\tau_{K_{si}}}\frac{P_{si}Y_{si}}{P_sY_s}}$$

The $TFPR_{si}$ is defined as follows:

$$TFPR_{si} = \frac{\sigma}{\sigma-1}\left(\frac{MRPK_{si}}{\alpha_s}\right)^{\alpha_s}\left(\frac{MRPL_{si}}{1-\alpha_s}\right)^{1-\alpha_s}$$

$$= \frac{\sigma}{\sigma-1}\left(\frac{R}{\alpha_s}\right)^{\alpha_s}\left(\frac{w}{1-\alpha_s}\right)^{1-\alpha_s}\frac{(1+\tau_{K_{si}})^{\alpha_s}}{1-\tau_{Y_{si}}}.$$

Then,

$$\overline{TFPR_s} \triangleq \frac{\sigma}{\sigma-1}\left[\frac{R}{\alpha_s\left(\sum_{i=1}^{M_s}\frac{1-\tau_{Y_{si}}}{1+\tau_{K_{si}}}\frac{P_{si}Y_{si}}{P_sY_s}\right)}\right]^{\alpha_s}\left[\frac{w}{\left((1-\alpha_s)\sum_{i=1}^{M_s}(1-\tau_{Y_{si}})\frac{P_{si}Y_{si}}{P_sY_s}\right)}\right]^{1-\alpha_s}$$

$$= \frac{\sigma}{\sigma-1}\left(\frac{\overline{MRPK_s}}{\alpha_s}\right)^{\alpha_s}\left(\frac{\overline{MRPL_s}}{1-\alpha_s}\right)^{1-\alpha_s}.$$

Using these expressions, we can derive Equation 3.5:

$$TFP_s \triangleq \frac{Y_s}{K_s^{\alpha_s}L_S^{1-\alpha_s}} = \frac{\left[\sum_i^{M_s}\left(A_{si}K_{si}^{\alpha_s}L_{si}^{1-\alpha_s}\right)^{\frac{\sigma-1}{\sigma}}\right]^{\frac{\sigma}{\sigma-1}}}{\left(\sum_i^{M_s}L_{si}\right)^{1-\alpha_s}\left(\sum_i^{M_s}K_{si}\right)^{\alpha_s}}$$

$$= \frac{\left[\sum_i^{M_s}\left(A_{si}\frac{1-\tau_{Y_{si}}}{(1+\tau_{K_{si}})^{\alpha_s}}\right)^{\sigma-1}\right]^{\frac{1}{\sigma-1}}}{\left(\sum_i^{M_s}\frac{1-\tau_{Y_{si}}}{1+\tau_{K_{si}}}\frac{P_{si}Y_{si}}{P_sY_s}\right)^{\alpha_s}\left(\sum_i^{M_s}(1-\tau_{Y_{si}})\frac{P_{si}Y_{si}}{P_sY_s}\right)^{1-\alpha_s}}$$

$$= \left[\sum_i^{M_s}\left(A_{si}\frac{\overline{TFPR_s}}{TFPR_{si}}\right)^{\sigma-1}\right]^{\frac{1}{\sigma-1}}.$$

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