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Essays on Small Firms' Finance and Asymmetric Information

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Resumen en Castellano

Mi tesis se centra en el análisis de los efectos de imperfecciones del mercado de crédito causada por información asimétrica y por regulaciones sobre contratos financieros óptimos, especialmente para las pequeñas empresas (empresas emprendedoras). En particular, se proponen modelos que pueden reproducir varios hechos estilizados en los mercados de crédito. Estos modelos proporcionan implicaciones adicionales comprobables.

El primer capítulo "Why are Small Firms more likely to use Convertible debt?" presenta un modelo de agente-principal bajo no verificabilidad de los rendimientos de las empresas. La no verificabilidad de los rendimientos de las empresas puede llevar las empresas a un defecto estratégica ex post. Las empresas con activos iniciales ilíquidos y heterogéneos necesitan fondos para invertir en un proyecto cuya rentabilidad depende del fondo invertido y del esfuerzo de las empresas, que no es observable para los financieros. Se demuestra que los contratos de deuda convertibles pueden mitigar la ineficiencia ex post. Por otra parte, se generaliza el modelo en dos direcciones: el riesgo moral y la aversión al riesgo de las empresas. En ambos casos, se demuestra el comportamiento óptimo de los contratos de deuda convertibles para las pequeñas empresas.

En el segundo capítulo "Non-monotonic Effect of Bankruptcy Exemptions on Small Firms' Finance", se propone un simple modelo de agente-principal. En muchos trabajos empíricos, se examina el vínculo entre el marco jurídico y las finanzas de las empresas. Las empresas (los agentes) no conocen la calidad de los proyectos que han elegido, sin embargo, los bancos pueden obtener la calidad de los proyectos de las empresas mediante la monitorización costosa. La precisión de tal monitorización depende de la intensidad de monitorización empleada por los bancos. Nuestro modelo demuestra que, en equilibrio, el préstamo obtenido por las empresas, así como la tasa de interés bruta, se determinan a través de dos canales: (1) Efecto directo: un efecto de las exenciones de bancarrota directamente sobre los contratos de equilibrio de la deuda, y (2) Efecto indirecto: un efecto de la exención de la bancarrota indirectamente sobre los contractos de equilibrio de deuda a través de la intensidad de monitorización de los bancos. Por la combinación de ambos efectos, el modelo produce una relación no monótona entre exenciones de bancarrota y el tamaño del préstamo de equilibrio de las empresas, así como la tasa de interés de equilibrio bruto, como se muestra también en los datos.

En el tercer capítulo "Borrowing Decisions in the Presence of Credit Market Imperfections and Heterogeneous Risk", se muestra que las decisiones de financiamiento de los empresarios no sólo dependen de las restricciones de crédito, sino también de la demanda de seguro de consumo de los empresarios. La literatura anterior destaca el impacto de las imperfecciones del mercado de crédito en la decisión de endeudamiento de los empresarios. En mi modelo, las actitudes heterogéneas de los empresarios al riesgo interactúan con dos fuentes distintas de las imperfecciones del mercado de crédito - responsabilidad limitada y riesgo moral - y estas imperfecciones resultan en distintas decisiones de financiamiento. En particular, cuando los empresarios se ven limitados por una restricción de responsabilidad limitada, su endeudamiento y por lo tanto, el capital invertido aumenta con su riqueza inicial. Por otro lado, en la presencia de riesgo moral, los empresarios se ven limitados por una restricción de compatibilidad de incentivos, y piden prestado menos y por lo tanto, eligen un menor capital a medida que aumenta su riqueza inicial.

Dissertation Abstract

My thesis focuses on analyzing the effects of credit market imperfections caused by asymmetric information or regulations on optimal financial contracts, especially for small firms (entrepreneurial firms). In particular, I propose models that can replicate several stylized facts in credit markets. These models provide further testable implications.

Chapter 1 “Why are Small Firms more likely to use Convertible Debts?” presents a principal-agent model under non-verifiability of firm returns. Non-verifiability of firm returns may lead to firms’ ex-post strategic default. Firms with heterogeneous initial illiquid assets need fund to invest in a project whose return depends on the fund invested and firms’ effort, which is unobservable to the financiers. We show that convertible debt contracts can mitigate ex-post inefficiency. Moreover, we further generalize the model into two directions: moral hazard and risk aversion of the firms. In both cases, we prove the optimality of convertible debt contracts for small firms.

In Chapter 2 “Non-monotonic Effect of Bankruptcy Exemptions on Small Firms’ Finance”, a simple principal-agent model is presented. The link between law and firms’ finance are examined in many empirical works. Firms (agents) do not know the quality of the projects they have chosen, however, banks can learn the quality of the firms’ projects by costly screening. The accuracy of screening depends on banks’ screening intensity exerted. Our model demonstrate that, in equilibrium, the loan obtained by the firms, as well as the gross interest rate, are determined through two channels: (1) Direct Effect: an effect of bankruptcy exemptions directly on the equilibrium debt contracts, and (2) Indirect Effect: an effect of bankruptcy exemption indirectly on the equilibrium debt contracts via banks’ screening intensity. Combining both effects, our model yields a non-monotonic relation between bankruptcy exemptions and the equilibrium loan size of the firms as well as the equilibrium gross interest rate as shown in the data.

In Chapter 3 “Borrowing Decisions in the Presence of Credit Market Imperfections and Heterogeneous Risk”, I show that entrepreneurs’ borrowing decisions depend not only on the borrowing constraints, but also on the entrepreneurs’ demand for consumption insurance. Previous literature highlights the impact of credit market imperfections on the entrepreneurs’ borrowing decision. In my model, entrepreneurs’ heterogeneous risk attitudes interact with two different sources of credit market imperfections-limited liability and moral hazard-and these imperfections result in distinct borrowing decisions. Specifically, when entrepreneurs are restricted by a limited liability constraint, their borrowing and hence the invested capital increases with their initial wealth. On the other hand, in the presence of moral

hazard, entrepreneurs are constrained by an incentive compatibility constraint, and they borrow less and hence choose lower capital as their initial wealth increases.

Contents

Chapter 1

Why Are Small Firms More Likely to Use Convertible Debt?

Non-verifiability of firm returns may lead to firms' ex-post strategical default. Under non-verifiability of firm returns, a principal-agent model is presented in this paper. Firms with heterogeneous initial assets need funds to invest in a project whose return depends on the fund invested and firms' effort, which is unobservable to the financiers. We show that convertible debt contracts can mitigate ex-post inefficiency. Moreover, we further generalize the model into two directions: moral hazard and risk aversion of the firms. In both cases, we prove the optimality of convertible debt contracts for small firms.

1.1 Introduction

Consider a firm that needs to raise funds in order to invest in a project. In the literature of firms' choices of financing sources, it is well known that the conflicts between the firms[1] and their financiers may cause economic inefficiency. In the firms with debt financing, firms might choose to invest in too risky projects, or they might hide the cash flows and default on their debt even they are able to pay back. If the firms are risk neutral, debt financing can effectively prevent the owner-managers to shirk (Innes (1990)). On the other hand, if firms choose equity financing, they might exert too little effort (Dybvig and Wang (2002)). However, the firms do not have incentive to hide their cash flows. Therefore, whether a firm should choose debt or equity financing depends on which of the two incentive problems is relatively more severe.

We start with an environment in which firms exert observable effort and they are protected by the limited liability when they default. Firms' ex-post returns are observable to both firms' owner-managers and their financial claimants, but are not verifiable by a third party (i.e., court). It is possible that non-verifiable firm returns, which results in firms' strategic defaults, distort the investors' efficient lending decisions. We consider firms are heterogeneous in their initial assets. The firms' benefits of defaulting strategically varies with their initial assets. Firms with smaller initial assets are more likely to default strategically than those with larger initial assets. The reason is that small firms have little to lose if they default and file for bankruptcy. If this problem is serious, compared with debt financing, equity financing may be preferable. Afterwards, we relax the assumption of observable effort and consider unobservable effort. Under this setting, another agency problem arises with equity financing, the incentives of firms' owner-managers to exert efficient effort are distorted under equity financing.

Our goal is to derive the optimal contracts such that in equilibrium, firms exert efficient effort and there are no strategic defaults. In this paper, we show that convertible debt contracts are optimal. Such a contract gives the holders an unilateral right to convert the debt into equity at the predetermined time and price (conversion rate). Convertible debt has the properties that combine both debt and equity. Specifically, it can thought as a standard debt contract plus a call option to convert the debt into equity when the firm's return has greater upside potential. We use this feature of convertible debt and our result shows that the problem of firms' strategic defaults due to non-verifiable firm returns can be solved.

The available evidence about convertible debt shows that small firms (i.e., firms

[1] For simplicity, we assume firms' owners are also managers of the firms. Hence, we can ignore the conflict between the owners and the managers of the firms.

with smaller initial assets) and firms with higher profit-to-assets ratio are more likely to use convertible debt over standard debt than larger firms. Noddings, Christoph and Noddings (2001) analyze the trading of U.S. convertible debts and convertible preferred stocks in January 2000. They find that among the total of 311 companies that use convertible debts, 58% are micro or small firms. Kahan and Yermack (1998) show empirically that the issuance of convertible debts is negative significantly related to the firm size[2]. Lewis, Rogalski and Seward (1999) find that firms with higher profit-to-assets ratio are more likely to use convertible debt.

Our paper is related to the literature on optimal contracts when the credit market is imperfect. In particular, Innes (1990) shows that, under limited liability and moral hazard due to the firms' unobservable effort, standard debt contracts are optimal. Since firms are residual claimers under standard debt contracts, standard debt contracts give firms incentives to exert effort. However, Innes (1990) considers the environment with verifiable firms' cash flows. We instead, relax the assumption of verifiable firm returns and characterize the optimal contracts assuming that the firm returns are not verifiable.

Besides, this paper is also closely related to the literature of incomplete contracts pioneered by Hart (Hart (2001), Hart and Moore (2007)). Due to the non-verifiability of firm returns, contracts can not be contingent on the firms' returns. As Hart mentioned, incompleteness of contracts open the door to a theory of ownership. In a recent work by Fluck (2010), he mentions that when the firms' returns or the owner-managers' misbehavior can not be verified or are too costly to be verified by the court, there are at least two ways to accomplish the optimal contracts. One is to make the contract contingent on other verifiable terms. The other way is to grant the investors an unconditional control right. This right allows the investors to threat the firm's owner-manager to replace him/her or to liquidate the firm's assets even though the firm's return is in the upside. However, this threat only works if the project is long-term. In the last period of the project, the owner-manager can never be induced to make the repayment without defaulting. Moreover, this threat only is effective if the firm has substantial assets. For the firms with little assets, this threat is not effective since small firms have little to lose. Consequently, the financiers are not willing to lend to small firms even though small firms have projects with positive present values. This leads to an *ex-ante* inefficiency. Hence, in order to discourage firms' strategic default *ex-post*, *ex-ante* inefficiency must be sacrificed at least partially.

Another related strand of literature focuses on the optimal security design of

[2] In their paper, firm sizes are defined by measuring firms' total initial assets. In particular, micro-cap: smaller than 225 million, small-cap: between 225 million and 1.25 billion, medium-cap: bewteen 1.25 billion and 10.5 billion, large-cap: larger than 10.5 billion.

venture capital financing. It studies the agency problem caused by double moral hazard between the firms and the venture capitalists. Repullo and Suarez (RS, 2004) and Schmidt (2003) show that under a double moral hazard problem, the optimal contracts are convertible debt contracts. In RS's papers, first, convertible debt contracts solve the agency problem because convertible debt contracts allow the venture capitalists to share the firms' profit and hence provide the venture capitalists incentives to exert effort. Second, by using stage financing (which is commonly used in venture capital financing), the venture capitalists can threat the firm to stop providing them credits in order to induce the firms to exert effort. Schmidt (2003) shows that convertible debt contracts can induce both venture capital firms and venture capitalists to exert effort sequentially. In his model, both firms of all different size and the venture capitalists will only exert effort under convertible debt contracts.

In this paper, the impact of non-verifiable firm returns are crucial on determining the optimal contracts. We first construct a simple model to show that standard debt contracts are dominated by both equity and convertible contracts for small firms because small firms have incentives to default strategically if firms' returns are not verifiable. Furthermore, we generalize the model in two directions. First, we consider a moral hazard problem due to firms' unobservable effort. Second, instead of assuming risk-neutrality of both firms and their financiers, we consider risk aversion the firms.

In the former case (with moral hazard probelm), we derive the optimality of convertible debt contracts for small firms. This result is consistent with the empirical evidence (Noddings, Christoph and Noddings (2001) & Lewis, Rogalski and Seward (1999)). Moreover, under this setting, we show that the probability of using convertible debt is positively related with the firms' profit-to-asset ratio, which is also found in the data (Lewis, Rogalski and Seward (1999)).

In the latter (with risk-averse firms), our results suggest that, under certain conditions, in partucular, if the firms' utility function has a complete monotone first derivative and if the probability of failure of the project under equity contracts is lower than $\frac{1}{2}$, convertible debt contracts are optimal for small firms if firms are risk-averse. The reason is that small firms have incentives to default strategically under standard debt contracts. Hence, standard debt contracts are dominated. Moreover, under the condition of the probability of failure of the projects being low, convertible debt contracts dominates equity contracts because convertible debt achieves better risk sharing. As for large firms, since they do not have incentives to default strategically, both standard debt and convertible debt contracts are optimal.

We also discuss the case when both directions of generalizations exist at the same time. We conclude that the relation between moral hazard and the relative degree of risk aversion of the firms and the financiers importantly shape the optimal contracts. In particular, we conjecture that the result of the optimality of convertible debt contracts for small firms still holds even when considering the moral hazard problem and the risk aversion of the firms.

The outline of this paper is the following: In Section 2, we first analyze the benchmark model. Afterwards, we relax the assumption of verifiability of firm returns and characterize the optimal contracts. In Section 3 and Section 4, we generalize the model in two directions – moral hazard and risk aversion – and analyze the optimal contracts under each case. We further discuss the more general case when considering both directions together in Section 5. Finally, we conclude in Section 6.

1.2 The Model

In this paper, a principal-agent model is presented. Firms (agents) have an investment project of positive present value, but they do not have funds to finance the project. As a result, firms have to obtain the funds from the financiers, the principals. The assumptions of the model are:

Assumption 1 Firms and financiers are both risk neutral. The firms are heterogenous in terms of their initial assets[3] A, $A \in \left(0, \overline{A}\right)$, where $\overline{A}$ is sufficiently high. The financiers' opportunity cost of lending per unit is assumed to be exogenous, and for simplicity, $i = 1$.

Assumption 2 The investment project yields a random return

$$y = \begin{cases} \theta & \text{if the project succeeds} \\ 0 & \text{if the project fails} \end{cases}$$

The distribution of the realized returns is endogenous, depending on the funds B invested in the project and firms' effort e which is observable to the financiers. For simplicity, we assume that there are two levels of effort, $e \in \{e_H, e_L\}$. The cost is increasing in the effort with $c(e_L) = 0$ and $c(e_H) = c_H > 0$.

The probability of success of the project is denoted as $p(B, e)$, with $p'_B(B, e) > 0$ and $p''_B(B, e) < 0$ for any e, and $p(B, e_H) > p(B, e_L)$ for any B.

Assumption 3 The project returns of firms are observable to both parties (firms and their financiers), however, returns are not verifiable by a third party (e.g., court).

[3] We assume the firms' initial assets can be liquidated without any liquidation cost. Firms can liquidate (partially) their own assets and self finance. Under the assumption of no liquidation cost, the firms' are indifferent between self-financing or external financing.

Assumption 4 Firms' liability to debt, as well as the financiers' liability to the investment are limited. That is, if a firm obtains debt from the financier, once it defaults and files for bankruptcy, the financier (lender) liquidates the firm's assets up to the required repayment. Besides, the repayment to the financiers whether the project succeeds or not is non-negative.

Assumption 5 Financiers compete *a la* Bertrand.

Assumption 3 is crucial in this paper. In the following analysis, we first analyze the benchmark model in which Assumption 3 is ignored. In other word, in the benchmark model, we consider an environment in which firm returns are verifiable. Next, we analyze the effect of non-verifiability on the optimal contracts. Further, we generalize the model by relaxing Assumption 2 and Assumption 1, respectively. In particular, in the first generalized model, we consider that firms' effort is unobservable to the *financiers.* In the second generalized model, we consider risk averse firms and risk neutral *financiers*.

1.2.1 The benchmark case

In this section, we derive the optimal contract as a benchmark in the environment that both, firms' effort and project returns, are observable and verifiable. Afterwards, we focus on three types of contracts which are commonly used in reality: (1) standard debt contract, (2) equity contract and (3) convertible debt contract.

The optimal contract outcomes are the solution to the following problem:

$$\max_{B,e,s_1,s_2} p(B,e)(\theta - s_1 + A) + (1 - p(B,e))(-s_2 + A) - c(e)$$

s.t.

$$E\pi_l = p(B,e)s_1 + (1 - p(B,e))s_2 - B \geq 0 \tag{PC}$$

$$\theta - s_1 + A \geq 0 \tag{LL1}$$

$$-s_2 + A \geq 0 \tag{LL2}$$

where (s_1, s_2) is the repayment from the firm to the lender if the project succeeds or fails, respectively.

Lemma 1 $E\pi_l^* = 0$

Lemma 1 shows that, in equilibrium, the financier's participation constraint always binds due to Bertrand competition. The equilibrium borrowing amount $B^*(e_j)$ depends on effort e and it satisfies

$$p'_B(B^*(e_j), e_j) = \frac{1}{\theta}, \quad j = H, L$$

We further assume that it is optimal to exert high effort for all firms.

Assumption 6 Exerting e_H is optimal.

Assumption 6 implies that, in equilibrium, a firm's expected profit if exerting e_H is strictly higher than the expected profit if exerting e_L for any A:

$$p\left(B^*\left(e_H\right), e_H\right)\theta - B^*\left(e_H\right) + A - c_H > p\left(B^*\left(e_L\right), e_L\right)\theta - B^*\left(e_L\right) + A$$

Equivalently,

$$\theta > \frac{\left(B^*\left(e_H\right) - B^*\left(e_L\right)\right) + c_H}{p\left(B^*\left(e_H\right), e_H\right) - p\left(B^*\left(e_L\right), e_L\right)}$$

Proposition 2 *The optimal contract $(B, s_1, s_2|A)$ is a state-contingent contract. In equilibrium,*

(1) $B = B^*\left(e_H\right) = B^*$, where $p'_B\left(B^*, e_H\right) = \frac{1}{\theta}$

(2) Repayment schemes $\left(s_1^*, s_2^*|s_1 \leq \theta + A, s_2 \leq A, s_1^* = \frac{B^* - (1 - p(B^*, e_H))s_2^*}{p(B^*, e_H)}\right)$ are not unique,

Proposition 2 shows that the optimal borrowing amount B^* increases as θ increases.

Due to the risk-neutrality of the firms and the financiers, the optimal repayment scheme is indeterminate. According to Modigliani and Miller's (1958) theorem on the irrelevance of firms' financial structure, any equilibrium repayment scheme that satisfies Proposition 2, is optimal. Therefore, standard debt, equity and convertible debt contracts are all optimal in the benchmark case. Note that in this paper, we assume that the costs of signing different types of contracts are the same. Without loss of generality[4], we assume that the cost is equal to zero. If the costs were different, the contract with the lowest cost would be optimal.

Assumption 7 The cost of signing standard debt, equity and convertible debt contracts equals to zero.

In the following sections, we analyze these three types of contracts which are commonly used in reality –standard debt, equity and convertible debt contracts– and show that if the firm returns and effort are observable and verifiable, all three types of contracts are optimal.

Standard debt contract

A standard debt contracts $(B, r|A)$ specifies the firm's borrowing amount B and the corresponding interest rate r for a firm with initial asset A. In equilibrium, the

[4]As long as the costs of signing different types of contracts are equal, there is no difference between assuming cost = 0 or cost = c (>0), where c is constant.

contract $(B, r|A)$ satisfies the financier's participation constraint (PC),

$$p(B,e)\, Br + (1 - p(B,e)) \min(Br, A) - B \geq 0 \tag{PC}$$

the firms' limited liability constraints (LL1&LL2),

$$\theta - Br + A \geq 0 \tag{LL1}$$

$$-\min(Br, A) + A \geq 0 \tag{LL2}$$

and the firm's expected profit is maximized.

Proposition 3 *The equilibrium standard debt contract* $(B, r|A)$ *is optimal for all risk-neutral firms, where in equilibrium*

(1) $B = B^*$

(2) $r = 1$ *for* $A \geq B^*$*; and* $r = \frac{B^* - (1 - p(B^*, e_H))A}{p(B^*, e_H) B^*} > 1$ *for* $A < B^*$

For unconstrained firms (firms with initial assets $A \geq B^*$), in equilibrium, both limited liability constraints do not bind. Besides, they have enough initial assets to repay fully even if the project fails. Their financiers always obtain the full repayment, thus, their debt is secured, and the equilibrium interest rate for the unconstrained firms is equal to 1.

For constrained firms (firms with initial assets $A < B^*$), one limited liability constraint LL2 binds in equilibrium. This means that if the project fails, the firms go bankrupt, and their assets are liquidated by the financiers. The financiers cannot receive the full repayment once the project fails, thus, the equilibrium interest rate is higher than 1 in order to satisfy the financiers' participation constraint. Besides, the equilibrium interest rate for constrained firms decreases when the firm's initial asset increases.

Given that the firm returns are observable and verifiable by a third party, under standard debt contracts, firms do not default strategically (i.e., they do not default when the project succeeds). If the project succeeds, once the firms default, their assets will be liquidated and the financiers will still obtain full repayment as if the firms do not default because the firm returns will be verified and thus they have to repay fully.

Equity contract

An equity contract $(B, s|A)$ specifies the investment amount B that the financier invests and the share s $(0 < s < 1)$ of the firm('s value) that the financier obtains (at the end of the period). Specifically, the profit the financier obtains is $s\theta$ if the project succeeds, and sA if the project fails. In equilibrium, the contract $(B, s|A)$ for a given A satisfies the financiers' participation constraint (PC),

$$p(B,e)\,s\theta + (1 - p(B,e))\,sA - B \geq 0 \tag{PC}$$

the firms' limited liability constraints (LL1&LL2)

$$\theta - s\theta + A \geq 0 \tag{LL1}$$

$$sA + A \geq 0 \tag{LL2}$$

and the firms' expected profits are maximized.

Proposition 4 *The equilibrium equity contract $(B, s|A)$ given an initial asset A is optimal, where in equilibrium*
(1) $B = B^*$
(2) $s = \frac{B^*}{p(B^*,e_H)\theta + (1-p(B^*,e_H))A}$

Since the financier obtains his share of the firm's profit automatically, the financier's expected profit is the same whether the firm returns are verifiable or not.

Convertible debt contract

Definition 5 *Convertible debt contracts*
A convertible debt contract is a standard debt contract plus a call option which gives the financier a unilateral right to convert the debt to equity at a predetermined time and with a predetermined conversion rate.

A convertible debt contract $(B, r, \alpha|A)$ for a firm with initial asset A specifies the borrowing amount B and interest rate r and a predetermined conversion rate α $(0 < \alpha < 1)$. Specifically, the financier receives αy if the financier converts debt to equity. The equilibrium convertible debt contract maximizes the firm's expected profit subject to the financier's participation constraint (PC)

$$p(B,e)\max(Br, \alpha\theta) + (1 - p(B,e))\max(\min(A, Br), \alpha A) - B \geq 0 \tag{PC}$$

and the two limited liability constraints of the firm (LL1&LL2)

$$\theta - \max(Br, \alpha\theta) + A \geq 0 \tag{LL1}$$

$$-\max(\min(A, Br), \alpha A) + A \geq 0 \tag{LL2}$$

In equilibrium, if the project succeeds, firms have no incentive to default strategically because if they do so, the financiers can either simply convert the debt to equity and thus share the profit, or even if they do not convert, they can go to the court and verify the firms' return. Hence, the financiers can obtain the same repayment whether they convert or not, and the firms will not default strategically. If the project fails, in equilibrium, the constrained firms always default, and the financiers will not convert because $\alpha A < A$. As for the unconstrained firms, they have no incentives to default, and the financiers obtain the full repayment.

Proposition 6 *The equilibrium convertible debt contract* $(B, r, \alpha|A)$ *given an initial assets* A *is optimal, where*

(1) $B = B^*$ *for all firms*

(2) $r = 1$ *for* $A \geq B^*$*; and* $r = \frac{B^* - (1 - p(B^*, e_H))A}{p(B^*, e_H)B^*}$ *for* $A < B^*$

(3) $\alpha = \frac{B^*}{\theta}$ *for* $A \geq B^*$*; and* $\alpha = \frac{B^* - (1 - p(B^*, e_H))A}{p(B^*, e_H)\theta}$ *for* $A < B^*$

In the previous section, under verifiability of firm returns and risk-neutrality of both firms and their financiers, all three types of contracts: standard debt, equity and convertible debt contracts are optimal. Since under verifiability for all three types of contracts, firms have no incentives to default strategically. Besides, under risk-neutrality of both firms and their financiers, firms are indifferent among all three contracts since in equilibrium firms' expected profits are the same and are maximized under all three types of contracts.

Through the following sections in this paper, we assume that firm returns are observable to both the firms themselves and their financiers, but not verifiable by a third party (Assumption 3), which is crucial to our analysis.

1.2.2 Non-verifiable firm returns

Now, suppose that firms' project returns are not verifiable by a third party (Assumption 3). For unconstrained firms ($A \geq B^*$), all three types of contracts are optimal. The reason is that under standard debt contracts, unconstrained firms do not have incentives to default strategically. Unconstrained firms have enough assets such that even if they default, the financiers still obtain the full repayment B^*. Moreover, since convertible debt and equity contracts are both immune to ex-post strategic default, they are optimal under non-verifiability of firm returns.

For constrained firms, standard debt contracts are not optimal. The reason is that constrained firms have incentives to default strategically under standard debt contracts. Because of non-verifiability of firm returns and limited liability of the firms, if the project succeeds, a constrained firm's profit is θ if it defaults and its profit is $\theta - \frac{B^*-A}{p(B^*,e_H)}$ if it does not default. The firm's profit is higher if it defaults.

$$\underbrace{\theta}_{\text{default}} > \underbrace{\theta - \frac{B^* - A}{p\left(B^*, e_H\right)}}_{\text{not default}}$$

As a result, equilibrium standard debt contracts must mitigate the firms' incentives to default strategically. That is, if the project succeeds, the equilibrium repayment must not exceed the firms' initial assets A. Therefore, the equilibrium borrowing amount B equals the firms' initial assets and standard debt contracts are not optimal for constrained firms.

Equity contracts and convertible debt contracts are both immune to ex-post strategic default due to non-verifiability of firm returns. An equity holder shares the firm's return automatically. Convertible debt contracts grant the financiers a unilateral right to convert debt to equity and thus to share firms' return if the project succeeds.

Proposition 7 *Under non-verifiability of firm returns,*

(1) For unconstrained firms, standard debt, equity and convertible debt are all optimal

(2) For constrained firms, standard debt contracts are dominated. Equity and convertible debt contracts are optimal

So far, we have shown that under non-verifiability of firm returns, standard debt contracts are dominated by the other two types of contracts. However, it is not enough to show the optimality of convertible debt contracts for smaller firms. Nor is it enough to explain the fact that the probability of using convertible debt contracts is positively related to the firms' profit-to-assets ratios. In the following two sections, we only consider the environment with non-verifiable firm returns (under Assumption 3) through the whole following paper. Besides, we further relax Assumption 2 and then Assumption 1 each by each.

1.3 Generalized Model (1): Moral hazard

In this section, we generalize the model and relax the assumption of observable firm effort (Assumption 2). The moral hazard problem is generated by the dependence of the distribution of the project returns on the firms' effort choice, which is unobservable to the financiers. In order to provide incentives to the firms to exert high effort, it is necessary to let the firms bear some risk of the project. In the following analysis, we again focus on the three types of contracts and derive the equilibrium contracts under moral hazard.

1.3.1 Standard debt contract

The equilibrium standard debt contracts for unconstrained firms are the solution to the following problem:

$$\max_{B,e,r} p(B,e)(\theta - Br + A) + (1 - p(B,e))(-Br + A) - c(e)$$

s.t.

$$E\pi_l = p(B,e)Br + (1 - p(B,e))Br - B \geq 0 \quad \text{(PC)}$$

$$\theta - Br + A \geq 0 \quad \text{(LL1)}$$

$$-Br + A \geq 0 \quad \text{(LL2)}$$

$$(p(B,e_H) - p(B,e_L))\theta \geq c_H \quad \text{(IC)}$$

Proposition 8 *For $A \geq B^*$, the equilibrium standard debt contract $(B, r|A)$ is optimal,*
(1) $B = B^*$
(2) $r = 1$

The equilibrium standard debt contracts for unconstrained firms are exactly the same as the ones in Proposition 3. The unconstrained firms pay a fixed repayment B^* and keep the rest of the returns. Therefore, they have incentives to exert high effort, and the (IC) constraints do not bind.

As for constrained firms, as we mentioned before, the standard debt contracts in Proposition 3 will not be offered in equilibrium since the constrained firms have incentive to default strategically due to non-verifiability of firm returns. As a result, in equilibrium, standard debt contracts are clearly dominated by equity and convertible debt contracts.

1.3.2 Equity contract

If equity contracts are offered in equilibrium, the equilibrium equity contracts must solve the following problem:

$$\max_{B,e,s} p(B,e)(\theta - s\theta + A) + (1 - p(B,e))(-sA + A) - c(e)$$

s.t.

$$p\left(B,e\right)s\theta+\left(1-p\left(B,e\right)\right)sA-B\geq 0 \tag{PC}$$

$$\theta-s\theta+A\geq 0 \tag{LL1}$$

$$sA+A\geq 0 \tag{LL2}$$

$$\left(p\left(B,e_{H}\right)-p\left(B,e_{L}\right)\right)\left(\theta-s\left(\theta-A\right)\right)\geq c_{H} \tag{IC}$$

Proposition 9 *For $A\geq A_2$, the equilibrium equity contracts $(B,s|A)$ is optimal,*
(1) $B=B^*$
(2) $s=\frac{B^*}{p(B^*,e_H)\theta+(1-p(B^*,e_H))A}$
where

$$A_2=\frac{\theta\left(B^*-\left(\frac{c_H}{p(B^*,e_H)-p(B^*,e_L)}\right)p\left(B^*,e_H\right)\right)}{B^*+\left(\frac{c_H}{p(B^*,e_H)-p(B^*,e_L)}\right)\left(1-p\left(B^*,e_H\right)\right)}$$

For $A\geq A_2$, the equilibrium equity contracts are the same as the ones stated in Proposition 4. This shows that for the firms with higher $A\,(A\geq A_2)$, equity contracts are optimal. However, for the firms with smaller $A\,(A<A_2)$, there will be no equity contracts offered in equilibrium due to the (IC) constraints.

1.3.3 Convertible debt contract

The equilibrium convertible debt contracts are the solution to the following problem:

$$\max_{B,e,\alpha} p\left(B,e\right)\left(\theta-\max\left(Br,\alpha\theta\right)+A\right)+\left(1-p\left(B,e\right)\right)\left(0-\max\left(\min\left(A,Br\right),\alpha A\right)\right)-c\left(e\right)$$

s.t.

$$p\left(B,e\right)\max\left(Br,\alpha\theta\right)+\left(1-p\left(B,e\right)\right)\max\left(\min\left(A,Br\right),\alpha A\right)-B\geq 0 \tag{PC}$$

$$\theta-\max\left(Br,\alpha\theta\right)+A\geq 0 \tag{LL1}$$

$$-\max\left(\min\left(A,Br\right),\alpha A\right)+A\geq 0 \tag{LL2}$$

$$\left(p\left(B,e_H\right)-p\left(B,e_L\right)\right)\left[\left(\theta-\max\left(Br,\alpha\theta\right)+A\right)+\max\left(\min\left(A,Br\right),\alpha A\right)\right]\geq c_H$$

Proposition 10 *For $A\geq A_1$, the equilibrium convertible debt contract $(B,r,\alpha|A)$ is optimal,*
(1) $B=B^*$ *for all firms*
(2) $r=1$ *for* $A\geq B^*$*; and* $r=\frac{B^*-(1-p(B^*,e_H))A}{p(B^*,e_H)B^*}$ *for* $A_1\leq A<B^*$
(3) $\alpha=\frac{B^*}{\theta}$ *for* $A\geq B^*$*; and* $\alpha=\frac{B^*-(1-p(B^*,e_H))A}{p(B^*,e_H)\theta}$ *for* $A_1\leq A<B^*$

where

$$A_1 = B^* - p(B^*, e_H)\left(\theta - \frac{c_H}{p(B^*, e_H) - p(B^*, e_L)}\right)$$

Proposition 10 demonstrates that convertible debt contracts are optimal for any $A \geq A_1$ if firms' effort is unobservable.

We show that, for unconstrained firms, all three types of contracts are still optimal even if taking into account the moral hazard problem since the (IC) constraints do not bind for larger firms. For constrained firms, standard debt contracts are not optimal because the firms' have incentives to default strategically. Equity contracts and convertible debt contracts both solve this ex-post strategical default. However, here due to the moral hazard problem, equity contracts are not optimal for firms with initial assets A smaller than a threshold A_2. For a firm with small initial asset $A < A_2$, in order to induce it to exert high effort, the share s assigned to the financier can not be too high. However, if the share is not high enough, the financier's participation constraint (PC) is violated (i.e., financier has negative profits) because the share s is too low. As a result, in equilibrium, there is no equity contract offered to the firms with $A < A_2$. This means that there exist a trade-off between ex-post and ex-ante efficiency. In order to prevent ex-post inefficiency caused by unobservable effort, the ex-ante efficiency would be sacrificed, that is, small firms will not be able to obtain the funds to invest in projects with positive present value. Convertible debt contracts result in the same trade-off as equity contracts due to moral hazard. However, the ex-ante inefficiency is a less severe problem in convertible debt contracts than in equity contracts. Because in equilibrium, $\alpha < s$, there exists another threshold A_1 such that for a firms with $A_1 \leq A < A_2$, convertible debt contracts are optimal. We show $A_1 < A_2$ in the following lemma.

Lemma 11 $A_1 < A_2$

In summary, unconstrained firms have no incentives to default strategically ex-post even though the firm returns are not verifiable. Besides, unconstrained firms are not constrained by the incentive compatibility constraints even though there is a moral hazard problem due to unobservable firm effort. Therefore, for unconstrained firms, standard debt, equity and convertible debt contracts are all optimal. However, this result does not hold for the constrained firms. First of all, constrained firms always have incentives to default strategically if the firm returns are not verifiable. The solutions to this ex-post strategical default are either to let the financier be an equity holder or a convertible debt holder. In other words, both equity and convertible debt contracts can effectively solve the ex-post strategical default of the firm returns are not verifiable. However, equity contracts are dominated by convertible debt contracts for firms with initial assets $A_1 \leq A < A_2$ because there is no equity contract for a firm with $A_1 \leq A < A_2$ such that the incentive compatibility constraint of the firm and the financier's participation constraint are both satisfied at the same time. Therefore, while equity and convertible debt contracts are both optimal for constrained firms with $A \geq A_2$, for constrained firms with $A_1 \leq A < A_2$, only convertible debt contracts are optimal.

This result is consistent with the stylized facts that the probability of using convertible debt is negatively related with the firm size (Lewis, Rogalski and Seward (1999)). In particular, for small firms ($A_1 < A < A_2$), only convertible debt contracts are optimal.

Moreover, under moral hazard, we are able to explain another stylized fact (Lewis, Rogalski and Seward (1999)) – the positive relation between firms' profit-to-asset ratio and the probability of using convertible debt – in empirical evidence. Firms' profit-to-asset ratio is defined as

$$\text{profit-to-asset ratio } = \frac{p\left(B^*, e_H\right)\theta - B^* + A}{A}$$

If θ increases, B^* increases (see Proposition 2). The profit-to-asset ratio also increases,

$$\begin{aligned} \frac{\partial\left(\frac{p(B^*,e_H)\theta - B^* + A}{A}\right)}{\partial\theta} &= \frac{p\left(B^*, e_H\right)}{A} + \frac{p'_B\left(B^*, e_H\right)\theta - 1}{A}\frac{\partial B^*}{\partial\theta} \\ &= \frac{p\left(B^*, e_H\right)}{A} > 0 \end{aligned} \tag{1}$$

Moreover, if θ increases, the threshold A_1 decreases and A_2 increases.

Lemma 12 $\frac{\partial A_1}{\partial\theta} < 0$

Lemma 13 $\frac{\partial A_2}{\partial\theta} > 0$

Note that the change of the function form of $p\left(B, e\right)$ also affects the firms' profit-to-asset ratio. We further derive the relation between the threshold A_1, A_2 and the probability function of success $p\left(B, e\right)$ in the following lemma.

Lemma 14 *Suppose* $p\left(B, e_j\right)$ *is a homogeneous function with degree* n $(n < 1)$*. Let* $F\left(p\left(B, e_j\right)\right) = \widetilde{p}\left(B, e_j\right)$ *be a homothetic function, where* $F\left(p\left(B, e_j\right)\right)$ *is monotone increasing in* $p\left(B, e_j\right)$*.* A_1 *under* $p\left(B^*, e_j\right)$ *is higher than* $\widetilde{A_1}$ *derived under* $\widetilde{p}\left(\widetilde{B}^*, \widetilde{e_j}\right)$*, where* $j = H, L$

Lemma 15 *Suppose $p(B, e_j)$ is a homogeneous function with degree n ($n < 1$). Let $F(p(B, e_j)) = \widetilde{p}(B, e_j)$ be a homothetic function, where $F(p(B, e_j))$ is monotone increasing in $p(B, e_j)$. A_2 under $p(B^*, e_j)$ is lower than $\widetilde{A_2}$ derived under $\widetilde{p}\left(\widetilde{B}^*, \widetilde{e_j}\right)$, where $j = H, L$*

If the distribution of firms' initial assets is further assumed to be exogenous, we derive that the probability of a firm being constrained by limited liability increases as the firm's profit-to-asset ratio increases. Using the result from Lemma 12, Lemma 13, Lemma 14, Lemma 15 and equation (1) derived above, we show that the probability of a firm using convertible debt also increases as the probability of the firm being constrained increases.

Assumption 8 The distribution of firms' initial assets is exogenous

Proposition 16 *If the firms' effort is unobservable, the probability of a firm using a convertible debt contract increases as the firm's profit-to-asset ratio increases.*

The result of Proposition 16 comes directly from Lemma 12, Lemma 13, Lemma 14, Lemma 15 and equation (1). Besides, we have shown that for firms with initial assets $A \in [A_1, A_2]$, convertible debt contracts are optimal. The probability of a firm using convertible debt can be written as

$$prob\,(A \in [A_1, A_2])$$

First, from the results of Lemma 12, Lemma 13, and equation (1), we can conclude that the probability of a firm using convertible debt contracts increase as the firms' profit-to-assets ratio increases which is due to an increase in θ. The reason is that as θ increases, A_1 decreases and A_2 increases.

Second, if the firms' profit-to-assets ratios increase is due to an increase in the function $p(B, e_j)$ ($j = H, L$), from Lemma 14, Lemma 15, we conclude that A_1 decreases to $\widetilde{A}_1$ and A_2 increases to $\widetilde{A}_2$. Therefore,

$$prob(A \in [\widetilde{A}_1, \widetilde{A}_2]) > prob\,(A \in [A_1, A_2])$$

1.4 Generalized model (2): Risk aversion

In this section, we assume Assumption 2 holds and we generalize the model by relaxing Assumption 1. We consider risk averse firms and risk neutral financiers.

To compare the three types of contracts under the assumption of the firms being risk averse is more complicated. Note that the probability of success $p(B, e)$ is endogenous. In particular, the probability of success can be increased through a higher borrowing amount B and a higher effort e of firms. Under risk aversion of firms, different types of contracts may result in different equilibrium borrowing B as well as different e. This in turns affect the probability of success $p(B,e)$. Hence, at this stage it is not clear to determine which type of contracts is optimal under risk aversion of the firms without further assumptions.

In order to analyze this problem, we introduce the concept of " mixed risk aversion", which is defined in Caballé and Pomansky (1996). They consider that the distribution function of outcomes is endogenous and it can be influenced by agents' behavior. This is the concept so called self-protection (Ehrlich and Becker (1972)). Caballé and Pomansky (1996) show that if the firms' utility functions satisfy $(-1)^{n+1} U^{(n)} \geq 0$, the measurement of mixed risk aversion is monotonic with Arrow-Pratt risk aversion. In other words, if an agent is more risk averse than the other, we can also conclude that the agent is more mixed risk averse than the other. Moreover, they provide a comparative study which allows us to analyze our problem and to compare the three types of contracts.

Suppose equity contracts are offered in equilibrium, the equilibrium equity contracts solve the following problem

$$\max_{B,e,s} p(B,e)\,U(\theta - s\theta + A) + (1 - p(B,e))\,U(-sA + A) - C(e)$$

s.t.

$$p(B,e)\,s\theta + (1 - p(B,e))\,sA - B \geq 0$$

$$0 < s \leq 1$$

We denote the equilibrium equity contracts $\left(B^E, s^E|A\right)$ where B^E is the equilibrium borrowing amount and s is the equilibrium share of the firms' profits promised to the financiers. Since effort is observable, equilibrium equity contracts depend on the firms' effort choice. We further assume that it is optimal for firms to exert high effort e_H even under risk aversion.

In the previous analysis, we have shown that standard debt contracts are dominated by convertible debt contracts if the firms are constrained by limited liability. As for unconstrained firms, standard debt and convertible debt contracts are both optimal and they achieve the same equilibrium contract outcomes. Hence, In the following, we only need to compare convertible debt contracts with equity contracts.

Suppose convertible debt contracts are offered in equilibrium, they solve the following problem:

$$\max_{B,e,\alpha} p(B,e)\,U(\theta - \alpha\theta + A) + (1 - p(B,e))\,U(0 - (\min(-Br + A, \alpha A), 0)) - C(e)$$

s.t.

$$p(B,e)\,\alpha\theta + (1 - p(B,e))\min(-Br + A, \alpha A) - B \geq 0$$

$\left(B^{CD}, r^{CD}, \alpha^{CD}|A\right)$ denotes the equilibrium convertible debt contracts.

Equity contracts make the financiers bear more risk compared to convertible debt contracts. In particular, the difference of the firms' utility between good outcome (the project succeeds) and bad outcome (the project fails) is smaller under equity contracts. This in turns leads to the following result.

Lemma 17 $B^E < B^{CD}$

Lemma 17 demonstrates that under convertible debt contracts, risk-averse firms will choose a higher borrowing amount B^{CD}, which implies a higher probability of success of the project under convertible debt. The intuition is the following: the utility of a firm at bankruptcy is lower under convertible debt contracts than the utility under equity contracts given the same loan size. Therefore, due to the firm's risk aversion, the firm will choose a higher equilibrium loan size under convertible debt in order to decrease the probability of bankruptcy.

Caballé and Pomansky (2000) and Dachraoui et al. (2000) show that more mixed risk averse individuals choose higher self-protection or are more willing to pay more for lowering the probability of the bad outcome when this probability is low. Although they focus on comparing agents' with different degrees of risk aversion given the same type of contracts, their results shed some light on our result of Proposition 18

Proposition 18 *If the firms are mixed risk averse, and if* $p\left(B^E, e_H\right) > \frac{1}{2}$*(i.e,* $1 - p(B^*, e_H) < \frac{1}{2}$*), equity contracts are dominated by convertible debt contracts and thus, convertible debt contracts are optimal.*

The intuition of Proposition 18 is that if the probability of failure of the project is already low even under equity financing, which implies that the probability of failure is very low, this state (failure of the project) can be negligible. As a result, firms are better off if choosing convertible debt contracts, since under convertible debt contracts, the borrowing amount B^{CD} is higher than B^E, and thus the probability of success is also higher $\left(p\left(B^{CD}, e_H\right) > p\left(B^E, e_H\right)\right)$. Therefore, the firms' utility if the project succeeds is higher under convertible debt than under equity contracts.

1.5 Discussion

In the previous sections, we have generalized the model in the two directions given non-verifiability of firm returns each by each. In reality, it is plausible that both moral hazard problem and risk aversion exist at the same time.

From the analysis in Generalized Model (1), we have shown that for larger firms, in particular, for $A \geq B^*$, the firms' (IC) constraints do not bind in equilibrium. Therefore, under the assumption of both firms and their financiers being risk-neutral, all three types of contracts are optimal. As for firms with assets $A_2 \leq A < B^*$, both convertible debt and equity contracts are optimal. Standard debt contracts are dominated because the firms' incentive of strategical default. Finally, for the small firms with initial assets $A_1 \leq A < A_2$, only convertible debt contracts are optimal.

In the analysis in Generalized Model (2), we have shown that under some conditions, constrained firms prefer convertible debt contracts over other types of contracts (Proposition 18). For unconstrained firms, both standard debt and convertible debt contracts are optimal.

Combing both results from the analyses, if we consider an environment in which firms are risk averse and the firms' effort is unobservable, we conjecture that for unconstrained firms, both convertible debt and standard debt contracts are optimal, and equity contracts are dominated due to risk aversion of the firms. For constrained firms, standard debt contract are dominated because the constrained firms have incentives to default strategically due to non-verifiability firm returns. As a result, only convertible debt contracts are optimal since convertible debt on the one hand, induces the constrained firms to exert high effort, and on the other hand, achieves better risk sharing compared to equity contracts.

1.6 Conclusion

In this paper, we analyze optimal financial contracts of firms with heterogeneous initial assets. We start at building a simple model with both risk-neutral firms and financiers and we show that under non-verifiability of firm returns, small firms have incentives to default strategically under standard debt contracts. Equity and convertible debt contracts can prevent the ex-post strategical default.

Further, we generalize the model and consider two additional dimensions each by each: (1) moral hazard caused by firms' unobservable effort, and (2) risk aversion of firms or/and their financiers. In (1), only convertible debt contracts are optimal for small firms. Standard debt contracts and equity contracts are dominated because of ex-post strategical default and moral hazard, respectively. In (2), firms are risk averse and their financiers are risk neutral. Under some condition, in particular, if the probability of failure of the project is lower than $\frac{1}{2}$ under equity contracts, risk -averse firms will be better off if using convertible debt contracts since under this condition, firms prefer to choose higher borrowing amount B and thus attain a higher probability of success of the project. This argument is true for all firms if they all have the same level of risk aversion. Since for unconstrained firms, both standard debt contracts and convertible debt contracts achieve the same contract outcomes in equilibrium, hence, both standard debt and convertible debt contracts are optimal for unconstrained firms.

In sum, when we consider both (1) and (2) together, large firms, even under non-verifiability of firm returns, do not have incentives to default strategically. Moreover, large firms have higher initial assets, thus they have incentives to exert high effort. However, due to risk aversion, larger firms prefer standard debt and convertible debt contracts over equity contracts. On the other hand, small firms have incentives to default under non-verifiability of firm returns. Standard debt contracts are dominated by convertible debt and equity contracts. Moreover, due to the risk aversion of the firms, convertible debt contracts dominate equity contracts since convertible debt not only induces the constrained firms to exert high effort but only achieves better risk sharing. Therefore, the optimality of convertible debt for small firms is proved.

1.7 Appendix

Proof. Lemma 1

Suppose that $\left(\overline{B}, \overline{s_1}, \overline{s_2} | A\right)$ is an equilibrium contract and it yields a positive expected profit to the lender,

$$E\pi_l \left(\overline{B}, \overline{s_1}, \overline{s_2} | A\right) > 0$$

The other financier can offer another contract $(B', s_1', s_2' | A)$ where $B' = \overline{B}$, $s_1' = \overline{s_1} - \varepsilon$ and $s_2' = \overline{s_2} - \varepsilon$ and the lender still have non-negative expected profit:

$$E\pi_l \left(\overline{B}, \overline{s_1}, \overline{s_2} | A\right) > E\pi_l \left(B', s_1', s_2' | A\right) > 0$$

This contract $(B', s_1', s_2' | A)$ gives the firm higher expected return. Hence, $\left(\overline{B}, \overline{s_1}, \overline{s_2} | A\right)$ is not an equilibrium contract. By doing so, the equilibrium contract should satisfy $E\pi_l^* = 0$. ■

Proof. Proposition 2

The optimal contract solves the following problem

$$\max_{B,e} p\left(B,e\right)\left(\theta - s_1 + A\right) + \left(1 - p\left(B,e\right)\right)\left(-s_2 + A\right)$$

s.t.

$$E\pi_l = p\left(B,e\right) s_1 + \left(1 - p\left(B,e\right)\right) s_2 - B \geq 0 \tag{PC}$$

$$\theta - s_1 + A \geq 0 \tag{LL1}$$

$$-s_2 + A \geq 0 \tag{LL2}$$

First, we ignore the two (LL) constraints. In equilibrium, $E\pi_l = 0$. Hence,

$$p\left(B,e\right)\left(s_1 - s_2\right) + s_2 = B$$

Plugging this into the firm's objective function. For (s_1, s_2) satisfying (LL) and (IC), B and e solve

$$\max_{B,e} p\left(B,e\right)\theta - \left(p\left(B,e\right)\left(s_1 - s_2\right) + s_2\right) + A \equiv \max_{B,e} p\left(B,e\right)\theta - B + A$$

Under Assumption 6, in equilibrium, $e = e_H$. Therefore, the optimal borrowing amount

$$B^*\left(e_H\right) = B^*$$

and it satisfies

$$p'_B(B^*, e_H) = \frac{1}{\theta}$$

Since both firms and their financiers are risk-neutral, the optimal repayment scheme (s_1^*, s_2^*) is indetermined and it satisfies the following relation

$$s_1^* = \frac{B^* - (1 - p(B^*, e_H))\, s_2^*}{p(B^*, e_H)}$$

■

Proof. Proposition 3(*Standard debt contracts*)

The equilibrium standard debt contract is the solution to the following problem

$$\max_{B,e} p(B,e)(\theta - Br + A) + (1 - p(B,e))(-Br + A)$$

s.t.

$$p(B,e)\, Br + (1 - p(B,e))\, Br - B \geq 0 \tag{PC}$$

$$\theta - Br + A \geq 0 \tag{LL1}$$

$$-Br + A \geq 0 \tag{LL2}$$

First, we ignore LL1 and LL2 and solves the problem. Under Assumption 6, $e = e_H$, and thus we derive the equilibrium $B = B^*(e_H) = B^*$.

For unconstrained firms ($A \geq B^*$), both LL1 and LL2 do not bind. Equilibrium $r = 1$.

For constrained firms ($A < B^*$), LL2 binds. Hence, in equilibrium,

$$r = \frac{B^* - (1 - p(B^*, e_H))\, A}{p(B^*, e_H)\, B^*} > 1$$

All firms' expected profits are maximized. ■

Proof. Proposition 4 (*Equity contracts*)

The equilibrium equity contract is the solution to the following problem

$$\max_{B,e} p(B,e)(\theta - s\theta + A) + (1 - p(B,e))(-sA + A)$$

s.t

$$p(B,e)\, s\theta + (1 - p(B,e))\, sA - B \geq 0 \tag{PC}$$

$$\theta - s\theta + A \geq 0 \tag{LL1}$$

$$-sA + A \geq 0 \tag{LL2}$$

From Lemma 1, (PC) binds in equilibrium. Besides, due to Assumption 6, $e = e_H$, and thus, equilibrium $B = B^*$ where $p'_B(B^*, e_H) = \frac{1}{\theta}$. And the equilibrium share

$$s = \frac{B^*}{p(B^*, e_H)\theta + (1 - p(B^*, e_H))A}$$

Moreover, in equilibrium, for unconstrained firms, $s\theta \geq B^* > sA$ always holds if $\theta > \overline{A}$. We prove this result by contradiction:

(a) suppose $s\theta > sA > B^*$, in equilibrium, $E\pi_l = 0$. Hence, equilibrium

$$B = B^*$$

and

$$s = \frac{B^*}{p(B^*, e_H)\theta + (1 - p(B^*, e_H))A}$$

Since $s\theta > sA > B^*$, we have

$$\frac{B^*\theta}{p(B^*, e_H)\theta + (1 - p(B^*, e_H))A} > \frac{B^*A}{p(B^*, e_H)\theta + (1 - p(B^*, e_H))A} > B^*$$

This is equivalent to

$$\frac{\theta}{p(B^*, e_H)\theta + (1 - p(B^*, e_H))A} > \frac{A}{p(B^*, e_H)\theta + (1 - p(B^*, e_H))A} > 1$$

From second part of inequality, we have

$$\frac{A}{p(B^*, e_H)\theta + (1 - p(B^*, e_H))A} > 1$$

Hence,

$$p(B^*, e_H)(\theta - A) < 0$$

which contradicts with $\theta > A$

(b) suppose $B^* > s\theta > sA$, the same argument as above, in equilibrium,

$$B = B^*$$

and hence

$$s = \frac{B^*}{p(B^*, e_H)\theta + (1 - p(B^*, e_H))A} > 1$$

which is impossible.

Therefore, the equilibrium B^* and s must satisfy $s\theta > B^* > sA$ if $\theta > \overline{A}$. On the other hand, if $\theta \leq \overline{A}$, for $\theta \leq A \leq \overline{A}$, $s\theta < B^* \leq sA$ must hold in equilibrium. ■

Proof. Proposition 6 (*Convertible debt contracts*)

For unconstrained firms, the equilibrium convertible contract solves the following problem

$$\max_{B,e} p(B,e)(\theta - \max(\alpha\theta, Br) + A) + (1 - p(B,e))(-\max(\alpha A, Br) + A)$$

s.t.

$$p(B,e)\max(\alpha\theta, Br) + (1 - p(B,e))\max(\alpha A, Br) - B \geq 0 \tag{PC}$$

$$\theta - \max(\alpha\theta, Br) + A \geq 0 \tag{LL1}$$

$$-\max(\alpha A, Br) + A \geq 0 \tag{LL2}$$

In equilibrium, under Assumption 6, $e = e_H$ and thus, $B = B^*$. Hence, $r = 1$ for unconstrained firms ($A \geq B^*$). From the proof of Proposition 5, it is shown that equilibrium B^* must satisfy $\alpha\theta \geq B^* > \alpha A$ if $\theta > \overline{A}$. Therefore, if $\theta > \overline{A}$, the financier does not convert if the project fails, and is indifferent between converting or not converting if the project succeeds. In equilibrium, $E\pi_l = 0$ (Lemma 1). Thus, for unconstrained firms, equilibrium $\alpha = \frac{B^*}{\theta}$ if $\theta > \overline{A}$. If $\theta \leq \overline{A}$, $\alpha\theta < B^* \leq \alpha A$ must hold. Hence, the financier does not convert if the project succeeds, and is indifferent between converting and not converting if the project fails. Therefore, in equilibrium,

$$\alpha = \frac{B^*}{A}$$

For constrained firms, since $A < B^*$, $\alpha\theta \geq B^* > \alpha A$ always holds assuming $\theta > \overline{A}$. In equilibrium, the financier converts if the project succeeds and does not convert if the project fails and obtain the firm's total asset A. Therefore, in equilibrium,

$$\alpha = \frac{B^* - (1 - p(B^*, e_H))A}{p(B^*, e_H)\theta}$$

for constrained firms. ■

Proof. Proposition 7 (*Non-verifiable firm returns*)

Equity and convertible debt contracts are immune to non-verifiability. Hence, in equilibrium, both contracts implement the optimal contracts derived in Proposition 2. However, standard debt contracts are not optimal for constrained firms because constrained firms have incentives to default strategically. Therefore, under standard debt contracts, in equilibrium, $B = A$, and the constrained firm's expected profits is $p(A, e_H)\,\theta$.

We can show that

$$p(B^*, e_H)\,\theta - B^* + A > p(A, e_H)\,\theta$$

if and only if

$$\theta > \frac{B^* - A}{p(B^*, e_H) - p(A, e_H)}$$

For any constrained firm ($A < B^*$), $\theta > \frac{B^*-A}{p(B^*,e_H)-p(A,e_H)}$ if (1)$\frac{B^*-A}{p(B^*,e_H)-p(A,e_H)}$ is decreasing in A and (2) at $A = 0$, $p(B^*, e_H)\,\theta - B^* > 0$ holds

(2) is always true since we assume that the project is of net positive present value. And we show (1) also holds in the following:

$$\begin{aligned} & \frac{\partial \frac{B^*-A}{p(B^*,e_H)-p(A,e_H)}}{\partial A} \\ = & \frac{p'(A, e_H)(B^*, e_H - A) - (p(B^*, e_H) - p(A, e_H))}{(p(B^*, e_H) - p(A, e_H))^2} \\ < & \ 0 \end{aligned}$$

because

$$p'(A, e_H) > \frac{p(B^*, e_H) - p(A, e_H)}{B^* - A}$$

is always true for any $A < B^*$ since the function $p(.)$ is strictly concave. Therefore, (1) & (2) both hold and thus, $p(B^*, e_H)\,\theta - B^* + A > p(A, e_H)\,\theta$ is true for all $A < B^*$.

In conclusion, for $A < B^*$, standard debt contracts are dominated by equity and convertible debt contracts. ■

Proof. Proposition 8

The proof is the same as in Proposition 3. ■

Proof. Proposition 9

The proof follows Proposition 4 and (IC) bind for firms with $A < A_2$. ■

Proof. Proposition 10

The proof follows Proposition 6 and (IC) bind for firms with $A < A_1$. ■

Proof. Lemma 11 (two initial asset thresholds)

A_1 is the threshold under convertible debt contracts such that

$$(p(B^*, e_H) - p(B^*, e_L))\left(\theta - \frac{B^* - A_1}{p(B^*, e_H)}\right) = c_H$$

Thus,

$$A_1 = B^* - p\left(B^*, e_H\right)\left(\theta - \frac{c_H}{p\left(B^*, e_H\right) - p\left(B^*, e_L\right)}\right)$$

And A_2 is the threshold under equity financing such that

$$\left(p\left(B^*, e_H\right) - p\left(B^*, e_L\right)\right)\left(\theta - s\left(\theta - A_2\right)\right) = c_H$$

where

$$s = \frac{B^*}{p\left(B^*.e_H\right)\theta + \left(1 - p\left(B^*, e_H\right)\right)A}$$

Thus,

$$A_2 = \frac{\theta\left(B^* - \left(\frac{c_H}{p(B^*,e_H)-p(B^*,e_L)}\right)p\left(B^*, e_H\right)\right)}{B^* + \left(\frac{c_H}{p(B^*,e_H)-p(B^*,e_L)}\right)\left(1 - p\left(B^*, e_H\right)\right)}$$

Let

$$\theta - \frac{c_H}{p\left(B^*, e_H\right) - p\left(B^*, e_L\right)} = Q$$

We have

$$A_1 = B^* - p\left(B^*, e_H\right)Q$$

and

$$A_2 = \frac{\theta\left(B^* - p\left(B^*, e_H\right)Q\right)}{B^* + Q\left(1 - p\left(B^*, e_H\right)\right)} = \frac{\theta A_1}{B^* + Q\left(1 - p\left(B^*, e_H\right)\right)}$$

We can show that $A_2 > A_1$ if and only if

$$\frac{\theta A_1}{B^* + Q\left(1 - p\left(B^*, e_H\right)\right)} > A_1$$

That is, we have to show $\theta > B^* + Q\left(1 - p\left(B^*, e_H\right)\right)$.

$$\begin{aligned}
& \theta > B^* + \left(1 - p\left(B^*, e_H\right)\right)\left(\theta - \frac{c_H}{p\left(B^*, e_H\right) - p\left(B^*, e_L\right)}\right) \\
= \; & B^* + \theta - \frac{c_H}{p\left(B^*, e_H\right) - p\left(B^*, e_L\right)} - p\left(B^*, e_H\right)\left(\theta - \frac{c_H}{p\left(B^*, e_H\right) - p\left(B^*, e_L\right)}\right) \\
= \; & \theta + B^* - \left(1 - p\left(B^*, e_H\right)\right)\frac{c_H}{p\left(B^*, e_H\right) - p\left(B^*, e_L\right)} - p\left(B^*, e_H\right)\theta
\end{aligned}$$

Therefore, $A_2 > A_1$ as long as the following inequality holds.

$$\underbrace{p\left(B^*, e_H\right)\theta - B}_{+} + \underbrace{\left(1 - p\left(B^*, e_H\right)\right)\frac{c_H}{p\left(B^*, e_H\right) - p\left(B^*, e_L\right)}}_{+} > 0$$

Since by assumption, the project is of a positive net present value, $p\left(B^{*},e_{H}\right)\theta-B$ is always positive. The inequality above holds and thus $A_{2}>A_{1}$. ■

Proof. Lemma 12

We have

$$A_{1}=B^{*}-p\left(B^{*},e_{H}\right)\left(\theta-\frac{c_{H}}{p\left(B^{*},e_{H}\right)-p\left(B^{*},e_{L}\right)}\right)$$

and

$$\frac{\partial A_{1}}{\partial\theta}=-p\left(B^{*},e_{H}\right)+\frac{\partial A_{1}}{\partial B^{*}}\frac{\partial B^{*}}{\partial\theta}$$

Since under

$$\frac{p'\left(B^{*}.e_{H}\right)}{p\left(B^{*},e_{H}\right)}<\frac{p'\left(B^{*},e_{L}\right)}{p\left(B^{*},e_{L}\right)}$$

we have

$$\frac{\partial A_{1}}{\partial B^{*}}<0$$

Besides,

$$\frac{\partial B^{*}}{\partial\theta}=-\frac{\frac{\partial^{2}E\pi_{f}}{\partial B\partial\theta}}{\frac{\partial^{2}E\pi_{f}}{\partial B^{2}}}>0$$

As a result,

$$\begin{aligned}\frac{\partial A_{1}}{\partial\theta} &= -p\left(B^{*},e_{H}\right)+\underbrace{\frac{\partial A_{1}}{\partial B^{*}}}_{-}\underbrace{\frac{\partial B^{*}}{\partial\theta}}_{+}\\ &< 0\end{aligned}$$

■

Proof. Lemma 13

We have

$$A_{2}=\frac{\theta\left(B^{*}-\left(\frac{c_{H}}{p(B^{*},e_{H})-p(B^{*},e_{L})}\right)p\left(B^{*},e_{H}\right)\right)}{B^{*}+\left(\frac{c_{H}}{p(B^{*},e_{H})-p(B^{*},e_{L})}\right)\left(1-p\left(B^{*},e_{H}\right)\right)}$$

and

$$\frac{\partial A_{2}}{\partial\theta}=\underbrace{\frac{B^{*}-\left(\frac{c_{H}}{p(B^{*},e_{H})-p(B^{*},e_{L})}\right)p\left(B^{*},e_{H}\right)}{B^{*}+\left(\frac{c_{H}}{p(B^{*},e_{H})-p(B^{*},e_{L})}\right)\left(1-p\left(B^{*},e_{H}\right)\right)}}_{+}+\frac{\partial A_{2}}{\partial B^{*}}\underbrace{\frac{\partial B^{*}}{\partial\theta}}_{+}$$

Since

$$\frac{\partial A_{2}}{\partial B^{*}}=(B-M)\left(\frac{\partial M}{\partial B}p_{H}+(B-M)+p_{h}Mp_{H}'+p_{H}'M\right)>0$$

where

$$M=\frac{c_H}{p\left(B^*,e_H\right)-p\left(B^*,e_L\right)}$$

and we denote $p_H=p\left(B^*,e_H\right)$

Therefore, $\frac{\partial A_2}{\partial \theta}>0$ ■

Proof. Lemma 14

We have

$$A_1=B^*-p\left(B^*,e_H\right)\left(\theta-\frac{c_H}{p\left(B^*,e_H\right)-p\left(B^*,e_L\right)}\right)$$

And

$$\widetilde{A_1}=\widetilde{B}^*-\widetilde{p}\left(\widetilde{B}^*,\widetilde{e_H}\right)\left(\theta-\frac{c_H}{p\left(\widetilde{B}^*,\widetilde{e_H}\right)-p\left(\widetilde{B}^*,\widetilde{e_L}\right)}\right)$$

Since $F\left(p\right)=\widetilde{p}$ is a homothetic function, $B^*=\widetilde{B}^*$ and $e_j=\widetilde{e_j}$. Besides, $F'>0$, we have

$$\widetilde{p}\left(\widetilde{B}^*,\widetilde{e_H}\right)=\lambda^n p\left(B^*,e_H\right)>p\left(B^*,e_H\right)$$

where $\lambda>1$ Thus,

$$\begin{aligned}
& A_1-\widetilde{A_1} \\
= & \underbrace{\left(B^*-\widetilde{B}^*\right)}_{=0}-\theta\underbrace{\left(p\left(B^*,e_H\right)-\widetilde{p}\left(\widetilde{B}^*,\widetilde{e_H}\right)\right)}_{-} \\
& +\underbrace{\left[\left(p\left(B^*,e_H\right)\frac{c_H}{p\left(B^*,e_H\right)-p\left(B^*,e_L\right)}\right)-\left(\widetilde{p}\left(\widetilde{B}^*,\widetilde{e_H}\right)\frac{c_H}{p\left(\widetilde{B}^*,\widetilde{e_H}\right)-p\left(\widetilde{B}^*,\widetilde{e_L}\right)}\right)\right]}_{=0} \\
> & \ 0
\end{aligned}$$

Therefore, $A_1>\widetilde{A_1}$ ■

Proof. Lemma 15

We have

$$A_2=\frac{\theta\left(B^*-\left(\frac{c_H}{p(B^*,e_H)-p(B^*,e_L)}\right)p\left(B^*,e_H\right)\right)}{B^*+\left(\frac{c_H}{p(B^*,e_H)-p(B^*,e_L)}\right)\left(1-p\left(B^*,e_H\right)\right)}$$

$$
\begin{aligned}
\widetilde{A_2} &= \frac{\theta\left(\widetilde{B}^* - \left(\frac{c_H}{p\left(\widetilde{B}^*,\widetilde{e_H}\right)-p\left(\widetilde{B}^*,\widetilde{e_L}\right)}\right) p\left(\widetilde{B}^*,\widetilde{e_H}\right)\right)}{\widetilde{B}^* + \left(\frac{c_H}{p\left(\widetilde{B}^*,\widetilde{e_H}\right)-p\left(\widetilde{B}^*,e_L\right)}\right)\left(1-p\left(\widetilde{B}^*,\widetilde{e_H}\right)\right)} \\
&= \frac{\lambda^n\left[\theta\left(B^* - \left(\frac{c_H}{p(B^*,e_H)-p(B^*,e_L)}\right) p\left(B^*,e_H\right)\right)\right]}{\lambda^n\left[B^* + \left(\frac{c_H}{p(B^*,e_H)-p(B^*,e_L)}\right)\left(\frac{1}{\lambda^n} - p\left(B^*,e_H\right)\right)\right]} \\
&> A_2
\end{aligned}
$$

since

$$
\left(\frac{1}{\lambda^n} - p\left(B^*,e_H\right)\right) < \left(1 - p\left(B^*,e_H\right)\right)
$$

■

Proof. Proposition 16

The proof follows from Lemma 12, Lemma 13, Lemma 14 and Lemma 15. Since θ and probability function $p\left(B,e_j\right)$ are the sources that affect the risk-neutral firms' profit-to-asset ratios, If θ increases, combine Lemma 12 and Lemma 13, we can conclude that the probability of a firm using convertible debt contract increases if the firms' profit-to-asset ratio increases since A_1 decrease and A_2 increases.

If $p\left(B,e_j\right)$ becomes $\widetilde{p}\left(B,e_j\right)$, from Lemma 14 and Lemma 15, A_1, A_2 become $\widetilde{A}_1$, $\widetilde{A}_2$ respectively, and

$$
prob\left(A \in [A_1, A_2]\right) < prob\left(A \in \left[\widetilde{A_1}, \widetilde{A_2}\right]\right)
$$

Hence, in sum, an increase in firms' profit-to-asset ratio leads to an increase in the probability of a firm using convertible debt. ■

Proof. Lemma17

If firms are risk averse, optimal contracts must solve

$$
\max_{B,e,s_1,s_2} p\left(B,e\right) U\left(\theta - s_1 + A\right) + \left(1 - p\left(B,e\right)\right) U\left(-s_2 + A\right) - c\left(e\right)
$$

s.t.

$$
p\left(B,e\right) s_1 + \left(1 - p\left(B,e\right)\right) s_2 - B = 0
$$

$$
\theta - s_1 + A \geq 0
$$

$$
-s_2 + A \geq 0
$$

This problem is equivalent to

$$\max_{B,e,s_2} p(B,e)\,U\left(\theta-\frac{B-(1-p(B,e))\,s_2}{p(B,e)}+A\right)+(1-p(B,e))\,U(-s_2+A)-c(e)$$

s.t.

$$\theta-\frac{B-(1-p(B,e))\,s_2}{p(B,e)}+A\geq 0$$

$$-s_2+A\geq 0$$

We derive the first order conditions for both B and s_2:

$$\begin{aligned}
[B]: \quad & p'_B\left(U\left(\theta-\frac{B-(1-p(B,e))\,s_2}{p(B,e)}+A\right)-U(-s_2+A)\right) && (1.1)\\
& +pU'\left(\theta-\frac{B-(1-p(B,e))\,s_2}{p(B,e)}+A\right)\left(-\frac{\partial\left(\frac{B-(1-p(B,e))s_2}{p(B,e)}\right)}{\partial B}\right) && (1.2)\\
= \quad & 0 && (1.3)
\end{aligned}$$

$$\begin{aligned}
[s_2] \quad : \quad & pU'\left(\theta-\frac{B-(1-p(B,e))\,s_2}{p(B,e)}+A\right)\frac{\partial\left(\frac{B-(1-p(B,e))s_2}{p(B,e)}\right)}{\partial s_2} && (1.4)\\
& -(1-p)\,U'(-s_2+A) && (1.5)\\
< \quad & 0 && (1.6)
\end{aligned}$$

From equation 1.7, we can show that

$$-\frac{\frac{\partial^2 EU}{\partial B^2}}{\frac{\partial^2 EU}{\partial B\partial s_2}}>0$$

consequently, $\frac{\partial B}{\partial s_2}>0$.

Since s_2 under convertible debt is higher than s_2 under equity contracts, hence, $B^{CD}>B^{E}$ ■

Proof. Proposition18

We can show that given B^E, the firms' expected utility is higher under convertible debt contracts than under equity contracts, iff

$$\begin{aligned}
& p\left(B^E, e_H\right) U\left(\theta-\frac{B^E-\left(1-p\left(B^E, e_H\right)\right) A}{p\left(B^E, e_H\right)}+A\right) \\
\geq \;& p\left(B^E, e_H\right) U\left(\theta-\frac{B^E \theta}{p\left(B^E, e_H\right) \theta+\left(1-p\left(B^E, e_H\right)\right) A}+A\right) \\
& +\left(1-p\left(B^E, e_H\right)\right) U\left(-\frac{B^E A}{p\left(B^E, e_H\right) \theta+\left(1-p\left(B^E, e_H\right)\right) A}+A\right)
\end{aligned}$$

This is equivalent to

$$\begin{aligned}
& \frac{U\left(\theta-\frac{B^E-\left(1-p\left(B^E, e_H\right)\right) A}{p\left(B^E, e_H\right)}+A\right)-U\left(\theta-\frac{B^E \theta}{p\left(B^E, e_H\right) \theta+\left(1-p\left(B^E, e_H\right)\right) A}+A\right)}{U\left(-\frac{B^E A}{p\left(B^E, e_H\right) \theta+\left(1-p\left(B^E, e_H\right)\right) A}+A\right)} && (1.7) \\
< \;& \frac{\left(\theta-\frac{B^E-\left(1-p\left(B^E, e_H\right)\right) A}{p\left(B^E, e_H\right)}+A\right)-\left(\theta-\frac{B^E \theta}{p\left(B^E, e_H\right) \theta+\left(1-p\left(B^E, e_H\right)\right) A}+A\right)}{-\frac{B^E A}{p\left(B^E, e_H\right) \theta+\left(1-p\left(B^E, e_H\right)\right) A}+A} && (1.8) \\
< \;& \frac{1-p\left(B^{E, e_H}\right)}{p\left(B^E, e_H\right)} && (1.9)
\end{aligned}$$

The sufficient condition for the inequality above held is $p\left(B^{E, e_H}\right)>\frac{1}{2}$. Since if $p\left(B^{E, e_H}\right)>\frac{1}{2}$,

$$\begin{aligned}
& p\left(B^E, e_H\right) \underbrace{\left\{\begin{array}{c}
{\left[\frac{B^E \theta}{p\left(B^E, e_H\right) \theta+\left(1-p\left(B^E, e_H\right)\right) A}-\frac{B^E-\left(1-p\left(B^E, e_H\right)\right) A}{p\left(B^E, e_H\right)}\right]} \\
-\left(-\frac{B^E A}{p\left(B^E, e_H\right) \theta+\left(1-p\left(B^E, e_H\right)\right) A}+A\right)
\end{array}\right\}}_{(i)} \\
< \;& \left(-\frac{B^E A}{p\left(B^E, e_H\right) \theta+\left(1-p\left(B^E, e_H\right)\right) A}\right)+A
\end{aligned}$$

since term (i) is always negative if

$$p\left(B^E, e_H\right)>\frac{1}{2}$$

Therefore, inequality 1.7 holds. Besides, from Lemma 17, we have shown that in equilibrium, $B^E<B^{CD}$.

As a result, we have shown that given B^E, firms' expected utility is higher under convertible debt than under equity

$$EU^{CD}\left(B^{E}\right) > EU^{E}\left(B^{E}\right)$$

Together with the result $B^{E} < B^{CD}$ form Lemma 17, B^{CD} maximizes the firms' expected utility under convertible debt, we derive

$$EU^{CD}\left(B^{CD}\right) > EU^{CD}\left(B^{E}\right) > EU^{E}\left(B^{E}\right)$$

and the result is shown. ■

Chapter 2

Non-Monotonic Effects of Bankruptcy Exemptions on Small Firms' Finance

The link between law and firms' finance are examined in many empirical works. A simple principal-agent model is presented in this paper. Firms (agents) do not know the quality of the projects they have chosen, however, banks can learn the quality of the firms' projects by costly screening. The accuracy of screening depends on banks' screening intensity exerted. Our model demonstrate that, in equilibrium, the loan obtained by the firms, as well as the gross interest rate, are determined through two channels: (1) Direct Effect: an effect of bankruptcy exemptions directly on the equilbrium debt contracts, and (2) Indirect Effect: an effect of bankruptcy exemption indirectly on the equilibrium debt contracts via banks' screening intensity. Combining both effects, our model yields a non-monotonic relation between bankruptcy exemptions and the equilibrium loan size of the firms as well as the equilibrium gross interest rate as shown in the data.

2.1 Introduction

The link between law and firms' finance has been examined in a great amount of empirical works. In particular, in a series of previous works by Berkowitz and White, the impact of personal bankruptcy law on small business finance is emphasized.

Small businesses are the primary source of new jobs in the U.S. economy. From 1990 to 1995, businesses with less than 500 employees accounted for 76.5% of net new jobs (which is the net of job creation and job destruction that both are weighted by the firm-level employment growth rate). Moreover, small business are considered to play an important role in the economic growth. As seen in the following table, more than 50% of U.S. private nonfarm output is produced by small businesses. This evidence stresses the important role of small businesses in the U.S. economy.

However, small business entrepreneurs have fewer assets and depend primarily on debt financing. Specifically, 50.37% of small businesses' external finance is debt finance among all sources of finance. Commercial banks contribute 18.75% of debt finance, which is the primary category among the nine categories of debt finance. (see Berger and Udell (1998)). Since obtaining credits from banks is important for small business to invest in projects, the factors that affect small business' access to credit in the credit market play a crucial role, one of them being the bankruptcy law.

In the U.S., it is well known that individual and corporate bankruptcy procedures are separated. Moreover, the U.S. personal bankruptcy law not only applies to the consumers, but also to small businesses that are non-corporate. The reason is that, if a firm is non-corporate, its debt is the personal liability of the owner of the firm. The objective of the personal bankruptcy law is to offer a security to debtors when they are in financial distress (for instance, which is caused by a negative income shock, job loss, or health problem). Borrowers can choose between filing Chapter 7 and Chapter 13.

If the borrowers choose Chapter 7, all of their unsecured debts are discharged and they repay and give up all their assets beyond a certain level which is predetermined by the bankruptcy exemption. Also, they do not have any obligation of repaying the debt in the future after filing Chapter 7. In other words, their future earnings are completely exempt. In the U.S., the bankruptcy law is a federal law and the procedure is uniform across the country. However, each state has its own right to set the exemption level. Most states have several types of exemptions, such as, homestead(the borrowers/entrepreneurs' own resident house) exemption, equity in vehicles and in other goods, etc. In most of the states, the homestead exemption is the largest one.

If the borrowers choose to file Chapter 13, they have to propose to the court (as well as the creditors) their plans for repaying part or all of their debt in the future by using their future earnings, but the assets they own are all exempt. Compared to Chapter 7, Chapter 13 is less favorable to the small business owners. One reason is that under Chapter 7, small businesses' future earnings are all exempt, which gives them an opportunity of "fresh start". By contrast, Chapter 13 makes it more difficult for small businesses to start a new business. Also creditors are entitled to receive no less under Chapter 13 than under Chapter 7 (13 U.S.C. Section 1325(a)(4)) and hence, the exemption level might have similar effect on the credit market conditions no matter which bankruptcy chapter is chosen. This means that, even if small business entrepreneurs have chosen to file Chapter 13, the conditions of the credit market they face are still affected by the level of exemptions and the conditions are similar as if they file Chapter 7. The statistical data shows that around 70% of all bankruptcy filings occur under Chapter 7 (Hasan and Wang (2008)). Thus, in this paper, we assume that borrowers file for bankruptcy under Chapter 7.

As mentioned above, the objective of the personal bankruptcy law is to provide a security to the debtors when they are in financial distress. Hence, higher bankruptcy exemptions provide higher protection to the debtors. In other words, the higher the bankruptcy exemptions are, the smaller the creditors' rights are. If the exemption is high, the creditors are less protected when the debtors go bankrupt under since the creditors can only obtain the debtors' assets that exceed the exemption level.

Banks are the primary creditors in the credit market. They provide credit to fund the firms' investment projects. However, since the returns of an investment project are not certain and firms might use the credit unproductively, banks may not be able to get the repayment back. This affects banks' incentives to provide (full) credit to the entrepreneurs. As LaPorta et al. (1998, p.1114) pointed out:

> [C]reditors are paid because they have the right to repossess collateral. Without these rights, investors would not be able get paid, and therefore firms would not have the benefits of raising funds from these investors.

This suggests that the higher protection of creditors' right would give banks incentives to provide more abundant and cheaper credit to the entrepreneurs. However, others have pointed out that high protection of the creditors' rights may, on the other hand, induce creditors to make more risky loans. For instance, Posner (1992, p. 400) notes:

> Some states have generous household exemptions for insolvent debtors, others chintzy ones. In the former states, the risk of entrepreneurship is reduced because the cost of failure is less, but interest rates are higher because default is more likely and the creditor's position in the event of default is weaker. And note that higher interest rates make default all the more likely. Cutting the other way, however, is the fact that in the low-exemption states lenders' risk is less, which induces lenders to make

more risky loans, i.e., loans likelier to end in bankruptcy. It is therefore unclear as a theoretical proposition whether there will be more bankruptcies in the high-exemption states or in the low-exemption ones.

Hence, another role of banks also has to be brought into attentions: the role of screening. Banks fund a large number of investment projects in some specific sectors, so they (may) have considerable experience dealing with similar projects undertaken by firms in these sectors. Moreover, they (may) have more information about the general economic trend. Hence, banks might know the quality of the projects chosen by entrepreneurs better than entrepreneurs themselves (Manove, Padilla and Pagano (MPP 2001), Garmaise (2001)). As a result, screening by banks may be socially desirable. The banks' incentive to screen is low when their rights of possessing the firms' assets are better protected. In other words, if the exemption level is low, banks may lack incentives to screen.

There have been many empirical works on the effects of bankruptcy exemptions on the credit market. Gropp et al. (1997) analyze how cross-state differences in U.S. personal bankruptcy rules affect the supply and demand for household credit, using data from the 1983 Survey of Consumer Finances. They find that generous state bankruptcy exemptions reduce the amount of credit available to low-asset households (controlling for their observable characteristics) and increase the interest rates on automobile loans. Berkowitz and White (2000) estimate that small firms are 25% more likely to be denied credit if they are located in states with unlimited rather than low homestead bankruptcy exemptions. This literature might lead to the conclusion that the higher exemption causes more credit rationing. Thus, it might be concluded that in order to improve the efficiency of the credit market, a low (or even zero) exemption is desired.

However, Berkowitz and White (2004) provide evidence that there is a non-monotonic relation between the exemption levels and the loan size, the probability of credit rationing and the interest rate. This evidence suggests that higher exemption does not necessarily lead to more credit rationing. Berkowitz and White (2004) examine the effects of bankruptcy exemption levels on small firms' access to credit. Their data source is the 1993 National Survey of Small Business Finance (NSSBF), which contains a representative sample of U.S. nonfinancial, nonfarm, for-profit businesses with less than 500 employees (small businesses). Table 1 represents each state's exemption level in 1993. The personal property exemption is defined as the sum of the state's exemptions for cash and for equity in vehicles, plus the value of the wildcard exemption (an exemption that allows a debtor to apply a certain dollar amount to any type of property to make it entirely or partially exempt).

For states with unlimited homestead exemptions, Berkowitz and White set the homestead exemption equal to the maximum dollar value across all states, which is \$160,000. In order to capture the effect of the unlimited homestead exemption, they also introduce a separate dummy variable that equals one for states that have unlimited homestead exemptions. In their paper, the exemption level is treated as exogenous. Although each state can set their own exemption level, the data shows that between 1983 and 1993, only a few states changed their exemption levels each year, and the federal bankruptcy exemptions remained unchanged. Also, they find exemption levels do not appear to be correlated with state loan market or demographic characteristics.

Table 1: 1993 Bankruptcy Exemptions by States (source: Berkowitz and White (2004))

	Homestead Exemption ($)
Alabama	10,000
Alaska	54,000
Arizona	100,000
Arkansas	Unlimited
California	75,000
Colorado	60,000
Connecticut	15,000
D.C.	15,000
Delaware	15,000
Florida	Unlimited
Georgia	10,000
Hawaii	40,000
Iowa	Unlimited
Idaho	100,000
Illinois	15,000
Indiana	15,000
Kansas	Unlimited
Kentucky	10,000
Louisiana	15,000
Massachusetts	100,000
Maryland	0
Maine	15,000
Michigan	15,000
Minnesota	Unlimited
Missouri	8,000
Mississippi	150,000
Montana	80,000
North Carolina	20,000
North Dakota	160,000
Nebraska	20,000
New Hampshire	60,000
New Jersey	15,000
New Mexico	40,000
Nevada	95,000
New York	20,000
Ohio	10,000
Oklahoma	Unlimited
Oregon	20,000
Pennsylvania	15,000
Rhode Island	15,000
South Carolina	15,000
South Dakota	60,000
Tennessee	7,500
Texas	Unlimited
Utah	10,000
Virginia	10,000
Vermont	60,000
Washington	60,000
Wisconsin	40,000
West Virginia	15,000
Wyoming	20,000

Table 2: Effects of Bankruptcy Exemptions on Loan Size (source: Berkowitz and White (2004))

	Noncorporate Firms		Corporate Firms	
	Coefficient	p-value	Coefficient	p-value
Homestead exemption	−.0970*	.010	−.0382	.077
Homestead exemption^2	.000677*	.010	.000304	.051
Unlimited homestead exemption	−5.18*	.050	−2.34	..129
Personal property exemption	.126	.371	−.0802	.290
Personal property exemption^2	−.00082	.723	.00114	.376
Past bankruptcy	−7.56*	.034	−2.99*	.036
Past personal delinquency	−3.07*	.027	−1.93*	.014
Past business delinquency	1.08	.371	.730	.213
Owner's age	−.127*	.002	−.120*	.000
Firm's age	.0333	.434	−.0153	.519
Family owned	−2.74	.057	1.27*	.015
Female owned	−1.13	.247	−.782	.110
African-American owned	−5.26	.061	−10.04*	.000
Other minority owned	−10.7*	.000	−2.35*	.007
Employment (in log form)	1.68 *	.000	1.95*	.000
Profit/asset ratio	−.164	.109	−.173*	.017
Debt/asset ratio	1.02*	.016	.284	.158
Sales growth	.000927	.209	−.00851	.576
HHI > 1800	1.74*	.036	1.047*	.017
Years of bank relationship	−.0209	.737	−.00436	.898
Years of bank relationship missing	−9.99	.000	−5.89	.002
Checking account at bank lender	−1.363	.400	−1.34	.100
Number of lenders	2.83	.068	1.17*	.000
Intercept	2.28	.495	4.98*	.002
Number of observations	1,801		2,780	
Pseudo-R^2	.0485		.0220	
Log-likelihood	−2275.52		−6133.26	

Note: This table presents results from Tobit regressions explaining loan size (in log form) for noncorporate and corporate firms. Loan size for all firms that did not receive credit is censored at zero. p-values are given, and asterisks indicate statistical significance at the 95% level.

Table 3: Effects of Bankruptcy Exemptions on Interest Rates (source: Berkowitz and White (2004))

	Noncorporate Firms		Corporate Firms	
	Coefficient	p-value	Coefficient	p-value
Homestead exemption	.0824*	.009	.0367*	.033
Homestead exemption^2	−.000588*	.007	−.000271*	.030
Unlimited homestead exemption	4.91*	.025	1.70	.168
Personal property exemption	−.139	.233	.0652	.283
Personal property exemption^2	.0011	.565	−.000939	.362
Past bankruptcy	5.71*	.052	2.20*	.053
Past personal delinquency	2.03	.077	1.45*	.021
Past business delinquency	−.867	.392	−0.540	.250
Owner's age	.106*	.002	.0887*	.000
Firm's age	−.0257	.467	.0191	.317
Family owned	2.28	.058	−.905*	.030
Female owned	.501	.541	.513	.189
African-American owned	5.26*	.027	8.31*	.000
Other minority owned	9.10*	.000	1.88*	.007
Employment (in log form)	−1.35*	.001	−1.33*	.000
Profit/asset ratio	.111	.171	.139*	.017
Debt/asset ratio	−.507	.177	−.206	.205
Sales growth	−.00104	.102	.00952	.442
HHI > 1800	−1.18	.088	−.854*	.015
Years of bank relationship	.000923	.986	−.00365	.989
Years of bank relationship missing	−.480	.769	3.95*	.007
Checking account at bank lender	1.31	.328	.844	.196
Number of lenders	−2.43	.058	−.939*	.000
Intercept	15.2*	.000	13.1*	.000
Number of observations	1,801		2,779	
Pseudo-R^2	.0508		.0205	
Log-likelihood	−2140.96		−5808.21	

Note: This table presents results from Tobit regressions explaining the interest rate on loans for noncorporate and corporate firms. Interest rates for firms that were credit rationed are right censored 17%, which is above the maximum value of 16.5% in the sample. Asterisks indicate statistical significance at the 95% level.

Table 2 and Table 3 show the logit regression result of the impact of exemptions on the loan size as well as on the interest rate respectively. First, the homestead exemption has a negative (positive) significant effect on the loan size (interest rates). However, the predicted effect of the homestead exemption level on the loan size (interest rates) is non-monotonic: as the exemption level rises, the loan size (interest rates) first falls (rise), then rises (fall), and then falls (rise) again when the exemption level becomes unlimited.

Berkowitz and White's model does not explain this non-monotonic relationship between the exemption level and the loan size. The objective of this paper is to build a model that explains this stylized fact. In this paper, we develop a simple one pricinpal-one agent model of competitive banking in the credit market. In our model, investment projects are of different quality. Banks, which are the only lenders, are able to learn the quality of a project undertaken by a firm by exercising a costly screening. The idea is similar with MPP's paper which endogenises the screening decision of banks. The main difference between our paper and theirs is that in their model, the project size (investment of the project) is fixed. Thus, all borrowers obtain the same loan size if they are funded. Here, on the other hand, we allow the project size to be endogenous. This allows us to analyze the effect of the bankruptcy exemptions on the loan size. We find that the bankruptcy exemption has a non-monotonic affect on banks' optimal screening intensity. The banks' screening intensity in turn affects the optimal loan size positively. Hence, bankruptcy exemptions and optimal loan size displays a non-monotonic relation.

The rest of the paper is organized as follows: Section 2 describes the setup of the model. In Section 3, we derive the equilibrium debt contracts under perfect competition and analyze the effect of bankruptcy exemptions on the equilibrium contracts. Section 4 further discusses how the relation between bankruptcy exemptions and the equilibrium loan size changes if the screening technology is improved. Finally, we conclude in Section 5.

2.2 The Setup

2.2.1 Agents and environment

Consider a one-period principal-agent model in the credit market. Firms (agents) are endowed with homestead assets which value A in the market. Each firm has an opportunity to undertake an investment project. The assumptions of the model are:

Assumption 1 Firms' utility functions are quasi-linear. In particular, firms' utility is concave in their homestead assets. Specifically, we assume the utility of their homestead is $V(A)$ where $V(.) \geq 0$, $V'(.) > 0$ and $V''(.) < 0$.

Following from Assumption 1, in equilibrium, firms' homestead assets are A^* such that $V'(A^*) = 1$ and $V(A^*) > A^*$. As a result, firms do not have fund to finance the investment projects since the firms will not liquidate their homestead in order to invest in the projects. Therefore, firms have to obtain the funds from the banks (principals).

Assumption 2 Banks are risk neutral. The banks' opportunity cost of lending per unit is assumed to be exogenous, and for simplicity, $i = 1$.

Assumption 3 The investment projects are of two quality: good (g) or bad (b). A good-quality project yields an observable random return z with two possible outcomes,

$$z = \begin{cases} \theta & \text{if the project succeeds} \\ 0 & \text{if the project fails} \end{cases}$$

However, a bad-quality project always yields return 0.

Also, the distribution of the realized returns of a good-quality project is endogenous, and it depends on the funds B invested in the project. The probability of success of a good-quality project is denoted as $p(B)$, with $p'(B) > 0$, $p''(B) > 0$ for all B.

Assumption 4 The proportion of good-quality projects among all projects is $\frac{1}{2}$, which is exogenous and is known to both firms and the banks. Moreover, firms can not distinguish the quality of the project they have chosen. However, banks can obtain a (imperfect) report about the quality of the projects by screening. The report obtain by banks from screening takes two outcomes: $R = \{Rg, Rb\}$

The accuracy of screening depends on the screening intensity s exerted by banks, which is private information of banks. Banks' screening intensity s is not observable to the firms and hence not contractible. We denote $\Pr(Rg|g, s)$ as the probability of the banks can recognize a good-quality project by exerting s, and $\Pr(Rg|b, s)$ as the probability of the banks misbelieve a bad-quality project as a good one. As we mentioned above, $\Pr(Rg|g, s)$ is equal to one and $\Pr(Rg|b, s)$ is decreasing in s. In particular, the probabilities are written in the following:

$$\Pr(Rg|g, s) = 1$$

$$\Pr(Rg|b, s) = \pi(s) = e^{-s}$$

Where $\pi(s) = 1$ for $s = 0$, $\pi'(s) \leq 0$, $\pi''(s) > 0$ for $s \in (0, \overline{s})$ and $\lim_{s \to \overline{s}} \pi(s) > 0$, where $\overline{s}$ is sufficiently high.

The cost of screening is defined as $C(s) = c_f + cs$, where c_f is a quasi-fixed cost and cs is a variable cost.

2.2.2 Bankruptcy law

The realized returns of the project are assumed to be observable and verifiable. When a project yields return 0, firms are not able to repay their debt and thus, they file for bankruptcy. Under Chapter 7, once a firm files for bankruptcy, their debt are all exempted. The bank liquidate partially the firm's initial assets $(A - E)$, and the firm keeps the part of the asset E with utility $V(E)$. Different levels of bankruptcy exemptions indicate different degrees of the protection of creditors' rights. In particular, higher level of bankruptcy exemptions imply lower degrees of the protection of creditors.

2.2.3 Discussion of Assumptions

Assumption 1: This assumption is equivalent to the fact that homestead assets are not perfect liquid. The reason is that since the assets are not perfect liquid, in practice, once the firms file for bankruptcy and their homesteads are liquidated, some fixed cost of liquidation, such as the commission to the trustee, must be borne. Therefore, the money received by the creditors' from liquidation must be lower than the value of the assets. Moreover, since the liquidation costs are fixed, the value of the assets is an increasing and concave function of the money received from liquidation by the creditors.

Assumption 3: As we mentioned in the introduction, banks are usually assumed to have a lot of experience in funding projects in some specific sectors. Moreover, they have better knowledge about the general economic trend (Manove, Padilla & Pagano (2001)). Hence, banks' role of screening projects should be considered.

Under **Assumption 4**, banks' screening intensity is unobservable to the firms, and hence not contractible. Therefore, if a firm obtains funds, since the firm does not observe the banks' screening intensity, the firms believes that the probability of the project is of good quality is still $\frac{1}{2}$, which is independent of banks' screening intensity.

2.2.4 Timeline and Competitive Equilibrium

The relationship between the firms and the banks is the following:
(1) Firms randomly pick a project, and the quality of the project is unknown to them.
(2) Given E, banks post debt contracts $(B, r|A^*)$ which depend on screening intensity s. Banks compete with each other by taking the debt contracts offered by other banks' as given.
(3) Each firm applies for at most one contract. In other words, if the application of

loan is rejected, firms simply do not have chances to invest.
(4) Banks choose their screening intensity and decide whether to approve the loans given the received report about the project quality. Since screening is unobservable to the firms, and hence, it is non-contractible.
(5) Funded project returns are realized. The firms repay their debt Br. In the case of failure to repay their debt, the firms file for bankruptcy and banks liquidate the firms' assets for recovering their credits.

Under perfect competition, the equilibrium debt contracts maximize firms' expected utility subject to banks' zero-profit condition. If banks do not screen projects at all, all firms are funded since they cannot distinguish good-quality projects from bad-quality projects. If banks screen, the funding decision of banks which depends on the received report after screening is: fund if $R = Rg$, and reject if $R = Rb$. The reason is intuitive, if after screening, banks do not distinguish funding decision depending on the report of screening, the screening is informative but costly. Hence, this would not happen in equilibrium.

In summary, a competitive equilibrium in the credit market is characterized by:
(a) a debt contract for given A^*, $(B^*, r^*|A^*)$
(b) a screening intensity $s^* \in [0, \overline{s}]$ exerted by banks
(c) a funding rule: funding a project if the report is good; rejecting a project if the report is bad.

2.3 A competitive credit market equilibrium

The funding rule states that banks only fund the projects if the received report says that the projects are good. Besides, since the screening intensity is unobservable to firms, the funded firms' expected utility is

$$\frac{1}{2}\left[p\left(B\right)\left(\theta - Br + V\left(A^*\right)\right) + \left(1 - p\left(B\right)\right)V\left(E\right)\right] + \frac{1}{2}V\left(E\right)$$

The equilibrium contracts maximize the funded firms' expected profit subject to banks' participation constraints. Therefore, an equilibrium contract is the solution to the following problem:

$$\max_{B,r,s} EU \;= \frac{1}{2}\left[p\left(B\right)\left(\theta - Br + V\left(A^*\right)\right) + \left(1 - p\left(B\right)\right)V\left(E\right)\right] + \frac{1}{2}V\left(E\right)$$

s.t.

$$E\pi = \frac{1}{2}\left[p(B)Br + (1-p(B))(A^* - E) - B\right] + \frac{1}{2}\pi(s)(A^* - E - B) - C(s) \geq 0 \qquad (\mathrm{PC}_B)$$

$$EU = \frac{1}{2}\left[p(B)(\theta - Br + V(A^*)) + (1-p(B))V(E)\right] + \frac{1}{2}V(E) \geq V(A^*) \qquad (\mathrm{PC}_F)$$

Lemma 19 $E\pi^* = 0$

Lemma 19 shows that under perfect competition, banks' participation constraints bind in equilibrium. This result can be shown by contradiction. From the binding participation constraints of banks, the equilibrium gross interest rate is

$$r^* = \frac{2C(s) + \pi(s)(B - A^* + E) - (1-p(B))(A^* - E) + B}{p(B)B}$$

Under perfect competition, the equilibrium screening intensity s^* should minimize the gross interest rate for any loan size B subject to firms' participation constraints given a bankruptcy exemption E.

Proposition 20 *(Equilibrium screening intensity)*

Equilibrium screening intensity s^* is

(1) $s^* = 0$ if $E \in [0, \underline{E}]$ or $E \in [\overline{E}, \infty)$

(2) $s^* > 0$ if $E \in (\underline{E}, \overline{E})$, where s^* satisfies $\pi(s^*) = \frac{2c}{B^* - A^* + E}$

Proposition 20 shows that if the bankruptcy exemption is low or high, in equilibrium, banks will not screen the firms' projects at all. However, if the bankruptcy exemption is at middle levels, banks will screen the firms' projects.

Whether banks screen or not depends on whether the benefits from screening can cover the cost of screening. If the bankruptcy exemption is low, it implies that even if the firms default and file for bankruptcy, since the exemption is low, banks still can recover most of the credits by liquidating the firms' assets. Therefore, the benefit from screening is small. However, banks have to spend a cost $C(s) = c_f + cs$ if they screen. If the quasi-fixed cost c_f is not too small, it is not worthy for the banks to screen the firms' projects. If the bankruptcy exemption is very high, due to the convexity of $\pi(s)$, banks' screening intensity should be very high if the banks screen. In turns, banks have to increase a lot the gross interest rate in order to break even. Therefore, in equilibrium, if banks screen, firms would not apply for the loan since the cost of credits (gross interest rate) is too high. Hence, the firms will not undertake the investment projects. As a result, banks will not screen when the bankruptcy exemption is very high.

Banks' decisions of screening are endogenous, consequently, the equilibrium loan size B not only depends on the exogenous bankruptcy exemptions, but also on the banks' screening decisions (which depends on bankruptcy exemptions). In the following analysis, we disentangle the total effect of bankruptcy exemption on the equilibrium loan size into two effects: one is Direct Effect and the other is Indirect Effect.

Direct Effect is the effect of bankruptcy exemption directly on the equilibrium loan size given banks' screening intensity unchanged. We demonstrate it in the following lemma.

Lemma 21 *(Direct Effect on B)*

Given a fixed $s = s'$,

$\frac{\partial B^*}{\partial E}|_{s=s'} < 0$ *for any E*

Lemma 21 shows that Direct Effect is always negative. In other words, an increase in bankruptcy exemptions leads to a decrease in the equilibrium loan size given a constant screening intensity of banks. The intuition of this result is that, an increase in bankruptcy exemptions increase the firms' utility if the firms go bankrupt. On the other hand, if the bankruptcy exemption increases, banks would raise the gross interest rate in order to break even. This in turn decreases the firms' utility outside of bankruptcy. Consequently, the marginal utility of loan size decreases as the bankruptcy exemption increases. Therefore, an increase in bankruptcy exemptions leads to a decrease equilibrium loan size if banks' screening intensity is constant.

However, the equilibrium loan size also depends on the banks' screening intensity which is determined by the bankruptcy exemptions. This is what we call Indirect Effect.

Lemma 22 *(Indirect Effect on B)*

$\frac{\partial B^*}{\partial s^*}\frac{\partial s^*}{\partial E} > 0$ *for all E*

Lemma 22 shows that Indirect Effect is positive. In other words, an increase in bankruptcy exemptions increases the banks' screening intensity and thus leads to an increase in the equilibrium loan size. The intuition of the result is the following: if the bankruptcy exemption increases, the loss of the banks becomes larger if the firms file for bankruptcy. Hence, under perfect competition, in order to keep banks' zero-profit condition hold, banks increase the screening intensity. As a result, the marginal utility of loan increases and thus the equilibrium loan size increases.

From Proposition 20 and Lemma 21, we can show that if bankruptcy exemptions are low or high, in equilibrium, an increase in bankruptcy exemptions certainly leads to a decrease in the equilibrium loan size.

Proposition 23 *If $E \in [0, \underline{E}]$ or $E \in \left[\overline{E}, \infty\right)$, an increase in bankruptcy exemption E leads to a decrease in the equilibrium loan size B^**

The result of Proposition 23 is straightforward. Since if the bankruptcy exemption is low ($E \in [0, \underline{E}]$) or is high $\left(E \in \left[\overline{E}, \infty\right)\right)$, banks do not screen any projects and thus $s^* = 0$. Therefore, there is no Indirect Effect of bankruptcy exemptions on equilibrium loan size. The total effect comes from Direct Effect. Following Lemma 21, the direct effect of bankruptcy exemptions on the equilibrium loan size is negative.

However, if the bankruptcy exemption is in the middle level, Direct and Indirect Effect coexist. Hence, total effect is ambiguous since they have opposite effect on the equilibrium loan size. The result depends on the magnitudes of the two effects. In the following Proposition, we show that if the bankruptcy exemption is relatively smaller in the middle level (i.e., $E \in [\underline{E}, E_1]$), Direct Effect dominates Indirect Effect. As the bankruptcy exemption continues increasing, in particular, if the bankruptcy exemption $E \in \left(E_1, \overline{E}\right)$, Indirect Effect dominates Direct Effect.

Proposition 24 *If equilibrium screening intensity $s^* > 0$, there exists a threshold E_1 such that an increase in bankruptcy exemptions leads to*

(1) a decrease in equilibrium loan size B^* if $E \in [\underline{E}, E_1]$

(2) an increase in equilibrium loan size B^* if $E \in \left(E_1, \overline{E}\right)$

Proposition 24 demonstrates that bankruptcy exemptions affect the magnitudes of the two effects. The result is driven by the concavity of the function $V(.)$ as well as the convexity of $\pi(s)$. When the exemption is in the middle level but still relatively small, an increase in the bankruptcy exemption leads to a large increase in the firms' utility at bankruptcy. Besides, the firms' utility outside of bankruptcy decreases a lot as well. As a result, the equilibrium loan size decreases since the marginal utility of loan is small. On the other hand, when the bankruptcy exemption keeps rising, the increase of the firms' marginal utility at bankruptcy becomes smaller as well as the decrease of the firms' marginal utility outside of bankruptcy. This in turns increase the marginal utility of loan when the bankruptcy exemption gets larger $\left(E \in \left(E_1, \overline{E}\right)\right)$. Consequently, the equilibrium loan size increases.

In sum, we combine the results from Proposition 23 and 24 , and we derive the relation between bankruptcy exemptions and the equilibrium loan size. This result is demonstrated in the following proposition.

Proposition 25 *(Total effect of bankruptcy exemptions on the equilibrium loan size)*

An increase in bankruptcy exemptions E *(when* $E \in [0, E_1]$*) first leads to a decrease in the equilibrium loan size* B^**. If the exemption continues increasing (*$E \in \left[E_1, \overline{E}\right]$*), the equilibrium loan size* B^* *increases as* E *increases, but finally, if the exemption continues rising (*$E \in \left[\overline{E}, \infty\right)$*), the equilibrium loan size* B^* *decreases as the bankruptcy exemption goes to infinity..*

The result of Proposition 25 is simply derived from combining the two results of Proposition 23 and 24.

As for the effect of bankruptcy on the equilibrium gross interest rate r^*, in the following proposition, we show that a non-monotonic relation between bankruptcy exemptions and the equilibrium gross interest rate also exists. In particular, the effect is symmetric but opposite to the effect of bankruptcy exemptions on the equilibrium loan size in Proposition 25.

Proposition 26 *(Total effect of bankruptcy exemptions on equilibrium gross interest rate)*

An increase in bankruptcy exemptions E *(when* $E \in [0, E_1]$*) first leads to an increase in the equilibrium gross interest rate* r^**. If the exemption continues increasing (*$E \in \left(E_1, \overline{E}\right)$*), the equilibrium gross interest rate* r^* *decreases as* E *increases, but finally, if the exemption continues rising (*$E \in \left[\overline{E}, \infty\right)$*), the equilibrium gross interest rate* r^* *increases as the bankruptcy exemption goes to infinity.*

We have analyzed the relation between bankruptcy exemptions and the equilibrium debt contracts by emphasizing the banks' role of screening beside of providing funds to the firms. Our model yields a non-monotonic relation between bankruptcy exemptions and the equilibrium loan size as well as the equilibrium gross interest rate, which is consistent with the data (Berkowitz and White (2004)).

In the following section, we further demonstrate that our model is not only able to explain the stylized fact which is found by Berkowitz and White (2004), but also offers other testable applications. In particular, we predict the relation between bankruptcy exemptions and equilibrium loan size if the screening technology is improved.

2.4 Discussion

2.4.1 An Improvement in Screening Technology

In this section, we discuss the effect of bankruptcy exemptions on the equilibrium loan size if the banks' screening technology has been improved. We consider three different types of improvements: two of them are the reduction in the cost of screening and the other is the improvement in the accuracy of screening.

A decrease in the quasi-fixed cost of screening

Suppose that the quasi-fixed screening cost c_f is reduced, it is obvious that banks will already start screening when the bankruptcy exemption is lower compared to the old threshold ($\underline{E}$) we derived. Proposition 20 shows that the quasi-fixed cost determines whether banks start screening or not when the bankruptcy exemption is low. Due to the reduction in c_f, it is straightforward that the new threshold $\underline{E}'$ will be smaller than the old threshold $\underline{E}$. However, the reduction of the quasi-fixed cost c_f does not affect the other threshold $\overline{E}$ as well as E_1, since they are independent of c_f. As a consequence, the effect of bankruptcy exemptions on the equilibrium debt contract is still non-monotonic as the analyzed above.

A decrease in the variable cost of screening

If the variable cost of per unit of screening decreases, that is, c decreases to c' ($c' < c$), the thresholds $\underline{E}$, $\overline{E}$ and E_1 we derived above all change. First of all, we can expect that the threshold $\underline{E}$ will decrease due to the reduction of total screening cost. Besides, $\overline{E}$ will increase because marginal cost of screening decreases and the marginal benefit of screening is unchanged for a given bankruptcy exemption. Therefore, in equilibrium, banks continue screening even $E > \overline{E}$. Specifically, if c reduces sufficiently, for $E \in \left[\overline{E}, \infty\right)$, it is still worthy to screen (i.e., $s^* > 0$). Thus, equilibrium loan size will increase if the bankruptcy exemption increases for any $E \in \left(\overline{E}, \infty\right)$.

In sum, an increase in the bankruptcy exemption first leads to a decrease (increase) in the equilibrium loan size (gross interest rate), and then leads to an increase (decrease) in the equilibrium loan size (gross interest rate) if the bankruptcy exemption continues increasing.

An improvement in the accuracy of screening

Now, we instead discuss about the case in which the banks' screening technology becomes more accurate. In particular, given the same screening intensity exerted, the banks can distinguish the projects of good quality from the ones of bad quality better. That is, $\widetilde{\pi}(s) < \pi(s)$ for any $s > 0$. Our conjecture is that it has the similar effect with the reduction in the per unit cost of screening. First, since now screening is more accurate given the same screening intensity, it is equivalent to say that for a given screening intensity, the cost of screening is lower. Hence, the threshold $\underline{E}$ decreases. Besides, the same argument is applied to the threshold $\overline{E}$ and $\overline{E}$ will increase. As a result, the effect of an increase in bankruptcy exemptions first decreases (increases) the equilibrium loan size (gross interest rate), and then increases (decreases) the equilibrium loan size (gross interest rate).

Beside of yielding the same non-monotonic relation between bankruptcy exemptions and equilibrium outcomes of debt contracts as found in Berkowitz and White (2004), our model offers a testable feature for further applications. In particular, it enables us to examine the relation between bankruptcy exemptions and equilibrium debt contracts as the screening technology progresses over time. The model predicts that if the screening technology is improved due to the reduction of variable cost, or the improvement of screening accuracy, there exists a U-shaped relation between bankruptcy exemptions and the equilibrium loan size. This result is consistent with the empirical work by Funchal and Araujo (2006), in which they use data from 1992 to 1997 from the Federal Deposit Insurance Corporation Statistics on Banking (FDIC) for small businesses and individuals loan and found a U-shaped relation between bankruptcy exemptions and the equilibrium loan size.

2.5 Conclusion

The impact of bankruptcy exemptions on the outcomes of debt contracts in a competitive credit market is analyzed in a simple principal-agent model. The banks' role of screening the projects of the firms are emphasized. We analyze the effect of homestead bankruptcy exemptions on the equilibrium loan size as well as the equilibrium gross interest rate. The model shows that an increase in bankruptcy exemptions at first leads to a decrease (increase), and then an increase (decrease), and then finally a decrease (increase) in the equilibrium loan size (equilibrium gross interest rate). Our model yields the same results as found in the empirical work by White and Berkowitz (2004).

An increase of bankruptcy exemptions affects the equilibrium debt contracts through two channels: (1) Direct Effect and (2) Indirect Effect. Direct Effect arises from the effect of an increase of bankruptcy exemptions directly decreasing the equilibrium loan size given banks' screening intensity as constant. The reason is that if the bankruptcy exemption increases and banks keep the screening intensity constant, in order to keep the banks' zero-profit condition hold, banks would increase the gross interest rate. This would decrease the loan size because of the increase of the cost of loan. Indirect Effect arises from the effect of an increase of bankruptcy exemptions indirectly increasing the loan size through banks' endogenous choice of their screening intensity. When the bankruptcy exemption increases, suppose a bank keeps the screening intensity constant, the gross interest rate would increase. However, another bank can simple choose a higher screening intensity and offer another debt contract with a lower gross interest rate and thus it takes over the market. Therefore, under perfect competition of banks, an increase in bankruptcy exemptions leads to an increase in banks' screening intensity. This in turns decreases the equilibrium gross interest rate and increases the equilibrium loan size. In sum, these two effects together shape the equilibrium outcomes of debt contracts. In this paper, we show that if the bankruptcy exemption is either small or large, Direct Effect dominates Indirect Effect. Therefore, a increase in bankruptcy exemptions decreases (increases) the equilibrium loan size (equilibrium gross interest rate). On the other hand, if the bankruptcy exemption is in the middle level, Indirect Effect dominates Direct Effect. As a result, an increase in bankruptcy exemptions leads to an increase (decrease) in the equilibrium loan size (equilibrium gross interest rate). Moreover, our model offers additional testable applications. In particular, our model conjectures that there is a U-shaped relation between bankruptcy exemptions and the equilibrium loan size changes if the banks' screening technology is improved due to a decrease in the variable cost of screening or an improvement in the accuracy of screening, which is consistent with the empirical work of Funchal and Araujo (2006).

2.6 Appendix

Proof. Lemma 19

Banks compete with other banks and choose screening intensity by taking other banks' contracts and other banks' screening intensity as given. Suppose that a debt contract $\left(B^{'}, r^{'}|A^{*}\right)$ given s^{*} is offered in equilibrium, and the bank earns a positive profits. However, another bank would offer another debt contract $(B'', r''|A^{*})$ given s^{*} such that $B' = B''$, but $r'' < r'$ but the bank still earns positive profits. Hence, it is clear that the bank which offers $(B', r'|A^{*})$ would not be able to attract any firms. By doing so, in equilibrium, given s^{*}, banks earn zero profit. ■

Proof. Proposition 20

Suppose that in equilibrium, $s^{*} > 0$, we can derive the equilibrium contracts $(B, r|A^{*})$ which make the banks earn zero profit

$$p(B)Br + (1 - p(B))(A^{*} - E) - B + \pi(s^{*})(A^{*} - E - B) = 2C(s)$$

However, if these contracts are offered in equilibrium, another bank can offer exactly the same contract with $s^{*} = 0$ such that the bank earns positive profits for some E.

Therefore, in equilibrium, $s^{*} > 0$ if and only if

$$2C(s^{*}) < (1 - \pi(s^{*}))(B - A^{*} + E)$$

which is equivalent to

$$\frac{2C(s^{*})}{1 - \pi(s^{*})} - B + A^{*} < E \tag{(1)}$$

where s^{*} satisfies

$$\pi(s^{*}) = \frac{2c}{B - A^{*} + E} = e^{-s^{*}} \tag{(2)}$$

we substitute s^{*} into equation (1), and we have

$$2c_f + 2c\ln(B - A^{*} + E) - 2c\ln 2c - B + A^{*} + 2c < E$$

Hence, we can find a threshold $E = \underline{E}$ such that $2c_f + 2c\ln(B - A^{*} + \underline{E}) - 2c\ln 2c - B + A^{*} + 2c = \underline{E}$. For $E \in [0, \underline{E}]$, the bank can earn positive profit with $s^{*} = 0$. Therefore, we show that in equilibrium, $s^{*} = 0$ for $E \in [0, \underline{E}]$.

Moreover, for $E \in \left[\overline{E}, \infty\right]$, in the following, we show that if $s^{*} > 0$ in equilibrium, $EU < V(A^{*})$. This implies that firms will not participate in the credit market.

$EU < V(A)$ if and only if

$$
\begin{aligned}
& p(B)\,\theta - (1+\pi(s^*))\,B && (2.1)\\
< \;\; & -(1-p(B)+\pi(s^*))\,(A^*-E) + (2-p(B))\,(V(A^*)-V(E)) + 2C(s^*) && (2.2)
\end{aligned}
$$

We substitute (2) into (3), and we can find a threshold $E = \overline{E}$ such that

$$
\begin{aligned}
& p(B)\,\theta - \left(1 + \frac{2c}{B-A^*+\overline{E}}\right) B \\
= \;\; & -\left(1-p(B)+\frac{2c}{B-A^*+\overline{E}}\right)\left(A^*-\overline{E}\right) + (2-p(B))\left(V(A^*)-V\left(\overline{E}\right)\right) + 2c_f + 2c\ln\left(B-A^*+\right.
\end{aligned}
$$

Moreover, we show that

$$
p(B)\,\theta - 2B \geq -(2-p(B))\,(A^*-E) + (2-p(B))\,(V(A^*)-V(E)) \quad \text{for all } E \in \left[\overline{E},\infty\right]
$$

■

Proof. Lemma 21

$$
\frac{\partial B}{\partial E}|_{s=s'} = \frac{-\frac{\partial^2 EU}{\partial B \partial E}}{\frac{\partial^2 EU}{\partial B^2}} = \frac{p'(B)\,(1-V'(E))}{-p''(B)\,(\theta+V(A^*)-A^*+E-V(E))} < 0
$$

since $1 - V(E) < 0$ and $p''(B) < 0$ ■

Proof. Lemma 22

We have

$$
\frac{\partial B}{\partial s^*} = \frac{-\frac{\partial^2 EU}{\partial B \partial s}}{\frac{\partial^2 EU}{\partial B^2}}|_{s=s^*} = \frac{-\frac{2c}{B-A^*+E}}{p''(B)\,(\theta+V(A^*)-A^*+E-V(E))} > 0
$$

and

$$
\frac{\partial s^*}{\partial E} = \frac{-\frac{\partial^2 EU}{\partial s \partial E}}{\frac{\partial^2 EU}{\partial s^2}}|_{s=s*} = \frac{1}{B-A^*+E} > 0
$$

Therefore, $\frac{\partial B}{\partial s^*}\frac{\partial s^*}{\partial E} > 0$ ■

Proof. Proposition 23

For $E \in [0,\underline{E}]$ and $E \in \left[\overline{E},\infty\right)$, $s^* = 0$ is shown is Proposition 20. Therefore, the equilibrium contracts solve the following problem:

$$
\max_{B,r} EU = \frac{1}{2}\left(p(B)\,(\theta - Br + V(A^*)) + (1-p(B))\,(V(E))\right) + \frac{1}{2}V(E)
$$

s.t.

$$E\pi = \frac{1}{2}\left(p\left(B\right)Br + \left(1-p\left(B\right)\right)\left(A^{*}-E\right)-B\right) + \frac{1}{2}\left(A^{*}-E-B\right) \geq 0 \quad ((\mathrm{PC}_{B}))$$

From Lemma 1, we know that banks' participation constraint binds in equilibrium. Therefore, we substitute r from the banks' participation constraint into the objective function. By taking the first derivative with respect to B, we have

$$p'\left(B\right)\left(\theta + V\left(A^{*}\right) - A^{*} + E - V\left(E\right)\right) - 2 = 0 \tag{2.3}$$

The equilibrium loan size with screening must satisfy the above equation.

By Envelope Theorem, we derive

$$\frac{\partial B^{*}}{\partial E} = -\frac{\frac{\partial^{2}EU}{\partial B \partial E}}{\frac{\partial^{2}EU}{\partial B^{2}}} = \frac{p'\left(B^{*}\right)\left(1-V''\left(E\right)\right)}{-p''\left(B^{*}\right)\left(\theta + V\left(A^{*}\right) - A^{*} + E - V\left(E\right)\right)} < 0$$

since $V'\left(E\right) > 1$ and $p''\left(B^{*}\right) < 0$. ■

Proof. Proposition 24

Equilibrium contracts must be the solution to the following problem,

$$\max_{B,r,s} EU = \frac{1}{2}\left(p\left(B\right)\left(\theta - Br + V\left(A^{*}\right)\right) + \left(1-p\left(B\right)\right)\left(V\left(E\right)\right)\right) + \frac{1}{2}V\left(E\right)$$

s.t.

$$E\pi = \frac{1}{2}\left(p\left(B\right)Br + \left(1-p\left(B\right)\right)\left(A^{*}-E\right)-B\right) + \frac{1}{2}\pi\left(s\right)\left(A^{*}-E-B\right) - C\left(s\right) \geq 0$$

$$EU = \frac{1}{2}\left(p\left(B\right)\left(\theta - Br + V\left(A^{*}\right)\right) + \left(1-p\left(B\right)\right)\left(V\left(E\right)\right)\right) + \frac{1}{2}V\left(E\right) \geq V\left(A^{*}\right)$$

Equilibrium B and s must satisfy:

F.O.C of B

$$p'\left(B\right)\left(\theta + V\left(A^{*}\right) - A^{*} + E - V\left(E\right)\right) - \left(1+\pi\left(s\right)\right) = 0 \quad ((1))$$

F.O.C of s

$$-2c - \pi'\left(s\right)\left(B - A^{*} + E\right) = 0 \quad ((2))$$

From Lemma 21 and 22, we obtain both Direct Effect and Indirect Effect. And we have show that

$$\underbrace{\left|p'(B)\left(1-V'(E)\right)\right|}_{(I)} \geq \underbrace{\frac{2c}{(B-A^*+E)^2}}_{(II)} \qquad \forall \underline{E} < E \leq E_1 \tag{(3)}$$

and

$$\left|p'(B)\left(1-V'(E)\right)\right| < \frac{2c}{(B-A^*+E)^2} \qquad \forall E_1 < E < \overline{E} \tag{(4)}$$

Therefore, we have to show $\left|p'(B)\left(1-V(E)\right)\right|$ and $\frac{2c}{(B-A+E)^2}$ cross only once at $E = E_1$ and moreover, equation (3) and (4) are satisfied.

Term (I) and term (II) are both decreasing in E, and we have to show that

$$\lim_{E\to\underline{E}} \left|p'(B)\left(1-V'(E)\right)\right| > \lim_{E\to\underline{E}} \frac{2c}{(B-A^*+E)^2}$$

and

$$\lim_{E\to\overline{E}} \left|p'(B)\left(1-V'(E)\right)\right| < \lim_{E\to\overline{E}} \frac{2c}{(B-A^*+E)^2}$$

Moreover, we know that since when $E \to \underline{E}$ and when $E \to \overline{E}$, equilibrium $s^* = 0$ form proposition 20. Hence, we have

$$\lim_{E\to\underline{E}} \frac{2c}{(B-A+E)^2} = \frac{1}{B-A^*+\underline{E}} = \frac{1}{2c}$$

and

$$\lim_{E\to\overline{E}} \frac{2c}{(B-A+E)^2} = \frac{1}{B-A^*+\overline{E}} = \frac{1}{2c}$$

Hence, we have to show that

$$2\left(V'(\underline{E})-1\right) > \frac{1}{2c}$$

and

$$2\left(V'\left(\overline{E}\right)-1\right) < \frac{1}{2c}$$

And we already know that $2\left(V'(\underline{E})-1\right) > 2\left(V'\left(\overline{E}\right)-1\right)$ since $V(E)$ is an increasing and concave function of E. As long as there exists a $c > 0$ such that

$$2\left(V'(\underline{E})-1\right) > \frac{1}{2c} > 2\left(V'\left(\overline{E}\right)-1\right)$$

We can show that there also exists a threshold $E = E_1 \in \left[\underline{E}, \overline{E}\right]$ such that $2\left(V'\left(E_1\right) - 1\right) = \frac{1}{2c}$, and equation (3) and (4) are satisfied. ■

Proof. Proposition 25

This result simply comes from combining the two results of Proposition 23 and 24. ■

Proof. Proposition 26

First, it is easy to show that if banks exert positive screening intensity $s^* > 0$ (when $E \in \left[\underline{E}, \overline{E}\right]$), in equilibrium, $B^* r^*$ is an increasing and concave function of B^*. Hence, $\frac{\partial(B^* r^*)}{\partial B^*} < r^*$ due to the concavity of $B^* r^*$. Therefore, the effect of bankruptcy exemptions on the equilibrium gross interest rate is the following:

$$
\begin{aligned}
& \frac{\partial r^*}{\partial E} \\
= \;& \frac{\partial\left(B^* r^*\right)\left(B^*\right)^{-1}}{\partial E} \\
= \;& \left(B^*\right)^{-1} \underbrace{\frac{\partial\left(B^* r^*\right)}{\partial B^*}}_{=0} \frac{\partial B^*}{\partial E} + B^* r^* (-1)\left(B^*\right)^{-2} \frac{\partial B^*}{\partial E} \\
= \;& \underbrace{\left(\frac{1}{B^*} \frac{\partial\left(B^* r^*\right)}{\partial B^*} - \frac{r^*}{B^*}\right)}_{-} \frac{\partial B^*}{\partial E}
\end{aligned}
$$

Therefore, from Proposition 24, we can show that $\frac{\partial r^*}{\partial E} > 0$ if $E \in \left[\underline{E}, E_1\right]$ and $\frac{\partial r^*}{\partial E} < 0$ if $E \in \left[E_1, \overline{E}\right]$.

Second, if banks do not screening $s^* = 0$ (when $E \in \left[0, \underline{E}\right]$ and $E \in \left[\overline{E}, \infty\right)$),

$$
\begin{aligned}
& \frac{\partial r^*}{\partial E} \\
= \;& \frac{\partial\left(B^* r^*\right)\left(B^*\right)^{-1}}{\partial E} \\
= \;& \left(B^*\right)^{-1} \frac{\partial\left(B^* r^*\right)}{\partial B^*} \frac{\partial B^*}{\partial E} + B^* r^* (-1)\left(B^*\right)^{-2} \frac{\partial B^*}{\partial E} \\
= \;& \left(\frac{1}{B^*} \frac{\partial\left(B^* r^*\right)}{\partial B^*} - \frac{r^*}{B^*}\right) \underbrace{\frac{\partial B^*}{\partial E}}_{-}
\end{aligned}
$$

As long as $\frac{\partial(B^* r^*)}{\partial B^*} < r^*$, $\frac{\partial r^*}{\partial E} > 0$.

$$
\frac{\partial\left(B^* r^*\right)}{\partial B^*} = \frac{2p\left(B^*\right) - 2p'\left(B^*\right) B^*}{p\left(B^*\right)^2} < \frac{\left(B^* - A^* + E\right) + B^* - \left(1 - p\left(B^*\right)\right)\left(A^* - E\right)}{p\left(B^*\right) B^*} = r^*
$$

which is equivalently to

$$\underbrace{\frac{B^*}{p\left(B^*\right)\left(2-p\left(B^*\right)\right)\left(\theta+V\left(A^*\right)-A^*+E-V\left(E\right)\right)}}_{(1)} > \underbrace{\frac{A^*}{B^*-A^*}}_{(2)} \qquad \text{((i))}$$

Term (1) is an increasing function of B^* and term (2) is a decreasing function of B^*. Moreover,

$$\lim_{B^*\to 0}\frac{B^*}{p\left(B^*\right)\left(2-p\left(B^*\right)\right)\left(\theta+V\left(A\right)-A+E-V\left(E\right)\right)} > 0 > \lim_{B^*\to 0}\frac{A}{B^*-A} = -1$$

. Therefore, equation (i) holds for all B^*. As a result, $\frac{\partial(B^*r^*)}{\partial B^*} < r^*$ and thus, $\frac{\partial r^*}{\partial E} > 0$ for $E \in [0, \underline{E}]$ and $E \in \left[\overline{E}, \infty\right)$. ■

Chapter 3

Borrowing Decisions in the Presence of Credit Market Imperfections and Heterogeneous Risk Attitudes

Previous literature highlights the impact of credit market imperfections on the entrepreneurs' borrowing decision. This paper shows that entrepreneurs' borrowing decisions depend not only on the borrowing constraints, but also on the entrepreneurs' demand for consumption insurance. In our model, entrepreneurs' heterogeneous risk attitudes interact with two different sources of credit market imperfections-limited liability and moral hazard-and these imperfections result in distinct borrowing decisions. Specifically, when entrepreneurs are restricted by a limited liability constraint, their borrowing and hence the invested capital increases with their initial wealth. On the other hand, in the presence of moral hazard, entrepreneurs are constrained by an incentive compatibility constraint, and they borrow less and hence choose lower capital as their initial wealth increases.

3.1 Introduction

Previous literature studies entrepreneurs' borrowing decisions under the assumption that entrepreneurs are risk neutral and face borrowing constraints. Two types of borrowing constraints are typically considered: the first one is an incentive compatibility (IC) constraint arising from moral hazard, and the other is a limited liability (LL) constraint. For example, Evans and Jovanovic (1989) consider an environment with limited liability where entrepreneurs are offered standard debt contracts. The main factor that affects entrepreneurs' borrowing decisions is their initial wealth. Specifically, entrepreneurs borrow more when their wealth increases. The differences between our model and Evan and Jovanovic's model is that we consider risk-averse entrepreneurs, and we do not restrict the contract space (we consider state-contingent contracts). Also, in our model entrepreneurs make both borrowing decisions and effort choices.

Aghion and Bolton (1997) model the borrowing/lending decisions in a dynamic environment with both moral hazard and limited liability. They assume a fixed capital investment and continuous effort choices. Again, the contracts considered are standard debt contracts. In this set-up, entrepreneurs with lower initial wealth have to borrow more in order to invest. Also, the fact that entrepreneurs only have to repay their debt when their investment project is successful distorts their incentive to exert effort. They conclude that optimal effort of an entrepreneur is increasing with his initial wealth. In our static model, we take into account limited liability and moral hazard separately (unlike Aghion and Bolton) in order to distinguish the different impact of each type of borrowing constraint on the entrepreneurs' borrowing decisions and their effort choices.

The two papers above consider risk-neutral entrepreneurs so they ignore the issue of risk management when entrepreneurs make their borrowing decisions. If entrepreneurs are risk-averse, risk management considerations become important in making borrowing decisions. Cressy (2000) proposed the idea of "risk rationing" which suggests that the reason for which some entrepreneurs are excluded from the credit market might be that the optimal contract offered to them embeds more risk than they are willing to bear. In general, when risk-averse entrepreneurs face a risky investment, they would like to manage the risk while making investment decisions by obtaining some insurance. The credit market provides entrepreneurs the ability to obtain credit as well as insurance. The importance of risk management for the borrowing decision of risk-averse entrepreneurs need not be negligible. In the corporate finance literature, for example, MacMinn (1987) and MacMinn and

Han (1990) show that obtaining insurance may increase the incentives to invest.[1][2]

In this paper, entrepreneurs have heterogeneous risk attitudes because of their heterogeneous initial wealth and the Decreasing Absolute Risk Aversion (DARA) property of their utility function. This heterogeneity generates a heterogeneous demand for insurance (that is, for consumption smoothing between states). I also assume that capital investment is divisible and there is no restriction on the form of contracts banks can offer (contracts are state-contingent)[3]. I further assume a perfectly competitive credit market (where banks are the only lenders). In this setting, risk-sharing between entrepreneurs and banks is considered as well as their borrowing/lending relation.

I consider three cases:

Case I: Entrepreneurs' effort is observable and contractible, and there is no limited liability constraint.[4]
Case II: Entrepreneurs' effort is observable and contractible, but there is limited liability constraint.
Case III: Entrepreneurs' effort is unobservable and there is no limited liability constraint.

The main results are summarized as follows: in a frictionless economy in which state-contingent contracts are feasible, banks and entrepreneurs share risk optimally. When the entrepreneur faces different credit constraints, his borrowing decision differs. In the presence of a LL constraint, the entrepreneur's borrowing and hence his capital investment is increasing in his initial wealth if the LL constraint is binding. If the LL is non-binding, the entrepreneur's capital investment is a non-increasing function of his initial wealth. That is, the relationship between an entrepreneur's initial wealth and his capital investment is an inverted U-shaped relation. Therefore, for constrained entrepreneurs, the result derived in Evan and Jovanovic still holds in

[1] MacMinn (1987) consider that bankruptcy cost and agency cost cause the value of an insured firm to be higher than a uninsured firm. Therefore, a firm has incentive to obtain insurance. MacMinn and Han take into account the moral hazard problem which is associated with limited liability. There exists a conflict of interest between debt holders and equity holders. This conflict may cause the equity holders to forgo the investment project which has positive present value. By obtaining insurance, they conclude that the conflict between these two parties can be eliminated and hence the investment decision would be optimal.

[2] Grace and Rebello (1993) and Rebello (1995) argue that the role of insurance is to signal the quality of firms to creditors in financial markets. Rebello (1995) also shows how insurance contracting affects firms' financial structure (the ratio of equity and debt used for financing an investment project) in the presence of adverse selection.

[3] Meh and Quadrini (2006) show that the availability of a state-contingent contract has important welfare consequences, and they find that those state-contingent contracts achieve higher welfare than the standard debt contracts.

[4] Case I serves as a frictionless benchmark for the following two cases in which credit market imperfections arise from two different sources.

our environment. As for optimal effort under the LL constraint, there are two cases. When the fraction of wealth that entrepreneurs can borrow is sufficiently small (the LL constraint is sufficiently tight), all entrepreneurs choose low effort. When the LL constraint is not so tight, however, poor and rich entrepreneurs exert low effort, but middle-income entrepreneurs exert high effort. In the environment with moral hazard, for entrepreneurs exerting high effort, borrowing and capital investment decrease with wealth. Also, in order to induce these entrepreneurs to exert effort, the contract must entail partial (instead of full) insurance. No insurance at all need in general not be optimal.[5]

The environment is described in *Section 2*. In *Section 3*, optimal contracts and entrepreneurs' optimal borrowing decisions are derived in each of the three cases. *Section 4* discusses the related literature and presents some empirical evidence supporting our results. I conclude in *Section 5*.

3.2 The Economy

We consider a small open economy with two types of agents: borrowers (entrepreneurs) and lenders (banks). We assume that entrepreneurs are risk-averse and heterogenous with respect to their initial wealth A. The economy is static (there is a single period).

Entrepreneurs have identical preferences with Decreasing Absolute Risk Aversion (DARA) represented by the Bernoulli utility function:

$$U(c, z) = \log c - \varphi(z)$$

where c is consumption and z is effort that the entrepreneur exerts. Consumption is simply the wealth of entrepreneurs at the end of the period. There are two possible effort levels: z_H (high) and z_L (low). The disutility of effort is higher with high than with low effort: $\varphi(z_H) > \varphi(z_L)$.

At the beginning of the period, the entrepreneur chooses a (perfectly divisible) capital investment $k \in [0, \infty)$ and effort $z \in \{z_L, z_H\}$. The total capital invested k is financed from two sources: the entrepreneur's own wealth A, and the debt B borrowed from banks; i.e., $k = A + B$. The outcome q of the project is stochastic, taking two possible values: $q = \theta_H$ (success) with probability $p(k, z)$ and $q = \theta_L$ (failure) with probability $1 - p(k, z)$. The probability of success has the following

[5] Shavell (1979) points out that moral hazard alone does not imply no insurance provision to the agent (entrepreneur)

functional form[6]:

$$p(k,z) = \frac{k^{\alpha} z^{1-\alpha}}{1 + k^{\alpha} z^{1-\alpha}}. \tag{3.1}$$

The function in (3.1) satisfies the Monotone Likelihood Ratio Property (MLR property):

$$\frac{prob\,(q = \theta_H | k, z_H)}{prob\,(q = \theta_L | k, z_H)} > \frac{prob\,(q = \theta_H | k, z_L)}{prob\,(q = \theta_L | k, z_L)} \qquad \forall k$$

and

$$\frac{prob\,(q = \theta_H | k, z)}{prob\,(q = \theta_L | k, z)} > \frac{prob\left(q = \theta_H | k^{'}, z\right)}{prob\,(q = \theta_L | k^{'}, z)} \qquad for\ k > k^{'}\ and\ each\ z$$

This means that for a given k, higher effort increases the probability of success. Similarly, for a given z, higher capital also increases the probability of success. Also, it is direct to verify that there exists a threshold $\widetilde{k}_{FB}$ such that:

$$\frac{\partial\left(p(k, z_H) - p(k, z_L)\right)}{\partial k} > 0 \quad \forall k \in \left[0, \widetilde{k}_{FB}\right] \tag{3.2}$$

and

$$\frac{\partial\left(p(k, z_H) - p(k, z_L)\right)}{\partial k} \leq 0 \quad \forall k \in \left[\widetilde{k}_{FB}, \infty\right)$$

In the range $k \in \left[0, \widetilde{k}_{FB}\right]$, the difference of probability of success between exerting high and low effort is increasing in k. That is, a higher k implies a higher contribution of high effort to the probability of success. Consequently, for that region, k and z are complements. For $k > \widetilde{k}_{FB}$, this difference of probability of success is decreasing in k. We shall restrict attention to the first region by choosing the parameters z, α appropriately.

Banks offer state-contingent contracts to maximize expected profit (they are risk-neutral). The credit market is perfectly competitive. We assume that banks can offer variable loan sizes. Banks require no collateral, as entrepreneurs with low wealth can choose a smaller loan size. These variable loan sizes allow banks to offer different contracts to entrepreneurs with different initial wealth (which is observable). Finally, each bank has the outside option of lending to other banks at the exogenous riskless interest rate i.

[6] This is as in Paulson et al.(2006)

3.2.1 Discussion of Assumptions

The critical assumptions in this model are the DARA utility function, and the functional form of the production technology (3.1). The first assumption has been tested extensively in the empirical literature. For example, the empirical study of Friend and Blume (1975) supports DARA. Moreover, experimental tests like Gordan et al. (1972) and Kroll, Levy et al. (1988a) strongly support DARA. The second assumption implies that the probability is an increasing function of capital and effort ($p_k \geq 0$, $p_z \geq 0$) and has decreasing increments and positive cross increments up to the threshold $\tilde{k}_{FB}$ ($p_{kk} \leq 0$, $p_{zz} \leq 0$ and $p_{kz} \geq 0$). These assumptions are standard. Paulson et al. (2006) derive the probability function (3.1) from the standard Cobb-Douglas production function, and normalize it to take values in $[0, 1)$. As mentioned, this probability function satisfies MLRP which is a standard assumptions in the principal agent literature (Holmström, 1979, Milgrom, 1981).

3.2.2 Financial Contracts

We assume that banks observe the entrepreneurs' initial wealth (as well as the realization of the project's outcome). Hence, banks can offer contracts which are contingent on A and q.

In the environment with full information and no limited liability constraint (Case I), a contract specifies a level of capital and effort, as well as the entrepreneur's consumption in each state (z, k, c_1, c_2). This case will serve as a benchmark for the cases in which entrepreneurs face financial constraints. We consider two kinds of financial constraints. The first one is an *Incentive Compatibility* (IC) *Constraint*, which arises from the moral hazard problem when effort is unobservable. The second is a *Limited Liability* (LL) *Constraint*.

Under moral hazard, effort is unobservable, so contracts cannot be contingent on z. Contracts specify the capital amount k, and the entrepreneur's consumption in each state, c_1 and c_2. In order to induce high effort, banks must offer contracts satisfying the following IC constraint:

$$\begin{aligned} & p(z_H, k) \log c_1 + (1 - p(z_H, k)) \log c_2 - \varphi(z_H) \\ \geq \; & p(z_L, k) \log c_1 + (1 - p(z_L, k)) \log c_2 - \varphi(z_L) \end{aligned} \tag{3.3}$$

To model limited liability, I follow Evans and Jovanovic (1989), and Paulson et al. (2006), and assume that entrepreneurs can only borrow up to a borrowing limit that is some multiple of their initial wealth. Specifically, the maximum amount that entrepreneurs can borrow is $(\lambda - 1) A$ where λ is exogenous and satisfies $\lambda > 1$.

Hence, the equilibrium contract has to satisfy the limited liability constraint:

$$k = A + B \leq \lambda A \tag{3.4}$$

3.3 Equilibrium Contracts

3.3.1 Full Information Frictionless Benchmark

With full information (e.g. observable effort), banks offer contracts which specify an effort level and a capital amount along with a consumption schedule, c_1 and c_2. Since entrepreneurs are risk averse, the competitive equilibrium contract provides full insurance to entrepreneurs ($c_1 = c_2$).

The entrepreneur expected utility from contract (z, k, c_1, c_2) is :

$$\max_{k,z \in \mathbb{R}_+ \times \{z_H, z_L\}} p\,(k,z) \log c_1 + (1 - p\,(k,z)) \log c_2 - \varphi\,(z) \tag{3.5}$$

where $p\,(k,z)$ is given by (3.1). We focus only on the entrepreneurs who borrow. The bank's expected profit from a contract is:

$$p(k,z) R_1\,(c_1) + (1 - p(k,z))\, R_2\,(c_2) - (k - A)\, i$$

where i is the riskless interest rate, $R_1(c_1) = \theta_H - c_1$ and $R_2(c_2) = \theta_L - c_2$. That is, R_1, R_2 are the repayments when the project is successful and when it fails, respectively.

Competition among banks implies that for a given A, the equilibrium contract maximizes the entrepreneur's expected utility subject to the bank's zero profit constraint (i.e., it is optimal)

$$\max_{k,z,c_1,c_2} \{p\,(k,z) \log c_1 + (1 - p\,(k,z)) \log c_2 - \varphi\,(z)\}$$

s.t.

$$p\,(k,z)\,(\theta_H - c_1) + (1 - p\,(k,z))\,(\theta_L - c_2) - (k - A)\, i \geq 0. \tag{3.6}$$

First of all, it is direct to show that banks make zero profits in equilibrium, and that the equilibrium contract entails full insurance; i.e.,

$$c_1^{FB} = c_2^{FB} = p\,(k,z)\,\theta_H + (1 - p\,(k,z))\,\theta_L - (k - A)\, i.$$

Substituting c_1 and c_2 in (3.5), the previous optimization problem reduces to:

$$\max_{k,z\in\mathbb{R}_+\times\{z_H,z_L\}}\left\{\log\left[\theta_L-(k-A)\,i+p\,(k,z)\,(\theta_H-\theta_L)\right]-\varphi\,(z)\right\}$$

The first-order condition for capital k satisfies:

$$\frac{\partial p\,(k,z_j)}{\partial k}\,(\theta_H-\theta_L)=i,\ \text{ for } j=L,H.$$

This condition gives the optimal capital amount as a function of effort, $k_H^{FB}=k\,(z_H)$ and $k_L^{FB}=k\,(z_L)$. We now study which level of effort is optimal for a given A.

Lemma 27 *Under MLR property, condition 3.2 and* $k^{FB}\,(z_H)<\widetilde{k}_{FB}$, $k^{FB}\,(z_H)>k^{FB}\,(z_L)$

Proof. See Appendix. ■

Lemma 27 provides conditions for a monotonic relation between effort and capital investment.

The following proposition shows entrepreneurs' optimal choices of capital and effort with respect to their initial wealth A.

Proposition 28 *(First-Best effort, capital and borrowing)*

Suppose the conditions in Lemma 27 hold. Then there exists a threshold A_{FB} *such that*

(a) entrepreneurs with $0\leq A\leq A_{FB}$ *choose high effort , capital* k_H^{FB} *and borrowing* $B^{FB}\,(A)=k_H^{FB}-A$

(b) entrepreneurs with $A>A_{FB}$ *choose low effort, capital* $k_L^{FB}(<k_H^{FB})$ *and borrowing* $B^{FB}\,(A)=k_L^{FB}-A$

Proof. See Appendix. ■

Proposition 28 says that entrepreneurs with low wealth have higher capital investment and effort than entrepreneurs with high wealth. Poor entrepreneurs ($A<A_{FB}$) exert high effort because with DARA the marginal benefit from exerting high effort is higher when wealth is lower (it decreases with wealth). Note that, with full information, entrepreneurs' heterogeneous wealth has no influence on their capital decision. The only determinant of the capital decision is the entrepreneurs' choice of effort (see figure 3-1 and figure 3-2). Entrepreneurs with initial wealth $A\in[0,A_{FB}]$ exert high effort and hence choose a higher capital investment k_H^{FB}. On the other hand, entrepreneurs with initial wealth $A>A_{FB}$ exert low effort and choose a lower capital investment k_L^{FB}.

The following corollary provides conditions that ensure that $A_{FB}>0$, so some of the entrepreneurs with lower wealth choose high effort.

Corollary 29 *Suppose*

$$\varphi\left(z_H\right) - \varphi\left(z_L\right) < \log \frac{p_H\theta_H + \left(1 - p_H\right)\theta_L - k_H i}{p_L\theta_H + \left(1 - p_L\right)\theta_L - k_L i},$$

where we denote $p_H = p\left(k_H, z_H\right)$ *and* $p_L = p\left(k_L, z_L\right)$. *Then* $A_{FB} > 0$.

Proof. See Appendix. ■

Corollary 29 says that the threshold A_{FB} is positive if exerting high effort is not too costly relative to exerting low effort, and if the ratio $\frac{p_H}{p_L}$ or $\frac{\theta_H}{\theta_L}$ is large enough. (Otherwise, A_{FB} is negative and every entrepreneur exerts low effort regardless of his initial wealth).

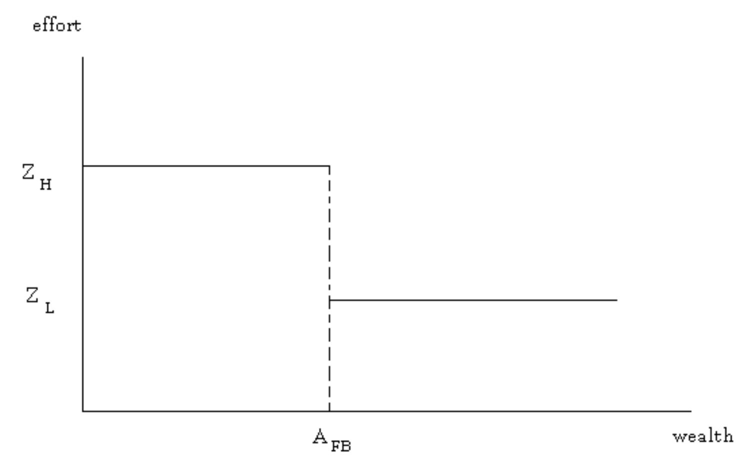

Figure 3-1: Optimal effort under full information

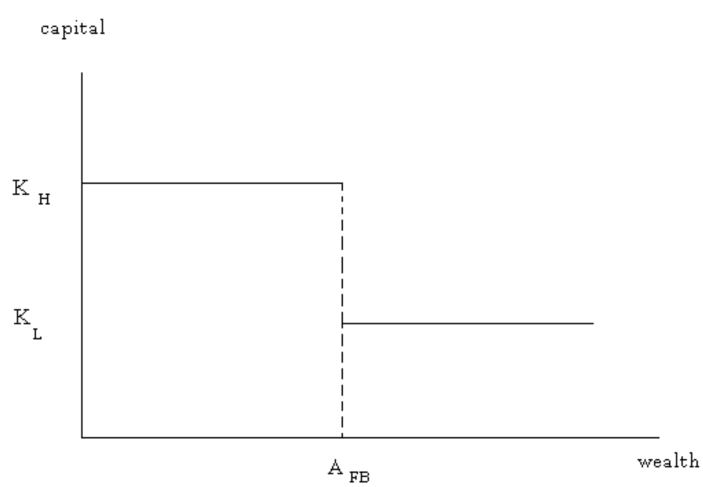

Figure 3-2: Optimal capital under full information

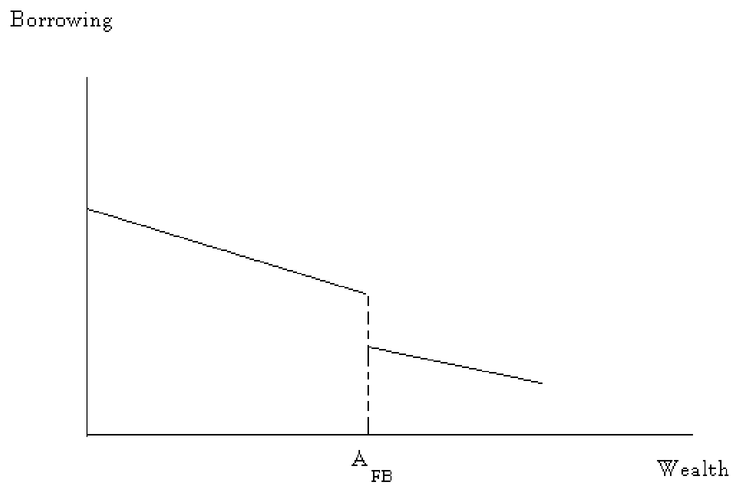

Figure 3-3: Optimal borrowing under full information

3.3.2 Limited Liability

After studying the full information frictionless environment, now we study an environment with limited liability. A competitive equilibrium is as in the previous section except that now (in addition to the zero-profit constraint) contracts must satisfy an extra limited liability (LL) constraint:

$$k \leq \lambda A \text{ with } \lambda > 1$$

Again, the zero profit constraint is binding and, since effort is observable and entrepreneurs are risk-averse, the equilibrium contract provides full insurance; i.e.,

$$c_1^{LL} = c_2^{LL} = p(k, z)\,\theta_H + (1 - p(k, z))\,\theta_L - (k - A)\,i$$

The first thing to note is that for entrepreneurs whose first-best level of capital $k^{FB}(z_j)$ satisfies $k^{FB}(z_j) < \lambda A$, $j = H, L$, and the limited liability constraint does not bind. The optimal contract in this case is the same as in the full information benchmark. However, for entrepreneurs with $k^{FB}(z_j) > \lambda A$, the limited liability constraint does bind, and in this case, $k^{LL}(z_j) = \lambda A$ and the borrowing is $B^{LL}(z_j, A) = (\lambda - 1)\,A$

Proposition 30 *(optimal capital under limited liability)*

Entrepreneurs are constrained by LL if their initial wealth satisfies: $A < A_{k,LL} = \frac{k^{FB}(z_j)}{\lambda}$, *and are unconstrained otherwise.*

Note that it is richer entrepreneurs who are unconstrained (because LL is more likely to be slack when A is higher and k is lower). Assuming $A_{k,LL} < A_{FB}$, the threshold for effort and capital investment is the same as in the full information case A_{FB}. As shown in Figure 3-4, the optimal capital is a function of A when entrepreneurs are constrained by limited liability. For lower wealth levels the LL constraint binds, so the optimal capital is λA, and increases with A up to some level $A_{k,LL}$ For $A > A_{k,LL}$, the LL constraint does not bind and so the optimal capital is the first best one.

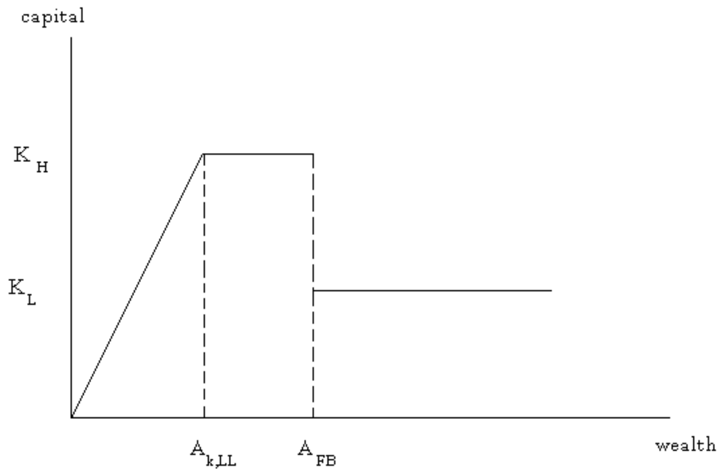

Figure 3-4: Optimal capital under limited liability a)

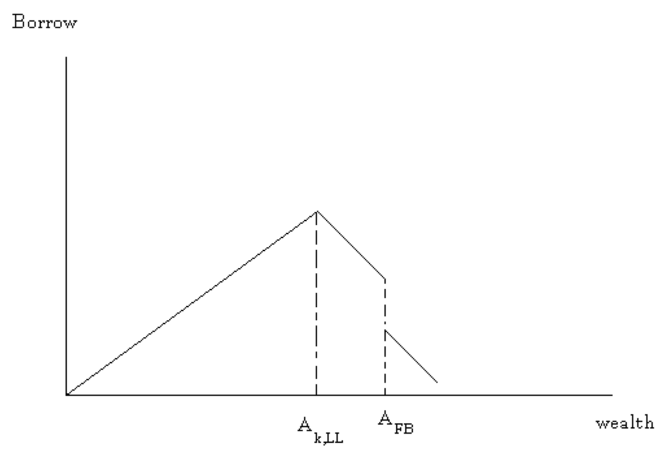

Figure 3-5: Optimal borrowing under limited liability a)

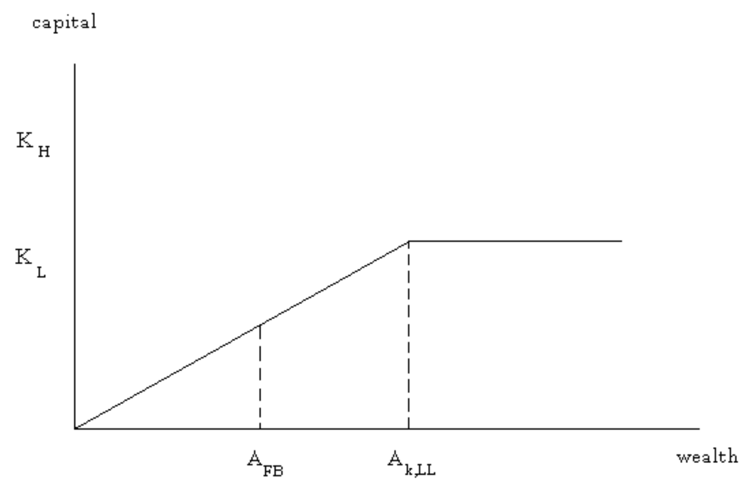

Figure 3-6: Optimal capital under limited liability b)

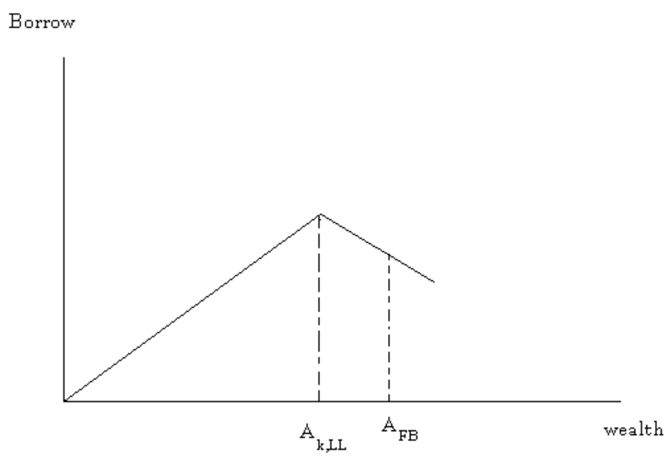

Figure 3-7: Optimal borrowing under limited liability b)

The following lemma characterizes the threshold $\widehat{A}_{z,LL}$ for the optimal effort level of the constrained entrepreneurs.

Lemma 31 *There exists a threshold $A_{z,LL}$ that satisfies*

$$\log \frac{\theta_L - (\lambda A_{z,LL} - A_{z,LL})\, i + p\,(z_H, \lambda A_{z,LL})\,(\theta_H - \theta_L)}{\theta_L - (\lambda A_{z,LL} - A_{z,LL})\, i + p\,(z_L, \lambda A_{z,LL})\,(\theta_H - \theta_L)} = \varphi\,(z_H) - \varphi\,(z_L)$$

such that

(a) $A_{z,LL} \leq A_{k,LL}$, *if* $A_{FB} > A_{k,LL}$,

(b) $A_{z,LL} > A_{k,LL}$, *otherwise,*

Proof. See Appendix. ■

In the following proposition, we derive entrepreneurs' optimal choices of effort under limited liability.

Proposition 32 *(optimal effort under limited liability)*
If $A_{FB} \geq A_{k,LL}$,
(a1) constrained entrepreneurs with wealth $A \in [0, A_{z,LL}]$ *exert low effort.*
(a2) constrained entrepreneurs with wealth $A \in (A_{z,LL}, A_{k,LL})$ *exert high effort.*
If $A_{FB} < A_{k,LL}$,
(b) All entrepreneurs exert low effort whether they are constrained or not.

Proof. See Appendix. ■

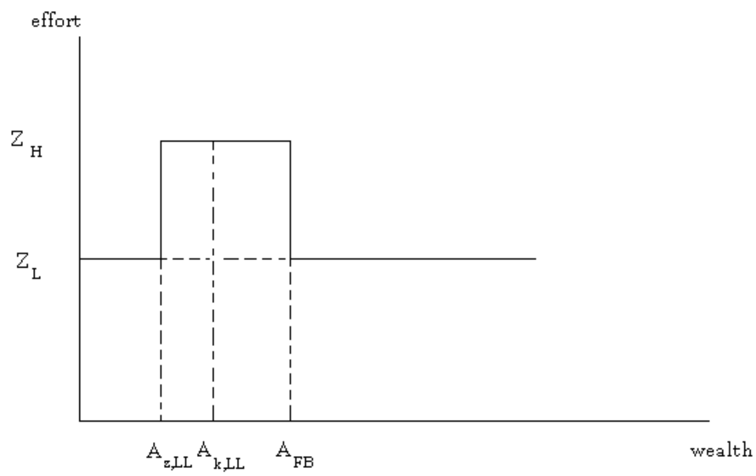

Figure 3-8: Optimal effort under limited liability a)

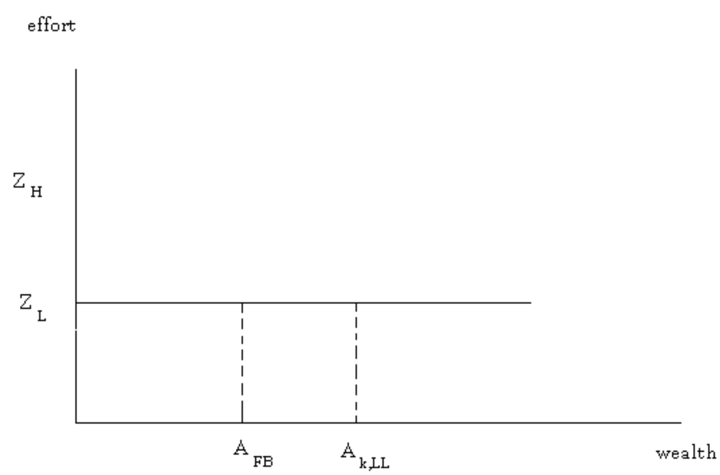

Figure 3-9: Optimal effort under limited liability b)

Figure 3-8 shows that in the case with $\widetilde{A}_{FB} \geq \overline{A}_{k,LL}$, constrained entrepreneurs exert low effort when they are very poor $(A < \hat{A}_{z,LL} < \overline{A}_{k,LL})$ but exert high effort when their initial wealth lies in some middle range. Richer individual are unconstrained entrepreneurs $(A \geq \bar{A}_{k,LL})$ and they always exert low effort.

All entrepreneurs are still fully insured. The richer unconstrained entrepreneurs' level of capital is optimal. However, for poor entrepreneurs $(A \in [0, \overline{A}_{k,LL}])$, capital is inefficiently low (they obtain less capital than they desire). The binding LL constraint reduces the benefit of exerting high effort. In particular, for very poor entrepreneurs $(A \in \left[0, \widehat{A}_{z,LL}\right],\ \widehat{A}_{z,LL} < \overline{A}_{k,LL})$ this reduction is large enough to make them to choose low (instead of high) effort. This in turn implies that the capital investment of these individuals is inefficiently low. Individuals with $A \in [A_{z,LL}, A_{k,LL}]$ continue to choose z_H and the capital $k^{FB}(z_H)$ even if they have less incentives to do so now compared to the first best effort.

In summary, if $A_{FB} \geq A_{k,LL}$ (the LL constraint is less tight), some of the entrepreneurs that exerted high effort in the full-information case now exert low effort because the binding limited liability constraint reduces the incentives to provide high effort. On the other hand, if $A_{FB} < A_{k,LL}$ (the LL constraint is tighter), all entrepreneurs choose to exert low effort z_L since they do not have desired capital amount $(k^{FB}(z_j) > \lambda A)$, and hence, the benefits of exerting high effort is less than the costs of doing so.

3.3.3 Moral Hazard

Suppose now that effort is unobservable. We shall restrict to levels of wealth $A < A_{FB}$ for which the optimal level of effort is high. For $A \geq A_{FB}$, the optimal level of effort is low and the IC constraint does not bind, so the contract coincides with the first-best contract. Contracts specify a consumption schedule (c_1, c_2), and a loan size $B\,(B = k - A)$. The difference with the frictionless benchmar is that contracts must now satisfy the incentive compatibility (IC) constraint (3.3).
For $A < A_{FB}$, the equilibrium contract with moral hazard $\left(k^{MH}, \widehat{c_1}, \widehat{c_2}\right)$ maximizes the entrepreneurs expected utility subject to the banks' zero profit constraint and the IC constraint:

$$\max_{k,c_1,c_2} p(k, z_H) \log c_1 + (1 - p(k, z_H)) \log c_2 - \varphi(z_H)$$

s.t.

$$p(k, z_H)(\theta_H - c_1) + (1 - p(k, z_H))(\theta_L - c_2) - (k - A)\, i \geq 0$$

and

$$\begin{aligned} & p(k, z_H) \log c_1 + (1 - p(k, z_H)) \log c_2 - \varphi(z_H) \\ \geq \; & p(k, z_L) \log c_1 + (1 - p(k, z_L)) \log c_2 - \varphi(z_L) \end{aligned}$$

The following proposition characterizes the equilibrium contract with moral hazard for wealth $A \in [0, A_{FB}]$.[7]

Proposition 33 *For $A < A_{FB}$, both the zero-profit and the IC constraint bind.*

Proof. See Appendix ■

Proposition 34 *(optimal consumption, capital, borrowing and effort under moral hazard)*

For every $A \in [0, A_{FB}]$

(a) The optimal consumption schedule $\left(\widehat{c}_1^{MH}, \widehat{c}_2^{MH}\right)$ entails partial insurance since $\frac{\widehat{c}_1^{MH}}{\widehat{c}_2^{MH}} > 1$

(b) Optimal capital k^{MH} and borrowing B^{MH} are both decreasing in A. Also, wealthier entrepreneurs bear higher risk: $\frac{\widehat{c}_1^{MH}}{\widehat{c}_2^{MH}}$ increases with A.

Proof. See Appendix. ■

For $A < A_{FB}$, entrepreneurs become less risk averse when wealth increases, and so their demand for insurance decreases. Hence, the richer the entrepreneurs, the harder it is to provide them with incentives to exert effort by making them bear part of the risk (since they care less about risk). This is why as A increases, the entrepreneur bears more investment risk himself. In other words, banks use a higher-power incentive scheme for the richer entrepreneurs, which means that the consumption schedule depends more on the realized project outcomes. Moreover, the optimal capital investment under moral hazard k^{MH} and hence the borrowing B^{MH} $\left(= k^{MH} - A\right)$ are decreasing when the entrepreneur's initial wealth increases.

In summary, when effort is unobservable, the full-insurance contract is not incentive compatible. In this case, a partial insurance contract is offered. The degree of the partial insurance is determined by the entrepreneur's wealth, which affects his risk attitude and hence his demand for the consumption insurance. The higher the entrepreneur's wealth A, the lower the amount of insurance that the contract must entail to induce the entrepreneur to exert high effort.

[7] The assumption that utility is separable in effort is key to show that the IC constraint binds. This need not be the case with non-separable utility (see Bennardo and Chiappori 2002).

3.4 Related Literature

As mentioned in the introduction, there is a literature that studies the impact of credit market imperfections on entrepreneurs' borrowing decisions. Evans and Jovanovic (1989) construct a static model of risk neutral entrepreneurs with heterogenous initial wealth. Entrepreneurs are offered standard debt contracts. They conclude that under limited liability constraint, entrepreneurs borrow more when their initial wealth increases. In this paper, we consider that entrepreneurs are risk averse. State-contingent contracts instead of standard debt contracts are offered, and this allows entrepreneurs to obtain optimal insurance on consumption in each state. Moreover, we consider not only the impacts of limited liability constraint on entrepreneurs' decisions of borrowing but also entrepreneurs' effort choices. Our results show that for constrained entrepreneurs, the borrowing amount is increasing as wealth increases, which is the same as shown in Evan and Jovanovic. Moreover, we find that if the LL constraint is less tight (λ is larger), there is an inverted U-shape relationship between entrepreneurs' capital investment and their wealth as well as the relationship between entrepreneurs' effort choices and wealth. On the other hand, if the LL constraint is sufficiently tight (λ sufficiently small), all entrepreneurs exert low effort and capital investment is a non-decreasing function of entrepreneurs' initial wealth.

Aghion and Bolton (1997) consider risk neutral entrepreneurs in a dynamic environment with moral hazard and a limited liability constraint. They consider again standard debt contracts, and assume a fixed capital investment and continuous effort choices. In their environment, the poor entrepreneurs need to borrow more in order to invest and it is harder to induce these entrepreneurs to exert effort since limited liability does not allow banks to punish them when their performance is poor. Their main result is that entrepreneurs' effort is increasing as wealth increases. In this paper, we consider moral hazard and limited liability separately in order to distinguish the impacts of two different borrowing constraints on risk averse entrepreneurs' borrowing decision and their effort choices. Also, we endogeneize the choice of capital investment (which is assumed divisible). Finally, to take into account the entrepreneurs' demand for insurance, we introduce risk aversion and consider state-contingent contracts. Our results show that in the environment with moral hazard alone, the capital investment is decreasing in wealth. As for optimal effort choices, IC constraint is binding for poorer entrepreneurs but not binding for richer entrepreneurs.

In the literature of corporate finance, firm/entrepreneurs' borrowing decisions are an important issue. Entrepreneurs' borrowing decisions and hence their capital structure can affect firms/entrepreneurs' production decisions. Modigliani and Miller (1958) show that the capital structure is irrelevant for firm's production decision when the capital market is perfect. However, this result is challenged when the capital/credit market imperfections are considered. When the capital/credit market is imperfect, the capital structure does matter. Beside of the credit market imperfections, the demand of entrepreneurs' insurance is also a source that affects entrepreneurs' borrowing decisions[8]. When the credit/capital market is imperfect, equilibrium contracts provide only partial insurance. Due to this partial consumption insurance, the entrepreneur's borrowing and investment decision is distorted.

3.5 Conclusions

This paper shows that when entrepreneurs differ in their wealth and their preferences are characterized by DARA, their borrowing decisions and capital investment decisions depend not only on the borrowing constraints, but also on their demand for consumption insurance.

First, we study an environment with a limited liability constraint and find that relatively poor entrepreneurs are constrained from borrowing and their optimal capital increases with their initial wealth. Specifically, if the LL constraint is sufficiently slack (λ is sufficiently large), there is an inverted U-shape relationship between entrepreneurs' capital investment/effort and their wealth. In this case, since effort is observable, banks can offer contracts which are contingent on effort and still provide risk-averse entrepreneurs full insurance through lending. Consequently, entrepreneurs' heterogeneous risk attitudes do not play an important role in their borrowing decisions.

We also study an environment with moral hazard. There, poor entrepreneurs are constrained from borrowing (in other words, the optimal capital is lower than the first-best capital amount). Besides, the optimal capital and borrowing are decreasing when the initial wealth increases. Moreover, because effort is unobservable, banks offer contracts which provide only partial insurance to the entrepreneurs (since the full insurance is not feasible anymore). The degree partial insurance entailed by the contract in turn depends on the entrepreneur's wealth. Entrepreneur with lower wealth are more risk averse, so it is easier to induce them to exert high effort. For these entrepreneurs, both the degree of insurance and the amount of capital are higher. However, for the richer entrepreneur (who cares less about insurance), the bank delegates more responsibility of the outcomes of the project to the entrepreneur by providing them less insurance and lower capital. That is, with moral hazard,

[8] Gobert (2001) considers the long term financial relationship between firms and financiers in an environment with full information. And he stresses that financiers not only can provide credit but also insurance to the firms. Leland (1998) focuses on the importance of both firms' capital structure and their risk management strategy.

entrepreneurs' heterogeneus risk attitudes matter in their borrowing decision.

In summary, entrepreneurs' needs for insurance in this paper is induced by their risk aversion, which influence their borrowing decisions, the optimal capital choices and effort choices. The other factors which also affect entrepreneurs' borrowing decisions are the borrowing constraints arisen from credit market imperfections. In the case of limited liability, these two factors are independent. The optimal borrowing decision depends only on the entrepreneurs initial wealth (but it does not depend on the level of risk aversion). However, in the case of moral hazard, these two factors are not independent anymore and they interact with each other. Higher wealth means lower risk aversion, and higher risk aversion makes it harder to provide incentives to exert effort. Consequently, the optimal borrowing decision differs because of entrepreneurs' heterogeneous risk attitudes.

3.6 Appendix

Proof. (Lemma 27)

a) By setting $\frac{\partial p}{\partial k}(z_H, k) = \frac{\partial p}{\partial k}(z_L, k)$, we have $\frac{\alpha k^{\alpha-1} z_H^{1-\alpha}}{\left(1+k^\alpha z_H^{1-\alpha}\right)^2} = \frac{\alpha k^{\alpha-1} z_L^{1-\alpha}}{\left(1+k^\alpha z_L^{1-\alpha}\right)^2}$. Hence, $k = \left(\frac{z_H^{\frac{1}{2}}}{(z_H z_L)}\right)^{\frac{1-\alpha}{\alpha}}$ and

$$\frac{\partial p}{\partial k}(z_H, k) = \alpha \left(\frac{z_H^{\frac{1}{2}}}{(z_H z_L)}\right)^{\frac{-(1-\alpha)^2}{\alpha}} z_H^{1-\alpha} \left(1 + \left(\frac{z_H^{\frac{1}{2}}}{(z_H z_L)}\right)^{1-\alpha} z_H^{1-\alpha}\right)^{-2}$$

This function is increasing in z_L:

$$\frac{\partial \left(\alpha \left(\frac{z_H^{\frac{1}{2}}}{(z_H z_L)}\right)^{\frac{-(1-\alpha)^2}{\alpha}} z_H^{1-\alpha} \left(1 + \left(\frac{z_H^{\frac{1}{2}}}{(z_H z_L)}\right)^{1-\alpha} z_H^{1-\alpha}\right)^{-2}\right)}{\partial z_L} > 0$$

Therefore, I can bound it by substituting z_L by z_H, and the following condition is derived for any $z_L < z_H$:

$$\alpha \frac{z_H^{(1-\alpha)(3-\alpha)/2\alpha}}{\left(1 + 2 z_H^{-\frac{1}{2}(1-\alpha)} + z_H^{-(1-\alpha)}\right)} < \frac{\alpha \left(\frac{z_H^{\frac{1}{2}}}{(z_H z_L)}\right)^{\frac{-(1-\alpha)^2}{\alpha}} z_H^{1-\alpha}}{\left(1 + \left(\frac{z_H^{\frac{1}{2}}}{(z_H z_L)}\right)^{1-\alpha} z_H^{1-\alpha}\right)^2} < \frac{i}{\theta_H - \theta_L}$$

b) As in part a), I derive $\widetilde{k} = \left(\frac{z_H^{\frac{1}{2}}}{(z_H z_L)}\right)^{\frac{1-\alpha}{\alpha}}$ which satisfies $\frac{\partial p}{\partial k}\left(z_H, \widetilde{k}\right) = \frac{\partial p}{\partial k}\left(z_L, \widetilde{k}\right)$. And for $k \leq \widetilde{k}$, $\frac{\partial p}{\partial k}(z_H, k) \geq \frac{\partial p}{\partial k}(z_L, k)$. On the other hand, for $k > \widetilde{k}$, $\frac{\partial p}{\partial k}(z_H, k) < \frac{\partial p}{\partial k}(z_L, k)$. ■

Proof. (Proposition 28)

a) the entrepreneur chooses optimal effort by comparing the expected utility of exerting high effort with that of exerting low effort. First of all, the expected utility of exerting high effort is

$$\log\left(p_H \theta_H + (1 - p_H)\theta_L - (k(z_H) - A)i\right) - \varphi(z_H)$$

and that of exerting low effort is

$$\log\left(p_L\theta_H + (1-p_L)\,\theta_L - (k\,(z_L) - A)\,i\right) - \varphi\,(z_L)$$

Therefore, if

$$\begin{aligned} & \log\left(p_H\theta_H + (1-p_H)\,\theta_L - (k\,(z_H) - A)\,i\right) - \varphi\,(z_H) \\ \geq\ & \log\left(p_L\theta_H + (1-p_L)\,\theta_L - (k\,(z_L) - A)\,i\right) - \varphi\,(z_L) \end{aligned}$$

the entrepreneur chooses to exert high effort and low effort otherwise. From above, we derive a threshold $\widetilde{A}$ by setting

$$\begin{aligned} & \log\left(p_H\theta_H + (1-p_H)\,\theta_L - (k\,(z_H) - A)\,i\right) - \varphi\,(z_H) \\ =\ & \log\left(p_L\theta_H + (1-p_L)\,\theta_L - (k\,(z_L) - A)\,i\right) - \varphi\,(z_L) \end{aligned}$$

Thus,

$$\widetilde{A} = \frac{\theta_L\,(1-\triangle\varphi) - i\,[k_H - \triangle\varphi k_L] + (\theta_H - \theta_L)\,[p_H - p_L\triangle\varphi]}{\triangle\varphi - 1}$$

Entrepreneurs with $0 \leq A \leq A_{FB}$, they exert high effort. On the other hand, for those with $A > A_{FB}$, they exert low effort.

b) as for optimal capital, there is an one-to-one relation between effort and capital. Once the entrepreneur chooses his optimal effort, he simultaneously decides optimal capital which is a function of his effort. Therefore, entrepreneurs with $0 \leq A \leq A_{FB}$ choose optimal capital $k\,(z_H)$ and those with $A > A_{FB}$ choose $k\,(z_L)$. By choosing parameters properly to make Lemma 1a) hold, we have $k\,(z_H) > k\,(z_L)$.

■

Proof. (Corollary 29)

From the previous proposition, we have

$$A_{FB} = \frac{\theta_L\,(1-\triangle\varphi) - i\,[k_H - \triangle\varphi k_L] + (\theta_H - \theta_L)\,[p_H - p_L\triangle\varphi]}{\triangle\varphi - 1}$$

where $\triangle\varphi = e^{\varphi(z_H) - \varphi(z_L)}$. In order to have $A_{FB} > 0$, we need

$$\theta_L\,(1-\triangle\varphi) - i\,[k_H - \triangle\varphi k_L] + (\theta_H - \theta_L)\,[p_H - p_L\triangle\varphi] > 0$$

which is equivalent to

$$\varphi\,(z_H) - \varphi\,(z_L) < \log\frac{p_H\theta_H + (1-p_H)\,\theta_L - k_H i}{p_L\theta_H + (1-p_L)\,\theta_L - k_L i}$$

■

Proof. (Lemma 31)

a) as A_{FB} is the threshold deciding optimal effort and $A_{k,LL}$ is the threshold such that the entrepreneur's wealth exactly equals to $\frac{k}{\lambda}\left(=A_{k,LL}\right)$. Thus, if $A_{k,LL} \leq A_{FB}$, the following holds:

$$\log \frac{\theta_L - \left(\lambda A_{k,LL} - A_{k,LL}\right) i + p\left(z_H, \lambda A_{k,LL}\right)\left(\theta_H - \theta_L\right)}{\theta_L - \left(\lambda A_{k,LL} - A_{k,LL}\right) i + p\left(z_L, \lambda A_{k,LL}\right)\left(\theta_H - \theta_L\right)} \geq \varphi\left(z_H\right) - \varphi\left(z_L\right)$$

Moreover, $\log \frac{\theta_L-(\lambda A-A)i+p(z_H,\lambda A)(\theta_H-\theta_L)}{\theta_L-(\lambda A-A)i+p(z_L,\lambda A)(\theta_H-\theta_L)}$ is an increasing function of A. Therefore, $A_{z,LL}$ must not be bigger than $A_{k,LL}$ ($A_{z,LL} \leq A_{k,LL}$), where $A_{z,LL}$ satisfies

$$\log \frac{\theta_L - \left(\lambda A_{z,LL} - A_{z,LL}\right) i + p\left(z_H, \lambda A_{z,LL}\right)\left(\theta_H - \theta_L\right)}{\theta_L - \left(\lambda A_{z,LL} - A_{z,LL}\right) i + p\left(z_L, \lambda A_{z,LL}\right)\left(\theta_H - \theta_L\right)} = \varphi\left(z_H\right) - \varphi\left(z_L\right)$$

b) same way to prove it as in part a) ■

Proof. (Proposition 32)

(a1), (a2) for the case with $A_{FB} \geq A_{k,LL}$, since from Lemma 2, we have $A_{z,LL} \leq A_{k,LL}$ such that it satisfies

$$\log \frac{\theta_L - \left(\lambda A_{z,LL} - A_{z,LL}\right) i + p\left(z_H, \lambda A_{z,LL}\right)\left(\theta_H - \theta_L\right)}{\theta_L - \left(\lambda A_{z,LL} - A_{z,LL}\right) i + p\left(z_L, \lambda A_{z,LL}\right)\left(\theta_H - \theta_L\right)} = \varphi\left(z_H\right) - \varphi\left(z_L\right)$$

and $\log \frac{\theta_L-(\lambda A-A)i+p(z_H,\lambda A)(\theta_H-\theta_L)}{\theta_L-(\lambda A-A)i+p(z_L,\lambda A)(\theta_H-\theta_L)}$ is an increasing function of A, we derive

$$\log \frac{\theta_L - \left(\lambda A - A\right) i + p\left(z_H, \lambda A\right)\left(\theta_H - \theta_L\right)}{\theta_L - \left(\lambda A - A\right) i + p\left(z_L, \lambda A\right)\left(\theta_H - \theta_L\right)} \leq \varphi\left(z_H\right) - \varphi\left(z_L\right) \quad \forall A \in [0, A_{z,LL}]$$

and

$$\log \frac{\theta_L - \left(\lambda A - A\right) i + p\left(z_H, \lambda A\right)\left(\theta_H - \theta_L\right)}{\theta_L - \left(\lambda A - A\right) i + p\left(z_L, \lambda A\right)\left(\theta_H - \theta_L\right)} > \varphi\left(z_H\right) - \varphi\left(z_L\right) \quad \forall A \in (A_{z,LL}, A_{k,LL}]$$

Therefore, we have the results.

b) if $A_{FB} < A_{k,LL}$, constrained entrepreneurs with $A < A_{k,LL}$ have

$$\log \frac{\theta_L - \left(\lambda A - A\right) i + p\left(z_H, \lambda A\right)\left(\theta_H - \theta_L\right)}{\theta_L - \left(\lambda A - A\right) i + p\left(z_L, \lambda A\right)\left(\theta_H - \theta_L\right)} < \varphi\left(z_H\right) - \varphi\left(z_L\right)$$

And unconstrained entrepreneurs with $A \geq A_{k,LL}$ ($> A_{FB}$) have

$$\log \frac{\theta_L - \left(k_H^{FB} - A\right) i + p\left(z_H, k_H^{FB}\right)(\theta_H - \theta_L)}{\theta_L - \left(k_L^{FB} - A\right) i + p\left(z_L, k_L^{FB}\right)(\theta_H - \theta_L)} < \varphi(z_H) - \varphi(z_L)$$

Therefore, they all choose to exert low effort. ■

Proof. (Proposition 33)

a) suppose IC constraint binds but the bank's zero-profit constraint does not. The equlibrium contract is (c_1, c_2, k). First, we can rewrite the binding IC constraint as follows:

$$\frac{c_1}{c_2} = e^{\frac{\varphi(z_H) - \varphi(z_L)}{p(k,z_H) - p(k,z_L)}} = g > 1$$

Rearrange the equation above and substitute $c_1 = gc_2$ into the bank's profit function. And because the bank's zero-profit constraint does not bind, we have

$$p(k, z_H)(\theta_H - gc_2) + (1 - p(k, z_H))(\theta_L - c_2) - (k - A) i > 0$$

which is equivalent to

$$p(k, z_H)\theta_H + (1 - p(k, z_H))\theta_L - c_2(1 + (g - 1) p(k, z_H)) > (k - A) i$$

therefore, another bank can offer a new contract $\left(c_1'', c_2'', k''\right)$ such that $c_2'' = c_2 + \varepsilon$, $c_1' = c_2' g = (c_2 + \varepsilon) g$ and $k' = k$. By doing this, the bank still earn positive expected profit and it can take over the whole market and also the entrepreneur is better off. Thus, each bank keeps undercutting the profit by doing so. At the end, zero-profit condition is satisfied in the equilibrium.

b) suppose the bank's zero-profit constraint binds but IC does not. The equilibrium contract is (c_1, c_2, k). The zero-profit condition is equivalent to

$$c_1 = \frac{p(k, z_H)\theta_H + (1 - p(k, z_H))\theta_L - (k - A) i}{p(k, z_H)} - \frac{1 - p(k, z_H)}{p(k, z_H)} c_2$$

Since IC does not bind: $\frac{c_1}{c_2} > e^{\frac{\varphi(z_H) - \varphi(z_L)}{p(k,z_H) - p(k,z_L)}} = g > 1$, by substituting zero-profit condition to IC constraint and rearrange it:

$$c_2 < \frac{\frac{p(k,z_H)\theta_H + (1 - p(k,z_H))\theta_L - (k-A)i}{p(k,z_H)}}{g + \frac{1 - p(k,z_H)}{p(k,z_H)}} = Q$$

Hence, banks can offer another contract $c_2' = c_2 + \varepsilon < Q$ and $c_1' = c_1 - \frac{1 - p(k,z_H)}{p(k,z_H)} \varepsilon$

and $k^{'} = k$ so that the entrepreneur has higher expected utility:

$$\begin{aligned} & p(z_H, k) \log c_1' + (1 - p(z_H, k)) \log c_2^{'} - \varphi(z_H) \\ > \; & p(z_H, k) \log c_1 + (1 - p(z_H, k)) \log c_2 - \varphi(z_H) \end{aligned}$$

The inequality above holds iff

$$\frac{\log \frac{c_1}{c_1^{'}}}{\log \frac{c_2^{'}}{c_2}} < \frac{1 - p(k, z_H)}{p(k, z_H)}$$

Moreover, since we know that $k < k^{FB} \in \left[0, \overline{k}\right]$ such that $k^{\alpha} z_H^{1-\alpha} < 1$ for any $k \in \left[0, \overline{k}\right]$. Thus, $p(z_H, k^*) < \frac{1}{2}$ and $\frac{1-p(k^*, z_H)}{p(k^*, z_H)} = N > 1$. Then we have $\log \frac{c_1}{c_1^{'}} < N \log \frac{c_2^{'}}{c_2} \rightarrow \frac{c_1}{c_1^{'}} < \left(\frac{c_2^{'}}{c_2}\right)^N \rightarrow \frac{c_1}{c_1 - N\varepsilon} < \left(\frac{c_2+\varepsilon}{c_2}\right)^N$

c) Suppose that IC constraint and zero-profit condition both do not bind

From a), we know that banks would keep undercutting the profit till there is no positive profit. And hence the result follows by b), IC also has to bind. ■

Proof. (Proposition 34)

a) since IC has to bind at the equilibrium

$$(p(z_H, k) - p(z_L, k)) (\log \widehat{c_1} - \log \widehat{c_2}) = \varphi(z_H) - \varphi(z_L)$$

we have $\log \frac{\widehat{c_1}}{\widehat{c_2}} = \frac{\varphi(z_H) - \varphi(z_L)}{p(z_H, k) - p(z_L, k)} > 0$. Hence, $\frac{\widehat{c_1}}{\widehat{c_2}} > 1$.

b) in the equilibrium, the shadow price of banks' zero-profit condition $\delta\,(> 0)$ and IC constraint $\mu\,(> 0)$ are follows:

$$\frac{1}{\delta} = p_H \theta_H + (1 - p_H) \theta_L - (k - A) i$$

$$\mu = \frac{(g-1) p_H (1 - p_H)}{(p_H - p_L)(1 + (g-1) p_H)}$$

We check the relation between entrepreneurs' wealth and these two shadow prices of the constraints:

$$\begin{aligned} \frac{\partial \mu}{\partial A} = \; & \frac{(p_H - p_L)(1 + (g-1) p_H) \left[\frac{\partial g}{\partial k} p_H (1 - p_H) + \frac{\partial p_H}{\partial k} (g-1)(1 - 2p_H)\right]}{(p_H - p_L)^2 (1 + (g-1) p_H)^2} \\ & + \frac{(g-1) p_H (1 - p_H) \left[\left(\frac{\partial p_H}{\partial k} - \frac{\partial p_L}{\partial k}\right)(1 + (g-1) p_H) + (p_H - p_L)\left(\frac{\partial g}{\partial k} p_H + (g-1)\frac{\partial p_H}{\partial k}\right)\right]}{(p_H - p_L)^2 (1 + (g-1) p_H)^2} \end{aligned}$$

We have: (i) if $p_H \leq \frac{1}{2} \left(\equiv k^{\alpha} z_H^{1-\alpha} \leq 1\right)$, then $\frac{\partial \mu}{\partial A} > 0$, and (ii) if $p_H > \frac{1}{2} \left(\equiv k^{\alpha} z_H^{1-\alpha} > 1\right)$,

$\frac{\partial \mu}{\partial A} < 0$. Moreover, at equilibrium we have $\frac{c_1}{c_2} = \frac{(1-p_H)\mu(p_H-p_L)+p_H(1-p_H)}{p_H\mu(p_H-p_L)+p_H(1-p_H)}$. Therefore,

$$\frac{\partial \frac{c_1}{c_2}}{\partial \mu} > 0$$

We combine the conditions we derived above and also (IC) constraint, $\frac{\partial \frac{c_1}{c_2}}{\partial A} > 0$ and hence $\frac{\partial k}{\partial A} < 0$ ■

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