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## LIMITED LIABILITY IN BUSINESS GROUPS

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### Abstract

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We consider a model in which a holding company has to decide whether to finance an investment project in a subsidiary. The project can be financed either through internal capital or through debt. The subsidiary's manager has private information on the quality of the project and has empire-building preferences. When bankruptcy is costly for the subsidiary's manager, the choice between internal and external financing is part of an optimal mechanism that induces truthful revelation of the information. The first best solution can be approached if the cost of bankruptcy for the manager is high enough.

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# 1 Introduction

The capital budgeting process is at the heart of the functioning of the modern corporation. As it is by now well understood, what makes the problem non-trivial is the existence of informational asymmetries and incentive problems. Managers often have superior information about the profitability of existing investment projects. If their interests differ from the ones of the shareholders, then appropriate incentive mechanisms have to be put in place in order to achieve an efficient allocation of resources (Harris and Raviv, 1996).

It is commonly assumed that managers prefer sometimes to finance investment projects with negative NPV, either because they may have empire-building preferences or because more funds allow them to exert less effort (Stein, 2003; Khanna and Tice, 2001; Harris and Raviv, 1996). Thus, unless countervailing incentives are put in place, managers will have a tendency to invest too much. Various mechanisms have been studied in the literature to address the overinvestment problem. The most common mechanisms use an initial capital spending limit for the subsidiary, monetary incentives and auditing to verify the information given by the subsidiary's manager (Harris, Kriebel and Raviv, 1982; Harris and Raviv, 1996). The capital structure also plays an important role as a managerial incentive mechanism (see e.g. Dessí and Robertson, 2003). Most of the research on internal capital markets has overlooked the interaction with external capital markets, and in this paper we want to study the issues appearing when multiple sources of financing, external and internal, can be used in the budgeting process when the holding company has limited liability with respect to the subsidiary's obligations and the subsidiary's manager has also limited liability in his compensation contract. This occurs in a business group, which is formed by legally independent firms but it has an internal market that allocate capital among the member firms (Almeida and Wolfenzon, 2005).

Stulz (1990) has analyzed debt financing as a method to limit the overinvestment problem. In his model debt forces the firm to pay a fixed amount of cash. This reduces overinvestment when the firm has a lot of cash, but induces under-investment in the opposite case. We consider another rationale for using debt as part of an optimal governance mechanism. In our model a profit-maximizing controlling shareholder (CS) controls a subsidiary. The manager of the subsidiary has private information about the quality of an investment project, and has 'empire-building' preferences. Internal capital however is controlled by CS, which has to decide (possibly after collecting information from the subsidiary's manager) whether to implement the in-

vestment. CS also decides how to finance the investment project, whether using internal capital or issuing debt. Notice that the decision to finance the investment project is ultimately taken by the profit-maximizing CS; there is no reason to use debt in order to reduce the amount of cash available to CS, as it happens in the Stulz model, since CS has no empire-building preferences. Debt is used because bankruptcy is costly for the subsidiary's managers, for example because it may destroy their firm-specific human capital or it may have a negative impact on their reputation. Since bankruptcy is more likely when the quality of the subsidiary's investment project is low, debt financing reduces the incentive rents that CS needs to pay to the subsidiary's manager. Notice that CS may choose debt financing even in cases in which there would be enough internal cash to finance the investment project without resorting to external capital markets.

Bankruptcy is also costly for CS, although the cost may be different from the one suffered by the subsidiary's manager (see Stulz, 1999, for a discussion of the costs induced by the possibility of bankruptcy), so that debt will only be used when the reduction in managerial rents is substantial. Notice also that debt will not be used if it is possible to provide incentives to the manager in less costly ways (for example through performance contingent pay). However, there will be a role for debt whenever there are restrictions on other methods of providing incentives<sup>1</sup>; in particular, we show that the choice of internal vs. external financing still plays a role if a 'limited liability' restriction is put on the set of feasible compensation contracts.

We emphasize that, in our model, debt is a more effective disciplinary tool when bankruptcy is more costly for the subsidiary's manager. Therefore, when CS and the subsidiary are legally separate entities (as it is the case in a business group) it is better to have the subsidiary to issue the debt. This is the case despite the fact that issuing debt may be cheaper for the holding company.

The literature on external finance in business groups is limited. One interesting paper is Bianco and Nicodano (2004). They consider a model in which a controlling shareholder has to decide whether to issue debt through a holding or a partially owned subsidiary. They focus on asymmetric infor-

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<sup>1</sup>As Stein (2003) observes, "*problems in internal capital markets may be exacerbated when divisional managers both: i) have a strong incentive to maximize their own division's capital allocation as opposed to profits; and ii) are powerful relative to the CEO -i.e., have valuable specific human capital (either expertise or internal political clout), and so can threaten to disrupt the firm's activities.*". In these cases, using debt may be the best instrument available to provide incentives, since other threats (such as dismissal by the CEO) are not credible.

mation between the controlling shareholder and the lenders, and ignore the role of debt as an incentive instrument, which is instead the focus of this paper.

The article is structured as follows. The next section presents the model. In section 3 we discuss the role of external debt in providing incentives when monetary incentives cannot be given to the subsidiary's manager. The analysis is then generalized in section 4, where we discuss the optimal mechanism when CS can use both external debt and direct monetary incentives. Section 5 contains the conclusions, and the appendix collects the proofs of our results.

## 2 The Model

A firm (holding company) has a subsidiary and an amount of cash  $M$  to invest. The subsidiary has no physical assets but it has access to an investment project. The project requires an investment of  $I$  at time 0, and it yields a stochastic gross return  $R$  at time 1; we will assume  $M \geq I$ , so that internal financing is always possible. The gross return  $R$  takes value  $V$  with probability  $p$ , and 0 with probability  $(1 - p)$ . The value of  $p$  is only known to the managers of the subsidiary. However, it is common knowledge that  $p$  can take either the value  $p_1$  or  $p_2$ , with  $p_1 < p_2$ , and that the probability of a value  $p_i$  is  $\theta_i \in (0, 1)$ .

Since the subsidiary has no cash, if the investment project is enacted it has to be financed. Financing can either come through internal capital markets or through a bank. We assume that the subsidiary is a legally separate entity, so any financing by the bank has to be repaid with funds generated by the investment project<sup>2</sup>. We also assume that bankruptcy has a fixed cost  $c$  for the subsidiary's manager and  $k$  for CS.

The subsidiary's manager receives private benefits from running the project which are proportional to the gross return of the project  $R$ . When she receives an expected salary of  $w$ , the expected utility when the project is enacted and financed with internal funds is:

$$U_{Int} = bE(R) + w$$

where  $b > 0$  is the coefficient that measures the private benefits. The expected utility when the project is enacted and it is financed with external

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<sup>2</sup>We consider only subsidiary-issued debt as a form of financing. In our model debt is useful only if bankruptcy is costly for the subsidiary's manager. CS has the money to finance the project without using external financing, so there is no point in issuing debt or equity.

funds is:

$$U_{Ext} = bE(R) + w - c \Pr(\text{fail})$$

where  $\Pr(\text{fail})$  is the probability of bankruptcy and  $c$  the fixed cost of bankruptcy for the subsidiary's manager.

We assume that the subsidiary's manager can obtain a utility of zero outside the subsidiary, so each scheme of compensation and financing proposed by the CS has to give the manager this level of utility.

CS has to decide whether or not to enact the project, and in case it is enacted whether to finance it through internal or external funds. The goal of CS is to maximize the expected profit of the subsidiary, net of the salary paid to subsidiary's managers. The risk free interest rate is  $r \geq 0$ .

As a preliminary observation, we notice that under full information, the project is funded with internal funds whenever  $pV - (1+r)I - w \geq 0$ . Notice further that since  $b > 0$ , we can set  $w = 0$ , which implies that a project receives internal funding if and only if  $p \geq \frac{(1+r)I}{V}$ , the standard 'positive NPV' rule. At last, we observe that under complete information external markets are never used, since CS prefers to avoid bankruptcy.

When there is asymmetric information over  $p$  CS can set up a direct revelation mechanism in order to learn the value of  $p$ . The mechanism receives as input a message  $\hat{p}$  by the subsidiary's manager, and delivers as output a probability of internal financing,  $\alpha_I(\hat{p})$ , a probability of external financing,  $\alpha_E(\hat{p})$ , and a salary level.

The compensation for the manager may depend on the announcement  $\hat{p}$ , on whether the investment project is enacted and, if it is enacted, on the outcome of the investment project and on the way in which the project is financed. In principle, compensation may depend on the financing decision, but given risk neutrality by the manager we can consider without loss of generality a single compensation function  $W(R, \hat{p})$ . The expected salary when  $p$  is the true state of the world and  $\hat{p}$  is announced will be denoted as  $w(p, \hat{p})$ , where

$$w(p, \hat{p}) = (1-p)W(0, \hat{p}) + pW(V, \hat{p}),$$

and we will also use the notation  $w(p) = w(p, p)$ . As a last piece of notation, let us denote by  $r_D$  the interest rate on debt which is paid when the subsidiary asks for external funds. For the moment, we take the value of  $r_D$  as given, and we will later endogenize this value.

The expected profit of CS for a given mechanism  $(\alpha_I, \alpha_E, w)$  is:

$$B = \sum_{i=1}^2 [\alpha_I^i (p_i V - (1+r)I) + \alpha_E^i (p_i (V - (1+r_D)I) - (1-p_i)k) - w^i] \theta_i$$

where we have simplified the notation using  $\alpha_I^i = \alpha_I(p_i)$ ,  $\alpha_E^i = \alpha_E(p_i)$  and  $w^i = w(p_i)$ .

If we denote by

$$\begin{aligned} H_I^i &= p_i V - (1+r)I \\ H_E^i &= p_i (V - (1+r_D)I) - (1-p_i)k \end{aligned}$$

the expected return of enacting a project of type  $i$  with internal ( $H_I^i$ ) and external ( $H_E^i$ ) financing respectively, the expected profit can be written as:

$$B = \sum_{i=1}^2 [\alpha_I^i H_I^i + \alpha_E^i H_E^i - w^i] \theta_i$$

The mechanism has to satisfy the participation and incentive compatibility constraints, given by

$$U(p_1) = b(\alpha_I^1 + \alpha_E^1) p_1 V + w^1 - \alpha_E^1 (1-p_1) c \geq 0$$

$$U(p_2) = b(\alpha_I^2 + \alpha_E^2) p_2 V + w^2 - \alpha_E^2 (1-p_2) c \geq 0$$

$$b(\alpha_I^2 + \alpha_E^2) p_2 V + w^2 - \alpha_E^2 (1-p_2) c \geq b(\alpha_I^1 + \alpha_E^1) p_2 V + w(p_2, p_1) - \alpha_E^1 (1-p_2) c;$$

$$b(\alpha_I^1 + \alpha_E^1) p_1 V + w^1 - \alpha_E^1 (1-p_1) c \geq b(\alpha_I^2 + \alpha_E^2) p_1 V + w(p_1, p_2) - \alpha_E^2 (1-p_1) c;$$

Furthermore, feasibility requires

$$\alpha_I^i \geq 0; \quad \alpha_E^i \geq 0; \quad \alpha_I^i + \alpha_E^i \leq 1.$$

In order to make the problem interesting we assume  $H_I^2 > 0 > H_I^1$ . Moreover, we concentrate attention on the case

$$p_2(1+r_D) \geq (1+r),$$

that is, the interest rate  $r_D$  must be high enough to give at least the same profitability as the risk-free interest rate when only good projects are financed. This condition implies  $H_I^2 > H_E^2$ . As a last remark, observe that  $H_E^2 < 0$  implies  $H_E^1 < 0$ .

### 3 Absence of Monetary Incentives

We first analyze the restricted problem in which monetary incentives cannot be used, and the financing policy is the only way to provide incentives to the manager. This case is simpler and it helps us in highlighting the role played by debt financing in incentive provision. The problem is

$$\max_{\alpha_I^1, \alpha_E^1, \alpha_I^2, \alpha_E^2} [\alpha_I^1 H_I^1 + \alpha_E^1 H_E^1] \theta_1 + [\alpha_I^2 H_I^2 + \alpha_E^2 H_E^2] \theta_2 \quad (1)$$

s.t.

$$U(p_i) = b(\alpha_I^i + \alpha_E^i) p_i V - \alpha_E^i (1 - p_i) c \geq 0 \quad i = 1, 2$$

$$b(\alpha_I^2 + \alpha_E^2) p_2 V - \alpha_E^2 (1 - p_2) c \geq b(\alpha_I^1 + \alpha_E^1) p_2 V - \alpha_E^1 (1 - p_2) c$$

$$b(\alpha_I^1 + \alpha_E^1) p_1 V - \alpha_E^1 (1 - p_1) c \geq b(\alpha_I^2 + \alpha_E^2) p_1 V - \alpha_E^2 (1 - p_1) c$$

$$\alpha_I^1 \geq 0; \alpha_E^1 \geq 0; \alpha_I^2 \geq 0; \alpha_E^2 \geq 0$$

$$\alpha_I^1 + \alpha_E^1 \leq 1; \alpha_I^2 + \alpha_E^2 \leq 1$$

The solution to this problem, depending the value of the parameters, may be pooling or separating. Notice that when the solution is pooling, the incentive compatibility constraints can be ignored.

We may have three types of pooling solutions:

1. Never invest, that is  $\alpha_E^1 = \alpha_I^1 = \alpha_E^2 = \alpha_I^2 = 0$ . In this case the value of the objective function is 0 and the individual rationality constraints are always satisfied.
2. Always invest with external funds,  $\alpha_E^1 = \alpha_E^2 = 1$  and  $\alpha_I^1 = \alpha_I^2 = 0$ . The value of the objective function in this case is

$$\Omega_{Ext} = H_E^1 \theta_1 + H_E^2 \theta_2$$

and the individual rationality constraint for type  $p_1$  is  $b p_1 V - (1 - p_1) c \geq 0$ .

3. Always invest with internal funds,  $\alpha_E^1 = \alpha_E^2 = 0$  and  $\alpha_I^2 = \alpha_I^1 = 1$ .  
The value of the objective function in this case is

$$\Omega_{Int} = H_I^1 \theta_1 + H_I^2 \theta_2$$

and the individual rationality constraints are always satisfied.

When  $bp_1V - (1 - p_1)c \geq 0$ , so that individual rationality is not an issue, and a pooling solution is chosen, then the choice is made comparing the values  $0$ ,  $\Omega_{Ext}$  and  $\Omega_{Int}$ . Call  $\Omega = \max\{0, \Omega_{Ext}, \Omega_{Int}\}$  the highest value of the objective function attainable with a pooling solution.

This value will have to be compared with the value that can be obtained under separation. Separation is impossible if  $\alpha_I^2 = 1$ , since in that case type  $p_1$  is better off announcing  $p_2$ . In order to reduce incentives to lie for type  $p_1$ , it turns out to be optimal (see appendix) to set  $\alpha_E^2 > 0$

When  $bp_1V - (1 - p_1)c \geq 0$ , it is necessary to set  $\alpha_E^2 = 1$  in order to make sure that incentive constraints are satisfied. However, this implies that a manager observing  $p_1$  can obtain a strictly positive utility. Thus, the optimal solution requires to give to the manager that level of utility. Absent monetary compensation, the only way to give utility to a manager is to implement the project with positive probability.

If the project is implemented with internal funds then the probability  $\alpha_I^1$  has to satisfy

$$\alpha_I^1 bp_1V = bp_1V - (1 - p_1)c \quad \rightarrow \quad \alpha_I^1 = \frac{bp_1V - (1 - p_1)c}{bp_1V}$$

Define  $\sigma = \frac{bp_1V - (1 - p_1)c}{bp_1V}$ . Then the value of the objective function when a separating solution is implemented is

$$\Upsilon = \sigma H_I^1 \theta_1 + H_E^2 \theta_2$$

**Proposition 1** *When  $p_1bV - (1 - p_1)c > 0$  the possible solutions are described as follows.*

1. *If  $\Omega > \Upsilon$  then a pooling solution is chosen. The optimal pooling structure is obtained comparing the values  $0$ ,  $\Omega_{Ext}$  and  $\Omega_{Int}$ .*
2. *If  $\Upsilon \geq \Omega$  then the separating solution  $\alpha_I^1 = \sigma$ ,  $\alpha_E^1 = 0$ ,  $\alpha_E^2 = 1$  and  $\alpha_I^2 = 0$  is optimal.*

The condition  $p_1 bV - (1 - p_1) c > 0$  is equivalent to assuming a low value of  $c$ , the bankruptcy cost for the manager. As previously observed, this makes sure that the participation constraints are always satisfied, no matter what is the policy chosen by CS. The counterpart is that debt financing, although it reduces the expected utility of the manager, cannot entirely eliminate the agency problem. More precisely, it is impossible to get a separating equilibrium in which only the good projects are enacted. Instead, whenever a separating equilibrium is the optimal solution, good projects will be implemented with probability 1 but financed with debt. This reduces the utility we have to give to the subsidiary's manager in the case  $p_1$ . However, since we are considering the case  $p_1 bV - (1 - p_1) c > 0$ , we have to give a strictly positive utility to a manager of type  $p_1$  since otherwise separation would not occur. This is obtained by setting  $\alpha_I^1 > 0$ , that is, allowing for some inefficient investment. The lowest value of  $\alpha_I^1$  inducing separation is  $\sigma$ , and it is increasing with  $p_1$ . Notice also that the lowest value of  $p_1$  compatible with the condition  $p_1 bV - (1 - p_1) c > 0$  is  $\underline{p}_1 = \frac{c}{bV+c}$ . At this value we have  $\sigma = 0$ .

The conditions described in Proposition 1 depend on the value  $r_D$ . We may therefore ask whether the conditions are satisfied in the case in which the capital market uses the fact that the firm is asking for debt financing as a signal about the quality of the project. In other words, can the conditions that make a certain solution optimal still be satisfied when the value of  $r_D$  is determined endogenously in a competitive capital market?

If we call  $\Pr(p_i|D)$  the conditional probability assigned to  $p_i$  by the bank when the firm asks for debt financing, then we have:

$$\Pr(p_1|D) = \frac{\theta_1 \alpha_E^1}{\theta_1 \alpha_E^1 + \theta_2 \alpha_E^2} \quad \Pr(p_2|D) = \frac{\theta_2 \alpha_E^2}{\theta_1 \alpha_E^1 + \theta_2 \alpha_E^2} \quad (2)$$

whenever  $\theta_1 \alpha_E^1 + \theta_2 \alpha_E^2 > 0$ . If  $\theta_1 \alpha_E^1 + \theta_2 \alpha_E^2 = 0$  (that is, the firm asks for debt financing with probability zero) then we can assign arbitrarily the belief  $\Pr(p_1|D)$ . Looking at the pooling solutions in which external financing is not used, if we set  $\Pr(p_1|D) = 1$ , so that  $r_D$  is obtained solving  $p_1(1 + r_D) = (1 + r)$ , then the conditions for optimality appear to be compatible with this value of  $r_D$ .

If the firm asks for debt financing with positive probability, the rate  $r_D$  is determined by the condition

$$(p_1 \Pr(p_1|D) + p_2 \Pr(p_2|D))(1 + r_D) = (1 + r)$$

which, using (2), becomes

$$(p_1\theta_1\alpha_E^1 + p_2\theta_2\alpha_E^2)(1 + r_D) = (\theta_1\alpha_E^1 + \theta_2\alpha_E^2)(1 + r),$$

and, in the case  $\alpha_E^1 = \alpha_E^2 = 1$  (pooling solution with debt financing)

$$(p_1\theta_1 + p_2\theta_2)(1 + r_D) = (1 + r). \quad (3)$$

Observe that  $(p_1\theta_1 + p_2\theta_2)(1 + r_D)$  is the unconditional expected value of lending a monetary unit at the rate  $r_D$ . One of the conditions which have to be satisfied to make this solution optimal is

$$H_I^1\theta_1 + H_I^2\theta_2 \leq H_E^1\theta_1 + H_E^2\theta_2,$$

that is, external financing is better than internal financing. When  $r_D$  is given by (3), the condition is equivalent to

$$\theta_1 p_1 + \theta_2 p_2 \geq 1,$$

which is impossible. We conclude that the pooling solution with external financing is not possible when  $r_D$  is determined endogenously.

Consider now the solution in which the two types are separated. In this case the firm asks for debt financing only when the probability is  $p_2$ , so that  $r_D$  is given by the condition  $p_2(1 + r_D) = (1 + r)$ . Thus the interest rate on the debt is low. This creates a problem, since debt financing becomes especially attractive when the project has a low probability of success. Specifically, the condition  $\Upsilon > \Omega_{Ext}$ , which implies  $H_E^1 < 0$  may not be satisfied. In order to make sure that debt financing is not optimal when  $p_1$  is observed, the cost of bankruptcy for CS must be sufficiently high. Specifically, the condition  $H_E^1 < 0$  becomes equivalent to  $(1 - p_1)k > p_1V - \frac{p_1}{p_2}(1 + r)I$ . Only when such condition is satisfied, a separating equilibrium becomes possible.

Consider now the case of high bankruptcy cost for the manager, that is  $p_1bV - (1 - p_1)c < 0$ . The main difference with respect to the case of low  $c$  is that the participation constraint for the manager who observes  $p_1$  need not be automatically satisfied. In fact, if we set  $\alpha_E^1 = 1$  the individual rationality constraint is violated. This implies that the only possible pooling solutions are:

1. Never invest, that is  $\alpha_E^1 = \alpha_I^1 = \alpha_E^2 = \alpha_I^2 = 0$ . In this case the value of the objective function is 0 and the individual rationality constraints are always satisfied.

2. Always invest with internal funds,  $\alpha_E^1 = \alpha_E^2 = 0$  and  $\alpha_I^2 = \alpha_I^1 = 1$ . The value of the objective function in this case is

$$\Omega_{Int} = H_I^1 \theta_1 + H_I^2 \theta_2$$

and the individual rationality constraints are always satisfied.

3. Invest with both external and internal funds, setting the probability of external financing strictly lower than one.

Let us discuss this last case. As we are looking for a pooling solution, the incentive compatibility constraints are satisfied. If CS wants to use external financing, the maximum value of the probability of using debt is determined by the individual rationality constraint for the manager in case  $p_1$ , that is

$$b(\alpha_I^1 + \alpha_E^1) p_1 V - \alpha_E^1 (1 - p_1) c \geq 0$$

$$\alpha_E^1 \leq \alpha_I^1 \frac{p_1 b V}{(1 - p_1) c - p_1 b V}.$$

Thus, CS chooses the maximum value of  $\alpha_E^1$  possible and, taking into account the feasibility constraint  $\alpha_E^1 + \alpha_I^1 \leq 1$ , the solution in this case is

$$\alpha_E^1 = \frac{p_1 b V}{(1 - p_1) c} \quad \alpha_I^1 = \frac{(1 - p_1) c - p_1 b V}{(1 - p_1) c}.$$

If we define  $\beta = \frac{(1 - p_1) c - p_1 b V}{(1 - p_1) c}$  and  $\xi = \frac{p_1 b V}{(1 - p_1) c}$ , this pooling solution will be  $\alpha_I^1 = \alpha_I^2 = \beta$ ,  $\alpha_E^1 = \alpha_E^2 = \xi$ . The value of the objective function in this case is

$$\Omega_{Mix} = \beta \Omega_{Int} + \xi \Omega_{Ext}$$

Therefore, the solution with mixed financing can only be optimal if  $\Omega_{Ext} > \Omega_{Int}$ .

Consider now a separating solution in which a subsidiary doesn't invest when  $p_1$  is the state of the world and it does when the state is  $p_2$ . If  $\alpha_E^2 = 0$  (as in the optimal solution with complete information) the incentive compatibility constraint for  $p_1$  is not satisfied. Therefore, to assure that the manager is telling the truth, there must be a positive probability of external financing in the state  $p_2$ . We choose the minimal value of  $\alpha_E^2$ , since we have

made the assumption  $H_E^2 < H_I^2$ , and we obtain:  $\alpha_I^1 = \alpha_E^1 = 0$ ,  $\alpha_I^2 = \beta$ ,  $\alpha_E^2 = \xi$ . The value of the objective function will be:

$$\Phi^2 = (\beta H_I^2 + \xi H_E^2) \theta_2. \quad (4)$$

For further reference, we also define

$$\Phi^1 = (\beta H_I^1 + \xi H_E^1) \theta_1. \quad (5)$$

**Proposition 2** *If  $p_1 bV - (1 - p_1) c < 0$  then:*

1. *The solution  $\alpha_E^1 = \alpha_I^1 = \alpha_E^2 = \alpha_I^2 = 0$  is optimal when  $\Phi^2 \leq 0$  and  $\Omega_{Int} \leq 0$ .*
2. *The pooling solution with internal financing  $\alpha_E^1 = \alpha_E^2 = 0$ ,  $\alpha_I^1 = \alpha_I^2 = 1$  is optimal when:  $\Omega_{Int} \geq 0$ ,  $\Omega_{Int} \geq \Phi^2$ ,  $\Omega_{Int} \geq \Phi^1$  and  $\Omega_{Int} \geq \Omega_{Ext}$ .*
3. *The pooling solution  $\alpha_I^1 = \alpha_I^2 = \beta$ ,  $\alpha_E^1 = \alpha_E^2 = \xi$  is optimal when  $\Phi^1 \geq 0$  and  $\Omega_{Ext} > \Omega_{Int}$ .*
4.  *$\alpha_I^1 = \alpha_E^1 = 0$ ,  $\alpha_I^2 = \beta$  and  $\alpha_E^2 = \xi$  is optimal only if  $\Phi^1 \leq 0$ ,  $\Phi^2 \geq \max\{0, \Omega_{Int}\}$ .*

In this case the value of  $c$  is high and therefore any solution with  $\alpha_E^1 = 1$  would violate the participation constraint for the manager of type  $p_1$ .

However, only in this situation we can reach a separating equilibrium where the subsidiary doesn't invest in  $p_1$  and it does in  $p_2$ , using only the way of financing as an incentive.

At the same time, if we suppose  $H_E^2 > 0$  the 'never invest' solution is never optimal, since a positive profit can be attained by inducing separation, that is,  $\Phi^2$  will be always positive.

The expected profit of this separating solution can be written as:

$$\Phi^2 = [p_2 V - (\beta(1+r) + \xi p_2(1+r_D)) I - \xi(1-p_2)k] \theta_2 \quad (6)$$

Observe that when  $(1+r) = p_2(1+r_D)$  (this will occur in a separating equilibrium in which the subsidiary of type  $p_2$  asks for external financing and capital markets are perfect) the expression (6) simplifies to

$$\Phi^2 = [(p_2 V - (1+r) I) - \xi(1-p_2)k] \theta_2$$

namely, the expected profit is equal to the first best minus the expected cost of bankruptcy. If we also assume  $k = 0$  (bankruptcy is not costly for

CS), then the first best can be reached, and wages are not needed to provide incentives.

The conclusion is that monetary incentives are necessary only when capital markets are not perfect or bankruptcy costs for CS are high<sup>3</sup>. If  $k > 0$ , then the profit will be greater the smaller the value of  $p_1$  (if  $p_1$  is high a pooling equilibrium can be a better option), the greater the value of  $p_2$ , and the greater  $c$ .

We now discuss the conditions for the separating mechanism to be optimal. In a separating mechanism, debt financing has to occur with positive probability, since otherwise type  $p_1$  would claim to be  $p_2$ . As we have discussed before, we will choose the lowest value of  $\alpha_E^2$  that satisfies the incentive compatibility constraint for  $p_1$ . Thus, debt financing must occur with an intermediate probability. Let  $\alpha_I^2 = \beta$ ,  $\alpha_E^2 = \xi$  be the probabilities of financing with internal funds and debt, respectively, under the optimal mechanism. Then the condition

$$\Phi^1 \leq 0$$

or, equivalently,

$$H_E^1 \xi + H_I^1 \beta \leq 0 \tag{7}$$

requires that when  $p = p_1$  it is not profitable to invest. If this value were positive, then the pooling solution with probabilities  $(\beta, \xi)$  of financing would be better than the separating solution.

On the other hand, the inequality

$$0 \leq \Phi^2$$

is always satisfied if  $H_E^2 > 0$ . In this case, this solution is always profitable. Remember that this was the reason why a ‘never invest’ solution is not chosen in this case. If  $H_E^2 < 0$ , this doesn’t always occur. Finally, we have

$$\Phi^2 > \Omega_{Int}$$

which says that the profit obtained under separation is better than the one obtained under pooling.

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<sup>3</sup>As it is usual in this type of literature, we are assuming that Headquarters is able to implement the promised decisions once the information is disclosed (the commitment assumption). If not, the separating equilibrium is impossible in the case  $k = 0$ , since Headquarters would have incentives to deviate from the equilibrium and to finance with debt the project of type  $p_1$ .

Looking now at the conditions for the optimality of the pooling solutions, we observe that when  $H_E^2 < 0$  the ‘never invest’ solution becomes again possible. One of the conditions for the optimality of this solution is  $\Phi^2 \leq 0$ , that is, the separating solution is not optimal. The other condition is

$$\Omega_{Int} < 0,$$

and it implies that it is not preferable to implement a pooling equilibrium with internal financing.

Another possible solution is always to invest with internal financing. The condition

$$\Omega_{Ext} \leq \Omega_{Int},$$

means that the pooling equilibrium with internal financing is preferable to the one that uses external financing. Observe that this last pooling solution is not possible because it doesn’t satisfy the participation constraint for type  $p_1$  (we have seen that there is no solution with  $\alpha_E^1 = 1$ ); the condition however implies that a mixed solution with positive probability of external financing will be always worse than a solution in which financing always occurs with internal funds.

The condition  $\Omega_{Int} \geq 0$  says that the expected profit must be positive, while the condition

$$\Phi^1 \leq \Omega_{Int},$$

means that the pooling solution is better than the separating solution when investment is made only when  $p = p_1$ . This condition will be always satisfied when  $H_E^2 < 0$ . Similarly, the condition

$$\Phi^2 \leq \Omega_{Int}$$

states that the pooling solution with internal resources must be better than the separating solution in which investment only occurs when  $p = p_2$ .

The last possibility is a pooling solution that uses external and internal funds in both states. This is optimal when

$$\Phi^1 \geq 0$$

or, equivalently, ,

$$H_E^1 \xi + H_I^1 \beta \geq 0.$$

That is, the expected profit of investing with these probabilities in  $p_1$  is positive (if not, the separating solution would be better). This condition is precisely the contrary to the one we have obtained in the separating solution (7). In this case, it must be convenient to invest in  $p_1$  with debt, despite having to invest also with internal resources to compensate the manager.

For this condition be satisfied we need  $H_E^1 > 0$ , which is possible only when  $H_E^2 > 0$  and  $k$  is not too high. The other condition for this equilibrium to be optimal is:

$$\Omega_{Ext} > \Omega_{Int}$$

that is, the pooling equilibrium with external financing must be preferable to the one with internal financing. Such pooling solution is not possible, because it wouldn't satisfy the participation constraints. But, if this condition is satisfied, the pooling equilibrium using a positive probability of external financing will be better than the pooling equilibrium with internal financing.

As we have seen in the last part, this condition cannot be fulfilled if the value of  $r_D$  is endogenous.

## 4 Monetary Incentives

If there is no restriction on the compensation schemes that can be used, and in particular if it is possible to pay a negative salary to the subsidiary's manager in some contingencies, then it is always possible to reach the first best, given that both the principal and the agent are risk neutral. This first best, as we have discussed before, consists of financing with internal resources whenever the NPV of the project is positive. Then, in the first best  $\alpha_I^2 = 1$  and  $\alpha_E^2 = \alpha_E^1 = \alpha_I^1 = 0$ . Let  $W_V^I$  be the salary paid when the project has obtained as result  $V$  in the case  $p_2$  and financing is internal. If we take this values of  $\alpha$  as given and put:

$$W_V^I(p_2) = -bV$$

and the rest of variables of wage equal to zero, then all the incentive compatibility and participation constraints are satisfied.

The only difference with the complete information solution is the necessity of paying a negative wage to give incentives to the managers to tell the truth. If this is not allowed, then in general obtaining the first best won't be possible, and it will be necessary to find a second best policy.

Consider now the case of 'limited liability', that is, wages must always be non-negative. This implies that the cost of bankruptcy for the manager

cannot be imitated internally. In this case there is no reason to pay a positive wage when the manager announces  $p_2$ . The main incentive problem is precisely to avoid that a manager who observes  $p_1$  announces  $p_2$ ; paying a positive expected salary when the manager announces  $p_2$  can only worsen the problem. Then, the only salary which will be positive, will be  $w(p_1) > 0$ , and, moreover, it will only be paid in the case in which the manager announces  $p_1$  and the project is not enacted. Therefore, we can limit the analysis to salary schemes that only pay a quantity  $w$  in case of announcement  $p_1$ . Observe that  $w(p_2, p_1) = w(p_1)$  (the salary paid to the manager when he says that the project has a probability of positive profit  $p_1$  and in fact is  $p_2$ ).

Furthermore, we can concentrate our analysis on the cases in which monetary incentives are used in order to achieve a separating solution. When a pooling solution is selected, then there is no need to pay any positive wage, since we do not need to provide incentives to differentiate the behavior of high and low types.

We can show that the ‘never invest’ solution is optimal under exactly the same circumstances than in the case of the absence of monetary incentives. The comments made before also apply. Then, we will limit ourselves to study the situations where there is investment.

The cases in which a separating solution with monetary incentives is optimal are discussed in the next two propositions. We first consider the case of low cost of bankruptcy.

**Proposition 3** *If  $p_1bV - (1 - p_1)c > 0$  the possible solutions are:*

1.  $\alpha_I^1 = \alpha_E^1 = 0, \alpha_I^2 = 1, \alpha_E^2 = 0, w^1 = p_1bV$  is optimal if  $H_I^1 < -p_1bV, H_E^1 < -(p_1bV - (1 - p_1)c), H_I^2\theta_2 - \theta_1p_1bV > 0$  and  $(H_I^2 - H_E^2)\theta_2 - \theta_1(1 - p_1)c > 0$
2.  $\alpha_I^1 = \alpha_E^1 = 0, \alpha_I^2 = 0, \alpha_E^2 = 1, w^1 = p_1bV - (1 - p_1)c$  is optimal if  $H_I^1 < -p_1bV, H_E^1 < -(p_1bV - (1 - p_1)c), H_E^2\theta_2 - \theta_1(p_1bV - (1 - p_1)c) > 0$  and  $(H_E^2 - H_I^2)\theta_2 + \theta_1(1 - p_1)c > 0$

The two possibilities consist of only investing in the case  $p_2$ . Taking into account that we are going to pay the manager a wage in the case  $p_1$ , there is no point in investing in this case, since  $H_I^1 < 0$ . That is, if we choose to pay a salary in  $p_1$  it is not necessary to provide further incentives to the manager by investing in this case.

Suppose first  $H_E^1 > 0$ . When is it optimal to set  $\alpha_E^1 > 0$ ? The only possibilities for that are a pooling equilibrium, that doesn't need monetary

incentives (analyzed in the previous section) or a separating equilibrium where the investment is financed with internal resources in the case  $p_2$  and with external resources in the case  $p_1$ . To satisfy the incentive compatibility constraint for  $p_1$  we need  $w^1 \geq (1 - p_1)c$ .

If we set  $w_1$  to the lowest possible level (that is,  $(1 - p_1)c$ ) then the incentive compatibility constraint for  $p_2$  is

$$p_2bV \geq p_2bV + (p_2 - p_1)c.$$

Since  $p_2 > p_1$  this expression cannot be satisfied. Therefore, this separating equilibrium is not possible.

Moreover, if we consider the possibility of paying also a monetary incentive in  $p_2$  we obtain the inequality

$$(1 - p_1)c \leq w^1 - w^2 \leq (1 - p_2)c.$$

Since  $(1 - p_1) > (1 - p_2)$  there is no value of  $w^1 - w^2$  for which the inequalities are satisfied. As we have pointed out before, paying a monetary incentive when the type of project is  $p_2$  would give more incentives to the subsidiary's manager for not telling the truth when the project is  $p_1$ . Therefore, when monetary incentives can be used and the cost of them are cheaper, the subsidiary will never invest in the case in which the probability of success is  $p_1$ .

In the case of choosing  $\alpha_I^2 = 1$ , observe the conditions simply result from a comparison with the cost of using other alternatives to provide incentives. For instance,  $H_I^1 < -p_1bV$  means that the cost of using a salary  $w^1 = p_1bV$  is smaller than the loss if we invested in  $p_1$  with internal financing; the cheaper way to provide incentives is then the wage. Similarly, the condition  $H_E^1 < -(p_1bV - (1 - p_1)c)$  compares the loss of investing in  $p_1$  with external resources with the salary that would have to be paid in this case (this is smaller than  $p_1bV$ , since the manager is deterred from falsely announcing  $p_2$  by the disutility suffered under debt financing). The condition  $H_I^2\theta_2 - \theta_1p_1bV > 0$  says that the expected profit has to be positive, and the last condition is simply the comparison between the profit obtained with this option ( $H_I^2\theta_2 - \theta_1p_1bV$ ) and the one that would be obtained with external financing in  $p_2$  ( $H_E^2\theta_2 - \theta_1(p_1bV - (1 - p_1)c)$ ). This last condition can be written also as:

$$H_I^2\theta_2 \geq H_E^2\theta_2 + \theta_1(1 - p_1)c$$

That is, the profit of investing with internal financing in  $p_2$  must be greater than the profit of investing with external financing in  $p_2$  plus the saving on monetary incentive costs.

In the case of  $\alpha_E^2 = 1$  the comments are the same, but with the indexes changed.

When is a separating solution better than a pooling solution? The pooling equilibrium with internal financing is preferred to the separating equilibrium with monetary incentives when

$$H_I^1 > -p_1bV$$

Similarly, the pooling solution with external financing is preferred to the separating solution if

$$H_E^1 > -(p_1bV - (1 - p_1)c)$$

We have already seen that the condition

$$(H_E^2 - H_I^2) \theta_2 + \theta_1(1 - p_1)c > 0$$

(for  $\alpha_E^2 = 1$  to be optimal) is just the comparison between the profit in this situation and the profit if we opted for  $\alpha_I^2 = 1$ . The term  $\theta_1(1 - p_1)c$  shows the savings that external financing allows in the incentive costs; these savings are realized when the project is of type  $p_1$ , this is the reason why the term is multiplied by its respective probability  $\theta_1$ . When we endogenize  $r_D$  (as we have seen before,  $p_2(1 + r_D) = 1 + r$ ) the inequality simply becomes

$$(1 - p_1)c\theta_1 > (1 - p_2)k\theta_2,$$

namely, the external financing will be preferred to the internal one when the saving in the incentive cost is higher than the expected cost of bankruptcy for CS.

The next proposition deals with the case of high  $c$ .

**Proposition 4** *If  $p_1bV - (1 - p_1)c < 0$  the possible solution using monetary incentives are:  $\alpha_I^1 = \alpha_E^1 = 0$ ,  $\alpha_I^2 = 1$ ,  $\alpha_E^2 = 0$ ,  $w^1 = p_1bV$  will be optimal when:  $H_I^1 < -p_1bV$ ,  $H_E^1 < (1 - p_1)c - p_1bV$*

The conditions for the optimality of the trivial solution are the same as before, and we have also already discussed the conditions for the optimality of the case  $\alpha_I^2 = 1$ . In this case we never have  $\alpha_E^2 > 0$  since monetary incentives are less expensive than using debt (this differs from the case in which  $c$  is low).

## 5 Conclusions

As we have seen, there are cases in which a firm may look for external financing, despite the fact that internal funds are available and external funds are more costly than the internal ones. This occurs when external debt is used to provide incentives to subsidiary's managers.

We have analyzed the different cases which may arise. When the cost of bankruptcy for the subsidiary's manager is small and monetary incentives are not allowed, then a separating solution is not possible. Therefore, the firm would invest in all projects (or none) depending on what is more profitable in expected terms. We find two cases where external financing is used: In the first case, financing always comes from debt. This case will be preferable when the interest on debt is low or when the fraction of the investment lost in case of bankruptcy ( $\frac{k}{7}$ ) is low. However, if we take an endogenous value of  $r_D$ , the pooling equilibrium with internal financing will be always preferable, so we conclude that a pooling equilibrium with external financing is possible only with an exogenous value of  $r_D$ . The second case consists on using external financing only when  $p = p_2$ , and investing in  $p_1$  with a probability smaller than 1 using internal financing. This case will be chosen for small values of  $p_1$ , because the probability of inefficient investment will be very close to zero. When  $r_D$  is determined endogenously, this solution turns out to be possible only for a sufficiently high value of  $k$ .

When the cost of bankruptcy for the subsidiary's manager is high enough, we can reach an equilibrium where only good projects are enacted, without using monetary incentives. The way to provide incentives to the manager will be precisely the existence of a positive probability of being financed with debt. If  $r_D$  is calculated endogenously and the bankruptcy cost for CS is zero, then the first best is reached because in this case the cost of debt will be equal to the cost of internal resources for the CS. Therefore monetary incentives are needed only when capital markets are imperfect or bankruptcy costs for CS are high. However, this is only true if we can assume that CS can commit not to deviate from their decision once the information about the project is obtained. The reason of this is that if bankruptcy costs for CS are zero, they have incentives to deviate from the equilibrium and to finance with debt the project of probability of success  $p_1$ . When CS can't commit, it will only be able to assure that they are not going to invest with debt in the bad project if bankruptcy cost for them is high enough. If bankruptcy cost for CS is not zero, the obtained profit with this equilibrium won't still be equal to the first best and it will be higher the lower is the value of  $p_1$  (if  $p_1$  is high, perhaps a pooling equilibrium will be preferable), and the higher

is the value of  $p_2$  and  $c$ .

When monetary incentives are available the first best can be reached simply making the manager pay the equivalent of its personal benefit when he announces a good project. This however requires a negative salary. If limited liability is assumed, then we can obtain a separating equilibrium where the low profitability projects will never be enacted and the manager receives a wage in this case. Depending on the cost of this incentive, it will be preferable to use it or simply apply the results obtained previously (using a pooling solution and invest also in the case  $p_1$ ). In the case of low bankruptcy cost for the subsidiary's manager, this separating equilibrium can be realized with internal or external financing. If the value of  $r_D$  is determined endogenously, external financing will be preferred to internal financing when the saving in the cost of the monetary incentive is higher than the expected bankruptcy cost of CS. However, if the bankruptcy cost for the subsidiary's manager is high, external financing with monetary incentives will never be selected, because in this case there is no saving of incentives cost.

## Appendix

**Proof of proposition 1.** If  $p_1 bV - (1 - p_1)c > 0$  then the individual rationality constraint of type  $p_1$  is always satisfied and can be ignored. We are going to solve the problem under the hypothesis that incentive compatibility constraint for  $p_2$  is satisfied with a strict inequality, so the associated Lagrange multiplier is zero. Defining the Lagrangian as:

$$\begin{aligned} L = & \alpha_I^1 H_I^1 \theta_1 + \alpha_E^1 H_E^1 \theta_1 + \alpha_I^2 H_I^2 \theta_2 + \alpha_E^2 H_E^2 \theta_2 + \\ & + \lambda [p_1 bV (\alpha_I^1 + \alpha_E^1 - \alpha_I^2 - \alpha_E^2) - (\alpha_E^1 - \alpha_E^2) (1 - p_1) c] + \\ & + \mu_I^1 \alpha_I^1 + \mu_E^1 \alpha_E^1 + \gamma_1 (1 - \alpha_I^1 - \alpha_E^1) + \\ & + \mu_I^2 \alpha_I^2 + \mu_E^2 \alpha_E^2 + \gamma_2 (1 - \alpha_I^2 - \alpha_E^2) \end{aligned}$$

The first order conditions are:

$$\begin{aligned} \alpha_I^1 & : H_I^1 \theta_1 + \lambda p_1 bV + \mu_I^1 - \gamma_1 = 0 \\ \alpha_E^1 & : H_E^1 \theta_1 + \lambda (p_1 bV - (1 - p_1)c) + \mu_E^1 - \gamma_1 = 0 \\ \alpha_I^2 & : H_I^2 \theta_2 - \lambda p_1 bV + \mu_I^2 - \gamma_2 = 0 \\ \alpha_E^2 & : H_E^2 \theta_2 - \lambda (p_1 bV - (1 - p_1)c) + \mu_E^2 - \gamma_2 = 0 \end{aligned}$$

These equations and the complementary slackness conditions have to be satisfied at an optimal point.

**Claim 5** *There is no solution with  $\lambda = 0$ .*

**Proof.** If  $\lambda = 0$  then

$$\gamma_1 = \theta_1 H_I^1 + \mu_I^1$$

As  $H_I^1 < 0$ , this implies  $\mu_I^1 > 0$  and  $\alpha_I^1 = 0$ . Moreover:

$$\gamma_2 = \theta_2 H_I^2 + \mu_I^2 > 0$$

so  $\alpha_I^2 + \alpha_E^2 = 1$ . Subtracting the condition relative to  $\alpha_E^2$  to the condition of  $\alpha_I^2$ , we obtain:

$$\mu_E^2 = \mu_I^2 + \theta_2 (H_I^2 - H_E^2) > 0.$$

Since  $\theta_2 (H_I^2 - H_E^2) > 0$ , we have  $\alpha_E^2 = 0$ . Now observe that if  $\alpha_I^2 + \alpha_E^2 = 1$ ,  $\alpha_E^2 = 0$  and  $\alpha_I^1 = 0$ , the incentive compatibility constraint cannot be satisfied. ■

The claim implies that at an optimal point the incentive compatibility constraint for  $p = p_1$  will be always satisfied as equality. Define now

$$\sigma = \frac{p_1 bV - (1 - p_1) c}{p_1 bV}.$$

Then Claim 5 implies that at any solution point

$$\alpha_I^1 - \alpha_I^2 = \sigma (\alpha_E^2 - \alpha_E^1) \quad (8)$$

We will say that a solution is *pooling* if  $\alpha_I^1 = \alpha_I^2$  and  $\alpha_E^1 = \alpha_E^2$ . The next result shows that if any of the two equalities is true then the other also has to be true.

**Claim 6**  $\alpha_E^1 = \alpha_E^2 \iff \alpha_I^1 = \alpha_I^2$ .

**Proof.** Follows immediately from (8). ■

When we restrict attention to pooling solutions, the incentive compatibility constraint is automatically satisfied. The assumption  $p_1 bV - (1 - p_1) c > 0$  implies that individual rationality is also always satisfied. Therefore the problem is simply

$$\max \quad \alpha_I (H_I^1 \theta_1 + H_I^2 \theta_2) + \alpha_E (H_E^1 \theta_1 + H_E^2 \theta_2)$$

subject to feasibility constraints, where  $\alpha_I$  and  $\alpha_E$  are the common probabilities for the two types. If we call  $\Omega_{Int} = H_I^1 \theta_1 + H_I^2 \theta_2$  and  $\Omega_{Ext} = H_E^1 \theta_1 + H_E^2 \theta_2$  then obviously the optimal pooling solution will be  $\alpha_I = \alpha_E = 0$  if  $\max \{\Omega_{Int}, \Omega_{Ext}\} \leq 0$ , it will be  $\alpha_I = 1$  and  $\alpha_E = 0$  if  $\Omega_{Int} \geq \max \{0, \Omega_{Ext}\}$  and  $\alpha_I = 0$  and  $\alpha_E = 1$  if  $\Omega_{Ext} \geq \max \{0, \Omega_{Int}\}$ .

The problem is therefore whether the solution is pooling or separating. Define  $\Omega = \max \{0, \Omega_{Int}, \Omega_{Ext}\}$  the maximum value attainable under a pooling solution. The next claim establishes the structure of a separating solution. Claim 6 implies that at a separating solution we must have both  $\alpha_E^1 \neq \alpha_E^2$  and  $\alpha_I^1 \neq \alpha_I^2$ .

**Claim 7** *The solution  $\alpha_I^1 = \sigma, \alpha_E^1 = 0, \alpha_E^2 = 1$  and  $\alpha_I^2 = 0$  is optimal when  $\sigma H_I^1 \theta_1 + H_E^2 \theta_2 > \Omega$ .*

**Proof.** Substituting  $\alpha_I^1$  from (8) in the incentive compatibility constraint we can rewrite the objective function as

$$\alpha_I^2 (H_I^1 \theta_1 + H_I^2 \theta_2) + \alpha_E^2 (\sigma H_I^1 \theta_1 + H_E^2 \theta_2) + \alpha_E^1 (H_E^1 - \sigma H_I^1) \theta_1$$

Since  $\sigma H_I^1 \theta_1 + H_E^2 \theta_2 > \Omega$ , at any solution we must have  $\alpha_I^2 = 0$  and  $\alpha_E^2 = 1$ . From 8 this in turn implies

$$\alpha_I^1 = \sigma (1 - \alpha_E^1).$$

Now notice that  $\sigma H_I^1 \theta_1 + H_E^2 \theta_2 > \Omega$  implies  $\sigma H_I^1 \theta_1 + H_E^2 \theta_2 > H_E^1 \theta_1 + H_E^2 \theta_2$ , which in turn implies  $(H_E^1 - \sigma H_I^1) < 0$ . Thus, at any optimal point  $\alpha_E^1 = 0$  and  $\alpha_I^1 = \sigma$ . ■

**Proof of proposition 2.** The Lagrangian is:

$$\begin{aligned} L = & \alpha_I^1 H_I^1 \theta_1 + \alpha_E^1 H_E^1 \theta_1 + \alpha_I^2 H_I^2 \theta_2 + \alpha_E^2 H_E^2 \theta_2 + \\ & + \lambda_1 [p_1 bV (\alpha_I^1 + \alpha_E^1) - \alpha_E^1 (1 - p_1) c] \\ & + \lambda_2 [p_1 bV (\alpha_I^1 + \alpha_E^1 - \alpha_I^2 - \alpha_E^2) - (\alpha_E^1 - \alpha_E^2) (1 - p_1) c] + \\ & + \mu_I^1 \alpha_I^1 + \mu_E^1 \alpha_E^1 + \gamma_1 (1 - \alpha_I^1 - \alpha_E^1) + \\ & + \mu_I^2 \alpha_I^2 + \mu_E^2 \alpha_E^2 + \gamma_2 (1 - \alpha_I^2 - \alpha_E^2) \end{aligned}$$

The first order conditions are:

$$\begin{aligned} \alpha_I^1 & : H_I^1 \theta_1 + (\lambda_1 + \lambda_2) p_1 bV + \mu_I^1 - \gamma_1 = 0 \\ \alpha_E^1 & : H_E^1 \theta_1 + (\lambda_1 + \lambda_2) (p_1 bV - (1 - p_1) c) + \mu_E^1 - \gamma_1 = 0 \\ \alpha_I^2 & : H_I^2 \theta_2 - \lambda_2 p_1 bV + \mu_I^2 - \gamma_2 = 0 \\ \alpha_E^2 & : H_E^2 \theta_2 - \lambda_2 (p_1 bV - (1 - p_1) c) + \mu_E^2 - \gamma_2 = 0 \end{aligned}$$

We first observe that there can be no solution with  $\alpha_E^1 = 1$ , since in that case the participation constraint for  $p_1$  wouldn't be satisfied. Furthermore, this participation constraint gives an upper limit for  $\alpha_E^1$ , given by  $\alpha_E^1 \leq \frac{p_1 bV}{(1-p_1)c - p_1 bV} \alpha_I^2$ .

Let us denote as  $\beta = \frac{(1-p_1)c - p_1 bV}{(1-p_1)c}$  and  $\xi = \frac{p_1 bV}{(1-p_1)c}$ , and  $\Phi^2$  and  $\Phi^1$  as in 4 and 5. We have the following result.

**Claim 8** *The solution  $\alpha_I^1 = \alpha_E^1 = 0$ ,  $\alpha_I^2 = \beta$  and  $\alpha_E^2 = \xi$  is optimal only if  $\Phi^1 \leq 0$ , and  $\Phi^2 \geq \max\{\Omega_{Int}, 0\}$ .*

**Proof.** Under the proposed solution the first order conditions yield

$$\begin{aligned} \frac{\mu_I^1}{p_1 bV} & = \frac{-H_I^1 \theta_1}{p_1 bV} - (\lambda_1 + \lambda_2) \\ \frac{\mu_E^1}{(1-p_1)c - p_1 bV} & = \frac{-H_E^1 \theta_1}{(1-p_1)c - p_1 bV} + (\lambda_1 + \lambda_2) \end{aligned}$$

The positivity conditions will be satisfied if and only if

$$\frac{H_E^1}{(1-p_1)c - p_1bV} \leq \lambda_1 + \lambda_2 \leq \frac{-H_I^1}{p_1bV}$$

which is possible only if

$$\beta H_I^1 \theta_1 + \xi H_E^1 \theta_1 \leq 0.$$

The conditions  $\gamma_2 \geq 0$ ,  $\mu_I^2 = \mu_E^2 = 0$  yield

$$\frac{-H_E^2 \theta_2}{(1-p_1)c - p_1bV} \leq \lambda_2 = \frac{(H_I^2 - H_E^2) \theta_2}{(1-p_1)c} \leq \frac{H_I^2 \theta_2}{p_1bV}.$$

Therefore we need

$$\max \left\{ \frac{-H_E^2 \theta_2}{(1-p_1)c - p_1bV}, 0 \right\} \leq \frac{(H_I^2 - H_E^2) \theta_2}{(1-p_1)c} \leq \frac{H_I^2 \theta_2}{p_1bV}.$$

Operating, we obtain:

$$-H_E^2 p_1 b V \leq H_I^2 ((1-p_1)c - p_1 b V). \quad (9)$$

This condition is equivalent to

$$\beta H_I^2 \theta_2 + \xi H_E^2 \theta_2 \geq 0.$$

(Notice that if  $H_E^2 > 0$  this condition is always satisfied.)

With this value of  $\lambda_2$ , the value of  $\lambda_1$  must satisfy:

$$\frac{H_E^1 \theta_1}{(1-p_1)c - p_1bV} - \frac{(H_I^2 - H_E^2) \theta_2}{(1-p_1)c} \leq \lambda_1 \leq \frac{-H_I^1 \theta_1}{p_1bV} - \frac{(H_I^2 - H_E^2) \theta_2}{(1-p_1)c}$$

And the following condition must be satisfied:

$$\frac{-H_I^1 \theta_1}{p_1bV} - \frac{(H_I^2 - H_E^2) \theta_2}{(1-p_1)c} \geq 0$$

which can be expressed as:

$$\xi H_E^2 \theta_2 + \beta H_I^2 \theta_2 \geq H_I^1 \theta_1 + H_I^2 \theta_2$$

■

**Claim 9** *The pooling solution with internal financing:  $\alpha_E^1 = 0, \alpha_E^2 = 0, \alpha_I^1 = 1$  and  $\alpha_I^2 = 1$  is optimal when  $\Omega_{Int} \geq \max\{0, \Omega_{Ext}, \Phi^2, \Phi^1\}$ .*

**Proof.**  $\mu_I^i = 0, \mu_E^i \geq 0, \gamma_i \geq 0$  for  $i = 1, 2$ .

As the first  $IR$  constraint is positive,  $\lambda_1 = 0$ . So, we obtain:

$$\begin{aligned}\frac{\mu_E^1}{(1-p_1)c} &= \frac{(H_I^1 - H_E^1)\theta_1}{(1-p_1)c} + \lambda_2 \\ \frac{\gamma_1}{p_1bV} &= \frac{H_I^1\theta_1}{p_1bV} + \lambda_2 \\ \frac{\mu_E^2}{(1-p_1)c} &= \frac{(H_I^2 - H_E^2)\theta_2}{(1-p_1)c} - \lambda_2 \\ \frac{\gamma_2}{p_1bV} &= \frac{H_I^2\theta_2}{p_1bV} - \lambda_2\end{aligned}$$

Therefore, to be the parameters positive:

$$\max\left\{\frac{-(H_I^1 - H_E^1)\theta_1}{(1-p_1)c}, \frac{-H_I^1\theta_1}{p_1bV}\right\} \leq \lambda_2 \leq \min\left\{\frac{(H_I^2 - H_E^2)\theta_2}{(1-p_1)c}, \frac{H_I^2\theta_2}{p_1bV}\right\},$$

which can be written as:

$$H_I^1\theta_1 + H_I^2\theta_2 \geq 0$$

$$H_I^1\theta_1 + H_I^2\theta_2 \geq \beta H_I^2\theta_2 + \xi H_E^2\theta_2$$

$$H_I^1\theta_1 + H_I^2\theta_2 \geq \beta H_I^1\theta_1 + \xi H_E^1\theta_1$$

This condition is included in the first one when  $H_E^2 < 0$ .

$$H_I^1\theta_1 + H_I^2\theta_2 \geq H_E^1\theta_1 + H_E^2\theta_2$$

■

**Claim 10** *The solution  $\alpha_I^1 = \alpha_I^2 = \beta, \alpha_E^1 = \alpha_E^2 = \xi$  will be optimal in the case:  $\Phi^1 \geq 0, \Phi^2 \geq 0$  and  $\Omega_{Ext} \geq \Omega_{Int}$ . This will be only optimal if  $H_E^2 > 0$ .*

**Proof.** The optimality of this solution requires:  $\mu_E^i = 0$ ,  $\mu_I^i = 0$ ,  $\gamma_i \geq 0$ ,  $i = 1, 2$ .

The resulting conditions are:

$$\begin{aligned} \frac{-H_I^1 \theta_1}{p_1 b V} &\leq \lambda_1 + \lambda_2 = \frac{(H_E^1 - H_I^1) \theta_1}{(1 - p_1) c} \leq \frac{H_E^1 \theta_1}{(1 - p_1) c - p_1 b V} \\ \frac{-H_E^2 \theta_2}{(1 - p_1) c - p_1 b V} &\leq \lambda_2 = \frac{(H_I^2 - H_E^2) \theta_2}{(1 - p_1) c} \leq \frac{H_I^2 \theta_2}{p_1 b V} \end{aligned}$$

As  $\lambda_1 \geq 0$  and  $\lambda_2 \geq 0$  (the IR and IC constraints are satisfied with equality) we obtain the following conditions:

$$\begin{aligned} \max \left\{ \frac{-H_I^1 \theta_1}{p_1 b V}, 0 \right\} &\leq \frac{(H_E^1 - H_I^1) \theta_1}{(1 - p_1) c} \leq \frac{H_E^1 \theta_1}{(1 - p_1) c - p_1 b V} \\ \max \left\{ \frac{-H_E^2 \theta_2}{(1 - p_1) c - p_1 b V}, 0 \right\} &\leq \frac{(H_I^2 - H_E^2) \theta_2}{(1 - p_1) c} \leq \frac{H_I^2 \theta_2}{p_1 b V} \end{aligned}$$

Observe that, as  $H_I^1 < 0$ , it won't be necessary to compare it with zero. In this way, this condition can be written as:

$$\frac{-H_I^1}{p_1 b V} \leq \frac{(H_E^1 - H_I^1)}{(1 - p_1) c} \leq \frac{H_E^1}{(1 - p_1) c - p_1 b V}$$

Or, what it is the same:

$$\begin{cases} \beta H_I^1 \theta_1 + \xi H_E^1 \theta_1 \geq 0 \\ H_E^1 - H_I^1 > 0 \\ H_E^1 > 0 \end{cases}$$

As we have assumed that  $H_I^1 < 0$ ,  $H_E^1$  must be positive for the first condition be satisfied. Then, the last condition is not necessary. In the same way, if  $H_E^1 > 0$ , and  $H_I^1 < 0$ , then necessarily,  $H_E^1 - H_I^1 > 0$ .

We can summarize the conditions in:

$$\beta H_I^1 \theta_1 + \xi H_E^1 \theta_1 \geq 0$$

$H_E^1$  can not be positive if  $H_E^2 < 0$ . Then, for this solution be optimal,  $H_E^2 > 0$ .

The second condition can be written as:

$$\beta H_I^2 \theta_2 + \xi H_E^2 \theta_2 \geq 0$$

If we assume  $H_E^2 > 0$  this condition is not needed, because this will be always positive.

Moreover, as  $\lambda_2 = \frac{(H_I^2 - H_E^2)\theta_2}{(1-p_1)c}$

$$\lambda_1 = \frac{(H_E^1 - H_I^1)\theta_1}{(1-p_1)c} - \frac{(H_I^2 - H_E^2)\theta_2}{(1-p_1)c} = \frac{H_E^1\theta_1 + H_E^2\theta_2 - (H_I^1\theta_1 + H_I^2\theta_2)}{(1-p_1)c} \geq 0$$

or, what is the same:

$$H_E^1\theta_1 + H_E^2\theta_2 \geq H_I^1\theta_1 + H_I^2\theta_2$$

■

**Claim 11** *The trivial solution  $\alpha_E^1 = \alpha_I^1 = \alpha_E^2 = \alpha_I^2 = 0$  can only be optimal if  $H_E^2 < 0$ . In this case, the optimality conditions are:  $\Phi^2 \leq 0$ ,  $\Omega_{Int} \leq 0$*

**Proof.** The conditions having into account that  $\gamma_1 = \gamma_2 = 0$  will be:

$$\begin{aligned} \frac{\mu_I^1}{p_1bV} &= \frac{-H_I^1\theta_1}{p_1bV} - (\lambda_1 + \lambda_2) \\ \frac{\mu_E^1}{(1-p_1)c - p_1bV} &= \frac{-H_E^1\theta_1}{(1-p_1)c - p_1bV} + (\lambda_1 + \lambda_2) \\ \frac{\mu_I^2}{p_1bV} &= \frac{-H_I^2\theta_2}{p_1bV} + \lambda_2 \\ \frac{\mu_E^2}{(1-p_1)c - p_1bV} &= \frac{-H_E^2\theta_2}{(1-p_1)c - p_1bV} - \lambda_2 \end{aligned}$$

Therefore, the positivity conditions will be satisfied if:

$$\begin{aligned} \frac{H_I^2\theta_2}{p_1bV} &\leq \lambda_2 \leq \frac{-H_E^2\theta_2}{(1-p_1)c - p_1bV} \\ \max \left\{ \frac{H_E^1\theta_1}{(1-p_1)c - p_1bV}, 0 \right\} &\leq \lambda_1 + \lambda_2 \leq \frac{-H_I^1\theta_1}{p_1bV} \end{aligned}$$

We observe in the first condition that if  $H_E^2 > 0$ , the trivial solution is impossible.

If  $H_E^2 < 0$ , then the conditions will be the following:

$$\begin{aligned} \frac{H_I^2\theta_2}{p_1bV} &\leq \frac{-H_E^2\theta_2}{(1-p_1)c - p_1bV} \\ \max \left\{ \frac{H_E^1\theta_1}{(1-p_1)c - p_1bV}, 0 \right\} &\leq \frac{-H_I^1\theta_1}{p_1bV} \end{aligned}$$

This conditions can be written as:

$$\begin{aligned}\xi H_E^2 \theta_2 + \beta H_I^2 \theta_2 &\leq 0 \\ \xi H_E^1 \theta_1 + \beta H_I^1 \theta_1 &\leq 0\end{aligned}$$

The first condition reminds us that  $H_E^2$  must be negative. The last condition is not necessary because we have assumed that  $H_I^1 < 0$  and  $H_E^1$  is always smaller than  $H_E^2$ .

Moreover, as we know the values between  $\lambda_2$  is, we obtain that:

$$\lambda_1 \leq \frac{-H_I^1 \theta_1}{p_1 bV} - \lambda_2 < \frac{-H_I^1 \theta_1}{p_1 bV} - \frac{H_I^2 \theta_2}{p_1 bV} = \frac{1}{p_1 bV} (-H_I^1 \theta_1 - H_I^2 \theta_2)$$

So, other condition for  $\lambda_1 \geq 0$  will be  $-H_I^1 \theta_1 - H_I^2 \theta_2 \geq 0$  or, what is the same:

$$H_I^1 \theta_1 + H_I^2 \theta_2 \leq 0$$

In the same way:

$$\begin{aligned}\lambda_1 &\geq \frac{H_E^1 \theta_1}{(1-p_1)c - p_1 bV} - \lambda_2 > \frac{H_E^1 \theta_1}{(1-p_1)c - p_1 bV} - \frac{-H_E^2 \theta_2}{(1-p_1)c - p_1 bV} \\ &= \frac{H_E^1 \theta_1 + H_E^2 \theta_2}{(1-p_1)c - p_1 bV}\end{aligned}$$

We can show that  $H_E^1 \theta_1 + H_E^2 \theta_2 < 0$ , because  $H_E^2 < 0$  and, therefore,  $H_E^1 < 0$ . So, this condition is not restrictive.

■

The conditions don't overlap again.

**Proof of proposition 3.** The individual rationality constraint can be ignored because it is always satisfied since  $p_1 bV - (1-p_1)c > 0$ .

We can pose the Lagrangian:

$$\begin{aligned}L &= \alpha_I^1 H_I^1 \theta_1 + \alpha_E^1 H_E^1 \theta_1 - w^1 \theta_1 + \alpha_I^2 H_I^2 \theta_2 + \alpha_E^2 H_E^2 \theta_2 + \\ &\quad + \lambda [p_1 bV (\alpha_I^1 + \alpha_E^1 - \alpha_I^2 - \alpha_E^2) - (\alpha_E^1 - \alpha_E^2) (1-p_1)c + w^1] + \\ &\quad + \mu_I^1 \alpha_I^1 + \mu_E^1 \alpha_E^1 + \gamma_1 (1 - \alpha_I^1 - \alpha_E^1) + \delta w^1 \\ &\quad + \mu_I^2 \alpha_I^2 + \mu_E^2 \alpha_E^2 + \gamma_2 (1 - \alpha_I^2 - \alpha_E^2)\end{aligned}$$

And the first order conditions:

$$\begin{aligned}
\alpha_I^1 & : H_I^1 \theta_1 + \lambda p_1 bV + \mu_I^1 - \gamma_1 = 0 \\
\alpha_E^1 & : H_E^1 \theta_1 + \lambda(p_1 bV - (1 - p_1)c) + \mu_E^1 - \gamma_1 = 0 \\
\alpha_I^2 & : H_I^2 \theta_2 - \lambda p_1 bV + \mu_I^2 - \gamma_2 = 0 \\
\alpha_E^2 & : H_E^2 \theta_2 - \lambda(p_1 bV - (1 - p_1)c) + \mu_E^2 - \gamma_2 = 0 \\
w^1 & : -\theta_1 + \lambda + \delta = 0
\end{aligned}$$

The analysis of the first order conditions produce the following results:

**Claim 12** *There is no solution with  $\lambda = 0$ .*

**Proof.** If  $\lambda = 0$  then the condition of  $w^1$  implies  $\delta = \theta_1 > 0$ , that is  $w = 0$ . Then, the system is identical to the case where no salary is paid and we can apply the result (5). ■

As the incentive compatibility constraint is always satisfied with equality, the problem can be expressed as:

$$\begin{aligned}
\max_{\alpha_I^1, \alpha_E^1, \alpha_I^2, \alpha_E^2} & (\alpha_I^1 H_I^1 + \alpha_E^1 H_E^1 - (((\alpha_I^2 + \alpha_E^2) - (\alpha_I^1 + \alpha_E^1)) p_1 bV - (\alpha_E^2 - \alpha_E^1) (1 - p_1) c)) \theta_1 \\
& + (\alpha_I^2 H_I^2 + \alpha_E^2 H_E^2) \theta_2
\end{aligned}$$

s.t.

$$((\alpha_I^2 + \alpha_E^2) - (\alpha_I^1 + \alpha_E^1)) p_1 bV - (\alpha_E^2 - \alpha_E^1) (1 - p_1) c \geq 0$$

$$\alpha_I^i \geq 0; \quad \alpha_E^i \geq 0; \quad \alpha_I^i + \alpha_E^i \leq 1,$$

where the first inequality guarantee that salary will be positive. We write the Lagrangian as:

$$\begin{aligned}
L & = \alpha_I^1 H_I^1 \theta_1 + \alpha_E^1 H_E^1 \theta_1 + \alpha_I^2 H_I^2 \theta_2 + \alpha_E^2 H_E^2 \theta_2 + \\
& + (\theta_1 - \delta) [p_1 bV(\alpha_I^1 + \alpha_E^1 - \alpha_I^2 - \alpha_E^2) - (\alpha_E^1 - \alpha_E^2) (1 - p_1) c] + \\
& + \mu_I^1 \alpha_I^1 + \mu_E^1 \alpha_E^1 + \gamma_1 (1 - \alpha_I^1 - \alpha_E^1) \\
& + \mu_I^2 \alpha_I^2 + \mu_E^2 \alpha_E^2 + \gamma_2 (1 - \alpha_I^2 - \alpha_E^2)
\end{aligned}$$

and the system of first order conditions result:

$$\begin{aligned}
H_I^1 \theta_1 + (\theta_1 - \delta) p_1 b V + \mu_I^1 - \gamma_1 &= 0 \\
H_E^1 \theta_1 + (\theta_1 - \delta) (p_1 b V - (1 - p_1) c) + \mu_E^1 - \gamma_1 &= 0 \\
H_I^2 \theta_2 - (\theta_1 - \delta) p_1 b V + \mu_I^2 - \gamma_2 &= 0 \\
H_E^2 \theta_2 - (\theta_1 - \delta) (p_1 b V - (1 - p_1) c) + \mu_E^2 - \gamma_2 &= 0
\end{aligned}$$

**Claim 13** *The solution is  $\alpha_I^1 = \alpha_E^2 = \alpha_I^2 = \alpha_E^1 = 0, w = 0$  if  $\Omega_{Int} \leq 0, \Upsilon \leq 0$  and  $\Omega_{Ext} \leq 0$*

**Proof.** The first order conditions generate the following system:

$$\begin{aligned}
\frac{\mu_I^1}{p_1 b V} &= -\frac{H_I^1 \theta_1}{p_1 b V} - \theta_1 + \delta \\
\frac{\mu_E^1}{(p_1 b V - (1 - p_1) c)} &= -\frac{H_E^1 \theta_1}{(p_1 b V - (1 - p_1) c)} + \delta - \theta_1 \\
\frac{\mu_I^2}{p_1 b V} &= -\frac{H_I^2 \theta_2}{p_1 b V} - \delta + \theta_1 \\
\frac{\mu_E^2}{(p_1 b V - (1 - p_1) c)} &= -\frac{H_E^2 \theta_2}{(p_1 b V - (1 - p_1) c)} - \delta + \theta_1
\end{aligned}$$

$$\begin{aligned}
\delta &\geq \frac{H_I^1 \theta_1}{p_1 b V} + \theta_1 \\
\delta &\geq \frac{H_E^1 \theta_1}{(p_1 b V - (1 - p_1) c)} + \theta_1 \\
\delta &\leq -\frac{H_I^2 \theta_2}{p_1 b V} + \theta_1 \\
\delta &\leq -\frac{H_E^2 \theta_2}{(p_1 b V - (1 - p_1) c)} + \theta_1
\end{aligned}$$

Therefore,

$$\max \left\{ \frac{H_I^1 \theta_1}{p_1 b V}, \frac{H_E^1 \theta_1}{(p_1 b V - (1 - p_1) c)} \right\} \leq \delta - \theta_1 \leq \min \left\{ -\frac{H_I^2 \theta_2}{p_1 b V}, -\frac{H_E^2 \theta_2}{(p_1 b V - (1 - p_1) c)} \right\}.$$

As  $\delta - \theta_1 \leq 0$  (if not,  $\mu_I^1$  and  $\mu_I^2$  couldn't be positive), we obtain exactly the same conditions as in the case without monetary incentives (??) and we obtain the conclusion. ■

**Claim 14** *The solution is  $\alpha_I^1 = \alpha_E^1 = 0$ ,  $\alpha_I^2 = 1$ ,  $\alpha_E^2 = 0$ ,  $w^1 = p_1 bV$  if  $H_I^1 \leq -p_1 bV$ ,  $H_E^1 \leq -(p_1 bV - (1 - p_1)c)$ ,  $H_I^2 \theta_2 - \theta_1 p_1 bV \geq 0$  and  $(H_I^2 - H_E^2) \theta_2 - \theta_1(1 - p_1)c \geq 0$*

**Proof.** The first order conditions knowing that  $\mu_I^1 \geq 0$ ,  $\mu_I^2 = 0$ ,  $\mu_E^i \geq 0$ ,  $\delta = 0$ ,  $\gamma_1 = 0$ ,  $\gamma_2 \geq 0$ ,  $i = 1, 2$ , are:

$$\begin{aligned}\mu_I^1 &= -H_I^1 \theta_1 - \theta_1 p_1 bV \\ \mu_E^1 &= -H_E^1 \theta_1 - \theta_1 (p_1 bV - (1 - p_1)c) \\ \gamma_2 &= H_I^2 \theta_2 - \theta_1 p_1 bV \\ \mu_E^2 &= (H_I^2 - H_E^2) \theta_2 - \theta_1(1 - p_1)c\end{aligned}$$

The sign of the parameters will be satisfied if the proposed conditions are fulfilled. ■

**Claim 15** *The solution is  $\alpha_I^1 = \alpha_E^1 = 0$ ,  $\alpha_I^2 = 0$ ,  $\alpha_E^2 = 1$ ,  $w^1 = p_1 bV - (1 - p_1)c$  if  $H_I^1 \leq -p_1 bV$ ,  $H_E^1 \leq -(p_1 bV - (1 - p_1)c)$ ,  $H_E^2 \theta_2 - \theta_1 (p_1 bV - (1 - p_1)c) \geq 0$  and  $(H_E^2 - H_I^2) \theta_2 + \theta_1(1 - p_1)c \geq 0$*

**Proof.** The parameters must have the following values:  $\mu_I^i \geq 0$ ,  $\mu_E^1 \geq 0$ ,  $\mu_E^2 = 0$ ,  $\delta = 0$ ,  $\gamma_1 = 0$ ,  $\gamma_2 \geq 0$   $i = 1, 2$ .

$$\begin{aligned}\mu_I^1 &= -H_I^1 \theta_1 - \theta_1 p_1 bV \\ \mu_E^1 &= -H_E^1 \theta_1 - \theta_1 (p_1 bV - (1 - p_1)c) \\ \gamma_2 &= H_E^2 \theta_2 - \theta_1 (p_1 bV - (1 - p_1)c) \\ \mu_I^2 &= (H_E^2 - H_I^2) \theta_2 + \theta_1(1 - p_1)c\end{aligned}$$

We obtain the conditions from this system. ■

**Proof of proposition 4.** We have to add to the previous Lagrangian the incentive rationality constraint for  $p_1$ :

$$\begin{aligned}L &= (\alpha_I^1 H_I^1 + \alpha_E^1 H_E^1 - w) \theta_1 + (\alpha_I^2 H_I^2 + \alpha_E^2 H_E^2) \theta_2 + \\ &+ \lambda_1 [p_1 bV (\alpha_I^1 + \alpha_E^1) - \alpha_E^1 (1 - p_1)c + w^1] \\ &+ \lambda_2 [p_1 bV (\alpha_I^1 + \alpha_E^1 - \alpha_I^2 - \alpha_E^2) - (\alpha_E^1 - \alpha_E^2) (1 - p_1)c + w^1] + \\ &+ \mu_I^1 \alpha_I^1 + \mu_E^1 \alpha_E^1 + \gamma_1 (1 - \alpha_I^1 - \alpha_E^1) + \delta w^1 \\ &+ \mu_I^2 \alpha_I^2 + \mu_E^2 \alpha_E^2 + \gamma_2 (1 - \alpha_I^2 - \alpha_E^2)\end{aligned}$$

The first order conditions are:

$$\begin{aligned}
\alpha_I^1 & : H_I^1 \theta_1 + (\lambda_1 + \lambda_2) p_1 b V + \mu_I^1 - \gamma_1 = 0 \\
\alpha_E^1 & : H_E^1 \theta_1 + (\lambda_1 + \lambda_2) (p_1 b V - (1 - p_1) c) + \mu_E^1 - \gamma_1 = 0 \\
\alpha_I^2 & : H_I^2 \theta_2 - \lambda_2 p_1 b V + \mu_I^2 - \gamma_2 = 0 \\
\alpha_E^2 & : H_E^2 \theta_2 - \lambda_2 (p_1 b V - (1 - p_1) c) + \mu_E^2 - \gamma_2 = 0 \\
w^1 & : (\lambda_1 + \lambda_2 - \theta_1) + \delta = 0
\end{aligned}$$

**Claim 16** *The trivial solution  $\alpha_E^1 = \alpha_I^1 = \alpha_E^2 = \alpha_I^2 = 0$  is optimal only if:  $\beta H_I^2 \theta_2 + \xi H_E^2 \theta_2 \leq 0$ ,  $\beta H_I^1 \theta_1 + \xi H_E^1 \theta_1 \leq 0$ ,  $H_I^1 \theta_1 + H_I^2 \theta_2 \leq 0$*

**Proof.** It is the same solution as the obtained before without using monetary incentives(11). It is proved in the same way. ■

**Claim 17** *The solution  $\alpha_I^1 = \alpha_E^1 = 0$ ,  $\alpha_I^2 = 1$ ,  $\alpha_E^2 = 0$ ,  $w^1 = p_1 b V$  will be optimal when:  $H_I^1 \leq -p_1 b V$ ,  $H_E^1 \leq (1 - p_1) c - p_1 b V$ ,  $H_I^2 \theta_2 - \theta_1 p_1 b V \geq 0$  and  $H_I^2 \theta_2 \geq H_E^2 \theta_2 + \theta_1 (1 - p_1) c$*

**Proof.** The parameters must take the following values:  $\mu_I^1 \geq 0$ ,  $\mu_I^2 = 0$ ,  $\mu_E^i \geq 0$ ,  $\lambda_2 \geq 0$ ,  $\lambda_1 = 0$ ,  $\delta = 0$ ,  $\gamma_1 = 0$ ,  $\gamma_2 \geq 0$ ,  $i = 1, 2$ .

$$\begin{aligned}
\lambda_2 & = \theta_1 \\
\mu_I^1 & = -H_I^1 \theta_1 - \theta_1 p_1 b V \\
\mu_E^1 & = -H_E^1 \theta_1 - \theta_1 (p_1 b V - (1 - p_1) c) \\
\gamma_2 & = H_I^2 \theta_2 - \theta_1 p_1 b V \\
\mu_E^2 & = (H_I^2 - H_E^2) \theta_2 - \lambda_2 (1 - p_1) c = (H_I^2 - H_E^2) \theta_2 - \theta_1 (1 - p_1) c
\end{aligned}$$

■

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