UNIVERSIDAD CARLOS III DE MADRID working papers

Working Paper 11-24 (07)
Business Economics Series
November 2011

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CAN WE CURB RETAIL SALES VOLATILITY THROUGH MARKETING MIX ACTIONS?*

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Abstract

Sales uncertainty is a central problem for marketing management. Marketers tend to focus on expected sales, rather than short-term time-varying oscillations. With long supply-chain streams, the Bullwhip effect can turn retail sales volatility into a major problem for upstream companies. While it has been recognized that conditional expected sales change through time (for a review see Dekimpe and Hanssens, 2000), marketers have not yet started to modeling explicitly time variation of sales' conditional variances. In this paper we focus on this issue, modeling and forecasting time-varying retail sales and marketing mix volatility and their crossed effects within brand, and between competitive brands. We analyze up to 6 product categories sold by Dominick's Finer Foods, finding volatility and co-volatilities in all of them. We discuss managerial implications for brand management and competitive strategy.

Keywords: Sales, Volatility, Bullwhip effect, Marketing mix.

* Research funded by two research projects, by the Comunidad de Madrid and the Spanish Government.

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1 Introduction

Do marketing managers care about sales volatility? Admittedly, they tend to focus on sales trends and overlook short-term fluctuations. But the volatility of sales may have an important effect on operational costs, through the requirement of inventory investments for avoiding stock-out in case of demand peaks (see Holt et al. 1960, Bo 2001), stock that will be redundant with demand falls, leading to excessive inventory and inefficient production with a row of financial costs. For example, the demand variability also expands costs in human resources due to hiring, training, dismissals and possible payoffs of employees.

Recently, Capgemini Consulting company has conducted an annual global supply chain study that covers 300 leading companies from various sectors across Europe, the US and Canada, Asia-Pacific and Latin America. The 2011 study reveals that 40% of respondents answer that demand volatility is the number one business driver. Additionally, a 2010 online survey prepared by Edge Research for IBM-Sterling Commerce shows that managing sales volatility and risk is one of the top priorities for the majority of the respondents (the survey is based on a screened panel of 301 sales, IT and supply chain corporate decision makers). The analysis of real time demand is the most prominent information "black hole" among companies.

In practice, managers usually rely on estimations of future expected (mean) sales conditionally on historic data, but the magnitude of deviations (oscillations) around that mean can evolve over time and finally become even a direr threat itself. The oscillating (volatile) demand is often magnified when the product is brought to customers through long *distribution channel* and *supply chain* streams. Down-stream firms increase (reduce or stop) orders under high (low) demand adapting their inventory buffer, but moving up-stream the demand oscillations are magnified as companies try to fulfill the demand of its predecessor in the chain (a phenomenon put forward by Forrester, 1961, and explained by Sterman 1989). In Supply Chain Management this is known as the *Bullwhip effect* as the upstream order fluctuations remind a cracking whip. Several managerial strategies have been developed to reduce it (for a review see Lee, Padmanabhan, and Whang, 1997, 2004), but they typically require a strong channel coordination with shared information and a corporative focus on sales, which either is not always feasible nor convenient. Some authors suggests the use of better demand forecasts. Hanssens (1998) shows that retailer sales data forecasts are a useful benchmark from the manufacturer's perspective, but he focuses on expected (mean) manufacturer orders and consumer sales rather than volatility (variance). Failure to manage demand oscillations leads to a poor customer relationship, lower loyalty, and often penalties from failed contract fulfillment and distrust between supply chain members.

Acknowledging that marketing managers must keep track of time-varying market volatility, in this paper we study sales and marketing mix dynamics considering both, conditional mean and covariance-matrix of sales and marketing mix of marketed brands. In order to do that we introduce some notation and motivate the use of this approach. Let us consider a stationary process $\{X_t\}_{t\in\mathbb{Z}}$ where X_t is a random vector in $\mathbb{R}^d$, with first moments $\mu = E[X_t]$, $H = Var[X_t]$ positive definite. The vector X_t contains sales of complementary/substitutive brands sold in the same market and the marketing mix variables related to the studied brands (all variables in logarithmic differences to ensure stationarity). Denote by $\Im_t$ the past information available up to time t. As the observations are generally dependent, the use of information in $\Im_t$ can improve the quality of the forecast of X_t. Let us denote the conditional models by $\mu_t = E[X_t|\Im_t]$, and $H_t = Var[X_t|\Im_t]$. The classical time series models (e.g., VAR, VARMA, VARMAX, and their extensions for integrated processes) are focused on the specification and estimation of μ_t, assuming that $H_t = H$ for all periods of time, which is appropriate when there is no volatility. But if that is not the case, H_t provides a much better intuition about sales fluctuations than H. To show this, recall that from the variance equation analysis, $H = E[H_t] + Var[\mu_t]$, implying that $H \geq E[H_t]$ (i.e., $(H - E[H_t])$ is positive definite) so that the risk is on average overrated if we use a marginal or static variance, forcing companies to have oversized safety stocks regularly. Moreover, $H \geq E[H_t]$ is compatible with reverse situations $H \leq H_t$ in some scenarios, suggesting that the safety stocks determined from H can be occasionally too short for insurance against the stock-out risk.

Note also that fluctuations in μ_t are relatively easy to forecast. They can be caused by seasonality (modeled with deterministic dummies, or modeled by seasonal unit roots leading to more realistic stochastic approaches), or they can be caused by business cycles (that can be modeled with sinusoidal deterministic trends, or a more flexible linear ARMA type model with complex roots leading to stochastic cycles). Expected sales fluctuations can be anticipated, and companies can adapt the production-inventory policies to fit the forecast. The actual risk is caused by deviations $(X_t - \mu_t)$ that cannot be anticipated previously, but which average magnitude is measured by H on the whole, and by H_t for each specific period of time. Our central objective is the study of H_t. As the vector X_t includes sales and marketing mix variables for several competitors in the same market, estimating H_t we study the crossed co-volatilities of marketing-mix on retail sales. The results show that the marketing mix can be used to curb volatility whilst increasing expected levels in μ_t. We also analyze crossed effects among competitors, via mean and variance, and how this can be used as a competitive advantage.

We organize this article as follows: after presenting the models that we will consider for the empirical problem at hand, we discuss the market panel-data which we use to estimate each firm's volatility dynamics. We then elaborate on our modelling approach for fitting the model to the data. Next we provide the analysis of the market for several categories, with a managerial discussion of marketing mix impact on sales volatility, and competitive effects between brands. We end up with a discussion of our contributions and managerial implications.

2 The Model

The marketing sales response literature has not paid too much attention to volatility problems. While modeling volatility of sales in a particular brand seems important, the co-movements of sales and marketing mix decisions over different brands is relevant from a strategic perspective, and this is why we consider multivariate co-volatility models including all marketed brands simultaneously. There is an extensive literature on modeling volatility in financial time series since the introduction of the Generalized Autoregressive Conditional Heteroskedasticity (GARCH) univariate model by Engle (1982) and Bollerslev (1986). These models have been extended to multivariate time series. In this paper we combine features from classical time series models for the analysis of conditional means with recent models for conditional variances, to analyze sales and marketing mix dynamics. In particular, we consider that sales and marketing mix variables define a stochastic $\mathbb{R}^d$-vector process $\{X_t\}_{t\in\mathbb{Z}}$ satisfying that

$$\begin{aligned} X_t &= \mu_t + u_t, \\ u_t &= H_t^{1/2}\varepsilon_t, \end{aligned}$$

where $E[u_t|\Im_t] = E[\varepsilon_t|\Im_t] = 0$, $Var[\varepsilon_t|\Im_t] = I$. Typically $\mu_t = E[X_t|\Im_t]$ is defined by a VAR or a vector ARMA model (or a more sophisticated model dealing with features such as seasonality, unit roots, etc.) This is widely used in marketing (see Dekimpe and Hanssens, 2000), and we do not delve into the analysis of the conditional mean. For the conditional heteroskedasticity, we assume that $H_t = Var[X_t|\Im_t]$ follows a $BEKK(p,q,1)$ model (where BEKK stands for Baba-Engle-Kraft-Kroner) given by

$$H_t = \widetilde{\omega}\widetilde{\omega}' + \sum_{j=1}^{q} \widetilde{A}_j u_{t-j} u_{t-j}' \widetilde{A}_j' + \sum_{j=1}^{p} \widetilde{B}_j H_{t-j} \widetilde{B}_j', \tag{1}$$

where $\widetilde{\omega}$ is a lower triangular $\mathbb{R}^{d\times d}$ matrix and $\widetilde{A}_j$, $\widetilde{B}_j$ are $\mathbb{R}^{d\times d}$ matrices. The model was introduced in Baba, Engle Kraft and Kroner (1991), and Engle and Kroner (1995). This model is a multivariate generalization of the $GARCH$ process guarantying that H_t is positive definite. When the matrices $\widetilde{A}_j$, $\widetilde{B}_j$ are diagonal, and the only nonzero elements are those associated to square elements $\{u_{jt-1}^2\}$, then we have a process X_t which conditional covariances are constant, and only the conditional variances evolve. When this is not the case, it means that X_t has covolatility (the level of volatility in one element X_{jt} affects volatility in other element X_{it}).

Denote by vec the operator that stacks the column of a matrix, and $vech$ the vector-half operator which stacks the lower triangular portion of a matrix (on and below the main diagonal). The vec operator satisfies that $vec(ABC) = (C' \otimes A)\ vec(B)$, where $\otimes$ denotes the Kronecker product of two matrices. In order to handle the BEKK model, we can rewrite (1) using the vec operator as follows

$$vec(H_t) = (\widetilde{\omega} \otimes \widetilde{\omega})\ vec(\ I) + \sum_{j=1}^{q} \left(\widetilde{A}_j \otimes \widetilde{A}_j\right)\ vec\left(u_{t-j}u_{t-j}'\right) + \sum_{j=1}^{p} \left(\widetilde{B}_j \otimes \widetilde{B}_j\right)\ vec\left(H_{t-j}\right).$$

The dimension of $vec\left(H_t\right)$ is d^2. Since the matrices involved in this representation are symmetric, we can reduce the dimension. Using the vector-half operator *vech* we rewrite (1) as

$$\begin{aligned} h_t &= w + \sum_{j=1}^{q} A_j \ vech\left(u_{t-j}u_{t-j}'\right) + \sum_{j=1}^{p} B_j \ h_{t-j} \\ &= w + \alpha\left(L\right) \ vech\left(u_t u_t'\right) + \beta\left(L\right) \ h_t. \end{aligned} \tag{2}$$

where $h_t = vech\left(H_t\right)$ has dimension $d\left(d+1\right)/2$, and we have used the matrices $w = D_d^{+}\left(\widetilde{\omega} \otimes \widetilde{\omega}\right) D_d vec\left(\ I\right)$, $A_j = D_d^{+}\left(\widetilde{A}_j \otimes \widetilde{A}_j\right) D_d$, $B_j = D_d^{+}\left(\widetilde{B}_j \otimes \widetilde{B}_j\right) D_d$, with D_d the $d^2 \times d\left(d+1\right)/2$ duplication matrix defined by the property $vec(H) = D_d \ vech\left(H\right)$ for any symmetric $d \times d$ matrix H (i.e., D_d contains some columns from the identity matrix $I_{d^2 \times d^2}$ extracting elements in $vec\left(H\right)$ coming from the lower triangle of H) and D_d^{+} denotes its Moore Penrose inverse. For the last equation (2) we have used a compact notation with matrix polynomials $\alpha\left(L\right) = \sum_{j=1}^{q} A_j L^j$ and $\beta\left(L\right) = \sum_{j=1}^{p} B_j L^j$ in the lag operator L.

3 Data Description

We use store-level scanner data made available by the James M. Kilts Center, University of Chicago, from *Dominick's Finer Foods*, the largest grocery retailer in the Chicago market. The database includes all weekly sales, shelf price, possible presence of sales promotions (coupons, bulk buy, or a special sale), retail margin, and daily store traffic, by individual item (referenced by UPC) for more than 25 product categories, and collected for 96 stores operated in the Chicago area over a period of more than seven years from 1989 to 1997. For the analysis, we aggregate the weekly sales data across stores, computing also the average price. We also compute a continuos promotion variable defined as the percentage of stores implementing any sales promotion. We perform our empirical analysis using six different 'fast moving consumer product categories' (products are sold quickly and at relatively low cost): cheese, refrigerated juice, laundry detergent, toilet tissue, paper towel and toothpaste. As can be seen in Table 1, for cheese and refrigerated juice categories, we consider two brands with the highest market share, forming 80% and 82% of the total category volume respectively whereas for laundry detergent, toilet tissue, paper towel and toothpaste categories we focus on the top three selling brands constituting 70%, 66%, 60% and 73% of the market, respectively.

Table1. Description of six categories used in the application

Category	Number of analyzed brands	Total number of brands in the category	Market Share of the analyzed brands
Cheese	2	12	80%
Refrigerated juice	2	7	82%
LaundryDetergent	3	14	70%
Toilet tissue	3	10	66%
Paper towel	3	13	60%
Toothpaste	3	13	73%

Table 2 provides more details on the analyzed brands in each category. In the **cheese** category, the two competing brands Dominick's and Kraft do not differ much since their average prices, promotions and sales are very close to each other. Also, the standard deviations of those variables show very small difference. In the **refrigerated juice** category, the two brands, Minute Maid and Tropicana, are similar in terms of average prices. The prices of those brands do not vary too much, i.e. the standard deviations are very close to each other. However, the sales of Tropicana almost doubles that of Minute Maid. For the **laundry detergent** we observe that brand's promotion intensity, are close to each other on average as well as their variabilities. The prices differ across the three brands. This difference may be perceived as signals of quality. This difference may make the brands differentiate them from the competitors. Similarly, in the **toilet tissue** category, brands' average promotions as well as the variability of the brand's promotions do not differ much, but prices are different and have different volatilities. Regarding the **paper towel** category, the average prices differ across the brands Bounty, Scott and Dominick's. Scott does more promotion on average, but we see almost no difference in the promotions variability. In the **toothpaste** category, the average prices of the three brands are very close to each other. Aquafresh has the highest price variability and the lowest average sales, while Crest has the moderate price variability, but the highest average sales. The average promotion differs across brands, but the level of the promotion is low.

Table 2. Descriptive statistics of the analyzed brands

Category	Variable	Mean	Median	Maximum	Minimum	Std. Dev.	Observations
Cheese	*Sales Dominick's*	104123.20	101624.00	264441.00	53745.00	26753.70	392
	Price Dominick's	2.10	2.11	2.71	1.26	0.21	392
	Prom Dominick's	92.24	97.67	100.00	0.00	20.09	392
	Sales Kraft	138445.50	122791.00	543061.00	83550.00	53543.57	392
	Price Kraft	1.99	1.95	4.10	0.90	0.28	392
	Prom Kraft	93.81	97.67	100.00	0.00	15.31	392
Refrigerated Juice	Sales Minute Maid	38690.04	23664.00	263612.00	13651.00	35673.52	396
	Price Minute Maid	2.14	2.17	3.49	1.06	0.35	396
	Prom Minute Maid	69.93	94.12	100.00	0.00	40.51	396
	Sales Tropicana	65726.83	48877.50	271965.00	26883.00	40931.44	396
	Price Tropicana	2.47	2.55	3.44	1.21	0.35	396
	Prom Tropicana	82.64	97.42	100.00	0.00	31.10	396
Laundry detergent	*Sales Wisk*	7339.28	4841.50	52357.00	1967.00	7061.49	396
	Price Wisk	5.31	5.24	8.88	2.81	0.87	396
	Prom Wisk	65.11	92.96	100.00	0.00	42.59	396
	Sales All	8332.82	5368.50	133703.00	2265.00	12171.34	396
	Price All	4.51	4.55	7.05	2.48	0.56	396
	Prom All	67.96	94.12	100.00	0.00	40.92	396
	Sales Tide	26318.69	20320.50	135839.00	10586.00	19925.67	396
	Price Tide	6.20	6.29	9.22	3.43	0.85	396
	Prom Tide	68.04	94.12	100.00	0.00	41.12	396
Toilet Tissue	*Sales Scott*	86464.14	56745.50	2062849.00	26163.00	155969.67	384
	Price Scott	0.70	0.66	1.88	0.25	0.20	384
	Prom Scott	47.67	56.89	100.00	0.00	44.17	384
	Sales Charmin	37143.97	19470.50	478101.00	9436.00	58301.06	384
	Price Charmin	2.11	2.13	3.14	0.70	0.51	384
	Prom Charmin	40.30	4.22	100.00	0.00	44.37	384
	Sales Northern	31854.61	18358.00	314957.00	10125.00	38232.68	384
	Price Northern	1.71	1.67	3.45	0.82	0.39	384
	Prom Northern	49.55	61.76	100.00	0.00	46.12	384
Paper Towel	*Sales Bounty*	34452.86	29840.50	163198.00	17202.00	17121.08	388
	Price Bounty	1.42	1.44	2.68	0.75	0.24	388
	Prom Bounty	39.72	2.33	100.00	0.00	44.20	388
	Sales Scott	22764.20	20314.00	112534.00	6115.00	15312.97	388
	Price Scott	1.35	1.28	2.28	0.76	0.30	388
	Prom Scott	61.15	86.04	100.00	0.00	42.96	388
	Sales Dominick's	23822.99	18030.50	208968.00	1346.00	24810.48	388
	Price Dominick's	0.76	0.71	2.31	0.34	0.21	388
	Prom Dominick's	40.29	35.92	100.00	0.00	40.49	388
Toothpaste	*Sales Aquafresh*	3746.22	3159.00	20727.00	1642.00	2171.50	398
	Price Aquafresh	2.24	2.31	3.24	1.05	0.50	398
	Prom Aquafresh	36.43	1.18	100.00	0.00	43.25	398
	Sales Colgate	9082.61	8297.00	25196.00	4926.00	3383.23	398
	Price Colgate	2.21	2.25	2.60	1.56	0.24	398
	Prom Colgate	49.72	48.54	100.00	0.00	44.31	398
	Sales Crest	12176.40	11528.00	49820.00	6769.00	4144.69	398
	Price Crest	2.30	2.37	2.80	1.18	0.32	398
	Prom Crest	43.10	29.48	100.00	0.00	43.99	398

4 Empirical analysis

Our model building approach consists of six main steps: In step 1, we perform preliminary analysis that includes exploratory data analysis, and the analysis of the time series' levels. In step 2, we estimate consistently a model for the conditional mean (typically a VAR model parameters by OLS). Then, in step 3, we explore the existence of volatility in the data, specifying and estimating a BEKK model for the residuals using a Maximum Likelihood Estimation (MLE). In step 4 we compute preliminary estimations for the parameters of the BEKK model. Finally, in step 5, we improve the efficiency of the estimators. We simultaneously estimate the VAR and BEKK parameters by full Gaussian Maximum Likelihood, using the consistent estimates from step 2 and 4 as initial values for the Newton Method (this choice is crucial given the high dimension of the problem). In Step 6 we analyze the estimation output and perform specific tests of independence and Granger Causality.

Step 1: Preliminary Analysis: We perform the standard exploratory data analysis. Then, we study the usual properties such as stationarity and cointegration, involving inspection of data graphs, Auto Correlation Functions (ACF), crossed, and partial autocorrelations. We determine to take natural logarithm for all variables**.** For most of the brands in all categories, we observe that the ACFs decay very slowly which is typical of a nonstationary time series. This conclusion is also consistent with the results of the Augmented Dickey-Fuller, Phillips-Perron and Kwiatkowski–Phillips–Schmidt–Shin (KPSS) unit root tests. Given the evidence, we opt for taking one difference of all variables in logarithms, which can be interpreted as growth ratios of the original series from period to period. Next, we conduct Johansen's cointegration test to study whether the integrated variables are cointegrated (i.e., if they have a long-run equilibrium in levels). Cointegration would imply the specification of a Vector Error Correction (VEC) model instead of a VAR model for variables in differences. For all categories, we accept the null hypothesis that the variables are not cointegrated. Note that in general these tests (unit root and cointegration tests) do not take into account conditional heteroskedasticity, and the output is somewhat exploratory, but it confirms the graphic analysis suggestions. Therefore, we proceed to estimate a VAR model for variables in logarithmic first differences. More complex models could be used if the inspection of the data shows evidence of other alternative specifications such as VARMA or VECM.

Step 2: Conditional mean analysis: Let us denote X_t the vector of log-differenced variables. Here we focus on the analysis of $\mu_t = E\left[X_t|\Im_t\right]$. We model the dynamic interactions among the variables through a VAR model (including all variables as endogenous). We choose the optimal lag length of the VAR model to be 1 based on the visual inspection of the ACFs of the first differenced log series. We also compute the information criteria (commonly used in the marketing literature, see Dekimpe and Hanssens, 1999; Pauwels et al. 2004). Schwarz information criterion (SIC) suggests one lag for all categories. As a result, we specify a VAR(1) model $\mu_t = \Pi X_{t-1}$. We estimate $\widehat{\Pi}$ by OLS, minimizing

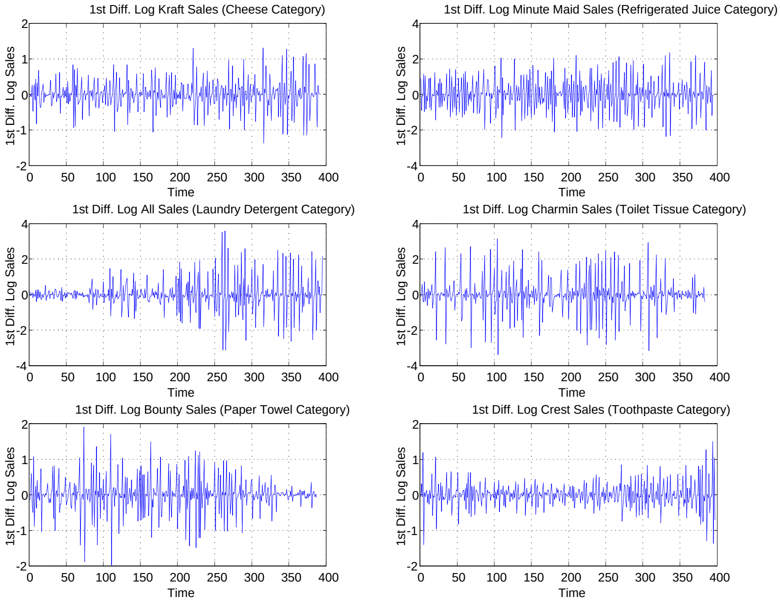
$$Q\left(\Pi\right) = tr\left\{\sum_{t=2}^{T}\left(X_t - \Pi X_{t-1}\right)'\left(X_t - \Pi X_{t-1}\right)\right\} \tag{3}$$

where tr denotes the trace. The solution is $\widehat{\Pi}' = \left(\sum_{t=2}^{T} X'_{t-1}X_{t-1}\right)^{-1}\sum_{t=2}^{T} X'_{t-1}X_t$. We also obtain the residuals $\widehat{u}_t = X_t - \widehat{\Pi}X_{t-1}$ to be used as a preliminary tool for volatility analysis.

Step 3: Volatility Modeling: Before carrying out our volatility model estimation, we explore the presence of volatility in our data. We first study the volatility of the residual series independently (univariate analysis), and then study the appropriate multivariate BEKK model:

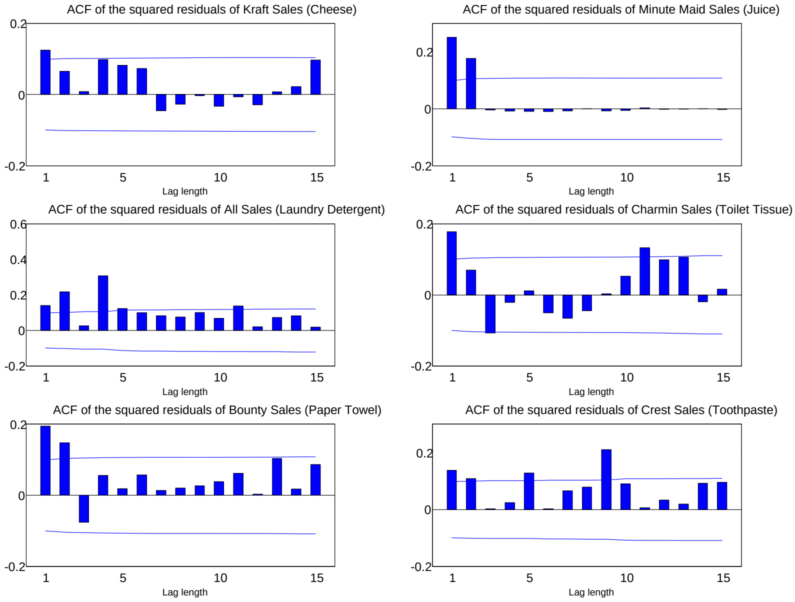
(i) *visual inspection of the sales plots:* As an example, Figure 1 shows the first differenced logarithm of one brand for each category. The plots show that in general the volatility is higher at some periods of time than others, indicating that the conditional variance is not constant over time.

Figure 1. First Differenced Logged Sales for some brands



(ii) *ACFs of the squared OLS residuals from the VAR(1) model:* We find substantial evidence of Autoregressive Conditional Heteroscedasticity (ARCH) effects as judged by the autocorrelations of the squared residuals. As can be seen from Figure 2, even though the magnitude of the autocorrelations sometimes are small after lag 1 or 2, the ACF plots of the squared residuals of sales variables show the presence of autocorrelation patterns. This suggests the existence of (Generalized) Auto Regressive Conditional Heteroskedasticity (ARCH/GARCH models).

Figure 2. ACFs of the squared residuals of sales for some brands



(iii) *ARCH test:* We tested formally the hypothesis of conditional heteroskedasticity applying Engle's (1982) ARCH test. The null hypothesis is that there is no autocorrelation in the squared residuals (and therefore no ARCH effect). For all brands (with two exceptions: brand Wisk in the laundry detergent category and brand Colgate in the toothpaste category), we reject the no-ARCH hypotheses, supporting our findings in (i) and (ii).

Finally, we study multivariate co-volatility relationships and specify a full BEKK model. In order to decide how many lags to be included, we use the ACFs and partial ACFs of the squared residuals. If we define $\eta_t = vech\,(u_t u_t') - h_t$, then we can write model (2) as a $VARMA(r,p)$ with $r = \max(p,q)$, given by

$$vech\,(u_t u_t') = w + (\alpha\,(L) + \beta\,(L))\ \ vech\,(u_t u_t') - \beta\,(L)\,\eta_t + \eta_t,$$

where η_t are martingale differences, and if X_t has four order moments $E\,[\eta_t \eta_t'] = Var\,[vech\,(X_t X_t')] - E\,[h_t h_t']$. The model is covariance stationary if and only if the roots of $|\alpha\,(L) + \beta\,(L)| = 1$ lie outside the unit circle, which usually occurs when $(\alpha\,(1) + \beta\,(1))$ has eigenvalues with modulus smaller than one. We also assume that p, q are as small as possible given that the matrices A_r and B_p have full rank, and the polynomials $(I - (\alpha\,(L) + \beta\,(L)))$ and $(I - \beta\,(L))$ have neither unit roots nor common left factors other than unimodular ones. The $VARMA(r,p)$ representation shows that we can identify p, q with the classical tools. If we estimate μ_t with standard time series methods (i.e. without taking care of the heteroskedasticity), and we can use the residuals $\widehat{u}_t = (X_t - \widehat{\mu}_t)$ to estimate autocorrelation functions for $vech\,(\widehat{u}_t \widehat{u}_t')$, which can be used to determine an appropriate p, q orders. In our case, inspection of sample autocorrelations for $vech\,(\widehat{u}_t \widehat{u}_t')$, subsequent estimation of the identified models, and implementation of a diagnosis process, leaded us to accept that a BEKK(1,1,1) model is an appropriate choice for all product categories.

Step 4: Preliminary BEKK model estimation: The estimation of the volatility model, similar to that of a univariate GARCH. We denote by θ the parameter vector of the model, the matrices $w = w\,(\theta)$, $A_j = A_j\,(\theta)$, $B_j = B_j\,(\theta)$ are functions of θ (in practice the components of θ are precisely the entries in these matrices). The parameters θ can be estimated by conditional pseudo maximum likelihood, i.e. minimizing

$$-2T \cdot L\,(\theta) = \sum_{t=1}^{T} \Big(\ln |H_{t,\theta}| + (X_t - \mu_t)'\ \ H_{t,\theta}^{-1}\ \ (X_t - \mu_t) \Big).$$

Results for the asymptotic properties of the estimator have been studied by Jeantheau (1998) and Comte and Lieberman (2003). In order to simplify its computation, once we have estimated μ_t using (3), we replace μ_t by $\widehat{\mu}_t$ in the likelihood function. This estimation is consistent, but inefficient as it is based on inefficient OLS estimations for the VAR model.

Step 5: Simultaneous estimation of the VAR and BEKK parameters to improve efficiency. We consider the estimated parameters from VAR(1) and BEKK(1,1) models as initial values, and use them in the full likelihood function to estimate all parameters together in order to achieve asymptotic efficiency. Therefore, including the parameters in μ_t in the vector θ we minimize

$$-2T \cdot L(\theta) = \sum_{t=1}^{T} \left(\ln |H_{t,\theta}| + \left(X_t - \mu_{t,\theta}\right)' \ H_{t,\theta}^{-1} \ \left(X_t - \mu_{t,\theta}\right) \right),$$

using the Newton-Raphson method from the preliminary estimators. Using Step 2 and 4 estimations as initial point is crucial for ensuring convergence given the high computational effort caused by the large dimension of the parameters.

Step 6: Inference analysis

We applied the analysis described above to the full vector of sales and marketing mix actions (price, promotion) for all the selected leader brands on each of the six categories. In all the cases, the estimations are globally significant, and their signs and magnitudes are as expected.

The dimension of the tables with the estimators is too large, and we do not report them in detail (they can be provided from the authors upon request). The dynamic structure of the volatility can be visualized using appropriate impulse response functions. Notice that we can expand (2) as

$$h_t = (I - \beta(1))^{-1} w + \Psi_{p,q}(L) \ vech\left(u_t u_t'\right).$$

where

$$\Psi_{p,q}(L) = (I - \beta(L))^{-1} (\alpha(L)) = \sum_{j=1}^{\infty} \Psi_j L^j,$$

the coefficients $\{\Psi_j\}$ can easily computed, they can be interpreted as an impulse-response function explaining the effect of previous unexpected changes of $vech\left(u_t u_t'\right)$ over current covolatility levels h_t. In the particular case of a BEKK(1,1,1) we have

$$\begin{aligned} h_t &= (I - B_1)^{-1} w + \Psi_{1,1}(L) \ vech\left(u_t u_t'\right) \\ \Psi_{1,1}(L) &= (I - B_1 L)^{-1} A_1 L = \left(\sum_{j=0}^{\infty} B_1^j L^j \right) A_1 L = \sum_{j=0}^{\infty} B_1^j A_1 L^{j+1}, \end{aligned}$$

so that

$$\begin{aligned} \Psi_0 &= 0, \Psi_1 = A_1, \ \Psi_2 = B_1 A_1 \\ \Psi_j &= B_1^{j-1} A_1 = B_1 \Psi_{j-1}, \quad j \geq 2. \end{aligned}$$

Inversion of the *vech* operator leads to an infinite BEKK expansion. Figures depicting coefficients in the matrices Ψ_j provide a visual description of the volatility (or covolatility) transmission of random shocks. Some of these graphs are shown in the main results section.

Furthermore, we can obtain much more insightful features from the conditional maximum likelihood estimations by testing conditional independence and Granger causality hypotheses. Consider a partition of X_t two groups of variables X_1 and X_2, then we can study the crossed effects between the different parts. In particular, we study the exogeneity and the independence of marketing mix (price and promotions) and sales within the context of a brand. A similar analysis is carried out for several competitors (e.g. the crossed relationship between sales of a brand and marketing mix of a competitor). From the VAR model, $X_t = \Pi X_{t-1} + u_t$, we partition in two blocks:

$$\begin{pmatrix} X_{1t} \\ X_{2t} \end{pmatrix} = \begin{pmatrix} \Pi_{11} & \Pi_{12} \\ \Pi_{21} & \Pi_{22} \end{pmatrix} \begin{pmatrix} X_{1t-1} \\ X_{2t-1} \end{pmatrix}.$$

If we only consider **mean-dependence**, it is sufficient to test some of the following hypotheses:

- If $\Pi_{12} = 0$ holds (i.e., Π is block-triangular) with Π_{21} significant, then there is Granger causality from X_{1t} to X_{2t}.
- If $\Pi_{12} = 0$, $\Pi_{21} = 0$ holds, (i.e., Π is block-diagonal), then X_{1t} and X_{2t} are independent conditionally on the past.

In order to test $H_0 : \Pi_{12} = 0$, (e.g. to test Granger causality of the marketing mix of a brand on the sales of the same/other brand) one can consider a Wald test

$$T \cdot vec(\widehat{\Pi}_{12}) \quad \left[Var(vec(\widehat{\Pi}_{12}))\right]^{-1} \quad vec(\widehat{\Pi}_{12}) \to \chi^2_k$$

where $Var(vec(\widehat{\Pi}_{12}))$ is the block- component (1,2) of the Maximum Likelihood Estimator covariance matrix (we estimate this matrix using a standard HAC estimator), and k is the number of tested parameters in Π_{12}. Tests for conditional independence are analogous, using estimators for Π_{12} and Π_{21}, and it can be applied for example to test independence between brands. The standard causality tests, including the presented Wald test consider just in-mean effects. Note that with conditional heteroskedasticity, the standard Granger causality tests cannot be used as the concept involves causality in both mean (VAR) and variance (BEKK) equations.

Note that any test based on the parameters of Π ignores the conditional variance dependences. Under volatility patterns, we should also pay attention to the conditional covariance model, testing if the appropriate parameters in $\widetilde{A}_j$ and $\widetilde{B}_j$ in (1) are zero. Consider, for example, the matrix $\widetilde{A}_1$. If the sub-matrix $\widetilde{A}_{12} = 0$ the conditional variance of X_{1t} does not depend on X_{2t} which is a requeriment for exogeneity. This is obvious computing the symmetric matrix

$$\begin{pmatrix} \widetilde{A}_{11} & 0 \\ \widetilde{A}_{21} & \widetilde{A}_{22} \end{pmatrix} \begin{pmatrix} u_{1,t-1} \\ u_{2,t-1} \end{pmatrix} \begin{pmatrix} u_{1,t-1} \\ u_{2,t-1} \end{pmatrix}' \begin{pmatrix} \widetilde{A}_{11} & 0 \\ \widetilde{A}_{21} & \widetilde{A}_{22} \end{pmatrix}'$$

and noticing that the element $\widetilde{A}_{11} u_{1t-1} u'_{1t-1} \widetilde{A}'_{11}$ does not depend on X_{2t}, and analogously for the coefficients in the matrices $\widetilde{B}_j$. Therefore, an exogeneity Wald test in this context is given by,

$$T\left(vec(\widehat{\Pi}_{12}), vec(\widehat{\widetilde{A}}_{12}), vec(\widehat{\widetilde{B}}_{12})\right)' \left[Var\left(vec(\widehat{\Pi}_{12}), vec(\widehat{\widetilde{A}}_{12}), vec(\widehat{\widetilde{B}}_{12})\right)\right]^{-1} \left(vec(\widehat{\Pi}_{12}), vec(\widehat{\widetilde{A}}_{12}), vec(\widehat{\widetilde{B}}_{12})\right).$$

If both $\widetilde{A}_{12}$ and $\widetilde{A}_{21}$ are zero there is block independence between the conditional variance of X_{1t} and X_{2t}. Therefore, for testing full conditional independence the Wald test is a quadratic form including the estimators

$$\left(vec(\widehat{\Pi}_{12}), vec(\widehat{\Pi}_{21}), vec(\widehat{\widetilde{A}}_{12}), vec(\widehat{\widetilde{A}}_{21}), vec(\widehat{\widetilde{B}}_{12}), vec(\widehat{\widetilde{B}}_{21})\right).$$

If we are not interested in mean effects, but just in co-volatility, we would compute a Wald test with the estimators $\left(vec(\widehat{\widetilde{A}}_{12}), vec(\widehat{\widetilde{A}}_{21}), vec(\widehat{\widetilde{B}}_{12}), vec(\widehat{\widetilde{B}}_{21})\right)$.

Summarizing, we consider total independence (exogeneity) test for all the parameters, a **mean**-independence (exogeneity) test using the VAR parameters, and a **variance**-independence (exogeneity) test using the BEKK parameters.

5 Main Results

Upon our conditional maximum likelihood estimation for the complete model with conditional mean (VAR model) and variance (BEKK model), we compute the Wald tests, and discuss the results of Granger exogeneity[1] for the marketing mix, and the independence[2] tests of marketing mix variables and sales (all measured as

[1] Exogeneity test example: we test if marketing mix of brand A is independent of its sales (in our context conditional mean and variance do not depend on sales), and not viceversa. Put it differently, causality is one-directional that goes from marketing mix to sales.

[2] Independence test example: we test the block independence between marketing mix and sales of brand A. In other words, neither of them affects the other, through expectations nor variances.

logarithmic growth rates). First, we discuss the results of the exogeneity and independence tests of marketing mix variables and sales for each brand separately, and then across brands. Note that given the observed volatility, all standard parametric inferences based on VAR model would be erroneous, as their usual tests do not account for conditional heteroskedasticity.

5.1 Within-Brand Analysis

For each brand, we particularly test the exogeneity of the marketing mix of the brand from its sales, and the independence between the sales and the marketing mix of the same brand. The results show that, for all brands in all categories (laundry detergent, toilet tissue, toothpaste, paper towel, cheese and refrigerated juice), **we do reject** the exogeneity of marketing mix hypotheses and we **do** also **reject** the strongest conditional independence hypotheses with a 95% of confidence (meaning that for each brand, the empirical evidence supports that sales means and variances depend on previous sales and marketing mix actions, and vice versa marketing mix actions are set based on previous sales and marketing actions). Table 3 contains a summary of these tests.

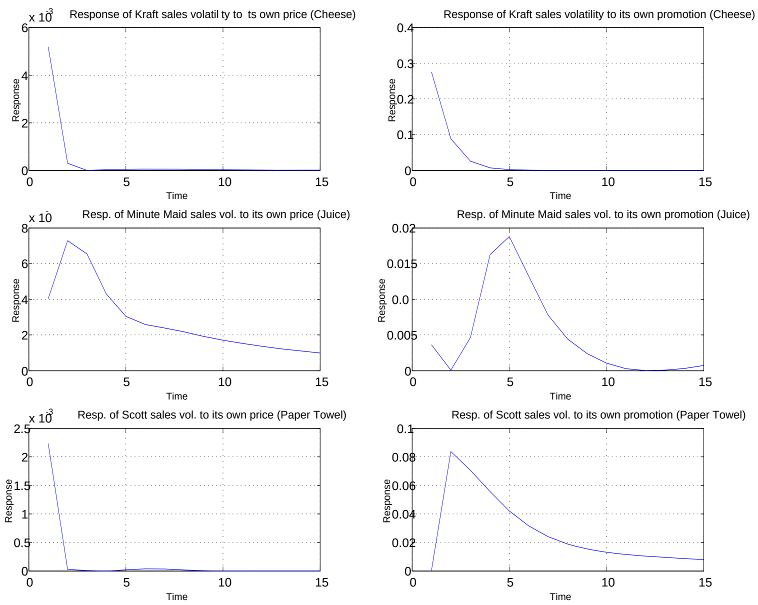
Table 3. Within-Brands Wald tests analysis

Category	Brand	Null hypothesis	Wald Test	d.f.	Chi-square Critical Value at 5%
Cheese	Dominicks	Marketing mix exogeneity	565,0	6	12,59
		Conditional independence	3992,7	12	21,03
	Kraft	Marketing mix exogeneity	441,0	6	12,59
		Conditional independence	5309,2	12	21,03
Refrigerated juice	Minute Maid	Marketing mix exogeneity	7383,7	6	12,59
		Conditional independence	29949,0	12	21,03
	Tropicana	Marketing mix exogeneity	7556,7	6	12,59
		Conditional independence	114770,0	12	21,03
Laundry Detergent	Wisk	Marketing mix exogeneity	1096,0	6	12,59
		Conditional independence	5409,2	12	21,03
	All	Marketing mix exogeneity	1088,0	6	12,59
		Conditional independence	12103,0	12	21,03
	Tide	Marketing mix exogeneity	3064,9	6	12,59
		Conditional independence	10246,0	12	21,03
Toilet Tissue	Scott	Marketing mix exogeneity	513,1	6	12,59
		Conditional independence	2970,7	12	21,03
	Charmin	Marketing mix exogeneity	90,0	6	12,59
		Conditional independence	910,6	12	21,03
	Northern	Marketing mix exogeneity	262,0	6	12,59
		Conditional independence	7584,2	12	21,03
Paper Towel	Bounty	Marketing mix exogeneity	1316,7	6	12,59
		Conditional independence	3574,5	12	21,03
	Scott	Marketing mix exogeneity	2456,4	6	12,59
		Conditional independence	15463,0	12	21,03
	Dominicks	Marketing mix exogeneity	593,3	6	12,59
		Conditional independence	3892,9	12	21,03
Toothpaste	Aquafresh	Marketing mix exogeneity	8689,0	6	12,59
		Conditional independence	20994,0	12	21,03
	Colgate	Marketing mix exogeneity	885,1	6	12,59
		Conditional independence	3335,7	12	21,03
	Crest	Marketing mix exogeneity	965,5	6	12,59
		Conditional independence	2462,6	12	21,03

We have also performed a narrowed version of the analysis, testing for conditional independence and exogeneity *particularized for the conditional mean* or for *the conditional variance* separately. In this setting, all but one null independence hypotheses are rejected when they are carried our just for VAR and for BEKK parameters, in line with the joint tests reported in Table 3, with just an exception: when we focus just on the VAR parameters, we accept the mean-independence between the marketing mix and sales of Northern in the Toilet Tissue category [4.8 (0.308)][3], nevertheless focusing on the BEKK parameters the null hypotheses of volatility independence of Northern's sales and marketing mix is rejected.

In order to display estimation results and to show the impact of a unit shock to a marketing mix element on sales volatility over time, we use the impulse-response analysis. Because of the space limitation we do not provide all of them. As an example, Figure 3 shows the volatility impulse-response function (VIRF) plots for cheese, refrigerated juice and paper towel categories. Notice that for Kraft brand in the Cheese category, increasing price and promotion growth rate have a positive impact on the sales growth volatility although the effect decays in few periods. For Minute Maid brand juices, the effect is longer for prices than for promotions, whereas for Scott Paper Towel promotions have a longer effect. Recall that all variables are in logarithmic differences, meaning that for the in-levels series the impact is permanent.

Figure 3. Within-brands volatility Impulse Response Functions



5.2 Between-Brand Analysis

For all categories, we test the exogeneity of the focal brand's marketing mix from the competitors' sales, the independence between the focal brand's marketing mix and the competitors' sales, the exogeneity of the focal brand's marketing mix from the competitors' marketing mix, the independence between the focal brand's marketing mix and the competitor's marketing mix, the exogeneity of the focal brand's sales from the competitors' sales, and the independence between the focal brand' sales and the competitors' sales.

When we consider jointly the VAR and BEKK model parameters in the Wald test, we find **significant crossed effects** for all brands in the all categories. We rejected conditional independence between the sales of all competitors, and we also rejected block conditional dependence between sales and marketing mix for all pairs of competitors, see Table 4. If we consider just exogeneity (unidirectional effects), the results are analogous with a few exceptions. For example, in the Cheese category we accept that Dominick's sales are independent from Kraft's sales [4.2 (0.2407)], but the opposite effect is rejected suggesting that Dominick's is a leader and Kraft is a follower in this market, regardless of the fact that Kraft average sales are slightly

[3]The first value is the Wald test statistic and the second value in parenthesis is the corresponding p-value. From now on, we will show the results of the rejected hypotheses in this this format.

larger (see Table 2). Both use their marketing mix as a competitive tool, since the block-independence between their marketing mix is rejected [6055.3 (0.0001)], also the exogeneity is rejected for any of them. Similar conclusions can be drawn for other product categories.

We have also narrowed the analysis to just mean or just variance dependence. Most conditional independence and exogeneity tests for volatility are rejected in all categories with few exceptions. Reciprocally, if we only focus on the conditional mean parameters, most conditional independence and exogeneity tests are also rejected, which is well established in the sales response models literature. The volatitlity analysis can shed some additional insights. For example, in the Cheese category (in spite the fact that we rejected that the sales of brand Kraft are independent from Dominick's sales), if we focus just on the volatility parameters we accept it [2.2 (0.3329)] (we reject for mean [204 (0.0001)]), indicating that the leadership of Dominick's matters in terms of volatility rather than average patters. Summarizing, the competitive effects are transmitted either through mean or variance, but usually both effects are relevant.

We can depict some Between-Brands effect using volatility impulse-response functions. For example, Figure 4 shows VIRF plots for laundry detergent category. Notice that a unit shock to the promotion change of the brand Wisk leads to increase in the sales growth volatility of the brand All, and a unit shock to brand Tide's price growth rate generates an increment on the sales growth volatility of All. Since all variables are in logarithmic differences, for the in-levels series the impact is permanent. An emergent conclusion is that promotional actions can be used to increase sales volatility of a competitor, which eventually can lead to a cost increment, and therefore to a competitive advantage but aware competitors could apply a similar strategy. This suggests that some commercial wars could be triggered by co-volatilities, rather than by the effects on average sales.

Figure 4. Between-brands co-volatility Impulse Response Functions

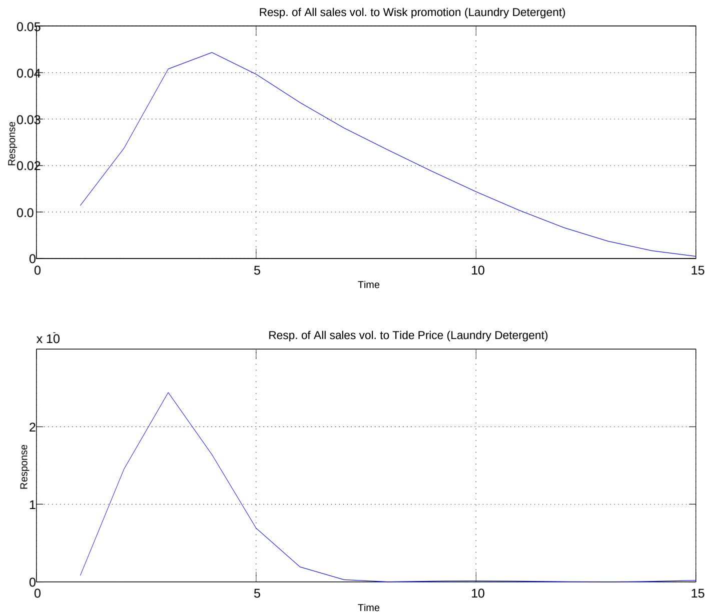


Table 4. Between-Brands Wald tests analysis

Category	Null hypothesis (Block conditional independence)	Wald Test	d.f.	Chi-square Critical Value at 5%
Cheese	Between Dominicks marketing mix and Kraft sales	4004,7	12	21,03
	Between Dominicks marketing mix and Kraft marketing mix	6055,3	24	36,42
	Between Dominicks sales and Kraft sales	591,6	6	12,59
	Between Dominicks sales and Kraft marketing mix	2297,7	12	21,03
Refrigerated juice	Between Minute Maid marketing mix and Tropicana sales	7117,9	12	21,03
	Between Minute Maid marketing mix and Tropicana marketing mix	37997,0	24	36,42
	Between Minute Maid sales and Tropicana sales	626,6	6	12,59
	Between Minute Maid sales and Tropicana marketing mix	19314,0	12	21,03
Laundry Detergent	Between Wisk marketing mix and competitors (All and Tide) sales	30939,0	24	36,42
	Between Wisk marketing mix and competitors (All and Tide) marketing mix	88156,0	48	65,17
	Between Wisk sales and competitors (All and Tide) sales	12564,0	12	21,03
	Between Wisk sales and competitors (All and Tide) marketing mix	20854,0	24	36,42
	Between All marketing mix and competitors (Wisk and Tide) sales	610400,0	24	36,42
	Between All marketing mix and competitors (Wisk and Tide) marketing mix	270940,0	48	65,17
	Between All sales and competitors (Wisk and Tide) sales	13794,0	12	21,03
	Between All sales and competitors (Wisk and Tide) marketing mix	31793,0	24	36,42
	Between Tide marketing mix and competitors (Wisk and All) sales	34891,0	24	36,42
	Between Tide marketing mix and competitors (Wisk and All) marketing mix	254000,0	48	65,17
	Between Tide sales and competitors (Wisk and All) sales	6978,8	12	21,03
	Between Tide sales and competitors (Wisk and All) marketing mix	65035,0	24	36,42
Toilet Tissue	Between Scott marketing mix and competitors (Charmin and Northern) sales	14614,0	24	36,42
	Between Scott marketing mix and competitors (Charmin and Northern) marketing mix	100470,0	48	65,17
	Between Scott sales and competitors (Charmin and Northern)sales	3729,0	12	21,03
	Between Scott sales and competitors (Charmin and Northern) marketing mix	14181,0	24	36,42
	Between Charmin marketing mix and competitors (Scott and Northern) sales	533520,0	24	36,42
	Between Charmin marketing mix and competitors (Scott and Northern) marketing mix	88358,0	48	65,17
	Between Charmin sales and competitors (Scott and Northern) sales	2947,3	12	21,03
	Between Charmin sales and competitors (Scott and Northern) marketing mix	15391,0	24	36,42
	Between Northern marketing mix and competitors (Scott and Charmin) sales	12160,0	24	36,42
	Between Northern marketing mix and competitors (Scott and Charmin) marketing mix	119460,0	48	65,17
	Between Northern sales and competitors (Scott and Charmin) sales	918,7	12	21,03
	Between Northern sales and competitors (Scott and Charmin) marketing mix	21128,0	24	36,42
Paper Towel	Between Bounty marketing mix and competitors (Scott and Dominicks) sales	19483,0	24	36,42
	Between Bounty marketing mix and competitors (Scott and Dominicks) marketing mix	104160,0	48	65,17
	Between Bounty sales and competitors (Scott and Dominicks) sales	2310,5	12	21,03
	Between Bounty sales and competitors (Scott and Dominicks) marketing mix	27106,0	24	36,42
	Between Scott marketing mix and competitors (Bounty and Dominicks) sales	33632,0	24	36,42
	Between Scott marketing mix and competitors (Bounty and Dominicks) marketing mix	142900,0	48	65,17
	Between Scott sales and competitors (Bounty and Dominicks) sales	5243,2	12	21,03
	Between Scott sales and competitors (Bounty and Dominicks) marketing mix	15374,0	24	36,42
	Between Dominicks marketing mix and competitors (Bounty and Scott) sales	39155,0	24	36,42
	Between Dominicks marketing mix and competitors (Bounty and Scott) marketing mix	97228,0	48	65,17
	Between Dominicks sales and competitors (Bounty and Scott) sales	4168,0	12	21,03
	Between Dominicks sales and competitors (Bounty and Scott) marketing mix	16443,0	24	36,42
Tooth Paste	Between Aquafresh marketing mix and competitors (Colgate and Crest) sales	10639,0	24	36,42
	Between Aquafresh marketing mix and competitors (Colgate and Crest) marketing mix	42411,0	48	65,17
	Between Aquafresh sales and competitors (Colgate and Crest) sales	5946,1	12	21,03
	Between Aquafresh sales and competitors (Colgate and Crest) marketing mix	34889,0	24	36,42
	Between Colgate marketing mix and competitors (Aquafresh and Crest) sales	135270,0	24	36,42
	Between Colgate marketing mix and competitors (Aquafresh and Crest) marketing mix	56618,0	48	65,17
	Between Colgate sales and competitors (Aquafresh and Crest) sales	3864,9	12	21,03
	Between Colgate sales and competitors (Aquafresh and Crest) marketing mix	9563,3	24	36,42
	Between Crest marketing mix and competitors (Colgate and Aquafresh) sales	20942,0	24	36,42
	Between Crest marketing mix and competitors (Colgate and Aquafresh) marketing mix	72844,0	48	65,17
	Between Crest sales and competitors (Colgate and Aquafresh) sales	5374,4	12	21,03
	Between Crest sales and competitors (Colgate and Aquafresh) marketing mix	18314,0	24	36,42

6 Conclusions

Sales data often have a high level or temporal aggregation which disguises their volatility. The use of relatively short time aggregation windows, such as weekly, daily, and even hourly for internet sales, allows marketers to capture short term fluctuations impacting production and stock management. In turbulent markets, it is possible to find volatility even with data aggregated over larger time windows, such as monthly and quarterly sales. A closer analysis of sales volatility may lead to better management of distribution and supply chain relationships, creating long-term competitive advantages for marketers.

In this paper we analyzed the presence of volatility in weekly retail sales and marketing mix data. We build a VAR model for the conditional mean and a BEKK model for the conditional variance, and use the estimated parameters to study conditional independence and exogeneity using Wald tests. We observe significant dependence in all categories for most brands, either in mean, variance or both. The volatility impulse response analysis shows the impact of marketing mix changes (price or promotions) over sales growth volatility, either for own marketing mix or a rival's action. One possibility to alleviate the sales growth variability could be to lower the rate of change in promotional intensity. Also, the retailer may choose more stable price policy because price fluctuations may result in stockpiling behavior of the customers which in turn leads to sales volatility (Lee et al., 1997).

A managerial implication of this research is the fact that marketing mix (at least, price and promotional actions) can be a useful tool for product and brand managers to curb volatility for smoothing out eventually the Bullwhip effect at the retail source level. Lower price and promotional growth rates lead to less volatility in sales growth. Managers should balance the positive effects on expected sales, and the negative effects on volatility. The article complements the work by Hanssens (1998) in which better expected sales data forecasts is proposed as an instrument to handle Bullwhip effects.

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