

# List pricing and discounting in a Bertrand–Edgeworth duopoly<sup>☆</sup>

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## A B S T R A C T

List, or retail, pricing is a widely used trading institution where firms announce a price that may be discounted at a later stage. Competition authorities view list pricing and discounting as a procompetitive practice. We modify the standard Bertrand–Edgeworth duopoly model to include list pricing and a subsequent discounting stage. Both firms first simultaneously choose a maximum list price and then decide whether to discount, or not, in a subsequent stage. We show that list pricing works as a credible commitment device that induces a pure strategy outcome. This is true for a general class of rationing rules. Further unlike the dominant firm interpretation of a price leader, the low capacity firm may have incentives to commit to a low price and in this sense assume the role of a leader.

### JEL Classification:

L0  
L1  
L4  
L11  
L13

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Capacity constrained models  
Mixed strategies  
Pure strategies  
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## 1. Introduction

List, or retail, pricing<sup>1</sup> is a widely extended trading institution where firms post prices for some period of time. A List Price is understood to be the retail price of a good as stated in a catalogue, or price list; often subject to discounts.<sup>2</sup> List prices can be posted by wholesalers or sellers alike. They indicate firm commitment to a price for a given period of time. List prices cannot be increased for the duration of the commitment. However, they can be lowered through offering of discounts. Further, competition authority permits discounts to list prices and views them as competition enhancing. Large retailing chains such as Sears and Roebuck and Montgomery Ward sell appliances at their regular prices much of the time, but often have sales when the price is reduced by 25%.<sup>3</sup> Other examples abound. For example, the price catalogue for Ikea stores, or Amazon online. Prices

in Ikea catalogues have an expiry date, however, Ikea is free to offer discounts on these prices at any time. They cannot however raise the list price once they commit to it.

The U.S. Department of Justice screens the use of list pricing as a possible collusion facilitating device. Of special interest are practices where list pricing, and discounting, information is shared among firms.<sup>4</sup> In a recent decision, Judge Posner discussed the role of list prices versus transaction prices in the High Fructose Corn Syrup matter. (295 F.3d 651: 2002 U.S. App.). He noted that even if most customers do not pay list prices, list prices may have an impact on transaction prices and thus fixing list prices may have an effect on competition. The FTC is also of the opinion that list prices can provide a means of reaching consensus and observing prices thus facilitating coordinated action.<sup>5</sup>

In this paper we capture such a pricing institution. Firms first post a maximum list, or retail, price that can be subsequently discounted in a later stage. We consider a Bertrand–Edgeworth duopoly where prices are determined simultaneously in two stages. In the first stage, both firms announce list, or retail, prices simultaneously. In the second stage, firms may discount from these (list) prices. Our pricing institution is similar to the one described above. We show that there exists a subgame

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<sup>1</sup> The US Federal Trade Commission views list, or retail, pricing analogously.

<sup>2</sup> Freedictionary.com. Similar definitions are found in other dictionaries.

<sup>3</sup> Varian (1980, p. 658).

<sup>4</sup> See, for example, the information exchange program studied by the U.S. Department of Justice with respect to publishing list price information, as proposed by the accounting firm HIOB (at <http://www.usdoj.gov/atr/public/busreview/211191.htm>).

<sup>5</sup> See, Scheffman and Coleman (2003).

perfect equilibrium in which both firms play pure strategies and that this equilibrium payoff dominates any other subgame perfect equilibrium. Our results hold under quite general assumptions about the rationing mechanism. We also demonstrate that there is no subgame perfect equilibrium of the list pricing game in which firms play mixed strategies in the discounting stage. One should, however, note that our pure strategy results hold as long as firms are not very different.

The intuition behind our result is simple. In a Bertrand–Edgeworth equilibrium a firm may set a price such that its rival obtains higher profits from selling to the residual demand (rather than setting an undercutting price). This price gives the rival a monopoly on the residual demand. By committing to a low list price a firm indicates to its rival that it can act as a monopolist on the residual demand in the subsequent discounting stage. In this sense the list pricing institution acts as a facilitating collusion device between the firms.<sup>6</sup> There are some examples that suggest the empirical relevance of this type of pricing behavior in concentrated industries with a single dominant firm (see for instance *Sorgard, 1997*).

Our paper yields pricing outcomes which are similar to those of *Deneckere and Kovenock (1992)*. They analyze a price leadership model in a duopolistic market where the firms choose the timing of their price announcements, maximizing total discounted profits. In their game, once announced, prices cannot be changed. Firm 1 announces its price at the beginning of an even index while firm 2 announce its price at the beginning of an odd index. Thus *Deneckere and Kovenock (1992)* impose price commitment, whereby one firm must eventually set its price first, as they are interested in the timing of the commitment. Our model is conceptually different as firms have an option to commit through list pricing but they may choose, or not, to do so. In this sense our paper is closer to that of *Kreps and Scheinkman (1983)* who find that in the subgame perfect equilibrium firms will choose capacity such that they will play pure strategies in the pricing stage.

Applying list pricing and discounting we obtain a pure strategy equilibrium. This is interesting because one of the drawbacks of these models has been the existence of a mixed strategy equilibrium.<sup>7</sup> Many do not consider mixed strategies as a satisfactory explanation of pricing behavior by firms. For example, *Shubik and Levitan (1980)* consider them as an “interesting extension of the equilibrium that is somewhat hard to justify.” *Dixon (1987)*, meanwhile, finds them “implausible,” while *Friedman (1988)* finds it “doubtful that the decision makers in firms shoot dice as an aid to selecting output or price.”

In the paradigmatic case of a duopoly<sup>8</sup> the alternatives to the mixed strategy solution have involved models that assume sequential timing of firm moves. This is the approach that is followed in *Shubik and Levitan (1980)*, *Deneckere and Kovenock (1992)*, and *Canoy (1996)*. We provide an alternative to the sequential timing hypothesis by analyzing a natural extension of a Bertrand–Edgeworth model for which pure strategy equilibrium always exists. Our model does not provide an alternative solution concept to the mixed strategy Nash equilibrium but, it yields the prediction that randomization by firms is not equilibrium behavior. It

does so with a straightforward extension of the classical model. Further, we generalize some of the results of the Bertrand–Edgeworth literature which were only known to hold for the classical one stage pricing game.

The paper is structured as follows: In *Section 2* we present the basic model of a price setting duopoly with capacity constraints and specify a general residual demand function. In *Section 3* we analyze the pricing equilibria of our list-pricing game and compare it to the equilibrium of the single stage pricing game. In *Section 4* we explore the relationship between list pricing and price leadership. *Section 5* concludes.

## 2. Residual demand in a Bertrand–Edgeworth duopoly

In the classical Bertrand–Edgeworth competition firms set prices under the realization that rivals may not be able to supply all the demand at those prices.<sup>9</sup> Once prices are announced, market demand is distributed between the firms according to some specified rationing rule. The rationing rule represents some underlying consumer behavior and is assumed to be either efficient, or proportional.<sup>10</sup>

Consider a market with two firms,  $i$  and  $j$ , that produce a homogeneous good. Firm  $i$  incurs constant marginal costs of production,  $c_i$ . Marginal costs may differ across firms. Firms face capacity restrictions  $k_i$ ,  $0 < k_i \leq D(c_i)$ . The aggregate market demand,  $D(p)$ , is continuous and results in a strictly concave function,  $(p - c_i)D(p)$ . There exists a  $p^0 > 0$  such that  $D(p)$  is positive, downward sloping, and twice differentiable on  $(0, p^0)$  and zero for  $p \geq p^0 > 0$ . Let  $P(q)$  denote the inverse demand function. Associated with the demand function, and firm capacity, we can define a firm’s monopoly price<sup>11</sup>  $p_i^M = \operatorname{argmax}_p (p - c_i) \min(D(p), k_i)$ .

Given a vector of prices  $p \in \mathfrak{R}_+^2$ , set by the firms, we now discuss how much firm  $i$  sells in the market.

$$q_i(p_i, p_j) = \begin{cases} \min[k_i, D(p_i)] & p_i < p_j \\ \min[k_i, \max(0, D(p) - I_i \cdot k_j)] & p_i = p_j = p \\ \min[k_i, R(p_i, p_j, k_j)] & p_i > p_j \end{cases}$$

where,  $R(p_i, p_j, k_j)$  represents a general residual (or contingent) demand function and is defined only for  $p_i \geq p_j$ . The residual demand function is determined by how the rationing of excess demand is modeled. Using the notation in *Deneckere and Kovenock (1996)*,  $I_i$  is an indicator that takes value of 1 if  $c_i > c_j$ , or  $c_i = c_j$  and  $i = 2$ , and takes value of 0 if  $c_i < c_j$ , or  $c_i = c_j$  and  $i = 1$ .

The Bertrand–Edgeworth literature has used one of two specifications of residual demand: proportional or efficient. Suppose that consumers have a unitary demand, that firm  $j$  undercuts firm  $i$ ,  $p_i > p_j$ , and that firm  $j$  cannot meet all its demand,  $D(p_j) > k_j$ . The proportional (or Beckmann) residual demand specification results from the hypothesis that each potential consumer of firm  $j$  has an equal probability of being served. The residual demand facing the high priced firm is then given by,

$$R_B(p_i, p_j, k_j) = \max\left(D(p_i) \left(1 - \frac{k_j}{D(p_j)}\right), 0\right)$$

The efficient, or surplus maximizing, residual demand specification assumes that low priced goods are allocated to consumers with the

<sup>6</sup> In a different context *Holt and Scheffman (1987)* analyze list pricing as a facilitating practice device.

<sup>7</sup> One way of avoiding this non-existence problem is the mixed strategy solution concept. *Maskin (1986)* proves existence of a mixed strategy equilibrium for very general specifications of the Bertrand–Edgeworth model.

<sup>8</sup> The mixed strategy outcome is not particularly troublesome when the number of firms in the industry is large. *Allen and Hellwig (1986a)* and *Vives (1986)* show, under different assumptions on the rationing function, that as the number of firms in a Bertrand–Edgeworth model grows the mixed strategy equilibrium converges in distribution to the competitive equilibrium. In this sense, *Allen and Hellwig (1986b)*, while considering the non-existence of a pure strategy equilibrium a “drawback of the Bertrand–Edgeworth specification,” argue that in the large numbers case randomization in prices is “in some sense unimportant” as firms will set prices close to the competitive price with very high probability. The competitive result is robust to a change in the equilibrium concept. *Dixon (1987)* and *Borgers (1992)* obtain convergence to the competitive equilibrium using the  $\varepsilon$ -equilibrium and iterated elimination of dominated strategies solution concepts, respectively.

<sup>9</sup> For a description of Bertrand–Edgeworth competition see *Vives (1999)*, page 123.

<sup>10</sup> Efficient rationing is used in *Levitan and Shubik (1972)*, *Kreps and Scheinkman (1983)*, *Vives (1986)*, and *Deneckere and Kovenock (1992)*. Proportional rationing is used in *Beckmann (1965)*, *Allen and Hellwig (1986a–b)*, *Dasgupta and Maskin (1986)*, *Davidson and Deneckere (1986)* (this last paper also has some results for a general class of rationing functions) and *Deneckere and Kovenock (1992)*.

<sup>11</sup> We assume that the low cost firm’s monopoly price lies above the marginal cost of the high cost firm. Otherwise the efficient firm always sets its monopoly price with the high cost firm acting on the residual demand.

highest valuation for the good.<sup>12</sup> Under this assumption the high priced firm has residual demand,

$$R_E(p_i, p_j, k_j) = \max(D(p_i) - k_j, 0)$$

Proportional and efficient rationing are but two of the many reasonable specifications of residual demand. For instance, one may assume that a proportion  $1-\lambda (>0)$  of the low priced firm's capacity is allocated randomly among potential buyers. The remaining capacity, meanwhile, goes to unsatisfied high valuation consumers. This would then result in residual demand for the high priced firm of,

$$R_\lambda(p_i, p_j, k_j) = \max\left(\lambda(D(p_i) - k_j) + (1 - \lambda)D(p_i) \left(1 - \frac{k_j}{D(p_j)}\right), 0\right).$$

This function belongs to a class of residual demand functions for which our results hold. They can be characterized by imposing the following restrictions on the function  $R(p_i, p_j, k_j): A \rightarrow \mathfrak{R}_+$  where,  $A = \{(p, p', k') \in \mathfrak{R}^3: p \geq p' \geq 0, k' \geq 0\}$

1.  $R(p_i, p_j, k_j)$  is continuous and twice continuously differentiable.
2. When  $R(p_i, p_j, k_j) > 0$ , it is strictly decreasing in  $p_i$ .
3. When  $R(p_i, p_j, k_j) > 0$  then  $R(p_i, p_j, k_j)(p_i - c_i)$  is strictly concave in  $p_i$ .
4.  $\max(0, D(p_i) - k_j) \leq R(p_i, p_j, k_j) \leq \max(0, \min(D(p_j) - k_j, D(p_i)))$ .
5. When  $R(p_i, p_j, k_j) > 0$  it is strictly decreasing in  $p_j$ .

Properties (1),(2), and (3) guarantee that the residual demand function inherits certain regularity properties from the demand function. In order to understand property (4) consider what happens as  $p_j$  gets arbitrarily close to  $p_i$ . In this case, the number of consumers of the low priced firm with a reservation price below  $p_i$  becomes arbitrarily small and the residual demand function is  $D(p_i) - k_j$ . With respect to the right hand side simply note that the low priced firm may never sell more than  $k_j$  units of the good.<sup>13</sup>

Property (5) refers to the fact that if firm  $j$  (the low price firm) lowers its price,  $p_j$ , more consumers enter the market. Note that, this reduces the proportion of firm  $j$ 's output that is allocated to high valuation consumers. This in turn increases residual demand for the high price firm  $i$ . Thus, firm  $i$ 's profits will rise as firm  $j$  lowers its price. The effect of firm  $j$  lowering its price on profits of firm  $i$  plays an important role in our results.

Further, it must be noted that the efficient residual demand is not included in the class of rationing functions we consider since it violates property (5). On the other hand our results do hold for functions that approximate efficient residual demand, i.e. very small  $\lambda$ , as  $R_\lambda(p_i, p_j, k_j)$  verifies properties 1–5 for any  $0 \leq \lambda < 1$ .

### 3. The list pricing game

In our extension of the classic model, firms simultaneously set prices in two stages. In the first stage, each firm  $i \in \{1, 2\}$  sets a list price,  $p_i^l$ . In the second stage, firms are allowed to offer a discount on this price. The discounted price,  $p_i^d (\leq p_i^l)$ , has to be less than or equal to the list price set in the first stage. The discounted price is offered to all consumers.<sup>14</sup> Given  $p_i^d$  consumers make their purchasing decisions

<sup>12</sup> It should be noted that although efficient rationing maximizes consumer surplus (for a particular capacity constrained firm), it does not maximize total consumer surplus. Given capacities and prices, if the high priced firm can meet all of its residual demand proportional rationing leads to greater total consumer and total surplus than efficient rationing.

<sup>13</sup> Properties (1), (2) and (4) are proposed by Davidson and Deneckere (1986) for a "reasonable rationing function."

<sup>14</sup> In real world situations, firms can offer discounts to a subset of consumers, i.e. price discriminate (based on volume, repeat purchases etc.). We do not consider this possibility.

according to  $q_i(p_i^d, p_j^d)$ . For simplicity, we do not consider list prices greater than  $p^0$ .

The game is solved backwards. In the final (discounting) stage we show some conditions under which firms set the competitive price (Theorem 1). Extending previous results for proportional and efficient residual demand we characterize all the mixed strategy equilibria in the discounting subgame (Theorem 2). Then, in the first stage, list prices are analyzed. We first show the existence of a pure strategy equilibrium in the classical Bertrand–Edgeworth game (Theorem 3 and Corollary 2). When the pure strategy equilibrium of the classical game does not exist, we show that any subgame perfect equilibrium of the list pricing game involves no mixed strategies in the discounting stage (Theorem 4 and Corollary 3). Finally we prove the existence of a pure strategies equilibrium (different from the competitive price) for the full game (Theorem 5). In sub-section 3 we present an example where we compute the equilibria for the list pricing game.

#### 3.1. The Discounting subgame

Firms can discount over their posted list price in the first stage. In the discounting stage firms set a discounted price that is less than, or equal, to the list price. We first verify the existence of an equilibrium to each discounting subgame given any pair of price ceilings  $p_i^l \geq 0$ . Applying Theorem 5 from Dasgupta and Maskin (1986), and using the tie breaking rule, it is straightforward to prove that the discounting subgame has a (mixed) Nash equilibrium for any  $(p_1^l, p_2^l)$ .<sup>15</sup>

We first consider the possibility of reaching a discounting subgame where the list prices induce a pure strategy equilibrium. A pure strategy equilibrium that does not involve the competitive price can always be induced in the list price stage if firm  $i$  sets its list price such that it is not undercut (and firm  $j$  sets a list price that is high enough). In fact, any pair of list prices  $p_i^l > p_j^l$ , such that firm  $i$  will act on the residual demand, if firm  $j$  sets a discounted price of  $p_j^d$ , i.e.  $\min(D(p_j^d), k_j)(p_j^d - c_j) \leq \max_{p \in [p_j^d, p_j^l]} R(p, p_j^d, k_j)(p - c_j)$ , induce a pure strategy equilibrium in the discounting stage,  $(\hat{p}_i^d, \hat{p}_j^d)$ . In this equilibrium, firm  $j$  does not discount its list price and firm  $i$  acts on the residual demand, setting a discounted price of  $\hat{p}_i^d = \arg\max_{p \in [p_j^d, p_i^l]} R(p, p_j^d, k_j)(p - c_i)$ .

A well known result of the Bertrand–Edgeworth literature is that the only candidate for a pure strategy equilibrium is the competitive price (see Arrow, 1951). The condition under which the Arrow result applies in our model is discussed in Theorem 1. We show later that this condition is never met in equilibrium unless the classic Bertrand–Edgeworth game has a pure strategy equilibrium.

Define the competitive price of our game as  $p^c = \max\{c_1, c_2, P(k_1 + k_2)\}$ . The following Theorem characterizes some list prices of a game where the only possible equilibrium in pure strategies is the competitive price. This Theorem will be used later to show that in some of the list pricing games the unique pure strategy equilibrium is the competitive price. Specifically, we show (Corollary 2) the existence of a pure strategy equilibrium in the classical Bertrand–Edgeworth game.

**Theorem 1.** Let  $p_i^l \geq p_j^l \geq p^c$ , if

$$\min(D(p_j^l), k_j)(p_j^l - c_j) > \max_{p \in [p_j^d, p_i^l]} R(p, p_j^l, k_j)(p - c_i)$$

then the only candidate for a pure strategy equilibrium in the subgame following  $(p_i^l, p_j^l)$  involves both firms setting the competitive price.

**Proof.** Suppose a pure strategy equilibrium to the discounting game, i.e.,  $(\hat{p}_i^d, \hat{p}_j^d)$ , exists. If  $\hat{p}_i^d < \hat{p}_j^d$ , this implies that  $\hat{p}_i^d = p_j^l$ . Contrarily, firm  $i$  would want to raise its price. But this contradicts  $p_i^l \geq p_j^l$ . Suppose on

<sup>15</sup> Also see Deneckere and Kovenock (1996).

the other hand  $\hat{p}_j^d < \hat{p}_i^d$ , this then implies  $\hat{p}_j^d = p_j^l$ . Further, in order for firm  $i$  to not have incentives to undercut firm  $j$  it must be the case that

$$\min(D(p_j^l), k_i)(p_j^l - c_i) \leq \max_{p \in [p_j^l, p_i^l]} R(p, p_j^l, k_j)(p - c_i)$$

This, however, leads to a contradiction. We then have that both firms set the same discounted price. In that case the equilibrium is competitive (or else, at least one firm will have an incentive to undercut its rival).

We now consider the possibility of reaching a subgame where list prices induce a non-degenerate mixed strategy equilibrium. Given the list prices set in the first stage  $(p_1^l, p_2^l)$ , a firm's strategy in the discounting subgame is defined by a (possibly degenerate) probability measure  $\mu_i^d$  on  $[c_i, p_i^l]$ . Let the minimum and the maximum of the support of  $\mu_i^d$  be denoted by  $\underline{p}_i^d$  and  $\bar{p}_i^d$ , respectively. Given any two strategies  $(\mu_i^d, \mu_j^d)$ , a firm's expected profits in the discounting stage will be denoted by  $\pi_i(\mu_i^d, \mu_j^d)$ . For any  $p_j^l \geq p^c$ , denotes by  $p_i^E$  the price that can be set by firm  $i$  such that firm  $j$  is indifferent between undercutting firm  $i$  and, acting on the residual demand. That is,

$$p_i^E = \min \left\{ p : \min(D(p), k_j)(p - c_j) = \max_{x \in [p, p_j^l]} R(x, p, k_i)(x - c_j) \right\}$$

We refer to  $p_i^E$  as the Edgeworth price. It is trivial to show that  $p_i^E \geq p^c$ .<sup>16</sup> The Edgeworth price will be very useful in order to characterize the equilibria that arise in the pricing subgames that we study. It should be noted that  $p_i^E$  is weakly increasing with  $p_j^l$ . For values of  $p_j^l$  close to the competitive price,  $p_i^E$  increases with  $p_j^l$ . For a high enough value of the list price of firm  $j$ ,  $p_i^E$  attains a maximum value. Let us denote this value by  $\bar{p}_i^E$ .

The next result characterizes some of the properties of a non-degenerate mixed strategy equilibrium in the discounting stage. Property i) shows that the lower bound of the price support is the same for both firms and above the market clearing price. Property iii) implies that there is a firm  $h$  that, when setting the highest price in its support, is undercut with certainty. Note that, a discounting subgame where the list prices are set at  $p^0$  is equivalent to the classical one stage pricing game. This Theorem thus generalizes some of the results of the Bertrand–Edgeworth literature which are known to hold in the case of the efficient and proportional residual demands to a more general class of residual demand functions.

**Theorem 2.** *Given,  $(p_1^l, p_2^l)$  and  $\min\{p_1^l, p_2^l\} > \max\{p_1^E, p_2^E\}$ . If a non-degenerate mixed strategy equilibrium,  $(\mu_1^d, \mu_2^d)$ , to the discounting subgame exists, then*

- $\underline{p}_1^d = \underline{p}_2^d = p^d$
- $\pi_i(\mu_i^d, \mu_j^d) = \min(D(p^d), k_i)(p^d - c_i)$  for any  $i \in \{1, 2\}$
- for one of the two firms  $h \in \{1, 2\}$ :

$$\pi_h(\mu_h^d, \mu_{-h}^d) = \int_{p^d}^{\bar{p}_{-h}^d} R(\bar{p}_h^d, p, k_{-h})(\bar{p}_h^d - c_h) d\mu_{-h}^d(p)$$

**Proof.** See Appendix.

As a Corollary to Theorem 2 we will prove that the lower bound of the support of the mixed strategy equilibrium is below the highest Edgeworth

price. This result is important since it implies that the high Edgeworth price firm would be better off committing to its Edgeworth price and, have the other firm acting on the residual demand (than in any discounting game that has a non-degenerate mixed strategy equilibrium). The proof is based on the fact that (by Theorem 2) in a mixed strategy equilibrium there is a firm  $h$  which sets a price of  $\bar{p}_h^d$  and is undercut by its rival with certainty. Setting this price, firm  $h$ 's payoffs are not certain. They are greatest when its rival sets a price of  $p^d$ . Thus, expected profits of firm  $h$  are

strictly less than  $R(\bar{p}_h^d, p^d, k_{-h})(\bar{p}_h^d - c_h)$ . If firm  $-h$  were to set a price sufficiently close to  $p^d$  with certainty, then firm  $h$  would best respond acting on the residual demand. This in turn implies that the Edgeworth price of firm  $-h$  must be greater than  $p^d$ .

**Corollary 1.** *Given,  $(p_1^l, p_2^l)$ , such that  $\min\{p_1^l, p_2^l\} > \max\{p_1^E, p_2^E\}$ . If a non-degenerate mixed strategy equilibrium to the discounting game exists, then  $p^d < \max\{p_1^E, p_2^E\}$ .*

**Proof.** By Theorem 2 there is a firm  $h$  for whom,

$$\min(D(p^d), k_h)(p^d - c_h) = \int_{p^d}^{\bar{p}_{-h}^d} R(\bar{p}_h^d, p, k_{-h})(\bar{p}_h^d - c_h) d\mu_{-h}(p)$$

- Given the continuity of the residual demand function, by the Mean Value Theorem we have that for some  $\underline{p}^d < z < \bar{p}_{-h}^d$ ,

$$\int_{p^d}^{\bar{p}_{-h}^d} R(\bar{p}_h^d, p, k_{-h})(\bar{p}_h^d - c_h) d\mu_{-h}(p) = R(\bar{p}_h^d, z, k_{-h})(\bar{p}_h^d - c_h) .$$

- By Property (5) of the residual demand function we have,

$$\min(D(p^d), k_h)(p^d - c_h) < R(\bar{p}_h^d, p^d, k_{-h})(\bar{p}_h^d - c_h) .$$

- That is,  $p_{-h}^E > p^d$  and, therefore,  $\max\{p_1^E, p_2^E\} > p^d$ .

### 3.2. The Full Game

We now characterize the subgame perfect equilibrium of the list pricing game. In part (i) of Theorem 2 we show that if the Edgeworth price of both firms coincides with the competitive price then any subgame perfect equilibrium of the list pricing game involves both firms setting the competitive price. Note that, if the competitive price is equal to the highest unit cost, then the high cost firm faces zero residual demand. Facing zero residual demand it will be indifferent between setting any price as its sales will always be zero. Then, any strategy such that the other firm (low cost) does not want to raise its price, will be an equilibrium. It is then straightforward to see that all these subgame perfect equilibria are payoff equivalent (part (ii) of Theorem 2).

**Theorem 3.** *Given the rationing rule, suppose that  $(k_1, k_2, c_1, c_2)$  are such that for every  $(p_1^l, p_2^l)$ ,  $\max\{p_1^E, p_2^E\} = p^c$ . Then,*

- i) there exists a subgame perfect equilibrium of the list pricing game where both firms set a discounted price of  $p^c$ .
- ii) any other subgame perfect equilibrium is payoff equivalent to i) and involves all quantities being sold at  $p^c$ .

**Proof.** Note that, a firm  $i$  can always guarantee itself profits of  $\min(D(p^c) - I_i k_j, k_i)(p^c - c_i)$  by setting the competitive price in the list pricing and the discounting stage. This implies that any list price below the competitive price is strictly dominated. For this reason, in a subgame perfect equilibrium, no firm will set a price below  $p^c$ .

Below we prove that is a firm set  $p^c$  in the discounting stage then the other firm's best response is setting  $p^c$ .

<sup>16</sup> Further assumptions on the residual demand function would be needed to determine which firm will have the highest Edgeworth price. It is straightforward to see that if the residual demand function is of the type  $R_\lambda$  then  $k_i \geq k_j, c_i \geq c_j$  implies  $p_i^E \leq p_j^E$ . That is, the low capacity/cost firm has a higher Edgeworth price than the high capacity/cost firm. Note that, if  $k_i > k_j$  and  $c_i < c_j$ , then either firm could have a higher Edgeworth price.



• First, consider the case where  $p^C = \max\{c_1, c_2\} > P(k_1 + k_2)$ . Let us suppose that  $c_j > c_i$  [or  $c_j = c_i$  and  $i = 1$ ]. Then it must be the case that firm  $j$  has zero residual demand at any price set by firm  $i$  above  $c_j$  (note that, because  $\max\{p_1^E, p_2^E\} = p^C = \max\{c_1, c_2\}$ , we have  $p_i^E \leq c_j$ , which means that firm  $j$ 's profits acting on the residual demand should be the same as when it undercuts, which are non-positive). This implies that  $k_i \geq D(c_j)$ .

If firm  $i$  sets a price of  $p^C$ , firm  $j$ 's residual demand will be zero. In that case, firm  $j$  could set any price, but only  $p_j^d = p^C$  (or any strategy that deters firm  $i$  from raising its price, could be an equilibrium). Firm payoffs are the same for all these strategies.

If firm  $j$  sets  $p_j^d = p^C$ , then  $\min(k_i, D(p^C))(p^C - c_i) \geq \max_{p \in [p^C, p_j^d]} R(p, P(k_1 + k_2), k_j)(p - c_i)$  (note that  $p_j^d = p^C = \max\{p_1^E, p_2^E\} \geq p_j^E$ ). If  $p_j^E < p^C$ , then firm  $i$  will always undercut. If  $p_j^E = p^C$ , then

$$\min(k_i, D(p^C))(p^C - c_i) > R(p, P(k_1 + k_2), k_j)(p - c_i)$$

for any  $p > p^C$  (directly applying properties (3) and (4) of the residual demand). Then firm  $i$ 's best response would be setting  $p_i^d = p^C$ .

• Now, consider the case where  $p^C = P(k_1 + k_2)$ . We then have that,

$$k_i(P(k_1 + k_2) - c_i) \geq \max_{p \in [P(k_1 + k_2), p_j^d]} R(p, P(k_1 + k_2), k_j)(p - c_i)$$

$$\text{as } p^C = \max\{p_1^E, p_2^E\} = P(k_1 + k_2).$$

Furthermore, by property (4) of the residual demand we have,

$$k_i(P(k_1 + k_2) - c_i) = R(P(k_1 + k_2), P(k_1 + k_2), k_j)(P(k_1 + k_2) - c_i)$$

This along with property (3) yields,

$$k_i(P(k_1 + k_2) - c_i) > R(p, P(k_1 + k_2), k_j)(p - c_i)$$

for any  $p > P(k_1 + k_2)$ , which proves the desired result.

This proves that setting the competitive price, or if  $p^C = \max\{c_1, c_2\} > P(k_1 + k_2)$  firm  $i$  playing  $p_i^d = p^C$  and firm  $j$  playing any strategy that deters firm  $i$  from raising its price, are equilibrium strategies (part (i) above). Payoff from these equilibria are equivalent to the payoff when both firms play  $p^C$  and all quantities are sold at this competitive price. Then, to prove part (ii) (above), we only need to show that there are not any other equilibria.

Note that if  $p_i^d = p^C$ , for some firm  $i$ , then it will set the competitive price in the discounting stage. By the previous argument, firm  $j$  will best respond by setting the competitive price also (or, for the case where  $p^C = \max\{c_1, c_2\} = c_h$ , firm  $h$  playing any strategy that deters firm  $-h$  from raising its price).

On the other hand, suppose that for both firms,  $p_i^d > p^C$ . Then, any hypothetical nondegenerate mixed strategy equilibrium will have (by Corollary 1)  $p^d < \max\{p_1^E, p_2^E\}$ . That is, the lower bound of the support lies below the highest price (and, therefore, below  $p^C$ ). Then a deviation to the competitive price is profitable. Thus, no nondegenerate mixed strategy equilibrium will exist.

If a nondegenerate mixed strategy equilibrium does not exist then there must be at least a pure strategy equilibrium (a discounting subgame always has a Nash equilibrium for any  $(p_1^d, p_2^d)$ ). If  $p_i^d > p^C$  for both firms, then it must be that  $p_i^d \geq p_j^d > p_j^E$ , for some  $i \in \{1, 2\}$ . Applying Theorem 1 we know that the only candidate for a pure strategy equilibrium in the discounted stage involves both firms setting the competitive price. Thus we obtain the desired result.

The intuition behind the result is that if both firms set the competitive price then no firm has an incentive to raise its price (since the characterization of the Edgeworth price implies that the profits, when undercutting, are equal to the maximum profits that can be obtained acting on the residual demand, i.e.,  $\min(k_i, D(p^C))(p^C - c_i) = \max_{p \in [p^C, p_i^d]} R(p, p^C, k_j)(p - c_i)$ ). To see that this is the only possible equilibrium, note that by Corollary 1, when  $\max\{p_1^E, p_2^E\} = p^C$  and  $p_i^d > p^C$ , for some  $i$ , a mixed strategy equilibrium to the discounting subgame does not exist.

The Bertrand-Edgeworth pricing game can be seen as a discounting game where the list prices are set arbitrarily high. Then, by Theorem 3, it is clear that if  $\max\{p_1^E, p_2^E\} = p^C$  the Bertrand-Edgeworth model has a pure strategy equilibrium. On the other hand, by Theorem 1, we have that the only candidate for a pure strategy equilibrium in a Bertrand-Edgeworth model is the competitive price. However, if the Edgeworth price of a firm  $i$  is greater than the competitive price (i.e.,  $p_i^E = \max\{p_1^E, p_2^E\} > p^C$ ) then firm  $j$  will have an incentive to deviate from this equilibrium. This leads to our next result which characterizes when the one stage pricing game has a pure strategy equilibrium.

**Corollary 2.** *The Bertrand-Edgeworth model has a pure strategy equilibrium if and only if  $\max\{p_1^E, p_2^E\} = p^C$ .*

*Theorem 3 and Corollary 2 imply that when the Bertrand-Edgeworth model has a pure strategy equilibrium the addition of a list pricing stage is innocuous in the sense that it leads to the same prices in equilibrium.*

*We will now characterize the equilibria of the list pricing game when a pure strategy equilibrium of the Bertrand-Edgeworth game does not exist. As mentioned earlier, this is observed when one of the firms has an Edgeworth price greater than the competitive price. In the next Theorem we prove that all subgame perfect equilibria must involve pure strategies in the discounting stage.*

**Theorem 4.** *Given the rationing rule, suppose that  $(k_1, k_2, c_1, c_2)$  are such that for every  $(p_1^d, p_2^d)$ ,  $\max\{p_1^E, p_2^E\} > p^C$ . Then, any subgame perfect equilibrium of the list pricing game must involve  $p_i^d = p_i^E$ , for some  $i$ .*

**Proof.** It is clear that in equilibrium the list price is greater than, or equal to, the Edgeworth price, i.e.  $p_i^d \geq p_i^E$ . As shown earlier, it is in no firm's interest to commit to a price below its Edgeworth price. On the other hand, if  $p_i^d > p_i^E$  for both firms, then from Theorem 1 we know that the only candidate for a pure strategy equilibrium in the discounting game involves both firms setting the competitive price. But from Corollary 2, both firms setting  $p^C$  in the discounting game can not be an equilibrium, if  $\max\{p_1^E, p_2^E\} > p^C = p_i^d > p_i^E$  for both firms. Therefore, no pure strategy equilibrium exists in the discounting game.

If a pure strategy equilibrium does not exist, this implies that there is a mixed strategy equilibrium in the discounting game. Below we will prove by contradiction that such an equilibrium where firms play  $(\hat{p}_1^d, \hat{p}_2^d)$  (in the list pricing stage) and  $(\hat{\mu}_1^d, \hat{\mu}_2^d)$  (a mixed strategy in the discounting stage), with  $\hat{p}_i^d > p_i^E$  for both firms, cannot exist.

Suppose that firm  $i$  has the highest Edgeworth price, i.e.  $p_i^E = \max\{p_1^E, p_2^E\}$ . Then by Theorem 2 -(ii) firm  $i$ 's profits are given by  $\pi_i(\hat{\mu}_i^d, \hat{\mu}_j^d) = \min(D(\hat{p}^d), k_i)(\hat{p}^d - c_i)$ . Given that  $\hat{p}^d < p_i^E$  (Corollary 1), a deviation in which firm  $i$  plays a discounted price equal to  $p_i^E$ , and firm  $j$  acts on the residual demand, would be profitable (firm  $i$ 's profits would be  $\min(D(p_i^E), k_i)(p_i^E - c_i)$ ). It is easy to see that if firm  $i$  deviates, setting  $p_i^d = p_i^E$  in the list price stage, the induced equilibrium in the discounting stage involves firm  $i$  setting a price  $p_i^E$  (with firm  $j$  acting on the residual demand). This, however, contradicts that  $(\hat{p}_1^d, \hat{p}_2^d)$  and  $(\hat{\mu}_1^d, \hat{\mu}_2^d)$  is a subgame perfect equilibrium.

From Theorem 4 we know that a firm sets a list price equal to its Edgeworth price. This implies that its rival,  $j$ , has no incentive to undercut and will set a (list price) in equilibrium which allows it to maximize its profits acting on the residual demand. This is formalized in the following Corollary.

**Corollary 3.** If  $\max\{p_1^E, p_2^E\} > p^C$ , any subgame perfect equilibrium of the list pricing game must involve  $p_i^L = p_i^d = p_i^E$ , and  $p_j^L \geq p_j^d = \arg\max_{p \in [p_i^L, p_i^E]} R(p, p_i^L, k_i)(p - c_j)$ .

Note that by Corollary 3 no subgame perfect equilibrium of the list pricing game will involve mixed strategies in the discounting stage.

In the following Theorem we prove that at least one subgame perfect equilibrium to the list pricing game exists. In this subgame perfect equilibrium, which we denote by  $\hat{e}$ , firm  $i$  sets its list price equal to the highest possible Edgeworth price ( $\bar{p}_i^E$ ) and does not discount. The other firm  $j$ , meanwhile, sets its list price arbitrarily high and acts as a monopolist on the residual demand in the discounting stage.

**Theorem 5.** Let  $\bar{p}_i^E \geq \bar{p}_j^E$ , then  $\hat{e} \equiv ((\hat{p}_i^L, \hat{p}_j^L), (\hat{p}_i^d, \hat{p}_j^d))$ , such that  $\hat{p}_i^L = \hat{p}_i^d = \bar{p}_i^E$  and  $\hat{p}_j^L \geq \hat{p}_j^d = \arg\max_{p \in [\bar{p}_i^E, p_i^L]} R(p, \bar{p}_i^E, k_i)(p - c_j)$ , is a subgame perfect equilibrium of the list pricing game.

**Proof.** If  $\bar{p}_i^E = p^C$ , this implies that  $\max\{\bar{p}_1^E, \bar{p}_2^E\} = p^C$ . We then obtain the desired result by applying Theorem 3.

On the other hand suppose that  $\bar{p}_i^E > p^C$ . Let us define  $p_j^R = \arg\max_{p \in [\bar{p}_i^E, p_i^L]} R(p, \bar{p}_i^E, k_i)(p - c_j)$ . In this case, the proposed equilibrium ( $\hat{e}$ ) yields the following profits:

$$\hat{\pi}_i = \min(k_i, D(\bar{p}_i^E))(\bar{p}_i^E - c_i)$$

$$\hat{\pi}_j = R(p_j^R, \bar{p}_i^E, k_j)(p_j^R - c_j) = \min(k_j, D(\bar{p}_i^E))(\bar{p}_i^E - c_j)$$

Note that, as firm  $i$  is playing its Edgeworth price, firm  $j$ 's profits are the same acting on the residual demand or undercutting, i.e.,  $R(p_j^R, k_j)(p_j^R - c_j) = \min(k_j, D(\bar{p}_i^E))(\bar{p}_i^E - c_j)$ .

Below we prove by contradiction that any deviation from the proposed equilibrium is unprofitable.

Suppose that a firm can profitably deviate. It must then involve setting a list price greater than  $\bar{p}_i^E$  in the first stage. Let this deviation be  $\tilde{e} \equiv ((\tilde{p}_i^L, \tilde{p}_i^L), (\tilde{p}_i^d, \tilde{p}_i^d))$ . If firm  $h$  sets a different list price, the discounted prices (for both firms) will change accordingly. Now, the discounted prices induced could be a pure, or a mixed, strategy.

Let the firms play pure strategies in the discounting stage in response to  $(\tilde{p}_i^L, \tilde{p}_i^L)$ . We know by Theorem 1 that the only candidate for a pure strategy equilibrium in the discounting stage, for  $(\tilde{p}_i^L, \tilde{p}_i^L)$ , will be  $\tilde{p}_i^d = \tilde{p}_i^L = p^C$ . Given that  $\bar{p}_i^E > p^C$ , both firms lose under the deviation.

Now, suppose that firms play mixed strategies,  $(\mu_i^L, \mu_j^L)$ , in the discounting stage earning expected profits,  $(\hat{\pi}_i, \hat{\pi}_j)$ . Then, from Theorem 2 we have that profits for both firms will be  $\pi_i(\mu_i^L, \mu_j^L) = \min(D(\tilde{p}_i^L), k_i)(\tilde{p}_i^L - c_i)$ . By Corollary 1,  $\tilde{p}_i^L < \bar{p}_i^E \leq \bar{p}_i^E$ . Again, this implies that the deviation is unprofitable. This then leads to a contradiction.

When  $\arg\max_{p \in [p_i^L, p_i^E]} R(p, p', k_i)(p - c_j)$  weakly increases with  $p'$  (for instance when the residual demand is proportional) then it is easy to show that  $\hat{e}$  is the unique equilibrium. However, when the best response price of a firm, acting on the residual demand, decreases with its rival's price, there could be additional equilibria with firm  $i$  setting  $p_i^L = p_i^d = p_i^E < \bar{p}_i^E$ , and firm  $j$  acting on the residual demand, such that  $p_j^L \geq p_j^d = \arg\max_{p \in [p_i^L, p_i^E]} R(p, p_i^L, k_i)(p - c_j)$ . These equilibria would involve lower payoffs than  $\hat{e}$  for both firms.

In the following Corollary we show that firm profits under the proposed equilibrium,  $\hat{e}$ , are greater than those in the equilibrium of the classic Bertrand–Edgeworth game.

**Corollary 4.** Firm profits in the equilibrium of the Bertrand–Edgeworth game are lower than under the proposed equilibrium with list prices,  $\hat{e}$ .

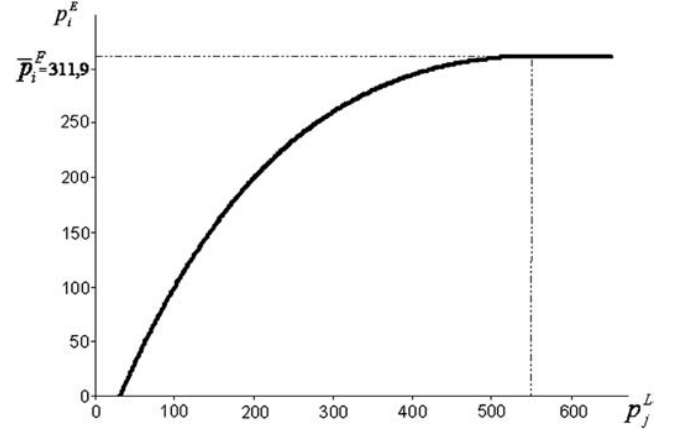


Fig. 1. Relationship: Edgeworth and List Price.

The proof of this Corollary is straight forward. By Theorem 2 and Corollary 1 the lower bound of the support of any mixed strategy equilibrium (in the Bertrand–Edgeworth game) will be lower than the highest Edgeworth price. This implies that expected profits under randomizing will be smaller than under the list pricing equilibrium,  $\hat{e}$ .

The equilibrium,  $\hat{e}$ , is similar to the one obtained in Gelman and Salop (1983). They analyze a game of entry in a market by a capacity constrained firm, where the entrant must commit to a price to which the incumbent best responds. Note, Gelman and Salop (1983) refer to the entrants low price (small size) strategy as “judo economics”. In the equilibrium of our model,  $\hat{e}$ , the low pricing strategy is followed by the firm with the highest Edgeworth price. Without any additional assumptions on the residual demand function it is not straightforward to prove a direct relation between the low price strategy and capacity. If we assume that the residual demand is given by  $R_\lambda(p_i, p_j, k_j)$  for any  $\lambda < 1$ , and arbitrarily high list prices, then it is easy to prove that, for  $c_i \leq c_j$ ,  $k_i < k_j$  implies  $\bar{p}_i^E > \bar{p}_j^E$ . In this case, the low capacity firm follows the low pricing strategy. There are several examples of the empirical validity of this type of “judo economics” pricing behavior which are given in Gelman and Salop (1983), and Sorgard (1997).

### 3.3. An example

Consider two symmetric<sup>17</sup> firms with capacity  $k_i = 400$  and unit variable costs  $c_i = 100$ , for  $i = 1, 2$ , competing for market demand  $D(p) = 1000 - p$ . The competitive price will then be  $p^C = \max\{c_1, c_2, P(k_1 + k_2)\} = 200$ , where both firms sell their entire capacity.

Let firm  $i$  set a price,  $p_i$ . Setting a lower price, firm  $j$ 's profits, would then be  $(p_j - c_j) \min\{k_j, D(p_j)\}$ . These profits are represented in Fig. 1. Firm  $j$ 's best response would be to undercut firm  $i$  earning profits  $(p_i - \varepsilon - c_j) \min\{k_j, D(p_i - \varepsilon)\}$ , if  $p_i \leq 600$ . If  $p_i > 600$  then its best response would be  $p_j = 600$ , obtaining a profit of 200,000.

Consider a proportional residual demand specification,  $R_B(p_j, p_i, k_i) = \max\{D(p_j) \left(1 - \frac{k_i}{D(p_i)}\right), 0\}$ . Given  $p_i$ , firm  $j$ 's profits acting on the residual demand will be  $\max_{p \in [p_i, p_i^E]} \left[ \max\{D(p) \left(1 - \frac{k_i}{D(p_i)}\right), 0\} (p - c_j) \right]$ , which

are proportional to the profits obtained from undercutting (depending on  $k_j$  and  $p_i$ ) (see Fig. 1 for the cases where  $p_i = 200$  and  $p_i = 311.9$ ). It is easy to show that, acting on the residual demand, firm  $j$ 's best response will be setting  $p_j = \min\{550, p_i^L\}$  if  $p_i \leq 550$ , and  $p_j = p_i$  if  $p_i > 550$ . If  $p_i \geq 600$ , firm  $i$  will serve the entire demand and firm  $j$ 's profits will always be zero. Then,

<sup>17</sup> We solve the symmetric case due to simplicity. The example can be extended to asymmetric costs and/or capacities.

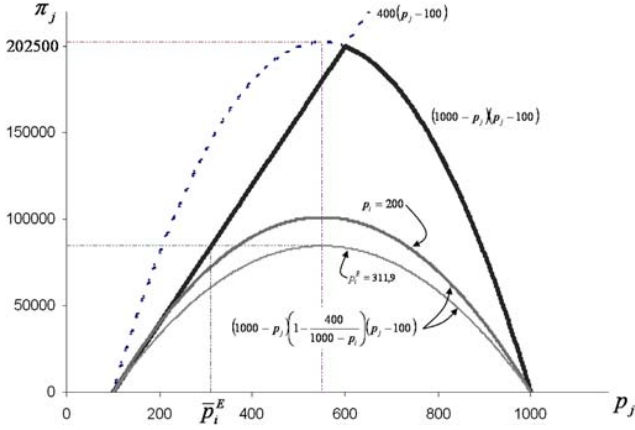


Fig. 2. Profits firm j.

given firm  $i$ 's price,  $p_i$ , firm  $j$ 's maximum profits acting on the residual demand will be:

$$\pi_j^{R^*} = \begin{cases} (1000 - p_j^i) \left(1 - \frac{400}{1000 - p_i}\right) (p_j^i - 100) & \text{if } p_i < p_j^i < 550 \\ 202500 \left(1 - \frac{400}{1000 - p_i}\right) & \text{if } p_i \leq 550 \leq p_j^i \\ (1000 - p_i) \left(1 - \frac{400}{1000 - p_i}\right) (p_i - 100) & \text{if } 550 < p_i < 600 \text{ and } p_i < p_j^i \\ 0 & \text{if } 600 \leq p_i < p_j^i \end{cases}$$

The Edgeworth price for firm  $i$ ,  $p_i^E$ , is the highest price that firm  $i$  can set such that firm  $j$  is indifferent between undercutting firm  $i$  or acting on the residual demand (see Fig. 1). If  $p_j^i \geq 550$ , the Edgeworth price for firm  $i$  is obtained from  $(p_i^E - 100)400 = 202500 \left(1 - \frac{400}{1000 - p_i^E}\right)$ , that is,  $\bar{p}_i^E = 311.9$ . If  $p_j^i < 550$ , the Edgeworth price solves  $(p_i^E - 100)400 = (1000 - p_j^i) \left(1 - \frac{400}{1000 - p_i^E}\right) (p_j^i - 100)$ , that is,  $p_i^E = 425 + \frac{(1100 - p_j^i)p_j^i - \sqrt{1476 \cdot 10^8 - (400 - p_j^i)(700 - p_j^i)(1100 - p_j^i)p_j^i}}{800}$  (see Fig. 2).

Now, it is easy to show that for  $p_i^i$  and  $p_j^j$  sufficiently high (the classic Bertrand-Edgeworth game), price strategies such as  $p_i^i = p_j^j = p^C = 200$  are not a Nash Equilibrium. Note that  $p_i^E = p_j^E > 200$  so both firms has an incentive to increase their prices, acting on the residual demand (Corollary 2).

By Theorem 5, we know that  $p_i^i = p_j^j = p^E = 311.9$  and  $p_j^j \geq p_i^i = 550$  is a subgame perfect equilibrium of the list pricing game. Note that firm  $j$  has no incentive to undercut firm  $i$ , or to vary its price from 550. Firm  $i$ , would like to increase its discounted price, but it is limited by their own list price. Both firms obtain the same profit.

#### 4. List pricing and price leadership

Price leadership has been studied in the literature with endogenous determination of the timing of the moves, i.e., whether a firm prefers to act as a leader, or as a follower. In these models, a price, once set, cannot be changed regardless of how the rival responds. Even though, ex-post it would be in the leader's interest to change its price none of the papers explain the strong nature of this commitment. In this section we argue that list pricing may provide such a credible commitment mechanism in which price outcomes emerge that are similar to price leadership.

Hamilton and Slutsky (1990) propose a two stage framework to endogenize the timing of a duopoly game where each firm

chooses a strategy (which could be a price or quantity). They analyze two types of games, one with observed delay and the other with (action) commitment. In the former game firms announce at what time they will choose their action and commit to their decision. In the latter game firms commit to an action in one of the two periods before the market clears. Firms may choose their strategy in period 1, or wait till period 2. If a firm chooses a strategy in the first period and the other firm waits, it is informed of the strategy chosen by its rival.

The action commitment game of Hamilton and Slutsky has three subgame perfect equilibria. In one of them both firms commit in the first period to the simultaneous-move Cournot-Nash equilibrium quantities. In the other two, each firm waits and the other plays its Stackelberg leader quantity in the first period. They also show that only the Stackelberg equilibria survive elimination of weakly dominated strategies.<sup>18</sup>

We will now show that the final equilibrium (discounted) prices of our list pricing game are a sub-game perfect equilibrium in the endogenous timing framework (i.e. the action commitment game of Hamilton and Slutsky). In order to obtain our equivalence result it suffices to prove that in the Bertrand-Edgeworth game that we analyze the mixed strategy equilibrium is indeed dominated by a sequential game where the high Edgeworth price firm moves first.

**Theorem 6.** Given list prices for which  $p_i^E \geq p_j^E$  holds. Then, when no pure strategy equilibrium of the Bertrand-Edgeworth game exists, firm  $i$  has an incentive to move first in the discounting stage.

**Proof.** Suppose that for given list prices for which  $p_i^E \geq p_j^E$ , by Theorem 2 and Corollary 1 in any mixed strategy equilibrium of the discounting game (including the case where list prices are set arbitrarily high) the expected payoff of firm  $i$  is given by  $\min(D(\underline{p}^d), k_i)(\underline{p}^d - c_i)$ , for some  $\underline{p}^d < p_i^E$ .

Contrarily, if firm  $i$  moves first and sets price  $p_i^E$ , then firm  $j$  will set  $p_j^M(p_i^E, k_j)$ . Firm  $i$  then obtains profits of  $\min(D(p_i^E), k_i)(p_i^E - c_i)$ .

We then have that the subgame perfect equilibrium involves one firm moving first. We will now prove that if firm  $i$  moves first then it will set its Edgeworth price.

**Theorem 7.** In any equilibrium of the subgame in which firm  $i$  moves first it sets a price of  $p_i^E$  when no pure strategy equilibrium of the Bertrand-Edgeworth game exists.

**Proof.** Suppose firm  $i$  moves first. Then, if it sets a price above  $p_i^E$  it will be undercut by firm  $j$  in the second stage and its profits will be bounded by  $\hat{\pi}_i = (D(\hat{p}) - k_j)(\hat{p} - c_i)$ . Where,

$$\hat{p} = \arg \max_p (D(p) - k_j)(p - c_i)$$

By Theorem 2, if firm  $i$  deviates to simultaneous play in the second stage, it will obtain expected profits of  $\hat{\pi}_i^* = \min(D(\underline{p}^d), k_i)(\underline{p}^d - c_i)$ . We will now prove that  $\hat{\pi}_i < \hat{\pi}_i^*$ . Let  $\hat{\pi}_i \geq \hat{\pi}_i^*$ . Then, it must be the case that  $\hat{p} > \underline{p}^d$ . Suppose that, firm  $i$  deviates from its mixed strategy and sets a price of  $\hat{p}$ . Given that the equilibrium is nondegenerate,  $\mu_j(\underline{p}^d, \hat{p}) > 0$ . The payoffs of firm  $i$  from this deviation will be bounded below by  $R(\hat{p}, p', k_j)(\hat{p} - c_i)$  where,  $p' \in [\underline{p}^d, \hat{p}]$ . By properties 4 and 5 of the residual demand function  $R(\hat{p}, p', k_j)(\hat{p} - c_i) > (D(\hat{p}) - k_j)(\hat{p} - c_i)$ .

Thus,  $\hat{\pi}_i < \hat{\pi}_i^*$ . From this we can conclude that if firm  $i$  moves first it will choose a price less than or equal to  $p_i^E$ . Given that, for firm  $i$  any price below  $p_i^E$  is dominated by  $p_i^E$ , we obtain the desired result.

Finally, it is straightforward to see that if  $p_j^E > p_i^E$ , and there exists a subgame perfect equilibrium where firm  $j$  leads, then it is dominated

<sup>18</sup> In another paper van Damme and Hurkens (1996) show that playing simultaneously is subgame perfect in the Hamilton-Slutsky timing game only if none of the players has an incentive to move first.



for both firms by the equilibrium where firm  $i$  leads. We have proved that in our model the sequential-timing and list-pricing solutions to the nonexistence of a pure strategy equilibrium are equivalent. The difference is that while in the sequential-timing models firms are not allowed to change their price (once it is chosen), in our list pricing approach firms can discount. Our result is obtained under a weaker assumption that reflects a pricing institution that is widely prevalent.

## 5. Conclusion

We analyze the widely used pricing institution of list pricing and discounting. Under the list pricing institution firms post a list price in the first stage of the game and can offer a discount on this price in the second stage. List prices are assumed to be price commitments for some duration of time. Our pricing structure is motivated by the real world situation where firms first announce list prices (valid for a given period of time) which they can later discount. The price catalogue of Ikea stores is an example along these lines. Ikea announces the list price of its goods which are valid for a certain duration of time. Ikea, however, is free to offer discounts on these list prices at a later stage. Department stores, retailers and wholesalers regularly inform consumers about prices through catalogues.

We model a two stage game where firms first, simultaneously, post list prices and then decide whether they want to discount in the second stage. We show that any subgame perfect equilibrium of the list pricing game involves firms playing pure strategies in the discounting stage. In this equilibrium, a firm sets a list price equal to its discounting price, such that it will not be undercut by its rival. The other firm, meanwhile, acts on the residual demand. We characterize some of the properties of a non-degenerate mixed strategy equilibrium in the discounting stage. As the Bertrand–Edgeworth game can be seen as a special case of the list pricing game (for arbitrarily high list prices), these results also apply to the one stage model. Further, we define a generalized residual demand function for which our results hold. The proportional and efficient rationing rules are special cases of this general specification.

We also show that at least one of these equilibrium exists. In this equilibrium, one firm commits to a low price, thereby signalling to its rival that it can act as a monopolist on the residual demand. Our result suggests that the traditional one-stage pricing Bertrand–Edgeworth models may overstate the competitiveness of an oligopolistic industry (Deneckere and Kovenock, 1992 make a similar point). Credible commitment to a price by a firm can enforce a pure strategy outcome.

Our results may also have some relevance in explaining persistent price dispersion. There exists a large empirical literature that supports persistent price dispersion in market selling homogenous goods.<sup>19</sup> We obtain price dispersion for a duopoly both under symmetric, and asymmetric, costs. The subgame perfect equilibrium of the list pricing game is always in pure strategies in the discounting stage. It would be interesting to see if the results of our model extend to  $n$  firms with asymmetric costs.

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<sup>19</sup> Empirical evidence points out that price dispersion is persistent. See among others, Clay, Krishnan and Wolff (2001), Clay, Krishnan, Wolff and Fernandes (2002), Baye and Morgan (2004), Baye, Morgan and Scholten (2004) and Clay, Smith and Wolff (2004).

## Appendix A

The following two Lemmas will prove useful in order to prove Theorem 2.

**Lemma 1.** *In a mixed strategy equilibrium to the discounting subgame, if  $\mu_i([\hat{p}, \hat{p} + \varepsilon]) > 0$  for any  $\varepsilon > 0$  and  $\pi_i(p)$  is right continuous at  $p = \hat{p}$ , then the expected profit of the equilibrium is given by  $\pi_i(\hat{p})$ .*

**Proof.** If  $\mu_i(\hat{p}) > 0$  the proof is trivial. Now, consider that  $\mu_i(\hat{p}) = 0$  and suppose  $\pi_i(\hat{p}) - \pi_i^* = C$ , where  $C < 0$ , and  $\pi_i^*$  is the expected payoff of the mixed strategy equilibrium. Since  $\mu_i([\hat{p}, \hat{p} + \varepsilon]) > 0$  for any  $\varepsilon > 0$ , it must be the case that  $\pi_i(\hat{p}) = \pi_i^*$  for some  $p \in (\hat{p}, \hat{p} + \varepsilon)$ .

Contrarily, by right continuity of  $\pi_i(p)$  at  $p = \hat{p}$ , we have that for any  $\delta > 0$ , there exists an  $\varepsilon > 0$ , s.t.  $0 < p - \hat{p} < \varepsilon$ , implies  $|\pi_i(\hat{p}) - \pi_i(p)| < \delta$ . Take  $\delta = \frac{C}{2}$ , and we reach a contradiction with  $\pi_i(\hat{p}) - \pi_i^* = C$ .

**Lemma 2.** *In a mixed strategy equilibrium to the discounting subgame, if firm  $i$  has positive measure at a price  $\hat{p}, p^c < \hat{p} \leq p_i^c$  then*

- If  $c_j > c_i$  [or  $c_j = c_i$  and  $j = 2$ ] then  $\mu_j([\hat{p}, \hat{p} + \varepsilon]) = 0$  for small enough  $\varepsilon > 0$ .
- If  $c_j < c_i$  [or  $c_j = c_i$  and  $j = 1$ ] then  $\mu_j([\hat{p}, \hat{p} + \varepsilon]) = 0$  for small enough  $\varepsilon > 0$ .

**Proof.** For any  $p < \hat{p}$  we may write the expected profits expression as

$$\begin{aligned} \pi_j(p) &= \left( \mu_i([\hat{p}, \hat{p}]) + \mu_i([\hat{p}, p^0]) \right) \min(k_j, D(p)) (p - c_j) \\ &\quad + \int_{[c_i, \hat{p}] - [p, \hat{p}]} R(p, z, k_i) (p - c_j) d\mu_i(z) \\ &\quad + \mu_i(p) \min(k_j, \max(0, D(p) - I_j k_i)) (p - c_j) \end{aligned}$$

Taking limits from the left

$$\begin{aligned} \lim_{p \rightarrow \hat{p}^-} \pi_j(p) &= \mu_i([\hat{p}, p^0]) \min(k_j, D(\hat{p})) (\hat{p} - c_j) \\ &\quad + \int_{[c_i, \hat{p}]} R(\hat{p}, z, k_i) (\hat{p} - c_j) d\mu_i(z) \end{aligned}$$

Now, consider the expected profits when  $p > \hat{p}$

$$\begin{aligned} \pi_j(p) &= \left( \mu_i([\hat{p}, p^0]) - \mu_i([\hat{p}, p^0]) \right) \min(k_j, D(p)) (p - c_j) \\ &\quad + \int_{[c_i, \hat{p}] \cup (\hat{p}, p)} R(p, z, k_i) (p - c_j) d\mu_i(z) \\ &\quad + \mu_i(p) \min(k_j, \max(0, D(p) - I_j k_i)) (p - c_j) \end{aligned}$$

Taking limits from the right

$$\begin{aligned} \lim_{p \rightarrow \hat{p}^+} \pi_j(p) &= \mu_i([\hat{p}, p^0]) \min(k_j, D(\hat{p})) (\hat{p} - c_j) \\ &\quad + \int_{[c_i, \hat{p}]} R(\hat{p}, z, k_i) (\hat{p} - c_j) d\mu_i(z) \end{aligned}$$

We also know that

$$\begin{aligned} \pi_j(\hat{p}) &= \mu_i([\hat{p}, p^0]) \min(k_j, D(\hat{p})) (\hat{p} - c_j) \\ &\quad + \int_{[c_i, \hat{p}]} R(\hat{p}, z, k_i) (\hat{p} - c_j) d\mu_i(z) \\ &\quad + \mu_i(\hat{p}) \min(k_j, \max(0, D(\hat{p}) - I_j k_i)) (\hat{p} - c_j) \end{aligned}$$



First, we prove that  $\mu_j(\hat{p}, \hat{p} + \varepsilon) = 0$  for both firms. We have

$$\pi_j(\hat{p}) - \lim_{p \rightarrow \hat{p}^+} \pi_j(p) = \mu_i(\hat{p}) \left( \min(k_j, \max(0, D(\hat{p}) - I_j k_i)) - R(\hat{p}, \hat{p}, k_i) \right) (\hat{p} - c_j)$$

Note that since  $\hat{p} > p^c$ , and by property (4) of the residual demand function,  $R(\hat{p}, \hat{p}, k_i) = \max(0, D(\hat{p}) - k_i) < k_j$ .

If  $c_j < c_i$  [or  $c_j = c_i$  and  $j = 1$ ],  $I_j = 0$ , then  $\pi_j(\hat{p}) > \lim_{p \rightarrow \hat{p}^+} \pi_j(p)$ . Then  $\varepsilon > 0$ , such that  $\pi_j(p) < \pi_j(\hat{p})$  for  $p \in (\hat{p}, \hat{p} + \varepsilon]$ . This implies  $\mu_j(\hat{p}, \hat{p} + \varepsilon) = 0$ .

If  $c_j > c_i$  [or  $c_j = c_i$  and  $j = 2$ ],  $I_j = 1$ , then it can be shown that  $\pi_j(\hat{p}) = \lim_{p \rightarrow \hat{p}^+} \pi_j(p)$ . Further, we know that  $\lim_{p \rightarrow \hat{p}^+} \pi_j(p) - \lim_{p \rightarrow \hat{p}^-} \pi_j(p) < 0$ . Then for  $\varepsilon > 0$ , sufficiently small,  $\delta > 0$ , such that  $\pi_j(p) < \pi_j(\hat{p} - \delta)$  for any  $p \in (\hat{p}, \hat{p} + \varepsilon]$ . This implies  $\mu_j(\hat{p}, \hat{p} + \varepsilon) = 0$ .

Second, we have

$$\pi_j(\hat{p}) - \lim_{p \rightarrow \hat{p}^-} \pi_j(p) = \mu_i(\hat{p}) \left( \min(k_j, \max(0, D(\hat{p}) - I_j k_i)) - \min(k_j, D(\hat{p})) \right) (\hat{p} - c_j)$$

For  $c_j > c_i$  [or  $c_j = c_i$  and  $j = 2$ ],  $I_j = 1$ . Since  $\hat{p} > p^c$ , then  $D(\hat{p}) - k_i < \min(k_j, D(\hat{p}))$ . That is,  $\pi_j(\hat{p}) < \lim_{p \rightarrow \hat{p}^-} \pi_j(p)$ , which implies that there exists a  $p < \hat{p}$  that gives firm  $j$  higher expected profits than  $\hat{p}$ , thus  $\mu_j(\hat{p}) = 0$ .

**Proof.** [Proof of Theorem 2-i] Suppose firms have different lower bounds for their support, thus  $\underline{p}_i^d < \underline{p}_j^d$ .

i) Consider the case  $\underline{p}_j^d \leq \underline{p}_i^M$ , we then have that, for firm  $i$ , a pure strategy  $\lambda \underline{p}_i^d = (1 - \lambda) \underline{p}_j^d$ , for any  $0 < \lambda < 1$ , dominates any strategy in  $[\underline{p}_i^d, \lambda \underline{p}_i^d + (1 - \lambda) \underline{p}_j^d]$ . On the other hand, by definition of support, we have that  $\mu_i([\underline{p}_i^d, \underline{p}_i^d + \varepsilon]) > 0$  for any  $\varepsilon > 0$ . This implies that  $\underline{p}_i^d = \underline{p}_i^l$  and  $\mu_i(\underline{p}_i^l) = 1$ . It must then be the case that

$$\min(k_j, D(\underline{p}_i^l)) (\underline{p}_i^l - c_j) \leq \max_{p' \in [\underline{p}_i^l, \underline{p}_j^l]} R(p', \underline{p}_i^l, k_i) (p' - c_j)$$

and thus firm  $j$  maximizes its profits by playing  $\arg \max_{p' \in [\underline{p}_i^l, \underline{p}_j^l]} R(p', \underline{p}_i^l, k_i) (p' - c_j)$ .

This then contradicts the assumption that firms are playing nondegenerate mixed strategies in equilibrium.

ii) Now, suppose  $\underline{p}_j^d > \underline{p}_i^M$  then  $\underline{p}_i^M$  dominates any other price for firm  $i$ . Thus, the best response of firm  $i$  involves playing a pure strategy  $\underline{p}_i^d = \min\{\underline{p}_i^M, \underline{p}_i^l\}$ . If  $\underline{p}_i^d = \underline{p}_i^l$ , then by the previous argument, firm  $j$  must be playing the pure strategy  $\arg \max_{p' \in [\underline{p}_i^l, \underline{p}_j^l]} R(p', \underline{p}_i^l, k_i) (p' - c_j)$ .

Contrarily, if  $\underline{p}_i^d = \underline{p}_i^M$ , firm  $j$  faces a residual demand of zero, and the best response of firm  $j$  must involve undercutting  $\underline{p}_i^d$ , which contradicts  $\underline{p}_i^d < \underline{p}_j^d$ .

**Proof.** [Proof of Theorem 2-ii] From Lemma 2, if  $c_j > c_i$  [or  $c_j = c_i$  and  $j = 2$ ] and  $\mu_i(\underline{p}^d) > 0$ , then  $\mu_j([\underline{p}_j^d, \underline{p}^d + \varepsilon]) = 0$ , for small enough  $\varepsilon > 0$ . Then we have  $\underline{p}_j^d > \underline{p}^d$ , which contradicts statement i). Therefore,  $\mu_i(\underline{p}^d) = 0$ .

Besides, from Lemma 2, if  $c_j < c_i$  [or  $c_j = c_i$  and  $j = 1$ ] and  $\mu_i(\underline{p}^d) > 0$ , then  $\mu_j([\underline{p}_j^d, \underline{p}^d + \varepsilon]) = 0$ , for small enough  $\varepsilon > 0$ . But we know from before that  $\mu_j(\underline{p}^d) = 0$ , then  $\mu_j([\underline{p}_j^d, \underline{p}^d + \varepsilon]) = 0$ , which contradicts statement i). Therefore,  $\mu_i(\underline{p}^d) = 0$ .

Finally,  $\pi_i(p)$  is continuous in  $\underline{p}^d$ . Applying Lemma 1 we obtain the desired result.

**Proof.** [Proof of Theorem 2-iii] Define  $\underline{p} = \max\{\underline{p}_1^d, \underline{p}_2^d\}$ . Suppose that  $\mu_i(\underline{p}) > 0$  for some  $i$ . Then, if  $c_i < c_j$  [or  $c_i = c_j$  and  $i = 1$ ], from Lemma 2,  $\mu_j(\underline{p}) = 0$ , and thus  $h = i$ . Contrarily, if  $c_i > c_j$  [or  $c_i = c_j$  and  $i = 2$ ], and  $\mu_j(\underline{p}) = 0$ , then  $h = i$ . But if  $\mu_j(\underline{p}) > 0$ , leads to a contradiction (Lemma 2).

Now, suppose  $\mu_i(\underline{p}) = 0$  for both  $i \in \{1, 2\}$ . Then  $\pi_i(p)$  is continuous in  $\underline{p}$ . It must then be the case that  $\underline{p} = \underline{p}_i^d$  for some  $i$ , thus  $h = i$ .

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