Comment

Comment on “Bertrand and Walras Equilibria under Moral Hazard”

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In a fundamental contribution, Prescott and Townsend (1984) have shown that the existence and efficiency properties of Walrasian equilibria extend to economies with moral hazard and exclusive contracts. Recently, in this *Journal*, Bennardo and Chiappori (2003) have argued that Walrasian equilibria may (robustly) fail to exist when the class of moral hazard economies in Prescott and Townsend's work is generalized to allow for aggregate, in addition to idiosyncratic, uncertainty, if preferences are nonseparable in consumption and effort. In this comment, we show that such a claim is incorrect and that the existence and efficiency properties of Walrasian equilibria remain valid in the setup considered by Bennardo and Chiappori.

We briefly describe the moral hazard economy considered by Bennardo and Chiappori (2003). There is a continuum of ex ante identical individuals with measure one and a single consumption good. Individuals are affected by both an aggregate and an idiosyncratic endowment shock. Specifically, there are two aggregate states, $s = 1, 2$, and two idiosyncratic states, $\sigma = a, b$. The individual's endowment y_s^σ is higher

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in aggregate state $s = 1$ than in $s = 2$ for each realization of the idiosyncratic state σ: $y_1^\sigma > y_2^\sigma$. Similarly, y_s^σ is higher in idiosyncratic state $\sigma = a$, no matter what the aggregate state s is: $y_s^a > y_s^b$. Idiosyncratic shocks are independently and identically distributed across individuals and are independent of the aggregate shock. The probability of each aggregate state s is exogenous and is denoted by λ and $1 - \lambda$ for $s = 1$ and 2, respectively. On the other hand, the probability of idiosyncratic state σ depends on an effort e supplied by the individual prior to the realization of uncertainty (both aggregate and idiosyncratic). Effort can be high or low; the set of effort levels is $E \equiv \{e_l, e_h\}$. Higher effort raises the probability of the high-endowment idiosyncratic state. Let the probability of state $\sigma = a$ be $P(e_h) = P$ when effort is high and $P(e_l) = p < P$ when effort is low. While the realization of uncertainty is publicly observable, an individual's effort is not.

Individuals have von Neumann–Morgenstern preferences described by the (state-independent) Bernoulli utility function $u: \Re_+ \times E \rightarrow \Re$. For each e, $u(\cdot, e)$ is twice continuously differentiable, strictly increasing, and strictly concave with $\lim_{c\to 0} \partial u(c, e)/\partial c = \infty$. Effort is costly, so $u(c, e_l) > u(c, e_h)$ for all $c \in \Re_+$.

To establish our result, we first need to briefly lay out the structure of markets.

Commodities.—The commodities traded are *insurance contracts*. An insurance contract specifies an effort level and a bundle of state-contingent net trades. This specification is allowed to be random. As in Prescott and Townsend (1984), the set of possible consumption levels in any state is assumed to be a finite set $C \subset \Re_+$ with n elements, and maximal element $\bar{c} \gg \max_{s,\sigma} y_s^\sigma$. When aggregate state s is realized, the set of possible net trades (contingent on the two idiosyncratic states) is then $Z_s = C^2 - \{(y_s^a, y_s^b)\}$. An insurance contract is described as a pair $\boldsymbol{x} = (\boldsymbol{x}_1, \boldsymbol{x}_2)$, where $\boldsymbol{x}_s$ is a probability measure on the finite set $Z_s \times E$, given by the vector $\{P_s(z_a, z_b, e)\}_{(z_a, z_b, e) \in Z_s \times E} \in \Re_+^{2n^2}$ satisfying

$$\sum_{(z_a, z_b, e) \in Z_s \times E} P_s(z_a, z_b, e) = 1 \quad \text{for each } s = 1, 2. \tag{1}$$

The terms $P_s(z_a, z_b, e)$ are probability weights on triples (z_a, z_b, e) of net trades in the two idiosyncratic states and effort in aggregate state s. The commodities traded by consumers are then elements $\boldsymbol{x}$ of $\Re_+^{4n^2}$, and $L \equiv \Re^{4n^2}$ denotes the commodity space.[1]

The interpretation of $\boldsymbol{x}$ is as follows. First, a lottery prescribes an effort

[1] An equivalent (though slightly more involved) analysis can be carried out when C is an infinite set (e.g., $C = \Re_+$), and the measure space $M(Z_s \times E)$ is then endowed with the weak-star topology (see, e.g., Jerez 2005). Our results extend to that case as well as to the case in which there is an arbitrary number of consumption goods and states (see also Rustichini and Siconolfi 2003).

level e for the individual. This lottery is given by the marginal of $\boldsymbol{x}_s$ with respect to e. Remember that effort is chosen prior to the realization of s, so this marginal must be independent of s, as stated in the following condition:

$$x_e \equiv \sum_{(z_a,z_b)\in Z_1} P_1(z_a,\ z_b,\ e) = \sum_{(z_a,z_b)\in Z_2} P_2(z_a,\ z_b,\ e). \tag{2}$$

Thus effort e is prescribed with probability x_e. Conditional on e, a second lottery specifies the individual's net trades in the two idiosyncratic states $(z_a,\ z_b)$, for every aggregate state s. This lottery is described by the probability distribution over $(z_a,\ z_b)$, conditional on e, implied by $\boldsymbol{x}_s$. Since effort is private information, the effort specification has to be understood as a prescription, which to be effective must satisfy appropriate incentive constraints (see below).

Conditional on the realization of aggregate state s, the expected utility of an individual who exerts effort e and realizes net trades $(z_a,\ z_b)$ is

$$v_s(z_a,\ z_b,\ e) \equiv P(e)u(y_s^a + z_a,\ e) + [1 - P(e)]u(y_s^b + z_b,\ e).$$

The expected utility from a contract $\boldsymbol{x}$ is then

$$\begin{aligned}\lambda(\boldsymbol{v}_1 \cdot \boldsymbol{x}_1) + (1-\lambda)(\boldsymbol{v}_2 \cdot \boldsymbol{x}_2) = \lambda &\sum_{(z_a,z_b,e)\in Z_1\times E} v_1(z_a,z_b,e)P_1(z_a,z_b,e)\\ &+ (1-\lambda)\sum_{(z_a,z_b,e)\in Z_2\times E} v_2(z_a,z_b,e)P_2(z_a,z_b,e).\end{aligned}$$

The incentive compatibility constraints require that, whenever $\boldsymbol{x}$ prescribes effort e, individuals prefer e rather than deviating to e'. It is immediate to verify that the incentive compatibility constraints can be equivalently written as follows:

$$\begin{aligned}&\lambda \sum_{(z_a,z_b)\in Z_1} v_1(z_a,z_b,e)P_1(z_a,z_b,e) + (1-\lambda)\sum_{(z_a,z_b)\in Z_2} v_2(z_a,z_b,e)P_2(z_a,z_b,e) \geq\\ &\lambda \sum_{(z_a,z_b)\in Z_1} v_1(z_a,z_b,e')P_1(z_a,z_b,e) + (1-\lambda)\sum_{(z_a,z_b)\in Z_2} v_2(z_a,z_b,e')P_2(z_a,z_b,e)\end{aligned} \tag{3}$$

for all $e,\ e' \in E$.

Admissible trades.—Since trades are assumed to be observable, any restriction on trades can be imposed. Following Prescott and Townsend (1984), the set of contracts $\bar{X}$ available for trade to any individual (with some abuse of language, her consumption set) is the set of incentive-compatible contracts, that is, the set of vectors $\boldsymbol{x} = (\boldsymbol{x}_1,\ \boldsymbol{x}_2) \in L_+$ satisfying (1), (2), and (3).

Feasible allocations.—We will look at symmetric allocations in which all individuals trade the same contract $\boldsymbol{x}$. By the law of large numbers, when all individuals exert effort e, a fraction $P(e)$ of them will end up in idiosyncratic state a; hence the total (per capita) use of resources in state s when the individuals' net trade is (z_a, z_b) is

$$r_s(z_a, z_b, e) \equiv P(e)z_a + [1 - P(e)]z_b.$$

A contract $\boldsymbol{x}$ satisfies the economy's resource constraints if the total net use of resources of such a contract is nonpositive in both aggregate states:

$$\boldsymbol{r}_s \cdot \boldsymbol{x}_s = \sum_{(z_a, z_b, e) \in Z_s \times E} r_s(z_a, z_b, e)P_s(z_a, z_b, e) \leq 0, \quad s = 1, 2. \tag{4}$$

Incentive efficient allocations.—A (symmetric) allocation $\boldsymbol{x}$ is *incentive efficient* if it maximizes the individual expected utility in the set of feasible allocations:

$$\max_{\boldsymbol{x} \in \bar{X}} \quad \lambda(\boldsymbol{v}_1 \cdot \boldsymbol{x}_1) + (1 - \lambda)(\boldsymbol{v}_2 \cdot \boldsymbol{x}_2)$$

$$\text{subject to } \boldsymbol{r}_s \cdot \boldsymbol{x}_s \leq 0, \quad s = 1, 2. \tag{5}$$

Problem (5) is a standard (finite-dimensional) linear program, with a nonempty feasible set.[2] Hence, an optimal solution exists. Note that the feasible set is convex (i.e., if contracts $\boldsymbol{x}$ and $\boldsymbol{x}'$ satisfy the incentive compatibility and resource constraints, so does any convex combination of these contracts).

Prices.—Prices are linear on the individuals' consumption set, that is, are linear in the probabilities. A price system is an element $\boldsymbol{\pi} = (\boldsymbol{\pi}_1, \boldsymbol{\pi}_2) \in L$, where $\boldsymbol{\pi}_s = \{\pi_s(z_a, z_b, e)\}_{(z_a, z_b, e)} \in \Re^{2n^2}$. The cost of a commodity bundle $\boldsymbol{x} \in L$ is then

$$\boldsymbol{\pi} \cdot \boldsymbol{x} = \boldsymbol{\pi}_1 \cdot \boldsymbol{x}_1 + \boldsymbol{\pi}_2 \cdot \boldsymbol{x}_2 = \sum_{s=1,2} \sum_{(z_a, z_b, e) \in Z_s \times E} \pi_s(z_a, z_b, e)P_s(z_a, z_b, e). \tag{6}$$

Firm intermediaries.—Following Prescott and Townsend (1984), as well as Bennardo and Chiappori (2003), we introduce firms with technology:

$$\mathcal{Y} = \{\boldsymbol{\nu} = (\boldsymbol{\nu}_1, \boldsymbol{\nu}_2) \in L: \ \boldsymbol{r}_s \cdot \boldsymbol{\nu}_s \leq 0, \ s = 1, 2\}. \tag{7}$$

Firms can offer any set of contracts as long as the total net payments required by the contracts offered are self-financing. The law of large numbers allows us to write the self-financing constraint in expected terms in each aggregate state. Since $\mathcal{Y}$ displays constant returns to scale, profits are zero in equilibrium and there is no loss of generality in assuming that there is a single firm.

[2] For example, the allocation in which individuals exert e_l with probability one and consume the expected value of their endowment in each aggregate state s is feasible.

Definition. A competitive equilibrium is a triple $(\boldsymbol{x}^*, \boldsymbol{\nu}^*; \boldsymbol{\pi}^*) \in L^3$ such that (i) $\boldsymbol{x}^*$ maximizes $\lambda(\boldsymbol{v}_1 \cdot \boldsymbol{x}_1) + (1 - \lambda)(\boldsymbol{v}_2 \cdot \boldsymbol{x}_2)$ over the set $\{\boldsymbol{x} \in \bar{X} : \boldsymbol{\pi}^* \cdot \boldsymbol{x} \leq 0\}$, (ii) $\boldsymbol{\nu}^*$ maximizes $\boldsymbol{\pi}^* \cdot \boldsymbol{\nu}$ over the set $\mathcal{Y}$, and (iii) markets clear, or $\boldsymbol{x}^* = \boldsymbol{\nu}^*$.

Condition i requires $\boldsymbol{x}^*$ to yield the highest utility to individuals among all admissible and budget-feasible contracts, and condition ii says that $\boldsymbol{\nu}^*$ is the profit-maximizing choice of the firm. Finally, condition iii says that aggregate demand for contracts by individuals equals supply by firms.

We now show that a competitive equilibrium always exists (in contrast to proposition 5 of Bennardo and Chiappori [2003]).

Theorem 1. A competitive equilibrium exists. In particular, any (symmetric) incentive-efficient allocation can be supported as a competitive equilibrium.

Proof. It is immediate to verify (see also lemma 3 in Bennardo and Chiappori [2003]) that, because $\mathcal{Y}$ displays constant returns to scale, equilibrium prices are such that, for each $s = 1, 2$,

$$\pi_s(z_a, z_b, e) = \beta_s r_s(z_a, z_b, e), \tag{8}$$

for some $\beta_s \geq 0$; that is, in each state s the price of net trades (z_a, z_b) with effort e must be either actuarially fair (proportional to the expected use of resources) or zero.

The rest of the proof relies on a constructive argument: for any solution $\boldsymbol{x}^E$ of the planner's problem (5), we find prices satisfying (8) that support $\boldsymbol{x}^E$ as a competitive equilibrium.

We first prove that at $\boldsymbol{x}^E$ the resource constraint must bind at least in one state s. Suppose that both constraints were slack. Let $\boldsymbol{x}^l$ be a deterministic contract specifying low effort and maximal consumption $\bar{c}$ with probability one (regardless of the realization of s and σ). Contract $\boldsymbol{x}^l$ is incentive compatible ($\boldsymbol{x}^l \in \bar{X}$) and strictly preferred to $\boldsymbol{x}^E$ by the individual. Hence, so is any convex combination of $\boldsymbol{x}^E$ and $\boldsymbol{x}^l$: $\boldsymbol{x}^\alpha = (1 - \alpha)\boldsymbol{x}^E + \alpha\boldsymbol{x}^l$ with $\alpha \in (0, 1]$. For α sufficiently small, $\boldsymbol{x}^\alpha$ also satisfies the resource constraints (4), so $\boldsymbol{x}^E$ cannot be a solution to (5).[3]

Consider then the case in which at $\boldsymbol{x}^E$ the resource constraint does not bind in one state, say $s = 1$. This is the case analyzed in proposition

[3] In general, any convex combination of a feasible contract $\boldsymbol{x}$ and $\boldsymbol{x}^l$ is strictly preferred to $\boldsymbol{x}$, so there is local nonsatiation within the set of feasible allocations. The reason is that deterministic contract $\boldsymbol{x}^l$, which gives maximal utility in the consumption set $\bar{X}$, is not feasible (i.e., $\bar{c} \gg \max_{s,\sigma} y_s^\sigma$).

5 of Bennardo and Chiappori (2003).[4] We claim that, when $\beta_1 = 0$ and $\beta_2 = 1$, the prices in (8) support $\boldsymbol{x}^E$ as a competitive equilibrium. Supporting prices must reflect the shadow cost of resources in states 1 and 2. This cost is given by the shadow price of the resource constraint in each state multiplied by the expected use of resources in that state. Because the resource constraint in $s = 1$ is slack at $\boldsymbol{x}^E$, its shadow price is zero. On the other hand, the constraint binds in $s = 2$, so its shadow price is positive and can be normalized to one. In sum, *prices are zero in* $s = 1$ *and actuarially fair in* $s = 2$, *and so the price of a contract is the expected use of resources in* $s = 2$.

When consumers face the prices in (8) with $\beta_1 = 0$ and $\beta_2 = 1$, their problem becomes

$$\max_{\boldsymbol{x}\in \bar{X}} \quad \lambda(\boldsymbol{v}_1 \cdot \boldsymbol{x}_1) + (1-\lambda)(\boldsymbol{v}_2 \cdot \boldsymbol{x}_2)$$

$$\text{subject to } \boldsymbol{\pi}^* \cdot \boldsymbol{x} = \boldsymbol{r}_2 \cdot \boldsymbol{x}_2 \leq 0. \tag{9}$$

It is then immediate to see that $\boldsymbol{x}^E$ is a solution to this problem. Since the resource constraint in $s = 1$ does not bind in the planner's problem (5), $\boldsymbol{x}^E$ is a local maximum of (9). Furthermore, the fact that the objective function is linear and the feasible set is convex in (9) implies that $\boldsymbol{x}^E$ is also a global maximum by the local-global theorem (Intriligator 1971, 75).

When the firm faces the prices in (8) with $\beta_1 = 0$ and $\beta_2 = 1$, profits are $\boldsymbol{\pi}^* \cdot \boldsymbol{\nu} = \boldsymbol{r}_2 \cdot \boldsymbol{\nu}_2$. So (7) implies $\boldsymbol{\pi}^* \cdot \boldsymbol{\nu} \leq 0$ for all $\boldsymbol{\nu} \in \mathcal{Y}$. Clearly, $\boldsymbol{x}^E \in \mathcal{Y}$, because $\boldsymbol{x}^E$ satisfies the resource constraints (4). Moreover, since at $\boldsymbol{x}^E$ (4) binds in $s = 2$, $\boldsymbol{\pi}^* \cdot \boldsymbol{x}^E = \boldsymbol{r}_2 \cdot \boldsymbol{x}_2^E = 0$. Thus $\boldsymbol{\nu}^* = \boldsymbol{x}^E$ is a profit-maximizing choice for the firm, and at this choice markets clear (condition iii of the definition of a competitive equilibrium holds). This proves our claim.

The argument is similar when both resource constraints bind at $\boldsymbol{x}^E$. Set β_1 and β_2 in (8) equal to the shadow prices β_1^E and β_2^E (both of which are now positive) of the resource constraints (4) at the solution $\boldsymbol{x}^E$ of (5). Again, $\boldsymbol{x}^E$ solves the consumer's problem (9) for this price system: since $\boldsymbol{x}^E$, β_1^E, and β_2^E satisfy the first-order conditions of (5), $\boldsymbol{x}^E$ solves the first-order conditions of (9) for the price system specified above when the shadow price of the budget constraint equals one (the Lagrangean

[4] Bennardo and Chiappori derive sufficient conditions for $\boldsymbol{x}^E$ to have this property (proposition 4) and show that there is an open set of economies that satisfy them. Intuitively, if consumption and leisure are complements and the marginal utility of consumption decreases fast enough with effort, there is a limit to the level of consumption such that agents are still willing to provide high effort. Hence, when the aggregate endowment in $s = 1$ is high enough, part of the aggregate endowment will not be consumed in that state.

functions of [5] and [9] have the same form). The rest of the argument is identical. QED

REMARK 1. In equilibrium, aggregate consumption is lower than the aggregate endowment in the high-endowment state $s = 1$ (i.e., there are resources not utilized in that state). However, there is no incentive-compatible and budget-feasible contract that provides the consumer a higher utility than $\boldsymbol{x}^E$ by allowing her to consume additional resources when $s = 1$ is realized. This claim is in contrast with the one in the proof of lemma 4 in Bennardo and Chiappori (2003). The authors argue that, if the price associated with consumption in state 1 were zero regardless of the effort level, the consumer could do better by buying a different contract $\boldsymbol{x}'$, where $\boldsymbol{x}'_1$ specifies low effort and a very high level of consumption with probability one, whatever the idiosyncratic state. Since $\boldsymbol{x}'$ is clearly not feasible, Bennardo and Chiappori concluded that $\pi_1(z_a, z_b, e)$ could not be zero at an equilibrium; the nonexistence result in proposition 5 then relies on such a claim. But this misses an important point: namely, that effort is chosen before the realization of the aggregate state; thus if $\boldsymbol{x}'_1$ specifies low effort with probability one, so must $\boldsymbol{x}'_2$ (eq. [2]). While a contract specifying low effort with probability one can provide a very high level of consumption if the high-endowment state $s = 1$ is realized, consumption in the low-endowment state $s = 2$ may have to be rather low. The consumer in fact needs to pay a positive price for the consumption goods received in state 2, and the price can be quite high—and the value of the endowment quite low—in $s = 2$ when the consumer exerts low effort.

Formally, if (as claimed by Bennardo and Chiappori) $\boldsymbol{x}'$ is feasible for the consumer, it must induce agents to exert low effort (incentive compatibility has to hold) and the budget constraint must be satisfied:

$$\boldsymbol{\pi}^* \cdot \boldsymbol{x}' = \boldsymbol{\pi}^*_1 \cdot \boldsymbol{x}'_1 + \boldsymbol{\pi}^*_2 \cdot \boldsymbol{x}'_2 = \boldsymbol{r}_2 \cdot \boldsymbol{x}'_2 \leq 0.$$

Also, if $\boldsymbol{x}'$ is strictly preferred to $\boldsymbol{x}^E$ by the consumer, so is any convex combination of $\boldsymbol{x}^E$ and $\boldsymbol{x}'$: $\boldsymbol{x}^\alpha = (1-\alpha)\boldsymbol{x}^E + \alpha\boldsymbol{x}'$ with $\alpha \in (0, 1]$. For any α, $\boldsymbol{x}^\alpha \in \bar{X}$ and satisfies the resource constraint in $s = 2$. Since at $\boldsymbol{x}^E$ the resource constraint in $s = 1$ is slack, if α is sufficiently small, the same is true at $\boldsymbol{x}^\alpha$. But this contradicts the fact that $\boldsymbol{x}^E$ is a solution to (5).

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