AN ECONOMIC ANALYSIS OF BANKING REGULATION

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Tesis Doctoral

Autor: F. JAVIER SUAREZ BERNALDO DE QUIROS
Director: RAFAEL REPULLO LABRADOR

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a mis padres
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CHAPTER 1

INTRODUCTION AND SUMMARY
Financial intermediation is one of the most heavily regulated activities in the economy. The existence and the design of the regulation which affects financial intermediaries and, more particularly, credit institutions (i.e. banks and savings banks) is a topic of intense debate. Apart from regulations related to the role of banks in the determination of the quantity of money and the operation of monetary policy, all countries have an important body of prudential regulation which submits credit institutions to numerous controls and restrictions, from licensing to capital standards, limits on risk-taking, supervision, closure rules and many others. The special preoccupation with solvency is the most distinctive feature of the prudential regulation of banks as compared with the regulation of other firms.

In the U.S., the safety net created after the Great Depression appeared to work remarkably well during nearly fifty years. Bank failures decreased from 4,000 in 1933 to 370 during the period 1934 through 1941 and declined still further from 1942 through 1980, with a total number of 198. This situation changed dramatically in the late 1970s and the 1980s. The failure of hundreds of savings and loan institutions caused the insolvency and reorganization of the Federal Savings and Loan Insurance Corporation. There were also many banks in trouble—in Texas, for example, nine of the ten largest bank holding companies were reorganized with public or outside assistance—. From 1987 through the end of 1990, the fund of the Federal Deposit Insurance Corporation declined from over $18 billion to approximately $9 billion (U.S. Department of the Treasury, 1991).

These facts have provided a powerful impetus for regulatory reform in the U.S. banking industry. Academics have broadly participated in the debate. Criticisms against the over-extended and risk-insensitive deposit insurance system have been predominant. It has also been argued that some old rules designated to protect banks from competition, such as limits on branching and interest rate ceilings, became barriers that
Introduction and Summary

prevented banks from adapting to new market conditions, thereby stimulating disintermediation and the loss of the banks' best customers.

Although financial crises have affected also European banking systems in the recent past -the Spanish banking system among them-, the debate on banking reform in the European Community (EC) has focused on the need for harmonization within an increasingly integrated Europe. The urgency of adapting very heterogeneous national regulatory system to the immediate liberalization of capital movements and the future competition of European credit institutions in a single market, has stimulated the dismantling of very rigid and complex regulations.

The difficulties in the harmonization process are mainly due to two reasons. First, the differences in the structure of the national financial sectors and, hence, in the roles played by banks in each country. Second, the priority given to monetary aspects, reflected in the design of the European System of Central Banks (see Chiappori et al, 1991). Nevertheless, some important fruits have been obtained, including the Second Banking Directive and the directives on own funds and solvency ratios.

At the world level, the increasing importance of international banking has also forced coordination. The most remarkable example is the 1988 Basle Accord on capital standards. This agreement set out the basis on which risk-weighted capital requirements should be determined, and inspired closely related directives by the EC one year later.

Despite its importance, the regulation of credit institutions has generated few theoretical developments. The literature is abundant, but dispersed and incomplete. The objective of this thesis is to provide a better understanding of the economic effects of prudential regulation. It does not deal with the design of the optimal regulation (a normative approach) but with the analysis of current regulatory devices and realistic proposals for reform (a positive approach). Still, neither the political process by which regulation is conceived nor the behavior of the agencies in charge of enforcing that regulation are examined in this
thesis.

Of course, incorporating the maximizing behavior of regulators and supervisors into the analysis would make it more complete, but at the cost of greater complexity. Generally, the advance of applied economic theory requires abstracting some points to concentrate in others. Banking regulation, in particular, is a wide and non-mature research field and focusing on a relatively small set of issues is obliged.

Notwithstanding, the scope of this thesis is not capricious. On the one hand, a good comprehension of the responses of economic agents to the introduction and reform of regulatory restrictions is a first step to provide a basis for both the welfare analysis and the political economy of banking regulation. On the other, the economics of bank behavior is relatively unexplored and can be enriched with concepts and methods already applied in other areas of economic theory.

Most economists accept the existence of several market imperfections which are closely related to the very nature of the banking activity. These imperfections provide a rationale for regulation although its final convenience should depend on comparing the gains from public intervention with the deadweight losses generated by agency costs and possibly distorting policy instruments.

First, modern theories of financial intermediation rely on informational problems. Asymmetric information is a characteristic of the relationship between borrowers and lenders. According to Diamond (1984), fixed costs linked to information acquisition together with diversification can justify why screening and monitoring are delegated by lenders to an intermediary. However, asymmetric information is also a characteristic of the relationship between the bank and its depositors, and may potentially counterbalance part of the economic gains derived from intermediation. Small depositors in particular have neither the incentives nor, probably, the competence to evaluate and monitor the quality and behavior of banks, and can be especially exposed to adverse selection and moral hazard phenomena.
Second, demand deposits may provide insurance services to depositors in a multi-period setting where they are subject to individual uncertainty on their liquidity needs but available investment projects are illiquid (Diamond and Dybvig, 1983). However, depositors' reactions to adverse information or beliefs on investment outcomes (and not only their response to genuine liquidity needs) take the form of deposit withdrawals, so costly bank runs may occur, even as a result of self-fulfilling expectations.

Third, given the links between financial institutions in financial and monetary markets, negative externalities associated to bank failures may exist (Goodhart, 1987).

The traditional goals of banking regulation are clearly related to these problems. Depositor protection, on the one hand, entails establishing rules in order to preserve or enhance the quality of financial services. Financial stability, on the other, demands limiting the incidence of bank failures and, in particular, avoiding bank panics, that may lead to failures even when the banks involved are initially solvent.

In most countries prudential regulation consists of combining implicit or explicit deposit guarantees and lender of last resort facilities with a variety of constraints on bank behavior, notably, limits on leverage, asset risk and off-balance sheet exposures. As a complement, supervisory agencies monitor the conduct and solvency of banks by submitting the institutions to reporting obligations, on-site examinations and sanctions.

In this context, deposit insurance can be thought of as a mean by which, on the one hand, depositors delegate the role of monitoring and disciplining banks to the banking authorities and, on the other, incentives for early withdrawal on a purely precautionary basis are removed. Accordingly, the regulatory problem could be explained in the following terms: since the risks of loss are transferred from depositors
to the deposit insurance agency (DIA), regulation (as opposed to the market) has to undertake the role of disciplining banks.

Deposit insurance, however, creates a moral hazard problem, since the insurer bears most of the losses in the event of failure, while the risk-taker obtains the rewards in the event of success. In some sense, the problem faced by the insurer of bank deposits is akin to that of the creditor of an indebted corporation, who faces a bankruptcy risk. But some differences exist. First, the DIA is a public agency that, when dealing with the creditor (i.e. the bank) substitutes regulatory rules for the clauses in a private contract. Second, informational asymmetries and the economic consequences of failures are qualitative and quantitatively more important in banking than in other economic activities.

In the literature on banking regulation, several strands can be distinguished.

First, standard portfolio selection models have been used to analyze the impact of capital requirements on bank behavior. These are the case of well-known papers by Kahane (1977), Koehn and Santomero (1980) and Kim and Santomero (1988), which follow Hart and Jaffee (1974) in modeling the bank as a risk-averse owner-manager. In that setup, however, investors are fully liable —bankruptcy does not exist— and both banks and their regulation lack clear economic justification. The incentives for a bank to take risk are just those of an ordinary risk-averse investor, except if some ad hoc assumption on the (low) degree of risk aversion by the intermediary is made. Thus, regulatory rules introduce additional constraints in the portfolio problem and bank decisions are distorted in exchange of no clear economic gain.

In such mean-variance framework, more stringent bank capital regulation (which reduces the risk associated with leverage) may cause the utility maximizing bank owner-manager to increase asset risk in order to compensate for the fall in the desired level of total portfolio risk. In some situations final portfolio risk may be even higher than
without regulation.

Neglecting limited liability when dealing with prudential issues—which are closely related to insolvency risk—has been recently criticized by Keeley and Furlong (1990). The problem of incorporating limited liability in portfolio selection models is partially addressed by Rochet (1992), although his treatment of short-sales (which are not discarded) and the exogeneity of bank capital (which is not a decision variable for banks) are not very satisfactory.

Second, option theorists have remarked the isomorphic relationship between options and a variety of very common financial arrangements such as corporate liabilities and loan guarantees. Following Black and Scholes (1973), equity can be seen as a call option on the assets of the firm with an exercise price equal to the nominal value of debt. Shareholders exercise their right to buy the assets when the firm's net worth is positive; otherwise, limited liability provisions apply and bankruptcy is declared. Similarly, when corporate debt is subject to credit risk, the payments which are not realized in the event of failure can be assimilated to the losses of the issuer of a put option on the assets of the firm with an exercise price that, again, is equal to the nominal value of debt.

Along these lines, as first noted by Merton (1977), if deposits are fully insured, the deposit insurance agency can be considered as the writer of a put option on bank assets with a strike price equal to the promised maturity value of deposits. The put option is sold to bank shareholders in exchange of a deposit insurance premium and the submission of the bank to capital and asset regulations. This put option represents essentially the limited liability of bankers.

Although this put is also implicit in standard debt contracts, what makes a difference in this case is the separation between the seller of the put (the DIA) and the holder of the debt contract (the depositors): credit risk falls on the insurer. In contrast with ordinary lenders, depositors do not demand higher rates of interest to less solvent banks,
except if bankruptcy and liquidation proceedings cause them some inconveniences, such as delay in reimbursement. Thus, market discipline is not exerted at all.

Merton's contribution suggested that, with an adequate valuation of the deposit insurance contract, the regulator could replace market discipline by actuarially fair deposit insurance premiums, i.e. premiums directly related to the discounted expected costs of bankruptcy to the DIA. Henceforth, most efforts have been devoted to obtain formulas for pricing deposit guarantees in different (and increasingly complex) environments (Merton, 1978; Ronn and Verma, 1986; Flannery, 1991; Allen and Saunders, 1993; Kerfriden and Rochet, 1993), and to test the actuarial fairness of the flat-rate premiums currently applied (Marcus and Shaked, 1984; Ronn and Verma, 1986; and many others).

In this literature, perfect information by the insurer is implicitly assumed and the use of fair deposit insurance premiums is conceived as a feasible way to solve the regulatory problem. Surprisingly, little effort has been made in justifying the purposed pricing schemes within models that explicitly formalize the behavior of banks. Discussion is limited in many cases to point out that in the absence of a proper pricing of deposit guarantees, banks will have an incentive for excessive risk-taking, because the value of the put option representing limited liability is increasing in leverage and asset risk (Furlong and Keeley, 1989).

Moreover, banking regulation might play a role in neutralizing these distortions or, at least in ameliorating their effects on the solvency of banks and the exposure of the deposit insurance system. But, in spite of having closed-form formulas for the valuation of the claims involved, this literature has paid small attention to realistic schemes such as ordinary capital requirements, limits on asset risk and the recently introduced risk-weighted capital-to-asset ratios.

With few exceptions, the effects of banking regulation on the behavior of banks have been analyzed in partial equilibrium settings
where competitive intermediaries invest in marketable assets whose prices are exogenously given. Neither the role of banks in financing real investment projects, nor general equilibrium issues, nor dynamic considerations related with bankruptcy and closure, nor the interaction between market power and risk-taking have been formally addressed.

Finally, some authors have recently focused their attention towards the design of optimal banking regulation: Pecchenino, 1992; Chan et al., 1992; Bensaid et al., 1993; Freixas y Gabillon, 1993. All of them coincide in explicitly dealing with informational asymmetries, but differ in the conception of banks, the assignment of economic value to banking activities, the definition of the regulator's objective function and some other details. These differences reflect the lack of a commonly accepted way to model the banking firm and its role in the economy, especially when the interest is centered on prudential issues.

The economic analysis of banking regulation can be enriched if the virtues of each of these strands of the literature are put together. In particular, this thesis applies the option valuation formulas which are typically used in the second strand in order to derive the objective function of banks under limited liability. On this basis, the effects of prudential regulation on bank behavior can be rigorously examined, providing the foundations to extend the analysis in the directions suggested by our critical review of the literature.

Although our treatment of limited liability is free from the critique to portfolio selection models, this thesis is not a re-statement of the mean-variance analysis of banking regulation, since the complexity of the problem in hand recommends focusing on the case where bankers are risk-neutral.

Limited liability introduces non-convexities in the bank's optimization problem, since bankers possess a residual claim on the bank's portfolio that yields positive rewards only in the upper tail of the distribution of the bank's net worth. If risk-aversion were introduced, the concavity of the corresponding expected utility function
could partially off-set the effects of the convexity in bankers' payoffs. But the final results would depend on the degree of risk-aversion. Presumably, for moderate degrees of risk-aversion, bank behavior would be biased towards risk as in the risk-neutral case analyzed in this thesis.

The content of the remaining chapters can be summarized as follows. Chapter 2 focuses on the behavior of perfectly competitive banks in a partial equilibrium setting. It contains the main elements upon which banks are modeled in successive chapters. Different types of regulation are examined, from the risk-based proposals to the risk-insensitive schemes that exist in many countries. The role of capital requirements in a perfectly competitive banking industry with flat-rate deposit insurance premiums is investigated. After exploring the possibilities of risk-weighted capital requirements, it evaluates the theoretical adequacy of the Basle Accord on capital standards.

In Chapter 3 banking regulation is analyzed in a general equilibrium setup where banks play an explicit role in evaluating and funding real investment projects. I show that in the long-run subsidization of risk-taking by a flat rate deposit insurance premium leads to over-investment in risky projects, though in the short-run capital market imperfections may account for the opposite result. Prudential regulation drives its general equilibrium effects through changes in intermediation margins, which, in turn, are determinants of the solvency of banks. The equilibrium effects may reinforce or weaken the direct effects. In particular, the equilibrium effects of an increase in the level of systematic risk and those of imposing tougher capital standards tend to reinforce the direct consequences typically pointed out in partial equilibrium analysis.

The last chapter examines the prudential regulation of banks in a dynamic model of bank behavior which allows for bankruptcy and closure. In a dynamic setting, the banker who goes bankrupt is likely to suffer losses related to future payoffs. Banking regulation contains special provisions for promoters and managers of banks which become insolvent.
Introduction and Summary

When the expected future rents are positive, the value of bank charters may be an important component of bankruptcy costs to bankers and may constitute an incentive to adopt prudent decisions despite the existence of a risk-insensitive deposit insurance system. Some authors have incorporated the charter as an exogenous bankruptcy cost in a standard one-period model, but charter values are intrinsically endogenous.

In Chapter 4 dynamic programming techniques allow us to obtain simultaneously the equilibrium value of a bank and the bank's optimal investment and financial policies. Risky policies are associated to low values of the bank. Under perfect competition closure rules are ineffective in disciplining banks because expected future rents, and hence the value of bank charters, are zero. Allowing for the exercise of market power, comparative statics provide insights into the fundamentals that influence both the value and decisions of a bank. Tough prudential regulation, high market power and a low risk-free interest rate generally elicit safe policies. So capital and asset regulations, on the one hand, and entry and closure rules, on the other, can be regarded as alternative ways to preserve solvency. These results are used to derive some policy implications for the current regulatory debate in the U.S. and Europe.
REFERENCES


Introduction and Summary


CHAPTER 2

DEPOSIT INSURANCE AND BANK BEHAVIOR:
A MODEL BASED ON OPTION THEORY
1. INTRODUCTION.

Banking regulation is traditionally intended to provide investor protection and to promote financial stability. Three are the rationales for these objectives. First, the nature of the banking business makes asymmetric information a characteristic of the relationship between the banks and their clients. Small depositors in particular have neither the incentives nor, probably, the competence to evaluate and monitor the quality and behavior of banks, and can be especially exposed to adverse selection and moral hazard phenomena. Second, since depositors' reactions take the form of deposit withdrawals and many bank assets are illiquid, costly bank runs may occur, even as a result of self-fulfilling expectations (Diamond and Dybvig, 1983). Third, given the links between financial institutions, negative externalities associated to bank failures may exist.

Depositor protection, on the one hand, requires establishing rules in order to preserve or enhance the quality of financial services. Financial stability, on the other, demands limiting the incidence of bank failures and, in particular, avoiding bank panics, that can lead to failures even when the banks involved are solvent.

We can think of deposit insurance as a mean by which, simultaneously, depositors delegate the role of monitoring and disciplining banks to the banking authorities and incentives for early withdrawal on a purely precautionary basis are removed. Deposit insurance, however, creates a regulatory problem: as it is based upon transferring the risks of loss from depositors to the deposit insurance agency (DIA), regulation (as opposed to the market) has to undertake the role of disciplining banks.

Banking practitioners and scholars have widely recognized that the
Deposit insurance system is particularly exposed to moral hazard on the part of banks' owners.¹ Bank shareholders operate within the domain of limited liability and, in the absence of regulation, have much to gain and little to lose from raising insured deposits at a close to risk-free rate and investing the proceeds in high risk portfolios. Whereas very high returns are appropriated by bank shareholders, very low returns cause bankruptcy and the intervention of the deposit insurance agency, that bears the costs.

Deposit insurance premiums and regulatory constraints on bank decisions not only affect the funding of the DIA, but also the capability and the incentives of banks to take risk. Prudential regulation may influence bank behavior, stimulate or restrain risk-taking, affect the rate at which deposits are supplied, and determine both the probability of failure of the banks and the financial condition of the deposit insurance system.

As first noted by Merton (1977), the deposit insurance agency can be considered as the writer of a put option on bank assets with a strike price equal to the promised maturity value of deposits. The put option is sold to bank shareholders in exchange of a deposit insurance premium and the submission of the bank to capital and asset regulations. From Merton's seminal contribution, option valuation formulas have been used to assess deposit guarantees for both theoretical and empirical purposes.²

In that literature, perfect information by the insurer is

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¹ See White (1989) and Berlin et al (1991) for an overview of this issue.
implicitly assumed and the use of fair deposit insurance premiums is conceived as a feasible (and adequate) way to discipline banks. Surprisingly, however, little effort has been made in justifying those schemes within models that explicitly formalize the behavior of banks. Moreover, in spite of having closed-form formulas for the valuation of claims that, as those of bank shareholders, incorporate limited liability (Black and Scholes, 1973), this literature has paid small attention to the schemes really applied in practice: ordinary capital requirements, regulatory limits to risk-taking and, more recently, risk-weighted capital-to-asset ratios.

On the contrary, these issues have been addressed by some authors using mean-variance models that neglect limited liability (Kahane, 1977; Koehn and Santomero, 1980; Kim and Santomero, 1988).\(^3\)

The purpose of this paper is to analyze the impact of prudential regulation on the behavior of a perfectly competitive bank in a simple setup: a two-period model where bank deposits are fully-insured and bankers possess limited liability. In contrast with the dominant approach to very particular regulatory regimes, my aim is to provide a unified setting to examine different types of regulation, from the perfect-information risk-based proposal to the risk-insensitive or partly risk-sensitive schemes that exist in many countries.

The paper is organized as follows. Section 2 formalizes the decision problem of the bank under a general regulatory framework. Section 3 illustrates the need for regulation by analyzing the behavior of the bank when it faces a flat-rate deposit insurance premium and no other regulation exists. The most traditional regulation, consisting of complementing flat-rate premiums with risk-insensitive capital standards and asset restrictions, is analyzed in Section 4. Section 5 discusses

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\(^3\) Keeley and Furlong (1990) go over this remark.
the risk-based regulatory proposal. Concluding remarks are contained in Section 6.

2. THE DECISION PROBLEM OF A BANK.

The model that I present in this section formalizes the decision problem of a perfectly competitive bank in a two-period economy \((t=0,1)\). The bank is conceived as the investment project of a group of investors called bankers. Bankers are risk-neutral, have an initial wealth \(W_0\) and can borrow or lend on personal account at the risk-free rate of interest, \(r\). Bankers' investment in bank capital, \(K_0\), is protected by the standard limited liability provision of equity contracts.

At \(t=0\) the bank raises insured deposits \(D_0\) and capital \(K_0\) in order to invest the funds in a portfolio of risky assets. The gross return of the portfolio of assets \(R(\sigma_0)\) is a random variable with expected value \((1+r)\) and a variance that increases with \(\sigma_0\). In order to obtain a convenient closed form for the bank's objective function, I assume that \(R(\sigma_0)\) is log-normally distributed:

\[
R(\sigma_0) = (1+r)\exp(\sigma_0 z - \sigma_0^2/2), \tag{1}
\]

where \(z\) is a standard normal random variable. \(F(z)\) and \(f(z)\) will respectively denote the distribution and density functions of \(z\). The decision of the bank entails choosing \(D_0, K_0\) and \(\sigma_0\).

At \(t=0\) the bank incurs in two different costs per unit of deposits: the deposit insurance premium \(p\) and some intermediation costs \(c\) which are link to the provision of transaction services to depositors. \(c\) is a constant, while the specification of \(p\) varies across different regulatory regimes. In general \(p\) may depend upon bank decisions. We assume \(p+c<1\), so that each unit of raised deposits provides \(1-p-c\) units of funds for investment. If \(A_0\) denotes the value of the portfolio of assets at \(t=0\), the bank's budget constraint imposes:
\[ A_0 = (1-p-c)D_0 + K_0. \]  

(2)

At \( t=1 \), once asset returns are observed, the net worth of the bank, \( N_1 \), is computed as the difference between the value of assets, \( A_1 \), and liabilities, \( D_1 \). The value of assets results from applying the stochastic return \( R(\sigma) \) to \( A_0 \):

\[ A_1 = R(\sigma)\[(1-p-c)D_0 + K_0\]. \]  

(3)

Liabilities are made up of promised payments to depositors (principal plus interest). Since deposits are fully insured, their interest rate, \( r_D \), does not depend on default risk. I assume that \( r_D \) is taken as given by the bank:

\[ D_1 = (1+r_D)D_0. \]  

(4)

Putting together (3) and (4), we can write:

\[ N_1 = R(\sigma)[(1-p-c)D_0 + K_0] - (1+r_D)D_0. \]  

(5)

Because of limited liability, bankers receive \( N_1 \) only when positive. If \( N_1 \geq 0 \), the bank liquidates the assets, pays off the depositors, and \( N_1 \) accrues to bankers. If \( N_1 < 0 \), the DIA closes the bank, pays off the depositors and, as receiver, liquidates the assets; payments to bankers are zero. Thereby, bankers' payoffs can be represented by:

\[ \max\{N_1, 0\}. \]  

(6)

As \( N_1 = A_1 - D_1 \), the payoffs implied by equation (6) can be interpreted as those of a (European) call option on the bank's portfolio of assets with strike price \( D_1 \) and maturity \( t=1 \) (see Figure 1). The bankers only
FIGURE 1

Bankers' payoff at t=1 as a function of the return on assets
exercise their right to buy the assets when its value at the maturity date, \( A_1 \), is greater than the pre-established price, \( D_1 \). Not to exercise the option is equivalent to declaring bankruptcy.

Expression (6) can be decomposed in order to show the impact of limited liability:

\[
\max(N_1,0) = N_1 + \max(-N_1,0).
\]  \( \text{(7)} \)

As Merton noted, bankers are not only the owners of the bank's net worth (the first term) but also of a put option representing the value of limited liability (the second term). Under deposit insurance, the writer of such a put option is the DIA, which is responsible for paying-off depositors upon bank default.\(^4\)

Bankers seek to maximize the expected value of their final wealth, \( W_1 \), which is made up of the proceeds from the bank plus the return (or payment) associated to their net lending position on personal account:

\[
W_1 = \max(N_1,0) + (1+r)(W_0 - K_0).
\]  \( \text{(8)} \)

Taking expectations in (8) and dividing by \((1+r)\), we can verify that maximizing the expected value of \( W_1 \) is equivalent to maximize the net present value of bankers' investment in the bank, \( V_0 \):

\[
V_0 = (1+r)^{-1}E[\max(N_1,0)]-K_0 = E[\max((1+r)^{-1}N_1,0)]-K_0.
\]  \( \text{(9)} \)

Thereby, \( V_0 \) will be the objective function in the bank's decision problem.

\(^4\) In absence of deposit insurance, depositors would be the writers of the put and deposits would not be safe.
Obtaining a closed form for $V_0$ as a function of the decision variables entails the computation of $E[\max((1+r)^{-1}N_1,0)]$. To begin with, define $\mu_D$ as the discounted value of the difference between the risk-free interest rate and the deposit rate, $(r-r_D)/(1+r)$. Next plug equation (1) into (5) and divide the result by $(1+r)$ in order to get:

$$(1+r)^{-1}N_1 = \exp(\sigma_0 z - \sigma_0^2/2)((1-p-c)D_0 + K) - (1-\mu_D)D_0,$$  \hspace{1cm} (10)

that depends on $r$ and $r_D$ only through $\mu_D$.

From equation (10), a value $\bar{z}$ can be defined such that $N_1 \geq 0$ if and only if $z \geq \bar{z}$:

$$\bar{z} = \frac{1}{\sigma_0} \left[ \log((1-\mu_D)D_0) - \log((1-p-c)D_0 + K) + \sigma_0^2/2 \right].$$  \hspace{1cm} (11)

So, we can write:

$$E[\max((1+r)^{-1}N_1,0)] = \int_{\bar{z}}^{\infty} (1+r)^{-1}N_1 f(z) dz,$$

and, substituting (10) into this integral and integrating by parts, we obtain:

$$\left[ \int_{\bar{z}}^{\infty} \exp(\sigma_0 z - \sigma_0^2/2)f(z) dz \right] \left[ (1-p-c)D_0 + K \right] - (1-\mu_D)(1-F(\bar{z}))D_0.$$  \hspace{1cm} (12)

As $f(z)$ is the density function of a standard normal random variable, $\exp(\sigma_0 z - \sigma_0^2/2)f(z)=f(z-\sigma_0 z)$. Then, introducing the change of variable $w=\sigma_0 z$ and defining $x$ as $\sigma_0 - \bar{z}$, the integral in equation (12) can be written as $F(x)$, so that plugging the result in equation (9) yields:

$$V_0 = F(x)[(1-p-c)D_0 + K] - (1-\mu_D)F(x-\sigma_0 D_0 - K_0],$$  \hspace{1cm} (13)

where
\[ x = \left(\frac{1}{\sigma_0}\right) \log[(1-p-c)D_0 + K_0] - \log(1-\mu_D) - \log(D_0) + \sigma_0^2/2]. \] 

(14)

From equation (13), we can deduct that the net present value of the bank to bankers is made up of one positive and two negative components: (i) the (positive) value of assets, (ii) the (negative) value of payments to depositors, and (iii) the (negative) initial investment of capital. Because of limited liability, the value of assets to bankers results from computing the mean of the truncated log-normal random variable that yields \( R(\sigma_0) \) in non-bankruptcy states and zero otherwise. The value of deposits is clearly the present value of promised payments to depositors, \((1-\mu_D)D_0\), times the probability of the bank being solvent, \( F(x-\sigma_0) \). Finally, the initial capital investment enters directly in the equation since it takes place at \( t=0 \), no matter the bank being or not solvent at \( t=1 \). Notice that \( F(x-\sigma_0) \) is an expression that allow us to assess the solvency of the bank.

For notational convenience, define \( k_0 \) as the net capital to deposits ratio:

\[ k_0 = \frac{[K_0-(p+c)D_0]}{D_0} \] 

(15)

where I refer to net capital as the result of subtracting costs incurred at \( t=0 \) from the capital initially invested in the bank. Now, substituting \((k_0 + p+c)D_0\) for \( K_0 \) in equations (13) and (14) and defining \( y_0 \) as the vector \((D_0,k_0,\sigma_0)\), we can re-write \( V_0 \) as follows:

\[ V(y_0) = \left[(1+k_0)F(x) - (1-\mu_D)F(x-\sigma_0) - (k_0 + p+c)D_0\right]. \] 

(16)

where

\[ x = \left(\frac{1}{\sigma_0}\right) \log(1+k_0) - \log(1-\mu_D) + \sigma_0^2/2. \]

(17)

If we take \( y_0 \) as the vector of decision variables, the problem of the bank is to choose the bank's size \((D_0)\), capital structure \((k_0)\) and portfolio risk \((\sigma_0)\) in order to maximize \( V(y_0) \).
At this point, some properties of this objective function can be remarked:

(i) \( x \), as defined in (17), does not depend on \( D_0 \). Thereby the probability of bankruptcy depends on financial structure and portfolio risk, but it does not depend on size. This result comes from constant returns to scale in intermediation (constant \( c \)) and perfect competition (constant \( \mu_D \)).

(ii) The derivative of \( V(y) \) with respect to \( x \) is zero. In order to prove the result, notice that given the form of the density function of a standard normal random variable,

\[
\frac{\partial F(x-\sigma_0)}{\partial x} = f(x-\sigma_0) = \exp(\sigma_0 x-\sigma_0^2/2)f(x).
\]

Thus, differentiating (16) we get:

\[
\frac{\partial V}{\partial x} = [(1+k_0)-(1-\mu_D)\exp(\sigma_0 x-\sigma_0^2/2)]f(x)D_0.
\]

But, from the definition of \( x \), \( \exp(\sigma_0 x-\sigma_0^2/2) \) is equal to \((1+k_0)/(1-\mu_D)\), so that the term in square brackets and, hence, the derivative are zero.

(iii) According to equation (7), the value of the bank can be decomposed as the sum of the value of an equivalent fully-liable bank, \( V_A(y_0) \), and the value of limited liability, \( V_B(y_0) \):

\[
V(y_0) = V_A(y_0) + V_B(y_0),
\]

where

\[
V_A(y_0) = (\mu_D-p-c)D_0, \quad (18)
\]

\[
V_B(y_0) = ((1-\mu_D)[1-F(x-\sigma_0)] - (1+k_0)[1-F(x)])D_0. \quad (19)
\]

The expression for \( V_A(y_0) \) can be obtained as a result of setting \( F(x) = F(x-\sigma_0) = 1 \) in equation (16), since a fully-liable bank would never
go bankrupt. The expression for $V_B(y_0)$ arises when $V_A(y_0)$ is subtracted from $V(y_0)$. $V_B(y_0)$ is an interesting re-statement, in terms of the notation used throughout this paper, of Merton's formula for valuing deposit guarantees (Merton, 1977). Clearly, $V_B(y_0)$ has as $V(y_0)$ the property of its partial derivative with respect to $x$ being zero.

(iv) Notice that $V_A(y_0)$ depends on $k_0$ and $\sigma_0$ only if $p$ does. With a constant or null $p$, $V_A(y_0)$ only depends on $D_0$. This means that, for the fully-liable bank, financial structure and portfolio risk are irrelevant as in the Theorem of Modigliani and Miller (1958). On the contrary, the contribution of limited liability is increasing and linear in $D_0$, and depends negative but non-linearly on $k_0$ and positive but non-linearly on $\sigma_0$. These properties are clearly related to the option-like returns of limited liability, which are higher the lower the (negative) conditional-on-bankruptcy net worth of the bank.

(v) Finally, the decomposition above allow us to write an expression for $V(y_0)$ that will very clearly show the contribution of $V_B(y_0)$ to the value of the bank and its potential effect on bank behavior:

$$V(y_0) = (\mu_D - c)D_0 + [V_B(y_0) - pD_0].$$ (20)

Although I will be more precise in next sections, the sort of regulatory devices analyzed in this paper consist of constraints upon bank decisions and transfers of funds between the bank and the DIA at $t=0$. The transfers of funds between the bank and the DIA are conducted through the deposit insurance premium, whereas the regulatory constraints on bank behavior (capital requirements and restrictions affecting the composition of bank portfolios) are included (together with the range of variation of the decision variables) in the definition of the set of feasible decision vectors.

If we denote the premium setting function by $p(y_0)$ and the set of feasible controls by $\Gamma \subset \mathbb{R}^3$, the decision problem of the bank can be formalized as
\[
\text{Maximize } V(y_0 \mid p=p(y_0)), \quad y_0 \in \Gamma.
\]

The properties of \( V(y_0 \mid p=p(y_0)) \) and the (regulatory) definition of \( \Gamma \) will be crucial in determining the behavior of the bank.

3. THE BEHAVIOR OF AN UNREGULATED BANK.

Assume that the deposit insurance premium is a constant \( \bar{p} \) and the feasible set \( \Gamma \) only specifies the range of variation of \( D_0, k_0 \) and \( \sigma_0 \).

In particular,

\[
\Gamma = \left\{ y_0 \in \mathbb{R}^3 \mid D_0 \geq 0, k_0 \geq (\bar{p}+c), \sigma_0 \geq 0 \right\},
\]

where the lower bound to \( k_0 \) comes from requiring \( K \geq 0 \) (see equation (15)), and \( \sigma \) stands for the volatility of the riskiest portfolio of assets in the economy. \( \sigma \) is finite provided that the returns on existing assets have finite variance and short-selling is forbidden.

From equation (16), we can compute the partial derivatives of \( V(y_0) \) when \( p=\bar{p} \):

\[
\frac{\partial V}{\partial D_0} - [(1+k_0)\mathcal{F}(x)-(1-\mu_0)\mathcal{F}(x-\sigma_0)-(k_0+\bar{p}+c)] \geq 0,
\]

\[
\frac{\partial V}{\partial k_0} = [\mathcal{F}(x)-1]D_0 \leq 0,
\]

\[
\frac{\partial V}{\partial \sigma_0} = (1-\mu_0)f(x-\sigma_0)D_0 \geq 0.
\]

It is immediate that \( V(y_0) \) decreases with \( k_0 \) and increases with \( \sigma_0 \), reflecting that the decisions of the bank on capital structure and portfolio risk are led by the limited liability component of its value (see equation (20)). The flat-rate deposit insurance system allows the
bank to appropriate this value, whatever its significance, by paying a constant rate of interest and a constant premium per unit of deposits. Accordingly, the optimal decision of the bank, \( y^* \), involves \( k^* = -(p+c) \) and \( \sigma_0^* = \overline{\sigma} \), so that both the lower bound to \( k_0 \) and the upper bound to \( \sigma_0 \) are binding.

Equation (22) implies that \( V(y_0) \) is a linear function of \( D_0 \) whose slope may be positive or negative. However, for the optimal values of \( k_0 \) and \( \sigma_0 \), the sign of (22) is positive, since the last term in (22) cancels out when \( k_0 = -(\overline{p}+c) \) whereas the others give the value (per unit of deposits) of the call option representing bankers' payoffs at \( t=1 \), which, by definition, cannot be negative:

\[
\frac{\partial V}{\partial D_0} \bigg|_{y^*_0} = [(1-\overline{p}-c)F(x)-(1-\mu_D)F(x-\overline{\sigma})] > 0.
\]

Consequently, for any meaningful values of \( p, c, \mu_D \) and \( \overline{\sigma} \), the bank wants to raise an infinite amount of deposits. The intuition is clear. For the uncapitalized bank, the flat-rate deposit insurance scheme comprises an opportunity to obtain unlimited profits: bankers' wealth at stake is zero, whilst the proceeds of investing insured deposits in risky assets are positive. Competition for deposits would lead \( \mu_D \) to \(-\infty\). In such conditions, no equilibrium can exist.

Instead of inspecting notably different setups -where, by means of introducing risk-aversion and market power into the analysis, we might force the conclusion that unregulated banking systems are compatible with equilibrium- I will stick to the benchmark case described above, examining whether alternative premium-setting functions or some regulatory constraints on bank decisions can solve the problem. As we will verify in the next section, the unboundedness of the solution of the bank's problem disappears -for some finite values of \( \mu_D \)- as soon as capital requirements or risk-based deposit insurance premiums are introduced.
4. THE TRADITIONAL RISK-INSENSITIVE REGULATION.

For many years, prudential regulation in US and Europe has been based on flat-rate deposit insurance premiums. Moreover, many central banks afforded implicit insurance to bank deposits in spite of the lack of formal arrangements regulating these guarantees and, consequently, the contribution of the banks to its funding. Still, most banking systems are subject to regulatory constraints that affect decisions on assets and liabilities. Capital requirements and restrictions on portfolio composition, off-balance-sheet operations, short-selling, sectorial and geographical concentration of lending and so forth have been broadly used. Fully risk-insensitive capital and asset regulations were dominant throughout the world until the introduction of the risk-based capital adequacy standards resulting from the Basle Accord (BIS, 1988) and the broader directives of the European Community (EC, 1989a, 1989b).

Nominally, the purpose of asset and liability regulations is to promote proper risk-taking by banks, reducing the incidence of failures. They aim, on the one hand, to limit the dead-weight losses associated with raising the public funds used to cover potential deficits of the DIA and, on the other, to ameliorate the residual external costs of bank failures to the financial system (already reduced by the existence of deposit insurance).

4.1. Introducing capital requirements and limits to asset risk.

In this section, deposit insurance premiums are assumed to be constant, \( p = \bar{p} \), and capital requirements are formalized as a lower bound to the capital ratio, \( k_0 \geq \bar{k} \), where \( \bar{k} > (\bar{p} + c) \). Restrictions on portfolio composition are represented by \( \sigma \) which is now assumed to be controlled by the regulator. Formally,

\[
\Gamma = \{ y_0 \in \mathbb{R}^3 \mid D_0 \geq 0, \, k_0 \geq \bar{k} - (\bar{p} + c), \, \sigma \geq \sigma_0 \geq 0 \},
\]

where \( \bar{k} \) and \( \bar{\sigma} \) are regulatory parameters. Risk-insensitiveness is
reflected by the fact that $\bar{p}$, $\bar{k}$ and $\bar{\sigma}$ are constants that, as such, do not depend on bank decisions. Note that imposing a minimum net capital to deposits ratio $\bar{k}$ is equivalent to imposing a minimum net capital to assets ratio $\gamma$, where $\gamma = \bar{k}/(1+\bar{k})$.

As regulatory constraints affecting decisions on financial structure and portfolio risk remain independent from each other, partial derivatives obtained in equations (22), (23) and (24) still guide the determination of the optimal solution to the problem of the bank. In particular, as $V(y_o | p = \bar{p})$ is decreasing in $k_o$ and increasing in $\sigma_o$, both the capital requirement and the limit to portfolio risk are binding, so that $k^* = \bar{k}$ and $\sigma^* = \bar{\sigma}$.

The key difference with respect to the unregulated case is that $\bar{k} + \bar{p} + c > 0$ and, then, there always exits a value $\mu_0^*$ such that the partial derivative of $V(y_o | p = \bar{p})$ with respect to $D_0$ is zero:

\[(1+\bar{k})F(x) - (1-\mu_0^*)F(x-\bar{\sigma}) - (\bar{k} + \bar{p} + c) = 0. \tag{25}\]

Notice that, on the one hand, as $\mu_0$ tends to 1, $F(x)$ and $F(x-\bar{\sigma})$ tend to 1 and, then, the left hand side of this equation goes to $(1-\bar{p}-c) > 0$. On the other hand, as $\mu_0$ tends to $-\infty$ (i.e. $r_0$ tends to infinity, $r$ being finite), the first and second terms in the left hand side tend to zero,

---

5 Suppose a net capital to assets requirement of the form:

\[\frac{K_o - (p+c)D_0}{(1-p-c)D_0 + K_0} \geq \gamma\]

Solving for $K_o$ we can write $K_o \geq [(p+c) + (1-\gamma)^{-1}\gamma]D_o$. From the definition of $k_o^*$, an equivalent inequality is $k_o^* \geq \gamma/(1-\gamma)$. So, setting $\gamma = \bar{k}/(1+\bar{k})$, we can obtain $k_o^* = \bar{k}$.
whilst the third is negative.\(^6\) Thus, by continuity, there exists a finite \(\mu_D^e\) that solves (24). Moreover, \(\mu_D^e\) is unique because the left hand side of (25) is increasing in \(\mu_D^e\).

Accordingly, the supply of deposits is infinite for \(\mu_D^e > \mu_D^e\), zero for \(\mu_D^e < \mu_D^e\) and any positive value \(D_0 \in (0, \infty)\) for \(\mu_D^e = \mu_D^e\). Therefore, as in the usual perfectly competitive firm with constant returns to scale, the zero profit condition given by equation (25) affords an implicit definition of a horizontal deposit supply curve.

Vertical movements of this horizontal supply curve can be obtained by totally differentiating (25):

\[
F(x-\sigma)d\mu_D^e - [1-F(x)]d\bar{k} + (1-\mu_D^e)f(x-\sigma)d\sigma - dp - dc = 0.
\]

The margin at which the supply of deposits is positive and finite, \(\mu_D^e\), increases with \(\bar{k}, \bar{p}\) and \(c\), whereas decreases with \(\bar{\sigma}\). This means that tighter regulation makes banking activity more costly so that the margin per unit of deposits which is necessary to compensate bank shareholders for their capital investment rises.

Figure 2 represents the deposit supply curve of the bank as a function of \(\mu_D\) and the variation of \(\mu_D^e\) with the parameters. Clearly, the equilibrium in the market for deposits could be fully characterized by adding the corresponding (downward sloping) demand curve for deposits.

\(^6\) Notice that \((1+\bar{k})F(x)\) tends to zero when \(\mu_D^e\) tends to \(-\infty\), because \(F(x)\) tends to zero. Observe also that \((1+\bar{k})F(x)-(1-\mu_D^e)f(x-\bar{\sigma})\) can never be negative, since by definition it is the value (per unit of deposits) of the call option that represents bankers' payoffs at \(t=1\). In addition, \((1-\mu_D^e)f(x-\bar{\sigma})>0\) for all \(\mu_D^e<1\). So if the limit of \((1-\mu_D^e)f(x-\bar{\sigma})\) were different from zero a contradiction would be obtained.
FIGURE 2

Deposit Supply under Risk-Insensitive Regulation
4.2. Fair flat-rate premiums.

Let us briefly examine the case in which the regulator—knowing that the optimal behavior of the bank involves $k^* = k$ and $\sigma^* = \sigma$, and leads to $\mu_D^e = \mu_D^e$—fixes the flat-rate deposit insurance premium so as to absorb the value of limited liability, $\mu_D^e = V_B(y^*)$. From equation (19) and given that $V_B(y)$ is linear in $D$, we can write:

$$\overline{p} = (1 - \mu_D^e)(1 - F(x - \sigma)) - (1 + k)(1 - F(x)).$$

If we plug this expression for $\overline{p}$ in equation (25), terms cancel out and a very simple solution for $\mu_D^e$ is obtained:

$$\mu_D^e = c$$

Intuitively, as the premium absorbs $V_B(y)$, the value of the banking activity is reduced to the profits of pure intermediation in deposits, $(\mu_D^e - c)D_0$ (see equation (20)). In equilibrium, where the supply of deposits has to be finite, perfect competition leads such profits to zero, yielding (27).

From this result we can deduct that if the flat premium is set below its fair value, the implicit subsidy is transferred to depositors, yielding an equilibrium intermediation margin which is smaller than the marginal cost of intermediation, $\mu_D^e < c$. The opposite result is obtained if the flat premium is greater than its fair value.

Equation (26) implies that the fair deposit insurance premium in this context is a decreasing function of $k$ and $\mu_D^e$ (i.e., of $c$), and an increasing function of $\sigma$. So, if the regulator wants the DIA to break even, a trade-off between flat premiums and complementary capital and asset regulations exists: the higher $\overline{p}$, the softer the required complementary constraints.

Notice that, with the ex-post fair deposit insurance premium, the
DIA can be funded on a *no subsidy nor profits basis*, but the behavior of the bank is still biased towards risk because no incentives are provided in order to correct the distortions of limited liability. On the contrary, solvency is promoted by imposing direct limits to the operative capacity of the bank.

For the fair $\tilde{p}$, the probability of survival of the bank has the following expression:

$$F(x-\sigma) = F\left(\frac{1}{\sigma}\left[\log(1+k)-\log(1-1l)+c\right]^{2/\sigma}\right).$$

Hence, the solvency of the bank increases with $k$ and $\mu^e_0$ (i.e. with $c$), and decreases with $\sigma$. Then, tighter regulation by constraining financial and investment decisions curtails the operative capacity of the bank, but enhances its solvency.

To sum up, fair flat-rate deposit insurance premiums do not correct the distortions created by limited liability. However, as the insurer anticipates that the insured bank will maximize $V(y_0)$ up to the limits imposed by $k$ and $\sigma$, she can establish $\tilde{p}$ so as to assure that the collected premiums equate the expected discounted value of her liabilities and deposits are provided at the marginal cost of intermediation. Any insured bank intending to pursue a more prudent policy (say, $k_0 > k$ or $\sigma_0 < \sigma$) would behave sub-optimally and could not pay as high an interest rate on deposits as those of its competitors. Actually, its optimal supply of deposits at the market intermediation margin $\mu^e_0$ would be zero.

5. THE RISK-SENSITIVE REGULATION.

5.1. The rationale for risk-based regulation.

The intuition underlying the risk-based regulatory proposal is very simple. Let regulatory restrictions affecting a bank being contingent on
its behavior. More concretely, define the premium setting function \( p(y_0) \) and the feasible set \( \Gamma \) in such a way that \( V_B(y_0) \) be absorbed by \( p(y_0)D_0 \) for all feasible \( y_0 \):

\[
V_B(y_0) - p(y_0)D_0 = 0 \quad \text{for all } y_0 \in \Gamma.
\] (28)

Then, the bankers will be indifferent among the financial structures and portfolio compositions in \( \Gamma \).

From equation (20), the bank’s objective function under the risk-sensitive regulation defined in (28) has the following expression for all \( y_0 \in \Gamma \):

\[
V(y_0 | p=p(y_0)) = (\mu_d - c)D_0.
\]

This means that the value of the bank to bankers comes from the profits from intermediation as for a fully-liaible bank. Clearly, as it does not depend on \( k_0 \) and \( \sigma_0 \), the Modigliani-Miller Theorem holds. On the other hand, the optimal supply of deposits is any positive and finite value for \( \mu_d = c \), infinite for \( \mu_d > c \) and zero for \( \mu_d < c \). Then \( \mu^e = c \).

Notice that the crucial difference between risk-insensitive and risk-based regulation is not in the form of the deposit supply, nor in the determination of \( \mu^e_d \) (which is \( c \) in both cases when flat-rate premiums are \( \text{fair} \)), nor in the fairness of the premiums, but in the (desirable) irrelevance of financial structure and portfolio risk that characterizes the latter but not the former. Thereby, we can conclude that when banking regulation provides the adequate incentives, limited liability and deposit insurance may be innocuous in the sense that, although bankruptcy is possible and depositors are insured, optimal bank decisions are the same as those of a (fictitious) fully-liaible bank.

5.2. Risk-based premiums versus risk-based capital requirements.

Variable deposit insurance premiums are the most direct way to implement
(28). No other regulation is needed in such a case. Setting $\mu_D = c$ in (19), the premiums can be fixed according to the following schedule:

$$p(y_0) = (1-c)[1-F(x-\sigma_0)] - (1+k_0)[1-F(x)]$$

and the feasible set can be that of an unregulated bank. The partial derivatives of $p(y_0)$ show that it is decreasing in $k_0$ and increasing in $\sigma_0$. $p(y_0)$ is decreasing in $c$ because when $c$ goes up the equilibrium intermediation margin rises, enhancing the solvency of the bank.

Figure 3 depicts the contour plot of function $p(y_0)$ in the $(k_0, \sigma_0)$ space, for $0<k_0 \leq 0.20$ and $0<\sigma_0 \leq 0.25$. In this figure, $c$ takes value 0.03, but the results are qualitatively identical for other values. The isopremium curves are clearly upward sloping, but the upper contour sets are not convex nor concave in general. Contour sets with higher $p(y_0)$ values can be reached moving up and to the left. The figure shows that the risk-based premium schedule is non-linear, so that linear schemes (sometimes included in practical proposals of banking reform) can hardly approximate it.

As an alternative to variable premiums, authors as Sharpe (1978), Ronn and Verma (1989), and Mullins and Pyle (1990) have suggested that, with a flat-rate premium $p = \bar{p}$, risk-based regulation could be conducted through risk-sensitive capital requirements. In our framework, these would have the form:

$$k_0 \geq g(\sigma_0),$$

where $g(\sigma_0)$ solves

$$\bar{p} = (1-c)[1-F(x-\sigma_0)] - (1+g(\sigma_0))[1-F(x)],$$

with

$$\bar{p} = (1-c)[1-F(x-\sigma_0)] - (1+g(\sigma_0))[1-F(x)],$$

(29)
FIGURE 3

Contours of $p(y_0)$ in the $(k_0, \sigma_0)$ space
$(c=0.03)$

$(p_1=0.0019, \ p_2=0.0228)$
\[
x = \frac{1}{\sigma_0} \{ \log[1 + g(\sigma_0)] - \log(1 - c) + \sigma_0^2/2 \}.
\]

Equation (29) defines an implicit function \( g(\sigma_0) \). Total differentiation in (29) shows that this function is increasing, since \( g'(\sigma_0) = (1-c)f(x-\sigma)/\{1-F(x)\} > 0 \). The function moves down when \( \bar{p} \) or \( c \) go up. Intuitively, as both asset risk and leverage rise the value of the deposit guarantee, the risk-based capital requirement reduces maximum allowed leverage in response to increases in \( \sigma_0 \) so as to assure \( V_B(y_0) = \bar{p}D_0 \) for all feasible \( y_0 \).

Imposing such a risk-based capital requirement is not, however, equivalent to charging variable premiums. Recall that with flat-rate premiums, the bank's objective function is decreasing in \( k \) and increasing in \( \sigma \). Suppose that the bank is subject to the constraint \( k_0 = g(\sigma_0) \). Then, from the definition of \( g(\sigma_0) \), the bank is indifferent between any \((k_0, \sigma_0)\) verifying \( k_0 = g(\sigma_0) \), i.e. any point belonging to the isopremium curve corresponding to \( \bar{p} \) in Figure 3. However, it clearly prefers any of these choices to any \((k', \sigma')\) such that \( k' > g(\sigma') \), i.e. any point below and to the right of this curve. Consequently, the risk-based capital requirement is always binding.

Whereas with risk-based premiums bankers are indifferent among all feasible choices of \((k_0, \sigma_0)\), with risk-based capital requirements indifference applies only to choices in a same isopremium curve, and those on the curve \( \bar{p} \) are the only optimal ones. By introducing an additional constraint in the decision problem, the capital requirement guarantees that the bank will not choose at once too high leverage and portfolio risk, but in exchange makes decisions involving low leverage and low risk \((k_0 > g(\sigma_0))\) being sub-optimal. Those decisions could have been chosen under risk-based premiums.

In spite of this rigidity, risk-based capital requirements are remarkably more flexible than the risk-insensitive regulation analyzed in Section 4, since under risk-insensitive deposit insurance premiums,
minimum capital ratios and upper bounds to \( \sigma_0 \), the bank makes \((k_0, \sigma_0) = (\bar{k}, \sigma)\). That is, in terms of Figure 3, the set of optimal decisions contains only one point: \((\bar{k}, \sigma)\). Note that if the flat premium \( \bar{p} \) is actuarially fair, the isopremium curve containing \((\bar{k}, \sigma)\) will correspond to \( p(y_0) = \bar{p} \).

Either if based upon deposit insurance premiums or solvency ratios, establishing a risk-sensitive regulation requires two necessary conditions. The first is to observe the decisions of the bank, the second is to be able to enforce relatively complex non-linear regulatory rules. In some informational and institutional contexts, these conditions clearly fail.

If the regulator cannot observe \( \sigma_0 \) or the timing of events is such that the bank chooses \( \sigma_0 \) once premiums and solvency ratios have been imposed, then premium schedules and capital requirements cannot not be contingent on \( \sigma_0 \), and the bank chooses the portfolio with the maximum feasible level of risk. Correcting such a moral hazard problem would probably require regulatory instruments that are not explicitly considered in this paper. Agency theory suggests that the solution to moral hazard might consist in modifying bankers' payoff, i.e. introducing ex-post transfers such as taxes on bank profits, penalties and prizes.\(^7\)

In practice, difficulties proceed from the need of compatibilizing the imperfect measurement of \( \sigma_0 \) by the regulator and the complexity of theoretic risk-based schedules with the precise legal terms in which risk-based regulation has to be dictated. As a consequence, the regulator may opt for risk-insensitive schemes as those of Section 4 or

\(^7\) John, John and Senbet (1991) examine corporate taxation as a possible way to induce safe decisions in a context of asymmetric information on bank actions.
simplified risk-based regulations as those of the Basle Accord on capital standards.

5.3. Risk-weighted capital requirements and the Basle Accord.

The approval of the Basle proposal on capital standards has meant the major worldwide change in banking regulation of the last decades. Most of the developed countries, including those of the G-10 and the European Community, have reorganized their capital adequacy rules in order to introduce risk-weighted capital requirements as a complement to flat-rate deposit insurance premiums. In a risk-weighted capital requirement, capital is related to a combination (usually a sum) of different asset or off-balance sheet exposures, weighted according to their relative riskiness.

This section analyzes a general class of risk-weighted capital requirements, which includes, as a particular case, those of the Basle Accord. After extending the model of the previous sections to be explicit on the composition of the bank's asset portfolio, I characterize a risk-weighted capital requirement designed to make the bank indifferent between assets with the same expected return but different levels of risk. Such solvency ratio would mimic the properties of the risk-based capital ratio defined in equation (29). The results suggest that linear risk-weighted ratios as those introduced in many countries might not provide an adequate trade-off between portfolio risk and leverage and, consequently, might bias the portfolio choice of the banks towards specific asset categories.

To obtain a closed form for the bank's objective function when a variety of assets is considered, it is very convenient to adopt a specification such that the gross return on the portfolio of assets is still log-normally distributed. Merton (1971, 1990) showed that in a continuous-time framework, whenever log-normality of gross returns for individual assets is assumed, the gross return on any portfolio with a constant composition between \( t=0 \) and \( t=1 \) will also be log-normally
Assume that the asset portfolio of the bank is composed of \( n \) different assets, \( i=1,2,\ldots,n \), whose gross returns, \( R_i \), are log-normally distributed:

\[
R_i = (1+r)\exp(\sigma_i z_i - \frac{\sigma_i^2}{2}),
\]

where \( z=(z_1,\ldots,z_n) \) is vector of standard normal random variables with zero mean, unit variance and a correlation matrix \( P=[\rho_{ij}] \). All the assets have an expected return \( (1+r) \) and \( \rho_{ij} \) can be interpreted as the correlation between the (log) returns on assets \( i \) and \( j \):

\[
\text{Var}[\log R_i, \log R_j] = (1+r)\sigma_i^2 \rho_{ij}.
\]

Denote the fraction of the bank's total assets invested in asset \( i \) at \( t=0 \) by \( w_i \), where \( 0 \leq w_i \leq 1 \) and \( \sum_{i=1}^{n} w_i = 1 \). Following Merton (1990), pp. 132-6, the bank's portfolio can be thought of as a composite asset with a log-normal gross return defined by:

\[
R(w) = (1+r)\exp[\sigma(w)z - \frac{\sigma^2(w)}{2}],
\]

where

\[
w=(w_1,\ldots,w_n),
\]

\[
\sigma^2(w) = w'\Sigma w,
\]

\[8\] In Merton's result, continuous-time is necessary in order to allow for a continuous rebalancing of the portfolio to some initial weightings \( w \). If such rebalancing did not take place, log-normality can still be used as an approximation. This approximation is commonly used for valuing options on a portfolio of stocks.
Finally, assume that $p=p^\circ$ and the capital requirement establishes
the constraint

$$k_0 \geq k(w), \tag{32}$$

where $k(w)$ is a function to be defined.

Constraint (32) may represent the type of minimum constant capital
to weighted-assets ratios commonly specified in actual regulation. If
weighted assets are $h(w)A_0$ and the minimum ratio is $\gamma$, the corresponding
capital to weighted-assets ratio is equivalent to (32) if

$$k(w) = (1 - \gamma h(w))^{-1} \gamma h(w).$$

Roughly speaking, the Basle proposal sets $\gamma = 0.08$ and defines a linear function $h(w) = \sum_{i=1}^{n} \alpha_i w_i$, where the $\alpha_i$ coefficients
are 0, 0.10, 0.20, 0.50 or 1 according to the risk category where each
asset (or off-balance-sheet exposure) is classified.

At this point, we can substitute $w$ for $\sigma_0$ in the vector of decision
variables and $\sigma(w)$ for $\sigma_0$ in the bank's objective function. The
optimization problem of the bank in this framework is as follows:

$$\begin{align*}
\text{Maximize} & \quad V(y_0 | p=p^\circ), \\
\text{subject to} & \quad y_0 \in \Gamma,
\end{align*}$$

where

$$\Gamma = \{ y_0 \in \mathbb{R}^{2 \times N} \mid D_0 \geq 0, k_0 \geq k(w), 0 \leq w_i \leq 1, \sum_{i=1}^{n} w_i = 1 \}. $$

---

9 See Footnote 5.
In order to analyze the solution to this problem, notice that the objective function is decreasing in $k_0$, as in previous sections. Hence, the capital requirement will be binding and we can substitute $k(w)$ for $k_0$ in the objective function. The resulting expression is linear in $D_0$ and depends on $w$ only through $k(w)$ and $\sigma(w)$.

Now, we are interested in characterizing a risk-weighted capital requirement $k(w)$ such that the optimal portfolio choice is indeterminate, as it would be for a risk-neutral fully-liable bank. From the definition of the risk-based capital requirement in section 5.2, we can simply make $k(w) = g(\sigma(w))$, where $g(\sigma)$ is the function implicitly defined in equation (29).

The partial derivative of $k(w)$ with respect to $w_i$ is:

$$\frac{\partial k}{\partial w_i} = \frac{dg(\sigma(w))}{d\sigma(w)} \cdot \frac{\partial \sigma(w)}{\partial w_i}$$

The first factor can be obtained differentiating in (29):

$$\frac{dg(\sigma(w))}{d\sigma(w)} = (1-\mu) \frac{f[x-\sigma(w)]}{1-F(x)} > 0.$$  

On the other hand, from the definition of $\sigma^2(w)$,

$$\frac{\partial \sigma(w)}{\partial w_i} = \frac{1}{2\sigma(w)} \cdot \frac{\partial \sigma^2(w)}{\partial w_i} = \frac{1}{\sigma(w)} \cdot [\sum_{j=1}^{n} \sigma_{ij} w_j]$$

Thus, we can write:

$$\frac{\partial k}{\partial w_i} = (1-\mu) \frac{f[x-\sigma(w)]}{1-F(x)} \cdot \sigma(w) \cdot \beta_{ir}(w), \quad (33)$$

where
Equation (33) says that the additional capital to be required per unit of deposits when the fraction of total assets invested in a particular asset increases must be proportional to the contribution of the asset to total portfolio risk. The contribution of asset $i$ to total portfolio risk is measured by $\beta_{iR}(w)$, i.e. the regression coefficient of asset $i$ returns on the returns of the bank's portfolio.

The risk-weighted capital requirement should penalize the investment in risky assets according to their relative riskiness, which cannot be measured on an asset-to-asset basis, but taking into account the covariance of the returns on an asset and the returns on the other assets as well as the composition of the bank's portfolio. Consider, for instance, an asset $i$ whose returns have a big variance but, for a given $w$, are negatively correlated with the bank's portfolio. Then, given this $w$, the risk-based capital requirement should be lower, the higher the investment in asset $i$. This is only, however, a local statement, since $\beta_{iR}(w)$ is increasing in $w_i$. Accordingly, if $w_i$ goes up, a point is reached where $\beta_{iR}(w)$ becomes positive and the sign of $\partial k / \partial w_i$ changes from negative to positive.

Classifying assets into broad categories of presumptive relative riskiness and assigning constant weights to each category is not a good approximation to what equation (33) suggests. That is true even if the returns of different assets are assumed to be uncorrelated. If this were the case, equation (33) would become:

$$
\frac{\partial k}{\partial w_i} = (1-\mu_x) f\{x-\sigma(x)\} \cdot \sigma(w) \cdot \tau_1(w) \cdot w_i,
$$

where $\tau_1(w)$ is the variance ratio $\sigma_i^2 / \sigma(w)^2$. It is worth noting that $w_i$ enters in the right hand side, so that ceteris paribus concentration in
a particular asset \( i \) would be penalized. Clearly, with uncorrelated assets, concentration impedes diversification and increases total risk.

From a practical viewpoint, quadratic weighting functions that were sensitive to the variances and covariances of asset returns would be more appropriate than linear weighting functions applied nowadays, specially after the approval of the Basle Accord.

6. CONCLUSIONS.

In the vast literature originated by Merton's (1977) discovery of the isomorphic relationship between deposit insurance and a put option, small attention has been paid to the behavior of banks. Actuarially fair deposit insurance pricing has been purposed, but no systematic analysis of bank decisions in different regulatory frameworks has been made. Notwithstanding, the option theoretic approach affords an adequate treatment of limited liability, which is typically neglected in mean-variance models of bank behavior.

In this paper, I have formalized the decision problem of a perfectly competitive bank in a two-period model where deposits are fully-insured and bankers have limited liability.

Bankers seek to maximize the expected value of their final wealth, which entails taking decisions so as to maximize the present expected value of their investment in the bank, i.e. final payoffs minus the initial investment of capital. Final payoffs equal the bank's net worth when positive, and zero otherwise.

The valuation of this option-like payoffs allow us to obtain a closed form for the bank's objective function. Bankers' problem can be written as consisting in choosing the bank's supply of deposits, capital structure and portfolio risk. Regulation enters as a set of constraints in the optimization problem. Deposit insurance premiums, capital
requirements and limits to the risk of bank assets are the three instruments considered in this paper.

When the only regulatory device is a flat-rate deposit insurance premium, the behavior of the bank is governed by the limited liability component of its value, and causes the collapse of the deposit insurance system. Profits are maximized by putting no capital, investing in the riskiest portfolio of assets and supplying an infinite amount of deposits.

When a minimum capital ratio is required, the limited liability component of the bank's value is no longer obtained for free, but in exchange of bankers' capital. A finite solution to the optimization problem can be found at an equilibrium intermediation margin that depends positively on the stringency of capital and asset restrictions, the deposit insurance premium and the cost of intermediation.

The difference between risk-sensitive and risk-insensitive regulation is not in the fairness of the deposit insurance premium (as often alleged in the literature) nor in the deposit supply schedule, but in their impact on the financial and investment decisions of banks. Risk-based premiums make the bank indifferent among alternative capital structures and levels of portfolio risk, and the Modigliani-Miller Theorem holds in contradistinction to risk-insensitive regulation, that yields maximum leverage and portfolio risk.

Direct restrictions on leverage and risk may limit the exposure of the deposit insurance system, but at the price of curtailing the operative range of the intermediary.

As an intermediate case in terms of flexibility, risk-based capital requirements allow the bank to choose within a specific schedule of capital ratios and portfolio risk. When the schedule is properly designed, the bank is indifferent between the points in it, but raising capital in excess of the amount required in accord with portfolio risk.
is never optimal.

In practice, the difficult observation and measurement of risk and the need for accuracy in actual regulation entangles the implementation of risk-sensitivity. Nevertheless, risk-weighted capital requirements recently introduced represent a remarkable attempt in that direction.

Our characterization of risk-weighted capital requirements suggests that linear weighting schemes as those contained in the Basle Accord are far from introducing an adequate trade-off between capitalization and portfolio risk. Risk-weighted solvency ratios which could reproduce the desirable properties of risk-based regulation should penalize the investment in risky assets according to their marginal contribution to the riskiness of the bank's portfolio. Such contribution should be measured by the regression coefficient of each asset's return on the returns of the bank's portfolio. Accordingly, weighting functions should be sensitive to the variances and covariances of asset returns, and not only to univariate risk measures as those considered in the Basle Accord.
REFERENCES


CHAPTER 3

BANKING REGULATION IN AN EQUILIBRIUM MODEL
1. INTRODUCTION.

Corporate finance literature has long recognized that limited liability distorts the investment decisions of indebted firms (Jensen and Meckling, 1976; Myers, 1977). In such a context, debt contracts typically incorporate special covenants designed to protect the lenders and, simultaneously, to avoid the inefficiencies linked to risk-shifting. In the banking industry, however, small depositors have neither the incentive nor, probably, the competence to collect information and to intervene into bank management. Moreover, their reactions take the form of deposit withdrawals, which, given the illiquid nature of most bank assets, can give rise to problems of contagious bank runs and panics (Diamond and Dybvig, 1983).

Deposit insurance is intended to protect small depositors that can be affected by asymmetric information problems and to promote financial stability that can be threatened by panics. However, as it eliminates depositors' discipline by transferring the risk of loss to the insuring agency, it may aggravate the risk-shifting incentives of depository institutions. Banking regulation is a way through which the insurer undertakes precautionary action in order to limit the harmful effects of opportunistic bank behavior.

Extant literature has widely referred to this issue as "the deposit insurance moral hazard problem". Deposit insurance has been modeled as a put option held by the bank's shareholders (Merton, 1977). From here, it follows that banks facing a flat-rate deposit insurance premium have an incentive to maximize leverage and asset risk. In particular, Kareken and Wallace (1978) argued that, when banks are perfectly competitive value maximizers and bankruptcy generates external (but not internal) costs, a flat-rate deposit insurance scheme causes distortions in the behavior of banks and leads to efficiency losses.
Nonetheless, following the corporate finance literature which examines risk-shifting behavior, the moral hazard problem affecting banks and the corrective impact of regulation have generally been analyzed in partial equilibrium settings, where single competitive banks invest in marketable assets with exogenously given prices. In Kareken and Wallace's complete markets setting, for instance, deposit insurance and even banks lack economic justification. Moreover, in absence of regulation, banks could obtain unlimited profits by taking funds from insured depositors at a risk-free rate and investing them in profitable risky assets. Clearly, however, such situation cannot be an equilibrium outcome. Furthermore, if we want the existence of banks to make sense, they should play some role in the allocation of resources.

The objective of this paper is to analyze banking solvency regulation in a general equilibrium setup where the role of banks in financing real investment projects is explicitly modeled and the rates of return on risky assets and deposits are endogenous. Attention is paid to the effects of regulatory changes on equilibrium variables, notably aggregate investment and interest rates. This paper does not deal with the design of the optimal regulation (a normative approach) but with the analysis of the effects of existing regulation (a positive approach). Comparative statics, however, provide the basis for the welfare analysis of regulatory reforms consisting of variations in the parameters that define current regulatory devices.

Real-world regulation rests on largely risk-insensitive deposit insurance premiums and capital requirements. Deposit insurance premiums provide funds to the deposit insurance agency (DIA), whilst capital requirements, by limiting the leverage of banks, increases the losses to shareholders in the event of failure and reduces the probability of bankruptcy and the exposure of the DIA.

This paper is related to Gennotte (1990) that has treated the impact of deposit insurance on the pricing of banking assets. It is also
related to the literature on credit rationing and investment under asymmetric information (Stiglitz and Weiss, 1981; De Meza and Webb, 1987) that has examined the impact of these financial phenomena on the real allocation of resources. This paper coincides with Romer (1985) in the general equilibrium approach to financial intermediation.

Some institutional features as full deposit insurance are taken for granted in my model. I also assume that insured deposits are the only outside source of funds for the banks, whereas inside capital is limited to the wealth of existing bankers. This assumption captures in a simple way the empirically supported idea that outside capital is specially costly to banks. Due to capital market imperfections, probably related to incentive problems caused by informational asymmetries and contracting costs, banks have difficulties to raise capital at a sensible cost, at least in the short run.

Actually, depending upon the relative scarcity of bankers wealth, this model generates different types of equilibrium. In one of them the capital endowment of bankers is not binding, whilst in the others bankers would like to have more capital in order to expand their activities. The first situation corresponds to either the case in which market imperfections in raising outside capital do not exist (or have been overcome) or that of a long-run equilibrium in which, despite of these imperfections, inside capital has been accumulated so as to soften the wealth constraint of the bankers (perhaps as the result of past extraordinary returns on existing capital). Clearly, the other types of equilibrium can represent short-run situations in which market imperfections impede raising the optimal amount of capital.

Reducing the discussion to the long-run (or perfect capital markets) equilibrium would notably simplify the analysis of the model but the alternative equilibria provide useful insights into non-trivial aspects of the debate on banking regulation. On the one hand, recent experiences of Japan and US banking systems in joining the capital adequacy standards set by the Bank for International Settlements show
that the difficulties of some banks to raise capital have led to reductions in asset growth. On the other hand, if banks could fund at a proper cost all their risky investments with outside capital, there would not be good reasons to maintain current universal banking and one could readily accept narrow banking proposals in which deposit-taking institutions invest in risk-free assets whereas capital funded investment banks provide finance to risky projects.

Our model has two dates (t=0,1) and three types of risk-neutral agents: entrepreneurs, bankers and depositors.

Entrepreneurs are the promoters of heterogeneous real investment projects. Their initial wealth is assumed to be zero, but their utility depends upon consumption at t=0. So, they want to start their projects in order to sell their enterprises in a stock market and obtain some profits. Such profits come from the difference between the value of the stock they issue and the initial investment in the projects.

Projects differ in their expected returns. However only the bankers have screening technologies to evaluate (at zero cost) their quality, so that they are the natural buyers in the stock market, otherwise affected by a serious problem of adverse selection. Bankers have an initial wealth invested in banking capital. Each banker allocates banking capital and deposits in the stock issued by entrepreneurs (risky assets) and in a riskless asset. The riskless asset gives an exogenous rate of return $r_B$.

Lastly, depositors have an initial wealth that can be invested in insured bank deposits or in the riskless asset. I assume that insured deposits provide some transaction and liquidity services to depositors so that they are willing to demand deposits even when their rate of
return is lower than the risk-free rate. In this static model, transaction and liquidity services are treated in a reduced form approach by introducing bank deposits in the depositors' utility function.

Summing up, banks perform two functions in this economy: first, they provide depositors some transaction-liquidity services associated to bank deposits; second, they finance risky investment projects.

Banks choose the composition of their asset portfolio once deposits are taken, intermediation costs and deposit insurance premiums are paid and the capital requirement is imposed and verified. Consequently, regulation is necessarily insensitive to risk and a moral hazard problem arises.

In this context, the role of the deposit insurance agency (DIA) is pretty mechanic and passive. The DIA monitors capital regulation, collects the deposit insurance premiums and, when a bank goes bankrupt, closes the bank, pays off depositors and liquidates the assets of the bank. The DIA does not necessarily break even. In order to close the model, we can assume that the DIA invests the premiums in the riskless asset at t=0, whereas its surpluses or deficits at t=1 are compensated with ex-post lump-sum transfers.

The main findings of this paper can be summarized as follows:

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1. I will refer to the "demand for deposits" instead of the more usual "supply of deposits" because deposits are conceived as assets issued by banks. On the contrary, risky assets are issued by entrepreneurs and demanded by the banks.

2. If the model were used for welfare analysis, a more careful treatment of the problem should take into account the shadow price of public funds, as in Laffont and Tirole (1986, 1993).
In equilibrium only the subset of highest quality real investment projects is funded. Regulatory parameters as well as the safe rate of return determine the marginal project undertaken. In the long-run equilibrium, the subsidization of risk-taking by the flat-rate deposit insurance scheme leads to over-investment in risky projects, in the sense that the expected return on the marginal project is smaller than the risk-free rate. Thereby, the economy is exposed not only to possibly unfair transfers of wealth between taxpayers and banks (as in Merton, 1977) or between taxpayers and the owners of investment projects (as in Gennotte, 1990), but to the costs of serious misallocation of resources. Bank competition transfers the subsidy to depositors and entrepreneurs, triggering off over-investment.3

- Tighter regulation generally has contractionary effects on the markets for risky assets and deposits, increasing equilibrium intermediation margins and reducing real investment.

- Even if solvency is the main concern of the regulator, the allocative consequences of prudential regulation cannot be neglected, because the movement of equilibrium intermediation margins affects the probability of failure of individual banks, reinforcing or weakening the direct effect on solvency of any regulatory reform.

The rest of the paper is organized as follows. Section 2 gives a detailed description of the model and the regulatory framework. Section 3 characterizes the behavior of banks and the equilibrium of the model.

3 In Gennotte’s model, banks face a heterogeneous population of investment projects with binomial returns. Projects are in limited supply and banks compete by bidding the price paid to the project’s initial owner. Projects differ in risk but not in expected return. His main finding is that the higher the risk of a project, the higher the equilibrium appreciation of its price, so that the original owners of the projects capture part of the deposit insurance subsidy.
Section 4 presents comparative static results referring to changes in the capital requirement, the flat-rate deposit insurance premium, the parameter of systematic risk which affects all projects, the capital endowment of the banking industry and the safe rate of return. Section 5 concludes.

2. DESCRIPTION OF THE ECONOMY.

2.1. Entrepreneurs.

There is a continuum of entrepreneurs, each of whom is the promoter of a risky investment project. Entrepreneurs have no initial wealth and their welfare depends exclusively on consumption at $t=0$. Thus, they want to organize and sell their projects at $t=0$ and to consume the proceeds in the same period. They do not play any role at $t=1$.

Formally, I consider a measure space of risky investment projects $(\mathbb{R}, \mathcal{B}, \lambda)$, where $\mathbb{R}$ is the set of labels of the projects, $\mathcal{B}$ is the $\sigma$-algebra of Lebesgue measurable subsets of $\mathbb{R}$, and $\lambda$ is the Lebesgue measure. Projects are heterogeneous and labels identify their quality. $\mathcal{B}$ can be thought of as the set of possible portfolios. Each project is assumed to require a unit investment to be undertaken, so that the measure of a portfolio represents the investment required at $t=0$ to obtain the return of the projects in it at $t=1$.

The gross return on a single project $a \in \mathbb{R}$ at date 1 is a random variable $\hat{R}(a)$ defined by:

$$\hat{R}(a) = \delta a^{\delta-1} \exp(\sigma \tilde{z} - \sigma^2/2),$$

where $0<\delta<1$ and $\tilde{z}$ is a standard normal random variable. For the sake of simplicity, I assume the existence of a single factor of systematic risk in the economy, $\tilde{z}$, that explains the stochastic returns of all the projects. Independent project-specific risk could be added, but it would
not modify the results, because given the infinitesimal size of every single project it would be perfectly diversified in any portfolio with positive measure.\(^4\) With this specification, \(\tilde{R}(a)\) is a log-normally distributed random variable with mean \(\delta a \delta^{-1}\) and log-sensitivity \(\sigma\) to variations in the risky factor. Note that \(\delta a \delta^{-1}\) decreases with \(a\). Log-normality will prove very useful in order to get an explicit closed form for the banks’ objective function.

For any portfolio of investment projects \(Q \in \mathcal{B}\), its return \(\tilde{R}(Q)\) can be obtained by integrating the function \(\tilde{R}(a)\) over \(Q\) with respect to \(\lambda\):

\[
\tilde{R}(Q) = \int_Q \tilde{R}(a) d\lambda(a) = \left[ \int_Q \delta a \delta^{-1} d\lambda(a) \right] \exp(\sigma^2 - \sigma^2/2).
\]

I assume that the quality of a project can be observed by the entrepreneur as well as by the bankers, who have the appropriate screening technology.\(^5\) Bankers compete in order to obtain the best projects. As systematic risk affects equally all projects, they must yield the same expected rate of return, whatever the preference of banks towards risk.\(^6\) Let \(r_A\) denote the (endogenous) rate of return on risky projects, and let \(P(a)\) represent the (also endogenous) market value of

\(^4\) This is consistent with modern theories of financial intermediation which rest partially upon diversification arguments (Leland and Pyle, 1977; Diamond, 1984).

\(^5\) Theories of financial intermediation remark the role of the intermediaries in collecting information. According to Diamond (1984) fixed costs linked to information seeking together with diversification can justify why screening and monitoring are delegated to an intermediary.

\(^6\) Bankers are risk-neutral, but limited liability may encourage risk-taking.
project $a \in \mathbb{R}_+$. Then we have:

$$P(a) = \frac{E[R(a)]}{1+\gamma_A} = \frac{\delta a^{\delta-1}}{1+\gamma_A}. \quad (1)$$

From the point of view of the entrepreneur, project $a$ is profitable only if $P(a)$ covers the unit investment which is necessary to carry it out. If $P(a)$ is greater than 1, the promoter can sell the future returns of the project, invest one unit in it and obtain a non-negative profit equal to $P(a)-1$. Therefore, entrepreneurs with $P(a) \geq 1$ sell their projects in the stock market.

Condition $P(a) \geq 1$ determines the set of projects which are undertaken for a given $r_A$:

$$Q(r_A) = \{ a \in \mathbb{R} \mid \delta a^{\delta-1} \geq (1+r_A) \} = \left[ 0, \left( (1+r_A)/\delta \right)^{-1/(1-\delta)} \right].$$

This gives the following investment function:

$$I(r_A) = \lambda(Q(r_A)) = \left( (1+r_A)/\delta \right)^{-1/(1-\delta)}. \quad (2)$$

Clearly, $I(r_A)$ is a decreasing function of the rate of return $r_A$.

For a given $r_A$, only the entrepreneurs with projects in $Q(r_A)$ sell stock. The aggregate market value of these projects is:

$$A(r_A) = \int_{Q(r_A)} P(a) d\lambda(a).$$

Substituting $P(a)$ from (1) into this equation and integrating over $Q(a)$ gives:

$$A(r_A) = \frac{1}{\delta} \left( (1+r_A)/\delta \right)^{-1/(1-\delta)}. \quad (3)$$

The aggregate (random) return of these projects is given by:
\[(1+r_A)A(r_A)\exp(\sigma^2/2). \quad (4)\]

From (2) and (3), entrepreneurs' profits (and consumption) at t=0 are:

\[A(r_A) - I(r_A) = (1-\delta) A(r_A). \]

So, entrepreneurs' consumption is a fraction \((1-\delta)\) of the funds obtained by selling their projects.

In section 3, I will characterize the equilibrium in the stock market by putting together equation (3) and the banks' demand for risky assets.

2.2. Depositors.

There is a representative risk-neutral depositor with an initial wealth normalized to unity that can be invested in the safe asset, B, or in bank deposits, D. His budget constraint is given by:

\[B + D = 1. \quad (5)\]

Deposits are completely insured (principal plus interest) by the deposit insurance fund, so they are riskless. In addition to their return, \(r_D\), they provide some transaction and liquidity services. These services are modeled in a reduced form approach by introducing a function \(U(D)\) that assigns to each amount of deposits the value at t=0 of the transaction-liquidity services.\(^7\) Thus, the depositor's objective function (in terms of utility at t=1) can be written as:

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\(^7\) In their famous article, Diamond and Dybvig (1983) provide an explicit modeling of this liquidity services in a multi-period setting with ex-post heterogeneous agents.
where $U(D)$ is assumed to be twice continuously differentiable, concave and increasing, with $U'(0)=1$ and $U'(1)=0$.

Substituting (5) into (6) gives the following objective function:

$$
(1+r_B) - (r_B-r_D)D + (1+r_B)U(D).
$$

From the properties of $U(D)$, the demand for deposits is positive for all $r_D>-1$, and for $r_D<r_B$ satisfies the following first order condition:

$$
(1+r_B)U'(D) = r_B-r_D.
$$

From here we obtain a deposit demand function which depends (negatively) on $(r_B-r_D)/(1+r_B)$. This term will be later defined as the bank intermediation margin on deposits, $\mu_D$. For $r_D=r_B$, all the wealth of depositors is invested in bank deposits.

### 2.3. Bankers.

Banks belong to a class of risk-neutral agents called bankers. A representative bank and a representative banker are considered. The banker has an initial wealth $K$ that is invested in banking capital. She also has a screening technology to evaluate (at zero cost) the quality of investment projects.

At $t=0$ the banker sets up the bank, which entails choosing first its deposits, $D_0$, and next its asset composition. In taking deposits, banks incur in two different costs at $t=0$, namely the deposit insurance premium and some intermediation costs. The intermediation costs are linked (again in a reduced form) to the provision of transaction and liquidity services to depositors. Deposit insurance premiums and costs are assumed to be linear functions of $D_0$, denoted $pD_0$ and $cD_0$. 
respectively.

Two kinds of assets are available to banks: risky assets -which are the shares in risky projects that have been analyzed in section 2.1- and the riskless asset. $A_0$ and $B_0$ stand respectively for the amounts invested in such assets. Then, the bank's budget constraint at $t=0$ has the following structure:

$$A_0 + B_0 + (p+c) D_0 = D_0 + K.$$  \hfill (8)

Banks have limited liability and deposits are fully insured. At $t=1$ asset returns are observed and the net worth of the bank, $\tilde{A}_1 + B_1 - D_1$, is computed, where $B_1$ and $D_1$ stand, respectively, for the values of the riskless asset and the deposits at $t=1$. If the net worth is negative, the DIA closes the bank, pays off depositors and, as receiver, liquidates the bank's assets. If positive, the bank liquidates its assets and pays off depositors. The residual worth accrues to the banker. Therefore, the value of the bank to the banker at $t=1$ is given by:

$$\tilde{V}_1 = \max(\tilde{A}_1 + B_1 - D_1, 0).$$

The banker is assumed to maximize her expected wealth at $t=1$, $E(\tilde{V}_1)$.

As stated by Black and Scholes (1973), many corporate claims under limited liability present option-like payoff functions. In this sense, $\tilde{V}_1$ can be interpreted as the payoff of a call option on the bank's risky portfolio with strike price $D_1 - B_1$ (see Figure 1). Given the stochastic properties of the returns of the risky projects -see equation (4)-, the value of the risky portfolio at $t=1$ is a log-normally distributed random variable:

$$\tilde{A}_1 = (1+r_A) A_0 \exp(\sigma \tilde{z} - \sigma^2/2).$$
FIGURE 1

Value of a bank as a function of the return on risky assets

\[ V_1 = \max(A_1 + B_1 - D_1, 0) \]
But then the computation of $E(\tilde{V}_1)$ leads to a simple variation of the standard Black-Scholes formula for the valuation of call options.\(^8,9\)

Let us define

$$V_1 = E(\tilde{V}_1) = \int_0^\infty \max(A_1 + B_1 - D_1, 0) g(A_1) dA_1,$$

(9)

where $g(\cdot)$ is the density function of $\tilde{A}_1$. When $D_1 > B_1$, several changes of variable make it possible to write the integral in equation (9) in terms of the distribution function of a standard normal random variable, $F(\cdot)$:\(^10\)

$$V_1 = (1+r_A) A_0 F(x) - (D_1 - B_1) F(x - \sigma),$$

(10)

where

$$x = \frac{1}{\sigma} \left[ \log[A_0/(D_1 - B_1)] + \log(1+r_A) + \sigma^2/2 \right].$$

The definition of $x$ is such that $\tilde{A}_1 + B_1 - D_1$ is positive when $\tilde{z} > \sigma - x$; therefore, $F(x - \sigma) = 1 - F(\sigma - x)$ stands for the probability of the bank being solvent. From equation (10), we can deduce that the expected value of

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\(^8\) The Black-Scholes formula can be obtained in a risk-neutral discrete-time setting precisely by assuming that the value of the underlying asset is log-normally distributed and calculating the expected present value of $\tilde{V}_1$.

\(^9\) Merton (1977) applied option theory to the valuation of the deposit insurance contract, opening an interesting line of research. Marcus and Shaked (1984), Ronn and Verma (1986) and Mullins and Pyle (1990) are good examples. Surprisingly, however, few behavioral models exploit the resulting valuation formulas (see Suárez, 1993).

\(^10\) See Suárez (1993) for a detailed derivation.
the bank to bankers comes from subtracting the expected value of its net debt from the expected value of its portfolio of risky assets. Because of limited liability, the value of this portfolio, \((1+r)A F(x)\), results from computing the mean of the truncated log-normal random variable that yields \(\bar{A}\) in non-bankruptcy states and zero otherwise. On the other hand, the expected value of the bank’s net debt is simply \((D-B)\) times the probability of the bank being solvent. Since the representative bank takes the deposit rate \(r_D\) as given, \(D = (1+r_D)D_0\). Similarly, \(B = (1+r_B)B_0\).

When \(D \leq B\) the bank cannot fail and \(V_1\) is simply the expected net worth of the bank:

\[
V_1 = (1+r)A_0 - (1+r_B)D_0 - (1+r_B)B_0.
\]  

(11)

Observe that (11) can be obtained by setting \(F(x)=F(x-\sigma)=1\) in (10).

Next, I will obtain an expression for the banker’s objective function in terms of the following two decision variables: the share of risky assets in the total assets of the bank, \(\alpha = A_0/(A_0+B_0)\), and the amount of deposits, \(D_0\). Using the budget constraint of the bank (8), we get:

\[
A_0 = \alpha[(1-p-c)D_0 + K],
\]  

(12)

\[
B_0 = (1-\alpha)[(1-p-c)D_0 + K].
\]  

(13)

Intermediation margins play a central role in the model. Define the intermediation margin on risky assets, \(\mu_A\), as the discounted value of the difference between the expected return on the risky asset and the safe rate of return, \((r_A - r_B)/(1+r_B)\), and define the intermediation margin on deposits, \(\mu_D\), as the discounted value of the difference between the safe rate of return and the deposit rate, \((r_B - r_D)/(1+r_B)\). The bank is assumed to be perfectly competitive so that it takes \(\mu_A\) and \(\mu_D\) as given.
As \( r_B \) is exogenously given, maximizing \( V_1 \) is equivalent to maximizing its present value, \( V_0 = \frac{1}{1+r_B} V_1 \). Then, from (12) and (13), and previous definitions, the objective function of the bank can be written as a function of the decision variables, \( \alpha \) and \( D_0 \):

\[
V(\alpha, D_0) = \left[(1-p-c)D_0 + K\right]\left[\alpha(1+\mu A)F(x)+(1-\alpha)F(x-\sigma)\right]-(1-\mu D_0)F(x-\sigma), \tag{14}
\]

where

\[
x \equiv \frac{1}{\sigma} \log \left(\frac{\alpha[(1-p-c)D_0 + K]}{(1-\mu D_0 - (1-\alpha)[(1-p-c)D_0 + K])}\right)^{(1+\mu A)+\sigma^2} \tag{15}
\]

The regulatory framework faced by the bank consists of the deposit insurance premium and a capital requirement. The portfolio decision of the bank, however, is taken once deposits have been raised, the deposit insurance premium has been paid and the capital requirement has been imposed and verified. As a consequence, neither the deposit insurance premium nor the capital requirement can be sensitive to the composition of the bank portfolio, and a moral hazard problem surges.

As a matter of fact, full risk-insensitiveness characterized banking regulation in many countries until the overall introduction of risk-based capital requirements resulting from the Basle Committee recommendations (Bank for International Settlements, 1988). But even the so-called Cooke ratios maintain a high degree of risk-insensitiveness, since assets of very different riskiness are included in wide single categories.

Assume, then, that \( p \) is a constant and that the capital requirement is defined as a minimum ratio \( k \) of net capital (capital at \( t=0 \) net of intermediation costs) to total assets:
Using the budget constraint of the bank, equation (8), we can rewrite (16) as an upper bound on $D_0$:

$$D_0 \leq \overline{D} = \left[ \frac{1-k}{k(1-p-c)+p+c} \right] K. \tag{17}$$

In this classic regulatory framework, the decision problem of the bank can be formulated as follows:

Maximize $V_0(\alpha, D_0)$

subject to: $0 \leq \alpha \leq 1$

$0 \leq D_0 \leq \overline{D}.$ \tag{18}

Notice that, while the range of variation of $\alpha$ comes directly from its definition, the upper bound for $D_0$ depends upon regulatory parameters. The analysis of this apparently simple optimization program will first lead to the characterization of the behavior of banks, and then the equilibrium outcome of the economy.

3. EQUILIBRIUM ANALYSIS.

This section is divided into three subsections. First, I analyze the behavior of banks. Four basic corner solutions arise because of non-convexities in the optimization problem (18). Each solution can be interpreted as one of four possible configurations of the banking business: universal banks, investment banks, mutual funds and narrow banks. Later on, equilibrium is characterized. The corner solutions lead to four possible types of equilibrium, depending upon the relative scarcity of bankers wealth (as compared with the investment
opportunities and for a given capital requirement). In particular, there is a type of equilibrium which is the one that one would expect to find in the long-run, when the rate of return on bank capital is not higher than the risk-free rate and, thus, new capital does not have an incentive to be accumulated in the banking industry. The last subsection discusses this long-run equilibrium.


Limited liability introduces non-convexities in the decision problem of the representative bank (recall Figure 1). It is well known that, other things equal, the value of the banker's call option over the bank's assets is an increasing function of the leverage of the bank and the volatility of its assets (Merton, 1977; Keeley and Furlong, 1990). Under deposit insurance and a classic regulatory framework, the banker may exploit her limited liability because neither the cost of deposits nor regulatory constraints impose higher costs on riskier actions.

In this model, however, intermediation margins are endogenous and, if they were sufficiently low, banks might prefer a conservative policy rather than the riskiest feasible one.

**PROPOSITION 1:** Under a classic regulatory framework, there are no pure interior solutions for $\alpha$ and $D_0$. Furthermore, $0<\alpha<1$ implies $D_0=0$ and $0<D_0<D$ implies $\alpha=0$.

**Proof:** See Appendix 1.

Proposition 1 is a direct consequence of the non-convexities mentioned above. As in any optimization program with corner solutions, we get the optimum by comparing the value of the objective function at every possible corner. Such a cumbersome task leads to Figure 2. The space of intermediation margins, $(\mu_A, \mu_D)$, is divided into four regions (denoted A, B, C and D), each corresponding to a pure corner solution.
FIGURE 2

The behavior of a bank in the regions of the \((\mu_A, \mu_D)\) space

B
(INVESTMENT BANKS)
\[ \alpha = 1 \]
\[ D_0 = 0 \]

A (UNIVERSAL BANKS)
\[ \alpha = 1 \]
\[ D_0 = D \]

C (MUTUAL FUNDS)
\[ \alpha = 0 \]
\[ D_0 = 0 \]

D (NARROW BANKS)
\[ \alpha = 0 \]
\[ D_0 = D \]
In region A, the solution to the problem of the representative bank is \( \alpha = 1 \) and \( D_0 = \overline{D} \). In region B, \( \alpha = 1 \) and \( D_0 = 0 \). In region C, \( \alpha = 0 \) and \( D_0 = 0 \). Finally, in region D, \( \alpha = 0 \) and \( D_0 = \overline{D} \). In the frontiers between regions (i.e.: \( \text{A} \cap \text{B}, \text{A} \cap \text{C}, \text{A} \cap \text{D}, \text{B} \cap \text{C} \) and \( \text{C} \cap \text{D} \)), the optimal decisions of the neighboring regions are equally optimal. Additionally, \( \alpha \in (0, 1) \) and \( D_0 = 0 \) is also optimal in \( \text{B} \cap \text{C} \), while \( \alpha = 0 \) and \( D_0 \in (0, \overline{D}) \) is optimal in \( \text{C} \cap \text{D} \). Appendix 2 contains the details of the derivation of the figure and, in particular, the definition of the curves that separate one region from another.

These results are easy to explain. The optimal \( \alpha \) depends basically upon \( \mu_A \); it is 1 when \( \mu_A \) is high (\( \text{A} \cup \text{B} \)) and 0 when \( \mu_A \) is low (\( \text{C} \cup \text{D} \)). Similarly, \( D_0 \) is \( \overline{D} \) when \( \mu_D \) is high (\( \text{A} \cup \text{D} \)) and 0 when it is low (\( \text{B} \cup \text{C} \)). The solution in region A corresponds to the case of universal banks (that take deposits in order to fund risky projects). Similarly, the solutions in regions B, C and D can be respectively related with investment banks (that invest own funds in risky assets), money-market mutual funds (that invest own funds in safe assets) and narrow banks (that take deposits in order to invest in safe assets).

The boundary between \( D_0 = 0 \) and \( D_0 = \overline{D} \), \( (\text{A} \cup \text{D}) \cap (\text{B} \cup \text{C}) \), is basically determined by intermediation costs. In particular, when \( \alpha = 0 \) is optimal, the bank accepts deposits only if \( \mu_D \) covers such costs (\( \mu_D \geq p + c \)). In contrast, when \( \alpha = 1 \), the reservation margin in deposits decreases as the margin in risky assets increases.

The boundary between \( \alpha = 0 \) and \( \alpha = 1 \), \( (\text{A} \cup \text{B}) \cap (\text{C} \cup \text{D}) \), reflects the attraction of risky assets for banks. When \( \mu_D \) is low and the optimal value of \( D_0 \) is 0, the criterion for the bank to invest or not in risky

\[11\] Note that in \( \text{A} \cap \text{B}, \text{A} \cap \text{C} \) and \( \text{A} \cap \text{D} \) the solution is either the one in A or the respective one in B, C and D, but not intermediate values of \( \alpha \) and \( D_0 \).
assets is simply given by $\mu_A \geq 0$. In contrast, when $\mu_D$ is greater than a certain critical value, there exists a set of intermediation margins $\mu_A < 0$ for which $D_0 = D$ and $\alpha = 1$ are optimal (the area of region $A$ that is under the horizontal axis in Figure 2).

This finding is crucial to understand one of the key results of this paper. Because of limited liability and the ability to borrow at a fixed rate, the bank may find attractive the investment in risky assets even when their expected rate of return is below the risk-free rate. If the probability of failure is positive, very high outcomes of $\hat{A}$ are to the benefit of the bank, whilst very low outcomes are to the detriment of the deposit insurance fund. High intermediation margins in deposits reduce the set of states in which the bank fails and, hence, ameliorate (but do not eliminate) the bias towards risky assets (see Figure 2).

### 3.2. Characterization of equilibrium.

An equilibrium for this economy is a pair of intermediation margins, $(\mu_A, \mu_D)$, such that: (i) agents maximize their objective functions subject to their budgetary and regulatory constraints, (ii) markets clear.

As $r_B$ is exogenously given (and, therefore, the supply of the riskless asset is perfectly elastic), we shall focus on the markets for risky assets and deposits. The asset supply function $A(r_A)$, defined in equation (3), and the deposit demand function $D(r_D, r_B)$, defined in equation (7), can be rewritten as functions of $\mu_A$, $\mu_D$, and $r_B$: $A(\mu_A, r_B)$ and $D(\mu_D)$. Then $(\mu_A, \mu_D)$ is a pair of equilibrium intermediation margins if $A(\mu_A, r_B) = A_0$ and $D(\mu_D) = D_0$.

**Proposition 2:** There cannot be an equilibrium in the region $(B \cup C \cup D \backslash A)$.

**Proof:** Suppose, to the contrary, that an equilibrium exists in $(B \cup C \cup D \backslash A$. Then either $\alpha = 0$ or $D_0 = 0$ would characterize the behavior of
the representative bank. \( \alpha=0 \) means \( A=0 \) and hence \( A(\mu_A, r) = 0 \). The supply of risky assets is zero only if \( \mu_A \) tends towards infinity. But in such case \( \alpha=0 \) is not an optimal choice for the bank (see Figure 2).

Similarly, \( D_0=0 \) means \( D(\mu_B) = 0 \), but the demand for deposits is zero only if \( \mu_D \) tends towards 1, and in such case \( D_0=0 \) is not an optimal decision for the bank.

From Proposition 2, equilibrium, if it exists, should take place in the interior of region \( A \), \( \text{Int}(A) \), or in its frontier, \( A \setminus \text{Int}(A) \). Intermediation margins in \( A \setminus \text{Int}(A) \) make the representative banker indifferent between at least two solutions (two possible configurations of the bank). The representative bank hypothesis must then be removed, because in equilibrium a fraction \( \gamma \) of the banking industry may choose one solution, while the remaining \( 1-\gamma \) may choose the other.\(^{12}\) In what follows, let \( D^* \) represent the aggregate banking supply of deposits and \( A^* \) the aggregate banking demand for risky assets. \( K \) will stand for the aggregate capital capacity of the banking industry. I will distinguish four types of equilibrium.

(i) Equilibrium of type I: \( (\mu_A, \mu_D) \in \text{Int}(A) \).

Any possible equilibrium in \( \text{Int}(A) \) implies \( \alpha=1 \) and \( D_0=\overline{D} \) for all banks (all banks are universal banks). Therefore, \( D^*=\overline{D} \) and \( A^*=\{(1-p-c)\overline{D}+K\}=\overline{A} \). The corresponding market clearing pair will be such that \( A(\mu_A, r_B)=\overline{A} \) and \( D(\mu_D)=\overline{D} \). Note the recursive nature of this equilibrium.

(ii) Equilibrium of type II: \( (\mu_A, \mu_D) \in A \cap B \).

Assume that a fraction \( \gamma \) of the banking industry chooses \( \alpha=1 \) and \( D_0=\overline{D} \), while the remaining \( 1-\gamma \) chooses \( \alpha=0 \) and \( D_0=0 \). Clearly, \( D^*=\gamma \overline{D} \) and \( A^*=\gamma(1-p-c)\overline{D}+K \). Furthermore, as \( \gamma \) varies, any allocation such that \( D^* \in [0, \overline{D}] \) and \( A^*=\gamma(1-p-c)D^*+K \) is compatible with the optimizing behavior

\(^{12}\) The argument could be extended to situations where more than two solutions are optimal, as in \( A \cap B \cap C \) and \( A \cap C \cap D \).
of banks. The market clearing pair will be such that:

\[ A(\mu_A, r_B) = (1-p-c)D(\mu_D) + K \quad \text{and} \quad D(\mu_D) \leq \bar{D}. \]  

(iii) Equilibrium of type III: \((\mu_A, \mu_D) \in A\cap C.\)

Assume that a fraction \(\gamma\) of the banking industry chooses \(A=1\) and \(D_0 = \bar{D}\), while the remaining \(1-\gamma\) chooses \(A=0\) and \(D_0 = 0\). Clearly, \(D_* = \gamma \bar{D}\) and \(A_* = \gamma \bar{A}\).

Using the definition of \(\bar{D}\) in (17), we get that \(\bar{A} = \gamma [(1-p-c)\bar{D} + K]\) equals \([1/(1-k)]\bar{D}\) and, hence, \(A_* = \gamma [(1/(1-k))]\bar{D}\). Then, the market clearing pair will be such that:

\[ A(\mu_A, r_B) = \frac{1}{1-k} D(\mu_D) \quad \text{and} \quad D(\mu_D) \leq \bar{D}. \]  

(iv) Equilibrium of type IV: \((\mu_A, \mu_D) \in A\cap D.\)

Suppose that a fraction \(\gamma\) of the banking industry chooses \(A=1\) and \(D_0 = \bar{D}\), while the remaining \(1-\gamma\) chooses \(A=0\) and \(D_0 = 0\). Clearly, \(D_* = \bar{D}\) and \(A_* = \gamma [(1/(1-k))]\bar{D}\). The market clearing pair will be such that:

\[ D(\mu_D) = \bar{D} \quad \text{and} \quad A(\mu_A, r_B) \leq \bar{A}. \]  

Table 1 summarizes the main features of these four types of equilibrium. The expected returns on banking capital exceed the risk-free rate in equilibria of types I, II and IV, while equal the risk-free rate in type III equilibrium. To see this, note that in type III equilibrium banks are indifferent between \((A,D_0) = (1,\bar{D})\) and \((A,D_0) = (0,0)\); with \((0,0)\) bankers invest all their wealth in the riskless asset, which yields \(r_B\). In the other types of equilibrium \((0,0)\) is no longer optimal. Therefore, in equilibria of types I, II and IV the expected return on bankers' wealth is greater than \(r_B\). For the sake of simplicity, we had previously assumed that bankers invested all their wealth in bank capital and, hence, that \(K\) was exogenously given. However, these results imply that the bankers do not regret this decision. In equilibrium, wealth invested in bank capital always yields at least the same expected return that a privately held portfolio of the safe asset.
TABLE 1

Description of Each Type of Equilibrium

Type I
- All banks are universal banks.
- Expected returns on banking capital exceed the risk-free rate.
- Over- or under-investment in risky assets are possible.

Type II
- Universal banks and investment banks co-exist.
- Expected returns on banking capital exceed the risk-free rate.
- Under-investment in risky assets.

Type III
- Universal banks and mutual funds co-exist.
- Expected returns on banking capital equal the risk-free rate.
- Over-investment in risky assets.

Type IV
- Universal banks and narrow banks co-exist.
- Expected returns on banking capital exceed the risk-free rate.
- Over-investment in risky assets.

The efficient allocation of wealth under perfect information in this economy would require \( \mu_A = 0 \). With \( \mu_A < 0 \), net economic gains could be obtained by denying finance to risky projects whose expected return is lower than \( r_B \) and investing these funds in the safe asset. Similarly, with \( \mu_A > 0 \), unfunded risky projects yield higher expected returns than the riskless asset and, except if all wealth were already invested in risky assets, net gains could be obtained by funding additional investment projects. These arguments lead to the over- and under-investment results in Table 1.

3.3. The long-run equilibrium.

Equilibrium of type III is in a sense the one that we might expect to find in the long-run. With perfect capital markets, the extraordinary
returns obtained by banks in the other types of equilibrium would attract new capital into the banking industry. In this paper, I have assumed, however, that banking capital is scarce. In fact, I have supposed that it is limited to the initial wealth of bankers, implicitly discarding the possibility of raising additional capital in the short-run. Notwithstanding, appealing to external finance is not necessary to argue that, in the long-run, banks can obtain additional capital, since excess returns on existing own funds can provide simultaneously the incentives and the funds for capital accumulation in the banking industry. In the next section, I will examine the effect of increases in $K$ on each type of equilibrium. The results support a very plausible convergence of the other types of equilibrium towards the type III. Accordingly, if the initial situation corresponded to an equilibrium of type I, II or IV, competitive banks would accumulate capital up to the point in which excess returns on own funds vanished.

In this long-run equilibrium $\mu_A$ is smaller than zero. Therefore the investment in risky assets exceeds the first best. Too many projects are financed and a clear distributional effect accompanies this efficiency loss. Risky projects are over-valued, to the benefit of entrepreneurs and the detriment of the deposit insurance fund. Contrary to models of deposit insurance pricing or banking behavior where $r^*_A$ and $r^*_D$ are exogenously given and rents from the limited liability put option are appropriated by banks as a simple wealth transfer, in this long-run equilibrium banking competition transfers the implicit subsidy to entrepreneurs causing both efficiency losses and distributional effects.\textsuperscript{13}

\textsuperscript{13} Informal discussion by regulators and other observers has long implicitly recognized both impacts, but the first formal statement is in Gennotte (1990). See also Gennotte and Pyle (1991).
4. COMPARATIVE STATICS.

Comparative static results in this model will show how changes in the capital endowment of banks \( K \), the capital requirement \( k \), the deposit insurance premium \( p \), the risk of real investment projects \( \sigma \), and the safe rate of return \( r_B \) affect the endogenous variables. We are interested in their impact on the equilibrium intermediation margins, the investment in risky projects, the aggregate supply of deposits, and the solvency of banks.

Notice that given the specification of the deposit demand function and the asset supply function, variations in the aggregate levels of investment in risky projects and deposits can be directly related (except when \( r_B \) varies) to changes in \( \mu_A \) and \( \mu_D \), respectively.

The effects on the solvency of banks can be examined by differentiating the probability of failure, \( q = F(\sigma - x) \), of those banks that take deposits in order to invest in risky projects (universal banks); the other banks are safe. In order to evaluate \( q \), notice that when \( \alpha = 1 \) and \( D_0 = D \) the expression for \( x \) reduces to:

\[
x = \frac{1}{\sigma} - [\log(1+\mu_A) - \log(1+k) - \log(1-\mu_D) + \sigma^2/2],
\]

so that \( q \) is decreasing in \( \mu_A \), \( \mu_D \) and \( k \), and increasing in \( \sigma \). As the equilibrium intermediation margins are endogenous variables, we can distinguish between the direct effect on \( q \) of a parameter \( w \), \( \partial q / \partial w \) (which is computed holding constant \( \mu_A \) and \( \mu_D \)), and the final effect, \( \partial q / \partial w \) (which takes also into account the changes in \( \mu_A \) and \( \mu_D \)). Comparing both effects is interesting because previous research has focused mainly on the direct effects, treating \( \mu_A \) and \( \mu_D \) as exogenous. Duplicated signs in the tables of results will appear when the indirect effects reinforce the direct ones.

Comparative statics results in this model are contingent on the type of equilibrium. Nevertheless, many results are robust. In type I
equilibrium, the exercise is recursive because $\bar{D}$ and $\bar{A}$ are determined by regulatory and structural parameters through the optimal balance sheet of the bank, while $\mu_A$ and $\mu_D$ are subsequently derived by solving the market clearing conditions $A(\mu_A, r_B) = \bar{A}$ and $D(\mu_D) = \bar{D}$. Differentiating the corresponding equations with respect to the parameters of interest and the endogenous variables leads to the results. In the other types of equilibrium, the exercise is not recursive, but follows the standard procedure.

Instead of describing the results for the four types of equilibrium, I will show first the results referring to the change in $K$, that support the convergence of the other types of equilibrium towards type III. Afterwards, I will focus on this long-run equilibrium, leaving the details of the others for Appendix 3.

4.1. The capital endowment of banks and the convergence result.

Only in type III equilibrium, the aggregate capital endowment of the banking industry is not binding. Bank capital yields no excess returns and bankers are, consequently, indifferent between investing their wealth in banks or holding a safe portfolio. Small changes in $K$ do not modify the equilibrium outcome. This is not the case in the other types of equilibrium, where the presence of $K$ in the equilibrium conditions reflects, on the one hand, that increases in $K$ enhance the capacity of the industry to take deposits and invest in risky assets, and, on the other, that at least one of these activities is profitable. In these types of equilibrium, bank capital obtains expected returns that exceed $r_B$.

To see how changes in $K$ modify the short-run equilibrium outcomes, notice that $K$ enters the equilibrium conditions but does not have any influence in the shape of the regions of bank behavior in the $(\mu_A, \mu_D)$ space (see Appendix 2). Thus, changes in $K$ move the pair of equilibrium margins over Figure 2, without changing the location of the frontiers.
In type I equilibrium, a rise in $K$ increases both $\bar{D}$ and $\bar{A}$, allows the expansion of pure banking activities and leads to smaller margins in both deposits and risky assets. Table 2 shows these effects as well as the direct and final impacts on the probability of failure. The first is null, but the second is clearly positive.

**TABLE 2**

**Changing the Capital Endowment of Banks ($K$)**

<table>
<thead>
<tr>
<th>Type of equilibrium</th>
<th>$\frac{d\mu_A}{dK}$</th>
<th>$\frac{d\mu_D}{dK}$</th>
<th>$\frac{\partial q}{\partial K}$</th>
<th>$\frac{dq}{dK}$</th>
</tr>
</thead>
<tbody>
<tr>
<td>I</td>
<td>$-$</td>
<td>$-$</td>
<td>0</td>
<td>$+$</td>
</tr>
<tr>
<td>II</td>
<td>$-$</td>
<td>$+$</td>
<td>0</td>
<td>$?$</td>
</tr>
<tr>
<td>IV</td>
<td>$-$</td>
<td>$-$</td>
<td>0</td>
<td>$+$</td>
</tr>
</tbody>
</table>

In type II equilibrium, the market clearing conditions summarized by $A(\mu_A, r_B) = (1-p-c)D(\mu_D) + K$ establish a positive relationship between the margins on deposits and risky assets. An increase in $K$ shifts this curve down to the right, which implies a lower $\mu_A$ and a higher $\mu_D$. The descent along the boundary between regions A and B reflects, on the one hand, greater investment in risky projects and, on the other, a lower supply of deposits. As $\mu_A$ and $\mu_D$ move in opposite directions, the final impact of $K$ on $q$ is ambiguous in this case.

Intuitively, in the initial situation, risky assets are sufficiently profitable to attract the investment of any additional amount of capital, but the intermediation margin on deposits is small enough to dissuade investment banks from becoming universal banks and using deposits to invest in risky assets. When $K$ goes up, investment rises. As investment rises, $\mu_A$ falls and the profitability of universal banks at prior $\mu_D$ deteriorates. The consequent rise in $\mu_D$ indicates that some universal banks become investment banks, abandoning the
deposit-taking activity.

In type IV equilibrium, market clearing conditions are summarized by \( D(\mu_D) = \bar{D} \). An increase in \( K \) augments \( \bar{D} \) and the aggregate supply of deposits. The resulting fall in \( \mu_D \) increases the probability of bankruptcy and the incentives to invest in risky assets. Thus some narrow banks become universal banks, lowering \( \mu_A \) and rising the investment in risky projects.

These results imply that increases in \( K \) lead the pair of equilibrium margins in any type of equilibrium towards the frontier between regions A and C, i.e. the locus where equilibrium of type III takes place (see the arrows in Figure 2). Therefore, as argued in section 3.3, the incentives to accumulate bank capital given by the excess returns on own funds in equilibria of types I, II and IV can lead to a convergence of these equilibria towards the long-run equilibrium, where bank capital yields an expected return equal to the risk-free rate.

4.2. Comparative statics in the long-run equilibrium.

As derived in Appendix 2, the boundary between regions A and C is the downward sloping curve defined by the following equation:

\[
\left[ \frac{1}{1-K} (1+\mu_A) F(x) - (1-\mu_D) F(x-\sigma) - \left( \frac{k}{1-k} + p + c \right) \right] \bar{D} = 0,
\]

whereas the market clearing conditions summarized in (20) establish:

\[
A(\mu_A, \mu_B) = \frac{1}{1-k} D(\mu_D),
\]

which defines an upward sloping curve in the \((\mu_A, \mu_D)\) space. Equilibrium of type III corresponds to the intersection between these curves and comparative statics can be diagrammatically obtained by analyzing the movement of both curves and the consequent shift in their intersection.
Total differentiation of these equations leads to:

\[ G(\mu_A, \mu_D; k, p, c, \sigma) = 0, \]
\[ + + - - + \]

and

\[ H(\mu_A, \mu_D; k, r_B) = 0, \]
\[ - + - - \]

respectively. The signs of the partial derivatives are written below the corresponding variables and parameters. Table 3 contains the results. Question marks stand for ambiguous signs and duplicated signs appear in the last column when the indirect effects on the probability of failure reinforce the direct ones.

**TABLE 3**

Comparative Statics in the Long-Run Equilibrium

<table>
<thead>
<tr>
<th>Parameter (w)</th>
<th>( \frac{d\mu_A}{dw} )</th>
<th>( \frac{d\mu_D}{dw} )</th>
<th>( \frac{\partial q}{\partial w} )</th>
<th>( \frac{dq}{dw} )</th>
</tr>
</thead>
<tbody>
<tr>
<td>k</td>
<td>?</td>
<td>+</td>
<td>-</td>
<td>-</td>
</tr>
<tr>
<td>p</td>
<td>+</td>
<td>+</td>
<td>0</td>
<td>-</td>
</tr>
<tr>
<td>( \sigma )</td>
<td>-</td>
<td>-</td>
<td>+</td>
<td>++</td>
</tr>
<tr>
<td>( r_B )</td>
<td>-</td>
<td>+</td>
<td>0</td>
<td>?</td>
</tr>
</tbody>
</table>

Changes in the capital requirement. A more stringent capital requirement has a clear contractionary impact on the market for deposits and lowers the probability of failure of risky banks.

An increase in \( k \) shifts \( G \) up to the right and \( H \) down to the right. The movement of \( G \) is justified by the fall in the profitability of universal banks that follows from the reduction of the deposit insurance
subsidy when leverage declines. The shift in H reflects that, with a higher capital ratio, less deposits are necessary to fund a given amount of risky assets: a greater $\mu_D$ is compatible with a given $\mu_A$. The net impact on $\mu_D$ is unambiguously positive, but the one on $\mu_A$ might be negative. In spite of this ambiguity, q can be shown to decrease.

**Changes in the deposit insurance premium.** A rise in p leads to greater intermediation margins and lowers the probability of bankruptcy.

For a given amount of deposits, an increase in p rises the payments to the deposit insurance agency, diminishing the funds available to invest in both assets. Thus higher p decreases the profitability of depository institutions, moving the frontier between regions with $D_0=0$ and $D_0=D$ to the right. The market clearing condition does not change. So, an increase in p rises intermediation margins, curtailing the funding of risky projects and the supply of deposits. Although p lacks a direct effect on q due to the specification in net terms of the capital requirement, the indirect effects are negative and lead to a smaller probability of failure.

**Changes in the risk of real investment projects.** Diagrammatically, increasing the volatility of the returns of risky investment opportunities (the systematic risk) shifts the boundary between regions A and C down to the left. Thereby, intermediation margins decline and the probability of bankruptcy goes up.

Intuitively, when available investment projects become riskier, the implicit subsidy to risk-taking rises and banking activity becomes more profitable. Banks get involved in strong competition both as suppliers of deposits and demanders of risky assets. At previous intermediation margins markets are no longer in equilibrium. The new equilibrium yields lower margins, further investment in risky assets and the expansion of deposits. According to (22), the probability of bankruptcy augments not only as a direct consequence of higher $\sigma$ but also because of reduced margins in both sides of the banks' balance sheet.
Changes in the safe rate of return. Changes in $r_B$ has non-symmetric effects on intermediation margins and may affect the solvency of banks, although, as expected, a rise in $r_B$ has a contractionary impact on investment and deposit supply.

Analyzing the effects of the change in $r_B$ is interesting, because it may account for monetary policy, whose transmission is influenced by financial intermediation. The analysis is notably simplified by the fact that the boundaries between the regions of Figure 2 are invariant to changes in $r_B$.

For a given pair of intermediation margins, higher $r_B$ reduces the set of profitable investment projects, whilst keeps constant the demand for deposits, and hence the funding capacity of the banking industry. The $H$ curve shifts down to the right so that previous intermediation margins violate the market clearing condition and re-establishing the equilibrium entails the simultaneous decrease of $\mu_A$ and increase of $\mu_D$. So, the rise in $r_B$ has a contractionary impact on both $A^*$ and $D^*$, though the effect on intermediation margins is not symmetric (recall that the supply of risky assets depends on both $\mu_A$ and $r_B$ so that the fall in investment is compatible with the decrease in $\mu_A^*$). The impact on $q$ is unclear, because $\mu_A$ and $\mu_D$ move in opposite directions.

4.3. Comparative statics in short-run equilibria.

Most of the results discussed for the long-run equilibrium are robust and the detailed description of the results for the other types of equilibrium will not be given here. The reader can find them in Appendix 3. Nevertheless, two particular comments are worthy.

First, out of the long-run equilibrium, the effect of a rise in $k$ on the market for risky assets is unambiguously contractionary, since the banking industry suffers capital shortage and cannot respond with an increase in $K$. So the effects of capital regulation on real investment
should be specially important when capital market imperfections impede bankers to raise capital. In this sense, recent experiences of Japan and US banking systems in implementing the capital standards set by the Bank for International Settlements indicate that the new requirements have been met at least in the short-run by cutting asset growth, as predicted here.

Second, in short-run equilibria the capital endowment of banks enters the equilibrium conditions. This means that market outcomes are influenced by capital shortage. In equilibrium of type I, for instance, all the industry takes deposits and invests in risky assets, so that \( D_0^* \) and \( A_0^* \) are as high as the availability of capital permits. The same is true for deposits in equilibrium of type IV. As a consequence, in short-run equilibria, changes in parameters such as \( \sigma \) and \( r_B \) -which do not modify the banks' capacity to take deposits or invest in risky assets- do not cause strong variations in \( \mu_A \) and \( \mu_D \). So, out of the long-run equilibrium, the effects of changes in systematic risk and the safe rate of return are generally lower than in the long-run equilibrium examined above.

5. CONCLUSIONS.

This paper has analyzed banking prudential regulation in an equilibrium model. In the model, banks play a role in financing heterogeneous risky investment projects whose original owners lack financial resources to undertake them. Banks can evaluate the quality of projects and, if adequate, they can provide funding for them. Banks have limited liability and deposits are fully insured. Deposits provide some transaction and liquidity services to depositors, so they are willing to accept rates of return below the risk-free rate.

Bankers invest all their wealth in bank capital and seek to maximize the expected present value of their investment subject to the regulatory constraints. Taking the rates of return as given, banks
choose the amount of deposits, \( D_0 \), and the share of risky assets in their portfolio, \( \alpha \). The behavior of banks is determined by intermediation margins on risky assets, \( \mu_A' \), and deposits, \( \mu_D' \). The optimal \( \alpha \) depends roughly on \( \mu_A' \), whilst the optimal \( D_0 \) depends upon \( \mu_D' \). Because of limited liability and the opportunity to borrow at a fixed rate, banks may find attractive the investment in risky assets even when their expected rate of return is below the risk-free rate.

Unlike in the previous literature, intermediation margins are endogenous. Four types of equilibrium can be identified. Each one corresponds to particular corner solutions to the problem of the representative bank that are linked to meaningful configurations of the banking industry, whose plausibility depends upon the scarcity of bank capital. However, there exists a type of equilibrium which is, in a sense, the one that we would expect to find in the long run, when the rate of return on bank capital is not higher than the risk-free rate and new capital does not have an incentive to enter or to be accumulated in the banking industry. In this long run equilibrium, the expected return on risky assets is lower than the risk-free rate and an over-investment result follows. Banking competition passes through the deposit insurance subsidy to financial markets, causing not only distributional effects but also deadweight losses.

Comparative static results show that the classical regulatory instruments and, in particular, capital requirements may be quite effective in solvency regulation. Its clear contractionary impact on the market for deposits yields a wider intermediation margin in deposits that reinforces the direct effect of lower leverage on the probability of bankruptcy. Nevertheless, care should be paid to the consequences of more stringent capital standards on real investment and on the provision of liquidity services via deposits, especially when, because of capital market imperfections, the banking industry suffers capital shortage and a credit crunch threatens.

Under a given risk-insensitive regulation, shifts in the systematic
risk $\sigma$ which affects investment projects may dramatically change the well-functioning, safety and soundness of the banking system. If, for instance, $\sigma$ goes up, the implicit subsidy to risk-taking rises and banking activity becomes more profitable, boosting competition for risky assets and deposits. Intermediation margins decline and the solvency of banks drops because of the twofold effects (direct and via equilibrium margins) of higher volatility of asset values.

This model may help to explain the recent solvency problems of the savings and loan institutions in U.S. In particular, the higher systematic risk of the late seventies and early eighties (higher $\sigma$) and the inadequate response of the regulator—which included shoring up book value net worth with a variety of accounting changes and reducing minimum regulatory capital requirements (decreasing $k$)—could joined short-term factors (poor realizations of $z$) to trigger off the debacle of the mid eighties. Detailed descriptions of the process given by the U.S. Department of the Treasury (1991) are consistent with the results predicted by this model (higher investment in risky assets and lower intermediation margins). Some empirical evidence also offers support to this interpretation (see Cole, McKenzie and White, 1992).
APPENDIX 1: Proof of Proposition 1.

The objective function of the representative bank (14) depends on $\alpha$ and $D_0$. When the investment in safe assets is great enough to guarantee the payment of the depositors' claims (i.e., when $(1+r_B)B_0 \geq (1+r_D)D_0$), the probability of failure of the representative bank $q$ is zero. Let us define $\bar{\alpha}(D_0)$ as the maximum $\alpha$ which implies a zero probability of failure for a given value of $D_0$. Using (13) and the definition of $\mu_D$, $\bar{\alpha}(D_0)$ can be defined by:

$$\bar{\alpha}(D_0) = \alpha \text{ s.t. } (1-\alpha)((1-p-c)D_0 + K) = (1-\mu_D)D_0.$$ 

$\bar{\alpha}(D_0)$ is decreasing and convex in $D_0$ and takes the value 1 when $D_0 = 0$. This function separates the region of the feasible set where $q = 0$ from the one where $q > 0$ (see Figure A1).

The proof of the result will be done in three steps. First, I will show that there is no solution $(\alpha', D')$ with $0 < \alpha' < 1$ and $\alpha' > \bar{\alpha}(D')$. Afterwards, I will prove that $(\alpha', D')$ with $0 < \alpha' < 1$ and $\alpha' \leq \bar{\alpha}(D')$ cannot be optimal, except with $D' = 0$. Finally, we will see that $(1, D')$ with $0 < D' < \bar{D}$ can be discarded too.

(i) Suppose that $(\alpha', D')$ with $0 < \alpha' < 1$ is a solution to the problem of the bank, and suppose also that $\alpha' > \bar{\alpha}(D')$ so that $q > 0$. The corresponding first order condition (FOC) implies:\n
$$\frac{\partial V_0}{\partial \alpha} \Big|_{\alpha'} = \left[ (1-p-c)D' + K \right] \left[ (1+\mu_A)F(x) - F(x-\sigma) \right] = 0. \quad (A1)$$

\[14\] Notice that $V_0$ as the Black-Scholes formula has the interesting property that its partial derivative with respect to $x$ is zero.
FIGURE A1

The feasible set and the probability of bankruptcy
Under (A1), the expression for \( V_0(\alpha', D') \) is

\[
V_0(\alpha', D') = \left[ (1-p-c)D' + K - (1-\mu_D)D' \right] F(x-\sigma)
\]

which should be greater than \( K \) and hence positive, since the bank could have chosen \((\alpha, D)_0 = (0,0)\) that yields \( V_0 = K \). Let \( f(x) \) be the density function of a standard normal distribution. The second partial derivative of \( V_0 \) with respect to \( \alpha \) can be written as:

\[
\frac{\partial^2 V_0}{\partial \alpha^2} = \left[ (1-p-c)D' + K \right] \left[ (1+\mu_A)f(x) - f(x-\sigma) \right] \frac{\partial x}{\partial \alpha}.
\]

At \( \alpha' \), we have:

\[
(1+\mu_A)f(x) - f(x-\sigma) = F(x-\sigma) \left[ f(x)/F(x) - f(x-\sigma)/F(x-\sigma) \right],
\]

since (A1) implies \((1+\mu_A) = F(x-\sigma)/F(x)\). Expression (A3) is negative, since \( f(\cdot)/F(\cdot) \) is the survival function of a standard normal random variable, which is a decreasing function. On the other hand, from the definition of \( x \) we can deduct that the sign of \( \partial x/\partial \alpha \) at \( D' \) is that of \((1-\mu_D)D' - (1-p-c)D' - K \), which from (A2) is negative. Consequently, \( \partial^2 V_0/\partial \alpha^2 \) is positive and, hence, \( \alpha' \) is not a maximum.

(ii) Suppose, next, that \( \alpha' = \bar{\alpha}(D') \) so that \( q' = 0 \). For a given \( D' \) and \( \alpha = \bar{\alpha}(D') \), \( V_0(\alpha, D') \) is a linear function of \( \alpha \). If \( \alpha' < \bar{\alpha}(D') \), then the following first order condition holds:

\[
\frac{\partial V_0}{\partial \alpha} = \mu_A = 0.
\]

and \( \bar{\alpha}(D') \) is also a solution to the problem of the bank. However, for any \( \alpha > \bar{\alpha}(D') \), we have \( F(x) < 1 \), \( F(x-\sigma) < 1 \) and \( F(x) > F(x-\sigma) \). Under (A4), the right partial derivative of \( V_0 \) at \( \bar{\alpha}(D') \) is strictly positive. Therefore, neither \( \bar{\alpha}(D') \) nor \( \alpha' \) can be a maximum, except when \( \bar{\alpha}(D') = 1 \) (i.e. \( D'=0 \)).
If \( \alpha' = \alpha(D') \), \( \frac{\partial V}{\partial \alpha} \) is non-negative at \( \alpha' \) and similar arguments can be used to prove that \((\alpha', D')\) cannot be a maximum except when \( D' = 0 \).

(iii) Solutions in the interior of the feasible set have been discarded above (see Figure A1); solutions in its East boundary has been discarded as well. Now I will show that solutions in the North boundary (excluding the corners) can also be discarded. Suppose that \((1, D')\) with \( 0 < D' < D \) is a solution to the problem of the bank. The corresponding first order condition implies:

\[
\left. \frac{\partial V}{\partial D} \right|_{D'} = (1+\mu_A)(1-p-c)F(x)-(1-\mu_D)F(x-\sigma) = 0. \tag{A5}
\]

On the other hand,

\[
\frac{\partial^2 V}{\partial D^2} = \left[ (1+\mu_A)(1-p-c)f(x)-(1-\mu_D)f(x-\sigma) \right] \frac{\partial x}{\partial D}. \tag{A6}
\]

As (A5) implies \((1+\mu_A)(1-p-c)F(x)/F(x-\sigma)=(1-\mu_D)\), the value of the first term in the right hand side of (A6) at \( D' \) equals:

\[
(1+\mu_A)(1-p-c) \left[ f(x)/F(x) - f(x-\sigma)/F(x-\sigma) \right] F(x). \tag{A7}
\]

Expression (A7) and \( \frac{\partial x}{\partial D} \) are strictly negative and, hence, \( D' \) is not a maximum.

Therefore, optimal solutions to the problem of the representative bank will be found either in the top right corner of the feasible set or in its West and South boundaries.\(\Box\)
APPENDIX 2: The behavior of banks in the \((\mu_A, \mu_D)\) space.

From Proposition 1, we can restrict our attention to the following sets of solutions: (i) \(\alpha=1\) and \(D_0=\overline{D}\), (ii) \(\alpha=1\) and \(D_0=0\), (iii) \(\alpha=0\) and \(D_0=0\), (iv) \(\alpha=0\) and \(D_0=\overline{D}\), (v) \(\alpha \in (0,1)\) and \(D_0=0\), (vi) \(\alpha=0\) and \(D_0 \in (0, \overline{D})\).

Under \(D_0=0\), \(V_0\) is a linear function of \(\alpha\), hence (v) is optimal only if (ii) and (iii) are also optimal, that is, when banks are indifferent between (ii) and (iii). Similarly, under \(\alpha=0\), \(V_0\) is a linear function of \(D_0\), hence (vi) is optimal when banks are indifferent between (iii) and (iv). Now I must compare (i), (ii), (iii) and (iv).

Consider first the optimal choice of \(\alpha\) for a given value of \(D_0\):

- If \(D_0=\overline{D}\), choosing between \(\alpha=1\) or \(\alpha=0\) involves the comparison of \(V_0(1, \overline{D})\) with \(V_0(0, \overline{D})\). The representative bank is indifferent between both if \(V_0(1, \overline{D})-V_0(0, \overline{D})=0\). So using the definition of \(V_0(\alpha, D_0)\) in (14) and noting that from (17) \((1-p-c)D+K=\frac{1}{l}(1+k)D\), we can obtain:

\[
(1-\mu_D)(1-F(x-\sigma))-\frac{1}{1-k}(1-(1+\mu_A)F(x))=0. 
\] (A8)

This equation defines an upward sloping curve, GG', in the space of intermediation margins. When \(\mu_D\) tends towards 1, \(\mu_A\) tends towards zero and the curve becomes flatter (see Figure A2).

- If \(D_0=0\), choosing between \(\alpha=1\) or \(\alpha=0\) involves the comparison of \(V_0(1,0)\) with \(V_0(0,0)\). The sign of \(V_0(1,0)-V_0(0,0)\) depends on \(\mu_A\). When \(\mu_A>0\) (i.e. above the horizontal axis), \(\alpha=1\) is better than \(\alpha=0\). The contrary occurs when \(\mu_A<0\) (i.e. below the horizontal axis).

So, if \(D_0=\overline{D}\) (\(D_0=0\)) the bank will choose \(\alpha=1\) or \(\alpha=0\) verifying if \((\mu_A, \mu_D)\) falls above or below GG' (the horizontal axis). As GG' is below the horizontal axis, three regions can be considered: (i) the region above the horizontal axis, (ii) the region below GG', (iii) the region between the previous two. In the first the optimal value of \(\alpha\) is 1; in the
FIGURE A2
second it is 0; in the third, the optimal value is 1 if \( D_0 = \bar{D} \) and 0 if \( D_0 = 0 \).

The next step is to characterize the optimal supply of deposits in each of these three regions:

(i) Region above the horizontal axis. As the optimal \( \alpha \) is 1, the bank will be indifferent between \( D_0 = \bar{D} \) and \( D_0 = 0 \) if \( V_0(1, \bar{D}) - V_0(1, 0) = 0 \). Noting again that form (17) \( (1-p-c)D + K = [1/(1+k)]\bar{D} \) and \( K = [1/(1+k)-(1-p-c)]\bar{D} \), the indifference condition can be written as:

\[
(1+p+c)(1-p-c) - (1-p-c)F(x, \sigma) = 0. \tag{A9}
\]

Equation (A9) defines a downward sloping curve, \( \text{HH}' \), in the space of intermediation margins. \( D_0 \) will be \( \bar{D} \) at points to the right of \( \text{HH}' \) and 0 at points to the left.

(ii) Region below \( GG' \). As the optimal \( \alpha \) is 0, the bank will be indifferent between \( D_0 = \bar{D} \) and \( D_0 = 0 \) if \( V_0(0, \bar{D}) - V_0(0, 0) = 0 \). Using (14) and (17) we have:

\[
(\mu_0 - p - c) = 0. \tag{A10}
\]

This equation defines a vertical straight line, \( JJ' \), in the space of intermediation margins. \( D_0 \) will be \( \bar{D} \) at points to the right of \( JJ' \) and 0 at points to the left.

(iii) Region below the horizontal axis and above \( GG' \). As the optimal value of \( \alpha \) is 1 if \( D_0 = \bar{D} \) and 0 if \( D_0 = 0 \), the optimal solution can be obtained by comparing \( V_0(1, \bar{D}) \) with \( V_0(0, 0) \). Indifference arises when \( (\mu_A, \mu_D) \) verifies \( V_0(1, \bar{D}) - V_0(0, 0) = 0 \), i.e.:

\[
\left[ \frac{1}{1-k} (1+p+c)F(x) - (1-p-c)F(x, \sigma) - \frac{k}{1-k} (1+p+c) \right] \bar{D} = 0. \tag{A11}
\]

Equation (A11) defines a downward sloping curve, \( KK' \), in the space of intermediation margins. \( D_0 \) will be \( \bar{D} \) at points to the right of \( KK' \) and 0
at points to the left.

Notice that HH' and KK' intersect at $\mu_A = 0$, while GG', JJ', and KK' intersect at $\mu_D = p + c$. Figure 2 can be straightforwardly derived from the results above. □
APPENDIX 3: Comparative static results.

An equilibrium of type II is a pair \((\mu_A, \mu_D)\) such that belongs to \(A \cap B\) (i.e. verifies (A9)) and verifies (19).\(^{15}\) (A9) defines a downward sloping curve in the space of intermediation margins, whilst (19) corresponds to an upward sloping curve. The equilibrium corresponds to the intersection between both curves. The effects of the change in a parameter can be diagrammatically analyzed by looking at the movements of the curves and the shift in their intersection. Implicit total differentiation of (A9) and (19) leads respectively to (A12) and (A13), where the signs of the partial derivatives are written below the corresponding variables and parameters (absent parameters do not affect the corresponding equilibrium condition).

\[
G_{II}(\mu_A, \mu_D; k, p, c, \sigma) = 0, \quad (A12)
\]

\[
H_{II}(\mu_A, \mu_D; k, p, c, K, r_B) = 0. \quad (A13)
\]

Similarly, an equilibrium of type III is defined by equations (A11) and (20), which lead to (A14) and (A15).

\[
G_{III}(\mu_A, \mu_D; k, p, c, \sigma) = 0, \quad (A14)
\]

\[
H_{III}(\mu_A, \mu_D; k, p, c, K, r_B) = 0. \quad (A15)
\]

\(^{15}\) I assume that inequality conditions such as \(D(\mu_B) \geq D\) hold with strict inequality, otherwise the equilibrium could be treated as an equilibrium of type I.
Finally, in an equilibrium of type IV, equilibrium conditions are (A8) and (21), thus,

\[ G_{IV}(\mu_A, \mu_B; k, p, c, \sigma) = 0, \]

and

\[ H_{IV}(\mu_A, \mu_B; k, p, c, K, r_B) = 0. \]

Obtaining the results of Tables 2, 3, and A1 is quite straightforward. Only the sign of \( dq/dk \) in type III equilibrium requires some additional computations in order to show that the direct effect dominates the ambiguous effect of \( \mu_A \) even in the most unfavorable case.
### TABLE A1

Comparative Statics

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<th>Parameter (w)</th>
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<th>( \frac{d\mu_D}{dw} )</th>
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REFERENCES


CHAPTER 4

CLOSURE RULES, MARKET POWER AND RISK-TAKING
IN A DYNAMIC MODEL OF BANK BEHAVIOR
1. INTRODUCTION.

It is well known that informational asymmetries between creditors and shareholders together with limited liability lead to excessive risk-taking by indebted firms (Jensen and Mecklin, 1976). This problem can be ameliorated by means of monitoring investment decisions, introducing special covenants in debt contracts and providing the adequate incentives in managerial contracts for managers to maximize the total value of the firm, instead of the value of equity. In the banking industry, however, small depositors have neither the incentive nor, probably, the ability to monitor and discipline bank management. Moreover, their reactions take the form of deposit withdrawals, which may lead to bank runs and eventual failures (Diamond and Dybvig, 1983). Bank failures may cause substantial costs generating the so-called systemic risk of banking.

Deposit insurance may therefore be considered as a way by which depositors delegate the role of monitoring and disciplining banks to the banking authorities (Dewatripont and Tirole, 1993). This delegation eliminates free-ri ding and solves the coordination problem underlying bank runs. In turn, it creates a regulatory problem, since regulation (as opposed to the market) has to deal with the moral hazard problem usually faced by creditors in other industries. As first noted by Merton (1977), the deposit insurance agency is the writer of a put option on bank assets at a strike price equal to the promised maturity value of deposits.

Static models of bank behavior under risk-insensitive deposit insurance show that higher mean-preserving portfolio risk and leverage are to the benefit of bank shareholders and to the detriment of the deposit insurance agency (see Furlong and Keeley, 1989). In these models, if the value of assets at the end of the period is smaller than
the value of liabilities, bankruptcy is declared. Then, the deposit insurance agency closes the bank pays off depositors and, as receiver, liquidates the assets of the bank. The payoff corresponding to bankers is zero and that is all. In a dynamic setting, the banker who goes bankrupt is likely to suffer losses related to future payoffs. Apart from reputational losses which would appear in other industries, banking regulation contains special provisions for promoters and managers of banks which become insolvent, so that they are typically excluded from business. Bankers need a charter to run a bank and, when supervisors intervene in bankruptcy procedures, the charter of the failed institution is either canceled after liquidation or transferred to a new holding company after a purchase and assumption transaction. As a consequence, the threat of loss of the value of the charter when the bank fails may act as a disciplinary device against risk-taking.

Of course, for this to be the case, the value of the charter has to be positive. It is not rare, however, that locational, informational and reputational rents surge in the normal course of the banking business, where switching costs and regulatory barriers to entry are very plausible sources of market power. Thus, in a dynamic setting, the stream of potentially positive future expected profits determines the cost of bankruptcy to the banker. The higher the present value of such stream, the lower the incentive to adopt risky short run decisions. Present and future profits depend on market power as well as on the regulatory constraints and macroeconomic factors affecting banks. From a prudential perspective, a clear regulatory trade-off may exist: providing market power to the industry makes it safer, but investors' and depositors' surpluses may decline. Moreover, the regulator should know to which extent capital and asset regulations can compensate for the deterioration of soundness in an increasingly competitive banking sector.

This paper examines the prudential regulation of banks taking into account the interactions between closure rules, market power and risk-taking that have been sketched above. Some authors have previously
analyzed the prudential implications of closure rules in banking, but from different, more partial or less formal perspectives. Davis and McManus (1991) examine the behavior of a risk-averse bank manager who faces an exogenous bankruptcy cost in a standard one-period model, focusing on the regulatory choice of the range of net worth values at which the bank is closed. In a similar setup, Mailath and Mester (1993) analyze discretionary closure within a game theoretic model, dealing with the determination of the closure decision by the authority. Banking charters are specifically mentioned in Marcus (1984) and Keeley (1990), who relate their value to market power. Their theoretical frameworks consist of simple one-period models where the charter is a shareholders' claim which is contingent on solvency and whose conditional-on-solvency value is essentially taken as given.

Endogenizing the value of the charter within an infinite horizon model which allows for bankruptcy and closure constitutes the main purpose of this paper. The optimization program presented in section 2.1 extends in a natural way the typical decision problem faced by a bank in a static setting to a dynamic one. While in the static model the bank chooses an investment decision once and for all, in the dynamic model it chooses a state-contingent sequence of investment decisions. In the simplest stationary version of the model, time indices can be dropped and the state can be summarized by an indicator variable which represents whether the bank remains open or has been closed by the authorities. As in the standard bankruptcy procedures, the bank is closed if its net worth at the end of a period is negative.

Dynamic programming techniques allow us to define the value function associated to the bank's problem and to obtain an implicit definition for the conditional-on-surviving equilibrium (net) value of the bank, \( v \) (section 2.2). Not surprisingly, \( v \) is the sum of current one-period profits and the equilibrium value of the banking charter (which, in turn, is the discounted value of \( v \) times the probability of the bank being closed at the end of the current period). Equivalently, \( v \) is the expected (net) present value of the stream of current and future
one-period profits.

Optimal policies depend on such equilibrium value of the bank. Under perfect competition, \( v \) is zero and the solution to the dynamic problem coincides with the one of the static problem: closure rules are ineffective in disciplining banks (section 2.3). With monopoly rents the dynamic problem is more interesting. And the easiest way to allow for rents is to assume that the bank is a (local) monopolist in the market for deposits (section 2.4).

With the elements previously introduced, section 3 discusses existence, uniqueness and comparative statics, deriving the effects on \( v \) of the capital requirement, the regulatory bound to the risk of bank assets, the degree of market power and the risk-free rate of interest. All these parameters may have a large influence on the value of the bank, which is the crucial determinant of bank risk-taking.

Section 4 analyzes the behavior of the bank as a function of these parameters. Optimal bank policies are characterized and the regulatory trade-offs are discussed. In section 5, I extend the model in order to deal with an alternative closure rule which allows for voluntary recapitalization by shareholders in the event of failure. Section 6 concludes with some policy implications for the current regulatory debate in the U.S. and Europe.

2. THE DYNAMIC OPTIMIZATION PROGRAM.

2.1. Banking charters, solvency and the dynamic program.

In this paper, a bank is conceived as the investment project of a group of shareholders called bankers. Bankers are risk-neutral, enjoy limited liability and are initially granted a banking charter. A banking charter is an official permission to keep the bank open and under the control of their shareholders. The charter is renewed at the beginning of each
period provided that the bank is solvent. If this is not the case, the bank is intervened and banking authorities assume control. From the viewpoint of the bankers, intervention is equivalent to closure, since entails the loss of the charter. Therefore, in analyzing the behavior of a bank I will indistinctly refer to intervention or closure.  

The sequence of events in any period \( t \) in which the bank remains open is as follows. At the beginning of period \( t \) the bank raises deposits \( D_t \) and capital \( K_t \) in order to invest in a portfolio of assets. The gross return of the portfolio of assets is a random variable \( R(\sigma_t) \), independently distributed across time, with \( E[R(\sigma_t)]=1+r \) and dispersion measured by \( \sigma_t \). \( r \) is the risk-free rate of interest. Shareholders' decision entails choosing \( D_t, K_t \) and \( \sigma_t \).

At the end of period \( t \), once asset returns are observed, the net worth of the bank, \( N_{t+1} \), is computed as the difference between the end-of-period-\( t \) value of assets and liabilities. The value of assets results from applying the stochastic gross return \( R(\sigma_t) \) to the initial investment \( D_t+K_t \). Liabilities are made up of promised payments to depositors (principal plus interest) and non-interest intermediation costs -whose payment is assumed to take place at the end of the period-. Liabilities are fully insured by a deposit insurance agency, so that their cost is independent of the default risk of the bank. They are modeled as an increasing function of \( D_t, C(D_t) \). Particular assumptions about the returns to scale in intermediation and the degree of market power posessed by the bank when taking deposits will determine the shape of this function.  

\[ 1 \text{ In practice, supervisors face a variety of alternative ways to resolve insolvencies and the one applied in each case depends upon considerations such as cost minimization and the preservation of confidence in the banking system (see Benston et al., 1986).} \]

\[ 2 \text{ In particular, a linear technology together with perfect competition} \]
the decision variables and the realization of $R(t)$:

$$N_{t+1} = R(t)(D_t + K_t) - C(D_t).$$

After computation of $N_{t+1}$, several possibilities arise. On the one hand, the bank may be liquidated by the bankers or intervened by the authorities; in this case the final payoff to shareholders is $\max(N_{t+1}, 0)$. Alternatively, the bank may remain open and under the control of the shareholders. Then they take decisions for the following period. In particular, by choosing $K_{t+1}$, the shareholders implicitly decide whether the bank pays a dividend ($K_{t+1} < N_{t+1}$) or raises more capital ($K_{t+1} > N_{t+1}$).

The dynamic problem of the bank is different from a sequence of static problems because of the existence of a charter whose renewal takes place according to a closure rule. I will consider the simple and realistic case in which banking authorities deny renewal and close the bank if its net worth at the end of a period is negative. Let the indicator variable $I_t$ represent whether the bank is open or closed at the beginning of period $t$:

$$I_t = \begin{cases} 0 & \text{if the bank is closed} \\ 1 & \text{if the bank remains open.} \end{cases}$$

Then the dynamics of closure under this rule can be formalized as follows:

$$I_t = I_{t-1} \cdot g(N_t),$$

in the market for deposits (i.e. a rate-taking behavior by the bank) would cause $C(D)$ to be linear.
where

\[ g(x) = \begin{cases} 
0 & \text{if } x < 0 \\
1 & \text{otherwise.} 
\end{cases} \]

With this specification, \( I_t \) takes value 0 in two different cases: first, when the bank has been previously closed, \( I_{t-1} = 0 \); second, when it becomes insolvent at the end of period \( t-1 \), \( N_t < 0 \). Otherwise, \( I_t \) takes value 1.

In terms of the dynamic program, the state variable is \( I_t \) and the vector of control variables is \( y_t = (D_t, K_t, \sigma_t) \). In each period, the bank is subject to static capital and asset regulations. On the one hand, a capital requirement obliges the bank to hold capital in excess of a certain fraction \( k \) of deposits, \( K_t \geq kD_t \). On the other, regulatory limits to risk-taking in the asset side (together with the level of systematic risk in the economy) determine an upper bound to the level of risk of bank portfolios, \( \sigma_t \leq \bar{\sigma} \). I assume that the informational and institutional context is such that banking regulation cannot or does not directly depend on \( \sigma_t \), although indirect constraints on portfolio composition, off-balance-sheet operations, short-selling, sectorial and geographical concentration and the associated surveillance techniques allow the regulator to influence the upper bound to \( \sigma_t \). Accordingly, the set of feasible controls of the dynamic program is specified as follows:

\[
\Gamma = \{ y_t = (D_t, K_t, \sigma_t) \in \mathbb{R}^3 \mid K_t \geq kD_t \text{ and } \sigma_t \leq \bar{\sigma} \}, \tag{1}
\]

where \( k \) and \( \bar{\sigma} \) are determined by the regulator. For the sake of simplicity, I will consider time-invariant \( k \) and \( \bar{\sigma} \) so that the regulatory framework is stationary.

The dynamic optimization problem solved by the bankers is:
Maximize \( \mathbb{E} \left[ \sum_{t=0}^{\infty} (1+r)^{-t} \psi(I_t, y_t) \right] \)

subject to \( y_t \in \Gamma \) \( t = 0, 1, 2, \ldots \)

\[ I_t = I_{t-1} \cdot g(N_t) \quad t = 1, 2, \ldots \]

\[ I_0 = 1 \]

where the state-dependent one-period profit function is given by:

\[ \psi(I_t, y_t) = \begin{cases} 
(1+r)^{-1} \max(N_{t+1},0) - K_t & \text{if } I_t = 1 \\
0 & \text{if } I_t = 0.
\end{cases} \] (2)

The bankers' problem can be interpreted as finding an optimal sequence of financial and investment decisions \( (D_t, K_t, \sigma_t) \) in an infinite discrete-time horizon. Risk-neutral bankers maximize the expected discounted value of the stream of cash-flows from their investment in the bank. If the bank is open, the investment generates an outflow of \( K_t \) at the beginning of period \( t \) and an inflow of \( \max(N_{t+1},0) \) at the end (discounting applies). If the bank is closed, cash-flows are zero.

The main difference between this program and the static one is that, while in the latter the bank ignores the closure rule, in the former the bank takes into account the impact of its present decisions on the probability of being closed, since closure hinders its shareholders from obtaining potentially positive future profits.\(^3\) When such profits are high, a clear incentive to adopt prudent short-run policies arises.

\(^3\) Provided that \( I_t = 1 \), the static program is: Maximize \( \mathbb{E}[\psi(I_t, y_t)] \), subject to \( y_t \in \Gamma \).
2.2. The value function.

Dynamic programming techniques allow us to define the value function associated to the bank's dynamic optimization problem:

\[ V(I_t) = \sup_{y_t \in \Gamma} E[\psi(I_{t}, y_t) + (1+r)^{-1}V(I_{t+1})]. \]  

This functional equation has a simple structure, since \( I_t \) takes only two values ("open" and "closed") and the value of a closed bank is zero. Then,

\[ V(I_t) = \begin{cases} 
0 & \text{if } I_t = 0 \\
\nu & \text{if } I_t = 1
\end{cases} \]

where \( \nu \) is the conditional-on-survival equilibrium value of the bank, which henceforth I will simply call the equilibrium value of the bank:

\[ \nu = \sup_{y_t \in \Gamma} E[\psi(I_{t}, y_t) + (1+r)^{-1}V(I_{t+1}) | I_{t+1} = 1]. \]

Observe that the stationarity of the problem implies that \( \nu \) does not depend on time.

Now, using expression (4) evaluated in period \( t+1 \) and the definition of \( I_{t+1} \), the conditional expected value of \( V(I_{t+1}) \) can be written as follows:

\[ E[V(I_{t+1}) | I_t = 1] = \text{Prob}[I_{t+1} = 1 | I_t = 1] \cdot \nu \]

Since the probability of remaining open in \( t+1 \) when the bank is open in \( t \) only depends on being solvent at the end of period \( t \), we have

\[ E[V(I_{t+1}) | I_t = 1] = \text{Prob}[N_{t+1} \geq 0] \cdot \nu. \]
Plugging (6) into (5) and denoting \( \text{Prob}[N_{t+1} \geq 0] \) by \( \Phi(y_t) \), we get:

\[
v = \sup_{y_t \in \Gamma} \{ E[\psi(1,y_t)] + (1+r)^{-1}\Phi(y_t)v \},
\]

This equation is fully time and state independent, so we can drop the time indices and leave:

\[
v = \sup_{y \in \Gamma} \{ \Pi(y) + (1+r)^{-1}\Phi(y)v \}, \tag{7}
\]

where \( \Pi(y) \) denotes the conditional-on-survival expected one-period profit of the bank, \( E[\psi(1,y)] \).

The set of optimal policies for a given \( v \) can be defined as follows:

\[
Y(v) = \arg\max_{y \in \Gamma} \{ \Pi(y) + (1+r)^{-1}\Phi(y)v \}. \tag{8}
\]

When the bank decides an optimal control at the outset of period \( t \), \( v \) is taken as given, since \( y \) does not affect the expected future value of the bank, but only the probability of it to remain open, \( \Phi(y) \). The higher the value of \( v \), the lower the incentives to adopt short-run decisions that could increase the likelihood of being closed.

2.3. One-period profits and the probability of being closed.

Notice that, if \( C(D) \) were linear and shareholders were able to raise as much capital as they wanted at an opportunity cost \( r \), the bank's one-period profit function, \( \Pi(y) \), would be homogeneous of degree 1 in \( D \) and \( K \), whilst the capital requirement inequality and the probability of survival \( \Phi(y) \) would be homogeneous of degree 0:

\[
\Pi(y) = (1+r)^{-1}E[\max(R(\sigma)(D+K)-C(D), 0)] - K,
\]
\[ \Phi(y) = \text{Prob}[R(\sigma)(D+K)-C(D) \geq 0]. \]

In such a situation, the value function would also be homogeneous of degree 1 in D and K. Degree-one homogeneity implies that either \( \Pi(y) \) is zero and, hence, \( v \) is zero for the optimal values of D and K (and, accordingly, the optimal scale of the bank is indeterminate) or the optimization problem lacks strictly positive and bounded solution for D and K.

Assume, for example, that \( C(D) = (1+r+D)D \), where \( r \) stands for the interest rate paid on deposits and \( c \) the non-interest intermediation costs per unit of deposits. It is well-known that the equilibrium outcome in a perfectly competitive industry facing constant returns to scale entails that prices are such that a zero-profit condition holds. Similarly, perfect competition between banks like the one described above would necessarily lead to an equilibrium deposit interest rate \( r_D \) such that \( \Pi(y) \) and \( v \) would be equal to zero. If \( v \) equals zero, however, the optimal control simply maximizes one-period expected profits (see equation (8)), thus ignoring the impact of its decisions on the probability of remaining open. With this equilibrium argument, we can state the following result.

**Proposition 1:** Under constant returns to scale in intermediation and perfect competition, closure rules do not affect bank decisions.

Previous literature has shown that the behavior of insured banks in static perfectly competitive frameworks is characterized by maximum leverage and volatility of the returns on assets (Furlong and Keeley, 1989) —which reach the bounds imposed by capital and asset regulations—. Moreover, implicit subsidies of the deposit insurance scheme pass through to prices, affecting the equilibrium returns on bank assets and liabilities (Gennotte, 1990) and the efficiency of investment decisions (Gennotte and Pyle, 1991; Suárez, 1993b). Proposition 1 means that closure rules as specified above are ineffective in lessening these problems. Besides, dynamic considerations are not relevant to solving
the optimization problem of the bank and, hence in such a context static models are valid to describe the impact of banking regulation.

In particular, the static model in Suárez (1993a) shows that under perfect competition the optimal profit maximizing policy involves $K/D=k$, $\sigma=\overline{\sigma}$ and a perfectly elastic supply of deposits at a rate $r^*_D$ (determined by a zero profit condition) which depends on $r$, $c$, $k$ and $\overline{\sigma}$.

Nevertheless, perfect competition may not be a reasonable hypothesis. Locational, informational and reputational rents may arise in the normal course of the banking business; switching costs and regulatory barriers to entry are possible sources of market power. For many years, direct controls on interest rates and commissions and regulatory restrictions on branching and geographical expansion imposed clear limits to competition. In fact, the structure of the banking sector in many countries is far from the idealized atomistic market structure where price-taking arises as a natural assumption.

Standard models of oligopolistic competition have long been applied to analyzing the banking industry (see Gilbert, 1984). More recently, several authors have successfully developed specific models of banking and financial intermediation inspired in modern industrial organization analysis (Matutes and Vives, 1992; Repullo, 1991).

The aim of this paper is to examine the effect of closure rules on bank behavior. Up to now we know that such effect is null if there is nothing to lose in the event of closure. An easy way to obtain positive rents in this model without dealing with complex strategic interactions between different banks is to assume that the bank exercises monopoly power in the market for deposits. So, I will analyze the case of a

4 Alternatively, positive rents might be obtained by assuming decreasing returns to scale in the intermediation technology, but this assumption
monopolistic bank in a simplified context where non-interest intermediation costs are zero.

2.4. Modeling market power.

Suppose that the bank has a local monopoly in the supply of deposits. Bank deposits provide transaction and liquidity services to depositors, who are consequently willing to demand deposits even if their rate of return \( r_D \) is smaller than the risk-free rate \( r \). Clearly, when \( r_D \) equals \( r \), depositors hold all of their financial wealth as deposits. When \( r_D \) is smaller than \( r \), the opportunity cost of holding deposits is positive, but the utility of liquidity services may compensate for it. Define the intermediation margin \( \mu \) as the difference between \( r \) and \( r_D \). If the marginal utility of liquidity services is decreasing in the amount of deposits, the demand for deposits will be a decreasing function of \( \mu \).

For notational convenience, the (exogenously given) financial wealth of potential depositors can be normalized to unity. Then, in terms of the inverse demand function for deposits, the behavior of depositors can be parameterized as follows:

\[
(1+r_D) = (1+r) D^\eta \quad \eta \geq 0, \quad 0 \leq D \leq 1,
\]

so that, taking logs, \( \mu = -\eta \log(D) \). Notice that \( \mu \) decreases as \( D \)

would be counter-factual. In a recent paper, McAllister and McManus (1993) argue that U-shaped cost functions traditionally estimated in banking (see Gilbert, 1984) were due to econometric problems. Once these problems are solved, there is strong evidence of increasing returns to scale for small banks and approximately constant returns for larger banks. On the other hand, considering the exercise of monopoly power in the market for loans would require a careful treatment of the implicit variety of risk levels associated to the choice of \( \sigma \).
increases, and is strictly positive for \( D < 1 \) and zero for \( D = 1 \). \( \eta \) is the semi-elasticity of \( \mu \) to changes in \( D \). The greater \( \eta \), the greater the degree of market power of the monopolist bank. Equation (9) yields the following simple specification for \( C(D) \):

\[
C(D) = (1+r) D^{\eta+1}.
\]

In order to obtain a convenient closed-form for \( \Pi(y) = E[\psi(1,y)] \), the following parameterization of \( R(\sigma) \) is assumed:

\[
R(\sigma) = (1+r)\exp(\sigma z - \sigma^2/2),
\]

where \( z \) is a gaussian white noise process. According to (10), \( R(\sigma) \) is a log-normally distributed random variable with expected value equal to \( (1+r) \) and standard deviation increasing with \( \sigma \). Let \( F(z) \) denote the cumulative distribution function of \( z \) and \( f(z) \) the corresponding density function.

Now, putting all of their components together, we have

\[
\psi(1,y) = \max(\exp(\sigma z - \sigma^2/2)(D+K)-D^{\eta+1},0) - K,
\]

\[
\phi(y) = \text{Prob}[\exp(\sigma z - \sigma^2/2)(D+K)-D^{\eta+1} \geq 0]
\]

(notice that the discount factor cancels out with the \((1+r)\) terms in \( R(\sigma) \) and \( C(D) \)). Since \( \exp(\sigma z - \sigma^2/2)(D+K)-D^{\eta+1} \geq 0 \) if and only if

\[
z \geq (1/\sigma)(\eta+1)[(D+K)-D^{\eta+1}] = w,
\]

we can write

\[
E[\psi(1,y)] = \int_{-\infty}^{+\infty} \max(\exp(\sigma z - \sigma^2/2)(D+K)-D^{\eta+1},0)f(z)dz - K
\]

\[
= \int_{w}^{\infty} \exp(\sigma z - \sigma^2/2)(D+K)-D^{\eta+1}f(z)dz - K.
\]
Now, from the normality of \( z \), \( E[\psi(1,y)] \) can be written in terms of the cumulative distribution function of a normal random variable. Integrating by parts and rearranging,

\[
\Pi(y) = E[\psi(1,y)] = F(x)(D+K) - F(x-\sigma)D^{\eta+1} - K, \tag{11}
\]

where

\[
x = \sigma - w = (1/\sigma)[\log(D+K) - (\eta+1)\log(D) + \sigma^2/2]. \tag{12}
\]

On the other hand,

\[
\Phi(y) = \text{Prob}[z \geq w] = F(x-\sigma). \tag{13}
\]

The first two terms in (11) represent, respectively, the value of assets and liabilities to the bankers. The first one is the product of the probability of the bank being able to pay off depositors at the end of the period, \( F(x-\sigma) \), times the conditional on solvency present expected value of assets, \( [F(x)/F(x-\sigma)](D+K) \).\(^5\) The second is the expected present value of payments to depositors, \( F(x-\sigma)D^{\eta+1} \). Expression (11) is akin to the Black-Scholes formula for the valuation of call options and has, as the latter, the interesting property of its partial derivative with respect to \( x \) being equal to zero.\(^6\)

---

\(^5\) The conditional on solvency present expected value of assets is the mean of a truncated log-normal variable (only non-bankruptcy values of \( z \) are relevant) and, hence, results from integrating \( \exp(\sigma z - \sigma^2/2) \) over \([w,\infty)\) with respect to \([1-F(w)]^{-1}f(z)dz=[F(x-\sigma)]^{-1}f(z)dz\).

\(^6\) In order to derive this result, notice that \( \partial F(x-\sigma)/\partial x=f(x-\sigma)=\exp(\sigma^2/2-\sigma x)f(x) \), write \( \partial \Pi(y)/\partial x=f(x)[(D+K)-\exp(\sigma^2/2-\sigma x)D^{\eta+1}] \), and use the definition of \( x \) in order to show that the term in square brackets is zero.
3. THE STEADY-STATE VALUE OF THE BANK.

In this section I use expressions (11) and (13) to show that the steady-state equilibrium value of the bank is well-defined in the sense that there exists a unique $v$ that solves equation (7). Later on, comparative static results on this $v$ will be derived.

3.1. Existence and uniqueness of $v$.

Let $H(v)$ be the auxiliary function defined by the right hand side of equation (7):

$$H(v) = \sup_{y \in \Gamma} \{ \Pi(y) + (1+r)^{-1}\Phi(y)v \}. \quad (14)$$

One technical lemma referring to the properties of the function $H(v)$ leads almost directly to Proposition 2, that states the desired result. The proof of the Lemma is given in the Appendix.

**Lemma 1:** For all $v \geq 0$, $H(v)$ is a continuous, increasing and positive function. Furthermore, its slope is smaller than 1.

**Proposition 2:** There exists an unique $v^* \geq 0$ such that $H(v^*) = v^*$.

**Proof:** We are looking for a single fixed point in $H(\cdot)$ (diagrammatically, a single intersection between the graph of $H(v)$ and the 45-degree line). Let $\bar{H}(v)$ be the function defined by the following expression:

$$\bar{H}(v) = \sup_{y \in \Gamma} \Pi(y) + (1+r)^{-1}v. \quad (15)$$

$\bar{H}(v)$ is linear in $v$ since $\sup_{y} \Pi(y)$ does not depend on $v$. $\sup_{y} \Pi(y)$ is finite because $\Pi(y)$ is a continuous function, $D$ and $\sigma$ are bounded, $\Pi(y)$ is decreasing in $K$ and $K$ has a lower bound in zero. Moreover, the slope of $\bar{H}(v)$ is smaller than one, because the discount factor is smaller than
one. Therefore, $H(v)$ cuts the 45-degree line at a single finite value of $v$, $\bar{V} = \frac{1+r}{r} \sup \Pi(y)$. Hence, using Lemma 1 and the fact that $H(v) = \bar{H}(v)$, we conclude that $H(v)$ also cuts the 45-degree line. Furthermore, the intersection takes place at single point $v^*$, to the left of $\bar{V}$ (see Figure 1).\textsuperscript{7}

Notice that $v^*$ is strictly positive when $\sup \Pi(y)$ is strictly positive (i.e. when the bank has market power) and zero when $\sup \Pi(y)$ is zero (i.e. under perfect competition).

3.2. Comparative statics.

Given the properties of $H(v)$ and the uniqueness of $v^*$, comparative static results can be obtained by examining how changes in the regulatory and structural parameters move the graph of $H(v)$ in the neighborhood of $v^*$. Upward shifts increase the steady state value of the bank, whereas downward shifts reduce it. Comparative statics on $v^*$ are interesting for two reasons. First, they provide insights into the fundamentals that determine the value of a chartered regulated bank. Some models of bank behavior include exogenous charter values, but the value of a charter is intrinsically endogenous. Second, the optimal policy of the bank depends critically upon $v$ (see equation (8)) and comparative statics will allow us to analyze in section 4 the impact of structural and regulatory parameters on bank risk-taking.

In order to study the movements of $H(v)$, we should take into account the underlying optimization process. If we denote by $y(v)$ a policy which is optimal for a given $v$ and define the function $G(y,v)$ as $\Pi(y) + (1+r)^{-1} \Phi(y)v$, the function $H(v)$ can be written as $G(y(v),v)$. The

\textsuperscript{7} The auxiliary function $\bar{H}(v)$ allow us to discard the possibility of $H(v)$ approaching asymptotically to the 45-degree line but never intersecting it despite of having a slope smaller than one.
FIGURE 1

Existence and uniqueness of $v^*$
change in a parameter may generally affect both the function $G(y,v)$ and its first argument $y(v)$. Comparative statics is, however, notably simplified by the fact that $y(v)$ is chosen so as to maximize $G(y,v)$ on the feasible set $\Gamma$.

Table 1 contains the comparative statics for changes in the capital requirement $k$, the regulatory limit on the level of risk $\sigma$, the risk-free rate $r$ and the degree of market power $\eta$.

<table>
<thead>
<tr>
<th>Parameter</th>
<th>Sign of $dv^*/dw$</th>
<th>Implicit assumption</th>
</tr>
</thead>
<tbody>
<tr>
<td>$k$</td>
<td>$-$</td>
<td>$k$ is binding$^+$</td>
</tr>
<tr>
<td>$\sigma$</td>
<td>$+$</td>
<td>$\sigma$ is binding$^+$</td>
</tr>
<tr>
<td>$r$</td>
<td>$-$</td>
<td>none</td>
</tr>
<tr>
<td>$\eta$</td>
<td>$+$</td>
<td>none</td>
</tr>
</tbody>
</table>

$^+$Otherwise, $dv^*/dw=0$.

Parameters $k$ and $\sigma$ do not affect $G(y,v)$ directly, but the feasible set $\Gamma$ and perhaps $y(v)$. Only when the constraints that they define are binding at $y(v)$, a small change in $k$ or $\sigma$, can change $y(v)$ and then $H(v)$. When the capital requirement is binding, higher (lower) $k$ reduces (widens) $\Gamma$ and causes $H(v)$ and, then, $v^*$ to be smaller (greater). When it is not, small variations in $k$ are innocuous.

Similarly, if the upper bound to portfolio risk is binding, greater $\sigma$ can make $H(v)$ and $v^*$ to rise, whilst smaller $\sigma$ will have the opposite effect.
Parameters $r$ and $\eta$ affect directly $G(y,v)$ but not $\Gamma$. Thus, small changes in them cannot change the components of $y(v)$ which represent a corner solution. On the other hand, the envelope theorem ensures that the interior components of $y(v)$ may vary, but their variation will only have (negligible) second order effects on $G(y,v)$. Therefore, only the direct effects of $r$ and $\eta$ on $G(y,v)$ are relevant. These effects can be computed as the corresponding partial derivatives of $G(y,v)$ at $y(v)$. From equations (11) and (13):

$$\frac{\partial G(y,v)}{\partial r} = - (1+r)^{-2} \phi(y)v < 0,$$

$$\frac{\partial G(y,v)}{\partial \eta} = - [F(x-\sigma)D^{\eta+1} + (1/\sigma)(1+r)^{-1}f(x-\sigma)v]\log(D) > 0.$$

Notice that $\log(D) < 0$, since $0 \leq D \leq 1$.

On the one hand, increases in $r$ cause the present value of future expected profits to fall and, hence, reduce $H(v)$ and the value of the bank. On the other hand, when the degree of market power increases, potentially wider intermediation margins and higher expected profits account for a greater value of $H(v)$ and $v^*$.

These results support the intuitive idea that regulatory reforms that reduce the operative capacity of the bank cannot do bankers good and are consistent with the usual reluctance of the banking industry to accept harder regulatory constraints, except when they enhance directly or indirectly the degree of market power and the rents of the existing institutions. Note, however, that such reluctance would not make sense in a perfectly competitive environment, where the equilibrium value of a bank would be zero anyway.

4. THE EFFECTS OF REGULATION ON BANK RISK-TAKING.

This section is devoted to analyze the impact of structural and regulatory parameters on bank risk-taking. First, I will characterize
the behavior of the bank for all possible values of $v$. Because of non-convexities introduced by limited liability, the bank will follow one of two distinct types of policy: a safe policy (where $\sigma=0$ and the optimal capital structure is indeterminate) or a risky policy (where $\sigma=\bar{\sigma}$ and the capital requirement is binding), the type that dominates depending on $v$. Later on, I will focus on the equilibrium policies -i.e. the policies which correspond to the equilibrium value of the bank $v^*$-, in order to determine the impact of the structural and regulatory parameters of the model on the equilibrium type of policy and the solvency of the bank. Finally, I will discuss the implications of the model for the design of banking regulation.


The set of optimal policies for a given value of $v$ is defined as follows:

$$Y(v) = \arg\max_{y \in \Gamma} \{ \Pi(y) + (1+r)^{-1}\Phi(y)v \}.$$ 

The optimization problem underlying this definition has corner solutions for $\sigma$. This means that the solutions found in static models under perfect competition also appear in this dynamic monopolistic framework. The difference, however, is that in this context the riskiest policy is not necessarily the best one. Briefly, according to the value of $v$, the bank decides to be safe or to be risky.

**Lemma 2:** There is no interior solution for $\sigma$. Depending on parameters, $\sigma(v)$ may be either 0 or $\bar{\sigma}$. Moreover, a policy involving $\sigma=\bar{\sigma}$ can only be optimal if the capital requirement is binding, $K=kD$.

**Proof:** See the Appendix.

Lemma 2 implies that policies with $0<\sigma<\bar{\sigma}$ whatever $k$, or with $\sigma=\bar{\sigma}$ and $K>kD$ will never be optimal. Intuitively, the bank decides either to be safe in order to preserve its future rents or to exploit the deposit
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The optimal policies are the best of $Y_S(v)$ and $Y_R(v)$.

The best safe policies. Under the safe choice of $\sigma$ the bank cannot fail, the capital structure is irrelevant and being or not closed becomes a deterministic outcome. The profit function takes a very simple expression:

$$\Pi(y) = \Pi(D,K,\sigma) = D - D^{\eta+1}. \quad (16)$$

The conditional optimal supply of deposits is defined by the first order condition derived from (16), which leads to $D_S = (1/(1+\eta))^{1/\eta}$. This expression is always smaller than 1 when $\eta$ is strictly higher than zero. As neither $\Pi(D,K,\sigma)$ nor $\Phi(D,K,\sigma)$ depend on $K$, the bank will choose any $K=kD_S$. Notice that the best safe policies are independent of $v$. Finally, let

$$H_S(v) = \max_{y \in \Gamma} \{\Pi(D,K,\sigma) + (1+r)^{-1}\Phi(D,K,\sigma)v\} = \frac{\eta}{1+\eta} D_S + (1+r)^{-1}v.$$

The best risky policies. Under $\sigma=\tilde{\sigma}$ and $K=kD$, the probability of being closed at the end of a period is positive for all $D>0$. The conditional optimal supply of deposits $D_R(v)$ can be defined as

$$D_R(v) = \arg\max_{0 \leq D \leq 1} ((1+k)F(x)D - F(x-\tilde{\sigma})D^{\eta+1} - kD + (1+r)^{-1}F(x-\tilde{\sigma})v), \quad (17)$$

where

$$x = (1/\tilde{\sigma})(\log(1+k)-\eta \log D + \tilde{\sigma}^2/2). \quad (18)$$

insurance system by means of the highest feasible volatility and leverage. Accordingly, we can condition the analysis of optimal policies upon the choice of $\sigma$, characterizing first the best policies for $\sigma=0$, $Y_S(v)$ (the best safe policies), and then the best policies for $\sigma=\tilde{\sigma}$ and $K=kD$, $Y_R(v)$ (the best risky policies). Clearly, the unconditional optimal policies are the best of $Y_S(v)$ and $Y_R(v)$.
An unique interior solution to this problem does not necessarily exist. For one thing, the usual first order condition may yield a value of D which is greater than one. For another, even if it yields a single 0<D<1, the second order condition does not necessarily hold, so that we can have a minimum, a maximum or a saddle point. Non-convexities might lead in principle to corner solutions such as D=0 and D=1.

Even though computing \( D_R(v) \) will require, in general, numerical calculation, the following lemma states that for values of \( v \) such that the best risky policies are better than the safe policies, \( D_R(v) \) is greater than \( D_S \).

**LEMMA 3**: For any \( v \neq 0 \) such that \( \sigma(v) = \tau \), \( D_R(v) > D_S \).

**Proof**: See the Appendix.

Intuitively, if appropriating the subsidy to risk-taking which is associated to the deposit insurance system makes sense, the bank is willing to pay higher deposit rates than under the best safe policy, since the increase in the subsidy due to greater D pays for the fall in intermediation margins, the subsequent loss of monopoly rents and the increase in the probability of being closed.

Finally, notice that \( Y_R(v) \), in contrast to \( Y_S(v) \), may depend on \( v \), and let

\[
H_R(v) = \max_{0 \leq D \leq 1} \{ h(D, kD, \tau) + (1 + r) \Phi(D, kD, \sigma)v \}.
\]

**The unconditional optimal policies.** Now, by comparing \( H_S(v) \) with \( H_R(v) \) we can obtain the unconditional optimal policy for each \( v \). From previous results, the function \( H(v) \) defined in expression (14) is equal to \( \max(H_S(v), H_R(v)) \). Proposition 3 shows that the optimal policy depends crucially but in a simple and intuitive way on \( v \).
PROPOSITION 3: There exists a unique $\bar{v}>0$ such that the optimal policy is the safe policy for all $v<\bar{v}$ and the risky policy for all $v>\bar{v}$.

Proof: See the Appendix.

Figure 2 represents functions $H_S(v)$ and $H_R(v)$. As shown in the picture, the ordinate at the origin is higher for $H_S(v)$ than for $H_R(v)$, but the slope of $H_S(v)$ is greater than that of $H_R(v)$ given the positive probability of being closed (and losing the charter value) under the risky policy. As proved in Proposition 3, they intersect in a single point $\bar{v}$, that separates the values of $v$ at which each policy dominates.

4.2. Regulatory and structural determinants of bank risk-taking.

From Proposition 3 we can deduct that $H(v)$ is a continuous function with a kink in $\bar{v}$. Whether the equilibrium policy of the bank is safe or risky depends upon the relative position of $\bar{v}$ with respect to the fixed point of $H(v)$, $v^\ast$. Accordingly, I will consider three possible cases: $v^\ast<\bar{v}$, $v^\ast>\bar{v}$ and $v^\ast=\bar{v}$.

(i) $v^\ast>\bar{v}$. When the equilibrium value of the bank is relatively high, the bank chooses a safe policy, $y(v^\ast) \in Y_S(v^\ast)$. Capital and asset regulations are not binding and the probability of failing the solvency test is zero. As the bank is safe, small changes in the parameters are innocuous from a prudential point of view: the probability of failure is zero. The parameters $k$ and $\sigma$ do not affect the equilibrium value of the bank. On the contrary, $r$ and $\eta$ have the effects shown in Table 1.

(ii) $v^\ast<\bar{v}$. When the equilibrium value of the bank is relatively low, the bank chooses a risky policy, $y(v^\ast) \in Y_R(v^\ast)$. Both capital and asset regulations are binding and the probability of failing the solvency test is strictly positive. The solvency of the bank, measured by $F(x-\sigma)$, depends directly on the parameters $k$, $\sigma$ and $\eta$ and on the optimal supply of deposits. This, in turn, is affected by the parameters and by the equilibrium value of the bank, and is higher than with the safe policy. Finally, all the parameters $k$, $\sigma$, $r$ and $\eta$ have non-null
FIGURE 2

Choosing between best safe and risky policies
effects on the equilibrium value of the bank, as shown in Table 1 (see section 3.2). So the comparative statics on the solvency of the bank involves three terms:

$$\frac{\partial F(x-\sigma)}{\partial w} + \frac{\partial F(x-\sigma)}{\partial D} \cdot \frac{\partial D}{\partial w} + \frac{\partial F(x-\sigma)}{\partial D} \cdot \frac{\partial D}{\partial v} \cdot \frac{\partial v}{\partial w},$$  (19)

for $w=k, \sigma, r, \eta$. It is worth noting that the last two terms of this equation are neglected in static models with perfectly competitive banks (constant returns to scale and perfect competition would leave $D_R$ indeterminate, whilst the introduction of $v$ only makes sense in a dynamic model).

From equation (18), we deduce that the direct effect (the first term in (19)) is positive for $k$ and $\eta$ (recall that $D_R \leq 1$), negative for $\sigma$, and null for $r$. If the supply of deposits does not vary, higher capitalization, a smaller upper bound on portfolio risk and increased market power enhance the solvency of the bank.

The effects coming from the shift in the supply of deposits depend on the sensitiveness of $D_R(v^*)$ to $v^*$ and the parameters. If the equilibrium supply of deposits is unique and equals one, $\partial D_R/\partial w$ equals zero and these effects disappear. When the equilibrium supply of deposits is smaller than one, the signs of $\partial D_R/\partial w$ and $\partial D_R/\partial v$ are ambiguous. Some numerical examples show that the supply-of-deposits effects are generally small compared to the direct effects.

(iii) $v^* = \bar{v}$. When the equilibrium value of the bank equals the critical value $\bar{v}$, the bank is indifferent between the risky policies in $Y_R(v^*)$ and the safe policies in $Y_S(v^*)$. The situation under each of this

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8 Equation (19) holds for cases in which $D_R(v^*)$ is unique; otherwise, differentiating $D_R(v^*)$ makes no sense.
policies has been described in (i) and (ii). In this case, however, changes in \( k, \sigma, r \) and \( \eta \) may lead the bank to shift from safe to risky or vice versa.

For \( k \) and \( \sigma \), the result is immediate. Tighter regulation makes the bank to prefer a safe policy, since regulatory burdens reduce the value of the bank only when risky (remember Table 1). Diagrammatically, higher \( k \) and lower \( \sigma \) move downward the curve \( H_R(v) \), but do not alter \( H_S(v) \). Thus, \( v \) moves to the left, while \( v^* \) remains constant. Figure 3 illustrates this result.

Variations in \( r \) and \( \eta \) change the position of both \( H_R(v) \) and \( H_S(v) \) and diagrammatic analysis is not enough to clarify whether the risky or the safe policies dominate after the change. However, the results are unambiguous. Proposition 4 shows that an increase in the interest rate introduces an advantage for risky policies, since their profitability rests comparatively more on one-period profits and less on future discounted profits.

**PROPOSITION 4**: When \( v^* = \bar{v} \), small increases (decreases) in \( r \) will lead the bank to choose the risky (safe) policies.

*Proof:* See the Appendix.

On the other hand, the equilibrium value of a bank is the expected discounted value of current and future one-period profits. Greater market power (\( \eta \)) enhances the current and future profits of the bank under both the risky and the safe policies. Nevertheless, the benefit of greater \( \eta \) is higher the lower the supply of deposits (as an extreme case, when \( D \) equals one the benefit is zero). Accordingly, an increase in market power makes the safe policy better since, from Lemma 3, \( D_S \) is smaller than \( D_R(v^*) \).

**PROPOSITION 5**: When \( v^* = \bar{v} \), small increases (decreases) in \( \eta \) will lead the bank to choose the safe (risky) policies.
FIGURE 3

The effects of more stringent regulation
Proof: See the Appendix.

The results obtained in this section provide a clear understanding of how regulatory and structural parameters influence the safety of banks. A bank can suddenly switch from the safe policies to the risky policies as a result of sudden or accumulated changes in the economic environment where it operates. Small changes in the stringency of capital and asset regulations, in the macroeconomic conditions as reflected in the interest rate or the level of systematic risk (which is likely to affect $\bar{\sigma}$ for a given asset regulation), or in the degree of market power of incumbent banks can increase, for instance, the latent advantages of risky policies versus safe policies. If the changes continue over time, a point can be reach where the bank jumps from the latter to the former, whilst in the meantime the potential deterioration of solvency remains hidden. Some authors have argued that the market value of equity could be used to monitor the safety of banks and pricing deposit guarantees. With the previous results, however, situations can be identified in which the jump to risky policies may coincide with increases in the value of the bank, contributing to confuse the regulator. Assume, for example, that $\bar{\sigma}$ increases so as to make the risky policy better. The rise in $\bar{\sigma}$ does not change the value of the safe policy, but increases the value of the risky policy. Then, the shift to the risky policy entails an increase in equity value (higher $v^*$).

The results concerning the case $v^*=\bar{v}$ can be used to analyze the

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9 These arguments are strictly valid within the stationary version of the model that has been presented in this paper if subsequent changes in the parameters are fully unanticipated and taken to be permanent.

10 Marcus and Shaked (1984), and Ronn and Verma (1986), among others, have used option valuation formulas and equity values to infer the (underlying) value of bank assets.
effects of regulatory reforms when regulation applies uniformly to a set of heterogeneous banks, say the banks in a national banking industry. Assume that banks only differ in their degree of (local) monopoly power and, then, $\eta$ is distributed across banks in a certain fashion. For a given set of parameters $k$, $\sigma$ and $r$, the case $v^*=\bar{v}$ will correspond to a particular $\eta=\bar{\eta}$. According to previous results, banks with $\eta<\bar{\eta}$ will prefer a risky policy, whereas banks with $\eta>\bar{\eta}$ will prefer a safe policy. Higher $k$, lower $\sigma$ and lower $r$ imply a lower $\bar{\eta}$, reduce the set of banks which choose to be risky and, hence, the risk of banking industry as a whole. The effects are the opposite if capital and asset regulations are lessened and the interest rate goes up.

4.3. Regulatory trade-offs.

Banking regulation is traditionally intended to pursue two different objectives: investor protection and financial stability. Investor protection, on the one hand, is justified in a context of asymmetric information where depositors have neither the incentives nor probably the capability to evaluate and monitor the quality and behavior of banks. Avoiding the perverse incentives and distortions created by adverse selection and moral hazard is a rationale for the delegation of such functions in a regulator which, correspondingly, insures bank deposits. On the other hand, preserving financial stability relates to the threat of bank runs and, more generally, to the external costs of bank failures that include the breakdown of long-run relationships between banks and investors (and the associated loss of information), the risk of contagion and the disturbance of the payment system.

Providing full deposit insurance to depositors as in the model presented in this paper solves some of the problems noted above, but raises others. By transferring the risks of loss from depositors to the deposit insurance agency, it eliminates market discipline. Banking regulation now has to provide such a discipline. The design of banking regulation should take into account the following issues: (i) the role of banks in funding risky investment projects with potentially positive
net present value, (ii) the role of banks in providing liquidity and transaction services through deposits, (iii) the cost of public funds eventually used to cover the deficits of the deposit insurance system, (iv) the residual externalities associated with the failure of fully-insured banks.

In the model presented in this paper, the rate of return on risky assets has been taken as given and, for the sake of simplicity, I have assumed that the expected return on risky assets equals the risk-free rate, so that investing in risky assets gives no special advantages from a social point of view. Then issue (i) can be neglected. On the contrary, issues (ii) and (iv) provide good reasons for promoting a safe policy, while issue (ii) discourages regulations which increase the monopoly power of banks in the detriment of depositors' surplus. Thereby, in this context, an adequate regulation could rest on asset regulations that restrain banks from investing in risky assets and, as a limit case, reduce $\sigma$ to zero.

The first issue is, however, crucial and its omission can lead to erroneous policy recommendations. Fortunately, it can be addressed by informal relaxation of the assumption mentioned above. Note that in a general equilibrium framework the return on risky assets would be endogenous and would depend on the interaction of supply and demand. More concretely, we can think of a setup where projects of a certain class of risk exhibit different expected rates of return and banks of the type described in the model bid for them in a competitive fashion. In equilibrium a critical expected return would be obtained such that projects above it would be funded whilst projects below it would not. With several risk levels, the shape of the equilibrium expected return-risk schedule would probably depend upon the preference of banks for risk. If for a given uniform rate of return the banks preferred safe policies, equilibrium market clearing would possibly lead the return on safer assets to be lower and the return on riskier assets to be higher; in such case, the slope of the return-risk schedule would be positive, positive risk-premia would be paid on risky assets and under-investment
in risky projects (from the perspective of a risk-neutral planner) would result. To the contrary, if banks preferred a risky policy when the expected rates of return were the same, the slope would be negative, implying negative risk-premia and over-investment in risky assets.

In such a framework, the allocative consequences of banking regulation are likely to be very important and the regulator should weight issue (i) against the others. In particular, although reducing \( \sigma \) induces safer behavior and lessens the probability of bankruptcy, \( \sigma \) curtails the set of projects to be funded, no matter what their expected return be. Nevertheless, market power and the associated rents can incite chartered banks to be safer and increase the \( \sigma \) which is necessary to induce safe policies. A clear regulatory trade-off may emerge. Banking regulation by means of the chartering policy and the requirements for the creation of new banks or branches, among others, can create barriers to entry which protect and enhance the exercise of monopoly power. In my model, a higher \( \eta \) is an alternative to capital and asset regulations when the regulator tries to promote solvency, although it obviously lessens depositors welfare by decreasing the interest rate paid on deposits. Some authors had previously referred to this trade-off as one between efficiency and solvency. Further research could centre on this issue from an optimal regulation perspective à la Laffont-Tirole.\(^{11}\)

5. THE OPTION TO RECAPITALIZE AND THE EFFECTIVENESS OF CLOSURE.

Throughout this paper, I have considered the simple (but realistic) closure rule under which banking authorities deny renewal of the charter and close the bank if its net worth at the end of a period is negative. The threat of being closed in such a context has been proved to be an

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\(^{11}\) See Laffont and Tirole (1993) and, for an application to banking, Bensaid et al. (1993).
Closure Rules, Market Power and Risk-Taking

An effective way to induce the bank to be prudent when the present value of its future rents is sufficiently high. In this section, I show that this disciplinary effect of closure vanishes when an apparently minor change in the closure rule is introduced to allow for recapitalization by shareholders when the net worth of the bank at the end of a period is negative but they wish the bank to remain open.

Assume that banking authorities, instead of directly closing the bank when its net worth at the end of a period is negative, allow shareholders to inject new funds into the bank so as to afford promised payments to depositors and obtain the renewal of the charter. Authorities may find attractive the avoidance of closure and liquidation when recapitalization takes place.

Under this rule, shareholders have an option to retain the charter. The exercise price of such option is the additional capital (if any) that has to be raised in order to fully pay off depositors. In terms of the dynamic optimization program, the state variable, \( I_t \), becomes also a control variable, whilst previous period net worth, \( N_{t-1} \), becomes a state variable. \( I_t \) is chosen at the beginning of each period, once \( N_t \) is observed. Choosing \( I_t = 1 \) requires \( I_{t-1} = 1 \) (otherwise the bank would be closed) and entails keeping the charter and recapitalizing, i.e. injecting \(-N_t\) if \( N_t < 0 \), and 0 otherwise. On the contrary, choosing \( I_t = 0 \) implies refusing the option.

The dynamic optimization problem of the bank is:

Maximize
\[
\left\{ \sum_{t=0}^{\infty} (1+r)^{-t} \min(N_t,0) \cdot I_t + \phi(I_t, y_t) \right\}
\]
subject to
\[
\begin{align*}
I_t & \in (0,1) \\
I_t & \leq I_{t-1} \\
y_t & \in \Gamma \\
N_0 & = 0
\end{align*}
\]

\( t = 0, 1, 2, \ldots \)
where the constraint \( I_t \leq I_{t-1} \) states that if shareholders refuse the option on the charter at any period, the charter is lost forever. The definitions of \( \Gamma \) and \( \psi(I_t, y_t) \) are those given by equations (1) and (2) in section 2.1. Recall that \( \psi(I_t, y_t) \) depends on \( I_t \) but not on \( N_t \).

Keeping the same notation as above, and with \( S_t = (I_t, N_t) \) as the vector of state variables, dynamic programming techniques allow us to define the following value function:

\[
V(S_t) = \sup_{I_t \leq I_{t-1}} \left\{ \min(N_t, 0) \cdot I_t + \sup_{y_t \in \Gamma} E[\psi(I_t, y_t) + (1+r)^{-1} V(S_{t+1})] \right\}, \tag{20}
\]

Let \( v \) be the conditional-on-continuation value of the bank:

\[
v = \sup_{y_t \in \Gamma} E[\psi(I_t, y_t) + (1+r)^{-1} V(S_{t+1}) | I_t = 1], \tag{21}
\]

which is time and state independent. If \( I_t \) equals 1, shareholders pay \(-\min(N_t, 0)\) and get \( v \), whilst if \( I_t \) equals zero, the bank is closed and the expression into big braces in (20) takes value zero. Shareholders choose to maintain the charter whenever the costs of recapitalization are not higher than \( v \). So, the choice of \( I_t \) can be characterized as follows:

\[
I_t = I(S_t) = \begin{cases} 
1 & \text{if } I_{t-1} = 1 \text{ and } N_t \geq v \\
0 & \text{otherwise.} 
\end{cases} \tag{22}
\]

Taking into account the optimal choice of \( I_t \), the expression for \( V(S_t) \) is quite simple

\[
V(S_t) = \begin{cases} 
\min(N_t, 0) + v & \text{if } I_{t-1} = 1 \text{ and } N_t \geq v \\
0 & \text{otherwise.} 
\end{cases}
\]

This expression can be used to compute \( E[V(S_{t+1}) | I_t = 1] \):
\[ E[V(S_{t+1})|I_t=1] = \text{Prob}[N_{t+1} \geq 0] \cdot v + \text{Prob}[-v \leq N_{t+1} \leq 0] \cdot E[N_{t+1} + v|N_{t+1} \leq 0]. \]

On the other hand, from the definition of \( \psi(I_t, y_t) \),

\[ E[\psi(I_t, y_t)|I_t=1] = (1+r)^{-1} \text{Prob}[N_{t+1} \geq 0] \cdot E[N_{t+1} + v|N_{t+1} \geq 0] - K_t. \]

Thus, plugging the last two equations in (21), we have

\[ v = \sup_{y_t \in \Gamma} \{(1+r)^{-1} \text{Prob}[N_{t+1} + v \geq 0] \cdot E[N_{t+1} + v|N_{t+1} + v \geq 0] - K_t\}. \]

dropping the time indices and replacing \( N \) by \( R(\sigma)(D+K)-C(D) \) yields

\[ v = \sup_{y \in \Gamma} \{(1+r)^{-1} E[\max\{R(\sigma)(D+K)-C(D)+v, 0\}] - K\}. \tag{23} \]

According to (23), the continuation value of the bank is that of investing \( K \) in a call option which is written not only on bank assets, \( R(\sigma)(D+K) \), but on the sum of bank assets and the continuation value of the bank, \( v \). The amount of promised payments to depositors, \( C(D) \), is the strike price of such option.

Specifying \( R(\sigma) \) and \( C(D) \) as in section 2.4, the value of this option can be computed following the same steps that led to \( \Pi(y) \):

\[ v = \sup_{y \in \Gamma} \Pi(y, v) + (1+r)^{-1} \Phi(y, v)v, \]

where

\[ \Pi(y, v) = F(x)(D+K) - F(x-\sigma)D^{\eta+1} - K, \]

\[ \Phi(y, v) = F(x-\sigma), \]

\[ x = (1/\sigma)(\log(D+K)-\log[D^{\eta+1}(1+r)^{-1}v+\sigma^2/2]). \]
The definitions of $\Pi(y,v)$ and $\Phi(y,v)$ are different from that of $\Pi(y)$ and $\Phi(y)$ (in previous sections) because of the definition of $x$, that depends on $v$. Such dependence reflects that the value of the charter affects shareholders' decision on continuation.

The next result states that, at least for $r>\eta$, the optimal policies for the bank are risky policies in the sense that capital and asset regulations are binding. Moreover, since the result does not depend on the values of $k$ and $\bar{r}$, there are situations where these regulations are ineffective as a mean of inducing a safe policy.

**PROPOSITION 6:** When recapitalization is allowed and $r>\eta$, the optimal policies for the bank are risky policies, whatever the values of $k$ and $\bar{\sigma}$.

**Proof:** See the Appendix.

This result has to do with the form of the payoffs associated to one-period decisions. Under the simple closure rule of previous sections, shareholders win $N+v$ when $N$ is positive, and 0 otherwise. On the contrary, when recapitalization is allowed, shareholders win $N_t+v$ when $N_t+v$ is positive and 0 otherwise.

Figure 4 represents, for both cases, the (discounted) payoffs to shareholders at the end of a period as a function of the (discounted) gross return on bank assets at that date. As can be seen, the difference is in the range $\left[D^{\eta+1}-(1+r)^{-1}v, D^{\eta+1}\right]$ of asset returns. Whilst with the first closure rule the bank is closed whenever $R(\sigma)(D+K)$ is the interior of that interval, with the second rule shareholders recapitalize, paying $D^{\eta+1}-R(\sigma)(D+K)$ to keep $(1+r)^{-1}v$. The non-convexity of the payoff in the first case explains shareholders' aversion to risk when $v$ is high enough. Conversely, the convexity of the payoff function in the second case induces risk-loving behavior, except when $D^{\eta+1}-(1+r)^{-1}v$ becomes negative.

In practical terms, this section enters a caveat against a
FIGURE 4

Shareholders' payoffs at the end of a period and the option to recapitalize

WITHOUT RECAPITALIZATION

\[(1+r)^{-1}v \quad \text{without recapitalization} \]

\[D^{\eta+1} \quad \text{option to recapitalize} \]

\[R(\sigma)(D+K) \quad \text{payoff} \]

WITH RECAPITALIZATION

\[(1+r)^{-1}v \quad \text{with recapitalization} \]

\[D^{\eta+1} \quad \text{option to recapitalize} \]

\[R(\sigma)(D+K) \quad \text{payoff} \]
modification of the closure rule that might seem attractive for banking authorities. More concretely, once insolvency takes place, the supervisory agency may prefer the injection of capital by shareholders to its public involvement in the resolution of the crisis. These ex-post incentives to give to the shareholders an option to recapitalize may, however, go against the ex-ante need for inducing discipline with a (credible) threat of closure.

6. CONCLUSIONS AND POLICY IMPLICATIONS.

In this paper I analyze the prudential regulation of banks in a dynamic model of bank behavior which allows for bankruptcy and closure. The effectiveness of chartering and closure policies and their relationship with market power, capital and asset regulations and risk-taking can be formally settled. Given closure policies and the provisions that exclude from business the promoters and managers of banks which become insolvent, the value of bank charters is an important endogenous component of bankruptcy costs to bankers and may constitute an incentive to adopt prudent decisions.

Dynamic programming techniques allow us to obtain the conditional-on-surviving steady-state value of a bank, $v^*$, together with the bank's optimal investment and financial policies. Comparative statics on $v^*$ provide insights into the fundamentals that influence the value of a chartered bank. They show that tighter capital and asset regulations (when binding) are associated to a lower $v^*$, while smaller rates of interest and greater market power result in a higher $v^*$.

The equilibrium behavior of the bank depends crucially on $v^*$. Because of non-convexities derived from the limited liability of bankers, optimal policies may be of two distinct extreme types: safe or risky. The risky type of policy dominates when $v^*$ is lower than a critical value $\bar{v}$, whereas the safe type of policy is optimal when $v^*$ is higher than $\bar{v}$. Tough prudential regulation in general elicits the safe
policy, since high capital requirements and strong limitations on portfolio risk cause the value of a risky policy to be low, whilst they do not affect the value of a safe policy. Similarly, I have proved that high market power and low interest rates favor safe policies, that rest comparatively more on long-run profits and less on short-run opportunistic exploitation of the deposit insurance system.

From the results, we can deduce that capital and asset regulation, on the one hand, and entry and closure policies, on the other, are alternative ways to preserve the solvency of banks. This trade-off should be taken into account in the design of banking regulation.

Even though some empirical implications of this model seem to be somewhat extreme, several facts give support to the kind of bang-bang behavior predicted here. The recent widespread solvency problems of the savings and loan institutions in U.S., for instance, appeared after a long period of well-functioning of the deposit insurance system. In an environment of increased competition (smaller \( \eta \)) and financial deregulation (greater \( \sigma \)), the higher real interest rates of the late seventies and early eighties (greater \( r \)) joined some regulatory mistakes to trigger off the debacle in the mid eighties: in the initial phases of the crisis, the regulator shored up book value net worth with a variety of accounting changes and reduced the minimum regulatory capital requirements (thereby producing a smaller \( k \)). The inadequate response of the regulator included also excessive forbearance and delay in closing or intervening thrifts in trouble.

After a huge number of failures and enormous costs to tax-payers, many scholars and practitioners became interested in the topic and the U.S. government and banking authorities introduced reforms oriented to

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restore prudent behavior, which had been predominant during the post-war period. The safety net built after the Great Depression has been partially based upon branching and geographical restrictions and interest rate ceilings, which acted as legal guarantees of market power. Financial innovation during the seventies and eighties irrevocably damaged, however, such conception of the banking business. Nowadays, disintermediation and largely unregulated non-bank financial intermediaries absorb a notable fraction of depositors' wealth, whereas modern information technology gives the banks' best debtors direct access to capital markets. As a consequence, regulatory reform faces the difficult task of restoring solvency in a necessarily more competitive context. This probably means that the future design of prudential regulation will have to rest more on asset and capital restrictions (as the Basle Accord of 1988 and the recent Federal Deposit Insurance Corporation Improvement Act confirm), but it also calls for prudence in the management of the chartering policy and the way to confront future deregulatory pressures.

In a different context, after the Second Banking Directive of the European Community (1989), European banks are able to provide their services throughout the Community with a single banking license from their home country. This rule raises potential threats to financial stability. For one thing, stronger competition and the erosion of charter values could be expected. For another, with banks potentially competing at a European level, domestic chartering policies become ineffective as a mean of controlling the degree of market power of banking institutions under the jurisdiction of home country authorities. Restoring their effectiveness would require coordination between national regulators.

One cross-sectional implication of our model is that purely competitive banks are more likely to be risky than banks enjoying monopoly rents. If banks with different degrees of market power co-exist, a certain segmentation of the banking industry might take place. With U.S. data, Marcus and Shaked (1984) found that most banks in
their sample were safe, whilst the riskiest 5 percent of the institutions accounted for virtually the entire value of FDIC liabilities. Although not only attributable to differences in market power, informal evidence shows that small recently created banks use to be more involved in negligence and fraud and are more inclined to go bankrupt during banking crisis. If this were the case, chartering policies should take into account the potential market power of each applicant bank as well as the effects of entry upon global competition in the industry. In addition, it might be the case that supervisory practices should be sensitive to such segmentation, trying to exert control upon risk-taking by banks with less market power.

A different but very related topic is the so-called too-big-to-fail problem, whose existence has been widely recognized (for a recent empirical statement, see Boyd and Gertler, 1993). The systematic reluctance of banking supervisors to close banks that are considered too big to fail actually means that the closure rule is not in force for such banks. All the disciplinary effect potentially related to the rents of bigger banks is lost. Paradoxically, the authorities confer a guarantee of survival upon big banks for fear of causing severe troubles to the financial system and losing the value of the banks as going concerns. This guarantee makes the optimal policy of big banks to be risky and increases the costs of the deposit insurance system. Although preserving the value of the charter and avoiding the external costs of bankruptcy can make sense, rescue techniques should be designed so as to simultaneously discipline the bankers. Regulators should be allowed to take over banks which failed the established solvency tests, and the final payment to shareholders should only be the liquidation value of the net worth (intangible assets excluded), whatever the size, going-concern value and final destination of the insolvent bank. Accordingly, discipline would be preserved, while if rescued banks had positive going-concern values (so they might have a future under the control of new shareholders), the price paid for the institution by the successful bidders could partially or totally off-set the cost of the funds injected by the authorities to restore solvency.
APPENDIX

Proof of Lemma 1. For all $v \in \mathbb{R}_+$, the following properties are true:

(i) $H(v)$ is a continuous function. Let $\mathcal{F}$ be the set defined as

$$\mathcal{F} = \{(y=(D,K,\sigma) \in \mathbb{R}^3 \mid 0 \leq D \leq 1, \ kD \leq K \leq \bar{K}, \text{ and } \sigma \geq \sigma)\},$$

where $\bar{K} > k$ is a high enough upper-bound to $K$. As I will show in section 4.1, an additional constraint such as $K \leq \bar{K}$ in the bank's problem would never be binding. Therefore, the following alternative definition for $H(v)$ is possible:

$$H(v) = \sup_{y \in \mathcal{F}} \{\Pi(y) + (1+r)^{-1} \Phi(y)v\},$$

where $\mathcal{F}$ replaces $\Gamma$. As $\mathcal{F}$ is non-empty and compact and $\Pi(y) + (1+r)^{-1} \Phi(y)v$ is continuous in $y$ and $v$ (see equations (11) and (13)), the Theorem of the Maximum ensures that $H(v)$ is a continuous function (see Stokey and Lucas, 1989; p. 62).

(ii) $H(v)$ is an increasing function. Consider any $v$ and $v'$ such that $v \leq v'$, then

$$H(v) = \Pi(y(v)) + (1+r)^{-1} \Phi(y(v))v \leq \Pi(y(v)) + (1+r)^{-1} \Phi(y(v))v' \leq \Pi(y(v')) + (1+r)^{-1} \Phi(y(v'))v' = H(v').$$

(iii) $H(v)$ is positive. As $v$ and $\Phi(y)$ are positive, $(1+r)^{-1} \Phi(y)v$ is positive, so that $H(v) \geq \sup_0 \Pi(y)$. But $\sup_0 \Pi(y)$ is always non-negative, since $y = (0, 0, 0) \in \Gamma$ and $\Pi(y) = 0$.

(iv) The slope of $H(v)$ is smaller than 1. Even though $H(v)$ may be non-differentiable, I can prove that $[H(v+h) - H(v)]/h$ is smaller than one for all $v \in \mathbb{R}_+$ and $h > 0$. In fact,
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\[ H(v+h) = \sup_{y \in \Gamma} (\Pi(y) + (1+r)^{-1} \Phi(y)(v+h)) \leq \sup_{y \in \Gamma} (\Pi(y) + (1+r)^{-1} \Phi(y)v) + \sup_{y \in \Gamma} ((1+r)^{-1} \Phi(y)h) \leq H(v) + (1+r)^{-1}h. \]

Thus, \([H(v+h) - H(v)]/h = (1+r)^{-1} < 1.\]

**Proof of Lemma 2:** The bank seeks to maximize:

\[ G(y,v) = F(x)(D+K) - F(x-\sigma)D^{\eta+1} - K + (1+r)^{-1}F(x-\sigma)v. \quad (A1) \]

where

\[ \frac{\partial G}{\partial \sigma} = f(x-\sigma)[D^{\eta+1} - (1+r)^{-1}(x/\sigma)v] \quad (A2) \]

and

\[ \frac{\partial^2 G}{\partial \sigma^2} = -f'(x-\sigma)[D^{\eta+1} - (1+r)^{-1}(x/\sigma)v]x/\sigma^2 + (1+r)^{-1}f(x-\sigma)[\log(1+k) - \eta \log(D)]v/\sigma^3. \quad (A3) \]

If an interior solution for \( \sigma \) existed, \( 0 < \sigma(v) < \bar{\sigma} \), (A2) would have to be zero, while (A3) would have to be negative. However, when (A2) equals zero, the first term in (A3) is zero, whilst the second is always positive. Thus, (A3) is positive, contradicting the necessary second order condition for a maximum. Consequently, \( \sigma(v) \) may be either 0 or \( \bar{\sigma} \), but not \( 0 < \sigma(v) < \bar{\sigma} \). Now, let us examine the choice of \( K \) when \( \sigma(v) = \bar{\sigma} \). From equation (A1), the first order condition associated to an interior solution for \( K \) is:

\[ [F(x)-1] + (1+r)^{-1}f(x-\sigma)\frac{v}{\sigma(D+K)} = 0, \quad (A4) \]

whereas, the necessary second order condition for a maximum is:

\[ \frac{f(x)}{\sigma(D+K)} - (1+r)^{-1}f(x-\sigma)(x/\sigma)\frac{v}{\sigma(D+K)^2} \leq 0 \]
From (A4) we can substitute $1-F(x)$ for $[\sigma(1+r)(D+K)]^{-1}f(x-\sigma)v$ in the left hand side of this inequality. Reordering, we arrive at:

$$\frac{1-F(x)}{\sigma(D+K)} \left[ \frac{f(x)}{1-F(x)} - x \right]$$

which is positive, since $f(x)/(1-F(x))$ is the hazard function of a standard normal random variable, that is greater than $x$ for all $x>0$. Then, the second order condition does not hold and no interior solution for $K$ can exist. If an optimal policy entails $\sigma=\bar{\sigma}$, the optimal $K$ is $kD$. The choice of $\sigma=\bar{\sigma}$ and an infinite $K$ makes no sense because the value of $G(y,v)$ at $(D,K,\bar{\sigma})$ when $K$ tends to infinity, whatever $D$, can be attained with the same $D$, any finite $K'$ and $\sigma=0.\sigma$

**Proof of Lemma 3:** The definition of $D_R(v)$ in equation (17) can be re-written in the following way:

$$D_R(v) = \arg\max_{0\leq D \leq 1} \{J(D)+Q(D)\},$$

where

$$J(D) = D - D^{\eta+1} + (1+r)^{-1}v$$

and

$$Q(D) = [1-F(x-\bar{\sigma})][D^{\eta+1}-(1+r)^{-1}v] - (1+k)[1-F(x)]D.$$ 

By definition, $J(D)$ attains an unique global maximum at $D_S$, is increasing at $D<D_S$ and decreasing at $D>D_S$. On the other hand, as $\sigma(v)=\bar{\sigma}$, Lemma 3 implies that the capital requirement is binding. Then, $G(y,v)$ -defined in (A1)- is decreasing in $K$ at $K=kD_R(v)$:

$$\frac{\partial G}{\partial K} = [F(x)-1] + (1+r)^{-1}f(x-\bar{\sigma})[\bar{\sigma}(1+k)D_R(v)]^{-1}v < 0.$$  \hspace{1cm} (A5)

This inequality will allow us to prove that $Q(D)$ is increasing in $D$ at $D_R(v)$:
\[
\frac{\partial Q}{\partial D} = (\eta+1)[1-F(x-\sigma)]D \eta + (1+k)F(x) - \eta(1+r)^{-1}f(x-\sigma)(\sigma D)^{-1}v.
\]

But, from (A5),

\[
(1+r)^{-1}f(x-\sigma)[\sigma D(v)]^{-1}v < (1+k)[1-F(x)].
\]

Then, at \(D_R(v)\),

\[
\frac{\partial Q}{\partial D} > (\eta+1)[1-F(x-\sigma)]D_R(v) \eta + (1+k)F(x) - \eta(1+k)[1-F(x)]
\]

\[
= (\eta+1)[1-F(x-\sigma)]D_R(v)^\eta-(1+k)[1-F(x)]) + (1+k)[1-F(x)] > 0.
\]

(Notice that the term in braces is the value of deposit guarantees per unit of deposits \((1+r)^{-1}E[\max(-N,0)]/D\), which is, by definition, positive). Now we can prove the result. Suppose, on the contrary, \(D_R(v) < D_S\). Then both \(J(D)\) and \(Q(D)\) are increasing at \(D_R(v)\) whereas values of \(D\) higher than \(D_R(v)\) are feasible. This contradicts the definition of \(D_R(v)\).

**Proof of Proposition 3:** Using the same arguments as in Lemma 1, we can show that \(H_S(v)\) and \(H_R(v)\) have the same properties as \(H(v)\) (i.e. they are continuous, increasing, positive and with a slope smaller than one). Now, in order to prove the result, I will show that \(H_S(v)\) and \(H_R(v)\) have an unique intersection at a point \(v > 0\). Equation (A5) shows the partial derivative of \(G(y,v)\) with respect to \(\sigma\). On the one hand, \(\partial G/\partial \sigma > 0\) at \(v = 0\), so \(\sigma = 0\) cannot be optimal for \(v = 0\) and \(H_S(0) < H_R(0)\). On the other hand, the sign of \(\partial G/\partial \sigma\) is the sign of \(D^\eta(1+r)^{-1}(x/\sigma)v\), which is maximum for \(D = 1\) and \(K = kD\). Then, there exists a value \(\hat{v} = (1+r)((1/2)+\log(1+k)/\sigma^2)^{-1}\) such that \(\partial G/\partial \sigma < 0\) for all \((D,K,\sigma)\in\Gamma\) and \(v > \hat{v}\) (recall the definition of \(x\) in equation (12)). Therefore, \(H_S(v) > H_R(v)\) at any \(v > \hat{v}\). Thus, since \(H_S(v)\) and \(H_R(v)\) are continuous, they intersect at least at one point \(v = \overrightarrow{v}\). Moreover, the intersection is unique because the slope of \(H_S(v)\) is greater than that of \(H_R(v)\) for all \(v: \phi(y) = 1 > \phi(y')\) for any \(y \in Y_S(v)\) and \(y' \in Y_R(v)\). (Notice that either the envelope theorem (in interior
solutions) or the fact that \( \Gamma \) does not depend on \( v \) (in corner solutions) ensure \( \frac{dH}{dv} = \frac{\partial H}{\partial v} = \phi(y) = 1 \) for any \( y \in Y(y) \) and \( \frac{dH}{dv} = \frac{\partial H}{\partial v} = \phi(y) < 1 \) for any \( y \in Y(v) \). Therefore, \( H_s(v) \leq H_r(v) \) for \( v \neq \bar{v} \) and \( H_s(v) \geq H_r(v) \) for \( v \approx \bar{v} \), and the result follows.

**Proof of Propositions 4 and 5:** Lemma 4 (below) provides necessary and sufficient conditions under which a type of policy dominates the other after the change in a parameter. Intuitively, for the risky type of policy to dominate, the upward (downward) movement of \( H_r(v) \) at \( v^* \) has to be great (small) as compared with the upward (downward) movement of \( H_s(v) \); otherwise, the safe type of policy dominates.

**LEMMA 4:** When \( v^* = \bar{v} \) and a small increase (decrease) in a parameter \( w = k, \sigma, r, \eta \) takes place, the risky (safe) policies will dominate the safe (risky) policies if and only if the following condition holds:

\[
\frac{\partial H_r}{\partial w} \geq \left[ 1 + \frac{1 - \phi(y)}{r} \right] \frac{\partial H_s}{\partial w}. \tag{A5}
\]

Otherwise, the safe (risky) policies will dominate the risky (safe) ones.

**Proof:** This proof is based in a geometrical argument which hinges upon a linearization of \( H_r(v) \) and \( H_s(v) \) around their intersection at \( v = v^* \). Figure A1 depicts in augmented scale a case in which the vertical movement of \( H_r(v) \) and \( H_s(v) \) (as a result of a change in a parameter \( w \) is such that indifference between risky policies in \( Y_r(v) \) and safe policies in \( Y_s(v) \) remains. Graphically, the situation is characterized by a sufficiently great vertical displacement of \( H_r(v) \), \( \overline{AC} \), as compared with the displacement of \( H_s(v) \), \( \overline{AB} \). Clearly \( \overline{AC} = DF = DG - FG = [1 - \partial H_r / \partial v] \overline{AC} \). Similarly, \( \overline{AB} = DE = DG - EF = [1 - \partial H_s / \partial v] \overline{AB} \). So, solving for \( \overline{AC} \) in the second equation and substituting back in the first, we get:

\[
\overline{AC} = [1 - \partial H_s / \partial v]^{-1} [1 - \partial H_r / \partial v] \overline{AB}
\]

Now, if \( \overline{AC} \) is approximated by \( (\partial H_r / \partial w)dw \) and \( \overline{AB} \) by \( (\partial H_s / \partial w)dw \), the result is:
\[ \frac{\partial H}{\partial w} = (1 - \frac{\partial H}{\partial v})^{-1} \frac{\partial H}{\partial v} \partial H / \partial w \]

If \( \frac{\partial H}{\partial w} \) were greater than in the case depicted in Figure 1, the new intersection of \( H_R(v) \) and \( H_S(v) \) would take place to the right of \( \hat{v} \) and the risky policies in \( Y_R(v) \) would dominate the safe ones in \( Y_S(v) \).

Expression (8) arises when \( \frac{\partial H_S}{\partial v} \) and \( \frac{\partial H_R}{\partial v} \) are computed:

\[ (1 - \frac{\partial H}{\partial v})^{-1} \frac{\partial H}{\partial v} = \left[ \frac{r}{1+r} \right]^{-1} \left[ \frac{r+1-\phi(y)}{1+r} \right] = 1 + \frac{1 - \phi(y)}{r}. \]

**Proof of Proposition 4:** From the envelope theorem, the impact of \( r \) on the optimal risky and safe policies can be ignored and simple differentiation leads to \( \frac{\partial H_R}{\partial r} = -(1+r)^{-2} \phi(y)v^* \) and \( \frac{\partial H_S}{\partial r} = -(1+r)^{-2} v^* \), with \( y \in Y_R(v^*) \). As \( \phi(y) < 1 \), the term in square brackets of condition (A5) is greater than one and condition (A5) holds:

\[ -(1+r)^{-2} \phi(y)v^* \geq -(1+r)^{-2} \left[ 1 - \frac{1 - \phi(y)}{r} \right] v^* \Leftrightarrow \phi(y) < 1 < 1 - \frac{1 - \phi(y)}{r}. \]

**Proof of Proposition 5:** As in the proof of Proposition 4, the elements in condition (A5) have to be computed. If \( D_R(v^*) \) equals one (corner solution), \( \frac{\partial H_R}{\partial \eta} \) equals zero, whereas \( \frac{\partial H_S}{\partial \eta} \) is positive, then the result is clearly true. If \( D_R(v^*) \) is smaller than one (interior solution), we have:

\[ \frac{\partial H}{\partial \eta} = -[(x - \tilde{\sigma})D_R(v^*)]^+ + (1+r)^{-1} f(x - \tilde{\sigma})v^*/\tilde{\sigma}]\log D_R(v^*) > 0 \]

and

\[ \frac{\partial H}{\partial \eta} = -[D_S^{\eta+1}]\log D_S > 0. \]

Now, from the first order conditions associated to the optimal choice of \( D_R(v^*) \) and \( D_S \) and the conditions \( H_R(v^*) = v^* \) and \( H_S(v^*) = v^* \), the terms in brackets can be re-written, leading to:
\[ \frac{\partial H}{\partial \eta} = -(1/\eta) \left[ \frac{\tau+1-\Phi(y)v^*}{\tau+1} \right] \log D(v^*) \]

and

\[ \frac{\partial H}{\partial \sigma} = -(1/\eta) \left[ \frac{\sigma-v^*}{\sigma} \right] \log D(v^*) \]

Then, recalling that \( D_R(v^*) > D_S \) and applying Lemma 4, the result follows.\( \Box \)

**Proof of Proposition 6.** Let \( G(y,v) \) be \( \Pi(y,v) + (1+r)^{-1} \Phi(y,v)v \) and notice that the partial derivative of \( G(y,v) \) with respect to \( x \) is zero. Note also that, under \( D^\eta - (1+r)^{-1}v > 0 \), \( \partial G(y,v)/\partial \sigma \) is positive and \( \partial G(y,v)/\partial K \) is negative so that the bank would adopt risky policies. On the contrary, under \( D^\eta - (1+r)^{-1}v \leq 0 \), we have \( F(x) = F(x-\sigma) = \Phi(y,v) = 1 \), so the bank would be safe and would choose a safe policy as those described in section 4.1. In order to prove the result, I will show that when \( r > \eta \) the best safe policies, \( ye Y_s \), entail \( D^\eta - (1+r)^{-1}v_s > 0 \) so that they cannot be optimal. For any \( ye Y_s \), \( G(y,v) = \eta/(1+\eta) D + (1+r)^{-1}v_s \), where \( D_s = [1/(1+\eta)]^{-\eta/(1+\eta)} \). Then, the value \( v_s \) that solves \( v = G(y,v) \) for all \( ye Y_s \) can be computed: \( v_s = (1+r)(\eta/r)(1+\eta)^{-\eta/(1+\eta)} \). We can easily check that condition \( D^\eta - (1+r)^{-1}v_s \leq 0 \) holds if and only if \( r \leq \eta \).\( \Box \)
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